Do Abstract Mathematical Axioms About Infinite Sets Apply To The Real, Physical Universe?

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Abstract

In mathematics, if one starts with the infinite set of positive integers, P, and want to compare the size of the subset of odd positives, O, with P, this is done by pairing off each odd with a positive, using a function such as P=2O+1. This puts the odds in a one-to-one correspondence with the positives, thereby, showing that the subset of odds and the set of positives are the same size, or have the same cardinality. This counter-intuitive result ignores the "natural" relationship of one odd for every two positives in the sequence of positive integers; however, in the set of axioms that constitute mathematics, it is considered valid. In the physical universe, though, relationships between entities matter. For example, in biochemistry, if you start with an organism and you want to study the heart, you can do this by removing some heart cells from the organism and studying them in isolation in a cell culture system. But, the results are often different than what occurs in the intact organism because studying the cells in culture ignores the relationships in the intact body between the heart cells, the rest of the heart tissue and the rest of the organism. In chemistry, if a copper atom was studied in isolation, it would never be known that copper atoms in bulk can conduct electricity because the atoms share their electrons. In physics, the relationships between inertial reference frames in relativity and observer and observed in quantum physics can't be ignored. Furthermore, infinities cause numerous problems in theoretical physics such as non-renormalizability. What this suggests is that the pairing off method and the mathematics of infinite sets based on it are analogous to a cell culture system or studying a copper atom in isolation if they are used in studying the real, physical universe because they ignore the inherent relationships between entities. In the real, physical world, the natural, or inherent, relationships between entities can't be ignored. Said another way, the set of axioms which constitute abstract mathematics may be similar but not identical to the set of physical axioms by which the real, physical universe runs. This suggests that the results from abstract mathematics about infinities may not apply to or should be modified for use in physics.

Infinite set (system)-infinite subset (subsystem) size comparison in abstract mathematics

Set theory is considered by many to be the theoretical foundation of mathematics (Bagaria, 2020), and the pairing-off method first developed by Cantor (1878) plays a major role in set theory. The pairing-off method uses a bijective function that lets the elements of one set be put into a one-to-one correspondence with the elements of another set. When the two sets whose elements form a one-to-one correspondence are infinite, the sets are said to have the same cardinality. So, if one starts with the set of all positive integers, P {1,2,3,4,5...}, and you want to compare the size of the subset of odd positive integers, O {1,3,5...}, with P, this can be be done by using a bijective function such as P=2O+1. Use of this function shows that there is a one-to-one correspondence between each odd and each positive integer as follows:

If O=1, the positive integer is 3
If O=3, the positive integer is 7
If O=5, the positive integer is 11
If O=7, the positive integer is 15
...

As can be seen, the gap between the odd integer and its corresponding positive integer keeps getting larger. However, because these are both countably infinite sets, this one-to-one correspondence can keep going forever, thereby showing that the sets are of the same size or cardinality. One might object and say that this thought experiment-derived result ignores the inherent, "natural" relationship in the set of positive integers of one odd positive integer for every two integers (e.g., (**1**,2), (**3**,4), (**5**,6), (**7**,8), (**9**,10)...) and therefore may create experimental artifacts. While the author agrees, it is considered by all mathematicians to be valid within the set of axioms that constitute abstract mathematics. Fair enough.

System-subsystem and system-system relationships in the real, physical universe

In the physical universe, system-subsystem and system-system relationships are of critical importance, and ignoring them in experiments is not valid and can cause incorrect results, or experimental artifacts. Two common sense examples of this are as follows. In chemistry, if one were to study a single copper metal atom in isolation, one might think that copper does not conduct electricity. However, this would be an experimental artifact because it is well known that copper in bulk, such as in a wire, does conduct electricity. This is because relationships between neighboring copper atoms let them share their electrons, which allows the electrons to flow more freely when a voltage is applied to the metal. In geology, if someone studied carbon from near the surface of the earth, that person would incorrectly conclude that this is the only form carbon could take without realizing that this carbon could eventually become a diamond if left for a longer period of time deep in the earth due to the high pressure and temperature found there. Thus, system-subsystem relationships cannot be overlooked in these chemistry and geology examples.

This also goes for fundamental physics. In special relativity, for instance, the length of an object or the rate of time's passage in inertial reference frame A as seen by an observer in inertial reference frame B will change depending on the velocity of A relative to B (Elani, 2020). Obviously, the relationship of one physical reference frames to another is of physical importance, and ignoring it can produce incorrect results. In the commonly accepted Copenhagen interpretation of quantum mechanics, the observer-observed relationship is of great importance. The observer or measuring apparatus interacts with and can affect the observed (Faye, 2019) which suggests that these two parts of the universe are physically related. Finally, the demonstration of seemingly instantaneous, or non-local, interactions between entities that once were close together but are now far apart (reviewed in Berkovitz, 2016) suggests an intimate and instantaneous relationship between these systems even if they are separated.

Additional evidence of the importance of system-subsystem and system-system relationships in the physical universe can be provided by the presence of experimental artifacts or biases when these relationships are ignored. Physics employs extensive use of advanced mathematics and computer simulations, which often require that a part of the system being studied is simplified, or "idealized", to simplify the mathematics or reduce computational demands. It is argued that this idealization and simplification is analogous to ignoring significant aspects of the subsystem-system interaction, just as the cell culture system does in biology, and can lead to artifacts. Indeed, such idealization-related artifacts are widely known in general relativity, cosmology and astrophysics (Barausse et al., 2021; Gueguen, 2019), condensed matter physics (Langmann, 2019; Savin, 2016, Walet, N. R. and Moore, M.A., 2013) and many other specialties in physics.

Beyond chemistry and physics, relationships between subsystems and their parent systems are of fundamental importance in biology as well. For instance, if one wants to study the function of heart cells, a common experimental method for doing this is to remove the heart from the organism, and then separate out the different kinds of cells from the heart. The cells of interest are then cultured in Petri dishes and studied in this in vitro (outside the whole organism) setting. However, while very valuable, it is widely known that the results in cell culture are often different than what occurs in the intact organism (Vetrees, 2009) because studying the cells in culture ignores the relationships in the intact body between the heart cells, the rest of the heart tissue, the rest of the organism and the environment. It also introduces conditions not present in the natural system (Pamies and Hartung, 2017). This results in expermental artifacts, which are experimental results that don't occur in the original system but are instead due to the expermental method used. Biologists always know that the experimental results found from studying something in vitro in cell culture are not necessarily representative of what occurs in the whole organism and must therefore be confirmed in the whole organism. In sum, the physical relationships between one part of a system and the overall system from which it came cannot be ignored in biology.

System-system relationships are also of critical importance in biology. As one of the main centers of metabolism in the body, the liver affects and interacts with many other organs. Insulin, produced by the pancreas, travels through the blood and affects glucose uptake by muscle and fat tissue. Pheromones produced by one animal affect the behavior of other animals. Plants, animals and microbes all interact with each other and the environment within an ecosystem. To ignore relationships between neighboring or communicating systems in biology is the same as not fully understanding how biological systems function.

In sum, it seems clear that system-subsystem and system-system relationships are of key importance and cannot be overlooked in the real, physical universe and that removal of a subsystem from a system drastically alters the results compared to the overall system-subsystem whole.

Do the abstract mathematical axioms about infinite sets and infinities cause problems in studies of the real, physical universe?

As described above, system-subsystem relationships can be ignored in the pairing-off method in the mathematics of infinities, but they cannot be ignored in the real, physical universe without the risk of producing incorrect results. Nor can the relationships between neighboring systems be ignored. Physics is the study of the real, physical universe, and mathematics is used extensively in physics. So, should the abstract mathematical results about infininte sets and infinities, with their built-in ignoring of the relationships in a sequence of numbers, really be used without change in physics? I suggest the answer is no and that their use may need to be modified for studies in physics.

That theoretical physics has problems with infinities is widely recognized. One of the earliest of these problems is, of course, Zeno's Paradoxes of motion. One of these, the Dichotomy Paradox, suggests that motion is impossible because for someone to move from point A to B requires that they always first have to move halfway from where they were to where they're going. This process continues forever, making movement of any distance impossible (reviewed in Huggett, 2019). The most common solution is that motion only occurs over unit sized, or finite, distances (Cote, 2013) and times (Huggett, 2019). But, this seems to be contrary to the requirement for continuous space in relativity (Dowker, 2014). So, while of less importance, solutions to Zeno's Paradoxes seem to conflict with relativity. Problems with infinities in classical physics have been discussed by Van Bendegem (1992). More relevant to modern physics is that infinities are abundant in quantum field theory and necessitate the troublesome renormalization technique to remove them (Nicolai, 2009). While this is now accepted, general relativity and, therefore, quantum gravity are non-renormalizable (Doboszewski and Linnemann, 2018; Hossenfelder, 2013). Another issue with infinities is the non-renormalizability of inflationary cosmology (Fumagalli, J. et al., 2020). The problems with infinities in physics is such that Tegmark even suggests that physicists look for ways of doing away with them (Tegmark, 2015). Overall, it seems possible that the use of the mathematics of infinities may cause problems and incorrect results and that their use should be modified in fundamental physics.

Conclusions

System-subsystem relationships may be ignorable in the set of axioms that constitute abstract mathematics as described above for the pairing-off method applied to the positive integers, but the evidence presented here makes it clear that neither they nor system-system relationships can be overlooked in the real, physical universe., and if they are, this can cause incorrect results. Indeed, the mathematics of infinities are widely known to cause problems in modern theoretical physics. This suggests that at least in the most fundamental of sciences, physics, the use of the pairing-off method and the mathematics of infinities may need to be modified.

An objection to this argument might be that physics has made great progress while using the mathematics of infinities, so why change? While this is true, theoretical physics seems to be at a standstill of late, and this is partly due to problems with infinities. The renormalization technique to remove infinities from equations in quantum physics was problematic for many years, and, as described, some fundamental physics theories such as general relativity, quantum gravity and inflationary cosmology are now plagued by non-renormalizable infinities so much so that some physicists wish to remove infinities from physics altogether (Tegmark, 2015).

It might also be said that the sequence of positive integers (1, 2, 3, 4, 5...) is nothing like the real universe, so the pairing-off method and the mathematics of infinities should not be a problem in physics. However, as shown above, infinities do cause problems in physics. Also, the pairing-off method is at the heart of set theory, which is at the heart of mathematics, and mathematics is used extensively in physics and thus, the problems with infinities cannot be ignored. Second, locations in space and moments in time have built-in spatial and temporal relationships to their neighboring locations and moments, respectively, in a very analogous way to the relationship between neighboring odd and even integers in the sequence of positive integers.

There is precedent for saying that although things can be mathematically valid, they may not occur in nature. For example, changing the sign of the time variable in physics equations is often mathematically fine, and the equation still works. But, time does not seem to run backwards in the real, physical universe.

While it is beyond the scope of this paper, there are other methods for measuring the relative sizes of sets besides pairing-off that may be of use in physics. One method is calculating the “measure” of a set. This is more of an intuitive, and physically meaningful, way of comparing the sizes of sets and subsets. Specifically, the Lebesgue measure can be used to compare the sizes of the sets in one-, two-, three- and n-dimensional spaces (Knapp, 2005) such as those found in the physical universe. Another way may be by using the concept of natural density, which is the density of a set within the set of natural numbers. The natural density of odd numbers relative to all the natural numbers is ½, for instance.

In conclusion, suppose a physicist submits a paper about the relationships found in a physical system but studies this system by destroying these built-in, inherent relationships, and then ignores the possibility that the results of this experimental processing may have been different than those that would have been obtained in the original system. I suggest that this paper would be, or should be, questioned. And, yet the mathematics of infinities is based on doing just this and is used extensively in physics. Given the known problems of infinities in physics, it seems reasonable that the mathematics of infinities should perhaps be modified, at least in fundamental physics.

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