nected even if they display temporal gappiness. But if the t_1 and t_4 stages are gen-identical, as are the t_1 and t_5 stages, then given transitivity, so must the t_4 and t_5 stages be gen-identical. But in fact the t_4 stage is definitely *not* a stage of the same object as the t_5 stage. Consider, first, that these stages share no parts in common. Secondly, the later stage at t_5 does not derive from the earlier stage at t_4 by way of gradual parts' replacement. Finally, and most importantly for the Causal theory, the t_4 stage is not causally connected in any way to the t_5 stage. This last fact alone guarantees that the Causal theory will judge that these stages are not stages of the same object. But, as we have seen, transitivity considerations require that these stages be gen-identical. The non-branching condition as formulated above will not protect the Causal theory from *this* violation of transitivity since in this example not more than one stage *at the same time* stands in relation R to an earlier stage.

What this type of case shows is that an adequate nonbranching clause must be formulated to exclude more than the possibility of simultaneous gen-identical stages (which are non-R-related). This case shows that given the possibility of gaps, there can exist branching causal paths, even though there are no simultaneous competing candidates for gen-identity with an earlier stage. The nonbranching rule should read as follows: x and z are gen-identical only if there does not exist some stage y such that x is R-related to y but z is not R-related to y, or such that z is R-related to y but x is not R-related to y. This version of the nonbranching rule is broader in scope than the earlier version considered and is sufficient to handle the kind of case described here.[1]

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[1] I would like to thank the Editor for his helpful suggestions.

ON OMNISCIENCE AND A 'SET OF ALL TRUTHS': A REPLY TO BRINGSJORD

By PATRICK GRIM

IN a number of pieces I have tried to present what I see as quite basic Cantorian problems for any set of all truths and for omniscience (see [4], [5], [6], [7]). In 'Grim on Logic and Omniscience' [2] Selmer Bringsjord suggests two escape routes. I think it can be shown that neither of these succeeds.

I

Bringsjord takes the power set axiom to be at the core of the Cantorian difficulties at issue, and the first escape he offers is simply to do without it: to adopt an alternative set theory such as ZF-Power in which such an axiom doesn't appear. 'Such a move vitiates, in one blow, the cottage industry that Grim has helped develop out of the power set axiom.' (p. 187)

This is not the first time such an escape has been suggested: in [13] Christopher Menzel proposed ZF-Power in particular as a solution for Cantorian problems plaguing a set of all truths appropriate to any given possible world.

Not too surprisingly, sacrifice of the power set axiom results in quite major technical limitations. At a single blow, for example, we are exiled forever from Cantor's paradise. Bringsjord, though fully aware of the major technical limitations at issue, thinks they're something we simply have to live with: an unfortunate cost of the need to maintain a set of all truths.

The truth of the matter, however — as I tried to indicate in my reply to Menzel ([5]) — is that even sacrifice of the power set axiom isn't *enough* to escape Cantorian difficulties regarding a set of all truths. We'd have to sacrifice significantly more.

Here the argument is as follows.

Let us suppose we *did* have a set T of all truths within ZF-Power. Then by the axiom schema of separation, carried over directly from ZF, we would have as a subset of T a set of all truths satisfying a particular condition $B(x)$. As long as our basic language is rich enough to express the notion that a truth t is about a topic c, it appears, one such condition will be 'x is about a set of truths.' By separation, then, we would have a set C of all truths about sets of truths.

More formally, using '$\mathrm{A}xy$' to indicate that x is about y, our condition 'x is about a set of truths' is:

$$\exists y \forall z (z \in y \supset z \in T . \& \ \mathrm{A}xy).$$

The axiom schema of separation, taken directly from ZF, is

$$\forall z_1 \ldots \forall z_n \forall a \exists y \forall x (x \in y \equiv x \in a \ \& \ B(x)),$$

where $z_1, \ldots, z_n$ are the free variables of $B(x)$ other than x, and the only restriction on $B(x)$ is that it does not contain y as a free variable (see for example [11], p. 175). Using T for a in this schema and our condition above for $B(x)$ would give us a set C of all truths about sets of truths:

$$\forall x (x \in C \equiv x \in T \ \& \ \exists y \forall z (z \in y \supset z \in T . \& \ \mathrm{A}xy)).$$

The existence of a set C, however, would give us a reductio. C would, in particular, be larger than T. For consider any one-to-one function f from T into C, mapping truths onto truths concerning sets of truths, and consider further a set c' of all truths which do not belong to the sets their assigned element is about. Here using '$a(f(x))$' to indicate the set (or union of sets) $f(x)$ is about,

$$x \in c' \equiv x \in T \,\&\, x \notin a(f(x)).$$

Clearly C will contain some truth about c' just as it contains some truth regarding any set of truths. But by familiar reasoning f can assign no element of T to any truth about c'. C is larger than T.

One branch of the reductio, then, is this: given a set T of all truths, it appears, there would be a subset C larger than the set T of which it is a subset.[1]

Another branch is this. Each element of C is a truth. There are thus more truths than elements of T, and thus T cannot, as assumed, be a set of *all* truths.

As an escape from the Cantorian difficulties of C, then, Bringsjord's appeal to ZF-Power proves insufficient. Something above and beyond the power set axiom would have to be sacrificed. Perhaps additional restrictions on the axiom schema of separation are called for. Perhaps the language of the system must exclude or restrict the notion of truths about sets of truths—though since there clearly *are* truths about any sets of truths this would seem far from satisfactory. Or perhaps we should simply conclude that there is no set of all truths.

II

The second escape Bringsjord offers, with an eye to saving omniscience in particular, is to challenge the following definition of omniscience:

Df. 1 $\forall x(x \text{ is omniscient} \supset \forall p(p \text{ is true} \equiv K_x p))$.

That such a definition is worthy of challenge I have no doubt; as indicated in [3] this definition clearly won't do as an adequate characterization of a traditional God's knowledge, and for that reason I have standardly avoided it. One problem is this: that a being might qualify as omniscient on such a definition and yet hold any number of *false* beliefs. What such a definition requires is that all truths be known by an omniscient being, and that all things known by such a being be true. Since falsehoods believed by such a being are neither truths nor things known, however, such a definition puts no effective restrictions on the false beliefs of an omniscient being.[2]

[1]Note that this is not a problem for sets in ZF-Power in general: the argument relies essentially on the supposition that T is a set of all truths.

[2]In order to qualify as omniscient and yet hold false beliefs, of course, a being B would have to hold contradictory beliefs, and would moreover have to know that his beliefs *were* contradictory. Were such a being so flawed as to shamelessly

This is not however the challenge that Bringsjord presses against Df. 1. His attempt is rather to defend omniscience by proposing that it be redefined as follows:

Df. 2 s is omniscient $=_{\mathrm{df}} \forall p(\Diamond K_s p \equiv K_s p)$,

claiming that

> it's not unreasonable to maintain that there are certain true propositions God can't *possibly* know, such as what it's like to be ignorant, finite, shortsighted, etc...I think it's safe to say that there well *may* be propositional knowledge lurking here...it may very well be, for example, that I know that being a finite creature is like [...], where [...] can be filled in in such a way that a genuine proposition is denoted. ([2], p. 188)

One peculiarity here is of course that Bringsjord's defence of omniscience is on the grounds that there are true propositions which God does not and cannot know. Surely that alone would seem enough to compromise a traditional notion of omniscience.

There are several further problems that are bound to plague such an approach as well, however.

Putting the details of Bringsjord's definition aside for a moment, consider simply the kind of being he is trying to invoke here: a divine being which knows, well, not *everything*, but everything metaphysically suitable to such a being; a being which knows everything except, say, those propositions knowable only from the shortcoming, mess-making perspective of finitude.

A major difficulty is that even such a radically re-conceived 'omniscient' being would fall victim to the kind of Cantorian argument at issue. Here for convenience I will again phrase the argument in terms of a power set, though the lesson of section I is that it might also be phrased without it.

Consider the set of all truths that a divine being such as Bringsjord intends *would* know – the set T of all p such that $K_s p$, where we are careful to exclude supposedly propositional self-knowledge of finitude, shortcoming, mess-making, and the like.

Consider further all subsets of T, elements of the power set $\mathscr{P}T$. For each such subset there will be a distinct set-theoretical truth – a truth to the effect that a chosen truth T_1 *is* a member of that set, for example, or a truth to the effect that it is not. By Cantor's theorem, then, there will be more set-theoretical truths regarding membership in subsets of T than there are truths in T: there will be set-theoretical truths that T leaves out.

But set-theoretical truths are certainly *not* the kinds of shortcoming, mess-making propositions that are knowable only from a

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recognize that he holds contradictory beliefs, however, he could nonetheless qualify as omniscient in the sense defined; nothing in the definition requires that an omniscient being avoid even blatantly obvious contradiction. For further discussion, however, see [9], esp. pp. 35–7.

perspective of finitude; these are precisely the kinds of truths all of which Bringsjord's being *would* be expected to know. The lesson of the Cantorian argument, however, is that given any set of all truths any being knows, there will be truths it does not – truths knowledge of which would moreover be perfectly appropriate the sort of being Bringsjord intends. If Cantorian difficulties pose a problem for a traditional notion of omniscience, then, they pose just as much of a problem for Bringsjord's. The escape fails.

Here the argument has been constructed in terms of the reconception of an omniscient being Bringsjord has in mind, however, rather than the one he strictly defines. Df. 2 itself faces further difficulties.

Within Bringsjord's definition, God is intended to qualify as omniscient, even if he doesn't know all true propositions, on the grounds that he knows all that is possible for a being of his sort to know. Here Bringsjord quite consciously models his account on definitions of omnipotence in which God is claimed to be omnipotent on the grounds not that he can do literally anything, but that he can do all that is possible for a being of his sort to do. It's not particularly surprising, then, that Bringsjord's definition will also face many of the same difficulties that plague such definitions of omnipotence.

Consider in this light a being Necessary McIg, essentially such that he knows only that he is conscious. McIg, on the definition Bringsjord offers, would qualify as omniscient.[3]

Worse still, consider any being which is essentially such that it is non-conscious – here a boulder qualifies, perhaps, or tomato juice or the Pacific Ocean.[4] For any such being Bringsjord's definition would hold, and we'd be committed to the omniscience of tomato juice.

III

There *are* areas relevant to the Cantorian argument against a set of all truths or against omniscience, I think, that are well worthy of further critical scrutiny. One of these Bringsjord mentions in a footnote: the issue of set-theoretical semantics for propositional quantification.

[3]Necessary McIg is here patterned on Necessary McEar, a refinement in Bruce Reichenbach's [15] of an example that appears in Plantinga's [14]. Classics regarding this type of problem for omnipotence clearly include [10] and [12], and it continues to play a major role in for example [17] and [8]. A more sophisticated recent treatment of 'maximal omnipotence' in terms of an ability to accomplish 'shareable' tasks appears in C. Anthony Anderson's [1].

[4]Here I am obliged to L. Theresa Watkins for discussion. A similar problem posed for certain definitions of omnipotence by stones and other inanimate objects is noted by Swinburne in [16].

What Bringsjord claims to offer, however, are two ways of escaping the basic Cantorian argument. Neither of these succeeds.

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