Paradoxes of the Infinite Rest on Conceptual Confusion[[1]](#endnote-1)

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The purpose of this paper is to dissolve paradoxes of the infinite by correctly identifying the infinite natural numbers. Let us begin with the question: Which objects are the *infinite natural numbers*? I use italics to indicate that we are searching for objects to link up to the concept *infinite natural number*. Another way to think of the question is: How should numbers like 3, 17, and 105 (that is, the finite natural numbers) be extended into the infinite?

Here is an idea: the infinite natural numbers are the *infinite natural numbers* (though someday this point will be seen as banal, in our modern Cantorian age it is likely to be misunderstood). And where do we find infinite natural numbers? In a nonstandard model of the reals. So for example, Abraham Robinson (1996, p. 50) writes, “Thus any finite natural number is less than any infinite natural number [henceforth ‘fnn’ and ‘inn’, respectively].” Let us agree that the inns are the *inns*.

 Then note that nowhere in the world do we encounter the infinite. Thus the finite must be our guide to the infinite. Some conclusions follow. First, the inns behave very much like the fnns, which provides strong evidence that the inns are the *inns*. Second (and staying within the context of whole, natural numbers) any finite answer to the question -- How many? -- must be a fnn. So if we take seriously the idea that the finite should guide our thinking in the infinite realm, it follows that any infinite answer to the question -- How many? -- must be an inn. Once we recognize that the inns are the *inns* and that the finite must guide our thinking in the infinite realm, many (perhaps all) paradoxes of the infinite are dissolved. (Some of these ideas pertaining to the infinite numbers are explored in greater detail in Gwiazda (2012a) and (2012b).)

Let us consider three examples. James Thomson (1954) presented Thomson’s lamp, where a lamp begins on, and the button is pressed to toggle the lamp on and off at times 1/2, 3/4, 7/8, and so on…. The structure of the button presses is the structure of ω, that is, the structure of the positive integers. There is a first, a second, a third, a fourth button press, and so on. The question arises (Thompson tried to create a problem by asking): What is the state of the lamp after these infinitely many button presses? I think the correct response is to say, “That’s not an infinite number of button presses. If you are talking about infinitely many, you have to be talking about an inn.” Then note that any inn is even or odd. So if the button was pressed an even infinite number of times, then the lamp is in it starting state; if the button is pressed an odd infinite number of times, then the lamp in its opposite its starting state. When you actually talk about inns, not only is there no paradox, there is not even any confusion regarding the final state of the lamp. (This topic of infinite tasks is addressed in Gwiazda (2013b).)

Another paradoxical example is a Zeno sphere, which is an object that has a spherical shell of radius 1/2, and a shell of radius 3/4, a shell of radius 7/8 , and so on…. These shells go on approaching, but never reaching, a radius of 1. Questions arise: What happens when two Zeno spheres collide? What happens when you shine a light on a Zeno sphere? I do not believe that there are good answers to these questions. I also think the correct response is to say, “The Zeno sphere does not have an infinite number of shells. If you are talking about infinitely many, you have to be talking about an inn. And any Zeno sphere that has an inn of shells will have an outer shell, so there is no problem with collisions and light reflecting.” Again, no paradox or puzzle remains once we correctly identify the inns.

As a final example, consider the spaceship paradox. A spaceship travels one mile in an half an hour, another mile in a quarter of an hour, a mile in an eighth of an hour, a mile in a sixteenth of an hour, and so on…. The question is then: After these infinitely many trips, where is the spaceship? There is no good answer. But the correct response is to say “This is not an example of infinitely many trips. To talk about an infinite number of trips requires an inn. And if a spaceship makes an inn of trips of 1 mile, call the inn M, then the spaceship is M miles away.” Recognizing the correct conception of infinite number dissolves a number of, or perhaps all, paradoxes of the infinite.

 But why, it might be asked, can’t we ask about spheres with shells of structure ω colliding – that is, why can’t we investigate Zeno spheres? Or a supertask? Or the spaceship paradox as traditionally conceived? Because using ω as an infinite number is much like asking about what state a lamp is in after 3+2i button presses. You simply are not talking about natural numbers. And natural numbers are required in response to questions of how many. But, the objector may continue, ω at least seems to be like a natural number, unlike 3+2i. The trouble with the objector’s point is twofold. First, ω is not an inn. Second, ω does not exist in any actual determined sense. It is merely a potential infinity. See Gwiazda (2013a) and (2013b) for arguments to this conclusion. Zeno spheres are logically impossible; there can be no Zeno sphere sitting on a desk. This point will likely prove contentious, and so let me stress the first point. A finite answer (to the question: how many?) requires a fnn, and so too an infinite answer (to the question: how many?) requires an inn. (Also note that nothing hinges on working with whole numbers -- similarly a question asking about infinite distance must be an infinite real number -- again found in a nonstandard model of the reals.)

 I have presented several paradoxes of the infinite, and suggested that they are completely dissolved once we recognize the correct conception of infinite natural numbers (or distances). I suspect that most paradoxes of the infinite rest on this conceptual confusion. Though I have only gone through three paradoxes, the fact is that inns behave very much like fnns. Thus any paradox of the infinite that remained, I claim, would also be a paradox of the finite – and thus not really a paradox of the infinite. Put another way, infinite number would not be responsible for the paradox. Once we recognize the correct conception of infinite number, few paradoxes of the infinite remain. I conclude with a challenge: Present any paradox of the infinite, while referring to the correct conception of infinite number.

References

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Notes

1. This paper is a modified portion of a forthcoming paper “On Multiverses and Infinite Numbers.” [↑](#endnote-ref-1)