Unknowable Truths*

Zachary Goodsell
John Hawthorne
Juhani Yli-Vakkuri

September 26, 2024

In an anonymous referee report written in 1945,[1] Alonzo Church suggested a sweeping argument against *verificationism*, the thesis that every truth is knowable.[2] The argument, which was published with due acknowledgement by Frederic Fitch (1963) almost two decades later, has generated significant attention as well as some interesting successor arguments. In this paper we present the most important episodes in this intellectual history using the logic Church himself favoured, as presented in the paper which inaugerated modern higher-order logic, 'A formulation of the simple theory of types' (1940), and we present reasons for thinking that the arguments are less than decisive.

Church himself in the same referee report rejected the distribution principle that drives the original argument against verificationism on the grounds that 'it is an empirical fact that there are fools'. In less colorful language, he maintained that someone might know a conjunction but fail to deduce, and for that reason never come to know, each of its conjuncts.

We agree with Church that this is a strong objection to the original anti-verficationist argument. What's more, the distribution principle sits poorly with a popular principle of propositional granularity, *intensionalism*, which says that the necessary equivalence of propositions suffices for their identity. This view, now most associated with the work of Lewis (e.g., 1973) and Stalnaker (1976), is taken

*Thanks to Andrew Bacon, Peter Fritz, the Higher-Order Logic Reading Group at the Dianoia Institute of Philosophy at Australian Catholic University, and some anonymous referees for helpful comments

[1]Published in Salerno 2008.

[2]There is also the stronger thesis that it is necessary that every truth is knowable. Assuming the possibility of nobody knowing anything, this stronger thesis would commit one to the absurd conclusion that it is possibly possibly known that nothing is known.

seriously by many contemporary exponents of higher-order logic.[3] Intensionalism is also strongly suggested by Church's own discussion of the individuation of propositions.[4] The clash between the distribution principle and intensionalism further diminishes the dialectical force of the original anti-verificationist argument.

Subsequent anti-verificationist arguments due to Williamson (2000) and Chalmers (2011) have avoided the distribution principle, but will be shown to rely on equally tendentious assumptions. Nevertheless, we conclude by presenting some new arguments against verificationism that we believe to be dialectically effective given the background of Church's logic, with or without intensionalism. Moreover, we show the more compelling of these arguments to survive various weakenings of Church's logic.

1 Church

Let X be any property of propositions. We will say that X is *factive* (Fact) iff every proposition that has X is true (so factivity is a property of properties of propositions). Formally:

$$\text{Fact}\, X := \forall p \,.\, (Xp \to p)$$

We will say that X *distributes over conjunction* (Dist) iff, whenever a conjunction has X, each conjunct also has X.

$$\text{Dist}\, X := \forall pq \,.\, (X(p \wedge q) \to (Xp \wedge Xq))$$

[3]For sympathetic recent discussions of Intensionalism see Dorr (2016), Bacon (2018), Dorr, Hawthorne, and Yli-Vakkuri (2021), Fritz (2023).

[4]Church (1951) introduced a taxonomy of what were then called 'criteria of individuation of intensional entities' and what are now called principles about the 'grain' of propositions: Alternatives (0), (1), and (2).

Roughly, Alternative (2) permits the inference of the identity $\mathbf{P} = \mathbf{Q}$ from a proof of the biconditional $\mathbf{P} \leftrightarrow \mathbf{Q}$ from purely logical assumptions. (As noticed by Myhill (1958), it is important to exclude certain of Church's axioms—namely the axioms of Choice and Descriptions—from the 'purely logical' to avoid a slingshot argument which asserts, implausibly, that there are only two propositions.) Alternative (2), though not quite entailing intensionalism, has many of the same controversial consequences. Moreover, on one definition of $\Box$ which is standard in literature on higher-order logic, namely $\Box := (=\top)$, Alternative (2) and intensionalism are equivalent (see Bacon 2018).

The other two Alternatives (0) and (1), are now known to be incompatible with Church's higher-order logic. Myhill 1958 refutes Alternative (0), and Bacon and Dorr forthcoming refute Alternative (1).

We will say that X is a *verificationist property* (Ver) iff every truth possibly has X.

$$\text{Ver}\, X := \forall p \,.\, (p \rightarrow \Diamond X p)$$

Finally, say X is *trivial* (Triv) iff every truth has X.[5]

$$\text{Triv}\, X := \forall p \,.\, (p \rightarrow X p)$$

In 1945, writing as an anonymous referee of a paper that was never published, Alonzo Church communicated to Frederic Fitch a result that the latter reported in a different paper nearly two decades later (Fitch 1963):

Church's Theorem. No property of propositions that is (i) necessarily factive, (ii), necessarily distributes over conjunction, and (iii) non-trivial is a verificationist property:

$$(\Box\, \text{Fact}\, X \wedge \Box\, \text{Dist}\, X \wedge \neg\, \text{Triv}\, X) \rightarrow \neg\, \text{Ver}\, X$$

Church's theorem is a theorem of his higher-order logic supplemented with the minimal modal logic KT. The novel assumptions are (a) that $\Box$ is closed under Modus Ponens (the K axiom), (b) that $\Box$ is factive (the T axiom), and (c) a rule of necessitation that roughly speaking permits the inference of $\Box\mathbf{P}$ from a proof of $\mathbf{P}$[6] (the rule of necessitation requires a more subtle statement for logics which, like Church's, include theorems which are thought to express contingent truths).[7] See Appendix A.3.

Proof. Let X be any property of propositions that has properties (i), (ii), and (iii)

[5] We do not intend to capture any standard notion of triviality. Arguably, if every truth had X but only contingently, it would not be right to call X trivial in any usual sense. We choose this notion of triviality because the triviality of knowledge, in our defined sense, is the problematic consequence of verificationism according to Church's original argument.

[6] Bold symbols are metavariables, i.e., an instance of the rule of necessitation is as described, where $\mathbf{P}$ is replaced with any formula of the language.

[7] The axiom for ι asserts that if there is a unique X, then ιX is it. For example, since there is a unique King of England in 2024 (KoE), then by the axiom for ι, ιKoE is the King of England. Including the axiom for ι in the rule of necessitation would allow us to prove that *necessarily*, if there is a unique King of England, then ιKoE is it, or

$$\Box((\exists! y \,.\, \text{KoE}y) \rightarrow \text{KoE}(\iota\text{KoE})) \tag{1}$$

Since Charles is the unique King of England, this would falsely imply that Charles is the *only* person who could have been uniquely King of England. (The same concern arises for the axioms for ε.) Indeed, Myhill (1958) showed that including the axiom for ι in the rule of necessitation collapses any distinction between necessity and mere truth.

as specified in the statement of Church's theorem. By (iii), there is at least one true proposition that lacks X. Let p be that proposition. Since p is true and lacks X, $p \wedge \neg Xp$ is true. Suppose for a contradiction that it is possible that $X(p \wedge \neg Xp)$. By (ii) and KT modal logic it follows that it is possible that $Xp \wedge X\neg Xp$, and by (i) and KT modal logic it follows that it is possible that $Xp \wedge \neg Xp$, which is inconsistent in KT modal logic. Since it is not possible that $X(p \wedge \neg Xp)$, $p \wedge \neg Xp$, is a true proposition that necessarily lacks X, so there is a true proposition that necessarily lacks X. □

When we instantiate X in Church's theorem with the property of propositions *being known (at some time, by some agent) that*, we have a refutation of verificationism provided that being known satisfies conditions (i), (ii), and (iii). Does being known satisfy them?[8]

We are not going to question the necessary factivity of knowledge (condition (i)) or the non-triviality of knowledge (condition (iii)). (Of course, some theists will deny the non-triviality of being known: they maintain that every truth is known by God.) However, there are two good reasons to reject the view that knowledge necessarily distributes over conjunction (condition (ii)).

The first reason is pointed out by Church himself in his referee report on Fitch's manuscript:

> In spite of the plausibility of the preceding argument I think Fitch has a good defense (but only one). This defense is that there is no law of psychology according to which one who believes a proposition must believe all its logical consequences; on the contrary, historical counter-examples are well known. To be sure, one who believes a proposition without believing its more obvious consequences is a fool; but it is an empirical fact that there are fools. It is even possible that there might be so great a fool as to believe the conjunction of two propositions

[8]Some might think that being known cannot be substituted for 'X' in Church's theorem, because a proper regimentation of knowledge attributions takes being known (by someone, at some time) as a relation between a proposition and something else, such as a guise, a mode of presentation, or a sentence in the language of thought (see, e.g., Kaplan (1968)). When verificationism is reformulated in one of these frameworks, there will be values of X for which Church's theorem will make analogous trouble. For example, thinking of verificationism as the thesis that every true proposition is possibly known under some guise or other, we can take X to be the property of propositions *being known under some guise or other*. Being known under some guise or other is plausibly factive (because being known in any way should imply truth), and plausibly nontrivial (because some propositions seem not to even be entertained, let alone known, under any guise whatsoever). The concerns raised for the distributivity of knowledge in what follows also speak against the distributivity of being known under some guise or other. Hence the dialectic that follows cannot be bypassed by adopting one of these nuanced frameworks.

> without believing either of the two propositions; at least, an empirical law to the contrary would seem to be open to doubt. On this ground it is empirically possible that a might believe k' at time t without believing k at time t (although k' is a conjunction one of whose terms is k) (Church 2009: 14).

As Williamson later points out, it is also plausible that a non-fool could feature in a counterexample to the necessary distribution of knowledge over conjunction:

> [T]here is no form of inference which people can be relied on to carry out. Distraction or sudden death is always liable to intervene (Williamson 2000: 83).

It is plausible that we know many of the facts we know by accepting sentences that express them, and that at least in some cases we come to know a conjunction by accepting a sentence that expresses it before we infer and come to accept its conjuncts and thereby to come to know what they express. Distraction or sudden death may intervene from the outside, and foolishness may block the inference from the inside.[9]

The second reason to doubt that knowledge necessarily distributes over conjunction arises from the popular principle of intensionalism. Recall that intensionalism asserts the identity of necessarily equivalent propositions.

$$\forall pq\,.\,(\Box(p \leftrightarrow q) \rightarrow (p = q)) \tag{3}$$

By adding intensionalism to Church's higher-order logic with KT for $\Box$, we have it that, whenever p necessitates q, p is identical to the conjunction $p \wedge q$.

$$\forall pq\,.\,(\Box(p \rightarrow q) \rightarrow (p = (p \wedge q))) \tag{4}$$

[9]A referee has suggested a weakening of the distributivity of knowledge that is still strong enough to make trouble for verificationism, given the necessitation of the factivity of knowledge (i.e., $\Box\Box(\forall p\,.\,(Kp \rightarrow p))$). The weakened principle asserts that necessarily, if a conjunction is known, then it is possible that both conjuncts are known, i.e.,

$$\Box\forall pq\,.\,(K(p \wedge q) \rightarrow \Diamond(Kp \wedge Kq)) \tag{2}$$

We believe Church's and Williamson's arguments against distributivity also provide a good basis for rejecting this principle. A fool might know a conjunction, one conjunct of which would be false if they had tried to inferred that conjunct from the conjunction. Similarly, someone on the verge of death might know a conjunction with a conjunct that would be false if they lived long enough to infer it from the conjunction. In either setting, the weakened principle will be false, so we do not find it a promising fallback for those who reject distribution for the reasons given by Church and Williamson.

(Because, assuming $\Box(p \to q)$, $\Box(p \leftrightarrow (p \wedge q))$ may be derived with modal logic.) Hence, if p is known, then $p \wedge q$ is known because it is the same proposition, so if knowledge distributes over conjunction then q is also known.

Under intensionalism, then, the distribution principle implies that every necessary consequence of what is known is also known. Against this, it is natural to complain that various propositions we know necessitate propositions that we do not even have the means to entertain and so, it would seem, we do not know.

Worse still, consider the actuality operator @, which in Church's logic may be defined as the propositional function which maps every true proposition to an arbitrarily chosen tautology $\top$, and everything else (i.e., every false proposition) to a chosen contradiction $\bot$.[10] (This definition of @ is first articulated in David Kaplan's PhD thesis (1964: 150).) When supplemented with KT modal logic as above, we may easily prove the characteristic axiom of the logic of actuality, namely that everything true is necessarily actually true and everything false is necessarily not actually true.

$$\forall p \,.\, ((p \to \Box @ p) \wedge (\neg p \to \Box \neg @ p)) \tag{6}$$

While Church's logic provides a natural vindication of the actuality operator, acknowledging such an operator and accepting the characteristic axiom (6) by no means requires the full strength of Church's logic.[11]

Consider now the proposition α, defined as *everything that is actually true is is true*, or

$$\alpha := \forall p \,.\, (@p \to p) \tag{7}$$

α can be proved to be a truth that necessitates every truth, i.e.,

$$\alpha \wedge \forall p \,.\, (p \to \Box(\alpha \to p)) \tag{8}$$

We submit that α is known. We know it, because we know enough logic to prove the sentence 'α', and that sentence means α.[12] There may be some exceptional circumstances in which knowledge is not extended by competent deduction, but

[10] @ may be defined using Church's definite description operator ι as

$$@ := \lambda p \,.\, \iota q \,.\, ((p \to (q = \top)) \wedge (\neg p \to (q = \bot))) \tag{5}$$

[11] In particular, Bacon and Dorr (forthcoming) show that acknowledging the actuality operator is much weaker than acknowledging Church's definite description operator (ι), which is used in footnote 10 to define @ in Church's logic.

[12] As noted earlier, the logic needn't be as strong as Church's.

we see no reason to think they are present in this case.[13] But if so, then by intensionalism we know the conjunction of every truth with α, because α is identical to its conjunction with any truth. And if we know the conjunction of every truth with α, then the additional assumption of distribution over conjunction yields the triviality of knowledge (i.e., that every truth is known), without appealing to verificationism.[14]

There are thus eminently respectable reasons for questioning distribution. Since we are looking for a compelling argument against verificationism, it would be nice to have one that didn't use distribution as a premise.

2 Williamson

Timothy Williamson (2000: 284-85) has proposed an argument against verificationism which does not rely on the distribution of knowledge over conjunction. The key idea is that, regardless of whether knowledge is distributes over conjunction, being known is not *conjunctively trivial*, in the sense that there are some truths that not only fail to be known but also fail to be conjuncts of known conjunctions. Let's generalize this idea and say that a property of propositions X is conjunctively trivial (CTriv) just in case every truth is part of a conjunction that has X:

$$\text{CTriv}\, X := \forall p\, .\, (p \rightarrow \exists q\, .\, X(q \wedge p)) \tag{11}$$

Here now is Williamson's result, generalized to an arbitrary property of propositions X:

[13] Some will deny that the proposition α is known despite accepting the proof of the sentence 'α', arguing perhaps that the proof only generates knowledge that *'α' is a true sentence*. We find this line deeply unpromising, and moreover the verificationist has strong reason to push back on it. This is because, according to verificationism, α is possibly known, so α must in fact be known by the factivity of knowledge and the fact that α necessitates $\neg K\alpha$ if α is not known. But if our proof of the sentence 'α' has not generated knowledge that α, it is hard to believe that α is known at all.

[14] The weakened distributivity principle discussed in Footnote 9 also trivializes knowledge on the assumption that α is known and given intensionalism. Suppose that α is known and that an arbitrary proposition p is true. $\alpha \wedge p$ is known, so by the weakened principle we have

$$\Diamond(K\alpha \wedge Kp) \tag{9}$$

and hence

$$\Diamond(\alpha \wedge Kp) \tag{10}$$

by the necessary factivity of knowledge. But α is true and necessitates the negation of every falsehood, so Kp cannot be false. So we have shown that an arbitrary truth is known, which is the triviality of knowledge.

Williamson's Theorem. No property of propositions that is (i) necessarily factive and (ii) not conjunctively trivial is a verficationist property:

$$(\Box\,\mathrm{Fact}\,X \wedge \neg\,\mathrm{CTriv}\,X) \rightarrow \neg\,\mathrm{Ver}\,X$$

Williamson's result is impeccable in its own right. But it only gives us a plausible argument against verificationism insofar as it is plausible that being known is not conjunctively trivial. Once again intensionalism puts the issue in a new light. Given intensionalism, every property of α is conjunctively trivial, so if α is known, then being known is conjunctively trivial. And again, it is natural to think that we know α: the one-character sentence which we have been using to express α, namely 'α', is a theorem of higher-order logic, which we can and did prove; and on the basis of that proof, we go around confidently asserting the sentence 'α', the meaning of which is α). Unless we are mistaken to accept Church's higher-order logic, plausibly this is enough to know α.

3 Chalmers

Yet another alternative which avoids assuming knowledge distributes over conjunction has been proposed by David Chalmers (2011). He writes:

> Here 'A', 'E', 'K, '$\Box$', '$\Diamond$' stand for 'Actually', 'Someone entertains', 'Someone knows', 'Necessarily' and 'Possibly' [...] In addition, q is any (entertainable and expressible) proposition that no one actually entertains, while r is $\neg Eq$, the proposition that no one entertains q.
>
> 1. Ar
> 2. $Ar \rightarrow \Box Ar$
> 3. $\Box(K(r \leftrightarrow Ar) \rightarrow (r \leftrightarrow Ar))$
> 4. $\Box(r \rightarrow \neg K(r \leftrightarrow Ar))$
> 5. $\neg\Diamond K(r \leftrightarrow Ar)$
>
> (Chalmers 2011: 411)

(For the purpose of refuting verificationism, Chalmers' argument could be simplified to show that r (i.e., the proposition that q is not entertained) is a counterexample to verificationism. But his ambition is to show that there is an unknowable *a priori truth*, and r is plausibly not *a priori*.)

The least obvious step in the argument is in line 4, about which Chalmers writes

> Premiss (4) follows from two principles concerning entertaining: [(4.1)] entertaining a proposition requires entertaining its constituents, and [(4.2)] knowing a proposition requires entertaining it (*ibid*.).

The thought is that, since r is the proposition that q is not entertained, one would have to entertain q in order to entertain, and hence in order to know, r. So r cannot be true if it is known—and hence cannot be known at all by the necessary factivity of knowledge.

At the very least, premise 4.1 should entail the following: necessarily, if the proposition that p is X is ever entertained, then so is the proposition that p. That is,

$$\Box\forall X \,.\, \forall p \,.\, (EXp \to Ep) \tag{12}$$

(Chalmers relies specifically on the case where X is knowledge.)

Unfortunately, (12) is in no better shape than the distributivity of knowledge, given Church's higher-order logic. This is because, given that logic, the only properties E for which (12) holds are properties which either every proposition has or which no proposition has.

$$(\forall X \,.\, \forall p \,.\, (EXp \to Ep)) \to (\forall p \,.\, (Ep) \vee \forall p \,.\, (\neg Ep)) \tag{13}$$

(Indeed, (12) is arguably in worse shape because the controversial thesis of intensionalism plays no role in this argument, nor does the assumption of an actuality operator. In fact, the argument works in the austere logic H described in Appendix A.2.)

Proof. Consider the case where, for some proposition q, $X = \lambda p \,.\, q$ (i.e., the function that maps every proposition to q).[15] Then $Xp = q$, and hence $EXp = Eq$. So if (12) holds then

$$\forall q \,.\, \forall p \,.\, (Eq \to Ep). \tag{14}$$

Hence either every proposition has E or no proposition does. □

Thus, an argument which appeals to Chalmers' premise 4.1 cannot be a convincing rebuttal of verificationism.

Now of course Chalmers could avoid this assessment by rejecting Church's higher-order logic. But it would be nice to have an anti-verificationist argument that does not rely on the tendentious rejection of Church's higher-order logic.[16]

---

[15] See Appendix A.1.

[16] A referee has suggested an alternative reconstruction of Chalmers argument that replaces Premise 4.1 with (4.1*) $\Box\forall p \,.\, (E\neg Ep \to Ep)$, a premise 'which isn't necessarily tied to the plausibility of some general principle about constituency'. We think that (4.1*) would be difficult to motivate

4 Edgington

We have been looking at the history of anti-verificationist arguments that are in the spirit of Church's original. It's also worth considering a pro-verificationist retort offered by Dorothy Edgington (1985), which has been widely discussed. Her central idea is that much of the spirit of verificationism could be retained by giving up the standard knowability principle in favor of the more modest principle that the actualization of every truth is knowable:

$$\forall p \,.\, (p \rightarrow \Diamond K@p).$$

The idea is that the actuality operator @ obeys the following axiom:

$$\forall p \,.\, ((p \rightarrow \Box @p) \wedge (\neg p \rightarrow \Box\neg @p)) \tag{15}$$

Even if we grant distribution, there is no tweak on Church's argument that can be used to defeat this more modest version of verificationism, which we will call *Edgingtonian verificationism*.

Timothy Williamson (2000: 293–294) has raised one *prima facie* concern about Edgingtonian verificationism: propositions that we express using sentences of the form '@**P**' would not be graspable if things were not as they actually are on account of the unavailability of any means of getting a cognitive fix on the actual world in a counterfactual situation. He asks: 'If this actual world had not obtained, how could anyone have referred to it?' (Williamson 2000: 293). Williamson assumes

---

except by assuming that propositions are fine-grained in a way inconsistent with Church's higher-order logic (modal or not), let alone intensionalism—a feature it shares with Premise 4.1. Suppose, for example, that $\neg E$(snow is white) is necessarily equivalent to some proposition π of fundamental physics (where 'π' stands in for a sentence of the language of fundamental physics that expresses this proposition; strictly speaking the proposition itself cannot be said to be one of fundamental physics given intensionalism). Under intensionalism Premise 4.1* implies that $E\pi \rightarrow E$(snow is white), which would seem especially implausible in a scenario in which π is entertained only under the guise of the sentence 'π'. We can find similar cases without assuming intensionalism: suppose that $\neg E$(snow is white) is Chalmers' favorite proposition. Then it follows in Church's logic (without any additions) that

$$\neg E(\text{snow is white}) = \iota(\lambda p \,.\, p \text{ is Chalmers' favorite proposition}),$$

and it would follow by (4.1*) that

$$\Box(E\iota(\lambda p \,.\, p \text{ is Chalmers' favorite proposition}) \rightarrow E(\text{snow is white})),$$

which seems implausible given that $\neg E$(snow is white) might be entertained only under the guise provided by '$\iota(\lambda p \,.\, p$ is Chalmers' favorite proposition)'. (How could its being entertained only under that guise necessitate the entertaining of the *distinct* proposition that snow is white?)

that in order to believe $@p$, one first has to think of the propositional function $@$, which may be difficult in other possible situations where that function would not be particularly special.

Church's higher-order logic casts this line of thought into doubt. In general, one does not need to think of the function f in order to believe the proposition fp, since although f might be obscure and hard to think of, it might map p to a proposition which is not difficult to believe, e.g., the proposition that grass is green. This situation seems especially likely in the case of $@$, where, given the Kaplanian definition described in Section 1, $@p$ is the proposition $\top$ (which is presumably graspable, and indeed known) whenever p is true.

On this definition of $@$, Edgingtonian verificationism is plausible, but the reason why it is plausible is also a reason for thinking that it preserves none of the spirit of verificationism. For, on this definition, we have that if p is true, then $@p$ is $\top$.

$$\forall p \,.\, (p \rightarrow (\top = @p)) \tag{16}$$

Thus, Edgingtonian verificationism asserts only that

$$\forall p \,.\, (p \rightarrow \Diamond K\top), \tag{17}$$

which is equivalent to the uncontroversial claim that $\top$ is possibly known.

In light of this fact, Edgingtonian verificationism only has a hope of preserving some of the spirit of verificationism if '@' is understood in some other way. But it is a rather obscure matter what way that is supposed to be—what theoretical role could there be for this concept which is not captured by Kaplan's definition? Moreover, given intensionalism, it is straightforward to show that every property of propositions $@'$ satisfying (15) also maps every truth to $\top$, thus cutting off any alternative understanding of the constant on which Edgingtonian verificationism would turn out to be more interesting.

5 Some new results

So far we have looked at ideas whose interest has depended on either the rejection of Church's higher-order logic or on the rejection of intensionalism. In this section we will present two new arguments against verificationism whose interest survives the acceptance of both.

This is not to say the new arguments require the acceptance of both Church's logic and intensionalism: neither argument relies on intensionalism, and both can survive certain weakenings of Church's logic. These weakenings are described in technical detail alongside Church's logic in the appendices.

The most notable weakenings are as follows: the axiom of descriptions can be replaced in the first argument with a principle of class comprehension, and those axioms can be replaced in the second argument with the assumption of a true proposition that necessitates every truth (like α). The second argument also withstands a further important weakening. Like the literature just surveyed, we have thus far taken classical propositional logic for granted. However, many historical verificationists were sympathetic with intuitionistic logic (most notably Dummett (1959)), and some, following Tennant (1997 ch. 8, 2001), have argued that Fitch-style arguments are ineffective in intuitionistic setting. As it turns out, a version of our second argument is convincing even from an intuitionistic perspective.

5.1 Our first result

In what follows the class-abstract notation $\{p : \mathbf{P}\}$ is used for the class of propositions p for which $\mathbf{P}$. This is the propositional function which maps every proposition for which $\mathbf{P}$ holds to $\top$ and everything else to $\bot$. That is (writing the more familiar $q \in \{p : \mathbf{P}\}$ for $\{p : \mathbf{P}\}q$),

$$\forall q \,.\, ((q \in \{p : \mathbf{P}\} = \top) \vee (q \in \{p : \mathbf{P}\} = \bot)) \tag{18}$$

The existence of such a propositional function is guaranteed by the axiom of descriptions, specifically we may define

$$\{p : \mathbf{P}\} := \lambda p \,.\, \iota\lambda q \,.\, ((\mathbf{P} \rightarrow (q = \top)) \wedge (\neg\mathbf{P} \rightarrow (q = \bot))) \tag{19}$$

Alternatively, (18) may be justified more directly with a principle of class comprehension as described in Appendix A.4.

In particular, for a property of propositions X, $\{p : Xp\}$ is the class of propositions which are X. Although X may be a contingent property, $\{p : Xp\}$ will not be, since the proposition that q is a member of $\{p : Xp\}$, is either $\top$ or $\bot$, neither of which can be contingent in Church's logic augmented with KT modal logic.

Our first result is as follows, with the proof given at the end of this subsection:

Theorem 1 ($\mathsf{H}_\Box$ + Class comp.)**.** *For all properties of propositions X and Y, if (i) X is necessarily factive, (ii) every proposition in the class $\{q : Yq\}$ lacks X, and (iii) the proposition that every proposition in $\{q : Yq\}$ lacks X is a member of $\{q : Yq\}$, then X is not a verificationist property.*

In the case of knowledge, the theorem says: if verificationism is true, then, if there is a property of propositions X such that no member of $\{q : Xq\}$ is known, then the proposition that no member of $\{q : Xq\}$ is known is not a member of $\{q : Xq\}$.

Let's see why this might be problematic for the verificationist. Say a proposition is *printed** on a page if it is the meaning of a sentence that has been printed onto that page. Suppose that a printer, π, prints* exactly two propositions, one of which is false and the other of which is the proposition that every proposition in $\{p : \pi \text{ prints* } p\}$ is unknown. It seems very plausible that something like this has happened or will happen at some time in the history of the world, in such a way that the second printed* proposition is true but unknown. It might happen, for example, that no one ever gets to see what π prints*; or it might happen that some people do get to see what π prints* but without realizing that the first proposition printed* by π is unknown. If this does happen at some time in the history of the world, then by Theorem 1 verificationism is false.

The verificationist is poorly placed if they concede that this kind of scenario could easily happen but bet that it never in fact happens. That would be a risky bet. But they might reasonably claim that it could not easily happen. Of course, there could easily be a printer called 'Patty' that nobody ever thinks much about, and which produces exactly the following two sentences before being destroyed:

> $2 + 2 = 5$.
>
> Everything in $\{p : \text{Patty prints* } p\}$ is unknown.

If the propositions the Patty ends up printing* are $2 + 2 = 5$, that everything in $\{p : \text{Patty prints* } p\}$ is unknown, and nothing else, then it seems very likely that everything in $\{p : \text{Patty prints* } p\}$ is unknown, since the first proposition cannot be known on account of being false, and the second is not known since nobody gives Patty a second thought.

However, the verificationist could quite reasonably deny that the propositions printed* in the described scenario were the two just mentioned, in effect saying that the sentences Patty produces should not be interpreted disquotationally in figuring out which propositions were printed*. Such a move is already forced upon us in similar cases by Church's logic alone. Imagine an alternative case where Patty produces exactly the following sentence before being destroyed:

> Everything in $\{p : \text{Patty prints* } p\}$ is false.

If we insist that Patty printed* that everything in $\{p : \text{Patty prints* } p\}$ is false, then Patty must have printed* something true and something false, hence at least two things. So despite appearances, an extra proposition might sneak into those which Patty printed*, if we insist on the disquotational interpretation of the sentence that was produced.[17] The verificationist might hope, therefore, that Patty has not printed* a class of propositions which refute verificationism.

[17] This variant of the liar paradox is first formalized, as far as we know, by Hilbert and Ackermann (1928, §4).

However, special pleading about which propositions Patty has printed* is far too local a defensive measure. Let's say that property of propositions Y is *funky* just in case the proposition *every proposition in* $\{p : Yp\}$ *is unknown* is itself Y. The example with Patty is defused by calling into question whether Patty printed* that every proposition in $\{p$: Patty prints* $p\}$ is unknown, i.e., by calling into question whether the property of being printed* by Patty is funky. But even conceding that being printed* by Patty is not funky, it is easy to generate examples of funky relations. Take the printer in the office in which we are currently working—let's call it 'Billy'. Since Billy is currently unplugged, Billy is not now printing* the proposition that every proposition in $\{p$: Billy is not now printing* $p\}$ is unknown, the property of not being now printed* by Billy is funky. That funky property doesn't make trouble for verificationism, because it's not true that every proposition in $\{p$: Billy is not now printing* $p\}$ is unknown (Billy is not now printing* anything, so is not printing* the known truth that grass is green). But now we see what the verificationist must hope for. The verificationist must hope that for every funky property Y, some Y proposition is known. Why would anyone have any confidence at all in that?

Proof of Theorem 1. Suppose that there is some property of propositions, Y, such that every proposition in the class $\{q : Yq\}$ is not X:

$$\forall p \,.\, (p \in \{q : Yq\} \rightarrow \neg Xp), \tag{20}$$

and the proposition that everything in $\{q : Yq\}$ is unknown is also in $\{q : Yq\}$:

$$(\forall p \,.\, (p \in \{q : Yq\} \rightarrow \neg Xp)) \in \{q : Yq\}. \tag{21}$$

Verificationism for X will be refuted by showing that (20) is not possibly X. So suppose for *reductio* that it is possibly X, i.e.,

$$\Diamond X \forall p \,.\, (p \in \{q : Yq\} \rightarrow \neg Xp). \tag{22}$$

By the necessary factivity of X and modal logic, we may infer that possibly, (20) is both true and X,

$$\Diamond(\forall p \,.\, (p \in \{q : Yq\} \rightarrow \neg Xp) \wedge X\forall p \,.\, (p \in \{q : Yq\} \rightarrow \neg Xp)), \tag{23}$$

whence by the necessity of class membership, it is possible that both of these are so while (20) is also a member of $\{q : Yq\}$:

$$\begin{aligned}&\Diamond(\forall p \,.\, (p \in \{q : Yq\} \to \neg Xp)\\ &\wedge X\forall p \,.\, (p \in \{q : Yq\} \to \neg Xp)\\ &\wedge (\forall p \,.\, (p \in \{q : Yq\} \to \neg Xp)) \in \{q : Yq\}).\end{aligned} \tag{24}$$

But the first conjunct says every member of $\{q : Yq\}$ is not X, the second gives an example of an X proposition, and the third says it is a member of $\{q : Yq\}$—a contradiction. So (24) is false, and so X is not a verificationist property by *reductio*. □

5.2 Our second result

We think that the preceding reflections should at least trouble the verificationist. We will now present some further considerations that we find to be decisive. As a prelude, notice that it suffices for the falsity of verificationism that there is some truth p that necessitates p being unknown. It's not hard to think of classes of propositions that plausibly have this feature. Suppose, for example, that every actual agent is below a certain level of intellectual sophistication, l. If so, one would expect there to be many highly detailed true propositions p about the distribution of fundamental properties and relations such that (i) p necessitates that no agent has a level of intellectual sophistication greater than l, and (ii) p's being known necessitates that some agent has a level of intellectual sophistication greater than l. Considerations like these already by our lights make verificationism look rather hopeless. But there are certain classes of propositions for which it is especially clear that they necessitate their being unknown, and those propositions will play a starring role in the arguments that make use of our second result.

In what follows, we assume a significantly weakened version of Church's logic augmented with KT modal logic, $\mathsf{H}_\Box$ + Actual world. $\mathsf{H}_\Box$ + Actual world includes none of class comprehension, an actuality operator, or the axiom of descriptions, but asserts the existence of α (i.e., namely, the truth that necessitates all truths). This final assumption is significantly weaker than the previous assumptions, and is much less controversial in contemporary literature on higher-order metaphysics (see, e.g., Bacon and Dorr (forthcoming)).

Let's say that a proposition p is *dark* relative to a property of propositions X iff it is necessary that if the disjunction of p with α is X, then p is false:

$$\text{Dark}\, pX := \Box(X(p \vee \alpha) \to \neg p)$$

When a proposition p is dark relative to knowledge and is not actually known, then

$p \vee \alpha$

Our second result is as follows, with the proof given at the end of the subsection:

Theorem 2 ($\mathsf{H}_{\Box}$ + Actual world)**.** *For all verificationist and necessarily factive properties of propositions X, the disjunction of any proposition which is dark relative to X with α, is itself X:*

$$\forall X\,.\,((\mathrm{Ver}\,X \wedge \Box\,\mathrm{Fact}\,X) \rightarrow \forall p\,.\,(\mathrm{Dark}\,pX \rightarrow X(p \vee \alpha)))) \tag{25}$$

Let's see how this applies to knowledge. Say a proposition is simply *dark* (as opposed to dark relative to some X) iff it is dark relative to knowledge. In other words, a dark proposition is one that is not possibly true while its disjunction with α is known. By Theorem 2 and the necessary factivity of knowledge, it follows that, if any disjunction of a dark proposition with α is unknown, then verificationism is false.

In light of this, it is overwhelmingly plausible that there are disjunctions of the kind required to refute verificationism. Notice that a dark proposition can be generated by conjoining the proposition that nobody ever knows anything with any other proposition (although such conjunctions will not exhaust the dark propositions). For example:

> Nobody ever knows anything and the mass of the sun is exactly x kilograms.
>
> Nobody ever knows anything and the average temperature of Earth is exactly y Kelvin.
>
> Nobody ever knows anything and there are exactly n protons.
>
> Nobody ever knows anything and Uranium is exactly z times more abundant than Plutonium in the Milky Way.

These propositions are dark because, if their disjunction with α is known, then somebody does know something after all, namely the proposition just mentioned, in contradiction with the first conjunct which says nobody knows anything. So each has the property that necessarily, if its disjunction with α is known, then it is false, which is to say that each is dark.

Presumably it is possible for nobody to know anything—for example if there had been no life, and thus no knowers around to do any knowing. Moreover, the above propositions will have different truth-conditions for different values of x, y, n, and z, because the absence of life is presumably compossible with basically any possible mass of the sun, possible temperature of the Earth, possible number of protons, or possible ratio of Uranium to Plutonium in the Milky Way. Thus, even by the strict standard of intensionalism, there are many distinct propositions for different values of x, y, n, z in this range (perhaps there are even uncountably many, but the present argument does not rest on this).

If Verificationism holds, then by Theorem 2 the disjunction of each such proposition with α is known. And it is clear that (absent an omniscient being) many such propositions will never be known. They are simply too obscure, and there are far too many of them. So Verificationism is false.

Proof of Theorem 2. Since α is true, $p \vee \alpha$ is true for any p, so in particular if if X is a verificationist property then

$$\forall p \,.\, \Diamond X(p \vee \alpha) \tag{26}$$

If X is necessarily factive, then,

$$\forall p \,.\, \Box(X(p \vee \alpha) \to (p \vee \alpha)) \tag{27}$$

Now, if an arbitrary p is dark with respect to X, we have

$$\Box(X(p \vee \alpha) \to \neg p) \tag{28}$$

whence by (27) and normal modal logic we can conclude

$$\Box(X(p \vee \alpha) \to \alpha) \tag{29}$$

and then, by (26),

$$\Diamond(\alpha \wedge X(p \vee \alpha)) \tag{30}$$

Now suppose $\neg X(p \vee \alpha)$. Then since α necessitates every truth,

$$\Box(\alpha \to \neg X(p \vee \alpha)) \tag{31}$$

which is inconsistent with (30), so we have

$$\neg\neg X(p \vee \alpha) \tag{32}$$

and then $X(p \vee \alpha)$ by double negation elimination. Since p was arbitrary this completes the proof. □

5.3 Our second result in intuitionistic logic

The challenge for verificationism raised in the last subsection is not easily avoided in the setting of intuitionistic logic. The proof of Theorem 2 is intuitionistically valid except for the application of double negation elimination at the end, i.e., it is valid up to that point in the intuitionistic higher-order modal logic presented in Appendix A.8. Thus, even the intuitionist should accept what is derived up until that point, namely

$$\forall X \,.\, (\mathrm{Ver}\, X \wedge \Box\, \mathrm{Fact}\, X) \rightarrow \forall p \,.\, \mathrm{Dark}\, pX \rightarrow \neg\neg X(p \vee \alpha)) \tag{33}$$

Since the proof of Theorem 2 still works up to (33), the intuitionist can only preserve verificationism by affirming that for every dark proposition, its disjunction with α is not unknown. In particular, in relation to the aforementioned examples which were taken to refute verificationism in the classical setting, they must affirm sentences such as

> For every real number x, if the proposition *nobody ever knows anything and the mass of the sun is exactly x kilograms* is dark, then the disjunction of that proposition with α is not unknown.

As well as analogous conditionals concerning the temperature of Earth, the number of protons, and the proportion of Uranium to Plutonium in the Milky Way. Even the intuitionist must accept in each case that the propositions in question are dark for any value of the parameter, since they include the conjunct *nothing is known*. Hence they must accept:

(*) For every real number x, the proposition *either α or nobody ever knows anything and the mass of the sun is exactly x kilograms* is not unknown.

As well as analogous propositions concerning the other quantities. But the verificationist is in no position to accept this deeply implausible claim—for some values of x surely nobody will ever entertain, let alone know, the proposition in question. So the intuitionist is not in a better position to accept verificationism.

Dummett (2009) suggests that the verificationist can accept sentences like (*). He does so by suggesting the reinterpretation of a negated formula $\neg A$ as

> It is in principle impossible for us to be in a position to assert that A. (Dummett 2009, p. 52)

Thus reinterpreted, (*), asserts the not obviously false

(**) For every real number x, the proposition *either α or nobody ever knows anything and the mass of the sun is exactly x kilograms* is such it is in principle impossible for us to be in a position to assert that it is unknown.

(**) is not obviously false because it is difficult to produce specific counterexamples to (*). This is difficult because whenever one instantiates the sentence (*) with a particular name **x** of a real number, it is plausible that we do know the corresponding proposition by recognizing the sentence 'either α or nobody ever knows anything and the mass of the sun is exactly **x** kilograms' as a theorem of a suitable higher-order logic.

Similarly, it is difficult to produce counterexamples to the principle

(†) Every rock is not such that it is not ever thought of by anyone.

Because by producing a counterexample one would presumably have to think of it. And so, although this principle is obviously false (many rocks deep underground and on distant planets will not ever be thought of), it is much more plausible that

(††) Every rock is such that it is in principle impossible for us to be in a position to assert that it is not ever thought of by anyone.

If we adopt Dummett's proposal for the interpretation of 'not', (†) becomes plausible. This is not a good way of defending theories which imply that sentence, which is clearly false. It only shows Dummett's gloss on the interpretation of 'not' is a bad one—(†) *should* be implausible, given a moment's reflection on the number of rocks in the world and the lack of conceivable motivation for thinking of most of them.

The same goes for (*) and (**). Dummett proposes interpreting negation so that (*) means (**). But this is not a way of making (*) plausible—(*) is refuted by a moment's reflection; it only shows Dummett's gloss of 'not' is wrongheaded.[18] We should add that this is no argument that the intuitionist must accept classical logic, since the intuitionist is under no pressure to accept Dummett's reinterpretation of 'not'. Rather, it only shows that Dummett has not made the consequent of (33) plausible, so reaffirms the untenability of verificationism even for the intuitionistic logician.

6 Conclusion

While previous arguments in the spirit of Church's own (unendorsed) argument against verificationism were not decisive, there are convincing arguments ready to

[18]The examples show that what Dummett intends by 'not' is not strong enough. It is also too strong in some circumstances. For example, you might not be jumping up and down right now, but it is not in principle impossible for us to be in a position to assert that you are jumping up and down right now. For example, you could have chosen to jump up and down and to tell us about it.

Here is another reason why the gloss on 'not' is problematic. Everyone should be sure that it is in principle impossible to assert that no assertion is being made. But this seems like a terrible basis for being sure that it is not the case that no assertion is being made. In a quiet moment for humanity, uncertainty on this matter could be perfectly reasonable.

hand.

A Church's higher-order logic and variations

All systems considered here will be systems of higher-order logic based on Church (1940), and which has become standard in the literature. We use the notational conventions of Goodsell and Yli-Vakkuri (2024). For introductions to the topic, see Dorr, Hawthorne, and Yli-Vakkuri (2021, Ch. 0), Bacon (2023), Fritz and Jones (forthcoming).

A.1 Church's logic

Church's logic has the following important components, which are described informally (for precise statements see Church (1940), or Goodsell and Yli-Vakkuri (2024)):

Classical propositional logic Including every classical tautology with sentence letters replaced by formulae of the language of higher-order logic.

$\beta\eta$-equivalence The standard rule for λ-terms, which most importantly permit the substitution of $(\lambda \mathbf{x} \,.\, \mathbf{A})\mathbf{b}$ for $[\mathbf{b}/\mathbf{x}]\mathbf{A}$ in any context.

Classical quantified logic Standard introduction and elimination rules for $\forall$ and $\exists$.

Axiom of descriptions The description function ι maps every uniquely instantiated property to the instance thereof.

$$\forall X^{\sigma t} \,.\, ((\exists! y \,.\, Xy) \rightarrow X(\iota X)) \tag{34}$$

(note: there is to be a distinct description function constant ι_σ of type $\langle \sigma t \rangle \sigma$ for each type σ.)

In addition, Church's logic includes three further axiom schemata which are irrelevant to this paper:

Function extensionality Which asserts that functions which give the same value on every input are identical, i.e.,

$$\forall f g^{\sigma\tau} \,.\, (\forall x \,.\, (fx = gx) \rightarrow f = g) \tag{35}$$

Axiom of choice A strengthened form of the axiom of descriptions, asserting that the choice function ε maps every instantiated property (whether or not uniquely instantiated) to some instance thereof.

$$\forall X^{\sigma t} \,.\, ((\exists y \,.\, Xy) \rightarrow X(\varepsilon X)) \tag{36}$$

Infinity An axiom to the effect that there are infinitely many individuals.

A.2 H

For classical variations on Church's logic we begin with the logic H, which consists of **classical propositional logic**, **$\beta\eta$-equivalence**, and **classical quantified logic** (H is also used as a starting point by Bacon (2018), Dorr, Hawthorne, and Yli-Vakkuri (2021), and Bacon and Dorr (forthcoming), among others). Notably, it excludes the axiom of descriptions, as well as the presently-irrelevant axioms of function extensionality and choice.

A.3 Modal higher-order logic; $\mathsf{H}_\Box$

The standard modal logic KT is adjoined to H by the addition of three axioms and one rule of inference. The axioms are:

K Necessity distributes over the material conditional.

$$\forall pq \,.\, (\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)) \tag{37}$$

T Necessity is factive.

$$\forall p \,.\, (\Box p \rightarrow p) \tag{38}$$

Possibility definition To be possible is to be not necessarily not true.

$$\Diamond = \lambda p \,.\, (\neg\Box\neg p) \tag{39}$$

The rule of inference is as follows:

Rule of necessitation Given a derivation of **P** from no assumptions, infer $\Box$**P**.

$\mathsf{H}_\Box$ is H plus these components.

Adding KT modal logic to a stronger logic than H gives rise to an important choice point: shall we consider the new axioms to be necessarily true, or not? If not, then it will not be appropriate to extend the rule of necessitation to derivations that employ the new axioms. For all extensions of $\mathsf{H}_\Box$ considered here, we will *not* be extending the rule of necessitation in this way.

Notably, it is essential to exclude axioms like the axiom of descriptions from the application of the rule of necessitation. The axiom is intended to express a contingent truth. For example, one instance of the axiom of descriptions says that ι(King of England) is identical to the King of England, if there is a unique King of England. Clearly, since Charles is that man, ι(King of England) is identical to Charles. But it is hardly necessary that Charles is King of England if there is a unique King of England—it could have been William instead. That is, the axiom of descriptions (and for the same reason the axiom of choice) is only contingently true. It is, in the terminology of Kripke (1980), an *a priori* contingent sentence (supposing it to be true).

The same goes for the weakenings of the axiom of descriptions, class comprehension and actuality, also considered here.

A.4 Class comprehension

We employ a small variation on Church's (1940: 61) definition of class (which has the effect of simplifying the definition of class-membership):

Definition 1 (Class$_\sigma$)**.** A class is a $\top$-or-$\bot$-valued propositional function (recall that $\top$ is an arbitrarily chosen tautology, e.g. $\exists p \,.\, p$, and $\bot$ its negation):

$$\text{Class}_\sigma := \lambda X^{\sigma t} \,.\, \forall y \,.\, ((Xy = \top) \vee (Xy = \bot)) \tag{40}$$

Definition 2. Membership in a class, $y \in X$, is the same as instantiating that class:

$$\mathbf{a} \in \mathbf{B} := \mathbf{Ba} \tag{41}$$

(On Church's definition, a class is a 0-or-1-valued function, and membership is being mapped to 1 by the class.)

The proof of Theorem 1 relies on $\mathsf{H}_\Box$ plus

Class comprehension $\{x : \mathbf{P}\}$ is a class, and being a member of $\{x : \mathbf{P}\}$ is equivalent with being some x such that $\mathbf{P}$.

$$(\text{Class}\{x : \mathbf{P}\}) \wedge (\forall x \,.\, (\mathbf{P} \leftrightarrow x \in \{x : \mathbf{P}\})) \tag{42}$$

As mentioned in Section 5.1, class comprehension is an easy consequence of the axiom of descriptions given the definition

$$\{x : \mathbf{P}\} := \lambda x \,.\, \iota q \,.\, ((\mathbf{P} \to (q = \top)) \wedge (\neg\mathbf{P} \to (q = \bot))) \tag{43}$$

The converse does not hold. However, the main reasons for which the axiom of descriptions is controversial in contemporary literature in higher-order metaphysics apply also to class comprehension—see Bacon and Dorr (forthcoming).

A.5 Actuality

The characteristic axiom for the actuality operator, @, is as follows:

Actuality

$$\forall p \,.\, ((p \to \Box @ p) \wedge (\neg p \to \Box \neg @ p)) \tag{44}$$

In $\mathsf{H}_\Box$ plus class comprehension, this axiom follows from the definition $@ := \{p : p\}$, hence the actuality axiom is a theorem of Church's logic plus KT modal logic, on this definition and the definition of class abstracts.

A.6 Actual world

Theorem 2 relies on one assumption beyond $\mathsf{H}_\Box$:

Actual world α is true and necessitates every truth.

$$\alpha \wedge \forall p \,.\, (p \to \Box(\alpha \to p)) \tag{45}$$

The actual world axiom follows from the actuality axiom on the definition of α as the proposition that every true proposition is actually true:

$$\alpha := \forall p \,.\, (p \leftrightarrow @ p) \tag{46}$$

hence also from class comprehension and from the axiom of descriptions. However, the actual world axiom is much weaker than the actuality axiom in some significant respects that make it much less controversial in recent literature on higher-order metaphysics—see Bacon and Dorr (forthcoming).

A.7 Intensionalism

Intensionalism Necessarily equivalent propositions are identical.

$$\forall pq\,.\,(\Box(p \leftrightarrow q) \rightarrow p = q) \tag{47}$$

A.8 Intuitionistic logic

In Section 5.2, it is asserted that Theorem 2 works in an intuitionistic logic, besides the final application of double-negation elimination. The logic we have in mind is as follows. It consists of an intuitionistic version of $\mathsf{H}_\Box$, with intuitionistic propositional logic instead of classical propositional logic, and intuitionistic quantified logic for quantifiers of all types in place of classical quantified logic—see, e.g., Troelstra and Dalen (1988). The $\beta\eta$-equivalence rule is left unchanged.

The modal component of the logic is as follows (following standard presentations, e.g. Simpson 1994). In addition to the K and T axioms and the rule of necessitation, the logic includes the following axioms for $\Diamond$ (which is taken as a primitive rather than defined) in place of the definition of possibility as being not necessarily not true:

Possibility distribution Any necessary consequence of something possible is also possible.

$$\Box(p \rightarrow q) \rightarrow (\Diamond p \rightarrow \Diamond q) \tag{48}$$

Contradiction impossibility A contradiction is not possibly true.

$$\neg\Diamond\bot \tag{49}$$

Finally, the logic includes the actual world axiom (Appendix A.6). Note that in the intuitionistic logic described, the actual world axiom is not derivable from the axiom of descriptions. Nevertheless, the actual world axiom is independently plausible, and notably does not cause a collapse to classical logic (this may be confirmed by interpreting α as $\top$ in an extensional intuitionistic higher-order logic). It is also worth noting that for the purpose of problematizing verificationism about K, it would suffice to replace α with a sentence α_K with the axiom

$$\alpha_K \wedge \forall p\,.\,(\neg Kp \rightarrow \Box(\alpha_K \rightarrow \neg Kp)) \tag{50}$$

That is, α_K is a proposition that for each unknown proposition, necessitates it is unknown.

References

Bacon, Andrew (2018). “The Broadest Necessity”. In: *Journal of Philosophical Logic* 47.5, pp. 733–783.

— (2023). *A Philosophical Introduction to Higher-Order Logics*. Routledge.

Bacon, Andrew and Cian Dorr (forthcoming). “Classicism”. In: *Higher-order Metaphysics*. Ed. by Peter Fritz and Nicholas K. Jones. Oxford University Press.

Chalmers, David J. (2011). “Actuality and Knowability”. In: *Analysis* 71.3, pp. 411–419.

Church, Alonzo (1940). “A Formulation of the Simple Theory of Types”. In: *Journal of Symbolic Logic* 5.2, pp. 56–68.

— (1951). “A Formulation of the Logic of Sense and Denotation”. In: *Structure, Method, and Meaning*. Ed. by Paul Henle Paul, Horace M. Kallen, and Suzanne K. Langer. The Liberal Arts Press, pp. 3–24.

— (2009). “Referee Reports on Fitch’s "Definition of Value"”. In: *New Essays on the Knowability Paradox*. Ed. by Joe Salerno. Oxford University Press, pp. 13–20.

Dorr, Cian (2016). “To Be F Is To Be G”. In: *Philosophical Perspectives* 30.1, pp. 39–134.

Dorr, Cian, John Hawthorne, and Juhani Yli-Vakkuri (2021). *The Bounds of Possibility: Puzzles of Modal Variation*. Oxford: Oxford University Press.

Dummett, Michael (1959). “Truth”. In: *Proceedings of the Aristotelian Society* 59.1, pp. 141–62.

— (2009). “Fitch’s Paradox of Knowability”. In: *New Essays on the Knowability Paradox*. Ed. by Joe Salerno. Oxford University Press.

Edgington, Dorothy (1985). “The Paradox of Knowability”. In: *Mind* 94.376, pp. 557–568.

Fitch, Frederic (1963). “A Logical Analysis of Some Value Concepts”. In: *Journal of Symbolic Logic* 28.2, pp. 135–142.

Fritz, Peter (2023). *The Foundations of Modality: From Propositions to Possible Worlds*. Oxford: Oxford University Press.

Fritz, Peter and Nicholas K. Jones (forthcoming). “Higher-Order Metaphysics: An Introduction”. In: *Higher-Order Metaphysics*. Ed. by Peter Fritz and Nicholas K. Jones. Oxford University Press.

Goodsell, Zachary and Juhani Yli-Vakkuri (2024). “LF: a Foundational Higher-Order-Logic”. In: arXiv: `2401.11050 [math.LO]`.

Hilbert, David and Wilhelm Ackermann (1928). *Grundzüge der theoretischen logik*. J. Springer.

Kaplan, David (1964). “Foundations of Intensional Logic”. PhD thesis. UCLA.

— (1968). “Quantifying In”. In: *Synthese* 19.1-2, pp. 178–214.

Kripke, Saul A. (1980). *Naming and Necessity*. Harvard University Press.

Lewis, David K. (1973). *Counterfactuals*. Malden, Mass.: Blackwell.

Myhill, John (1958). “Problems Arising in the Formalization of Intensional Logic”. In: *Logique Et Analyse* 1.1, pp. 78–83.

Salerno, Joe (2008). *New Essays on the Knowability Paradox*. Oxford, England and New York, NY, USA: Oxford University Press.

Simpson, Alex K. (1994). “The proof theory and semantics of intuitionistic modal logic”. In.

Stalnaker, Robert C. (1976). “Propositions”. In: *Issues in the Philosophy of Language: Proceedings of the 1972 Colloquium in Philosophy*. Ed. by Alfred F. MacKay and Daniel D. Merrill. Yale University Press, pp. 79–91.

Tennant, Neil (1997). *The Taming of the True*. New York: Oxford University Press.

— (2001). “Is Every Truth Knowable? Reply to Williamson”. In: *Ratio* 14.3, pp. 263–280.

Troelstra, Anne and Dirk van Dalen (1988). *Constructivism in Mathematics, Vol 1*. ISSN. Elsevier Science.

Williamson, Timothy (2000). *Knowledge and its Limits*. Oxford University Press.