

Scientific Measurement as a Dynamic and Cognitive Integration. The case of Kepler's astronomy¹

Dr. Godfrey Guillaumin
Department of Philosophy
Universidad Autónoma Metropolitana –Iztapalapa,
Av. San Rafael Atlixco no. 186
Colonia Vicentina
México City 09340, México.
Office: +(52)-55-58-08-65-22
Fax: +(52)-55-56-88-42-96
E-mail: guillaumin.godfrey@gmail.com

Thanks to measurement, modern science has actually established itself.
Karel Berka (1983)

Abstract

Scientific measurement was a central factor behind the rise of modern science. Nevertheless, despite the epistemic and methodological importance of measurement for the development and growth of scientific knowledge during the modern epoch, few historical-philosophical studies seek to understand measurement's epistemic, methodological, and cognitive aspects. This paper aims to propose a historical-philosophical approach for scientific measurement in terms of an integration of mathematical, conceptual, and instrumental aspects. This integration was achieved after long periods of time, as result of “feedback” processes between these three aspects, where each other were mutually adjusted. I illustrate this dynamic and integrative notion of scientific measurement analyzing some of Tycho Brahe's and Johannes Kepler's determinations of celestial parameters.

Keywords

Scientific measurement, cognitive integration, iterative process, Tycho Brahe, Johannes Kepler,

1. Introduction

Lord Kelvin (1824-1907) stated “I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be” (Thomson, 1883). Historical studies support Kelvin's statement by documenting the importance of measuring in science. In a recent book, Jimena Canales observed that “Measurement practices and measurement-based science were often listed among the determining causes behind the great divide marking off modernity from other eras” (Canales, 2010, p. 207) because “the success of modern science was guaranteed by rapidly advancing measurement techniques” (Canales, 2010, p. 206). John Heilbron (1979, 1993) has studied how measuring allowed the development of imponderable fluid areas known as electricity and magnetism during the eighteenth and nineteenth centuries. Other works that are more chronological than analytical, including Joseph Keithley (1999), who studied the history of measurement in electricity, and Clifford Pickover (2008), who studied the history of the discovery of the laws of nature, show how measurement was fundamental in the discovery of laws.

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In addition to science historians, some philosophers have also studied measurement from the formal logic perspective, focusing on elucidating the nature of mathematics, the formal and axiomatic knowledge that measurement provides. This tradition studies the mathematical properties of counting and measuring and the relations between mathematical and world entities.² Under this approach, measurement is the assignment of abstract numbers (or numerals, according to a nominalist) to entities and events to represent their properties and relationships. This tradition can be traced in the works of Helmholtz *Counting and measuring* (1887); Hölder's *Die Axiome der Quantität und die Lehre vom Mass* (1901); Russell *Principles of mathematics* (1903); and *An account of the principles of measurement and calculation* (1928) from Campbell Physics³

Despite the importance of measurement for the development and growth of scientific knowledge, remarkably few historical-philosophical studies seek to understand measurement's epistemic, methodological, and cognitive aspects. Canales has a similar diagnosis of the situation: "Despite the fact that measurement is perhaps the most original feature of modernity [and consequently in modern science as well], *we have neglected to understand just how it works*" (Canales, 2010, p. 220, emphasis mine). A possible explanation of this situation is that the vast majority of historians and philosophers of science have focused on studying *theories* and estimated that measurement is simply a component or appendix of them; metaphorically, it is a mere quarry from which data are extracted. An interesting exception to this is Hasok Chang (2007), who manages to recognize historical and epistemological aspects of measuring temperature and the long process of thermometer construction (both *physical* and *conceptual*). Chang studies different epistemic, cognitive, and technical challenges that led to the creation of the thermometer through the eighteenth and nineteenth centuries and examines the conceptual, methodological, epistemic and technological problems overcome during the creation of the first thermometers, particularly creating a temperature scale, establishing the concepts of "temperature" and "heat", and developing standards for precision instruments. Chang shows that the knowledge of measuring temperature underwent a development that led to innovative thinking, meticulous experiments, bold conjectures, creating tools, and generating methodological standards.⁴

This paper aims to illustrate a historical-philosophical approach (or dynamic-conceptual) for *scientific measurement* that emphasizes epistemic, methodological, and cognitive aspects. This concept is based on three considerations: the first is that scientific measurement has gone through several historical stages, and each stage has specific features, some of which are shared with other stages. The historical development of scientific measurement has not been systematically studied, and no coherent idea of how the scientific measurement has been developed.⁵ This paper focuses on the early developmental stage of modern science, specifically astronomy. This historical period is characterized by a slow abandonment of ancient astronomy, without substituting another theory to replace it. The second consideration is that scientific measurement requires a *dynamic integration* for its development (in terms of coupling and self-adjustment) of three different types of resources: conceptual, mathematical, and instrumental. These resources are required to measure, and the measuring results will be wrong if any work improperly. This leads to an epistemological and cognitively interesting feature: measurement has three potential error sources. The third consideration is that such integration is a dynamic measurement *system*, not in the usual "metric system" sense established in the late eighteenth century in which importance lay in

² While the logical and formal aspects of measurement are important as a part of mathematics, here I will not consider this approach. My reasons for not considering are similar to what Toulmin has found regarding formal logical studies that prevail in the development of philosophy of science: "Lucid, erudite and carefully argued they may be; yet somehow they seem to miss the mark. It is not that the things that are said are untrue or fallacious, but rather that they are irrelevant: the questions that are so impeccably discussed have no bearing on physics. Meanwhile, the actual methods of argument physical scientists employ are only rarely examined" (Toulmin, 1953, p. 10).

³ A very detailed study of this tradition is found in José Diez (1997).

⁴ In a recent study, nevertheless, David Sherry maintains that Chang fails to explicate quantitative measurement: "Chang's gesture toward meaningful arithmetical operations offers a promising strategy, but he doesn't pursue it. [...] We need, at a minimum, an account of meaningful calculations involving temperature and heat" (Sherry, 2011, p. 511).

⁵ There are, however, studies on specific areas of measurement; mainly John Heilbron (1979,1993), Hasok Chang (2007), and David Sherry (2011) have made contributions to the study of measurements in electricity, magnetism and temperature.

standardizing measurement units, but rather in a cognitive system that integrates three different resources and allows us to find some features of the empirical physical world. This systemic approach contrasts with the operationalism concept of measurement in mere typical operations.

Given the strong normative emphasis of Lord Kelvin's famous phrase, I should explain that my analysis is not normative in defending the idea that all empirical sciences should perform measurements, or that measurement is the *only* means by which empirical knowledge becomes satisfactory. What I want to analyze is which elements of measurement changed (either through transformation or generation) to allow the abandonment of ancient science, specifically astronomy, and made the emergence of modern astronomy possible. The philosophy of science has been traditionally considered "theoretical change" for analyzing these changes. I propose exploring the role of measurement in that change. We see something unexpected; namely, in astronomy (but the pattern is most likely similar in other cases), that the change of ancient to modern astronomy depended on epistemic, methodological, conceptual and cognitive development of new (cognitive) measurement systems, and something more fundamental. Without these systems there is no way to transform and advance knowledge.

2. Towards an integrative notion of scientific measurement

"Measure" is an ambiguous word because it refers to both the measurement *process* and the *product* thereof. While this distinction is analytical, it is not in practice, *i.e.*, there can be no measurement *results* without some act or *process* of measuring, and vice versa. This simple reflection, which seems obvious, contains part of what an operationalist like Bridgman (1927, 1938) had argued about the meaning of concepts in metric measurement operations that had to be carried out.⁶ "Operations of measuring generate meaningful metric scientific concepts," might summarize the basic operationalist concept. There is something more fundamental in the relationship between the measurement *process* and the *result* it generates: the two aspects are mutually dependent epistemically, methodologically and cognitively.⁷ Different measurement processes thus generate different metric knowledge, *i.e.*, the results achieved metrically tell us about the measurement processes used. If this point is correct, as I will try to show, then one way to study the epistemic and cognitive content of scientific measurement is by beginning to understand the measurement process, which is actually complex.

Two authors will be particularly important in guiding the approach I want to develop: Ian Hacking (1983) and Karel Berka (1982). Hacking, in *Representing and Intervening*, claims that "what is so great about science is that it is a collaboration between different kinds of people: the speculators, the calculators, and the experimenters" (Hacking, 1983, p 248). This gives us a starting point to argue that scientific measurements *integrate* three different resources, hence the proposed name *integrative conception* of measurement, explained below. Considering the analysis of Faraday's theories of Faraday, Maxwell and others, Hacking argues, with C. W. F. Everitt, that the physical sciences have three activities: speculation, calculation and experiment (Hacking, 1983, p. 212-13). It is not, however, only to say that science is an enterprise in which different departments *work together*, but something more fundamental: the characteristic of the scientific method is to "put these two abilities [speculation and experimentation] in contact through use of a third human ability, which I have called joint and calculation" (*ibid.*). These three skills are different but relatively autonomous from each other and occasionally *integrate* themselves to generate a *single result*.

Berka's book, *Measurement. Its Concepts, Theories and Problems*, goes deeper into the issue of

⁶ Operationalism mainly emphasizes semantic aspects of the measurement, leaving aside the epistemic and cognitive ones.

⁷ In quantum mechanics it is known that the act of measuring changes what is measured. This is actually a limiting case of a common feature of any scientific measurement: measurement's results depend on the resources (concepts, mathematics, and instrumentation) with which the measurements were done. This approach does not lead to relativism or radical constructivism, which assumes that our measurements literally built the world. This approach ignores the fact that the number of parameters that can be potentially measured the same phenomenon is huge in comparison to the parameters that we actually measure, therefore, our measures capture, they do not build an emerging number of such parameters. In other words, in order to successfully measure certain parameters of a phenomenon we need (developing, using and / or transforming) certain resources metrics that correspond to *these* parameters but not others, and *mutatis mutandis* to measure *other* parameters of the *same* phenomenon.

measuring than Hacking by giving a full study on different aspects of measurement. Berka states “measurement encompass different aspects and components of an empirical and theoretical nature, which are *mutually conditioned* in a very complicated way.” (Berka, 1982, p. 14, emphasis mine). These aspects are from a practical and a theoretical nature. On the practical side are experimental preparation and execution, the appropriate choice of measurement operations, construction and use of measuring instruments, and the development and evaluation of the results of a measurement. On the theoretical side, says Berka, relevant problems are numerous, and they are related to the conceptualization of measurement objects and their relationships, to the demarcation of the basics of measurement theory and commensurability conditions, the elucidation of the relationship between empirical and mathematical aspects of measurement, and provide a general theory of measurement. Berka states that these aspects are not involved to the same degree, or in any particular case or theoretical analysis of measurement. This feature, mutually conditioned elements, is essential for understanding the process by which measurements are constituted by *dynamic integration* of these resources. To Berka, measurement is the interrelationship of theoretical and material elements and can only give good results in various areas if there is a dialectical relationship (or *epistemic iteration*, in Chang) between theory and practice, advanced methods and tools of measurement and a specific theory of the measured quantity, when used in a practical measurement technique or product, or between the hypothesis testing or verification of numerical laws and their theoretical justifications, which depend on a general theory of measurement, only if performed within the framework of basic research (Berka, 1983, p. 10). In scientific measurement, the theoretical and material elements including the teleological (the purposes for which measurements are made), are integrated. As does Hacking, Berka identifies a bridge in mathematics between material and theoretical elements: “The remarkable results which physics achieved by combining the experimental approach, based on measurement, with theoretical constructions, as well as by the natural connection between experience and theory that was mediated by mathematical means, have made a deep impression on other sciences and have become, especially now, the universal ideals of all sciences” (Berka, 1983, p. 9). Berka identifies the experimental access of physics to measurement by linking the theory and mathematical constructs or conceptual issues.

Both authors share the idea that the process of measurement in science is the *integration* of diverse type of elements. I understand “integration” in its most literal sense: to *dynamically compose* a whole based on the involvement of different components or resources. These resources sometimes do not exist and must be generated; at other times, a resource may have to be modified so the rest can be adjusted. Integration is thus primarily a *dynamic* process of mutual adjustment. I intend to establish a consequence derived from this integrative dynamic view. There is a point in the development of measurements where the adjustments are minimal and further development of some parts is no longer required. The knowledge generated by this measurement is robust and well supported at that time. Scientific measurement is supported on *theoretical* (concepts, mathematics), *materials* (tools, experiments) and *pragmatic* grounds (measurement procedures for the measurements). As shown below, this process is also the cognitive core that measurements provide to science. That was the case for some astronomical measurements at the beginning of the seventeenth century.

The first resource that integrates the measurement is what Hacking calls “*speculation*”, which means the *intellectual representation of something* interesting that is usually done through restructuring ideas that lead to a *qualitative* understanding of some general world characteristic. There are different ways of understanding intellectually what you want to measure, but not all of them are likely to assign numbers.⁸ Berka makes this point using the measurement objects’ conceptualization and relationships. Not any conception of what is to be measured is adequate for measuring. For example, the theme of movement was important during the seventeenth century, and a conceptual change of what was meant by “movement” was a key part for its measurement in planetary motion and the movement of the heavy bodies. Adequate conceptualization of which objects are targets of measuring is only performed through a

⁸ Berka states: “numeral values acquired through a specific process of measurement are, at most, the values from realm of rational numbers” (Berka, 1982, p. 4). The natural numbers are used for counting. This consideration is one way to distinguish between measuring and counting: counting is to assign natural numbers, meanwhile measuring is to assign rational numbers. An interesting study of the ontology behind the distinction between both types of numbers is Cooper and Humphrey (2010).

systematic process of conceptual transformation of *qualities* into *magnitudes*, which typically occurs through the processes of *abstraction* and *idealization*. While the literature on abstraction and idealization is huge, it is enough to characterize it as Nola and Irzick (2011, pp. 302 ff.) did, stating that abstraction from an object or event is performed when you select some properties and exclude others in connection with a particular theory. As the property selection depends on the theory accepted, abstracting different properties requires considering different theories. For idealization, it is not the case that some properties are considered by themselves and others are ignored, but the property picked out is conceived as “pure” without “imperfections” that are usually present. To successfully perform scientific measurements, appropriate abstractions and idealizations are required. The “adequate” and “appropriate” will depend, among other things, on the degree of precision to be carried out while running the processes of measuring;⁹ this will be analyzed in the case of Kepler.

The *speculation*, Hacking explains, is sometimes developed through “quantitative ornaments”, but it has ceased to be essentially *qualitative*, as in the old speculation that the distance travelled by a freely falling body to the earth changes with the square of the time it takes to fall, which is represented as $1/2gt^2$. However, the numerical value of the local acceleration of gravity, g , is not part of the initial speculation, but must be *set empirically*. Hacking's idea is that although speculation is given by equations and calculations, at the end of the day certain constants values of nature must be determined. Hacking's clarification is important because it indicates that quantitative way does not necessarily mean that the relevant parameters have been established through accurate measurement. There is another element in the example used by Hacking: that *no matter how convincing, categorical and quantitative the explanation is, it is unable by itself to establish the value of physical constants*. Determination is made by other means because while *some constants are implicitly in the speculation, its value is not*. To say that a property is qualitative is simply to say that it has not undergone the long process of abstraction and idealization that eventually leads to assigned (rational) numbers; this does not mean that the magnitudes (which are metrizable properties) are not qualitative in a different way. Hegel stated that “the relation of quality and quantity is reciprocal, that quality can become transformed into quantity just as much as quantity into quality, that, in fact, reciprocal action takes place” (Hegel *apud* Berka, 1982, p. 5). There is a cognitive value, which is qualitative, added to any measure properly achieved, simply because of the correct data that could be deduced from it. In such cases, the measurement is cognitively valuable for generating new information and/or knowledge. Measuring is not simple or roughly to quantify qualities, but rather to transform qualities (abstracted and idealized) into (cognitively) *relevant magnitudes*.¹⁰

The second element involved in measurement is the use of instruments.¹¹ Measurement instruments serve two functions: the first one is as a unit of measure. A knotted rope is a tool that serves as a unit for measuring length or an hourglass time measurement. The other function is a mean to obtain information that otherwise could not be obtained. The telescope is a good example in this case; the micrometer is a good one of the previous one. There are cases, however, where a single instrument fulfills

⁹ Lloyd has stressed an important distinction between *precision* and *accuracy*: “precision in the sense of ‘the degree of refinement with which an operation is performed or a measurement stated’ (as Webster puts it) and accuracy in the sense of ‘degree of conformity to some recognised standard value’ or ‘value accepted as true’” (Lloyd, 1991, p. 302, n. 3). I follow here such a distinction underlying the fact that “precision” involves a practical and an expertise character, while “accuracy” entails epistemic considerations; both are crucial aspects in scientific measurements. The important thing here is that high precision in a measurement does not warranty high accuracy in data generated by that measurement, but there is no high accuracy without high precision, as we will see in the case of Kepler.

¹⁰ Julia Annas (1975) made an analysis of Aristotle's idea that time is a number type, or more precisely, that the time is the number of the movement. This text is important because it shows the conceptual changes required to conceive what is measured. While Carl Boyer (1945) shows the sense in which Aristotle and Aquimedes developed a quantitative science without taking measurements. Boyer's text especially shows Hacking's point about that explanations are not necessarily quantitative metrics.

¹¹ Part of the philosophy of science has been focused on experimentation analysis in the seventeenth century, but the use of measuring instruments or measurement experiments has not been analyzed in detail. Hacking, for example, often do not distinguish on measuring the epistemic and cognitive differences using instruments, and the use of experiments. Rather Hacking often speaks generically of experimentation; here, he is referring to one of the three components that define modern science. Chang (2007) and Sherry (2011) are exceptions to this common situation; both studies analyze different aspects in the transformation of thermoscopes to thermometers.

both functions, as the barometer in the experiments of Torricelli, Pascal and Mersenne worked to detect a variation of atmospheric pressure and was used as the unit of measure. Almost all experiments conducted during the seventeenth century were experiments about measurements. Hacking argues that philosophers of science almost never say anything about the experiments or the use of technology or knowledge to change the world (Hacking, 1983, p. 149). He states that the silence about experimentation is strange considering that, since the Scientific Revolution of the seventeenth century, the experiment was officially declared the royal road to knowledge. Francis Bacon (1561-1626) taught that nature should not only be observed, but it was necessary to manipulate our world to learn its secrets.

Hacking is one of the few philosophers of science whose explicit purpose is to vindicate the role of experiments in modern science. For Hacking, experimentation has "its own life", and it not only displays different roles and relationships with theory, but there are also different types of experiments. The relationship between theory and experiment, says Hacking, differs in many developmental stages, and not all natural sciences go through the same cycles. Sometimes an experiment leads to a theory, in others the reverse; in some cases, the experimental results are clear and categorical but there is no theory available to explain them, and so on. "Some profound experimental work is generated entirely by theory. Some great theories spring from pre-theoretical experiment. Some theories languish for lack of mesh with the real world, while some experimental phenomena sit idle for lack of theory. There are also happy families, in which theory and experiment coming from different directions meet" (Hacking, 1983, p. 159). The experimental functions and their relationships with theories are many and varied.

The third component of the measurement is *articulation* and *calculation*. Hacking uses "articulation" as Kuhn gave its definition in normal science, *i.e.*, the theory is articulated to fit better the world. Though many speculations could explain the physical world, they are barely connected with the world in their initial states, and the *explanation* thus differs from the *connect*, which is a crucial distinction for understanding the role of bridge Hacking gives to the joint. To "connect" a statement to the world means that it is possible to deduce consequences of this statement that can be tested. The testing requires new experimental ideas and types of technology, as sometimes tests cannot be performed with what is at hand. Hacking makes a fundamental statement about this process:

[...] Kuhn's articulation must denote two kinds of things, the articulation of theory and the articulation of experiment. I shall arbitrarily call the more theoretical of these two activities 'calculation'. I do not mean computation, but the mathematical alteration of a given speculation, so that *one brings it into closer resonance with the world* (Hacking, 1983, p. 214, emphasis mine).

This idea is in the core of Hacking's (and Kuhn's) conception of measuring, and it requires a detailed analysis, presented below. It is interesting to note the similarity between what Hacking calls "calculation" and the idea of measurement in standard theory. For Brian Ellis, "measurement is the link between mathematics and science" (Ellis, 1968, p. 1), and he later adds that "all measurement involves the application of arithmetic" (*Ibid*, p. 4). For Russell "measurement of magnitudes is, in its most general sense, any method by which a unique and reciprocal correspondence is established between all or some of the magnitudes of a kind and all or some of the numbers, integral, rational, or real, as the case may be [...] In this general sense, measurement demands some one-one relation between the numbers and magnitudes in question — a relation which may be direct or indirect, important or trivial, according to circumstances" (1903, p. 176). N. R. Campbell argues that "measurement is the process of assigning numbers to represent qualities; the object of measurement is to enable the powerful weapon of mathematical analysis to be applied to the subject matter of science" (Campbell, 1957, p. 267-68). Hacking discusses how his tripartite distinction (speculation, instruments, and calculations) is compatible with traditional hypothetic-deductive theories like Campbell's. In the literature about measurement, there is at least a basic agreement: *a necessary condition of measurement is the assignment of numbers (or numerals to be precise) to a magnitude*. However, Hacking makes the most important point, in the transformation from qualitative to metrics disciplines, saying that there is no magnitudes to which to apply mathematics, but it requires a *conceptual transformation* of two ways: on the one hand, qualities

must be transformed conceptually into quantities by idealizations and abstractions,¹² and, on the other hand, *the available mathematics should be modified or created a new one if it is conceptually inadequate*. Hacking claims more than the mere *application* of arithmetic to the physical world, and this is more about *transforming* the speculation into mathematical terms that must be developed. Sometimes “it needed new mathematics to answer or even to ask the questions” (Hacking, 1983, p. 214). The articulation often transforms both the conceptual and the mathematical speculation. But sometimes the mathematics already existed but not the adequate concepts, as Sherry points out in the case of the transition from thermoscope to thermometer: “There is nothing mathematically new in the mixture problem, as ancient computations of balance points used an analogous principle. The only innovation consists in treating a quality (temperature) as a continuous quantity” (Sherry, 2011, p. 514); that innovation was a conceptual transformation.

Moreover, Hacking mentions that Kuhn had identified two articulations: one about theory (Hacking called “calculation”) and the other about experiment. Hacking does not analyze the articulation of experiments (or instruments) and its role in measurement. Hacking’s analysis of the experiment can be reduced to three points: 1) there are many types of experiments, each with different functions, 2) relations with theories are varied in different ways and both directions: there are even experiments without theory and vice versa; 3) one main experimental feature is the intervention and creation of phenomena. No item refers to how (if any) some experiments articulate speculations; in other words, Hacking does not speak of “the *experimental* modification of a speculation to be harmonized with the world.” Hacking may say that there is no such thing or simply that the experiments have several functions, one of which is to articulate. From my perspective, the other half of the story relies on the experimental articulation (which is better called instrumental).

During the seventeenth century, the areas that went from qualities to quantities necessarily went through a *dynamic integration* of the transformations of speculation, mathematics and instruments.¹³ These three resources were transformed and simultaneously integrated dynamically until modern disciplines were shaped by the laws of nature and successful predictions of unexpected events. Conceiving thus, the measurement process has epistemic and cognitive consequences important for developing modern science’s image. Following this approach, Hacking does not consider that *there is no “mathematical modification” really fertile and productive without “modifications of instruments”*, and there are at least two reasons for it. First, measuring instruments work as data providers that are usually beyond our natural system of perception. To be *reliable* suppliers requires transforming and refining them, otherwise the data could have significant deviations when making calculations. Second, we cognitively *think based on instruments*, which means that certain ideas were conceived through the instruments we use and we could hardly conceive that part of the world without them. In that sense, instruments in general, but measuring instruments are an extension of our cognition and minds.¹⁴ Based on these considerations, it can be argued that *performing measurements to investigate the empirical world requires the mathematical integration and instrumental modification of speculation. Such integration can take decades and necessarily requires the transformation, creation or refinement of empirical concepts, mathematical concepts (or systems) and instruments (or experiments)*. It is mandatory to achieve the integration required to fit these three elements mutually.¹⁵ Because the idea of integration is at the core of

¹² Sherry (2011) is a very interesting study of how temperature, which was traditionally considered a qualitative concept, became into a quantitative one.

¹³ It is interesting that Sherry detected the same phenomena that I am naming “dynamic integration” in the case of measurement of heat: “Unlike his predecessors, [Joseph] Black used reading from his device to achieve a *theoretical* understanding of thermal phenomena by *applying mathematics* to the readings. Surprisingly, Black’s mathematical techniques emerged long before the thermoscope. He melded this older, conceptual tradition with the experimental tradition that immediately preceded his discoveries” (Sherry, 2011, p. 512, underlined mine).

¹⁴ Bertoloni (2006) shows how cognitively speaking, some objects, instruments or experiments used in specific ways functioned as extensions of the human mind to *conceive* new ideas and sometimes to “modify experimentally” speculation.

¹⁵ A strong indication of such integration in the early stages of development of modern science was to establish to what extent the results of subsequent calculations derived from the measurements were consistent with the empirical data obtained either by the same measurements or independent measurements. From Herschel and Whewell, to Lakatos and Laudan, philosophy of science in general has shown the importance of successful predictions of unexpected events as a central epistemic virtue of any

what is to do appropriate, relevant and correct measurements, I consider appropriate to call this theory of making measurements the *integrative theory of scientific measurement*.

3. Instruments, geometry and measurement in ancient astronomy

I now analyze how the transformation from old into modern astronomy, in the early seventeenth century, occurred as integrative measurement process. I start by analyzing the transformation of astronomical instruments and the conceptual and mathematical transformation. Astronomy has always (and virtually in all cultures) used instruments to establish certain astronomical parameters, so using measurement instruments in seventeenth-century astronomy was not a new aspect. The novelty exits in two aspects: the first is the consistently increased precision of existing astronomical instruments to a level never observed before, and consequently new instruments were invented. The second is based on integrating these previously unknown tools with brand new concepts and mathematics. Let us examine the details.

In the early seventeenth century, the terms "instruments" and "experiments" did not mean what they do now. There is one important difference that has disappeared over time but is crucial for historical understanding as it depends largely on how instruments have transformed the measurement practices and empirical knowledge. Jim Bennett states that "from the seventeenth century instruments were conventionally classified into mathematical, optical and natural philosophical types" (Bennett, 1989, p. 105).¹⁶ The mathematical instruments "such as the dioptra and the astrolabe—and yes, even Galileo's proportional compass—were part of a long tradition, certified by usage and custom in the restricted realm of the mathematical subjects. The dioptra was based on the well-known principles of Euclidean and Ptolemaic optics; the astrolabe embodied the technique of mathematical projection of a sphere on a plane; and the proportional compass was based on the theory of proportions going back to Eudoxus" (van Helden, 1994, p. 9). Hipparchus (190-120 BC) improved the dioptra and created the astrolabe, and he attributed to Eratosthenes (276-194 BC) the creation of the armillary sphere (Evans, 1998). Greek positional astronomy used some "mathematical instruments" and incorporated geometry and trigonometry in their models of the cosmos, as well as records of celestial observations collected for more than five centuries by the Babylonians and Egyptians. However, "from the sources we now have in translation, it appears that the very accurate parameters of the motions of the planets, the sun, and the moon current the Mesopotamians after about 700 B.C. were the result not of accurate measurements but rather of very mediocre measurements continued over a long period of time. We must go to late antiquity, the period from Hipparchus to Ptolemy (ca. 150 B.C.—ca. A.D. 150), to find the first attempts at accurate and convenient measurement to solve specific astronomical problems. Even in the case of Ptolemy, however, it is obvious that he relied heavily on Mesopotamian sources" (Van Helden, 1983, p. 53). Though the *Almagest* was seen as the most complete astronomical model for over 1500 years, as well as a useful and true synthesis of contemporary astronomical knowledge, we must remember that "although Ptolemy's stated observations are not particularly good and in one case err by more than a degree, they do match his tables in an uncanny way. In fact, this situation prevails throughout the entire *Almagest*, which leads to the suspicion that something is going on that does not meet the eye [...]" (Gingerich, 1997, p. 17). The *Almagest* was not conceived as a research book in our sense, but more as a textbook that compiled the astronomical knowledge of the period influenced by Euclidean geometry. Gingerich explains the proper correspondence, when it exists, between Ptolemy's models with the data used in the following way: "I am inclined to think that [...] he may no longer have felt in necessary to find perfect observations, for he could simply have used his theory to judge how much observational error an observation contained, and he could the correct it accordingly" (*Ibid.*). Much has been written about whether Ptolemy falsified data,¹⁷

successful theory. Correspondingly, what these authors have failed to note is the fact that predictions, including striking phenomena, is one of the cognitive and epistemic consequences of the most important measurement.

¹⁶ The main tools developed and / or used during the seventeenth century natural philosophy and astronomy were the vacuum pump, the astrolabe, the barometer, calculating machines, the compass, the cross geometry, the microscope, the compass of proportion quadrant, the slide rule, the telescope, the thermoscope and pendulum clock. For development of mathematical instruments during this epoch see especially Bennett (2011).

¹⁷ In my book (2005, pp. 160 ff.), I analyze the recent controversy that has arisen regarding the Ptolemy's use of apparently misleading data.

but it is only necessary to emphasize that it was committed (not needed to) to develop a systematic program of detailed and precise measurements of celestial parameters. In brief, there were measurements in ancient astronomy integrating geometry, instruments and concepts, but qualitatively these resources had reached their cognitive limit, as they had reached their maximum development. One of these three had to reach another level of development to perform other types of measurements: in the case of astronomy, it was the instruments.

The astronomy of Ptolemy and Copernicus (1473-1543) was relatively the same in (unsophisticated) precision, the mathematics used in their models, and the instruments for observation. It can be stated that “Copernican astronomy was neither simpler nor more accurate than Ptolemaic astronomy” (Barker and Goldstain, 1988, p. 302). The revolutionary element of Copernicus was to place the earth moving and the sun still, not in the center, as suggested by Aristarchus of Samos (310-230 BC), but “in most respects his *De revolutionibus* (1543) follows Ptolemy’s *Almagest* so closely that he can equally well be regarded as the last great practitioner of ancient astronomy” (Thoren, 2003, p. 3). Copernicus inherited the astronomical data and observations made using the standards and instruments of his time. He did not develop new measurement tools, new mathematics to analyze data or new measurement techniques. For the accuracy of astronomical measurements, from ancient times until Copernicus, Van Helden argues that we must draw a distinction between *accuracy* and *convenience*. Precision in positional astronomy refers to two things: the accuracy of the astronomical theory predictions with observations, or the ability to detect angular distances. Accuracy is a cognitive element of measurement, as it mainly provides data on magnitudes. Convenience, for its part, considers other practical purposes of measurement than accuracy. An example of a vague but convenient astronomical instrument, according to John North, is the astrolabe: despite its imprecision, it was undoubtedly useful in practice to know the time (North, 1974). It can be said that, including ancient astronomy until Copernicus, data were more convenient than accurate.¹⁸

An observational consequence of the heliocentric system is that there must be some stellar parallax. Parallax is the angular displacement of the apparent position of a celestial body when viewed from two different places. Detecting a small parallax would have been direct observational evidence of the displacement of the Earth around the Sun, but the level of accuracy of the measuring instruments from the time of Copernicus was well below that required for this task. For angular distances, the accuracy of Copernicus’s measurements has been estimated between $1 / 8^\circ$ and $1 / 10^\circ$ (6'), which was almost ten times better than the accuracy of medieval astrolabes, but still not good enough to detect parallax. Without the development of new methods and measurement tools, Copernican theory would have remained a mere mathematical hypothesis.

3.1 Scientific measurement as *cognitive integration* in astronomy from the late seventeenth century. Tycho Brahe and Johannes Kepler

Copernicus made far-reaching conceptual changes in astronomy. What is usually forgotten in the history of astronomy is that without the transformation of the *instruments* and *mathematics*, his model would not have become widely accepted. Tycho Brahe (1546-1601) systematically developed an unprecedented program, in the history of astronomy, for more accurate astronomical measurements, and the mathematical mind of Johannes Kepler (1571-1630) finally *integrated* the new measures of the cosmos using mathematical transformations. In early 1570, Tycho recognized the need to build “a new astronomy based entirely on logic and mathematics, *without recourse to any hypothesis*. [He] agreed on the need for new and accurate observations *before attempting to explain* the celestial motions, and it is obvious that Tycho was aware of the need for good instruments to obtain those observations” (Hellman, 1974, p. 402, emphasis mine). Between 1576 and 1596, he had collected a large amount of data obtained with instruments that he had ordered,¹⁹ so precise that they made all astronomical observations from before the

¹⁸ To see in detail in which sense the data of Ptolemy and Copernicus are convenient rather than accurate, see Newton (1977), Evans (1998) and Gingerich (1997).

¹⁹ The instruments that Tycho ordered to be constructed and he used in the observatory on the island of Hven were the following: a metal azimuthal quadrant (1576) was one of the first instruments built in Hven. Tycho used it to observe the

sixteenth century obsolete, including those used by Copernicus (Blair, 1990). Estimations have shown that the range of accuracy of Tycho's instruments was between 30" to 50", or approximately 10 to 20 times more than those used by Copernicus (Chapman, 1983). Accurate data depended not only on the instrument. "The accuracy of the observations depended on the instruments and the care with which they were used. Although Tycho's were without magnification, error was minimized by their huge size and by the graduations carefully marked on them to facilitate angular measurements on the celestial sphere, altitudes, and azimuths. *Tycho checked instruments against each other and corrected for instrumental errors.* [...] He observed regularly and achieved an accuracy within a fraction of a minute of arc, an accuracy unsurpassed from the time of Hipparchus to the invention of the telescope" (Hellman, 1974, p. 405, emphasis mine). To make precise measurements, some methodological considerations must be considered. In his instruments, Tycho made two major technical advances: diagonal scales used in the arms of reading instruments, which allowed him to measure fractions of a degree without increasing the size of the observation instruments and improving the instruments through cracks in the alidade in sextants or quadrants, which decreased alignment errors. However, the major methodological development was an improved data collection method, repeating observations and getting more data for each element observed (Thoren 1973, Wesley 1978). "Tycho had an unprecedented concern for accuracy and a unique ability to detect and eliminate flaws in either observational procedure or mechanical design" (Thoren, 2003, p. 12). Tycho had transformed the *art* of making good astronomical observations, which was dominant mode of astronomical practices from the ancient astronomers to Copernicus, to a high-precision astronomical *science*. After Tycho, astronomical observations had acquired new precision and methodological and cognitive standards, indicating that he passed a "cognitive threshold" in observational astronomy that had never been so high in the history of astronomy.²⁰

Though Tycho's observations achieved accuracy, he lacked the proper understanding about the optical elements of observations, including refraction's causes and effects. Tycho was seriously considering refraction, and he even argued that Copernicus had his own latitude incorrect by over two minutes of arc due to ignoring the effects of refraction (Thoren, 2007, p. 195).²¹ However, Tycho himself did not understand properly the theory of refraction, "it is needless to say that the accuracy cannot be so great as Tycho fondly hoped, as the errors of observation would be increased by neglect of refraction and by his ignorance of the existence of aberration and nutation" (Dreyer, 1890, p. 351). Tycho did not theoretically understand the effects caused by refraction in astronomical measurements, but Kepler did. Moreover, although the accumulation of data with unprecedented accuracy was a remarkable achievement in itself, it did not estimate the cognitive, methodological and epistemic consequences required. Specifically, it lacked getting the correct ratios and proportions "hidden" among such data in the framework of a *theory of the heliostatic cosmos*, for which it was insufficient to continue adding data. It required a conceptual and mathematical analysis of these relations and proportions. Kepler made that analysis about different physical and astronomical phenomena: one of them is refraction, and the other is the motion of Mars.

Kepler met Tycho in early 1600 and slowly begins to gather data that Tycho had accumulated. Kepler made an early conceptual development on refraction from Tycho's data. In his measurements,

comet in 1577 and it has an estimated accuracy of 48.8 arc sec. A Calculating Globe (1580), which Tycho used to record the positions of stars that in 1595 summed up to a thousand and which also served to locate the azimuth coordinates and turn them into conventional celestial coordinates. An Armillary Sphere of 1.6 meters of diameter (1581). A triangular Sextant (1582) of 1.6 meters of radio. Armillary Great Circle (1585) three feet in diameter and whose accuracy was estimated at 38.6 arc seconds. A wooden mobile Quadrant (1586) of 1.6 meters. radio and whose accuracy was estimated at 32.3 seconds of arc. A mobile metal Quadrant (1588) whose accuracy was estimated at 36.3 seconds of arc.

²⁰ To get an idea of the cognitive threshold, it will be useful to consider the following: The angular diameter of the moon on average is about 30 minutes of arc (half a degree). The average estimated accuracy of the instruments of Tycho is about 30 seconds of arc, while in Copernicus' measurements the accuracy was 6 minutes. The precision reached by Tycho is out of range of that achieved not only at first glance but also of the astronomical instruments used before him apart from using methodological considerations to eliminate observation errors.

²¹ Tycho recognized the serious effects of refraction because they interfere with three very important astronomical parameters: 1) they affected the correct setting of the latitude of his observatory, 2) they introduced a radical change in the value that Tycho and everyone before him had been using to determine the obliquity of the ecliptic, and 3) they affected the establishment of the solar parallax. (Thoren, 2007, p. 226).

Tycho recognized that the Sun appeared apparently moved upward as it approached the horizon. He thought that this phenomenon was due to the refraction of light near the horizon and devoted a series of systematic measurements of the positions of the sun, moon and fixed stars. Comparing these measurements with predictions, he concluded that refraction was not detected above forty-five degrees in the cases of the Sun and the Moon, or twenty degrees for fixed stars and planets. Tycho's measurement data were taken to three different refraction tables, one for the Sun, another for the moon and the third for the fixed stars and planets (Van Helden, 1983, pp. 57-58). Data from the three tables were accurate but not joined conceptually or mathematically by Tycho. Kepler tried to remedy the situation of having three different refraction tables, and he attempted to integrate these data in *Paralipomena ad Vitellionem. Astronomia pars optica* (1604). He considered certain optical principles and argued that refraction is due to different the densities of the ether and atmosphere. He sought a mathematical relationship based on these principles that agreed with all measurements made from Tycho's three tables. Kepler was able to find a formula or law of refraction, which gave good results for calculating latitudes higher than ten degrees, though it had some errors from our perspective. With that mathematical relationship, Kepler built a single refraction table for all celestial objects, even more accurate than Tycho's. Kepler's analysis of Tycho's data joined measurements with mathematical and theoretical considerations to develop a law of refraction.²² Tycho had accurate data, but no mathematical or conceptual considerations for finding a mathematical proportion to the refraction.

Another problem existed with refraction and, in general, with the astronomical observations, which Kepler faced and solved. Astronomers measured the apparent distances between the fixed stars, planets and the edges of the Sun and the Moon with instruments and expressed those measurements in arcs of *visual angles* (*anguli visorii*). Human vision is not only involved in the measurements but is also a component of them, so "this whole enterprise in astronomy rests upon optical reasons" (Kepler, [1604] 2000, p. 321). This describes Kepler's consideration on the importance of the study of optics in two ways: a study of the nature of light as well as the functioning of human vision. This last topic has not been adequately evaluated in the development of astronomy (Hon, Giora, and Zik, Yaakov, 2009). Before any consideration, we must understand how Kepler conceived the astronomical practice. In the preface to his *Paralipomena ad Vitellionem. Astronomia pars optica*, (*Additional Aspects of Witelo. The optical part of astronomy*),²³ usually known simply as *Optics*, Kepler explains that for him, astronomy aims to study the *motion* of celestial bodies. This study has two parts: the first is the investigation of the *shapes* of the movements of the celestial bodies, and the second, derived from the first, is to investigate the *positions* of celestial bodies at any given time, whose main practical purpose was prediction. The first part is formed by geometry; the second by arithmetic. Three components comprise the first geometric part: the first one was called *mechanic* and has to do with using instruments for astronomical observations; the second was called *historical* and contains voluminous records to understand these observations; the third was called the *optical* component, and it involved the optical principles that include both the nature of light and the functioning of human vision. Kepler recognized that Tycho had greatly advanced the *mechanical* and *historical* aspects, both for the construction of instruments and the "24 books of the most meticulous observations of this sort, embracing the past 40 years or so" (Kepler, 2000, p. 13). The relationship of *optics* and astronomy was not only neglected but also completely misunderstood. Kepler's *Optics* can be considered as a theoretical reform of optical knowledge and the complement to astronomy required to achieve greater reliability in the data generated by measurements.

The main motivation for Kepler to develop a detailed analysis of the optics was to correct the optical knowledge that he inherited and was linked to measuring fundamental astronomical parameters. Astronomical practices depended entirely on optical reasoning, and the only way to guarantee a reliable link between the mental construct and physical reality of the heavenly bodies was to understand

²² A detailed study of the methodological considerations that Kepler used in the construction of his law of refraction is Buchdahl (1972).

²³ Donahue (1994) and Lindberg (1996) maintain that Kepler reach, in that book the highest theoretical peak of perspective tradition in optics, which begins with the Greeks and passed by the Arabs as Alhazen (965-1040) and medieval authors as Witelo (1230 - ca. 1300) of the thirteenth and fourteenth centuries. At the same time, Kepler opens the modern approach to optics, which topics are to understand three phenomena: how the eyes work, how refraction works and a new level of understanding of how lenses form images.

theoretically both the nature of light and the functioning of the human eye. Kepler thus argued that “supposing the place of the celestial body to be known with complete precision, throws the demonstration into difficulty: the nature of light, beset by the inconstancy of optical causes, does not always allow such precision of instruments” (Kepler, 2000, p. 6). One of the most common astronomical measurement methods since ancient times was what Kepler called the “theorem”. This was “to use a compass to measure the magnitudes of solar eclipses, the ratios of the diameters of the sun and moon, and the inclinations to the vertical of the circle drawn through the centers of the luminaries” (Kepler, 2000, p. 57). However, Kepler said that astronomers who used that method used to think (incorrectly) that they were “avoiding the inadequacy of the eyes, and avoiding the error which generally occurs in a bare estimation” (*Ibid.*). This assumption was one of the biggest mistakes made by astronomers and had contaminated many astronomical records. This unfortunate methodological situation prevented the realization of precise astronomical measurements, so Kepler states the following:

It is indeed well worth while here to see how much detriment would result from the ignorance of the proof of this theorem [method]. For since it escaped a number of authors, the result was that in believing in the theorem without restrictions they fell into a large error. For however many eclipses were observed in this way, they all had come out much greater in the sky than it appeared in the ray: all showed a much greater lunar diameter in the sky than in the ray. [...] *It is my hope in these pages to remove these considerable difficulties...* (Kepler, 2000, p. 57, emphasis mine).

Kepler's reform in optics and astronomy, which now are considered as two different areas, has one main pillar: his concern for the theoretical and methodological aspects of measuring high-precision astronomical data. From Kepler's perspective, a reform was needed in both astronomy and optics.

Such a reform needed to be performed from the core. One of the most routine, everyday and "simple" astronomy activities was to identify the position of the planets, and yet it presented significant difficulties “because of the eccentric, the planets appear either slow or fast. The cause is partly physical, partly optical. The physical part of the cause does not give the sense of vision a reason for error but also represents to the vision that which in fact occurs, [an account] of which is in the *Commentaries on the motions of Mars* [this book was later known as *Astronomia Nova*]" (Kepler, 2000, p. 321). Conversely, Kepler warned astronomers that they should not rely on their sense of sight while making measurements of the apparent diameter of the full Moon and Sun (Kepler, 2000, p. 298). Unlike Tycho Brahe, Kepler was convinced that because of large variations in visual capacity of each individual observer, it was impossible to establish accurate astronomical tables. Highly developed and accurate observation instruments were required, but light behavior must be first understood. Kepler said that "it cannot... be argued from this accident of the sense of sight to what happens outside of consideration of the sense of sight, nor can tables be established for the sake of the sense of sight, which represent neither the object itself nor the defects of all senses of sight. For the astronomer should not present anything other than those things that in actual fact occur. The sense of vision, however, we leave to the physicians to remedy” (Kepler, 2000, p. 298). Even the traditional instruments used for these observations were insufficient. He realized that, using a pinhole camera as instrument to observe and make measurements of the Sun's apparent diameter, the image just did not show the true diameter of the Sun or the Moon and the astronomer was easily fooled. Before using or redesigning the pinhole camera (as Kepler eventually did), a better understanding of the causes of these optical projections was required “to teach how to enter into a most certain procedure for measuring the quantities of eclipses” (Kepler, 2000, p. 298).²⁴ One method used by Kepler to investigate the varying distance between the Earth and the Sun was to measure the

²⁴ In the last chapter of his *Optics*, chapter 11, Kepler offers the construction details of an observation instrument built by him, and he incorporated the theoretical reforms on optics he had developed in the previous chapters. Such an instrument was called an "instrument of ecliptic" and consisted of the combination of a pinhole camera (which very precisely measured the apparent diameter of the Sun and the Moon) with a compass, traditionally used to establish the positions of astronomical bodies. Among other benefits, such an instrument was able to measure the image of the diameter of the sun projected on a screen with the appropriate theoretical adjustments derived from his principles of optics, and avoided the eye's natural distortions that are different in each observer (Kepler, 2000, p. 297). A detailed study of the problems, both theoretical and material, that Kepler had to overcome in building the "instrument of ecliptic" is that of Hon and Zik (2009).

apparent diameter of the Sun over a year; although this method yielded inconclusive results, it provided the opportunity to complete astronomical measurements with optical theory.

4. - Final remarks

Scientific measurements become knowledge through a dynamic and iterative process for integrating mathematics, instruments, and concepts. Ancient astronomy's transformation into modern astronomy required three different stages: the conceptual, instrumental, and mathematical stages. Copernicus conceptually modified ancient astronomy by placing the sun still and the earth moving. Tycho provided precision instruments with rich unpublished and precise astronomical data, but he did not conceptually reform astronomy. Kepler found various mathematical ratios and proportions for both refraction and planetary orbits. Measurements in modern astronomy handle the process of establishing values (with the desired accuracy) of many magnitudes and finding the correct proportions between them. This process integrates conceptual, mathematical, and instrumental aspects.²⁵ In positional astronomy, the only way to climb to new cognitive levels was the integrative modification of the measurement, in part because “the interaction between instrumentation and scientific exploration and discovery flowed in both directions: advances in instruments led to new discoveries (or highlighted specific properties), and this in turn influenced instrument design.” (Hackmann, 1989, p. 58-59). These three elements of the measurement, conceived as cognitive-integrative, relate to each other in a mutual interaction.

5.- Bibliography

Annas, Julia. (1975). Aristotle, Number and Time. *The Philosophical Quarterly* 25, no. 99 (April 1): 97-113.

Barker, Peter, y Bernard R. Goldstein. (1988). The role of comets in the Copernican revolution. *Studies In History and Philosophy of Science Part A* 19, no. 3 (September): 299-319.

Bennett, J. A. (1983). “A Viol of Water or a Wedge of Glass” in Gooding, Pinch, & Schaffer (eds.) *The Uses of Experiment*. Cambridge: Cambridge University Press.

_____. (2011). Early Modern Mathematical Instruments. *Isis*, 102: 697-705.

Berka, Karel. (1982). *Measurement: Its Concepts, Theories and Problems*. 1° ed. Netherlands: Springer.

Bertoloni, Domenico. (2006). *Thinking with Objects. The Transformation of Mechanics in the Seventeenth Century*. Baltimore: The Johns Hopkins University Press.

Blair, Ann. (1990). Tycho Brahe's Critique of Copernicus and the Copernican System. *Journal of the History of Ideas* 51, no. 3 (Julio 1): 355-377.

Boyer, Carl B. (1945). Quantitative Science without Measurement: The Physics of Aristotle and Archimedes. *The Scientific Monthly* 60, no. 5 (May 1): 358-364.

Bridgman, Percy Williams. (1927). *The Logic of Modern Physics*. New York: Macmillan.

_____. (1938). “Operational Analysis”, *Philosophy of Science* 5: 114–131.

Buchdahl, Gerd. (1972). Methodological aspects of Kepler's theory of refraction. *Studies In History and Philosophy of Science Part A* 3, no. 3 (November): 265-298.

²⁵ Different authors consider such integration in different contexts of their own discussion. Hackmann, for example, states: “the interaction between instruments, experiments, and the development of scientific concepts is extremely complex” (Hackmann, 1989, p. 33). Van Helden has the opinion that “If mathematical science must be based on a foundation of accurate measurements, it in turn makes demands for further accurate measurements. Once this mutually reinforcing relationship had been established, the demand for accurate instruments escalated” (Van Helden, 1983, p. 58). Hasok Chang says that “In epistemic iteration we start by adopting an existing system of knowledge, with some respect for it but without any result in the refinement and even correction of the original system. It is this self-correcting progress that justifies (retrospectively) successful courses of development in science [...]” (Chang, 2007, p. 6). And David Sherry (see note 14).

- Campbell, Norman Robert. (1957). *Foundations of Science: The Philosophy of Theory and Experiment*. New York: Dover Publications Inc.
- Canales, Jimena. (2010). *A Tenth of a Second: A History*. 1° ed. Chicago: University Of Chicago Press.
- Cooper, Gordon, y Humphry, Stephen. (2010). The ontological distinction between units and entities. *Synthese*. DOI 10.1007/s11229-010-9832-1.
- Chang, Hasok. (2007). *Inventing Temperature: Measurement and Scientific Progress*. 1° ed. Oxford: Oxford University Press.
- Chapman, Allan. (1983). The Accuracy of Angular Measuring Instruments Used in Astronomy Between 1500 and 1850. *Journal for the History of Astronomy* 14: 133-137.
- Diez, José. (1997). A Hundered Years of Numbers. An Historical Introduction to Measurement Theory 1887-1990. *Studies In History and Philosophy of Science*, 28: 167-185.
- Donahue, William H. (1994). Kepler's Invention of the Second Planetary Law. *The British Journal for the History of Science* 27, no. 1 (March 1): 89-102.
- Dreyer, J. L. E. (1890). *Tycho Brahe: A Picture of Scientific Life and Work in the Sixteenth Century*. Edinburgh: Adam & Charles Black.
- Ellis, B. (1968). *Basic Concepts of Measurement*. Cambridge: Cambridge University Press.
- Evans, James. (1998). *The History and Practice of Ancient Astronomy*. Oxford: Oxford University Press.
- Gingerich, Owen. (1997). *The Eye of Heaven: Ptolemy, Copernicus, Kepler*. Netherlands: Springer.
- Guillaumin, Godfrey. (2005). *El surgimiento de la noción de evidencia*. México: UNAM.
- Hacking, Ian. (1983). *Representing and Intervening*. Cambridge: Cambridge University Press.
- Hackmann, Willem. (1989). "Scientific Instruments: Models of Brass and Aids to Discovery", in Gooding, David, Trevor Pinch, y Simon Schaffer. 1989.
- Heilbron, John. (1979). *Electricity in the 17th and 18th centuries: a study of early Modern physics*. California: University of California Press.
- _____. (1993). *Weighing imponderables and other quantitative science around 1800*. California: University of California Press.
- Helden, Albert Van. (1983). "The Birth of the Modern Scientific Instrument, 1550-1700", in John Burke, *The Uses of Science in the Age of Newton*. California: University of California Press.
- _____. (1994). Telescopes and Authority from Galileo to Cassini. *Osiris* 9. 2nd Series: 9-29.
- Hellman, Doris, C. (1974). "Brahe, Tycho", in Charles Gillispie (ed.). *Dictionary of Scientific Biography*, vol 2, USA: Charles Scribner's Sons.
- Hon, Giora, y Zik, Yaakov. (2009). Kepler's Optical Part of Astronomy (1604): Introducing the Ecliptic Instrument. *Perspectives on Science* 17, no. 3: 307-345.
- Keithley, Joseph F. (1999). *The Story of Electrical and Magnetic Measurements: From 500 BC to the 1940s*. 1° ed. New York: Wiley-IEEE Press.
- Kepler, Johannes. (2000) [1604]. *Optics*. 1° ed. New Mexico: Green Lion Press.
- Lindberg, David C. (1996). *Theories of Vision from Al-Kindi to Kepler*. Chicago: University Of Chicago Press.
- Lloyd, G. E. R. (1991). *Methods and Problems in Greek Science*. Cambridge: Cambridge University Press.
- Newton, Robert R. (1977). *The Crime of Claudius Ptolemy*. USA: The Johns Hopkins University Press.
- Nola, R., & Irzik, G. (2011). *Philosophy, Science, Education and Culture*. Netherlands: Springer.
- North, John. (1974). The Astrolabe. *Scientific American*, January: 96-106.
- Pickover, Clifford. (2008). *Archimedes to Hawking: Laws of Science and the Great Minds Behind Them*. Oxford: Oxford University Press.
- Russell, B. (1903). *Principles of Mathematics*. Cambridge: Cambridge University Press.
- Sherry, David. (2011). Thermoscopes, thermometers, and the foundations of measurement. *Studies in*

History and Philosophy of Science 42: 509-524.

Thomson, William. Lord Kelvin (1883). Lecture "Electrical Units of Measurement" (3 May 1883), published in *Popular Lectures* Vol. I, p. 73.

Thoren, Victor. (1973). New Light on Tycho's Instruments. *Journal for the History of Astronomy* 4: 25-45.

_____. (2003). "Tycho Brahe", in Taton and Wilson, *Planetary Astronomy from Renaissance to the Rise of Astrophysics, Part A, Tycho Brahe to Newton*. Cambridge: Cambridge University Press.

_____. (2007). *The Lord of Uraniborg: A Biography of Tycho Brahe*. Cambridge: Cambridge University Press.

Toulmin, Stephen. (1953). *The Philosophy of Science. An Introduction*. London: Hutchinson University Press.

Wesley, Walter. (1978). The Accuracy of Tycho Brahe's Instruments. *Journal for the History of Astronomy* 9: 42-53.