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# Reduction and the Neighbourhood of Theories: A New Approach to the Intertheoretic Relations in Physics

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**Abstract** This paper proposes a classification of the intertheoretic relations in physics by bringing out the conditions for a relation of *reduction* which is eliminative, so that a theory reduced in terms of reductionism is superfluous in principle, and by distinguishing such a relation from another one based on comparison, which will be called *neighbourhood of theories*; the latter is a neighbouring relation between theories and is not able to support claims of eliminative reductionism. In the first part, it will be argued that this differentiation between neighbourhood and eliminative reduction permits an adequate classification of the intertheoretic relations in physics. By means of this differentiation, the second part discusses reductionism and shows that there are indeed some historical examples of reduction in the aforementioned sense, but that modern physical theories are typically only neighbouring.

**Keywords** Intertheoretic relations · Physics · Reduction · Reductionism

## 1 The Reduction of Physical Theories

It seems to be widely accepted that the complicated and miscellaneous intertheoretic relations in physics do not fit into a single scheme of reduction and that the interesting work to do is the investigation of special contexts. Nevertheless, the central question of this paper is the basic concern whether there are eliminative reductions in physics: are there physical theories that have been reduced to other theories, hence not needed any more for a complete description of the world and therefore superfluous? To answer that question requires taking into account that such a reduction is generally possible only *in principle*, so that a theory might still be in use for pragmatic reasons but might be eliminatively reduced as a matter of principle. The aim of the first part of this paper is to make clear the

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conditions for a reduction to be eliminative and to distinguish eliminative reductions from other intertheoretical relations. This investigation deals with examples taken from the context of gravitation. The second part is about reductionism itself. It analyses whether there are actually eliminative reductions in physics and for this purpose applies the resulting definitions of the first part to three examples: again to the case of gravitation, but also to the relation between classical and quantum mechanics and to the relation between phenomenological thermodynamics and statistical mechanics.

## 1.1 Direct and Indirect Reduction

Obviously, a reduction is eliminative if the reducing theory is suitable to explain all the phenomena explained by the reduced theory, which makes the reduced theory obsolete for a complete description of the world. The reducing theory should also be able to replace all the other merits of the reduced theory beyond explanation, e.g. description, prediction or introduction of an adequate vocabulary. However, the focus of this investigation is on explanation, and a reduction by means of explanation of phenomena is called *indirect reduction*. The disadvantage of indirect reductions is that the theories within such a reductive relationship may possibly have nothing in common: they could be able to explain the same phenomena just by chance. Hence, since Nagel (1961) reductions are commonly understood as *direct* relations between theories, as immediate relations between their laws and concepts. A reduction by means of direct relations between the laws and concepts of the involved theories is called *direct reduction* and does not by itself include the capability of the reducing theory to explain the phenomena explained by the reduced theory. Therefore, the question arises, whether direct reductions are eliminative, as the way of speaking in physics suggests that a theory is due to such direct relations 'contained as a special case' in another one. This will be discussed in the following: it will be shown that a reductive relation must indeed contain a direct part to avoid the reduction of entirely different theories, but that also an indirect part is needed for a reduction to be eliminative.

A direct reduction would be eliminative if the laws of the reduced theory could be logically deduced from the laws of the reducing one. This would be in accordance with the famous concept of reduction formulated in Nagel (1961), where the addition of some bridge laws to the laws of the reducing theory is postulated in order to connect the different terminologies of the theories—without bridge laws, a logical connection between the two theories would be impossible. However, with his thesis of incommensurability in the background, Feyerabend (1962) argues against the possibility of Nagelian bridge laws and against this concept of reduction altogether. But while logical connections between different theories are indeed impossible, direct relations can still be established against Feyerabend. But the question is, whether such direct relations are eliminative. To be able to give an answer to this question, we will have to take a closer look on Feyerabend's objections in this regard.

It is Feyerabend's main point that—ignoring trivial cases—different theories contradict each other, so that a logical deduction is ruled out a priori. This can be seen in two aspects. Firstly, we have to consider the *mathematical aspect* of the contradiction of laws. Secondly, there is the *conceptual aspect* of different terminologies: terms are defined in contradicting theories and are thus incommensurable. Now, the mathematical aspect will be briefly discussed, followed by the discussion of the aspect of terminology.

## 1.2 Comparing Theories: Mathematics

In physics, relations between laws of different theories are usually only approximative while deductions are in most cases impossible. Feyerabend's point against reduction as deduction is therefore really a matter of course but was explicitly aimed against Nagelian reduction. Physics deals with the impossibility of logical deductions by deriving laws only approximately with the help of limiting relations. The difference between (approximate) *derivation* and (logical) *deduction* is crucial to the aim of this paper, because only a deduction would be a direct relation between theories, which makes a (deduced) theory really superfluous. Despite this, according to a definition of Batterman (2012), a physical theory is reduced to another one if in a limiting process a characteristic dimensionless parameter of the reducing theory takes a specific limiting value and if on that condition the laws of the reduced theory can be *derived* from the laws of the reducing theory. On this note, Galileo's law of falling bodies is reduced to Newton's law of gravitation because the constant acceleration of the former can be derived from the increasing acceleration of the latter if the distance to earth of the falling body compared to the earth's radius is taken to be zero.

*But the law derived under this assumption is strictly speaking only valid for bodies lying on the earth's surface, while Galileo's law is about falling bodies.* Thus, the common mode of speaking that this derivation delivers approximate validity only for small distances covers the fact that we have *not (logically) deduced Galileo's original law* but rather *established a comparison between the two theories under certain circumstances* (in this case under the condition of small distances to the earth's surface): this is all we can say about 'approximate derivations' here. Such a comparison is indeed a direct relation between the two theories, but a relation between two independent theories with basically different claims about the same physical situation, which therefore are simply contradicting each other. Hence, this direct relation on its own does not establish an eliminative reduction: there is *no deduction* of the original law of Galileo from Newton's law of gravitation. The only direct link between these contradicting theories is a *comparison*, which is in general—as will be shown immediately—not necessarily a comparison between explanations of phenomena and which thus *on its own does not make any theory redundant*.

However, Galileo's law *is* superfluous, but not because of its approximate derivation, but rather because Newtonian physics can also describe falling bodies, in a similar manner as Galileo's law as has been shown in the comparative limiting process: Galileo's law is eliminatively reduced due to an *indirect reduction* based on the explanation of phenomena and *not* just due to a direct comparison with Newton's theory.

The difference between direct retentive comparisons and indirect eliminative reductions is not particularly striking if we consider simple cases like that of Galileo's law, where explanations of phenomena are compared. Therefore, this point has been widely overlooked in the debate on reduction. Of course, comparisons between physical theories are possible also *without* explanations of phenomena, for example between Newton's theory of gravitation and general relativity. The comparing relation which is possible in this case can be established between the mathematical *structures* of these theories. It is the claim of this paper that this relation on its own does not make Newton's theory redundant. In physics, the relation between these theories is often presented as a limiting process, which leads to the belief that Newton's theory of gravitation is contained in general relativity as a special case (cf. e.g. Misner et al. 1973, Section 17.4). In contrast to that, a mathematical relation between these theories in a more precise manner is developed for example in Scheibe (1999) within the structuralist metatheory, which was founded by Sneed (1971) and Stegmüller (1973) and applied to the problem of reduction e.g. by Moulines (1984). The structuralist metatheory formalizes physical theories in a way that differs from the formalizations of physics itself for the purpose of a rational reconstruction of these theories (cf. Schmidt 2008). For the sake of a precise comparing relation, Scheibe (1999) formulates Newton's theory and general relativity axiomatically according to the structuralist metatheory within a common superset as sets of models. These sets can be compared topologically with respect to a well-defined metric within this superset. Scheibe defines a common extension of the axiomatically formulated theories by varying the value of the velocity of light up to infinity, so that finite values represent the Minkowski space–time and an infinite value the Euclidean space (cf. l.c., 59–108 for the technical details). It is not the aim of this paper to analyse the technical details of the structuralist account of reduction (cf. e.g. Moulines 1984 in addition to Scheibe 1999), but to discuss the general consequences of the differentiation between logical deduction and approximate derivation in the context of the intertheoretic relations in physics. In this regard, Scheibe does not derive the original Newtonian equations—which would not lead to a deduction anyway—but rather proves the topological neighbourhood of solution sets of characteristic equations of the different theories in terms of the metric of a common topological superset and shows that this is all we can achieve as a relation between these theories in a mathematically adequate way. The physicist's derivation is actually nothing but a tentative form of such a topological comparing relation and *the claim of having shown that Newton's theory of gravitation is contained in general relativity as a special case is a rather loose mode of speaking, which hides the fact that a limiting process ultimately is nothing but a comparison between independent theories.* Newton's theory is not 'contained' in general relativity, but approximates the latter as an independent theory, which can be shown in the physicist's typical derivation or, more precisely, in topological comparing relations with the help of the structuralist metatheory.

However, Scheibe's aim of a complete structuralist 'reduction' of the whole Newtonian theory of gravitation has not been fully achieved, and would not have been a deduction anyway, but a subtle topological comparison between mathematical structures of independent theories reformulated in the structuralist metatheory. This comparison would indeed provide a demonstration of the topological neighbourhood of the two theories in the sense indicated above, but the possibility to compare mathematical structure does not include at all that the reducing theory is able to deal with the phenomena explained by the theory to be reduced as it is the case in the simple example of Galileo's law. For instance, these topological comparisons have to rely on assumptions as low velocity, low gravity and the like, while there may be phenomena explained by the theory to be reduced not being in line with these assumptions and therefore not being explained by means of these comparisons. As a matter of fact, there are only very few cases of explanations of concrete Newtonian phenomena by general relativity in an eliminative way (for example the explanation of the orbits of the planets by the Schwarzschild solution, cf. Scheibe 1999, 89–101). *While Galileo's law is eliminatively reduced to Newtonian gravitation because the latter can also explain falling bodies, there are in contrast many phenomena of gravitation explained by Newton's theory but lacking a description by means of a solution of the field equations of general relativity.* This will be discussed in more detail in the second part of this paper.

### 1.3 Comparing Theories: Terminology

The second of Feyerabend's objections concerns terminology and the incommensurability of the vocabulary of contradicting theories. In our context, the equation of motion in general relativity is the geodesic equation for neutral test particles whereas Newton's law of gravitation describes a force between two masses. Hence, we are concerned with two entirely different concepts and the *identification* of the Newtonian gravitational potential with Christoffel Symbols, which can be found in physics textbooks (cf. e.g. Misner et al. 1973, 415), connects concepts of different theories, *which are not identical* but—though not necessarily incommensurable—only comparable with each other. Even more concretely, in the example of the orbits of the planets, their description within the Schwarzschild solution deals with test particles without influence on the overall curvature and thus without gravitational masses, whereas their Newtonian description is based on forces just between these masses. Therefore, these concepts cannot be related by any simple identification and it has to be conceded that it is generally not possible to establish reductions via logical deduction with the help of identifying bridge laws.

But nonetheless theories need not be incommensurable. It is of course possible to compare the concepts of different theories, e.g. with the aid of the structuralist metatheory. But this is generally a difficult and not very straightforward comparison process: the terms of two theories can be related in special case studies which can prove for example that the Newtonian potential is related to (and in a sense neighbouring, passing asymptotically into) the Christoffel symbols in the topological way explained above. The precise structuralist elaboration of this in Scheibe (1999) does not identify these concepts, but compares the corresponding equations and shows that their solution sets are topologically neighboured with respect to a certain metric in a common superset of models of the theories (cf. l.c., 87–89). But such case studies of comparing concepts are no self-evident processes and again lead to a comparing relation rather than a deduction, and such a direct comparison on its own does not permit an eliminative reduction. As has been shown in the discussion of the mathematical aspect, a reduction is only eliminative if there is not only a direct comparing relation between the structures of the theories, but also an indirect reduction based on the explanation of phenomena. This result, which was obtained here using the example of the different theories of gravitation, will be briefly recapitulated twice in the second part by means of the examples of quantum versus classical mechanics and of phenomenological thermodynamics versus statistical mechanics.

## 1.4 Concepts of Reduction: Schaffner, Hooker and Bickle

By means of the differentiation between direct and indirect reduction, which can be found in Kemeny and Oppenheim (1956) already, the answer of Schaffner (1967) to the arguments of Feyerabend will now be discussed. Schaffner concedes that a deductive link between contradicting theories is impossible, but he nonetheless tries to establish an eliminative concept of reduction, which is explicitly supposed to be a direct relation because indirect reductions can be justified between completely alien theories. He thus suggests that a theory is not reduced to another one if its laws are logically deduced, but if it is possible to deduce a new *corrected theory* from the reducing theory, which is (as a direct relation) *strongly analogous* to the original theory to be reduced. This approach was extended by Hooker (1981), who additionally postulates that, in order to guarantee its deducibility, the corrected theory should be formulated in the reducing theory's vocabulary. Finally, Bickle (1998) made the concept of analogy, which remains rather vague in the accounts of Schaffner and Hooker, more precise with the help of the above-quoted structuralist metatheory, which again yields a subtle comparing procedure between mathematically formalized theories, or as Moulines (1984) states, a '[…] mathematical relationship between two sets of structures' (l.c., 55). In doing so '[…] this [structuralist] scheme of reduction does not require semantic predicate-by-predicate connections nor deducibility of statements' (Moulines 1984, 54). The structuralist approach to physical theories is also the basis of the comparing relations between theories in the aforementioned account of Scheibe (1999) and thus a fundamental constituent of the following definitions of this paper, but it does not—on its own—establish an appropriate concept of reduction, for Moulines also indicates that in a purely structuralist account of reduction '[w]e could have a reductive relationship between two theories that are completely alien to each other' (ibid., 55). This point is accentuated even more by Endicott (2001): 'For example, thermodynamics, hydrodynamics, and exchange economics might have the same formal structure, but they do not reduce to one another' (381).

Hence, the structuralist metatheory has to be combined with an autonomous concept of reduction as given e.g. in the following definitions of neighbourhood and reduction. But it can also be combined with Schaffner's concept of reduction via corrected theories, which indeed becomes more precise within the structuralist metatheory. Though Bickle (1998) might be too optimistic in calling his account the *New Wave Reduction*—'[…] Bickle's account is a model-theoretic [i.e. structuralist] version of Schaffner, not the new wave' (Endicott 2001, 382)—his account nonetheless delivers a precise differentiation of the concept of analogy between the original and the corrected theories in the Schaffnerian account of reduction: these analogies may be strong or weak (*smooth* or *bumpy*, as Bickle calls them), and in the end this is all Bickle wants to provide: 'I only intend this discussion to show that I can make quantitative sense of "amount of correction" that locates reductions on the smooth-to-bumpy spectrum' (Bickle 1998, 101).

A reduction in the Schaffner–Hooker–Bickle-sense is indeed eliminative. However, this is not a result of a comparison in the framework of the analogy relation, but of the construction of the corrected theory, which is—let the correction be smooth or bumpy—able to explain the phenomena explained by the (not corrected) theory to be reduced. Since the corrected theory, which is a strongly—smoothly or bumpily—analogous correction of the theory to be reduced, is by definition also logically contained in the reducing theory and even formulated within the latter's vocabulary, *these explanations are ultimately explanations by the reducing theory itself.*

But in order to carry out explanations by a reducing theory, there is no need for the digression via corrected theories. On the contrary, as Callender (2001) shows using the example of explanations of thermodynamic phenomena by statistical mechanics, this can even be obstructive (cf. the second part of this paper). Moreover, scientific practice deals with theory changes, but not with corrected theories in the spirit of the Schaffner–Hooker–Bickle-account of theory reduction. Such corrected theories do not exist and are only postulated to establish a concept of eliminative reduction based on deduction. Ultimately, comparisons between theories can also be established between the theories themselves without a corrected theory introduced between them. This can be seen in the above-quoted example of Scheibe (1999), who—like Bickle—relies on the structuralist account of theories. All in all, the endeavour of reduction is not easier with corrected theories: 'Any problems for classical bridge laws will therefore accrue to this newest new wave' (Endicott 2001, 387).

Different concepts cannot be identified by bridge laws but can only be compared with each other—by means of corrected theories or directly. Eliminative reductions on the other hand can only be established by indirect reductions via the explanation of phenomena by the reducing theory. While Bickle (2006)—in the context of the philosophy of mind—argues for his new-wave-conception of direct reductions via comparing relations with corrected theories as 'ruthless reductionism', Schaffner (2006) advocates small-sized indirect reductions in the context of biology, i.e. *explanations* of concrete phenomena by reducing theories without claiming to reduce whole theories: ‘The results [of these explanations] are like reductions, but I think they are better described as explanations, using that term as an alternative to reduction because the e-word does not carry the conceptual freight of various reduction models and is a more appropriate general context, within which to analyze what is actually occurring in the biomedical sciences’ (380). Such explanations can indeed result in eliminative (indirect) reductions if they are combined with the comparing relations of the traditional concepts of (direct) reduction (e.g. Nagel 1961; Schaffner 1967; Hooker 1981; Bickle 1998). But as Schaffner (2006) points out, robust eliminative reductions seem to be unusual and small sized explanations seem to be the business to follow instead:

> ‘The first thesis is that what have traditionally been seen as robust reductions of one theory or one branch of science by another more fundamental one are largely a myth. Although there are such reductions in the physical sciences, they are quite rare, and depend on special requirements. In the biological sciences, these prima facie sweeping reductions tend to fade away, like the body of the famous Cheshire cat, leaving only a smile… The second thesis is that the “smiles” that remain are fragmentary patchy explanations, and though patchy and fragmentary, they are very important, potentially Nobel-prize winning advances’ (378).

## 1.5 Reduction and Neighbourhood

Even if there are only a few examples, the *concept* of eliminative reduction can still be defined and, as has been shown, has to rely on a combination of direct and indirect reduction. The concept of reduction formulated by Schaffner (1967), Hooker (1981) and Bickle (1998) with its relation of analogy and its explanation of phenomena consists of a mixture of direct and indirect reduction—a mixture, which is possible also *without* corrected theories. Consequently, a concept of eliminative reduction based on such a mixture without corrected theories will now be defined.

In a first step, it seems to be appropriate to call most of the intertheoretic relations in physics a relation of *neighbourhood*: two independent and contradicting theories can actually be compared with each other, most adequately with the help of the structuralist metatheory as indicated above. In this approach, a common topological superset can be constructed, whose special metric permits topological comparisons of theories formulated axiomatically as sets of models. As a matter of fact, the construction of this superset presupposes that the involved theories describe the same physical phenomena. This condition cannot be formalized in the structuralist metatheory and has to be assumed on a primordial level (cf. Scheibe 1999, 72). But if two theories fulfil this condition, topological comparisons can show that they are neighbouring via approximate derivation and related concepts: the work of Scheibe (1999) quoted above demonstrates that the solution sets of the equations of the predecessor theory may asymptotically approximate the solution sets of the equations of the successor theory (e.g. Newton’s theory is approximating general relativity within the structuralist account—not vice versa) and accordingly the concepts of the predecessor theory may asymptotically pass into the concepts of the successor theory. This can be called the topological neighbourhood of solution sets of the equations of the respective theories and—according to this—the corresponding neighbourhood of their concepts. This leads to the following (asymmetrical) definition of neighbourhood:

**Definition 1** Neighbourhood of Theories: Two theories are *neighbouring* if (1) they deal with the same physical phenomena, if (2) mathematical and conceptual comparisons show that the solution sets of the equations of the predecessor theory are asymptotically approximating and hence topologically neighbouring the solution sets of the successor theory and if (3) accordingly the concepts of the predecessor theory are asymptotically passing into the concepts of the successor theory.

This intertheoretic relation could also be called ‘retentive reduction’, but since in physics reduction is mostly meant to be eliminative, the term ‘neighbourhood’ seems to be more adequate, because it does not evoke reductionist claims. It is an approximative but nonreductive relation between two independent theories with different claims about the same phenomena (this is meant by ‘contradicting theories’ in this paper). Furthermore, if two theories deal with differing scopes of phenomena, they can only be neighbouring with respect to their common domain.

In a second step, as a special case of neighbourhood, a concept of *reduction* can be defined on the basis of the explanation of phenomena: a physical theory can be regarded as reduced to another one if it is neighbouring the latter, if any phenomenon explainable by means of the theory to be reduced can be explained by the reducing theory and if some of the explanations by the reducing theory are more adequate:

**Definition 2** Reduction of Theories: A theory *reduces* to another one if (1) all phenomena explained by it are also explained by the reducing theory, if (2) the reducing theory explains some of these phenomena more adequately than the reduced theory and if (3) the two theories are neighbouring.

This indeed defines an eliminative concept of reduction, because a theory reduced according to this definition is redundant: ‘its’ phenomena are explained by another theory equally well or even better. Additionally, this is not a coincidence, but comprehensible by virtue of the neighbourhood of the two theories. Strictly speaking, however, the definition ought to be supplemented by the requirement that the reducing theory is also able to replace all the other merits of the reduced theory beyond explanation, e.g. description, prediction or introduction of an adequate vocabulary. Furthermore, an adequate account of reduction should also be able to account for shifts between explainable facts and contingent phenomena. But while Definition 2 refers basically to the explanation of phenomena according to the concept of indirect reduction introduced above, it can easily be augmented adequately. And finally, since the explanations of indirect reductions are characteristic explanations of physical phenomena and not pathological cases of the concept of explanation as the length of the shadow cast by a flagpole or males taking birth control pills, the definition above rests on the standard *DN*-model of explanation (cf. Gutschmidt 2009 for a more detailed discussion of these definitions and the other concepts of reduction mentioned above).

Let us now apply these two definitions to the theories of gravitation: First of all, it is certainly true that Galileo’s law of falling bodies and Kepler’s laws of planetary motion are in the framework of the well-known approximative relations neighbouring Newton’s theory of gravitation with respect to the typical gravitational phenomena. In addition to this, the quoted investigations of Misner et al. (1973) or Scheibe (1999) show that Newton’s theory of gravitation is neighbouring general relativity: Consider e.g. the Newtonian gravitational potential, which is passing asymptotically into the Christoffel symbols without being identical to them (cf. e.g. Scheibe 1999, 87–89, for a precise elaboration of this neighbourhood, cf. also the examples of neighbouring theories given in this paper’s second part). Moreover, Newton’s theory of gravitation is able to explain both, the phenomenon of falling bodies and that of the planets’ motion in the solar system, and in fact more precisely than Galileo's law of falling bodies or Kepler's laws of planetary motion respectively. Hence, in terms of the definition just introduced, these theories are eliminatively reduced to Newtonian gravitation.

But when it comes to the relation of Newton's theory of gravitation to general relativity, things are not that easy—it is doubtful whether we have a reduction beyond neighbourhood here. There are many phenomena explained by the former but lacking an explanation by the latter because nobody solved the field equations for these cases: at least at the moment there is no reduction in the sense of the definition just given. Moreover, this could be even a matter of principle: the second part of this paper introduces some arguments showing fundamental limitations of explanations by general relativity without the systematic use of Newtonian concepts.

This issue and similar questions concerning the relation between classical and quantum mechanics and that between phenomenological thermodynamics and statistical mechanics will now be discussed: it will be shown that these are—now and perhaps forever—examples of neighbourhood without reduction and that therefore the differentiation between neighbourhood and reduction proves to be fruitful.

## 2 Reductionism in Physics

According to the definition of the first part, a physical theory is eliminatively reduced to another if they are neighbouring and if the latter is able to explain all the phenomena explained by the former. Since this relation ought to show that the reduced theory is not needed anymore for a complete description of the world and is therefore superfluous—this is the claim of an eliminative reduction—the reducing theory must be able to explain all these phenomena completely on its own and particularly *without the systematic help of the theory to be reduced*. In contrast, the explanatory support of higher-level theories is allowed in *functional reductions* showing that lower-level theories are actually able to deal with higher-level phenomena. But if eventually these explanations of higher-level phenomena are not possible without substantial support of a special theory, this special theory is not eliminatively reduced by means of these explanations. Therefore, functional reductions are indirect reductions, but not necessarily eliminative. Furthermore, if the theory to be reduced is only applied heuristically to find some explanations, which are eventually possible without the reduced theory, we may say that it is eliminatively reduced. But it may be the case that the support of a putatively reduced theory is needed systematically—and the use of this theory may be well justified by a neighbourhood relation *between two independent theories*. In this case, obviously *the theory is still needed for a complete description of the world and therefore not eliminatively reduced in spite of being the neighbour of another theory.*

However, it might be plausible to consider explanation to be transitive and to claim that e.g. general relativity explains why Newton's theory of gravitation works and that Newton's theory explains in turn certain phenomena. But if general relativity is not able to explain these phenomena on its own, Newton's theory of gravitation is simply not superfluous, no matter if it is explained by another theory or not: if the theory to be reduced is systematically needed in order to carry out these explanations, it is not eliminatively reduced. Besides, an 'explanation' of a theory by another one after all can be nothing but the justification of a relation of neighbourhood between the two theories (leaving aside trivial cases).

With the differentiation between heuristic and systematic support in the background, three examples of neighbourhood without reduction will be sketched briefly in the following (in the space of just a few pages only some arguments for these non-orthodox anti-reductionist claims can be indicated: cf. Gutschmidt 2009 for the full elaboration of the examples in all their particulars).

## 2.1 Newton's Theory of Gravitation and General Relativity

The first example again concerns gravitation. For a start, it is important to point out that we are dealing with two independent theories: there is Newton's theory of gravitation, based on the concept of gravitational forces between two or more bodies, and there is general relativity with its field equations and its concept of gravity as a matter of curved space–time. There indeed exist limiting relations between the two theories (cf. e.g. Misner et al. 1973 or Scheibe 1999), but as has been demonstrated in the first part of this paper already, these relations are no deductions but just subtle comparing relations between independent theories, which merely show their neighbourhood. The simple point here is that *contradicting theories are not identical* but, despite comparing limiting relations, due to their entirely different concepts and laws *basically independent of each other*. Therefore, Newton's theory of gravitation is not eliminatively *contained* in general relativity as a special case and may hence not be applied systematically in explanations of phenomena if these explanations are supposed to show—in the framework of an indirect reduction—that Newton's theory of gravitation is redundant. Newton's theory would only be superfluous if there were genuine explanations by general relativity, i.e. solutions of the field equations, for all gravitational phenomena which Newton's theory can handle.

But Newton's theory of gravitation is used to explain many phenomena of gravitation lacking an exact solution of the field equations also describing them: while the two-body problem is directly solved by Newton's law of universal gravitation, it has (and as a matter of fact can have) only numerical solutions in general relativity. The interactions between the planets or even more complex formations as star clusters or spiral galaxies are handled with Newton's theory as well (neighbourhood to general relativity indicates that this might not be too bad), but there does not even exist a numerical solution of the field equations for them: all these phenomena are described by Newton's theory of gravitation and are too complex for the application of the field equations.

The Schwarzschild solution indeed describes the orbits of the planets in the solar system, and even better than Newtonian gravitation, but has to regard the planets as test particles without gravitational masses and is therefore as a matter of principle not able to calculate the interactions of gravitation between them. Except for these interactions, we can say that the Newtonian description of the orbits of the planets is reduced eliminatively to the Schwarzschild solution of general relativity. But these interactions as well as the other cases just mentioned are examples of phenomena, which are described by Newton's theory of gravitation and not by general relativity, which is why Newtonian gravitation as a whole is not eliminatively reduced to general relativity according to the definition of the first part. As Weinberg (1993) puts it: '[…] general relativity contributed very little to our understanding of the Solar System or the tides. We already knew enough to calculate planetary orbits with great precision, and general relativity did not help us with the major puzzles still outstanding (long-term stability, tidal dissipation), only with one tiny anomaly in the orbit of Mercury' (475–476)—at the moment, there are only very few explanations of Newtonian phenomena based on general relativity.

However, this may be the case only at the moment and it can be claimed that an eliminative reduction is possible *in principle*, for example by numerical solutions of the field equations. But because of the difficult and abstract character of the field equations in contrast to the high applicability of Newton's theory we may also question the possibility of such explanations. Leaving aside the problems of the complexity of the non-linear differential equations involved here, it has to be considered that a numerical simulation can only compute the temporal development of a system according to the field equations, whereas the initial values of the system cannot be described in terms of the field equations if there is no exact solution for that system—and that there is no exact solution in the cases just mentioned is a fact of mathematics. Hence, if for example in astrophysics the interaction of two black holes is described numerically, the simulation is started at a distance between them at which the system can be described by Newton's theory of gravitation (or, to be correct, by the so called post-Newtonian approximation, which is essentially based on Newton's theory of gravitation and is augmented by some extra terms approximating general relativity):

> 'Nevertheless, it is one of the remarkable consequences of general relativity that, during the orbital phase before coalescence, the black holes follow orbits that are described to first order by Newtonian gravity: their interaction when separated by a significant distance does not reflect the enormously strong gravity inside and near them. Only when they come within a few tens of gravitational radii do we require full general relativity to describe the dynamics. Before that, the post-Newtonian approximation—an asymptotic approximation to general relativity valid for small orbital velocity […] in gravitationally bound systems—provides a systematic approach to studying the orbital inspiral phase, where orbits shrink and lose eccentricity through the radiation of energy and angular momentum in gravitational waves' (Schutz 2004, 1).

The values obtained from that post-Newtonian description of the inspiral phase are then taken over for the calculation with the field equations:

> 'A numerical simulation *must* start with a representation of the black holes at some point late in their inspiral phase. *Since our knowledge of their location at this time is a result of solving the post-Newtonian approximation, we do not have a complete description of the spacetime metric at this initial time.* […] There is thus the possibility that the initial configuration for the numerical integration does not represent two black holes after a long inspiral phase' (9, emphasis added).

This describes the technical problem of how to change exactly from the Newtonian description to general relativity: 'But this problem is far from being solved, and until we have a better understanding of it, it will be difficult to trust any waveform predictions [of gravitational waves]' (10). In our context, this particularly shows that for systematic reasons the post-Newtonian approximation is indispensable for numerical solutions of the field equations in complex scenarios. Therefore, the Newtonian theory of gravitation is not eliminatively reduced to general relativity. It is of course a common practice in physics to explain phenomena by a mixture of theories, but since this practice might have systematic reasons, we should be very careful with reductionist claims.

Another reason against the possibility of numerical explanations without the help of Newtonian gravitation is the problem of the physical interpretation of numerical values: even in the *exact* Schwarzschild solution the mathematical result has to be compared with Newton's theory for the interpretation of a particular constant of integration as the system's central mass. In Misner et al. (1973), the Schwarzschild solution with the uninterpreted constant of integration is compared to its Newtonian equivalent in a region 'where the geometry is nearly flat' (607), which means that Newton's theory of gravitation can be applied. This comparison then shows: 'Consequently, *M* [the uninterpreted constant of integration] is the mass that governs the Keplerian motions of planets in the distant, Newtonian gravitational field—i.e. it is the star's "total mass-energy"' (ibid.). Therefore, in the case of non-exact, but *numerical* solutions of the field equations, physical interpretations of the calculated data depend on the systematic support of Newtonian concepts all the more. This can be shown by means of the example of the simulation of a binary system similar to the aforementioned black-hole-merging, which is discussed in Pollney et al. (2007):

> 'A time-integration of those equations is needed in order to compute the recoil and this obviously opens the question of determining an integration constant which is in practice a vector. Fortunately, this integration constant has here a clear physical meaning and it is therefore easy to compute. In essence it reflects the fact that *at the time the simulation is started, the binary system has already accumulated a non-vanishing net momentum as a result of the slow inspiral from an infinite separation*' (10, emphasis added).

The interpretation of numerical values obtained in a simulation of a binary system rests on the time *before* the simulation starts and is therefore not based on general relativity, but on the post-Newtonian approximation, which is valid in 'an infinite separation', and on Newtonian considerations concerning the momentum. Furthermore, the simulation is not only based on interpretations taken from the post-Newtonian approximation, but also evaluated by comparisons with it: 'We remark that a proper choice of this constant [the integration constant quoted above] is essential […] because it allows for a systematic interpretation of the results. Without it, in fact, […] a comparison with the PN [Post-Newtonian] prediction [is] impossible' (19). To sum up, since many gravitational phenomena cannot be described in terms of the field equations alone and due to the complexity and universality of the field equations, numerical solutions depend on an interplay with Newtonian concepts: initial values and the interpretation of numerical values are obtained by Newtonian descriptions. Physicists use a mixture of both theories for their explanations, which is a fact that corresponds to similar approaches in the domain between classical and quantum mechanics and phenomenological thermodynamics and statistical mechanics respectively, as will be shown in the following. Inasmuch as there are systematic reasons for these mixtures of theories, this fact matters to reductionism.

The examples discussed so far show that a description of the world of gravitation in terms of the field equations alone seems to be impossible in principle, even if numerical simulations are considered: there are the problems of the simulations' initial values and of the interpretation of numerical values. But since such a description would be necessary in order to prove the claim that Newton's theory is eliminatively reduced to general relativity, even the fundamental possibility of an eliminative reduction of Newton's theory of gravitation to general relativity can be put into question. Of course, it is not possible to show that such a reduction is fundamentally impossible, but the burden of proof is transferred to the reductionist's position, which has to show how all the phenomena mentioned can be explained by general relativity without the systematic support of Newton's theory. Such explanations are not needed in physics, because for explanations by a mixture of both theories it is legitimate to use Newton's theory of gravitation due to its neighbourhood to general relativity. But the fundamental possibility of such explanations by general relativity alone has to be shown in order to make a corresponding reductionist claim plausible.

All in all, this demonstrates that Galileo's law of falling bodies and Kepler's laws of planetary motion are reduced to Newton's theory of gravitation and that the Newtonian description of the orbits of the planets is reduced to the Schwarzschild solution of general relativity, but that Newton's theory of gravitation as a whole is merely neighbouring general relativity: these are two independent theories contradicting each other in their descriptions of gravitational phenomena and despite comparing limiting relations between them there are many phenomena explained by the former but—at least at the moment—not explained by the latter. And as the first part shows, without an explanation of these phenomena there is no eliminative reduction.

## 2.2 Classical and Quantum Mechanics

When it comes to the question whether classical mechanics is reduced to quantum mechanics according to the definition of the first part, the first problem to be considered is whether the former is contained in the latter as a special case. A positive answer seems to be a common belief in physics. But a closer look in the line of argument of the first part of this paper shows that classical mechanics is not logically contained in quantum mechanics, but as an independent theory only neighbouring the latter. This can be seen in at least three ways. Firstly, there are the famous *Ehrenfest theorems*, which are nothing but relations of *formal analogy between entirely different concepts*: they state that the mean values of the differential operators, which describe quantum mechanical objects, satisfy equations, which are formal analogues of the equations of the (exact) functions, which describe classical objects. Not only are mean values of differential operators not classical functions but there also is a wide difference between quantum and classical objects. Therefore, the Ehrenfest theorems establish a *comparing relation of formal analogy between two independent theories* and a closer investigation, e.g. with the help of the structuralist metatheory, might thus show the neighbourhood of these theories—but the Ehrenfest theorems surely do not show that classical mechanics is logically included in quantum mechanics. The second way concerns the argument of varying Planck's constant in order to establish classical mechanics as a special case of quantum mechanics. Varying constants of nature is generally a peculiar procedure and seems to be physically senseless—it can only be justified for the purpose of comparing different theories. But then the result obviously is not a logical deduction of a physical theory from another—either Planck's constant is zero or not—but merely a comparison between independent—and particularly contradicting—theories. In short, as it has been shown in the first part of the paper with the help of the example of the different gravitational theories, contradicting theories with different concepts may be derived from each another by means of asymptotic limiting relations, but not deduced. If the procedure of these derivations is analysed in a mathematically adequate (e.g. structuralist) way, it becomes clear that these derivations are nothing but subtle, topological comparisons between independent theories, which can show the neighbourhood of the respective theories at best (cf. e.g. Scheibe 1999, 163–250, for a detailed investigation of the relation between classical and quantum mechanics with the help of the structuralist metatheory and its topological supersets as discussed above in the case of gravitation—again, it is not the aim of this paper to analyse the technical details of structuralist reductions, but to discuss the general consequences of the differentiation between logical deduction and approximate derivation).

The third way to be discussed here is the programme of *decoherence,* which was developed in the last decades and shows in another way that the common belief about classical mechanics as being contained in quantum mechanics is mistaken. In this context, this belief is reformulated as follows:

> 'Most textbooks suggest that classical mechanics is in some sense contained in quantum mechanics as a special case, similar to the limit of small velocities in relativity. Then, for example, the center-of-mass motion of a macroscopic body would be described by a narrow wave packet, well localized in both position and momentum. The spreading of the wave packet according to the Schrödinger equation is indeed negligible for large masses, so that the Ehrenfest theorems seem to allow a derivation of Newtonian dynamics as a limiting case' (Joos 1996, 1–2).

This belief was refuted above by means of the arguments from the first part of this paper concerning the difference between logical deduction and approximate derivation: different concepts and contradicting descriptions of phenomena cannot be part of one reducing theory but belong to independent though, as the case may be, neighbouring theories. Another refutation of this common belief comes from the programme of decoherence, which stresses the interaction of macroscopic objects with their environment:

> 'It is now increasingly being realized that the conventional treatments of the classical limit are flawed for a simple reason: they do not represent any realistic situation. The assumption of a closed macroscopic system (and thereby the applicability of the Schrödinger equation) is by no means justified in the situations which we find in our present universe. Objects we usually call "macroscopic" are interacting with their natural environment in such a strong manner that they cannot even approximately be considered as isolated, even under extreme conditions. Large molecules, for example, are already "macroscopic" in this dynamical sense' (2).

It is beyond the scope of this paper to discuss these arguments of decoherence, but it has to be admitted anyway—against the common belief, especially among physicists—that *classical mechanics is not contained in quantum mechanics as a special case*: their relationship is, considering decoherence or not, much more complicated and can at best if anything be described as some kind of neighbourhood.

It is nonetheless possible that classical and quantum mechanics are not only neighbouring each other but that the former reduces to the latter in the sense of the definition of the first part, i. e. by means of an indirect reduction based on explanations of phenomena. But then quantum mechanics should be able to explain all the phenomena of classical mechanics and should particularly be able to explain them completely on its own, without the systematic help of classical mechanics. This really seems to be out of question and is only considered here as an hypothetical speculation for the sake of argument. An authentic quantum mechanical explanation of a macroscopic phenomenon would have to consider all particles involved and to solve the Schrödinger equation for all of them. If this were possible, this would also show that the macroscopic phenomenon is in a quantum state because all solutions of the Schrödinger equation allow for superpositions. However, nobody ever observed macroscopic objects in quantum states (aside from neutron stars and the like). Admittedly, the claim of the programme of decoherence is to show that macroscopic objects actually are in quantum states and that they only seem to be in a classical state due to measurement-like interactions with the objects' environment. This would also mean that we would have to solve the Schrödinger equation not only for the macroscopic phenomenon in question, but also for its environment. Since it is not clear, where this environment ends, we ultimately would have to solve the Schrödinger equation for the whole universe: 'Given that everyday macroscopic objects are particularly subject to decoherence interactions, this raises the question of whether quantum mechanics can account for the appearance of the everyday world […]. To put it crudely: if everything is in interaction with everything else, everything is generically entangled with everything else […]' (Bacciagaluppi 2012, Section 2.2).

On the other hand, there are exact solutions of the Schrödinger equation only for very few and simple cases like the hydrogen atom. Calculations in quantum mechanics therefore consists essentially in numerical solutions of the Schrödinger equation. But even in the micro-world of molecules these numerical solutions partly base on classical assumptions: The *WKB-method* and the *Born–Oppenheimer-Approximation* are only valid in semi-classical cases, where Planck's constant is neglected due to classical approximations and where larger collections of particles are considered not to be in quantum states, but in a statistical mixture of classical states. This is not only a heuristic simplification, but a necessary assumption, because larger molecules are in classical states already. Therefore, even numerical quantum explanations in the micro-world rest in a systematic way on classical mechanics. This is another example for explanations by a mixture of two theories as it was mentioned above in the case of the theories of gravitation. Using mixtures of theories is a typical method of the working physicist, which may be well justified in terms of the neighbourhood of the theories involved, but which should also lead to caution with regard to reductionist claims.

Nevertheless, as a matter of principle it is still possible to claim that numerical solutions of the Schrödinger equation for macroscopic objects (including their environment up to the whole universe) are possible some day. But as was already mentioned in the example of the numerical solutions of the field equations of general relativity, the problem of interpreting numerical values has to be considered. As Primas (1981) shows, even in quantum chemistry, dealing with numerical solutions of the Schrödinger equation for larger molecules, it is systematically necessary to use concepts of chemistry in order to be able to interpret the numerical values. Numerical solutions on their own are not able to reproduce chemical concepts which on the contrary are necessary to interpret these solutions. This is an argument similar to the necessary guidance of Newton's theory of gravitation in the case of the numerical simulations of general relativity. Therefore, the problem of interpreting numerical values is a problem in the micro-world of quantum chemistry already and will thus be a problem of calculations of classical macroscopic phenomena all the more. In general, numerical simulations require initial values and interpretations of numerical values, which, in the case of a quantum mechanical calculation of macroscopic phenomena, both can only be obtained by Newtonian descriptions, because a description of macroscopic phenomena in terms of quantum mechanics alone would only be possible as the result of just this calculation. Because of this circularity, numerical solutions of the Schrödinger equation for macroscopic phenomena are indeed impossible without Newtonian guidance. As in the case of the numerical solutions of the field equations, numerical solutions of the Schrödinger equation for macroscopic objects can only be obtained by an interplay with Newtonian theory. Hence, the latter is for fundamental reasons not eliminatively reduced to quantum physics, but systematically needed in explanations by a mixture of the two theories. This is no big deal and corresponds to the common practice in physics, but it was discussed here for the sake of argument. This discussion shows that classical mechanics is not eliminatively reduced to quantum mechanics. While there is no need for quantum mechanical explanations of macroscopic phenomena—everything is fine with classical mechanics—classical concepts in fact are indispensable in the micro-world of larger molecules.

An even more fundamental problem for the applicability of quantum mechanics to macroscopic objects, which would be necessary to show that classical mechanics reduces in principle to quantum mechanics according to Definition 2 of the first part, arises in the context of the *measurement problem.* As already mentioned, the successful application of the Schrödinger equation to macroscopic objects would imply that they are in quantum states, because all solutions of the Schrödinger equation allow for superpositions. Krips (2007) discusses an 'insolubility theorem' for this problem: '[…] by sticking to the Schrödinger linear dynamics we are stuck also with the result that at the end of the measurement process, there must be superpositions of macroscopically distinct states of the apparatus, and in general of a macro-system […]. And this result […] is contrary to experience, since, at the end of the measurement process, although we may be uncertain of the position of the pointer, the pointer itself is never in an indeterminate superposition of different positions' (Section 2). The discussion of the programme of decoherence above demonstrates the necessity to account for the object's environment in order to understand the classical appearance of the macroscopic world, what makes numerical calculations of macroscopic phenomena to a lost cause of unmanageable complexity. But over and above it seems to be doubtful, whether the programme of decoherence can handle the fundamental problem of the applicability of quantum mechanics to macroscopic objects at all, which is crucial for the question of the fundamental possibility of reducing classical mechanics to quantum mechanics according to the first part of the paper:

> 'We are left with the following choice, whether or not we include decoherence: either the composite system is not described by such a sum [superposition], because the Schrödinger equation actually breaks down and needs to be modified, or it is described by such a sum, but then we need to understand what that means, and this requires giving an appropriate interpretation of quantum mechanics. Thus, decoherence as such does not provide a solution to the measurement problem, at least not unless it is combined with an appropriate interpretation of the theory' (Bacciagaluppi 2012, Section 2.1).

Therefore, the programme of decoherence on its own does not solve the measurement problem, despite of its self-image—'Unfortunately, naive claims of the kind that decoherence gives a complete answer to the measurement problem are still somewhat part of the "folklore" of decoherence, and deservedly attract the wrath of physicists […] and philosophers […] alike' (ibid.)—, and it is hence a matter of principle that classical mechanics does not reduce to quantum mechanics according to the presented definition: the insolubility of the measurement problem depends on questions concerning the interpretation of quantum mechanics, but demonstrates above all that the Schrödinger equation cannot be applied to macroscopic objects, which is the essential requirement for a reduction according to Definition 2 of classical to quantum mechanics.

To sum up, a reduction of classical mechanics to quantum mechanics in the sense of the definition given in this paper's first part based on the explanation of phenomena seems to be out of question, even though this is not irrevocably proven and may depend on the notorious and still open questions of interpreting quantum mechanics. These questions concern especially the relation between classical and quantum mechanics. For that matter, the complementarity conception of the *Copenhagen interpretation* could in some sense be regarded as a special case of the concept of neighbourhood of theories as it is presented here.

Be that as it may, according to the discussion above it seems to be appropriate to call the relation between classical and quantum mechanics a relation of neighbourhood (given that the conditions of Definition 1 are fulfilled, what is not self-evident, cf. e.g. Scheibe 1999, 163–250, for a positive answer in structuralist terms) and not of reduction. Therefore, this differentiation seems to be adequate in this case too.

## 2.3 Phenomenological Thermodynamics and Statistical Mechanics

The last part of this paper analyses the example of the relation between phenomenological thermodynamics and statistical mechanics. While the discussion of the last two examples has shown that it is a common practice in physics to explain physical phenomena by a mixture of theories, the two theories of this last example are intertwined to *statistical thermodynamics* from the outset and shall be separated here for the sake of argument: thermodynamics on the one hand deals with macroscopic quantities of fluids or gases, and describes phenomena as heat transfer or phase transitions, whereas statistical mechanics deals with gases under the additional assumption that they are composed of molecules and tries to explain the same phenomena by means of mechanical laws on the basis of this assumption. In doing so, statistical mechanics copies thermodynamic concepts and reformulates them within its terminology. For example, the phenomenological concept of temperature is defined via equilibrium states and presupposes the second law of thermodynamics, whereas statistical mechanics introduces an analogue concept defined as the mean kinetic energy of the molecules of the amount of the respective gas.

This definition of temperature is just Nagel’s paradigmatic example of a bridge law, but also Feyerabend’s paradigmatic example of his attack to the concept of bridge laws. Feyerabend’s point is quite easy: these two theories contradict each other, because the second law of thermodynamics is not valid in statistical mechanics due to its statistical nature. More precisely, the two theories are contradicting with regard to the second law, because they make different claims about the transfer of heat from a body of lower temperature to a body of higher temperature, which is impossible according to thermodynamics but which can happen as the result of statistical fluctuations according to statistical mechanics (consider the scenario of Maxwell’s demon). But the thermodynamic definition of temperature rests just on the second law, for which reason the relation between the two concepts of temperature *cannot be an identification* (cf. Feyerabend 1962, 78). As Sklar (1999) puts it:

> ‘The regularities in which it [temperature in its statistical reformulation] appears will now be statistical regularities, and the association of these new laws with the traditional laws of thermodynamics will be less immediate than any simple identification of the latter with the former. Heat, for example, most certainly can, and does, flow from a colder to a warmer body in an isolated system in this new probabilistic framework’ (194–195).

Hence, we have to do with difficult correlations of two independent concepts and not with the deduction of thermodynamics from statistical mechanics by means of identifying bridge laws. These correlations are once more a solely comparison which does not make any theory redundant and which without explanation of phenomena can only establish the neighbourhood of two different theories as has been shown in the first part of this paper.

Another argument concerns the concept of entropy. On the one hand, we have its definition by Clausius as a quantity describing macroscopic heat transfers, on the other hand there is the statistical entropy defined by Boltzmann, which describes the number of the possible microscopic configurations of the molecules of the amount of gas in question. The relation between these two concepts is much more complicated than that between the two concepts of temperature and is not in any account an identification. In physics textbooks the Boltzmann entropy is introduced as the definition of the Clausius entropy, but this ignores the serious problems in establishing an appropriate relation between these two different concepts, which are defined in theories with contradicting claims about the phenomenon of heat transfer. The first problem is that there is '[…] a wide variety of "entropies" to correlate with the thermodynamic concept, each functioning well for the specific purposes for which it was introduced' (Sklar 1993, 354). Furthermore

> '[…] the choice of the appropriate statistical mechanical correlate of this kind for thermodynamic entropy is fixed by a multitude of considerations, including the additivity of entropy for independent systems in thermodynamics and, most importantly, the demand that the function be maximized for the equilibrium configuration of the microcomponents and that this configuration obey some kind of stationarity under the kinetic equation' (355).

Thus, the discussion of the concepts of temperature and entropy shows that phenomenological thermodynamics and statistical mechanics are two contradicting theories using different concepts and that all direct relations between these theories are not straightforward, but difficult correlations and by no means simple identifications. Therefore, these relations do not establish a direct reduction by means of a logical deduction but can only show the neighbourhood of these two theories. The neighbourhood of these theories, which are intertwined to statistical thermodynamics anyway, can again be demonstrated e.g. within the structuralist metatheory (cf. Scheibe 1999, 129–158), but to establish a reduction according to Definition 2 of the first part it is necessary to find (as an indirect relation) statistical mechanical explanations of thermodynamic phenomena. This problem again is a purely hypothetical speculation, since statistical thermodynamics is a well established theory, but it will be considered here for the sake of argument.

While it is quite easy to give a statistical reconstruction of the ideal gas law, there are many thermodynamic phenomena without a mechanical description, even if the heuristic help of thermodynamics via copying its concepts is considered, which would be legitimate if the resulting explanations are finally independent of thermodynamics (cf. the beginning of the second part of this paper about functional reductions and the distinction between heuristic and systematic support). On the contrary, as Callender (2001) shows, copying thermodynamic concepts in statistical mechanics proves to be obstructive to mechanical explanations (see below). But with heuristic help or not, there are no independent statistical explanations of central phenomena as the approach to equilibrium states, phase transitions, the universality of critical phenomena and, not least, the macroscopic phenomenon of irreversibility and increasing entropy. These phenomena and the problem of their explanation by statistical mechanics will now be discussed briefly.

Firstly, in phenomenological thermodynamics the description of phase transitions is based on the so-called thermodynamic potential as e.g. the potential of free energy, which has singularities at the critical temperatures of phase transitions. Therefore, in phenomenological thermodynamics phase transitions are defined as the singularities of the corresponding thermodynamic potentials. Statistical mechanics copies this conception by redefining thermodynamic potentials as partition functions of the possible micro-states of the system. But the problem is that partition functions of finite systems do not have singularities: 'SM [statistical mechanics] represents the abrupt phase-changes of a system as singularities of its partition function. But no partition function of a finite system can have these singularities; only infinite systems can' (Yi 2003, 1032). The common solution of this problem of explaining phase transitions in statistical mechanics is to take the 'thermodynamic limit', which means to assume that the system in question consists of an infinite number of particles: 'Mathematical physics avoids this result by taking the thermodynamic limit, for it is possible for systems with infinite *N* to display singular behavior for non-vanishing partition functions' (Callender 2001, 549). No real system consists of infinitely many particles, not even the whole universe does, and there is no such thing as an 'approximately infinite' system:

> 'However, even if the thermodynamic limit can be given a full philosophical justification, that justification cannot turn an infinity to a finite quantity. We can grant that it is often fine to substitute finite *N* with infinite *N* for the purposes of practical physics. *But if the system is really finite N, what we have until we say more is a mathematical proof that it cannot undergo a phase transition*' (550, emphasis added).

Therefore, copying the thermodynamic concept of phase transitions as singularities in statistical mechanics does not allow for their explanation. Of course, a numerical simulation of a finite number of particles might deliver an appropriate mechanical explanation of phase transitions. But this is not only highly improbable due to the complexity of the equations, which would have to deal with a really huge number of particles: 'The equations for actual systems are too difficult to solve. Indeed, this is the very reason why statistical mechanics uses singularities in the partition function as a way of studying phase transitions' (551, consider also the huge number of the Avogadro constant, which is defined as the number of particles in a single mole of a substance). Also the fundamental problem of interpreting numerical values has to be considered as in the cases of general relativity and quantum mechanics: in the case of simulating phase transitions, these values would have to be physically interpreted as phase transitions, what apparently would have to rely on criteria taken from phenomenological thermodynamics. However, the typical textbook explanation of phase transitions in statistical mechanics by means of partition functions can at best considered to be a special comparing relation to phenomenological thermodynamics under the fictional assumption of infinite systems and hence might establish a special kind of neighbouring relation between these two theories (similar to the fictional assumption of varying Planck's constant in order to establish a direct relation between classical and quantum mechanics), but it does not provide an appropriate explanation of phase transitions by statistical mechanics. This explanation is an explanation by *statistical thermodynamics*, which is a mixture of both theories and which contains substantial parts of phenomenological thermodynamics. For this reason, this explanation does not give rise to an eliminative reduction of thermodynamics to statistical mechanics. As in the other two cases presented in this paper, the working physicist typically uses mixtures of theories and should therefore be very cautious about reductionist claims.

Another example in this context is the universality of critical phenomena, which, according to Batterman (2002), can neither be explained by phenomenological thermodynamics nor by statistical mechanics alone but *needs the systematic help of both conceptions* within statistical thermodynamics: '[…] we cannot understand the universality of critical phenomena in fluids […] without asymptotically sewing statistical aspects of the behavior of the fluids' components onto singular thermodynamic structures (critical points). *These thermodynamic structures are necessary for a complete understanding of the emergent critical phenomena of interest*' (127, emphasis added). According to Batterman, an explanation of the universality of critical phenomena is particularly only possible with the systematic support of thermodynamics, which exceeds purely heuristic considerations. Therefore, phenomenological thermodynamics is not eliminatively reduced to statistical mechanics as Newton's theory of gravitation is not eliminatively reduced to general relativity and as classical mechanics is not eliminatively reduced to quantum mechanics: in all these cases the latter theory is not able to explain many phenomena without the systematic help of the—independent and not logically implied—first theory, and, as the problem of the universality of critical phenomena shows once more, not for pragmatic reasons, but as a matter of principle.

Furthermore, in the case of phenomenological thermodynamics and statistical mechanics, there is a problem even worse than that of the universality of critical phenomena: there is no statistical explanation of the macroscopic phenomenon of irreversibility and increasing entropy. Actually, a statistical explanation of the second law of thermodynamics seems to be impossible. Physics textbooks tell us that a systems state transforms to a more probable state, but they leave open the question, why the actual state of our world, according to this explanation, should have a low probability. The statistical explanation of increasing entropy leads to the question, why at present entropy is low and was even lower in the past, what is, according to the cited textbook explanation, highly improbable. Boltzmann assumed that we live at a time and at a place of the developing universe, which has low entropy due to a fluctuation. But modern cosmology shows that this explanation fails. Currently, physicists are studying cosmological arguments, which show that the entropy was—against probability—low at the beginning of the universe, or arguments from a certain interpretation of quantum mechanics, the *GRW-interpretation*, which prove the irreversibility of thermodynamic processes (cf. e.g. Hellman 1999). But an explanation of the macroscopic irreversibility by statistical mechanics alone seems to be impossible. Therefore, this is a phenomenon, which is explained by thermodynamics (by its second law with its non-statistical Clausius definition of entropy—*the problem of explaining increasing entropy is solely a problem of the reductionist approach to thermodynamics*) but non-accessible to statistical mechanics alone. If a statistic-mechanical explanation were possible, it would have to be amended by some cosmological or quantum theories, what seems not to be adequate for an explanation of simple thermodynamic phenomena: '[…] do I really have to go back to the big bang and some special kind of primordial low entropy cosmic state; or, alternatively, must I descend to random fluctuations in the evolving quantum wave function [according to the GRW-interpretation], in order to explain why my popsicle melted?!' (l.c., p. 209). Even if the reductionist claims that in the case of irreversibility a reduction of phenomenological thermodynamics to statistical mechanics *plus* cosmology or quantum mechanics would be possible, then she also has to show, how phase transitions and the universality of critical phenomena can be explained by this conglomerate. Again, it is not possible to prove that such explanations are fundamentally impossible, but the burden of proof is definitely transferred to the reductionist's position.

All in all, since the first part shows that there is no eliminative reduction without the explanation of all the corresponding phenomena, a reduction of thermodynamics to statistical mechanics according to Definition 2 is not yet achieved and seems to be impossible. This leads to a further example of neighbourhood without reduction.

## 3 Conclusion

Altogether, the investigation of the three examples in the second part shows that, according to the concepts introduced in the first part, modern physical theories are rather neighbouring each other than being reduced to more fundamental theories. There are only some historical examples of reductions according to Definition 2, which make reduced theories redundant. This is the case of the examples of Galileo's law of falling bodies, Kepler's laws of planetary motion, the Newtonian description of the orbits of the planets or the ideal gas law. In contrast, reductionist claims, which state that the modern physical theories considered here can be reduced eliminatively, are nothing but speculations about the plausibility of the fundamental possibility of explanations that may exist but have not yet been found. In physics, we are on a good way to demonstrate the neighbourhood of modern physical theories, what admittedly is critical in the case of classical and quantum mechanics though it would satisfy the common belief in physics. But at the moment, we are far away from an 'authentic' reduction of all physics to physics of a fundamental level. For that matter, all we can do is speculate. Yet I won't do that here, because—aside from concerns about the incitement of scientific development—there is nothing bad about the pluralistic picture, according to which a complete description of the physical world requires many neighbouring but independent theories, while only few of them have been reduced in the progress of science. In addition, pluralism needs not to imply disunity, because the neighbouring relation provides a connection between different theories, which is indeed weaker than an eliminative reduction would be, but which nonetheless may be strong enough to guarantee the unity of physics. And as it was mentioned above, physicists are used to explain phenomena by mixtures of theories anyway.

In summary, this paper demonstrates some limits of reductionism in physics (or at least indicates some arguments for its non-orthodox claims, cf. Gutschmidt 2009 for a full elaboration) and proposes at the same time, by means of the differentiation between neighbourhood and eliminative reduction, a new approach to the intertheoretic relations in physics.

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