Infinite Sets: The Appearance of an Infinite Set Depends on the Perspective of the Observer

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Abstract

Given an infinite set of finite-sized spheres extending in all directions forever, a finite-sized (relative to the spheres inside the set) observer within the set would view the set as a space composed of discrete, finite-sized objects. A hypothetical infinite-sized (relative to the spheres inside the set) observer would view the set as a continuous space and would see no distinct elements within the set. Using this analogy, the mathematics of infinities, such as the assignment of a cardinality to a set, depends on the reference frame of the observer thinking about them (the mind of the mathematician) relative to the infinite set. This reasoning may also relate to the differing views of space in relativity as continuous and in quantum mechanics as discrete.

The Appearance of Set N to Internal Observer O

What would an infinite set look like to internal and external observers? Consider a set, N, defined as containing an infinite number of discrete, finite-sized elements such as balls. One of these balls is defined as an internal observer, O. As such, O is of the same finite size scale as the balls. The balls extend outward in infinite numbers relative to any location and orientation of O. That is, wherever O is in the set and in whichever direction O is “looking”, the elements of the set extend without bounds the same potentially infinite distance in all directions relative to O. What does set N look like from internal observer O's perspective? Given the above:

- O sees the contents of the set as a discrete space made of finite-sized balls.
- O sees its size relative to the entire set as approaching, but never quite reaching, zero. It never reaches zero because O exists, and so its size can't be zero. Also, no matter how far O looks, it can never see an actually infinite endpoint of the set. From O’s viewpoint, the set is always potentially infinite and, thus, its size gets smaller and smaller relative to the whole set but never quite reaches zero. If O were ever able to see the whole set in its entirety, then O’s size would finally reach zero relative to that whole. Luckily, for O, that can’t happen!
- No matter how far O travels within its reference frame (e.g., inside set N), it will never reach the edge, boundary or “exit door” of the set due the unending nature of infinity.
- By definition of this set, relative to any location of element/observer O, the other elements progress radially away from it the same potentially infinite distance in all directions, and, thus, O would view the set as a potentially infinite sphere. It would be an odd sphere because the observer, or center point, O, can be at any point inside the sphere. Of course, elements within the sphere can never view the edge, so that, for them, the shape of the overall set only approaches that of an infinite sphere.
- O cannot “step outside” N and hope to be able to see the set as a whole. This is because O is of a different size scale than the set as a whole. So, even if O tried to step outside N, it would still be infinitesimally small relative to the set as a whole and would, therefore, still see set N as a potentially infinite-sized object. Thus, O is “trapped” in its reference frame, or “dimension”, inside the set.

In sum, relative to finite-sized internal observer O, set N would appear as a potentially infinite, spherical space composed of finite-sized, discrete elements.

The Appearance of Set N to External Observer P

Next, how would set N appear to a hypothetical, external observer P? Consider the same set N, defined above as having an infinite number of elements relative to any location and orientation of an element/observer, O, within the set. However, now assume that there is a second observer, P, outside this set and that P's size relative to O is actually infinite. That is, P is of the same size scale as the entire set N, which is actually infinite relative to O. Therefore, P views the entire set N itself as of finite size, which means that P can see set N in its entirety. Given this, then:

- If P's size relative to O is actually infinite, then O's size relative to P is infinitesimally small. Additionally, O's boundary that defines it and separates it from the other elements inside the set, is also infinitesimally small relative to P. The O elements still exist, by definition of set N, but they and their boundaries are of infinitesimally small size relative to the actually infinite observer P, and, thus, individual O elements and their boundaries become indiscernible to P. These elements, therefore, don't disappear, because they do exist, but instead merge into a continuous space from P's perspective. That is, P would observe the inside of set N as a continuous space, as opposed to O's view of it as a space filled with discrete elements.

- P can see the whole or entire amount of N and, thus, can see the edge or boundary of N, which means that set N, in its entirety, is seen as an existent, finite-sized, discrete thing by P.

- P cannot "step inside" N and hope to be able to see its elements as discrete. This is because P is of a different size scale than the elements inside the set. P's scale is the same as that of the entire set, that is, actually infinite relative to O. So, even if P tried to step inside N, it would still be infinitely big relative to the elements and would, therefore, still just see a continuous space. Thus, P is "trapped" in its reference frame, or "dimension", just as O is trapped in its reference frame inside the set.

Thus, relative to infinite-sized, external observer P, set N would appear as a finite-sized existent object with an internal continuous, smooth, infinitely divisible space. An important point is that these arguments don't prove the existence of an external observer; they just suggest how this observer, if it existed, would view the set.

Implications of Set N for Our Perceptions of the Universe

The first implication of the appearance of set N to different observers is that the appearance of something as either discrete or continuous depends on the perspective, or reference frame, of the observer relative to the something. In more concrete terms, suppose set N is our universe. This would assume that our universe is infinite in size, which is not known, but if it is, how does the appearance of set N to different observers relate to our perception of the universe from physical and mathematical perspectives?

First, from a physics perspective, one implication of set N is that the dichotomy between finite-sized internal observer O's view of set N as discrete and infinite-sized external observer P's view of set N as continuous is analogous to the dichotomy between the quantum physics-based view of space as discrete and quantized and the general relativity-based view of space as smooth and continuous, respectively (Kempf, 2010). This suggests that general relativity is formulated from the perspective of the infinitely big observer P outside the set and that quantum physics is formulated from the perspective of the finite size observer O inside the set. Because general relativity is very good at describing the universe at very large scales, and quantum physics is very good at describing it at very small scales, this analogy seems to fit. This suggests that perhaps general relativity and quantum physics cannot be unified by either trying to quantize relativity or relativize quantum physics because these two theories are based on conceptually-opposite perspectives. Instead, starting from first principles may be a better approach.

Next, humans seem to view our universe as being made of discrete elements like quarks, atoms, rocks, people, and even atoms of space-time in many models of quantum gravity. This suggests that our perspective is that of observer O within infinite set N. That is, we view the universe as being made of discrete chunks. If true, this would solve Zeno's Paradoxes of motion. One of these, the Dichotomy Paradox, suggests that motion is impossible because for someone to move from point A to B requires that they always first have to move halfway from where they were to where they're going. This process continues forever, making movement of any distance impossible (reviewed in Huggett, 2019). The most common solution is that motion only occurs over unit-sized, or finite, distances (Cote, 2013) and times (Huggett, 2019). While this would go against the the requirement for continuous space in relativity, it would correspond to the idea that we live inside set N in a universe made of discrete chunks, even at the most fundamental level.

Finally, the different appearances of set N suggest that it is is very important to use an internally consistent perspective throughout any mathematical description of reality. For instance, if a theory describes space-time as discrete, indicating that the scientist's perspective is similar to that of internal observer O, then it should ideally use the same perspective in its calculations, such as in its calculations of probabilities. Assuming a continuous, real number-like distribution of probabilities for location and time while also assuming a discrete space-time would mean that the theory is switching back and forth in its perspective of reality. This would seem to cause inconsistencies.

From the mathematics perspective, one implication of set N is that the cardinality we assign to an infinite set depends on the perspective, or reference frame, of the observer (e.g., the mind of the mathematician) relative to the set. For example, within infinite set N, observer O would assign the set's cardinality as equal to that of the set of integers. However, outside set N, observer P would assign it a cardinality equal to that of the real numbers. A second implication is that the perception of the integers as being a potentially infinite set of finite, discrete chunks (e.g., 1, 2, 3....) and the real numbers within an integral range (e.g., from 0 to 1, from 1 to 2, etc.) as being a continuum will vary depending on the perspective of the observer. That is, if an observer could decrease his or her size scale to that of the real numbers, they might appear as finite-sized and discrete elements instead of their usual external observer-based description as infinitesimally small. Additionally, a hypothetical external observer of infinite size would view the set of integers as a continuous, infinitely divisible space similar to how we observe the real numbers.

Conclusions

It is shown that the same infinite set will have different appearances depending on the perspective of the observer. An internal observer will view an infinite set of elements, such as set N, as a potentially infinite, spherical space composed of finite-sized, discrete elements. An external observer whose size is actually infinite relative to an element in the set will view the set as a finite-sized object containing a continuous, smooth, infinitely divisible space. The implications of these differing views of the same set for both physics and mathematics were discussed. Overall, these results suggest that the appearance of reality as discrete or continuous will depend on the perspective of the observer (ie, the scientist's mind) relativity to reality and that the perspective of the observer should be taken into account when using infinities in physics and mathematics.

References

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