What is a Contradiction?

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Abstract

The Law of Non-Contradiction holds that both sides of a contradiction cannot be true. Dialetheism is the view that there are contradictions both sides of which are true. Crucial to the dispute, then, is the central notion of contradiction. My first step here is to work toward clarification of that simple and central notion: Just what is a contradiction?

The notion of contradiction is far from simple, it turns out, and the search for clarification points up a menagerie of different forms of the Law of Non-Contradiction and Dialetheism as well. Might some of these at least be eliminated as trivially true or false—true or false by definition, perhaps—allowing us to concentrate on the more interesting forms?

Even the attempt to settle the easy cases raises a potential impasse in the dynamics of the debate—an impasse that can be expected to characterize the debate quite generally. The remainder of the paper is devoted to the question of whether that impasse might be broken.

Introduction: Dialetheism and the Law of Non-Contradiction

In the search for clarity, it is perhaps a bad omen that the central principle at issue goes under two apparently contradictory names: it is referred to as both the Law of Contradiction and the Law of Non-Contradiction. I’ll abbreviate that central principle as the LNC. Whatever you
call it, the classical source is Aristotle:

LNC1 ...the most indisputable of all beliefs is that contradictory statements are not at the same 
time true.

—Aristotle, *Metaphysics* Γ (c 350 BC), 1011b13-14

LNC2 Evidently then such a principle is the most certain of all; which principle this is, let us 
proceed to say. It is, that the same attribute cannot at the same time belong and not 
belong to the same subject and in the same respect; we must presuppose, to guard against 
dialectical objections, any further qualifications which might be added.

—Aristotle, *Metaphysics* Γ (c 350 BC), 1005b18-22

To Aristotle’s formulations we can add a few more recent representations:

LNC3 . . . the law of non-contradiction, 5(a ⊤ 5a).


LNC4 The law of contradiction asserts that a statement and its direct denial cannot be true 
together (‘not both p and not-p’) or, as applied to terms, that nothing can both be and not 
be the same thing at the same time (‘Nothing is at once A and not-A’).

LNC5 . . . the law of noncontradiction: nothing is both true and false.


LNC6  Thus there seems to be a role in dialogue for an expression whose significance is captured by the law of non-contradiction: by the principle that a proposition and its negation cannot both be accepted.

    —Huw Price (1990), 224.

The opposing position at issue here also has two names: it appears in the literature as both ‘Dialetheism’ and ‘Dialethism’, which complicates a word search. Whatever its spelling, the term was coined in 1981 by Graham Priest and a second wonderful philosophical logician. That second logician, as it happens, had two names as well: Richard Routley was also Richard Sylvan.

One formulation of Dialetheism puts the position in direct opposition to the LNC:

DIAL1  The view that the LNC fails, that some contradictions are true, is called dialetheism.


Here again we can list some other formulations:

DIAL2  . . . believing that inconsistent information or theories may be true. The view that some
are true has come to be called *dialetheism*, a dialetheia being a true contradiction.

—Graham Priest (2002)

DIAL3  The view that some claims are neither true or false is of ancient ancestry . . . The dual view that some claims are both true and false (dialetheism) is of equally ancient lineage.

—Graham Priest (1993), 35.

DIAL4 . . . *dialethism*, the thesis that a single proposition can be both true and false at the same time.


DIAL5  In standard logics, contradictions are always false; for diadethism, contradictions are both true and false.

—Peter Suber (2001)

My primary attempt here is simply to get clearer on the basic positions represented by Dialetheism and the LNC. Part of that attempt at clarification involves exploring prospects for settling the dispute in at least the easy cases. But I will stick to the ‘easy’ cases: I leave to others the larger task of solving the dispute in the harder cases and in general, if general solution there is to be.

It is plain that the central bone of contention is the status of contradictions. In order to make the dispute as clear as possible we need simply to make it clear what contradictions are and
what each side is claiming about them.

   As it turns out, however, that simple task is significantly harder than it looks.

1. What is a Contradiction?

   The following is a sampler of outlines of contradiction. These are drawn fairly randomly
from the logical and philosophical literature, but I have arranged them here quite deliberately in
order to make some points regarding their differences:

C1  Contradictories, or propositions one of which must be true and the other false. . .

    —Augustus DeMorgan (1846), p. 4.

C2  Contradictory negation, or contradiction, is the relation between statements that are exact
    opposites, in the sense that they can be neither true together nor false together—for
    example, ‘Some grass is brown’ and ‘No grass is brown.’


C3  Contradictories: Two propositions are contradictories if and only if it is logically impossible
    for both to be true and logically impossible for both to be false.


C4  Two statements are inconsistent with each other if they cannot both be true, and more
    specifically if the truth of one would entail the falsity (non-truth) of the other.
C5 Definition. A sentence is *contradictory* if and only if it is impossible for it to be true.


C6 Contradictory: The contradictory of a wff* (statement) $A$ is a wff* (statement) which must be false if $A$ is true and true if $A$ is false.

—Susan Haack (1978), p. 244.

C7 Contradiction: Wff* of the form ‘$A \land \neg A$’; statement of the form ‘$A$ and not $A$’.

—Susan Haack (1978), p. 244.

C8 . . . two formulae are *explicitly contradictory* if and only if one is of the form $q$ and the other of the form $\neg q \kappa$, that is, if one is the negation of the other.


C9 A contradiction consists of a pair of sentences, one of which is the negation of the other.

—Kalish, Montague, and Mar (1980), p. 18

C10 This case is called a *contradiction*; a formula of this kind is always false. We obtain such formulas by taking the negation of a tautology.

—Hans Reichenbach (1947), p. 36.
C11 A statement form which is false for all possible truth values of its statement letters is called a contradiction.


C12 Thus it is plain that every affirmation has an opposite denial, and similarly every denial an opposite affirmation. . . . We will call such a pair of propositions a pair of contradictories.


C13 Contradiction: the joint assertion of a proposition and its denial.


C14 A *contradiction* both makes a claim and denies that very claim.


C15 To deny a statement is to affirm another statement, known as the negation or contradictory of the first.

—W. V. O. Quine (1959), p. 9

C16 We would not say that a man could, in the same breath, assert and deny the same thing without contradiction.

To say of two statements that they are contradictories is to say that they are inconsistent with each other and that no statement is inconsistent with both of them. To say of two statements that they are contraries is to say that they are inconsistent with each other, while leaving open the possibility that there is some statement inconsistent with both. (This may be taken as a definition of ‘contradictory’ and ‘contrary’ in terms of ‘inconsistent’.)


Contradictory statements, then, have the character of being both logically exclusive and logically exhaustive.


A contradictory situation is one where both B and ~B (it is not the case that B) hold for some B.


As this assortment of quotations make clear, contradictions are spoken of in many ways. Let me try to tease out some of the most important differences:

Some authors define contradictions directly in terms of possibility of truth and falsity. In C1, for example, DeMorgan defines contradictions as pairs of propositions one of which must be true and the other false. In C2, Prior defines contradictions as pairs of statements that can be
neither true together nor false together, a definition echoed in Sainsbury=s C3. In C4 Wolfram outlines contradictions as pairs that cannot both the true, and the truth of one of which entails the falsehood of the other. Bonevac=s bald claim in C5 is that a sentence is contradictory if and only if it is impossible for it to be true, and part of Reichenbach=s outline in C10 is that a contradiction is always false. In C6, Haack defines a contradictory of a statement as one which must be true if it is false and false if it is true.

Because all of these definitions rely on a direct appeal to notions of truth and falsity I will term them semantic outlines of contradiction. It should be noted, however, that all except Reichenbach=s C10 offer a definition in terms of possibility, truth and falsity: here contradiction is portrayed as a matter of modal semantics.

A second class of definitions characterize contradictions explicitly in terms of form. Haack=s C7 is a common definition: that a contradiction is of the form ‘A & ~A’ or ‘A and not A’. Contradictions are pairs of formulae one of which is the negation of the other in Forbes (C8) and are pairs of sentences one of which is the negation of the other in Kalish, Montague, and Mar (C9). These I will term syntactic outlines of contradiction. Among syntactic outlines I also tend to include Reichenbach=s proposal in C10 that contradictions be identified with negations of tautologies and Mendelson=s definition of a contradiction as a statement form which is false for all possible truth values of its sentential components (C11), although it is clear that these definitions mix in generous portions of formal semantics as well.

A third approach to contradictions is in terms of assertion and denial. Here Aristotle again has the first word: in On Interpretation contradictions are defined as pairs of propositions consisting of an affirmation and the opposite denial (C12). In C13, Baruch Brody defines a
contradiction as the joint assertion of a proposition and its denial. Kahane’s C14 defines contradiction in terms of making and denying the same claim. An outline of contradictories in terms of assertion and denial is clear in Quine’s C15 and appears in both Strawson’s C16 and in Prior’s outline of the Law of Non-Contradiction in LNC4 above. Because defined in terms not of content or form but in terms of the acts of assertion and denial I will term these pragmatic outlines of contradiction.

A fourth and final approach I will term ontological. In the contemporary accounts listed, interestingly enough, an ontological approach appears only in Routley and Routley’s C19, though it is explicit in Aristotle’s LNC2 and makes a brief appearance in Prior’s LNC4. On an ontological outline, a contradiction would be neither a single statement nor a pair of statements, neither a proposition nor a pair of propositions, but a state of affairs. A contradictory state of affairs would be one in which something had a particular property and also an incompatible property, or in which something both had a particular property and lacked that property.

It should be noted that a distinction between full contradictions and mere contraries is possible in any or all of the approaches above. That distinction is explicit in Strawson’s C17. There a sufficient condition for contraries is that they are exclusive. Strawson’s defining characteristic for contradictions is that they are both logically exclusive and logically exhaustive (C18).

Evident in the literature, then, are at least four basic approaches to the notion of contradiction: semantic, syntactic, pragmatic, and ontological. We could complicate the picture still further by noting that some accounts speak of contradictions as pairs of sentences or statements (C1 through C4, C6, C8, C9, C12 and C17, for example), while others speak of
contradictions as single statements or propositions (C5, C7, C10, C11, C13 and C14). Often what is meant by the latter is a conjunction of contradictions in the former sense. But not always: in Bonevac’s C5, Reichenbach’s C10, and Mendelson’s C11 there is no demand that single-statement contradictions be conjunctive in form.

Evident as a trace element in the outlines above is a further complication regarding explicit and implicit contradictions. Explicit contradictions somehow wear their status—whatever that status might be—on their sleeves. It is sometimes said that contradictions are things it is irrational to accept or to believe, a view that is most plausible when it is only explicit contradictions that are at stake. Implicit contradictions are single statements or pairs which in some way imply, entail, or commit us to explicit contradictions down the line. That something is an implicit contradiction may thus be far from obvious. For even very simple systems it may indeed be formally undecidable whether a formula is contradictory in this sense, and any epistemic injunction to avoid contradiction thus becomes significantly harder to obey. To the extent that epistemic ‘ought’ implies epistemic ‘can’, the notion that we have violated epistemic obligations if we fail to avoid contradiction becomes correspondingly less plausible when it is implicit contradictions that are at stake.

Let me mention a further complication in order to put it aside. Some of the outlines of contradiction above are written in terms of contradictory sentences, some in terms of contradictory statements, and some in terms of contradictory propositions (further variations include wffs, as in Haack’s C7, formulas in Forbes’s C8 and Reichenbach’s C10, and claims in Kahane’s C14). Although there is widespread agreement that utterances or inscriptions, token sentences and types, statements, propositions, claims and assertions are quite different things,
there does not appear to be agreement on precisely what any of these different things are. It is clear that an outline of contradiction could be written in terms of any understanding of any of these.

One final complication that will come back to haunt us is that of negation. Essentially all of the accounts above employ negation in one way or another. But if there are different forms or different senses of negation, as is often held, there will be importantly different readings of each of these definitions.

Where we might have expected a single univocal notion of contradiction we find an enormous range. We have at least four basic forms of approach—semantic, syntactic, pragmatic and ontological—multiplied by (1) a distinction between implicit and explicit contradictions, multiplied by (2) contradictions as pairs or single statements, multiplied by (3) the number of distinctions between token sentences, types, statements, propositions, assertions and claims, with that in turn multiplied by (4) the number of senses of negation. On the most conservative of estimates, that gives us some 240 senses of contradiction.

Here I want to put many of these distinctions aside, however, concentrating only on the distinction between semantic, syntactic, pragmatic, and ontological approaches. The hope is that attention to this distinction may offer at least some hope in clarifying the LNC-Dialetheism dispute.

2. Resolving the Easy Cases

The Law of Non-Contradiction in some way forbids contradiction. Given four approaches to the basic notion of contradiction, however—semantic, syntactic, pragmatic, and
ontological—it is hardly surprising that there will be at least four Laws of Non-Contradiction.

Indeed there will be significantly more, because there are a number of relevant forms of forbidding. One might maintain that contradictions (in one of our many forms) are always false or impossibly true, for example—semantic forms of forbidding. Contradictions might be claimed to be incapable of belief, or at least of rational belief—a psychological form of ‘forbidding’. They might be pragmatically forbidden in the sense of being unassertable, or be communicatively out of bounds in being inherently incomprehensible or incapable of conveying information. They might be epistemically forbidden in being inherently unwarrantable, or might be rhetorically forbidden in being indefensible. It might be that they never obtain, and might be impossible that they obtain—ontological forms of ‘forbidding’. Even beyond the complications of senses of ‘contradiction’ it is clear that we have not a single LNC to deal with but a menagerie of alternative Laws forbidding in some sense contradictions in some sense.

Defined in opposition to LNC, Dialetheism can be expected to show a similar variety. The Dialetheic claim may be that some semantic contradictions, say, are semantically acceptable—‘true’, perhaps, or ‘not necessarily false’. The claim may on the other hand be that some syntactic contradictions are semantically acceptable. Or it may be that some syntactic contradictions are assertable, are psychologically believable, can be legitimately informative, are epistemically warrantable, rhetorically defensible, or actually obtain.

By attending to different approaches to contradiction and to different forms of ‘forbidding’ we have assured that we are dealing with a full conceptual deck. But there is a point at which the deck becomes too full to be manageable. Is there a way of resolving the LNC-Dialetheism debates at least to the extent of cutting down the number of serious contenders?
Aren’t there at least some easy cases on each side that can be usefully eliminated?

At first glance it appears that there are; that some of the easy cases can be settled immediately by definition. Definition settles these aspects of the dispute trivially, perhaps, but it settles them nonetheless, thereby clearing the way for harder work on harder cases.

On some of the standard outlines of contradiction above we can apparently make short work of the dispute. Consider, for example a form of the LNC that maintains that a contradiction cannot be true—a semantic forbidding of contradiction. Consider a corresponding form of Dialetheism that maintains that some contradictions are true. On at least some of the definitions offered, the dispute seems easily resolvable: one side definitely wins and one side definitely loses.

In C2, Prior defines contradictions as pairs of statements that can neither be true together nor false together. If conjunction is taken as ‘together’, Prior=s definition entails that a contradiction cannot be true. On that definition, then, the LNC is inviolable. Dialetheism, taken as the claim that a contradiction can be true, or that the LNC fails, comes out necessarily false. Here the definition of ‘contradiction’ alone serves to settle the matter. The same reasoning applies using Sainsbury=s C3, Wolfram=s C4, and Bonevac=s C5. On any of these definitions, it appears, the dispute is over and we can go home: the LNC wins and Dialetheism loses. Such an argument is offered with an emphasis on contradictions as opposed to subcontraries in B. H. Slater (1995):

One central fact is that contradictories cannot be true together—by definition.

[In footnote:] For the record, this definition is given, for instance, in Sainsbury 1991, p.
16.

Not just any semantic outline of contradiction will close the dispute so conveniently. In C1, DeMorgan defines contradictories as pairs of propositions one of which must be true and the other false. On Haack=s C6, contradictories are such that if one is true the other must be false. But neither C1 nor C6 specify that the conjunction of contradictories so defined—of a true statement and a false statement—cannot itself be both true and false. The proponent of the LNC and the Dialetheist might therefore concur on any of these semantic definitions and yet continue the debate. Although they agree on definition, the proponent of the LNC might maintain that the conjunction of true and false cannot itself be true, whereas the Dialetheist might maintain that it could be both true and false. Not surprisingly, disproof by definition depends on the definition; using these semantic definitions the issue is not immediately closed.

There are other definitions on which the dispute is quickly resolved, but in the other direction. Here Dialetheism wins and the LNC loses. In C8, Forbes gives a syntactical outline of contradictories as pairs of formulae one of which is the negation of the other. In C7, Kalish, Montague and Mar give a similar syntactical account in terms of pairs of sentences. Susan Haack=s outline in C6, written for statements rather than sentences, is that a contradiction is ‘of the form A and not A’.

Can a conjunction of sentences or statements of this defined form be true? Using normal English negation the answer is clearly ‘yes’. One set of examples involve indexicals: the conjunction of your statement ‘I am an Australian’ and my statement ‘I am not an Australian’, or the statement it takes the slow-talker days to make, ‘Today is Tuesday and today is not Tuesday.’
Other set of examples include those in which information is clearly conveyed by using predicates conjoined with their negations: ‘Well, he=s what you=d call handsome and he=s not what you=d call handsome. . .’ ‘Well, he=s in charge but he=s not in charge. . .’ Given a syntactical outline for contradiction of the form of C6, C7, or C8, nothing in the definition prohibits a contradiction from being true. As a matter of fact contradictions so defined seem quite often to come out true. Thus semantical forbiddings of contradictions syntactically specified seem to fail: Dialetheism triumphs and the corresponding LNC fails.

This easy triumph may seem nearly as cheap as the definitional victory for the LNC above, however. For it is on the grounds of precisely these kinds of examples that Strawson resists a syntactical definition, insisting that ‘...one cannot explain what a contradiction is just by indicating ... a certain form of words’ (Strawson 1952, p. 8). In Aristotle’s formulation of the Law of Non-Contradiction quoted as LNC2 above, Aristotle himself adds an immediate proviso: ‘...the same attribute cannot at the same time belong and not belong to the same subject and in the same respect; we must presuppose, to guard against dialectical objections, any further qualifications which might be added’ (Metaphysics Γ, 1005b 22). In On Interpretation Aristotle notes that identity of subject and of predicate must not be ‘equivocal’ in supposed contradictions, and adds ‘Indeed there are definitive qualifications besides this, which we make to meet the casuistries of sophists’ (On Interpretation, 17a 35). The danger is that the LNC that is defeated using easy examples against syntactical definitions is an LNC that no-one would have tried to defend anyway.

The fact that some semantical accounts give an easy victory for the LNC, we noted, need not entail that all semantical accounts do. Similarly, the fact that some syntactical accounts give
an easy victory for Dialetheism need not mean that the dispute is settled for all syntactical accounts. Many have attempted to add the further qualifications that Aristotle alludes to, striving for a class of fully specified and ‘eternal’ sentences. Or at least many have assumed that some such attempt could succeed. In that spirit it might be maintained that for sentences sufficiently filtered—sentences cleaned of indexicals and elliptical suggestions, perhaps, purified of any aspect of content which might vary with context—we could give a syntactical specification adequate to guarantee that any syntactically contradictory sentence is incapable of truth. It might alternatively be maintained that a definition of contradiction in terms of form, but form of something other than sentences, might do the trick. If the ‘A’ of our form specification represents not a sentence type but a ‘genuine proposition,’ it might be maintained, it will be impossible for conjunctions consisting of such propositions with their negations to be true.

At this point our conclusion is that there are indeed some easy cases that can be fruitfully eliminated, clearing the ground for more serious work in more serious areas. Given at least one type of semantic account of contradiction, victory for at least one form of the LNC seems definitionally assured. Given at least one type of syntactic account of contradiction, victory for a form of Dialetheism seems to be guaranteed. Interestingly enough, there is also a link to issues of decidability here—to questions of how one is to tell whether something is a contradiction. On simple syntactical definitions, it should be easy to tell at least explicit contradictions: one need only recognize conjunction and a negation in order to tell that a sentence has the form $A \varpi \sim A$. But those are precisely the kinds of definitions on which the LNC seems vulnerable and Dialetheism seems to triumph. The cases in which the LNC seems to win an easy victory, on the other hand, are cases in which contradictions are defined semantically (‘cannot possibly be true’).
or in terms of language-like abstract entities such as propositions. Here there is no clear way to
tell whether something is a contradiction or not. That we have no algorithm for impossible truth
follows immediately from limitative results in metamathematics: if we did, we would have an
algorithm for necessary mathematical truth, and that we know we cannot have. A strong
argument that there can be no algorithm for whether something is a proposition or not can be
made in terms of self-reference (see Grim 1991, ch. 1). What this suggests is that perhaps
definitional victory for the LNC is possible only where it is undecidable entities that are at stake
as ‘contradictions’. For decidable ‘contradictions’ it may always be Dialetheism that has the
upper hand.

Just as it turned out that the simple notion of contradiction is far from simple, however, it
turns out that even the easy cases of the LNC-Dialetheism dispute are far from easy. Even in the
easy cases the dynamic of the debate quickly arrives at an impasse.

3. The Impasse

The suggestion of the previous section is that there are multiple approaches to
contradiction, giving us a still greater multiplicity of approaches to the LNC and Dialetheism.
On some approaches, the victory is easy but trivial, in favor of the LNC. On other approaches it
is an easy but cheap victory for Dialetheism. Were that first account successful, we would at
least have eliminated some easy cases on each side, leaving a wealth of more important and
substantive work to be done.

Here a major difficulty arises, however. It is a difficulty which has arisen in LNC-
Dialetheism debates in the past, is predictable for debates in the future, and threatens to wipe
away even the small gains of our few easy conclusions. The difficulty appears at two levels in
the debate. It is evident at the bottom level of what is at issue: contradictions, and in particular
how contradictions are to be understood in terms of negation. But the difficulty appears with a
vengeance at the higher level of our own discussion as well, in the attempt to establish one
position and to exclude another even in the easy cases.

Here is the problem in the context of the higher debate:

By the first argument above, the LNC is vindicated by some definitions of contradiction
and Dialetheism is defeated. For if contradictions are by definition some species of sentence
which cannot be true, it must be that contradictions cannot be true.

That the LNC is vindicated by such an argument is clear. But that Dialetheism is thus
defeated may not be evident. For if the Dialetheist takes his position to be that the LNC is false,
and thus that there are propositions that are both true and false, he may well regard even the
firmest proof that the LNC is true to be insufficient to convince him that the LNC is not also
false. Since what he has maintained all along is that simply that the LNC is false, proof that it is
ture may be insufficient to convince him that he is wrong.

By the same token, the proponent of LNC may begin to regret early celebration over the
proof from definition. For what he wanted to argue was that the LNC was true and thereby that
Dialetheism was false. To have established that the LNC is true but to have failed to defeat a
Dialetheist opponent will seem not a cheap victory but no victory at all.

What this failure of the first easy argument seems to call for is a simple clarification of
positions. What the proponent of the LNC maintains is that the principle is true only, or true and
not false. His earlier mistake was to rest content with proof of truth, whereas he also needs proof
of non-falsity. What the proponent of Dialetheism maintains is that the principle is at least false, though perhaps it is both true and false.

Can the debate then proceed given that further clarification? Not as one might expect. For suppose that the proponent of the LNC got everything he wanted at this point: firm and emphatic proof, again by definition perhaps, that the LNC is true and not false. Surprisingly, that doesn’t take us any further. Since the Dialetheist maintains that there can be propositions that are both true and false, he may take even this further proof to be insufficient. What the Dialetheist maintains is that some contradictions may be true. That the proposition expressed by the LNC is both true and not false may thus seem insufficient to defeat him. For if a contradiction may be true, the fact that the LNC is not false need not entail that it is not false as well, and that it is false is after all what the Dialetheist has maintained all along.

We have reached a dialectical impasse reminiscent of the of the early Socratic dialogues. The problem isn’t that the proponent of the LNC hasn’t yet been clever enough to find an argument strong enough to defeat the Dialetheist. The problem is that no argument we can imagine him providing will be strong enough to circumvent the Dialetheist’s counter. Despite the fact that Dialetheism is sometimes defined in opposition to the LNC, as in DIAL1, the Dialetheist seems able to swallow cheerfully a proof that the LNC is true, or even that it is true and not also false, without in the least impugning his conviction that Dialetheism remains true. If he finds contradiction painless, we can hardly expect to force the Dialetheist to our conclusion ‘on pain of contradiction’.

Such an impasse is also reminiscent of Lewis Carroll’s ‘What the Tortoise Said to Achilles’ (1895). Achilles’ attempt there is to compel the Tortoise to accept a conclusion Z on
the basis of an argument from Euclid which moves from premises A and B to Z. The Tortoise accepts A and B, but refuses to accept Z unless an additional hypothetical premise is explicitly added: C, to the effect that if A and B are true, Z must be true. When C is added, however, the Tortoise refuses to accept the new argument as compelling Z without explicit addition of still another premise D: that if A and B and C are true, Z must be true. At each step the Tortoise insists that an explicit premise regarding the validity of a contested argument be added as a premise, giving another argument for which he makes the same demand. Within these constraints, it becomes clear that Achilles can offer the Tortoise no argument that will compel him to accept Z.

With a topic of contradictory negation rather than validity, it is the proponent of the LNC that is Achilles and the Dialetheist that is the Tortoise. The proponent of LNC offers a demonstration that the LNC is true. The Dialetheist insists that truth need not entail non-falsity, and that his position is simply that the LNC is false. The proponent of LNC then goes on to demonstrate that the LNC is true and not false. The Dialetheist points out that by his lights the fact that the LNC is not false need not entail that it is not false as well, and that that is after all his position. Whatever Achilles proves, if contradictions may be true the proof need not entail that the Tortoise’s position is wrong. And of course what the Tortoise has maintained all along is that contradictions may be true.

The dynamics of this impasse are evident in the literature.

Priest had at one time [concluded] that some sentences ‘are so contradictory as to take impossible values such as both true and false ({1,0}) and true only ({1})’. But he seems
not to have been alarmed, and one sees why. Dialethism is not simply a theory about contradictions; it requires the theorist himself to assert some, and the discovery that a sentence both must and cannot take exclusive values looks like just one more contradiction to take on board.

—Timothy Smiley (1993), 31

What are we going to do, charge the Dialetheist with contradicting himself? That, as Smiley notes, ‘sounds as futile as Brer Fox throwing Brer Rabbit into the brier-patch’ (Smiley 1993, 27). The fact that someone holds a position which makes it impossible to convince them of the contrary, however—a position which is thus in a sense ‘irrefutable’ or ‘indefeasible’—need not entail that their position isn’t deeply and terribly wrong. It may be that one cannot effectively be argued out of solipsism or nihilism or the view that all positions are false equally or equally true. That doesn’t make them true, and certainly shouldn’t convince us to become solipsists or nihilists.

Although impasse is all too possible in the LNC-Dialetheism debate, it should be noted that it is not inevitable. There are, to begin with, different forms of Dialetheism. Contrary to the implication of Peter Suber’s definition in DIAL5, there is no requirement that a Dialetheist believe that all contradictions are both true and false. Someone could fully qualify as a Dialetheist, believing some contradictions are true, even were they to hold that truth does entail non-falsity in the particular case of the LNC, for example. One could still be a card-carrying Dialetheist while holding that statements regarding the LNC are not in the class of those for which contradictions may be true. Against these forms of Dialetheism, a demonstration that the
LNC is true would constitute a full refutation. Were such a refutation to rely on a particular definition of contradiction, however—contradictions defined as statements the truth of which is impossible, for example, as in C1 through C4—such a Dialetheist might concede the LNC for contradictions so defined while maintaining that his position does hold for other and more interesting senses of the term.

In a debate that comes to impasse, a quite understandable reaction on the side of the LNC=s proponents will be that the Dialetheist is refusing to engage the real issue or is consistently side-tracking any real debate. The central principle that the LNC=s proponents take themselves to be defending is that contradictions cannot be true, and that in a sense of >cannot= that explicitly excludes the possibility that they might somehow be true as well. The LNC that its proponents want to defend is that any claim that contradictions are true must be false and false only, and that in a sense that excludes truth rather than leaving it as a lingering possibility that might be added later. By the LNC=s proponents= lights it thus appears that the Dialetheist fails even to engage the real principle at issue because he is constantly reading it as something weaker, constantly re-reading their ‘not’ as something other than the intended ‘not’, or their ‘false’ as something other than the sense of ‘false’ intended.

The cost of impasse can also be indicated from the other side. Dialetheism is conceived in denial of the LNC, and offers itself in opposition (DIAL1). But how is the Dialetheist to express his own position in a way which makes it clear what he is opposing? The threat here is that what the Dialetheist seeks to claim regarding the LNC will become as ineffable as rival positions he condemns on the basis of inexpressibility (see for example Priest, 1995).
Suppose that you say ‘β,’ and Priest replies ‘5β.’ Under ordinary circumstances you would think that he had disagreed with you. But then you remember that Priest is a dialetheist, and it occurs to you that he might very well agree with you after all—since he might think that β and 5β are both true. How can he indicate that he genuinely disagrees with you? The natural choice is for him to say ‘β is not true.’ However, the truth of this assertion is also consistent with β=s being true—for a dialetheist, anyway. . .


Priest has sometimes replied that the Dialetheist can make his opposition clear, not indeed by asserting any form of ‘5β’, but by simply denying β:

It is sometimes urged as an objection to dialetheism that dialetheists cannot express their own views. Notably, they cannot express α in such a way as to rule out 5α. . . .

. . . it is not clear that non-dialetheists can do any better as far as ruling out goes. If dialetheism is correct then, like it or not, no one can rule out 5α by asserting α. Maybe they would like to; but that does not mean they succeed. Maybe they intend to; but intentions are not guaranteed fulfillment.

[But] anyone (dialetheist or otherwise) can express themselves in such a way as to rule things out. They cannot rule out α by asserting 5α—or anything else. But they can simply deny α.

It is clear that Priest does take assertion and denial to be exclusionary, though he refuses a similar exclusion for any sense of ‘\(\beta\)’ and ‘\(5\beta\)’ (See also Priest 1993 and 1998).

The retention of pragmatic exclusion between assertion and denial seems a necessary foothold against the charge of Dialetheic inability to either champion or contest any position. But retention of that foothold is peculiar as well. It is unclear, to begin with, why the argument should stop at this point. If Dialetheism has so much going for it, why stop it short of assertion and denial? It is also unclear that exclusion can be restricted to the pragmatics of assertion and denial alone. Given that the Dialetheist can deny certain claims, including the LNC, what is the information that he conveys by his denial? If we accept his denial, what precisely is it that we have accepted? If we learn that he is right, what precisely is it that we have learned? All of these questions reflect the fact that a denial is intended to convey some content. But any content that inherits the exclusionary characteristics that Priest recognizes for denial will thereby have precisely the exclusionary characteristics he refuses to recognize for negation. For some outlines of the terms at issue, moreover, Dialetheism would demand extension to assertion and denial. For pragmatic notions of contradiction defined in terms of assertion and denial (as in C12 through C16) and for Dialetheism defined as an acceptance of contradictions, it turns out that even Priest would not qualify as a Dialetheist.

Here the road to clarity regarding the opposing principles at stake—indeed, the road to a treatment of those principles that can make it clear that they are genuinely understood and that they are genuinely in opposition—seems to force us to an examination of basics regarding assertion and negation. One might have thought that issues regarding negation could remain as the mere subject matter of the debate, at the same level as for example the distinction between
implicit and explicit contradictions. What the impasse makes clear is that the simplest issues of negation at the lowest level infect the character of the debate throughout.

4. ‘Not’

We think of concepts of negation, of assertion and denial, of truth and falsehood, as some of the simplest possible. Perhaps we are wrong about how simple they are. But it is clear that they are central in our conceptual toolkit and ubiquitous in our mental and linguistic life.

Their ubiquity makes the attempt to examine them philosophically all the more difficult. Precisely because they are so central in our conceptual toolkit it seems impossible to examine these concepts without using them. If we use them everywhere, in an attempt to conceptualize everything we come in contact with, we will of course have to use them in the attempt to understand them themselves. If we are to dig that deep, we will have to dig under the very ground we stand upon.

As an attempt to make some progress beyond the all too real possibility of impasse, let me attempt to outline just one elementary notion. Although I take it to be a form of negation, I don’t wish to claim any special status for this particular form: I don’t wish to claim that it is the common or English notion of negation, for example. I’m not sure there is a single common or English notion of negation, and if there is I am not sure that this is it. Perhaps I’m even wrong that what I want to introduce really counts as negation. None of that need prevent us from introducing a useful concept in the pursuit of clarity.

If I succeed in introducing the term I=m after, I think it will offer progress through the LNC-Dialetheism impasse sketched above. If I don’t succeed, I expect my failure to be
instructive as well.

Let me begin here:

Some things are genuinely impossible. There are things that cannot be.

One such case is that in which some properties are incompatible with others. If a line segment is less than two inches long, for example, it cannot also be more than three feet long—the properties *less than two inches long* and *more than three feet long* are incompatible. If something is uniformly red and of no other color, it cannot also be uniformly green and of no other color—the property of being *uniformly red only* excludes the property of being *uniformly green only*.

We can speak of such properties as *exclusive*: the possession of one excludes the possession of the other. But of course a given property need not exclude only one property. The property *less than two inches long* excludes a whole range of alternative properties: *more than one feet long, more than two feet long, more than three feet long...* All of those properties incompatible with a given property P we might think of as in the *exclusionary class* of P.

Our ability to conceive of things effectively is part of what allows us to navigate successfully in the world. It is generally to our advantage to be able to conceive of the properties that things actually have, and also to be able to conceive of the relations between those properties that actually hold. Given that we are the communicatively cooperative beings that we are, it is also to our advantage to be able to talk about these things.

Because some things are less than two inches long, the concept of being less than two inches long and the ability to say that something is less than two inches long both come in pretty handy. It also comes in handy, with reference to a proposed property P, to be able to say of
something that it has some property that excludes P: that P is in the exclusionary class of some property that the thing has. If it is proposed that the Empire State building is less than ten inches high, for example, it would be useful to be able to make it clear that it has some property that excludes its having the property of being less than ten inches high, even if we are not sure precisely what exclusionary property that happens to be (is the building 1250 feet tall? 1251 feet tall? Does it change in a stiff wind?).

I propose to use ‘not’ for this useful job. When I say that the Empire State building is not less than ten inches high, what I intend is that it has some property for which less than ten inches high is in the exclusionary class.

Let me add some immediate qualifications in order to cement the term ‘not’ as I wish to use it:

A. Implicit fields of application

Many terms apply sensibly only to limited ranges of things. Terms of emotion apply to people and animals, for example, but seem to lose their grip on stars and stones. Is Betelgeuse upset? Measurement in inches seems to lose its grip on emotions and special events.

What is true of individual terms also applies to conceptual families. The color terms form a family some aspects of which are captured in the familiar color wheel or color solid. Blue and green have a certain similarity, for example, but orange and purple are diametrically opposed. As long as we are speaking of something that can have a color, a strong denial that something is chartreuse may lead us to guess at its color starting at the other side of the color wheel. But if we are speaking of things to which color terms don’t apply—celebrations, sizes
and prices, for example—this oppositional web of color terms fails to apply as a whole.

The important point here is that properties and their exclusionary classes may together assume appropriateness classes of application. It may be that properties P and P’ both apply only to a certain class of things. With regard to that class, they are exclusive. With regard to other things, neither may apply.

Quite often we simply assume an appropriate class of application. It goes unsaid. That too proves useful for efficient communication, and I expect this kind of everyday implicit assumption to continue to be carried in the use of the term ‘not’. Could it be that properties P and P’ are exclusionary with regard to one assumed background class of things and not so with regard to another? That is at any rate a possibility that I don’t want to rule out.

B. Exclusive and exhaustive.

As specified, (1) being P and (2) not being P are exclusive. Nothing can be both. But nothing has been said that would indicate that (1) and (2) are exhaustive. Perhaps something can slip through the cracks, qualifying as neither having a property P nor having any property that is a member of its exclusionary class. Here the issue of implicit fields of application applies with full force. One way that being P and not being P might fail to be exhaustive is if both are bound to a particular field of application. If there are things to which neither applies, they would remain exclusive but not exhaustive. With regard to specifications for ‘not’, at least, exhaustiveness remains an open question.

In first trying to outline ‘not’ it is tempting to use pictures. That is indeed the course that the Routleys take in ‘Negation and Contradiction.’ But it soon becomes clear that the pictures
One is tempted to draw have some serious drawbacks.

One might start by trying to outline ‘not’ with a diagram like this one:

In the area marked A are those things that have a property P. In the area marked B are those that have properties exclusive of P. Never the twain shall overlap, just as in the diagram.

The issue of appropriate classes of application, however, leads us to complicate the diagram immediately. In order to represent the role of implicit appropriateness classes, we need our diagram to be but part of a larger whole:

If the properties at issue are color-terms, the central area represents those things to which color terms are appropriate. Events, sizes, and days of the week, inappropriate as subjects for color predication, lie in the outer area.

Even so, however, our diagram misrepresents. For the tidy line down the middle of the central area indicates that the property P, which holds for those things in A, and its exclusive class of properties which hold for things in B, are exhaustive within their appropriateness class.
All that ‘not’ demands is that it be impossible for something to be both P and not P. But our diagram imposes a further requirement of its own: that anything within the appropriateness class must be either P or not P.

It is tempting to try to compensate for this difficulty by complicating our diagram further. Here we add a central area in the middle:

Now areas A and B are exclusive, but the central area indicates that they need not be exhaustive within their appropriateness class.

Even so, however, our diagram still misrepresents. For it is still sketched with sharp lines, suggesting that there is a precise point in property space beyond which something ceases to be P though it has not yet become not P. The mere exclusiveness of ‘not’ need not commit us to sharp borders and false precision in this sense, and thus the diagram again imposes unintended specifications. ‘Bald’ is in the exclusionary class of ‘hirsute’, applied appropriately to human heads and sick dogs but not to colors, days of the week, or volumes of space. But there is neither a sharp line between ‘bald’ and ‘hirsute’ nor a sharp line between either of them and some ‘neither bald nor hirsute’ central space. In order to get a better diagram we would have to start blurring some of the lines.
C. Ontology, Semantics, and Pragmatics

As outlined, whether a property P and a property P equals are exclusive is an ontological matter, more specifically a matter of modal ontology: what is at issue is simply whether something can be both. ‘Not’ is at base ontological as well: something is not P if it has some property that excludes P. But semantics follows easily: To say that something is not P is to say that it has some property that excludes P.

Assertion and denial are aspects of pragmatics: things we do in saying, rather than themselves part of the content of what we say. When I deny that I am late or assert that I am sorry it is my being late or my being sorry that is denied or asserted.

I want to make it clear that ‘not’ lives in the realm of content rather than performance. I can assert that something is not P, which means that it has some property that excludes P. It thus has a semantic existence of its own: it is not merely a performative operator, as for example a marker of pragmatic denial might be.

5. The Promise of ‘Not’

Let us suppose, at least for the moment, that my attempt in the previous section has been successful: that I have succeeded in outlining ‘not’. What might that get us?

‘Not’ would allow the proponent of the Law of Non-Contradiction to make the central claim that he wants to, with the force he wants to make it. For what he wants to claim is essentially this:

LNC 7 Any and all contradictions are not true.
If ‘not’ has been successfully outlined, this is a claim that excludes:

DIAL6 Some contradictions are true.

This in turn would offer the possibility of breaking the dialectical impasse. For were the proponent of LNC7 to prove his claim, the Dialetheist could not cheerfully accept it and also insist on DIAL6, because to understand ‘not’ is to understand exclusion, and what LNC7 explicitly excludes is DIAL6.²

The promise of ‘not’ thus appears to be a debate in which the contestants can be genuinely at loggerheads. That would at least break the impasse so as to offer the possibility for a genuine debate between Dialetheism and the LNC.

Would it also offer a winner for the debate? At least a trivial winner in an easy case. For if we can successfully outline ‘not’, we might choose to define contradictions in any of the following ways:

Semantically:

C20 A contradiction is a statement such that it is not possible that it be true.

Syntactically:

C21 A contradictory pair consists of (1) a statement to the effect that something has property P and (2) a statement to the effect that that same thing does not have the
property P.

C22 A contradiction is a conjunction of a contradictory pair.

Ontologically:

C23 A contradictory situation would be one in which it is the case that something is P and also the case that that thing is not P.

Given any of these definitions of contradiction, among many possible variations, it would appear that the LNC in the form of LNC7 is indisputably true. It follows by definition that contradictions are not true in a sense which precludes impasse: the Dialetheist cannot recognize ‘not’ as outlined and maintain that they are true as well.

Interestingly enough, definition of contradiction in these terms is not without precedent. Here is Lewis Carroll’s definition:

C24 If we think of a certain Class, and imagine that we have picked out from it a certain smaller Class, it is evident that the Remainder of the large Class does not possess the Differentia of that smaller Class. Hence it may be regarded as another smaller Class, whose Differentia may be formed, from that of the Class first picked out, by prefixing the word ‘not’; and we may imagine that we have divided the Class first thought of into two smaller Classes, whose Differentiae are contradictory.

6. Prospects for Dialetheism

Given ‘not’, an easy case of the dispute would appear to be over. For that trivial case, at least, a form of the LNC wins by definition. What might those of Dialetheist bent say in response?

A. One option for the Dialetheist is to concede a minor battle and hold out for victory in a larger war. The victory for the LNC outlined above applies only to a particular form of definition for ‘contradiction’—that employing ‘not’—and only to a particular form of the LNC phrased in terms of that sense of contradiction. Any defeat for Dialetheism is thus only a very limited defeat.

We know that there are a range of rival definitions for contradiction and a range of corresponding senses of the LNC and Dialetheism. One option for the Dialetheist is to move on to more interesting senses of the term. I have been careful not to claim that ‘not’ represents an English ‘not’. Formulate a notion of contradiction in terms of an English ‘not’ instead and the argument from definition offered above won’t even touch it. Here the battle can continue to rage, although we can also expect often to face impasse.

B. There is another option for the Dialetheist which does not entail conceding even this small defeat. We’ve phrased our conclusion to this point as follows: Given ‘not’, a form of the dispute is over and a form of the LNC wins. At least one form of dyed-in-the-wool Dialetheist may insist that ‘not’ cannot be taken as given, despite my best attempts at careful outline.

Can one introduce ‘not’ as I have tried to above? One part of standard philosophical
practice says ‘yes’; as long as one is clear in definition and scrupulous in following one’s own stipulations, so the story goes, one can specify a term to mean what one wants it to mean (shades of Lewis Carroll a third time). I have resisted claiming ‘not’ to be the English ‘not’, and indeed have avoided putting weight on my assumption that it is a form of negation. Thus my rights regarding ‘not’ are simply the rights of stipulative definition.

A potential problem for standard philosophical practice, however, is the example of A. N. Prior’s ‘tonk’, a connective * stipulated such that \( a \mid a \ast b \) and \( a \ast b \mid b \) (Prior 1960). Here we seem to have a clearly stipulated use for a clearly stipulated connective, and yet all Hell breaks loose: allow * in that sense and everything follows. Clear stipulation may thus not be enough. Perhaps ‘not’ is akin to *, and should also be resisted in the same way.

I regard the notion of stipulative but nonetheless unacceptable definitions to be worthy of further exploration. The reason that Prior’s ‘tonk’ is so corrosive in deduction is clear: it is itself defined in terms of the deductive relation. Small wonder, then, that it has as consequence the legitimation of a particular (and particularly obnoxious) pattern of proof. Any term stipulated short of that overreaching grasp would not have the same effect.

A similar claim could be made for familiar forms of the ontological argument in which ‘God’ is defined in terms of the modal notion of necessity. Small wonder that even the possibility of applying such a term carries full necessities in consequence. Any term stipulated short of that overreaching modal presumption would not have the same effect (see Grim 1979, 1981, 1991).

The reason for resisting Prior’s ‘tonk’ and the Ontological Argument’s ‘God’, however, does not seem to apply to the outline for ‘not’ offered above. I am unable to find in that outline
the kind of overreaching grasp that clearly accounts for unhappy surprises in Prior’s ‘tonk’ and the ontological argument’s ‘God’. What these cases do seem to establish is that stipulative definition is not as harmless as it is sometimes taken to be. But it is also clear that stipulation can be innocent and convenient, and without the kind of structural analysis that is possible in the case of ‘tonk’ and the ontological argument there does not seem to be a structural objection to the stipulative introduction of ‘not’.

C. A third option for the Dialetheist is not to concede the outline of ‘not’ and swallow a small victory for the LNC in consequence, not to resist it as dangerous on the model of ‘tonk’, but simply to find my attempt at outline unsuccessful.3

Despite my disclaimer that ‘not’ is not to be taken as the English ‘not’, it is obvious that the outline above uses various forms of negation, including the English ‘not’, prominently and repeatedly in trying to get the idea across. If these forms of negation can be understood a particular way, it seems inevitable that ‘not’ can be understood a particular way. Given a dialetheic interpretation of all the various forms of negation in the outline, then, one might well end up with a dialetheic interpretation of ‘not’. The result could be that every claim made above is allowed, but without the concept of exclusion that is their main intent: all claims regarding ‘not’ are cheerfully accepted, together with all claims regarding what ‘annot be’, ‘impossibility’, ‘incompatibility’, and ‘exclusion’, but with the reserved right to cheerfully add ‘not’ forms of all of these to the mix as well.

The prospect of a dialetheic reinterpretation of ‘not’ is reminiscent of Huw Price’s dialogue:
Me: ‘Fred is in the kitchen.’ (Sets off for kitchen.)

You: ‘Wait! Fred is in the garden.’

Me: ‘I see. But he is in the kitchen, so I=ll go there.’ (Sets off.)

You: ‘You lack understanding. The kitchen is Fred-free.’

Me: ‘Is it really? But Fred=’s in it, and that=’s the important thing.’ (Leaves for kitchen.)


My attempt above was to outline a very basic term of semantic exclusion. But without some fundamental grasp of precisely that notion to begin with it seems quite possible that it cannot later be specified.

Is there some way of outlining such a basic term in order to guarantee that the intended sense of exclusion gets across? I think not. We know that we cannot accomplish the comparative task of outlining even the notion of number unambiguously, so as to avoid interpretation of our ‘numbers’ as merely the evens, for example. Quite generally, it appears, we cannot by specification alone target a unique reading for our specifications (see for example Putnam 1980). If semantic exclusion is not understood to begin with, what possible exposition could we rely on to nail it?

7. Conclusion

I hope that the general tour of the territory that I=ve offered compensates for the fact that there are so few conclusions that can yet be firmly drawn.
What is clear is that contradiction is spoken of in a variety of ways—semantic, syntactic, ontological and pragmatic, just for starters—which entail a variety of LNCs and Dialetheisms that may be in competition.

At that point it seems that we should be able to eliminate at least some of the contenders as trivially true or false—trivially true and false given particular definitions of ‘contradiction’, for example. Even in the apparently easy cases, however, it turns out that impasse is all too close. That impasse ultimately threatens the basic understanding of the very principles at issue, and thus threatens any conviction that there can be any real debate between Dialetheism and the LNC.

One way out, I’ve proposed, is the introduction of a stipulative and explicitly exclusionary ‘not’. Given such a ‘not’, some easy cases again become easy cases. More importantly, the fact of real disagreement can be expressed in a way that offers a prospect for real debate. Although some easy cases are settled simply by definition on each side, important questions regarding negation and contradiction more widely remain open for exploration.

That is the course that I would hope a Dialetheist to take. Given the positions at issue, however, I must admit that I see no way to make it compulsory. Dialetheists of at least some stripes may choose to reject the possibility of ‘not’ and anything of its exclusionary kind. All I can say is that those forms of Dialetheism seem less interesting to me; I don’t see how the prospect of impasse is then to be avoided, and such forms don’t seem to me to promise any deeper understanding of notions as central to our conceptual toolkit as is the notion of contradiction.

Notes
Or at least so it initially appears. I leave to section 3 the further subtlety that leads to impasse: that the Dialetheist might by his lights cheerfully accept the definitional vindication of the LNC and yet insist that Dialetheism is true as well.

Here ‘not’ appears with ‘true’, of course. Though the concept of truth may carry its own philosophical difficulties, ‘not’ is intended for the same job as before: something qualifies as ‘not’ P just in case it has some property that excludes P.

Some philosophers, Priest among them, have cited the paradoxes of self-reference as primary motivations for Dialetheism. Dialetheists of this particular motivational stripe may be forced to reject ‘not’ in one way or another, by hook or by crook, since it is clearly possible to create a self-referential paradox that employs it:

This sentence is not true.

If the paradox-motivated Dialetheist rejects LNC for ‘not’ on the grounds that it embroils us in the standard reasoning of the Liar, consistency (!) would seem to force him to reject an LNC for ‘not’ on the grounds that it gives us similar reasoning for this variant. For this point I am obliged to an anonymous referee.

References


Suber, Peter (2001), ‘Non-Contradiction and Excluded Middle,’ URL www.earlham.edu/~peters/courses/logsys/pnc-pem.htm