Logical Pluralism: Where the Conflict Really Lies

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Abstract

Recent years have seen a surge of attention to the problem of logical pluralism; most of which has been a reaction to Beall and Restall's account of logical pluralism as the existence of more than one equally correct semantic relation of logical consequence. The underlying thesis is that the indeterminacy of the notion of validity goes beyond what the inductive-deductive distinction can precisify. The notion of deductive validity itself is indeterminate as well and this indeterminacy has its roots in the indeterminacy of the more fundamental notion of *case*. *Cases* are what make the premisses and the conclusion of an argument true; the most notable example being Tarskian models for classical logic. Deductive validity is the preservation of truth across all *cases*. This paper argues that unless this account of logical pluralism is supplemented with an argument in favor of the equal legitimacy of the purported *cases* it becomes merely a semi-controversial exposition of how different logics can be generated.

Keywords: Logical Pluralism, Logical Consequence, Validity, Case

1. Introduction

Since the introduction of classical logic, various challenges have been brought up against it; both from those who see it as a correct yet inadequate enterprise and those who consider it to be incorrect in the first place. The original intent of the creators of classical logic was the analysis of mathematical arguments. Mathematicians practice some kind of reasoning while doing mathematics and classical logic is supposed to be a description of that practice. Given the necessary and timeless nature of mathematical truths, classical logic does not incorporate modality and temporality. Also, given its narrow scope and exclusion of non-mathematical arguments, classical logic does not possess the power to analyze non-mathematical arguments either, or even worse, it may not be fit to analyze mathematical arguments in the first place[1].

These criticisms have acted as an incentive for logicians to come up with a plethora of logical systems. It is natural for rivalry to arise in such a context. Is classical logic the one and only correct logic? Or should another logical system take its place? Is there even a constraint to adopt only one correct logic or is it possible for more than one logic to be true? The intuitive response to the last question seems to be that there could only ever be one correct logic. But intuition may not always be the best judge. Beall and Restall (2000) and subsequently Beall and Restall (2001, 2006) argued for a version of logical pluralism that relies on a semantic or model-theoretic interpretation of logical

[1] For more detail see Burgess (2009).

consequence[2]. According to this interpretation, an argument is valid if and only if there is no *case* in which the premisses are true and the conclusion false. The core of Beall and Restall's argument is that 'case' refers to an indeterminate concept; and depending on how a *case* is specified, there can be different validities or logical consequences. So for instance, if *cases* are complete, consistent, and have a non-empty domain then the logical consequence will be classical. In section 2 of this article we discuss their formulation of logical pluralism in detail.

Since its publication, Beall and Restall's proposal has come under attack from various perspectives. There are a family of objections which Caret (2017) calls the *collapse problem*. These objections maintain that Beall and Restall's thesis ultimately collapses into a form of logical monism (Keefe, 2014; Priest, 2001, 2006; Read 2006). There is also the problem of the truth-conditions of logical connectives and meaning variance; which deals with the problem of how the meaning of logical connectives stays the same across different logics while that of logical consequence changes. The list goes on (Beall & Restall, 2000, 2001). However, one area of weakness that we believe has been neglected, even though alluded to at some point, is the equal legitimacy of these purported *cases*. We argue that the ultimate explanation of legitimacy for *cases* is metaphysical.

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[٢] Later Restall (2014) argues for a proof theoretical reading of model theory; hence, making a proof theoretical case for logical pluralism using certain limitations on Gentzen's sequent calculus.

Beall and Restall's approach to the legitimacy of *cases*, roughly speaking, is their presumption of innocence or in this instance the presumption of legitimacy; i.e. *cases* are legitimate unless proven otherwise. As long as a logic can explain what its *cases* are and how they make sentences true it's free to roam. This, however, doesn't seem to line up with the history of the conflict between certain logics. Intuitionistic logic, for instance, was born out of the anti-realist conviction that mathematical objects are mental constructions. On Beall and Restall's account, intuitionists are employing constructive reasoning, as opposed to classical reasoning, for mathematical objects. What they fail to acknowledge is that intuitionistic logic takes constructive reasoning to be the only valid form of reasoning; i.e. constructions are the only legitimate instances of *case*. There is no legitimacy for classical *cases* from an intuitionistic perspective. It takes a bit more than what Beall and Restall offer to convince the intuitionist to take classical logic to be as correct as intuitionistic logic. There seems to be a need to argue for the equal legitimacy of *cases* if one intends to defend Beall and Restall's thesis. Later, we will lay out this problem in more detail.

2. Logical Pluralism

Logical pluralism has had its own proponents prior to Beall and Restall (2000). The most notable one is perhaps Carnap (1937); who defends a form of logical pluralism via linguistic pluralism or *the principle of tolerance*. Beall and Restall's account, however, is what Priest calls 'the most sustained defense of [logical] pluralism'

(Priest, 2006, p. 200). The significance of their account may be due to the fact that it tries to remain loyal to our basic intuitions regarding meta/logical concepts, yet make a case for pluralism. These basic intuitions involve two theses: The meaning invariance of logical constants and the common concept of logical consequence.

According to the meaning invariance thesis, the meaning of logical constants across different systems does not change. There is only one negation, conjunction, disjunction, etc. Carnap's pluralism negates the meaning invariance thesis. Corresponding logical constants in different logics, on his account, are merely homonymous. The disjunction in classical logic and intuitionistic logic merely look alike and sound alike but they have different meanings. This is a fairly important issue in the literature on pluralism. The implication of this for logical pluralism is that when different logics disagree they are in fact merely talking past each other and there is no substantial disagreement in play. When discussing the disagreement between consistent and paraconsistent logics regarding *the principle of explosion* Quine writes,

> My view of this dialogue is that neither party knows what he is talking about. They think they are talking about negation, '~', 'not'; but surely the notation seized to be recognizable as negation when they took to regarding some conjunctions of the form '*p*. ~*p*' as true, and stopped regarding

> such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject (Quine, 1970, p. 81).

On this view, there is no one single argument that is being disagreed upon. The opposing parties are talking past each other; they are talking about two different arguments. There is a classical conjunction and there is a paraconsistent conjunction; even though their homonymy gives rise to the illusion that they are talking about the same argument.

What Beall and Restall claim to have accomplished is that the meaning invariance thesis stands. The intuitionistic, relevantistic, and classical logician all talk about the same conjunction, negation, disjunction, etc. The hypothetical argument in question which is being disagreed upon by all parties is one and the same. Nevertheless, they disagree upon its validity. With regards to negation they write,

> $\sim A$ is true in x iff A is not true in x. Call this the classical negation clause. There are many good reasons for using a classical negation clause in constructing an account of truth in *cases*. The most obvious reason is the way we use negation, and the conditions under which negations are, in fact, true: $\sim A$ is true just when A is not true. This, one might say, is simply what 'not' means (B&R, 2000, p. 481).

The meaning of logical constants stays the same. The variable that generates different validities are *cases*. This seems to be congruent with our observation of the disagreement between different logics. There is a substantial disagreement and not what Quine calls a 'change of subject'. To what degree have they managed to establish this thesis falls beyond the scope of this article. For the sake of argument, let's assume they've successfully managed to establish the meaning invariance thesis. At the very least, it's what they claim to have done while still being able to preserve some form of pluralism; and that's what makes their formulation controversial and worth the attention it has gotten so far[3].

The second intuition is what Tarski calls the common concept of logical consequence (Tarski, 1983, p. 409). Tarski claims that our informal understanding of the notion of logical consequence plays a crucial role in its formal characterization. His characterization is what has come to be known as the model-theoretic understanding of logical consequence. This notion is characterized by the lack of any counterexamples. For any set Σ of premisses and φ a conclusion, $\Sigma \vDash \varphi$ (φ is a logical consequence of Σ) if and only if it's not possible for Σ to be true (every sentence in Σ to be true) and φ false.

Moreover, according to Tarski's account of logical consequence, there are also three more integral features that a

[3] Restall (2002, 2014) reaffirm the meaning invariance thesis.

relation needs to possess in order for it to be a relation of logical consequence: modality, formality, apriority[4]. The modal element is represented by the use of the term 'possible'. Not only is it not the case that Σ is true and φ false, but it's impossible for Σ to be true and φ false. Secondly, the logical consequence relation is a formal relation. It is the logical forms of the sentences of Σ and φ that determine whether the relation obtains or not. Thirdly, it's a priori. Our knowledge of $\Sigma \vDash \varphi$ is a priori and cannot be affected by empirical knowledge.

Beall and Restall base their formulation of logical pluralism on this understanding of logical consequence. They formulate the model-theoretic understanding with what they call the Generalized Tarski Thesis,

(GTT) $\Sigma \vDash \varphi$ iff in any *case* in which Σ is true is also a *case* in which φ is true.

This thesis is meant to capture the Tarskian idea that there should not exist any counterexamples. As longs as a relation conforms to (GTT) and is necessary, formal, and a priori it can be called a relation of logical consequence. This is the core of the model-theoretic account of logical consequence.

There is a peculiar aspect to (GTT) that makes it very interesting. Despite it being able to capture the core of the model-

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[4] Beall and Restall (2000) and cook (2010) do not include the epistemic element. Later in Beall and Restall (2006) normativity is added as the epistemic element. McKeon (2010) takes apriority to be the epistemic element.

theoretic notion, it contains an indeterminate concept; that of a *case*. Beall and Restall maintain that different logics specify *cases* differently; through which, different validities can be generated. There is no such thing as an absolute validity. Validity exists only relative to a specific logic. So to formulate (GTT) precisely,

(GTT_X) $\Sigma \vDash_X \varphi$ iff in any $case_x$ in which Σ is true is also a $case_x$ in which φ is true.

The argument from Σ to φ is $valid_x$ if and only if any $case_x$ in which Σ is true is also a $case_x$ in which φ is true. So it seems their pluralism rests upon different specifications of *cases*. In their own words,

> A logic is given by a specification of the *cases* to appear in (V)[5]. Such a specification of *cases* can be seen as a way of spelling out truth conditions of the claims expressible in the language in question (B&R, 2000, p. 477).

To make all of this clearer let's use Quine's example of paraconsistency,

(EFQ) For every φ, $A \wedge \sim A \vDash \varphi$

The classical logician accepts (EFQ) while the paraconsistent logician rejects it. The Quinean analysis suggests

[5] The (GTT) was named the (V) thesis in Beall and Restall (2000). Later in Beall and Restall (2006) they renamed it to (GTT); which is what it has come to be known as ever since.

that there are actually two different (EFQ)s that the two parties are talking about,

$(\text{EFQ})_c$ For every φ, $A \wedge_c \sim_c A \vDash \varphi$

$(\text{EFQ})_p$ For every φ, $A \wedge_p \sim_p A \vDash \varphi$

It is obvious that $(\text{EFQ})_c$ and $(\text{EFQ})_p$ do not represent one and the same argument. So unless the two parties determine what conjunction and negation really mean the problem remains. Once they succeed in doing so the problem is dissolved.

On Beall and Restall's analysis the conjunction and negation are the same in both arguments. It's the validity that varies,

$(\text{EFQ})^*_c$ For every φ, $A \wedge \sim A \vDash_c \varphi$

$(\text{EFQ})^*_p$ For every φ, $A \wedge \sim A \vDash_p \varphi$

And what makes these validities different is that the classical logician specifies *cases* as consistent, but the paraconsistent logician takes them to be inconsistent. $(\text{EFQ})^*_c$ says 'For every φ, in any consistent *case* that $A \wedge \sim A$ is true, φ is true too.'; while $(\text{EFQ})^*_p$ says 'For every φ, in any inconsistent *case* that $A \wedge \sim A$ is true, φ is true too.' There is no further question about the absolute validity of (EFQ). Classical validity and paraconsistent validity have equal rights to be deemed a relation of logical consequence.

In their seminal paper, Beall and Restall discuss four different *cases*: possible worlds, Tarskian models, situations, and constructions. Now, all four of these presumably have equal rights to act as *cases*, but not all of them retain the core features of the common concept of logical consequence. If *cases* are to be taken as possible worlds logical consequence will lose its formality. To cite their own example, the argument *a is red* ⊨ *a is colored* is valid if *cases* are possible worlds; for the very simple reason that in every possible world in which 'a is red' is true, 'a is colored' is true too. However, its validity does not hold in virtue of its logical form. The logical form of the argument is $Ra \vDash Ca$[6]; which is not a valid form of argument.

The other three, however, do leave the core features intact. Beall and Restall (2000) and subsequently Beall and Restall (2006) cover this issue extensively; Tarskian models for classical logic, situation semantics for relevantistic logic, and Kripke semantics for intuitionistic logic[7].

Situations are like bits or fragments of the world. Unlike Tarskian models, situations are incomplete. Thus, $A \vDash B \vee \sim B$ fails. It is possible for A to be true in situation s and for $B \vee \sim B$ not to be true in situation s. Furthermore, situations can be inconsistent as well; i.e., (EFQ) fails. For the argument $A \wedge \sim A \vDash B$, it is possible for $A \wedge \sim A$ to be true in situation s and for B not to be true in situation s.

In intuitionistic logic, constructions are incomplete as well. The Kripke semantics for intuitionistic logic is meant to

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[6] Rx: 'x is red' and Cx: 'x is colored'.

[7] A possible-world semantics for intuitionistic logic. For more details, see Kripke (1965).

model these constructions to imitate truth-conditional semantics. Constructions are about provability. In the argument $A \vDash B \vee \sim B$, it is possible for A to be true in construction c (provable) and for $B \vee \sim B$ not to be true in construction c (not provable). Both relevantistic and intuitionistic logics are paracomplete; i.e. they are not complete in the sense explained above.

To sum it up, there are two steps to successfully establish the kind of pluralism Beall and Restall are trying to defend,

(1) Different equally legitimate logics can be generated by plugging different *cases* into (GTT_X); provided the generated logical consequence relation retains the three core features of logical consequence: necessity, formality, apriority.

(2) The *cases* that generate the equally legitimate logics in (1) are themselves equally legitimate.

Beall and Restall argue for (1) in great detail. But (2) is not seriously addressed. It is alluded to in a sense when they're trying to respond to critics who blame their formulation for dissolving the issue of real disagreement between different logic. They say,

> Perhaps a more telling illustration arises within our own pluralistic ranks, and in particular on the issue of dialetheism, according to which contradictions may be true. Dialetheists maintain that there are arguments of the form $A \vee B, \sim A \vDash B$ which are not only invalid but which have

> true premises and an untrue conclusion. Now, while both of us agree that the given argument is invalid-there are *cases* in which the premises are true and the conclusion untrue (viz., inconsistent situations)-we disagree with each other on the issue of whether the actual world is a *case* in which the premises are true. One of us (JC) endorses dialetheism; the other (Greg) does not. Still, despite this disagreement within our own ranks neither of us has transgressed our pluralist commitments. The point of disagreement is a genuine one; however, it is an issue on which pluralism is neutral (B&R, 2000, pp. 488-489).

This paragraph is very interesting. It seems that Beall and Restall have no intention of defending (2) at all. Their pluralism is neutral with respect to it; i.e. it has nothing to offer with respect to the disagreement between dialetheists and non-dialetheists. But isn't that what logical pluralism was supposed to accomplish in the first place? Moreover, they see this disagreement as a genuine disagreement about how the world is. What's interesting about this is that their pluralism is not an argument for pluralism at all[8].

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[8] Perhaps this is why Beall (2018) claims to be both a logical pluralist and a logical monist! The one true logic is FDE (First-Degree Entailment); logics that are neither complete nor consistent. These different logics discussed in their pluralism are merely the result of applying certain restrictions to the one true logical consequence of FDE.

It's more of an exposition of the story behind different forms of validity rather than an argument for their equal status. Even though (1) is a necessary condition for defending pluralism, it is not sufficient. For pluralism to work (2) must be established as well; one has to demonstrate that the different *cases* that are being plugged into (GTT_X) stand on equal ground. We will pick up on these issues later. Before delving deeper into the problem with Beall and Restall's pluralism, it's imperative that we go through the history of logic to see where the conflict between different logics actually lies.

3. The real conflict

To shed light on the historical conflict between rival logics, here we discuss two different logics: intuitionistic and Free logics.

Mathematical intuitionism is the view that takes mathematics to be essentially about the mental constructions of the human mind. Intuitionists initially denigrated the role of language in mathematics to a mere medium through which mathematical constructions can be communicated. According to intuitionism, mathematics is primarily about mental activities performed by the human intellect. The epistemological *terminus a quo* of intuitionism is the notion of *the move of time*, which can be traced back to Kant's notion of time as pure a priori intuition. The founder of intuitionism, Brouwer (1948), started off from this point and began his phenomenological analysis of the nature of natural numbers and how they are constructed, which is often referred to as *the first act of intuitionism*. Then he proceeded to develop the rest of mathematics based on this basic intuition. All mathematical objects are created by the human mind through the first act, which would make these objects totally mind-dependent; in other words, they are mental objects. So intuitionists reject the existence of abstract mathematical objects which are mind-independent, non-spatiotemporal objects. Put differently, intuitionists are anti-realists in ontology. For Brouwer 'a mathematical statement is true only when a corresponding construction has been made' (Schlimm, 2005, p. 174).

The phenomenological analysis Brouwer makes rests on the premise that the human mind is capable of perceiving the continuity of time introspectively. In perceiving a single moment of this continuity, the human mind can also perceive this single moment fall into two separate moments, with one succeeding the other. This is what he calls the *two-onesness*. These moments are distinct, but at the same time they're continuous. This is how the mind constructs the numbers one and two. By repeating this process all natural numbers can be constructed. According to Brouwer, 'intuition is the abstract form of any perception of change' (Schlimm, 2005, p. 173). And this is how the intuition of time rips of every moment of its qualities and yields us the *bare two-oneness* or the *pure form*. This method constitutes the grounds for constructing the rest of mathematics, and that's why the temporal intuition is referred to as the *basal intuition* of mathematics.

There are a couple of points about Brouwer's intuitionism that need to be emphasized. First, from Brouwer's point of view, the construction process fully captures the essence of mathematics. Language is not an essential part of mathematics since it is not involved in this process at all. If it wasn't for its intermediary role, mathematical language would've already been dispensed with. Thus no attempt was made to come up with a semantical theory. Second, Brouwer reduced existence to constructability. In other words, for a certain mathematical object to exist it only needs to be constructed.

Even though for Brouwer language may not have been an essential part of mathematics, that doesn't mean it is impossible for the intuitionist to somehow accommodate language. Inessentiality is not a good reason to ignore the role language plays in mathematics. The intuitionist can be loyal to the distinction Brouwer makes between mathematics and mathematical language and still underscore the role language plays in mathematics.

Heyting (1956) made the first attempt to devise a formal semantics for intuitionistic logic. He was fully aware of the fact that the objective truth-conditional semantics of classical logic fails to capture the metaphysical anti-realism of intuitionism. So he proposed a replacement delineated in terms of proof-conditions, rather than truth-conditions. Despite his attempt, he shared the animosity Brouwer harbored towards the role of language and logic in mathematics.

Later, a major systematic linguistic turn in intuitionism was made by Michael Dummett. Dummett (1977, 1978) managed to accomplish an important task, he developed a semantical theory not only for mathematical language, but also for the rest of language. A semantical theory consistent with intuitionism. In his own words 'What I have done here is to transfer to ordinary propositions what the intuitionists say about mathematical propositions' (Dummett, 2001, p. 247). Dummett's semantical theory stems from Wittgenstein's later views; according to which the *use* of a proposition, rather than its truth-conditions, determines its meaning. Dummett invites us to compare truth with the formal rules of winning and losing in a board-game. There are formal rules for what is called 'win' and 'lose', rules that are defined in terms of the final positions of the participants in the game. These rules may be able to tell apart the winner from the looser, but they ignore an important aspect; namely, 'it is part of the concept of winning a game that a player plays to win' (Dummett, 2001, p. 230). Likewise, we need to take into account that 'it is part of the concept of truth that we aim at making true propositions' (Dummett, 2001, p. 230). Dummett strongly believed truth-conditional semantics failed to fulfill this aspect. Dummett represents the anti-realist neo-verificationist movement which has been a critique of metaphysical realism within the analytic movement.

It seems that underneath all the technicalities of Kripke semantics lies a great deal of metaphysical antirealism that Beall and Restall have not taken into account. Now, how does all this metaphysical jargon pertain to logic? The connection lies within the antirealism held by the intuitionist. To quote Brouwer himself,

> The long belief in the universal validity of the principle of the excluded third in mathematics is considered by intuitionism as a phenomenon of history of civilization of the same kind as the old-time belief in the rationality of π or in the rotation of the firmament on an axis passing through the earth. And Intuitionism tries to explain the long persistence of this dogma by two facts: firstly, the obvious non-contradictority of the principle for an arbitrary single assertion; secondly the practical validity of the whole of classical logic for an extensive group of simple everyday phenomena. The latter fact apparently made such a strong impression that the play of thought that classical logic originally was, became a deep-rooted habit of thought which was considered not only as useful but even as aprioristic (Brouwer, 1948, p. 94).

Dummett, too, writes with the same spirit 'classical mathematics employs forms of reasoning which are not valid on any legitimate way of constructing mathematical statements' (Dummett, 1978, p. 215).

It's fairly clear that intuitionists were vehemently against classical forms of reasoning. Their primary target, as Brouwer points out, is *the law of excluded middle* (LEM). For intuitionists, (LEM) rests upon metaphysical realism about mathematical objects and the mind-independent truth-value of mathematical sentences. The rejection of one horn does not imply the acceptance of the other. There is no independent mathematical realm out there in virtue of which this is guaranteed. (LEM) is true if and only if one of the horns can be proven; which in intuitionistic terminology equates with mental construction. Dummett simply generalizes this phenomenon and expands it include to non-mathematical domains.

Given all the metaphysical underpinnings of intuitionism, does it sound reasonable to simply assume both complete and paracomplete *cases* are equally justified? For the intuitionist, paracompleteness is rooted in his anti-realist metaphysics which he deems to be superior to realism. Yet, as we saw in the previous section, Beall and Restall portray this significant gap as a mere choice between two innocent options.

Let's take this even further by discussing another logic. The *cases* of classical logic, i.e. Tarskian models, are complete and consistent. The domain of objects is also non-empty. Free logic was devised by Lambert (1960) to get rid of the ontological assumptions of classical logic. Lambert sees his project as an extension of the eradication of existential assumptions that classical logic applied to Aristotelian logic. In Aristotelian logic, predicates should have non-empty extensions. Classical logic eradicated this assumption by including empty predicates. Lambert takes it one step further and eradicates the existential assumption of singular terms and the non-emptiness of domains from classical logic.

Free logic does meet the two criteria Beall and Restall put forward for *cases*. Their *cases* are similar to Tarskian models, except for the fact that they include empty domains, and they are very good at assigning truth-values to their sentences. The truth-assignment may change in different semantics, but one principle stays the same: it is possible for an empty domain to exist.

Are *cases* that include empty domains superior to those that don't? Proponents of Free logics would say yes. The ability of models that include empty domain far outweighs that of its opponents. It is superior in that it removes unnecessary, and even problematic, existential assumptions. It's seen as an advancement over classical logic as much as classical logic is an advancement over Aristotelian logic.

The historical account of the conflict between classical logic and the two rivals mentioned above suggests that the conflict between logics really lies at the level of *cases*; which Beall and Restall seem to be relatively liberal about. The intuitionist maintains that mental constructions are the only legitimate instances of *case*. And they are very adamant in their rejection of classical *cases* which assume realism about mathematical objects. Similarly, the proponents of Free logic maintain that the only legitimate *cases* are those that include empty domains. *Cases* that restrict their domains to non-empty ones are illegitimate in their opinion. They see the eradication of the ontological assumptions of classical logic as an advancement, and the *cases* they consider legitimate are superior to classical *cases*.

4. Concluding remarks

As we saw earlier, Beall and Restall's logical pluralism rests upon the notion of *case*. In light of their account, we should seek the real reason behind the conflict between rival logics in their choice of *cases*. As demonstrated in the previous section, historically speaking, there is a metaphysical dimension to the notion of *case*. For the intuitionist, the choice of *case* is based upon their metaphysical antirealism. For the proponents of Free logic, it is based upon the inclusion of empty domains and eradication of the existential assumptions of classical logic. Therefore, the real conflict between rival logics is a metaphysical conflict.

A genuine form of logical pluralism would convince the intuitionist to stay fully loyal to his metaphysical anti-realism, yet be able to incorporate classical reasoning at the same time; to be both fully intuitionistic and fully classical. This requires the full acceptance of both classical *cases* and intuitionistic *cases* at the same time. The same goes for Free logics. To regard both classical and Free logics as equally legitimate, one needs to regard both empty and non-empty *cases* as equally legitimate. Beall and Restall fail to offer any argument for the equal legitimacy of *cases* and thus fail to argue for a genuine form of logical pluralism.

At this point, we hope to have shown that in order for Beall and Restall's logical pluralism to succeed there needs to be an argument for the equal legitimacy of *cases*. And that at least in the case of intuitionistic and Free logics this argument falls within the domain of metaphysics[9]; which in and of itself is worth noting to the logician who prefers to bury his head in the sand of mathematical technicalities and ignore the fact that some of these conflicts originated from a metaphysical dispute.

The proponents of both of the abovementioned parties seem to have good reasons not to consider logical pluralism at all since it may have seemingly bizarre implications. For the intuitionist, metaphysical realism needs to be equally legitimate to metaphysical antirealism. For the Free logician, the existential assumption of the existence of non-empty domains should be rendered moot. These assertions sound highly implausible to the extent that one would just rather stay in his comfort zone and adopt some form of logical monism.

So, for Beall and Restall's logical pluralism to succeed there needs to be an argument in favor of some form of non-monism in ontology[10]. This forces the logical pluralist into the domain of metaontology. Mathematical objects either exist or they don't. The existential assumptions of classical logic are either

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٩ For another example of how one's metaphysics can play a role in his choice of logic see Priest (2014).

١٠ Prominent proponent of this view include Chalmers (2009), Hirsch (2002), and Yablo (1998).

true or they are not. On the level of ontology, questions of existence are dealt with; like the existence of mathematical objects and empty domain. On the level of metaontology, it is discussed that whether there are objective answers to these kinds of questions.

According to what has been discussed so far, a negative answer to the metaontological question of the existence of one single objective answer to ontological questions, would greatly benefit Beall and Restall's account of logical pluralism.

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