Application of "A Thing Exists If It's A Grouping" to Russell's Paradox and Godel's First Incompleteness Theorem

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Abstract

A resolution to the Russell Paradox is presented that is similar to Russell's "theory of types" method but is instead based on the definition of why a thing exists as described in previous work by this author. In that work, it was proposed that a thing exists if it is a grouping tying "stuff" together into a new unit whole. In tying stuff together, this grouping defines what is contained within the new existent entity. A corollary is that a thing, such as a set, does not exist until after the stuff is tied together, or said another way, until what is contained within is completely defined. A second corollary is that after a grouping defining what is contained within is present and the thing exists, if one then alters what is tied together (e.g., alters what is contained within), the first existent entity is destroyed and a different existent entity is created. A third corollary is that a thing exists only where and when its grouping exists. Based on this, the Russell Paradox's set R of all sets that aren't members of themselves does not even exist until after the list of the elements it contains (e.g. the list of all sets that aren't members of themselves) is defined. Once this list of elements is completely defined, R then springs into existence. Therefore, because it doesn't exist until after its list of elements is defined, R obviously can't be in this list of elements and, thus, cannot be a member of itself; so, the paradox is resolved. This same type of reasoning is then applied to Godel's first Incompleteness Theorem. Briefly, while writing a Godel Sentence, one makes reference to a future, not yet completed and not yet existent sentence, G, that claims its unprovability. However, only once the sentence is finished does it become a new unit whole and existent entity called sentence G. If one then goes back in and replaces the reference to the future sentence with the future sentence itself, a totally different sentence, G1, is created. This new sentence G1 does not assert its unprovability. An objection might be that all the possibly infinite number of possible G-type sentences or their corresponding Godel numbers already exist somehow, so one doesn't have to worry about references to future sentences and springing into existence. But, if so, where do they exist? If they exist in a Platonic realm, where is this realm? If they exist *pre-formed* in the mind, this would seem to require a possibly infinite-sized brain to hold all these sentences. This is not the case. What does exist in the mind is the system for creating G-type sentences and their corresponding numbers. This mental system for making a G-type sentence is not the same as the G-type sentence itself just as an assembly line is not the same as a finished car. In conclusion, a new resolution of the Russell Paradox and some issues with proofs of Godel's First Incompleteness Theorem are described.

The Russell Paradox

The Russell Paradox (Russell, 1903) considers the set, R, of all sets that are not members of themselves. On its surface, R seems to belong to itself only if it doesn't belong to itself. This is where the paradox comes from. Several ways to resolve this paradox have been proposed over the years starting with Russell's own method based on the theory of types (Russell, 1903; Russell, 1908). Here, a solution is proposed that is similar to that based on the theory of types but is instead based on the definition of why things exist as described previously by this author (Granet, 2001). In that work, it was proposed that a thing exists if it is a grouping tying "stuff" together into a new unit whole. This grouping is equivalent to a definition of what is contained within (e.g., what is tied together) the new existent entity. A corollary to this is that a thing, such as a set, does not exist until the stuff is tied together, or said another way, until what is contained within is defined. For instance, a set does not exist until after its elements are defined (e.g., tied together). The tying together, or complete definition, of the elements and the subsequent existence of the thing is represented by an enclosing surface or boundary, which in the case of a set, is denoted by the curly braces around the list of elements in the set. A second corollary is that after a grouping defining what is contained within is present and the thing exists, if one then alters what is tied together (e.g., alters the definition of what is contained within), the first existent entity is destroyed and a different existent entity is created. A third corollary is that a thing exists only where and when its grouping exists. Two seemingly identical things whose groupings are at different places or times are two different and distinct existent things. Now, in regard to the Russell Paradox, suppose one is writing a list of all the sets that are not members of themselves. When you're done writing the complete list, you wish to call that grouping of sets, set R. Based on the above reasoning, set R does not even exist until after the list of the elements it contains (e.g. the list of all sets that aren't members of themselves), and that are to be tied together, is defined. Once this list of elements is completely defined, R then springs into existence. Therefore, because it doesn't exist until after its list of elements is defined, R obviously can't be a member of itself; so the paradox is resolved. Additionally, you can't then put R back into its list of elements after the fact because if you did this, it would be a different list of elements, and it would no longer be the original set R, but some new set, R1. Overall, because set R can't be a member of itself, there is no paradox.

Another way of looking at this is via the idea of perspective, or reference frames. When writing the list of elements in a set, the mathematician's perspective is that of being inside the set. However, once the membership list is fully defined, the set springs into existence, and the mathematician's vantage point now shifts to the "outside" of the set, to a different reference frame, where he can see the set as a new unit whole and as being in existence. The existent set, as a whole, cannot then be pushed back into its original membership list because this list is in a different reference frame, internal to the set. It would be similar to trying to make your body be a component of itself. This suggests that the way we perceive sets, and, in general, things that exist, depends on the perspective, or reference frame, we're observing them from.

Since this was first published online (Granet, 2001), another author (Pleitz, 2018) independently used somewhat similar reasoning to resolve the Liar Paradox. As he says “Detailed semantico-metaphysical arguments show that in this dynamic setting, an object can be referred to only after it has started to exist. Hence the circular reference needed in the Liar paradox cannot occur, after all.”

Godel's First Incompleteness Theorem: Godel sentence

This same reasoning applies to Godel's first Incompleteness Theorem (Godel, 1931), which very roughly states that there will always be some statements made using a formal system of arithmetic, P, that are true but that can't be proven to be true based on system P. Godel's proof of this theorem (reviewed in Raatikainen, 2021) relies on converting all elements of the formal system (mathematical and logical symbols, variables, formulas, etc.) to a unique number, called a Godel number. He then substitutes a Godel number for a formula back into the formula itself. Here though, for the sake of clarity, I won't focus on Godel numbers but will instead use two natural language-based explanations for Godel's proof that make use of Godel sentences and diagonalization and will show that the same reasoning used to resolve the Russell Paradox reveals problems with both of these proofs.

A natural language paraphrasing (Rucker, 1995) for the layperson of the Godel sentence-based proof of the incompleteness theorem is:

1. "System P will never say that this sentence is true."
2. Now, call this sentence G for Godel.
3. Note that G is equivalent to: 'System P will never say G is true' ".

This seems to be saying that the formal system P can produce statements that it can't show to be true, I.e., Godel's theorem. But, a problem with this is as follows. G, which is defined as the *whole* sentence "System P will never say that this sentence is true." is only defined and, therefore, only exists after there is a whole sentence. That is, a sentence is a grouping that ties several individual, previously unrelated words together in a certain order to create a new unit whole called the “completed sentence”. Until these words are tied together in a certain order, the completed sentence does not exist. So, statement 1 makes a claim about something that doesn't even exist yet, which on its own seems non-sensical. Despite this, once the sentence exists, if you then go back in and replace (or substitute in the parlance of Godel's Incompleteness Theorem), the words “this sentence” that refer to the future, completed sentence with the future, completed sentence itself, you're creating a new and different grouping, or sentence, G1. In other words, if you do this, you'd be creating

G1: “System P will never say that (System P will never say that this sentence is true.) is true."

and not the original

G: 'System P will never say this sentence is true' ".

Looking at sentence G1, there's no longer a reference to a future, not yet completed sentence. But, what it seems to be saying is that:

System P will never say that (sentence claiming that system P will never say a sentence is true) is true.

This seems to be saying that System P will never be able to say that (the claim that it will never say a sentence is true) is itself true. This is almost the opposite of incompleteness and is in fact saying that System P can never prove that it's incomplete.

Stated differently, one might say that when reading statement 1, the mind of the reader, or observer, is in the reference frame that is inside the "construction site" where the sentence is being built and that part of that site is referring to a future, not yet existent, reference frame in which the whole sentence is fully constructed and actually exists. Then, when statement 1 is completed, the mind of the observer jumps to the outside of the sentence and can see it as a whole, newly existent, completed sentence and calls it sentence G. But, once that happens, the external observer can't then go back in and stick the completed sentence as a whole, G, back inside the internal reference frame after the fact without creating a totally different sentence. The reference to the future, not yet existent sentence is not the same as the future, existent sentence itself. The jumping back and forth between reference frames is what causes problems in the reasoning in both the Russell Paradox and the Godel sentence. Taken together, while it may be true that there is some sentence that cannot be proved by system P, the Godel sentence method cannot logically be used to prove this.

An objection to this argument is that the possibly infinite number of all possible G-type sentences derived from arithmetic system P, or their corresponding Godel numbers, already exist somehow, and so there is no "this sentence"-does-not-yet-exist (or self-reference) problem. But, where do these sentences or numbers exist? If the answer is a Platonic realm, can you provide evidence for, or point out, this realm? Until then, it's purely a faith-based argument. It could be true but can't be argued rationally. Alternatively, do the possibly infinite number of these sentences or numbers exist *pre-formed i*n the mind, and if so , wouldn't this require a mind of possibly infinite size, which humans don't have? 48,934,386,127,842. Did that number exist in your brain before reading it just now? No. Therefore, it seems unlikely that these sentences or numbers already exist either in a Platonic realm or in the mind. However, what does exist in the mind is a system, or method, for creating G-type sentences and/or Godel numbers. The system is composed of knowledge of how a formal language or logic system works, knowledge of mathematics and how the sequential character of the number line works, a visualization method, and language ability for saying things. This system is what exists in the mind; the pre-formed Godel sentences and numbers don't exist until this mental system creates them, and the system is not the same as the sentence itself, just as an automobile assembly line is not the same as a finished car. This is what is meant when it was said above that the reference to the future sentence G is not the same thing as future sentence G itself.

Others have told me that Godel sentences are exceptions to the idea that it's incorrect to replace a reference to a future not yet existent sentence with the future sentence itself once it exists because, and I'm paraphrasing their reasoning here, Godel sentences and Godel's proof don't depend on the semantic meaning of the words in the sentence and instead only depend on the overall string of symbols in the sentence as shown by the corresponding Godel number. In response, though, I would say that if Godel sentences are not talking about themselves as a meaningful sentence but only as a string of symbols, then:

1.) The unique Godel number of a sentence depends not only the letters, numbers and symbols in the sentence but on the sequence the letters, numbers and symbols are in, and the sequence they're in is directly related to the meaning of the sentence.

2.) If the theorem is only saying that a meaningless sentence can't be proved, what use is a meaningless sentence in a formal system of arithmetic? A somewhat similar objection has been raised by Johnstone (2003) who stated "If the Gödel sentence is not meaningful, then its assertion that it is not derivable is not meaningful."

An objection similar to the semantic-meaning/symbols-only objection might be that statements made using a formal system of arithemetic P are abstract constructons, so the time aspect doesn't matter. In other words, who cares if “this sentence” is a reference to a sentence that doesn't exist yet? However, if system P is atemporal, can it really be useful for anything? For instance, the logical symbol → meaning “implies” or “if-then” obviously implies a temporal process. The simple equation x +2 = 4 means that *when* you add 2 to x, you *get* 4. When you do something, then you get something is a temporal process. So, time and sequential processes matter in arithemetic. Given that they matter in arithemetic, they also matter in the formal system of arithmetic P. Thus, one can't ignore the fact that “this sentence” in statement 1 is a reference to a future, not yet existent sentence and not the future, not yet existent sentence itself.

Another more concrete way of thinking about the above is this. Suppose you're in the middle of making some product using components A1, A2, A3, A4 and a piece of paper saying “I am the completed version of this product”. Once you're done, call this product with the piece of paper in it product B. Then, if you could somehow stick finished product B back in place of the note, which seems impossible in the first place, you'd have a product with components A1, A2, A3, A4 and its future completed self B in the middle of it. This new product, called B1, is a different existent entity than product B. B is the product with the piece of paper in it, but B1 is the product with its complete self as one of its own components. These are two different things because the groupings tying things together are tying different things together. And, it doesn't matter that this example is of a physical product, and the Godel Sentence is an abstract sentence. The principle is exactly the same.

Godel's First Incompleteness Theorem: Diagonalization

The diagonalization argument for Godel's theorem can be summarized for the layperson (http://en.wikipedia.org/wiki/Gödel's_incompleteness_theorems) by using the following puzzle-like sentence

", when preceded by itself in quotes, is unprovable.", when preceded by itself in quotes, is unprovable.

Wikipedia explains the argument as:

"This sentence does not directly refer to itself, but when the stated transformation is made the original sentence is obtained as a result, and thus this sentence asserts its own unprovability. The proof of the diagonal lemma employs a similar method."

Quine (1966) refers to a similar sentence (“Yields a falsehood when appended to its own quotation” yields a falsehood when appended to its own quotation.) in his explanation of Godel's proof as discussed by Vidal-Rosset (2006).

The problem with this argument is as follows. First, suppose the below sentence is called G.

", when preceded by itself in quotes, is unprovable.", when preceded by itself in quotes, is unprovable.

What this sentence says is that after doing the transformation suggested, which is the part shown in red, the resulting sentence is unprovable. So, first off, this sentence is making a claim of unprovability about a not yet existent sentence, which on its own is non-sensical. Beyond this, though, the words in red fulfill the same role as “this sentence” in the Godel sentence example (above); they're a reference to a future not yet existent sentence and not that future sentence itself. So, let's do the transformation and get the future sentence that sentence G says will be unprovable. The transformation is to take the phrase inside the quotes, make a copy of it, put quotes around the copy and put the quoted copy before the phrase. After doing this, we get the transformed stuff, shown below in red, which is the sentence that G says is unprovable.

", when preceded by itself in quotes, is unprovable.", when preceded by itself in quotes, is unprovable (G says this is unprovable)

So again just as with the Godel sentence argument, this is saying that the transformed sentence in red, which claims its unprovability, is itself unprovable. This is the opposite of incompleteness and is in fact saying that this system can never prove that it's incomplete.

Others take sentence G

", when preceded by itself in quotes, is unprovable.", when preceded by itself in quotes, is unprovable.

to mean that you should take the phrase inside the quotes, make a copy of it, put quotes around the copy and put the quoted copy before the phrase, but then they ignore the part about sentence G saying the resulting sentence is unprovable. This seems incorrect.

The same objections to the Godel Sentence proof that were discussed above could also be made for the diagonalization argument. In other words, that the statements in red, or their corresponding Godel numbers, already exist somewhere, so there is no problem with a reference to a future not yet existent sentence. But, as above, do these parts in red exist in a Platonic realm, for which no evidence is present? Or, do their possibly infinite number of Godel numbers exist pre-formed in the mind, which would seem to require a mind of possibly infinite size? No. Again, the systems for creating these sentences or numbers exist in the mind, but the possibly infinite number of pre-formed sentences and numbers do not.

Taken together, the above reasoning suggests that these two informal proofs of Godel's first Incompleteness Theorem are problematic. This doesn't mean that the theorem is incorrect; it just means that neither the Godel sentence nor the diagonalization argument can be used to prove it.

Conclusions

In conclusion, in an accompanying paper (Granet, 2001), a thing was defined as existing if it was a grouping tying things together into a new unit whole and thereby defining what is contained within that unit whole. Until this grouping defining what is contained within is present, the thing does not exist. After the grouping defining what is contained within is present, and the thing exists, if one then alters the definition of what is contained within, the first existent entity is destroyed and a different existent entity is created. Using this definition of an existent entity, a new resolution of the Russell Paradox and some problems with the Godel sentence and diagonalization proofs of Godel's first Incompleteness Theorem are presented. In the Russell Paradox, the set R of all sets that are not members of themselves doesn't exist until after its elements are defined, and once they are, set R can't then be stuck back into its own membership list. The same reasoning applied to Godel's first Incompleteness Theorem means that references to future sentences that state their unprovability are not the same as the future sentence itself even if they use the same words in the same order. They're different things that exist at different times. And when the future sentence is substituted back into the original, it ends up asserting that its unprovability can't itself be proved. In regard to Godel's first Incompleteness Theorem, the problems with the proofs don't mean that the theorem is incorrect; they just mean that neither the Godel sentence nor the diagonalization argument can be used to prove it.

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