Abstract

The famous EPR article of 1935 challenged the completeness of quantum mechanics and spurred decades of theoretical and experimental research into the foundations of quantum theory. A crowning achievement of this research is the demonstration that nature cannot in general consist in noncontextual pre-measurement properties that uniquely determine possible measurement outcomes, through experimental violations of Bell inequalities and Kochen-Specker theorems. In this article, I reconstruct an argument from Niels Bohr's writings that the reality of the Einstein-Planck-de Broglie relations alone implies that no such properties can exist for momentum and position measurements, show how this argument responds to the challenge of EPR on general physical grounds, and advance that this reconstruction shows that and how Bohr's "complementarity" is a view of the objective content and logic of quantum theory.

Bohr on EPR, the Quantum Postulate, Determinism, and Contextuality

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Introduction

Niels Bohr's philosophical thought has variously been characterized as instrumentalist, positivist, subjectivist, anti-realist, and idealist, and sometimes dismissed as either an obviously unacceptable or hopelessly vague and confused specimen thereof.[1] In the past few decades, many have urged that Bohr's views are considerably more subtle and worthy of study than such blanket dismissals suggest, e.g. [Folse, 1989, p. 255], [Faye, 1991, pp. 233ff], and [Katsumori, 2011, xii].[2] Some have also offered competing categorizations of his views within philosophy of science to avert this charge, e.g. [MacKinnon, 1994], [Shomar, 2008], and [Maleeh and Amani, 2013]. This article sits squarely in both trends: it finds Bohr to be a realist of sorts whose thought speaks directly to contemporary issues in the foundations of quantum theory. In particular, while Bohr's language was often inflected with positivist turns of phrase, and while Bohr did periodically advance positivist-friendly claims, I find that he nonetheless offers materials for a valid argument in defense of the completeness of quantum mechanics in response to the famous EPR argument that does not itself rest on any suspect appeal to some untenable or dogmatically restrictive theory of meaning.[3] Rather, the acceptance of the reality of one physical principle that "[expresses] the essence of quantum theory" [Bohr, 1928/1985c, p. 148]—the "quantum postulate" of the reality of the Einstein-Planck-de Broglie relations—disputes the conclusion of the EPR argument on general grounds. This conclusion is that, in the EPR state, there are pre-measurement properties that uniquely determine any possible measurement outcome of momentum and position, which measurements can accordingly be thought to reveal the pre-measurement obtaining of those properties—call this principle "Pre-Existing Properties." This article thus agrees with Henry Folse that "[Bohr's] defense of the completeness of the quantum description is not based on a positivist rejection of all ontology of science, but on physical reasons expressed in the quantum postulate" [Folse, 1989, p. 271] and aims to spell this defense out in more detail.[4]

This reconstruction of Bohr's response to EPR is further notable because this response features remarkably instrumentalist-sounding language that has significantly contributed to the image of Bohr as an instrumentalist. As I exposit in Section 6, however, this language is actually an expression of the general consequence of the Einstein-Planck-de Broglie relations that the only possible truth-apt representation of atomic phenomena vis-à-vis measurements of position and momentum is the fundamentally statistical

[1]Mara Beller and Arthur Fine articulate this dismissal perhaps most influentially when they assert that only the acceptance of an "extreme positivist attitude" that "identif[ies] measurability and meaning" can salvage Bohr's reply to EPR [Beller and Fine, 1994, pp. 12, 14]. Similarly, Fine calls Bohr's response to EPR "virtually textbook neo-positivism" while voicing complaints about positivism as a theory of meaning [Fine, 1986, pp. 8, 34-35] and Richard Healey characterizes Bohr as offering a theory of meaning and complains that it rests on too vague and anthropocentric of foundations [Healey, 2012, pp. 1536, 1553]. See also e.g. [Beller and Fine, 1994, p. 9], [Fine, 1986, pp. 4-5, 29, 31, 34-35], [Beller, 1996, p. 196], [Beller, 1999, p. 152], and [Cushing, 1994, pp. 25, 29].

[2]See also [Feyerabend and Mckay, 1958].

[3]One such claim, following an argument that the quantum of action "sets a limit ... to the *meaning* we may ascribe to [information attainable by measurements]" is that "[w]e meet here in a new light the old truth that in our description of nature the purpose is not to disclose the real essence of the phenomena but only to track down, so far as it is possible, relations between the manifold aspects of our experience" [Bohr, 1929/1985b, p. 296].

[4]While I agree with Folse at this level of description, I disagree with his suggestion that the quantum postulate challenges the principle of Separability per se [Folse, 1989, p. 264]: on my reading, the quantum postulate can only challenge Separability indirectly, in that either non-Separability or the existence of superluminal causality follows from the failure of Pre-Existing Properties (implied by the quantum postulate) and the reality of EPR correlations. See fn 21 and Section 6.

description of transitions from pre-measurement states to measurement states, in the specific context of measurements of members of the EPR state. Accordingly, this article suggests viewing Bohr as more concerned to offer a perspicuous overview of the objective content of quantum theory than any instrumentalist philosophy of science per se.

I show how theorems modelled on the Kochen-Specker theorem as well as theorems premised on Bell locality each challenge the principle of Pre-Existing Properties and contrast Bohr's challenge from those of these theorems in Section 1. I exposit the EPR argument in Section 2 and a first pass of Bohr's response in Section 3. In Section 4, I argue that Bohr's response could only be interpreted as genuinely engaging with EPR if it is directly challenging Pre-Existing Properties for any measurements of position and momentum. In Section 5, I find materials for a valid argument against Pre-Existing Properties for position and momentum premised on the quantum postulate and key to understanding Bohr's pre-EPR conception of the "essential disturbance" of atomic phenomena by measurement. In Section 6, I show how this argument and correlate "disturbance" view make good sense of Bohr's response to EPR and clarify how this response interfaces with the premises of the EPR argument [Bohr, 1928/1985a, p. 216].[5] I close by clarifying that and how Bohr's "complementarity," as it emerges from this reconstruction, is a view of the content and logic of quantum theory premised on the quantum postulate that is worthy of further consideration.

1 No-Go Theorems vis-à-vis Noncontextuality and Determinism

There are a number of no-go theorems that establish the inconsistency of certain predictions of quantum mechanics with certain forms of hidden-variable completions of quantum mechanics. My concern here is to bring out that two influential forms of no-go theorems—that of the Kochen-Specker theorem and reasoning premised on Bell locality—challenge what I call Pre-Existing Properties and to preview my reading of Bohr's reasoning against Pre-Existing Properties by way of contrast.[6] To repeat: Pre-Existing Properties is the principle that measurement outcomes are uniquely determined by properties that obtain immediately prior to measurement.

The Kochen-Specker theorem shows that certain combinations of measurement outcomes of compatible observables predicted by quantum mechanics are inconsistent with (i) the assumption that "all [observables at issue] simultaneously have values [at all times], i.e. are unambiguously mapped onto real

[5]There are three main novelties of my elaboration of Bohr's response. First and primarily, Bohr's protracted concern with experimental arrangements does not reflect an appeal to instrumentalism, *pace* Beller and Fine, but rather reflects physical consequences of the quantum postulate. Second, Bohr's concession of the point that we can predict the position or momentum of the unmeasured particle in an EPR state with certainty is thinner than concession of the cogency of bare property ascription and is to be elaborated counterfactually. Third, the number of distinct physical situations / experimental arrangements at issue in considering the meaning of the EPR wavefunction is at least four.

[6]Thank you to an anonymous reviewer for suggesting that I explicitly situate this reconstruction of Bohr with respect to such no-go theorems.

numbers" [Held, 2022], (ii)(a) the rule that values associated with sums of compatible operators are sums of the values associated with the respective summed operators, and (ii)(b) the rule that values associated with products of compatible operators are products of the values associated with the respective operators.[7] To accept (i) is, in effect, to accept that the system has properties that uniquely determine any possible (observable) measurement outcome at all times. As Carsten Held shows [Held, 2022, pp. 21-24], (ii)(a) and (ii)(b) can be derived from (i) together with the premise that these physical properties of a quantum system are "noncontextual," that is, obtain independently of any measurement context. Kochen and Specker then show that certain predicted combinations of measurement outcomes of components of total angular momentum for some quantum states are incompatible with the rules (ii)(a) and (ii)(b), given the commutation relations quantum theory enforces on the respective operators [Kochen and Specker, 1967, pp. 61-73].[8] My concern is not with any of the details of the proof of this theorem, but simply to note that the sorts of properties it targets are plainly those postulated by Pre-Existing Properties: they are properties the system has on its own that are uniquely mapped to one real number that represents the outcome of a possible measurement. The strategy common to many theorems modelled after this theorem, moreover, is precisely to show that some combination of possible measurement outcomes for some state predicted by some fragment of quantum theory cannot be understood to be revealing self-standing properties of the system of interest that would uniquely determine any such measurement outcome. In other words, the strategy is to show that such measurement outcomes are incompatible with Pre-Existing Properties in the given measurement contexts.[9,10]

Bell's theorem and theorems modelled on it are concerned most directly with whether or not nature can be considered local in a precise mathematical sense given certain combinations of measurement outcomes on spatially separated parts of so many identically prepared physical systems. Specifically, the question is whether correlations in distant measurement outcomes can be explained by some possibly unknown distribution of conditions that obtain in the local neighborhoods of the respective measuring devices from run to run of an experiment. The sought-out sort of explanation is this: conditioned on some obtaining of the relevant difference-making local conditions λ from the set Λ of all possible such total conditions at the two respective measurement sites, for any pair of measurement outcomes a and b at sites 1 and 2 with settings s and t, respectively, the joint probability function of getting a positive result in both wings of the experiment $p_{a,b}(s,t|\lambda)$ can be factorized into the product of two independent probability functions in the two respective wings, $p_{a,b}(s,t|\lambda) = p^1_a(s|\lambda)p^2_b(t|\lambda)$. A given observed probability function $q_{a,b}(s,t)$ is then *locally explicable* if it results from some unknown weighted sum of factorizable joint probability

[7] By "compatible operators" I mean operators representing measurements of compatible observables.

[8] See [Held, 2022, pp. 8-9, 11-19] for a more accessible presentation of their reasoning.

[9] For example, Asher Peres and David Mermin show that the obtaining of such properties for all possible spin measurement outcomes for the singlet state of two electrons at all times is inconsistent with canonical commutation relations on spin angular momentum operators [Peres, 1990], [Mermin, 1990a]. See [Held, 2022] for an overview of such theorems.

[10] There are also attempts to generalize the Kochen-Specker theorem for noncontextual properties that do not uniquely determine measurement outcomes, e.g. [Kunjwal and Spekkens, 2015].

functions $p_{a,b}(s,t|\lambda)$, that is if $q_{a,b}(s,t) = \int_\Lambda f(\lambda)p^1_a(s|\lambda)p^2_b(t|\lambda)d\lambda$ for some $f(\lambda)$, $p^1_a(s|\lambda)$, and $p^2_b(t|\lambda)$. One then derives inequalities – Bell inequalities – that combinations of such observed, locally explicable probability functions must obey. Some of these inequalities are incompatible with predictions of quantum theory for certain combinations of measurements on certain multi-particle states. Such inequalities thus identify experiments that allow one to test both quantum theory and whether the experimental outcomes are locally explicable in the above sense.[11]

While Myrvold et al. rightly point out that "outcome determinism"—the principle that the local conditions in the two respective spatial regions of interest uniquely determine the outcome of any possible measurement taken in each region—is not strictly presupposed by this reasoning [Myrvold et al., 2021, p. 30], this principle is plainly and uncontroversially amongst those challenged by experimental violations of Bell inequalities. For this is just a special case of the factorized probability distributions $p^1_a(s|\lambda)$ and $p^2_b(t|\lambda)$ in which they are exclusively either 1 or 0 for every a, b, s, t, and λ. Theorems premised on Bell locality therefore point to possible experimental outcomes whose obtaining would challenge Pre-Existing Properties.[12] The essence of the Greenberger-Horne-Zeilinger (GHZ) theorem (originally presented as a cousin of Bell theorems) is the demonstration of the incompatibility of the existence of local properties that uniquely determine possible spin measurement outcomes on three different spin-1/2 particles in the GHZ state $|GHZ\rangle = \frac{|\uparrow\uparrow\uparrow\rangle+|\downarrow\downarrow\downarrow\rangle}{\sqrt{2}}$ with certain predicted combinations of measurement outcomes of those quantities given the commutation relations imposed on their corresponding operators in the quantum-mechanical formalism [Mermin, 1990b].[13] The theorem thus plainly presents possible experimental outcomes that would challenge Pre-Existing Properties in the context of those experiments.

My purpose in bringing out these uncontroversial points is to situate Bohr's response to EPR with respect to such no-go theorems. Like these theorems, Bohr gives us reason to reject Pre-Existing Properties. But where these theorems identify possible measurement outcomes compatible with some formal fragment of quantum theory but inexplicable by such properties, Bohr advances that accepting a single physical principle rules out Pre-Existing Properties straightaway for the quantities of position and momentum in any context, with lessons for what the quantum symbolism must represent in the first place and for the logic of that representation. Far from merely offering a hazy positivism, then, Bohr squarely concerned himself with the objective content of quantum theory. Moreover, he did so by bringing out the

[11]See [Bell, 2004] for the inaugural paper and [Myrvold et al., 2021] for an overview of these inequalities, various philosophical disputes concerning the reasoning involved, and an overview of some of the corresponding experimental work.

[12]Outcome determinism appears to be what Guy Blaylock calls "counterfactual definiteness" and advances as a premise of Bell theorems [Blaylock, 2010, p. 115]. This principle is the "assumption ... that we are allowed to postulate a single definite result [that would result from] an individual measurement even when the measurement is not performed." Blaylock does not mean, trivially, that any measurement outcome, were it to obtain, would obtain, but rather that there is only "one possibility for the potential result of a measurement." This is just to advance that some property of the system to be measured uniquely determines what result a corresponding measurement upon that system would give. However, I agree with Myrvold et. al. in holding that outcome determinism holds only for a special case of the sorts of local properties at issue, and so is not a premise from which the inequalities are derived *überhaupt* [Myrvold et al., 2021, p. 30]: the inequalities do not assume that $p^1_a(s|\lambda)$ and $p^2_b(t|\lambda)$ are exclusively either 1 or 0 for every a, b, s, t, and λ, which is precisely what Blaylock calls the principle of counterfactual definiteness and what I call Pre-Existing Properties.

[13]See [Mermin, 1990a] for a consideration of some logical relations between Bell theorems, the GHZ theorem, and theorems of the form of the Kochen-Specker theorem.

consequences of the postulate of the bona fide reality of one foundational principle for quantum theory vis-à-vis position and momentum, one that provides a physical reason deterministic noncontextuality should fail for measurements of these quantities. Accordingly, resources for the task Anton Zeilinger urges have been hiding in plain sight in the writings of one of quantum theory's founders all along, viz. the identification of an uncontroversial and universally-acceptable "foundational conceptual [physical] principle" through which we can conceptualize the content of quantum theory [Zeilinger, 1999, pp. 631-2].

2 EPR

At issue in the EPR article is whether quantum theory is complete in a specific sense. Namely, the issue is whether quantum theory is a "closed theory" of atomic phenomena, where

> [a] closed theory covers a limited or bounded domain of phenomena; it is a mathematically well-defined, consistent and ... perfectly accurate description of those phenomena [Ryckman, 2017, p. 116].

In short: the issue is whether quantum theory fails to have the resources to represent everything within the phenomena it purports to describe.

EPR attempt to establish that quantum theory is incomplete in this sense by arguing that not every "element of reality" can have a counterpart in that theory. Specifically, they argue directly that in certain situations, some particle has both a completely sharp position and completely sharp momentum, whereas quantum theory does not permit the assignment of simultaneously sharp values of position and momentum for any particle whatsoever owing to the commutation relations $[\hat{\mathbf{x}}_j, \hat{\mathbf{p}}_j] = i\hbar$ for any particle j, which formally codify the Heisenberg Uncertainty Relations (HUR) for these quantities.[14]

The direct argument for the existence of simultaneously sharp momentum and position of some particle relies on a certain sufficient condition for when there is an "element of reality"—the EPR criterion—applied to a certain state the construction of which the quantum formalism allows—the EPR state. The EPR criterion is this:

> If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity [Einstein et al., 1935, p. 777].

The EPR state is represented by the wavefunction

$$\Psi_{EPR}(\mathbf{x}_1, \mathbf{x}_2) = \int_{\text{all } \mathbf{p}-\text{space}} e^{\frac{2\pi i}{\hbar}(\mathbf{x}_1 - \mathbf{x}_2 + \mathbf{x}_0)\cdot \mathbf{p}} d^3\mathbf{p}\,, \tag{1}$$

which is a simultaneous eigenstate of $\hat{\mathbf{x}}_1 - \hat{\mathbf{x}}_2$ (eigenvalue $-\mathbf{x}_0$) and $\hat{\mathbf{p}}_1 + \hat{\mathbf{p}}_2$ (eigenvalue $\mathbf{0}$) [Einstein et al.,

[14] As Arthur Fine clarifies [Fine, 1986, pp. 32-33], the logical form of the actual argument given in the article is much more and needlessly complex, in that the more complex reasoning crucially relies on a direct argument for the existence of simultaneous sharp values of position and momentum of some particle, and the success of this argument would suffice for establishing the incompleteness of quantum theory in light of the simple observation that quantum theory does not permit assignment of such values for any particle.

1935, pp. 779-780]. For this state, for any measurement of the positions $\mathbf{x}_1$, $\mathbf{x}_2$ of particles 1 and 2, one will get results such that $\mathbf{x}_1 - \mathbf{x}_2 = -\mathbf{x}_0$, and for any measurements of momenta $\mathbf{p}_1$, $\mathbf{p}_2$ of particles 1 and 2, one will get results such that $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{0}$.

The argument then utilizes the EPR criterion and the EPR state by observing that if one has an EPR state, one could measure the position $\mathbf{x}_1$ of particle 1 and thus infer the position $\mathbf{x}_2$ of particle 2 ($= \mathbf{x}_0 + \mathbf{x}_1$); likewise, one could measure the momentum $\mathbf{p}_1$ of particle 1 and thus infer the momentum $\mathbf{p}_2$ of particle 2 ($= -\mathbf{p}_1$). A measurement on particle 1 certainly does not disturb particle 2, as particle 2 is space-like separated from particle 1. By the EPR criterion, it thus follows that both the (arbitrarily sharp) position and (arbitrarily sharp) momentum of particle 2 are elements of reality, the set of which quantum theory does not have the resources to represent. Therefore, quantum theory is incomplete in the relevant sense [Einstein et al., 1935, p. 780].

It has become clear that Einstein himself was at this time at least in part concerned with the incompatibility of quantum theory with the conjunction of two principles Don Howard has called "Separability" and "Locality," which are briefly and implicitly invoked in a small fragment of reasoning in the EPR article:

> ...since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system [Einstein et al., 1935, p. 779].

As Arthur Fine documents [Fine, 1986, pp. 35-36], Einstein first started making the premises implicitly invoked here explicit in a letter to Schrödinger about the EPR article dated June 19, 1935 with the "principle of separation" (*Trennungsprinzip*), which Fine aptly paraphrases as "the claim that whether a physical property holds for one of the particles does not depend on measurements (or other interactions) made on the other particle when the pair is widely separated in space" [Fine, 1986, p. 37]. As Howard observes [Howard, 1985, pp. 172-173, 179, 182 ff], this principle is actually the conjunction of two principles: "Separability" or the principle "...that any two spatially separated systems possess their own separate real states" and "Locality" or the principle "...that all physical effects are propagated with finite, subluminal velocities so that no effects can be communicated between systems separated by a space-like interval." Howard also documents articulation of both principles in Einstein's post-EPR correspondence concerning quantum mechanics [Howard, 1985, pp. 182ff].

The reasoning of the EPR article can be further elucidated by expositing what Bell famously emphasized, namely that determinism in the contexts of possible measurements on EPR states is a consequence rather than premise of this reasoning [Bell, 2004, p. 143]. In particular, determinism is a consequence of the following premises: (i) the postulated reality of the measurement outcomes as represented by wavefunction (1), i.e., the postulated reality of EPR correlations; (ii) the principle that probability distributions of actual and possible measurement outcomes obtain in virtue of the pre-measurement states of the systems being measured; (iii) Separability; and (iv) Locality. If, as per (i), possible measurements of (momentum, position) on particles 1 and 2 of an EPR state are perfectly (anti-)correlated, then, conditioned on the measurement of position or momentum actually being made on particle 1, the possible measurement outcome of position or momentum is settled with $p = 1$. As per (ii), the pre-measurement state of particle 2, conditioned on the actual measurement outcome of (position, momentum) on particle 1, settles the outcome of a possible (position, momentum) measurement on particle 2 with $p = 1$.[15] As per Separability, however, the pre-measurement state of particle 2 is independent of the pre-measurement state of particle 1, and as per Locality, it is not affected by the possible measurement procedures on particle 1. Therefore, the pre-measurement state of particle 2 of an EPR state alone deterministically settles the possible measurement outcomes of position and momentum of particle 2, i.e. particle 2 of an EPR state has properties of position and momentum that deterministically settle possible measurements of such quantities with arbitrary precision. *Mutadis mutandis*, the same reasoning starting from considerations of possible measurements of momentum and position on particle 2 of an EPR state shows that the above premises imply that particle 1 of an EPR state has properties of position and momentum that deterministically settle possible measurements of such quantities with arbitrary precision.[16] In sum: Pre-Existing Properties (see Introduction) for position and momentum in the specific context of EPR measurements is a consequence of Separability, Locality, and the postulated reality of EPR correlations.

3 Bohr's Response: A First Pass

Bohr's response seeks to respond to the challenge posed by the EPR argument by diagnosing "an ambiguity as regards the meaning of the expression 'without in any way disturbing a system' " in the statement of the EPR criterion. EPR plainly meant this expression in the sense of an interaction that changes the properties of particle 2 during the process of measuring either the position or momentum of particle 1. Bohr agrees that there is no disturbance in this "mechanical" sense, but takes it that there is still a disturbance in another, and so "non-mechanical" sense: "there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system.*" This purportedly overcomes the challenge to the completeness of quantum mechanics because "these conditions constitute an inherent element of the description of any phenomenon to which the term 'physical reality' can be properly attached" [Bohr, 1935, p. 700].

As it stands, the above is an admittedly obscure summary statement Bohr offers of his response to EPR after offering "simple, and in substance well-known considerations" about two variants of the double-slit experiment [Bohr, 1935, p. 699]. Understanding the meaning of the summary statement and attempting to see how the response engages with the EPR argument therefore requires reviewing these considerations.

In the first variant of the double-slit experiment, a diaphragm with a single slit is rigidly bolted, allowing no measurement of any momentum of the particle passing through it on its way to a double slit in a

[15]I take (ii) to be both commonsensical and unobjectionable (and entirely compatible with Bohr's reasoning), so I will simply tacitly assume it from now on. I flag it here just to make it explicit.

[16]Thank you to Guido Bacciagaluppi for helping me to clarify EPR's reasoning.

rigidly bolted diaphragm, behind which there is a screen. The second variant is identical except that the diaphragm of the first slit has a freedom of movement that allows us to measure the momentum of the particle passing through it, but at the expense of precision in any determinable location of where the slit is when the particle passes through it [Bohr, 1935, pp. 697-699].

One key point here is that only in the first experimental arrangement is there any interference characteristic of the original double-slit experiment, and in this arrangement, there is no question of any determination of the momentum of the particle when it passes through the first diaphragm.[17] Bohr thus urges that, in this context, while we have the freedom to choose whether to measure either the momentum or the position of the particle with the first diaphragm, in doing so we are not picking out one element of reality or another in one and the same "phenomenon" [Bohr, 1935, p. 698], but rather the choice is

> ...a rational discrimination between essentially different experimental arrangements and procedures... [Bohr, 1935, p. 699]

Bohr further claims that in neither experimental arrangement is there the possibility of making further determinations of respectively momentum or position of the particle passing through the first slit: "the renunciation in each experimental arrangement of the one or the other of the two aspects of the description of the physical phenomena" is forced by the nature of the objects of investigation themselves. It depends, namely,

> essentially ... on the impossibility, in the field of quantum theory, of accurately controlling the reaction of the object on the measuring instruments, i.e., the transfer of momentum in the case of position measurements, and the displacement in the case of momentum measurements [Bohr, 1935, p. 699].

These considerations purportedly generalize straightforwardly to the measurements of and inferences about elements of reality in the EPR state. On a first pass, the generalization appears to go as follows: in measuring the position of particle 1, we must use a rigid diaphragm, and by this procedure we lose our only basis for making predictions as to the behavior of particle 2 upon measuring its momentum; likewise, in measuring the momentum of particle 1, we must allow the diaphragm to move freely, and consequently lose our only basis for predicting the location of particle 2 [Bohr, 1935, p. 700].

[17] We can explain why there is no interference effect in the second arrangement if the HUR characterize the predictive significance of the initial state of the electron when localized in the first slit, as then the convolution of the spread in location of the first slit with the fringing pattern on the screen washes out the fringing pattern to give a non-fringed pattern on the screen [Bohr, 1949/1996, p. 357]. See [Uffink and Hilgevoord, 1985, pp. 938-939] and [Greenstein and Zajonc, 1997, pp. 86-88] for mathematical details. As I exposit from Bohr's pre-EPR writings in Section 5, the quantum postulate implies that the HUR indeed characterize the predictive significance of any initial conditions of momentum and position of any atomic entity.

4 The Quantum Postulate vis-à-vis (In)determinism as Key to Bohr's Response

There are two main lines of complaint lodged against Bohr's response, consideration of which help show that the only hope for reconstructing it as engaging with the EPR argument lies in reading Bohr as challenging Pre-Existing Properties via challenging determinism for position and momentum measurements on general grounds, and reading his conception of "non-mechanical disturbance" as in part expressing this challenge. First, it is unclear how Bohr's conception of how measurements "disturb" quantum systems applies to measurements on the EPR state in a way that engages with the EPR argument, even though Bohr takes this conception to be central to his response. Second—and here I agree with Bohr's critics but draw a different lesson from the point—, it is unclear how Bohr's reasoning can directly engage with either Separability or Locality. I will further argue that Bohr's response must challenge determinism in the context of possible measurements of the EPR state somehow to engage with the EPR argument at all; given that the response does not engage with Separability or Locality per se, it must therefore be a general challenge to determinism for position and momentum measurements, one that Bohr indicates follows from the quantum postulate or "the very existence of the quantum of action" (see Section 5).

Bohr's response certainly appears primarily to adduce the necessity of distinct experimental arrangements for distinct measurements in attempting to defuse the EPR argument, a point later Bohr insists was always key both to a proper view of the foundations of quantum theory and to responding to the EPR argument [Bohr, 1949/1996, pp. 355, 358, 362, 370, 373]. However, Bohr also appears to advance another point as key for the "problem of physical reality" at issue before ever turning to details of so many experimental arrangements:

> *the finite interaction between object and measuring agencies* conditioned by the very existence of the quantum of action entails—because of the impossibility of controlling the reaction of the object on the measuring instruments if these are to serve their purpose—the necessity of a final renunciation of the classical ideal of causality and a radical revision towards the problem of physical reality [Bohr, 1935, p. 697].

As we have just seen, this point is adduced as a reason that more information about the position or momentum is unobtainable in the two respective variants of the double-slit experiment, which Bohr takes to be key for responding to EPR. If this point is indeed key to this response, it should also bear on the experimental arrangements and states of matter EPR adduce. It is however unclear so far precisely how it does so, given a pressing disanalogy between the two respective pairs of experimental arrangements to which Bohr does not speak.

Namely, the two variants of the double-slit experiment differ in the local interaction that takes place between the first slit and the particles being measured, whereas the differences in measurement procedures on particle 1 of the EPR state is not a difference in how anything interacts with particle 2 of the

EPR state. Moreover, as Beller and Fine emphasize, despite Bohr professing to be identifying a "non-mechanical" sense of disturbance affected by the choice of measurement on particle 1, Bohr's language in discussing the two variants of the double slit experiment does appear to be mechanical, through appeal to the "impossibility ... of controlling the reaction of the object on the measuring instruments..."[18] The problem is simply that no such interaction takes place between any measuring device and particle 2 in the two possible EPR measurements. Bohr's own description of the sense of "non-mechanical disturbance" at issue—"an influence on the very conditions which define the possible types of predictions regarding the future behaviour of the system" [Bohr, 1935, p. 700]—does little to help: on its own, it seems simply to advance that we cannot ascribe both properties of position and momentum of arbitrary sharpness to particle 2 because we cannot actually infer both of their values at a given time from actual measurements on particle 1. Not only does this not engage with any of the premises of the EPR argument, it begs the question against the possibility of a hidden-variable completion of quantum mechanics along lines recommended by the EPR argument. If this is what Bohr means, Beller and Fine are right in their claim that Bohr evades the EPR argument through a "final – and somewhat forced – landing in positivism" [Beller and Fine, 1994, pp. 14, 29].[19] A reconstruction of Bohr's response according to which it genuinely engages with the EPR argument must therefore alternatively elaborate a "non-mechanical" conception of the sense in which measurements "disturb" quantum mechanical systems that is "conditioned by the very existence of the quantum of action"—that is, that follows from the quantum postulate (see Section 5).

A number of commentators further accuse Bohr of completely failing to respond to the EPR argument [Becker, 2018, p. 60], [Maudlin, 2018], [Bell, 2004, p. 156], [Norsen, 2006, pp. 287-292], or, more specifically, of begging the question by simply denying Separability or Locality [Stapp, 1991, p. 8], [Whitaker, 2006, p. 232-233], [Kumar, 2009, Ch. 13], [Fine, 2020]. I dispute all of these claims, but concede that it is far from clear how Bohr's reasoning could engage with either Separability or Locality *directly*: It might well take two different experimental arrangements in the neighborhood of particle 1 to *infer* respectively the momentum or the position of particle 2, but if one endorses Separability and Locality, it would still follow from the existence of EPR correlations that there are properties of momentum and position of arbitrary sharpness settling the outcomes of possible such measurements with arbitrary sharpness;[20] moreover, the mere fact that each measurement is an interaction between measuring device and measured entity challenges neither

[18] This language is also a continuation of pre-EPR language Bohr uses when discussing measurement in quantum theory [Beller and Fine, 1994, pp. 10-13].

[19] Beller and Fine also make this complaint in [Fine, 1986, pp. 4-5, 29, 31, 34-35], [Beller, 1996, p. 196], and [Beller, 1999, p. 152]. See also [Cushing, 1994, p. 29], [Redhead, 1987, p. 51], and [Kumar, 2009, Ch. 13] for similar complaints. Cf. [Whitaker, 2004, p. 1329].

[20] Thus simply pointing out that what is "predictable with certainty" is relative to an experimental context and suggesting on this basis that so, too, are the "candidates for real status," as Howard does on Bohr's behalf [Howard, 1979, p. 256], does not engage with any of EPR's premises. For the difference in experimental contexts is not a difference in the two physical situations in the neighborhood of particle 2, and hence, by Separability and Locality, should not make any difference as to what properties particle 2 has. The suggestion of Halvorson and Clifton that the elements of reality are those that are left invariant under some "relevant" group of symmetry transformations on the EPR state [Halvorson and Clifton, 2001, pp. 11-16], while formally illuminating, also does not engage with EPR's premises for the same reason. For the "relevant" group of symmetries is none other than that group of symmetries that (i) is applied to both particles and (ii) leaves the measured observable of particle 1 in a given experimental arrangement invariant. This amounts substantively to Howard's suggestion that the only "candidates for real status" are set by the experimental context of making one sort of measurement or other on particle 1.

Separability nor Locality, nor the reality of EPR correlations.[21]

Not only is it unclear how Bohr's response could directly engage with Separability and Locality, any successful response to the EPR argument would need to show that possible momentum and position measurements of the EPR state are not deterministically settled by that state—this is because such measurements include measurements of arbitrary precision of both position and momentum. Therefore, if possible momentum and position measurements of the EPR state are deterministically settled by that state, both particles have simultaneously sharp pre-measurement properties of momentum and position, and quantum theory would be incomplete in the relevant sense. Thus, any effective reconstruction of Bohr's response would need to dispute determinism of possible momentum and position measurements in this context. Given that Bohr's response does not directly engage with Separability or Locality per se, any such reconstruction would have to be a uniform response to the principle that possible measurement outcomes of momentum and position are deterministically settled by corresponding pre-measurement properties, and thus be a general challenge to Pre-Existing Properties for position and momentum, brought to bear on this particular case.

In fact, a look to the historical record gives us reason to suspect Bohr engaged directly with this principle in the lead-up to EPR. Tim Maudlin describes Einstein's pre-EPR reasoning against the completeness of quantum mechanics as follows:

> Einstein saw that the phenomena themselves—as distinct from Schrödinger's theory with its wavefunctions—did not require anything spooky. All you had to believe is that the electron was always in some precise location, of which we are ignorant, and takes a humdrum path from the source to the screen, causing a flash. But because quantum mechanics does not specify the location [at all times], accepting this picture demands rejecting the completeness of quantum mechanics [Maudlin, 2018].

Maudlin appears to be referencing Einstein's reasoning at the Solvay conference, where he advances that wavefunction realism appears to require a collapse of a wavefield incident on a whole screen to a point (small region) on the screen and thus that the point at which the electron becomes localized is in dynam-

[21] In a more recent article, Howard advances that Bohr's response to Einstein consisted in his "[embrace] of entanglement, seeing in it the roots of complementarity" and a rejection of Separability accordingly, and that Bohr has been vindicated through quantum theory being "well-confirmed" and taking entanglement to be "fundamental" [Howard, 2007, pp. 59, 69-70, 79-80, 84]. Specifically, Howard advances that Bohr holds "[t]hat instrument and object form an entangled pair," adducing as textual evidence Bohr's pre-EPR statement that "an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation" because "...the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected" [Bohr, 1927/1985c, p. 114]. Folse made essentially the same suggestion a few years prior [Folse, 1989, pp. 264ff].

This proposed reconstruction of Bohr's response, however, does not directly engage with Separability in the sense at issue, for the simple reason that the difference in the two measuring devices for the two possible measurements on particle 1 of the EPR state is not a difference that obtains upon preparation of either of the two particles; rather, both particles were identically prepared by a preparing device, namely in an EPR state. Further, the possible interactions at issue when measuring either particle 1 or particle 2 are localized to the spatially separated neighborhoods of each respective particle, and so a simple appeal to these possible interactions does not give any reason to think the two-particle system should be non-Separable. Hence any "entanglement" between measuring device and system measured cannot directly speak for the non-Separability of the two particles in the EPR state.

Moreover, as will become clear in Section 5, the quantum postulate does not even imply the reality of EPR correlations, so it cannot directly imply the non-Separability that would follow from them and the completeness of quantum theory. However, as we will see (Section 6), the quantum postulate does imply the failure of determinism for EPR measurements, which in turn implies either that the EPR state is non-Separable or that there is superluminal causality involved in EPR measurements *if* their outcomes are as predicted by wavefunction (1). If superluminal causality is a non-starter on relativistic grounds, the quantum postulate would thus imply that the EPR state is non-Separable indirectly, via the failure of determinism.

ical community with the entire spatial extent of the wavefield, contrary to the conjunction of Separability and Locality [Einstein et al., 1927/1985]. The alternative is to hold that immediately pre-measurement, the electron already had a determinate location, i.e. to assume Pre-Existing Properties holds in this situation for position. To avoid the same difficulty for all possible such position measurements, one must hold that the electron has a determinate location at all times, i.e. that it takes a "humdrum path from source to screen" and thus has completely determinate momentum and position at all times. For the very reason that Einstein presented this reasoning at the Solvay conference, however, the suggestion that Bohr never understood how Pre-Existing Properties challenges the completeness of quantum theory strains credulity, and the claim that Bohr "never came to grips" with Einstein's reasoning here is premature without sustained scrutiny [Maudlin, 2018]. This is because Bohr was famously in attendance when Einstein presented the above difficulty; furthermore, he represented this difficulty perfectly cogently in recalling his many exchanges with Einstein on the foundations of quantum theory:

> On account of the diffraction of the wave connected with the motion of the particle ..., it is under such conditions not possible to predict with certainty at what point the electron will arrive at the photographic plate, but only to calculate the probability that, in an experiment, the electron will be found within any given region of the plate. The apparent difficulty, in this description, which Einstein felt so acutely, is the fact that, if in the experiment the electron is recorded at one point A of the plate, then it is out of the question of ever observing an effect of this electron at another point (B), although the laws of ordinary wave propagation offer no room for a correlation between two such events [ZH: i.e., Schrödinger evolution does not tell us why the electron is observed at point A rather than at point B] [Bohr, 1949/1996, pp. 352-353].

On grounds of charity, then, we should expect that Bohr at one point or another targeted Pre-Existing Properties in defense of the completeness of quantum theory, and we have seen that his response can only be understood as engaging with the EPR argument at all if it offers a general argument against Pre-Existing Properties for measurements of position and momentum by challenging determinism in such contexts. We have also seen that a successful reconstruction of Bohr's response should find a "non-mechanical" conception of the sense in which measurements "disturb" quantum systems that is "conditioned by the very existence of the quantum of action"—that is, that follows from the quantum postulate (see below)—and that applies straightforwardly to EPR measurements. I now argue that Bohr's reflections on the quantum postulate in the lead-up to EPR indeed offer materials for a valid and entirely general argument against Pre-Existing Properties for position and momentum measurements premised on the quantum postulate.[22] I further dispute the contention of Beller and Fine that Bohr changes the sense of "disturbance" he adduces in responding to EPR from his pre-EPR disturbance view [Fine, 1986, p. 35], [Beller and Fine, 1994, pp. 10-14], [Fine, 2020]. Rather, as I show in the following, this view expresses the failure of Pre-Existing Properties for position and momentum implied by the quantum postulate, and Bohr articulates further consequences of this physical principle vis-à-vis measurements of position and momentum for the specific

[22] Karen Barad also reads Bohr as challenging this principle [Barad, 2007, p. 107]. However, my reconstruction will make more explicit how he challenges it and how this serves to defuse the challenge from EPR than Barad's discussion [Barad, 2007, pp. 269-275], which advances that the key issue is simply that there are distinct experimental arrangements [Barad, 2007, pp. 274-275]. As it stands, this is unsatisfactory for the reasons rehearsed above.

context of measurements of the EPR state precisely in his description of the "non-mechanical disturbance" that the choice of measurement procedure on particle 1 of the EPR state affects.

5 The Quantum Postulate and the "Essential Disturbance" of Phenomena

A key physical premise throughout Bohr's reflections on the foundations of quantum theory is the "quantum postulate," which "[expresses] the essence of quantum theory" [Bohr, 1928/1985c, p. 148]. Here, I focus primarily on Bohr's reflections on the quantum postulate in the Como lecture and other pre-EPR writings to get in view how he saw its relevance for quantum theory in the lead-up to EPR. I find that the quantum postulate is a physical postulate from which a failure of Pre-Existing Properties for position and momentum follows, a failure that Bohr's pre-EPR conception of the "essential disturbance" of atomic phenomena by measurement expresses [Bohr, 1927/1985b, p. 91].

In portions of the Como lecture itself, it is easy to read the quantum postulate simply as a commitment to the sort of instrumentalism Beller and Fine take to be central to Bohr's response to EPR, for example when Bohr writes that "an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation" because "the quantum postulate ... implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected" [Bohr, 1928/1985c, p. 148].[23] However, Bohr also writes that the quantum postulate is "symbolised by Planck's quantum of action," which is a physical constant that relates momentum and energy to characteristic frequencies f and wavelengths λ in the Einstein-Planck-de Broglie relations $E = hf$ and $\mathbf{p} = h\boldsymbol{\sigma} := h\frac{1}{\lambda}\hat{n}$ ($\hat{n}$ is the unit vector representing the direction of the momentum) [Bohr, 1928/1985c, pp. 148-149]. Put otherwise, the quantum postulate is in the first instance the postulated reality of the Einstein-Planck-de Broglie relations and in this sense the postulated reality of Planck's quantum of action. Importantly, these relations mean that a determinate momentum is associated with a determinate wavelength *simpliciter* and that a determinate energy is associated with a determinate frequency *simpliciter* or with an "elementary harmonic wave" of one wavelength or frequency (rather than with e.g. "plane-wave of wavelength λ multiplied by a Gaussian function f localizing an entity in space") [Bohr, 1928/1985c, p. 149].

I now argue that the quantum postulate implies an "ontological" understanding of the Heisenberg Uncertainty Relations (HUR) from which an entirely general and direct challenge to Pre-Existing Properties for position and momentum measurements follows.[24] Further and significantly for the purposes of interpreting Bohr's philosophical thought, his various instrumentalist-inflected turns of phrase, including the

[23]Thank you to Noah Stemeroff for pushing me on this point.
[24]See [Hilgevoord and Uffink, 2016] for an overview of different ways of understanding the HUR.

above quotation in the Como lecture and those in his response to EPR, can be seen as expressions of consequences of this conception of the HUR and thus as physical consequences of the quantum postulate for measurement contexts.[25]

Bohr correctly advances that it follows from the relations $E = hf$ and $\mathbf{p} = h\boldsymbol{\sigma}$ that we must represent any localized physical entity as having a spread in its momentum in accordance with the HUR [Bohr, 1927/1985a, pp. 76-78], [Bohr, 1927/1985b, pp. 92-93], [Bohr, 1927/1985c, pp. 117-118], [Bohr, 1928/1985c, pp. 149-150]. This is correct because the quantum postulate implies that any entity (that is in three-dimensional space and) that has a single determinate momentum must be a plane-wave, which is infinite in spatial extent.[26] Therefore, atomic entities that are localized in (three-dimensional) position-space to any degree (and thus any real such entities at all) must have a spread in their determinate momentum.[27] This spread in determinate momentum cannot be a weighted sum of potential measurement outcomes, as the infinite plane-waves with completely determinate momentum can never be the state of any atomic entity upon measurement. Rather, in order that a real (and thus localized) atomic entity be a localized atomic entity in the first place, *all* such values of momentum must jointly characterize the *actual* motion of that localized atomic entity—this is why a (massive) particle in general spreads out in space as it propagates (see Figure 1). The quantum postulate of the reality of the Einstein-Planck-de Broglie relations, therefore, implies that any localized (massive) atomic entities are characterized by "gradual[ly] spreading wave fields" [Bohr, 1927/1985b, p. 94].[28] One way to represent the necessary spread localized atomic entities must have in their momentum mathematically is with the relation $\Delta x \Delta \sigma_x \gtrapprox 1$ between the standard deviation Δx of a spatially localized function resulting from a weighted sum of elementary waves of various wavelengths and the spread in those weights $\Delta \sigma_x$ (i.e., the standard deviation of this function) [Bohr, 1927/1985a, p. 78], [Bohr, 1927/1985b, p. 93], [Bohr, 1927/1985c, p. 118], [Bohr, 1928/1985c, p. 149].[29] The reality of the quantum of action thus implies reciprocal relations of $\Delta x \Delta p_x \gtrapprox h$ between the spatial extent of any physical entity and the degree of spread in the momentum that entity has at all—that is, the quantum postulate implies the HUR, "ontologically" conceived.

We can thus already see how Bohr's reasoning provides a general defense against challenges to the completeness of quantum mechanics on grounds of the HUR, as the HUR objectively represent the reciprocal relation between the spatial extent and degree of spread in the momentum any atomic entity can have

[25] To Beller's credit, she also recognizes that the Como lecture primarily contains physical arguments as opposed to instrumentalist philosophy of science [Beller, 1999, pp. 117-134, esp. p. 123].

[26] Thank you to Sam Fletcher for pushing me to clarify this point. See [Norton, 2017] for an intuitive presentation of this issue.

[27] Within the formalism of quantum theory, we could say that only non-eigenstates of momentum can be spatially localized to any degree.

[28] To be precise, that the wave fields associated with massive particles spread out is a consequence *both* of this observation and the dispersion relation $\omega \propto k^2$ postulated to hold for such particles in order to ensure correspondence with the Newtonian relation $E_{\text{kinetic}} = \frac{p^2}{2m}$, given the relations $E = \hbar\omega$ and $p = \hbar k$.

[29] Bohr does not mark any approximation at work in the reasoning he references, but he appears to be referencing heuristic arguments from optics that are seen as approximate reasoning schemes when compared to the analysis of Fourier transforms from which the modern relation $\Delta x \Delta p_x \geq \frac{\hbar}{2}$ is derived. This accounts for the difference of a factor 2 in the relation Bohr writes and the modern relation. See [Uffink and Hilgevoord, 1985, pp. 925-928].

at all according to the quantum postulate. The reasoning should thus also provide an argument against Pre-Existing Properties for measurements of position and momentum, as this principle and the HUR stand directly opposed to one another in ways rehearsed in Section 2.

The general challenge to Pre-Existing Properties for such measurements is particularly clear for the simplest such measurement, namely that of the position of a particle after it propagates through a single slit. Consider: the slit has a finite half-width, a, and the bulk of the particle passes through the center of the slit. Hence, according to the quantum postulate, there is a spread in the momentum the particle has on the order of $\frac{h}{a}$, as for it to be localized in a region with half-width $\Delta x = a$, the spread in the wavenumbers characteristic of the particle's motion must have half-width of $\Delta\sigma \gtrapprox \frac{1}{a}$ [Bohr, 1927/1985a, p. 79], [Bohr, 1928/1985b, p. 190] (Cf. [Bohr, 1949/1996, pp. 353-354]). There is therefore no physical fact about the particle that will constrain the particle to just one position on the screen on which we subsequently measure it, and therefore no single path that can make exclusive claim to represent the future behavior of the particle in this initial state.[30] Rather, the particle's motion is characterized by a spreading wavefield incident on many possible sites of localization in the screen. Thus, even a particle with zero total momentum transverse to the slit has a non-zero chance of subsequently being localized in a region displaced with respect to the slit along that axis.[31] This illustrates why, at least in this context, we must replace the "continuous, causal description" of classical physics with "a *fundamentally* statistical mode of description" [Bohr, 1929/1985a, p. 244 - my emphasis].[32] See Figure 1.

[30]The only way to get an empirically determined path from slit to subsequent screen is therefore by placing a series of intermediate slits, or by replacing the slit with e.g. a metal block with a small path drilled through it and putting a screen on the opposite end of the block, such that we know that any particle that makes it to the screen must have done so by passing through known intermediate positions. But these would just be totally different physical situations, where interactions of any particle with these other physical systems forces any through-passing particle to be so constrained. See [Bohr, 1935, pp. 697-698] and cf. [Bohr, 1939/1996, pp. 311-312], [Bohr, 1948, p. 313], [Bohr, 1958/1996a, p. 391], and [Bohr, 1958/1996b, p. 419].

[31]We can accordingly understand why Bohr advances that the HUR characterize the predictive significance of measurement results on quantum systems [Bohr, 1927/1985c, pp. 122-123], [Bohr, 1928/1985c, p. 151], [Bohr, 1929/1985b, p. 296]. Cf. [Bohr, 1939/1996, p. 315].

[32]See also [Bohr, 1927/1985b, p. 92], [Bohr, 1929/1985a, p. 243], and [Bohr, 1929/1985b, p. 295]. Cf. [Bohr, 1939/1996, pp. 303-305, 311], [Bohr, 1948, pp. 313-314], [Bohr, 1949/1996, pp. 355-356], [Bohr, 1958/1996a, p. 391], and [Bohr, 1958/1996b, p. 419].

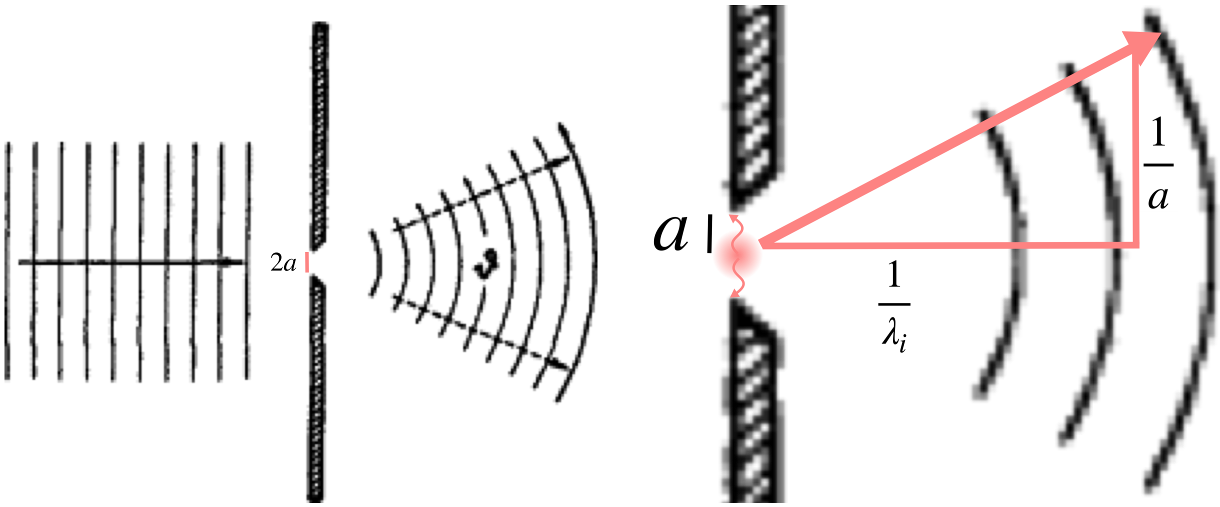

Figure 1: For a particle to be localized within a slit or hole in a screen, it must possess a spread in the momentum it has. These figures have been adapted from two original figures in Bohr's "Discussion with Einstein on Epistemological Problems in Atomic Physics" [Bohr, 1949/1996, p. 354] in two ways: the slit-width has been labelled; a triangle has been added whose legs show the ratio of the spread in wavenumbers characteristic of the particle's momentum transverse to the slit during and after propagating through it to the wavenumber characteristic of the incident particle's momentum orthogonal to the slit (which ratio thus specifies the angle ϑ characterizing the spread of the wavefield indicated in the original figure on the left), and these legs are also labelled accordingly.

The challenge to Pre-Existing Properties here is not merely that the supposed elements of reality at issue would happen not to be empirically ascertainable by us should they exist, but that it follows from the quantum postulate that there is objectively and in principle nothing in the initial conditions that can determinately settle one location at which the particle will be found on any finitely separated screen. The key observation for this challenge, implied by the quantum postulate, is that the localization of the atomic entity of interest to some known region requires a spread in the momentum that entity actually has. This point allows us to completely generalize the above reasoning to the scope required in the current argumentative context as follows: any real atomic entity is localized in three-dimensional space; by the quantum postulate, any degree of localization of an atomic entity in three-dimensional space involves an intrinsic spread in the momentum it has; this spread in momentum has physical significance for how the atomic entity exchanges momentum and means that it (its spreading wavefield) will be incident on many sites of possible localization at finite time after being localized to any degree; measurements of position and momentum of atomic entities involve (i) their becoming localized within one possible site of localization at a time finitely subsequent to their being localized to some degree and (ii) exchange of momentum of that particle with those systems; therefore, no pre-measurement state of any real atomic entity deterministically settles the outcome of possible measurements of position and momentum of that entity. The quantum postulate thus implies that Pre-Existing Properties for position and momentum is false in any possible such measurement context on general physical grounds.

As I now show, this reasoning also makes sense of Bohr's claim that "the quantum postulate implies that no observation of atomic phenomena is possible without their essential disturbance" and clarifies how he conceives of measurements of position and momentum [Bohr, 1927/1985b, p. 91]. Bohr's claim here cannot merely be that the measuring device is always in dynamical community with the system measured, for this would be true even if the quantum postulate were false. It must be some sort of "essential disturbance" that is unique to quantum theory and a clear consequence of the quantum postulate. Fortunately, Bohr does indeed offer an explicit contrast between the status of measurements in classical physics and in quantum theory in the Como lecture:

> As remarked by Heisenberg, one may even obtain an instructive illustration to [*sic*] the quantum theoretical description of atomic (microscopic) phenomena by comparing [the HUR] with the uncertainty, due to imperfect measurements, inherently contained in any observation as considered in the ordinary description of the natural phenomena. He remarks on that occasion that even in the case of macroscopic phenomena, we may say, in a certain sense, that they are created by repeated observations. It must not be forgotten, however, that in the classical theories any succeeding observation permits a prediction of future events with ever-increasing accuracy, because it improves our knowledge of the initial state of the system. According to the quantum theory, just the impossibility of neglecting the interaction with the agency of measurement means that every observation introduces a new uncontrollable element [Bohr, 1928/1985c, p. 152].[33]

Evidently, that every measurement introduces an "uncontrollable element" expresses the sense in which measurements "disturb" quantum systems.[34] This passage also unambiguously offers this claim as illustrative of the contrast of the HUR with imperfect knowledge in classical physics vis-à-vis the status of measurements in classical and quantum physics. Thus, the precise sense in which measurements "disturb" atomic systems is supposed to illustrate just this contrast. This contrast, moreover, is the following: while we can treat successive measurements as improving our knowledge of the initial state of the system in classical physics, this is not so in quantum theory.[35] As I now show, the impossibility of uniquely inferring pre-measurement states of atomic entities from measurements involving them follows from the above "ontological" understanding of the HUR that rules out Pre-Existing Properties.[36]

As we have seen, owing to the intrinsic spread in the momentum any atomic entity has, there is not a single path for any such entity to take. Hence, there is nothing about any initial state of any atomic

[33]See also [Bohr, 1927/1985c, p. 123] and [Bohr, 1929/1985b, p. 296].

[34]The statement that "the impossibility of neglecting the interaction with the agency of measurement means that every observation introduces a new uncontrollable element" for quantum systems echoes the claim that "...any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected" [Bohr, 1928/1985c, p. 148]. Not only does this latter statement replace the statement that "...the quantum postulate implies that no observation of atomic phenomena is possible without their essential disturbance" in an earlier draft of the Como lecture [Bohr, 1927/1985b, p. 91], it still serves to problematize "our usual description of physical phenomena[,] ... based entirely on the idea that the phenomena concerned may be observed without disturbing them appreciably" [Bohr, 1928/1985c, p. 148].

[35]Such inference is, for example, part of the logic of evidence of Newtonian gravitational theory, according to which the successive determinations of the positions as a function of time of the various gravitating bodies in the solar system allow us to compute their masses and hence uniquely infer their positions prior to the first set of measurements; these masses and relative positions are then a set of initial conditions from which the states determined in each successive measurement, as well as each state successive to those partially determined from the measurements taken, are forward-evolved. Thus the previous and future states can be inferred and predicted with ever-increasing accuracy in proportion to the precision and number of measurements of a succession of states, because each such state evolves continuously and deterministically out of the previous ones [Smith, 2014].

[36]This explains why the reasoning from the quantum postulate to this understanding of the HUR is presented in the section prior to one from which I have just quoted (titled "Measurements in Quantum Theory") and why the section from which I have just quoted focuses on the HUR.

entity that determines precisely in what site of possible localization it will subsequently be localized, and the wave field characterizing that entity spreads in accordance with the HUR such that it is incident on multiple such sites of possible localization in an experimental arrangement. But measurements of position or momentum involve an atomic entity becoming localized to one such possible site of localization. Thus, "[d]ue to the gradual spreading of the wave fields associated with the individuals[,] ... we must contemplate a proper reduction of the spatial extension of the fields after every new observation" [Bohr, 1927/1985b, p. 94]. Therefore, a measurement of position and or of momentum involves a discontinuous, indeterministic change of an atomic entity's position. Because localized entities always have a spread in their momentum in accordance with the HUR, there is a corresponding freedom in momentum that can be exchanged between two spatially adjacent systems, such that total momentum exchange is not deterministically settled by the pre-interaction state.[37] Measurements of position or momentum, however, involve the measured entity becoming spatially adjacent to the parts of the measuring device discontinuously, and so there will in general be such indeterministic momentum exchange upon every such discrete change of state. Measurements of position and momentum are thus in general also "accompanied ... by a finite change in the dynamical variables" [Bohr, 1928/1985c, p. 152].[38] Therefore, the quantum postulate implies that the momentum and position of any atomic entity upon measurement are not continuously and deterministically evolved from any pre-measurement state, and so makes it impossible to infer any pre-measurement state of any atomic entity uniquely from measurements on those states.

What Bohr means by phrases such as "measurement disturbs atomic phenomena," then, is the following: the quantum postulate implies that any result of any possible measurement of momentum and any result of any possible measurement of position must involve reinstantiation of both the measured entity's position and its momentum, in that these conditions are not continuously and deterministically evolved from a previous such set of initial conditions.[39]

6 Bohr's Response, Revisited

Bohr's pre-EPR conception of how measurements "disturb" atomic phenomena articulates the falsity of Pre-Existing Properties for any context of position or momentum measurement that follows from the quantum postulate. Accordingly, we should be able reconstruct Bohr's response anew as thus engaging with EPR by considering how this conception bears both on the two variants of the double slit experiment Bohr spends the bulk of his response discussing as well as on possible measurements of position and

[37] For example, if one were to place a movable slit instead of a screen beyond a rigidly bolted, preparing slit through which a particle propagates with zero motion transverse to the preparing slit, there will nonetheless be a spread in the degree to which the subsequent movable slit moves from run to run.

[38] In the Como lecture, Bohr only makes this point about position measurements, but the same is true of momentum measurements.

[39] I take it that this is also what Bohr means when he says that the quantum postulate implies "a resignation as regards causal space-time coordination of atomic processes," that a "rigorous definition of [a] system is no longer possible," and "the impossibility of causal space-time description of [the] phenomena," and so a limitation of "the classical physical ideas when applied to atomic phenomena" [Bohr, 1927/1985b, p. 91], [Bohr, 1928/1985c, p. 148].

momentum of the particles in the EPR state.

In the two variants of the double-slit experiment, there is a sense in which the rigidly bolted and freely moving diaphragms each in their own way mechanically disturb the particle that passes through the slit in them: they interact with the particle and change its momentum and position. The specific way in which they do so, according to the quantum postulate, is such that each event of the particle passing through them involves a discontinuous and indeterministic reinstantiation of the position and momentum of the particle. Because these states are qualitatively distinct in the two variants of the experiment, they have different propensities to give measurement outcomes of position and momentum after propagating through a double-slit in a subsequent diaphragm.[40]

Bohr's pre-EPR disturbance view, however, also implies precisely the "non-mechanical" disturbance affected by the choice of measurement on particle 1 Bohr expressed in his response to EPR. As we have seen, the quantum postulate implies that there are no noncontextual, deterministic pre-measurement properties of position and momentum, but rather wavefields with propensities to change their state indeterministically in measurement contexts.[41] Thus, just as for the two sorts of double-slit experiment, for the two possible choices of measurement procedure on particle 1, there are physically significant differences in how the atomic entity comes to be localized with respect to another physical system, and this localization cannot be the deterministic and continuous evolution of the pre-existing state of that entity. Hence in each case we are concerned with physically distinct phenomena, depending on which measuring device measures particle 1 for the EPR state. The same is true, *mutatis mutandis*, of any experimental arrangement for particle 2. Therefore, although Bohr neglected to articulate this point explicitly, the EPR state is involved in *at least four distinct phenomena*, depicted in Figure 2. Because the pre-measurement states of the two particles do not settle possible measurement outcomes of position and momentum deterministically but only have fundamentally statistical propensities to give such-and-such measurement outcomes, the only possible truth-apt representation of any atomic phenomena represented by wavefunction (1) is the fundamentally statistical representation of the probabilities of various occurrences in the four distinct sorts of experimental arrangement corresponding to the four distinct sorts of EPR phenomena. Therefore, the free choice of measuring position or momentum of particle 1 or 2 is really just a choice of which EPR phenom-

[40] For instance, because the HUR represent the reciprocal relation between the spatial extent of some atomic entity and the spread in momentum that entity has, for there to be a determinate amount of total momentum transfer from the particle to the movable slit such that one has measured the momentum of the particle, this interaction must have taken place over a larger region than just the slit-width, from which region the wavefield of that particle must accordingly propagate.

[41] When Bohr concedes that a measurement of the position or momentum of particle 1 of the EPR state "automatically determine[s]" the position or momentum of particle 2, therefore, he cannot mean that this measurement determines a noncontextual property that exists prior to and uniquely explains the measurement outcome of any hypothetical measurement performed on particle 2 [Bohr, 1935, p. 699]. This must instead be read counterfactually: if we had an EPR preparation procedure, and if we were to place a (position, momentum) measuring device that would allow the predicted measurement outcome for particle 2, it would register that outcome. This analysis therefore provides an argument for the meaninglessness of the inference of properties that obtain independently of any measuring context, just as Allen Stairs suggests [Stairs, 2011, p. 236]. It is however unclear why the counterfactual indicated here should itself be considered meaningless: Stairs simply asserts that this is "the best answer" for an indeterminist [Stairs, 2011, p. 236], but it is not clear why the counterfactual is meaningless just because no measurement reveals a noncontextual pre-existing property. Perhaps what Stairs has in mind is that the counterfactual situation would be a different total physical situation, and so the counterfactual is "meaningless" if its meaning is thought to be indicative of a noncontextual pre-existing property.

ena to consider: if we choose to measure the position of particle 1, we are considering either phenomena in which there is a certain correlation between this measurement result and the outcome of a position measurement on particle 2 (perfect correlation in the ideal case) or phenomena in which there is a spread of possible momentum measurement outcomes on particle 2, i.e. either phenomena of type 1 or phenomena of type 4 in Figure 2; if we choose to measure the momentum of particle 1, we are considering either phenomena in which there is a certain correlation between this measurement result and possible momentum measurement outcomes on particle 2 (perfect correlation in the ideal case) or phenomena in which there is a spread in the correlations between the location of this measurement and the outcomes of possible position measurements on particle 2,[42] i.e. either phenomena of type 2 or phenomena of type 3 in Figure 2. The choice of what measurement procedure to undertake on particle 1 is therefore not a choice that makes a difference for any interaction that changes the properties of particle 2 and hence is not a choice between two forms of "mechanical" disturbance of particle 2, but is rather a choice that affects "the very conditions which define the possible types of predictions regarding the future behavior of the system," in that it is a choice of which type of EPR phenomena to consider, amongst which we must indeed make different, fundamentally statistical predictions [Bohr, 1935, p. 700].[43] Without Bohr's pre-EPR conception of "disturbance" in view, this statement sounds like a question-begging appeal to instrumentalism, but with that conception in hand, we can now see that it expresses (albeit obscurely) these consequences of the quantum postulate.

[42]This spread is wider in proportion to the sharpness of the momentum measurement on particle 2: the sharper the momentum of particle 2 during measurement, the larger the region of space throughout which particle 2 must be spread during measurement.

[43]Accordingly, I agree with Thomas Ryckman that Bohr takes the key mistake of Einstein / EPR to lie in an appeal to a "well-defined used of the concept of 'state' " of an atomic entity independently of its relations to measuring devices [Bohr, 1939/1996, p. 313], [Ryckman, 2017, p. 152] and additionally urge that this mistake is more specifically the neglect of the general failure of Pre-Existing Properties for measurements of momentum and position implied by the quantum postulate.

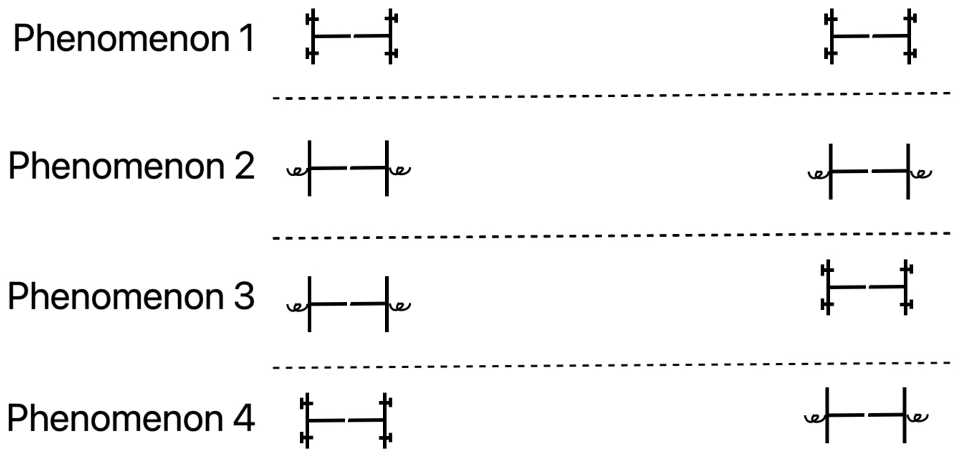

Figure 2: A schematic representation of four distinct sorts of phenomena all represented by the wavefunction (1), individuated by distinct experimental arrangements measuring particles 1 and 2 after the suitable preparation procedure, and hence different determinate physical situations not evolved continuously from the two particles that were prepared in the EPR state. Here, the symbols in "Phenomenon 1" represent slits in diaphragms rigidly bolted to a frame, which can thus measure position in that frame, and the symbols in "Phenomenon 2" represent slits in diaphragms that can move freely with respect to a frame, which can thus measure momentum in that frame.

In sum, then, Bohr's reasoning interfaces with the reasoning in EPR as follows. EPR argues from Separability, Locality, and the assumed reality of EPR correlations that Pre-Existing Properties holds for all possible momentum and position measurements of either particle in the EPR state. Bohr, in effect, argues that the quantum postulate implies that Pre-Existing Properties can never be true for position and momentum measurements. This interfaces with the premises of the EPR argument as follows: if one accepts the reality of the quantum of action, any pre-measurement states of position and momentum must be characterized by objective propensities to give such-and-such measurement outcomes, rather than as pre-existing properties that uniquely determine possible measurement outcomes; if EPR-correlated measurement outcomes obtain, the objective propensities for the parts of the system to be localized in their respective measuring devices in their respective ways are not independent of one another, but "choreographed" in giving these significantly correlated measurement outcomes[44]—thus they are either non-Separable joint propensities, or one measurement event superluminally affects the other (separate) propensity.[45]

This reconstruction clarifies Bohr's protracted concern with experimental arrangements in his response.

[44]In the context of Bell theorems, Bohr's reasoning thus recommends viewing the obtaining of correlated measurement outcomes of the form predicted for the EPR state as indications of a genuine failure of what is called outcome independence for such measurements on such states. See [Myrvold et al., 2021]. Thank you to Juliusz Doboszewski for suggesting I spell this out.

[45]Thus, on my reading, differences between later Einstein's argumentative strategy regarding the completeness of quantum theory vis-à-vis Separability and Locality and EPR's (e.g. that later Einstein explicitly marks his argument as independent of the eigenstate-eigenvalue link [Howard, 1985, p. 181], [Einstein et al., 1971]) do not change the basic picture of how Bohr's response interfaces with such concerns. For, any interesting differences of detail aside, the basic form of the reasoning is still that from correlated measurement outcomes on spatially separated systems, one can infer an incompatibility of the completeness of quantum theory with the conjunction of Separability and Locality, and Bohr's reasoning still interfaces with such reasoning precisely by showing that Separability or Locality must fail if there are correlated measurement outcomes of position and momentum on spatially separated particles and one accepts the reality of the quantum of action.

This concern does not reflect a hazy positivist conception of physics begging the question in favor of quantum orthodoxy. Rather, it reflects the entirely general consequence of the quantum postulate that distinct measuring processes are distinct progressions of determinate states of the entities of interest, rather than detections of noncontextual pre-existing properties. Accordingly, considering the sense in which "the quantum postulate implies that no observation of atomic phenomena is possible without their essential disturbance" [Bohr, 1927/1985b, p. 91] essentially involves considering distinct measuring processes: distinct measuring processes just are distinct phenomena with their own individuality, such that the introduction of more or alternative measuring devices is simply the consideration of another phenomenon, rather than further analysis of a qualitatively identical manifold of pre-measurement properties uniquely determining outcomes of corresponding measurements [Bohr, 1935, p. 697]. The quantum postulate implies this point no more and no less for the different possible measurement procedures on the EPR state than it does for the two variants of the double-slit experiment, which explains why Bohr thought that consideration of the former "does not actually involve any greater intricacies" than consideration of the latter [Bohr, 1935, p. 699].

One must however admit that Bohr's response itself misfired as a self-contained and transparent counter-argument to EPR, for the above reasoning was only elliptically presented in this piece, through a hasty and rough description of the consequences of the quantum postulate reviewed above ("the finite interaction between object and measuring agencies conditioned by the very existence of the quantum of action entails ... ") and subsequent review of various qualitatively distinct measuring procedures. In particular, Bohr failed to spell out the incompatibility of Pre-Existing Properties and the quantum postulate and thus why and in what sense the quantum postulate implies that the phenomena characteristic of every distinct measuring arrangement has its own individuality incompatible of further analysis by introduction of more or different measuring devices, and failed to indicate clearly how this observation applies to the experimental arrangements at issue in the EPR state [Bohr, 1935, pp. 697, 699].[46]

One can also rightfully complain that it is misleading and confusing to call Bohr's pre-EPR conception of the sense in which atomic phenomena are "essentially disturbed" by measurement any sort of "disturbance" in the first place. For the measuring device within which the entity of interest is localized is part of the physical situation that obtains during measurement, hence we cannot think of it as disturbing an independently defined physical state. As Bohr puts it a few years after responding to EPR, "...[the interaction during measurement] cannot be sharply separated from an undisturbed behaviour of the object" [Bohr, 1939/1996, p. 311]. The localization of an entity within a measuring device therefore itself belongs to the progression of physical configurations involving that entity. Measurements are thus elements of the course of the phenomena about which we have determinate information, which enables us to use

[46]There may well be other shortcomings of Bohr's response beyond those obscurities. For example, there could be problems with Bohr's discussion of experimental arrangements suited to study the EPR state, as Beller advances [Beller, 1999, p. 150]. These would not however affect the validity of the reasoning reconstructed here.

them as descriptions of the phenomena; more precisely, measurements are stages in the phenomena we have described by having determinate information about them. Precisely this complaint, however, makes good sense of Bohr's post-EPR conception of what the phenomena *are* and corresponding proposal that we "reserve the word 'phenomenon' for the comprehension of the effects observed under experimental conditions" [Bohr, 1939/1996, p. 316], for this dispenses with the notion of a continuous, deterministic evolution of physical states "disturbed by" our observations.[47,48] The language of "disturbance" is really only fitting if and insofar as one thinks there really should be an underlying manifold of physical properties continuously evolved into measurement states, but the quantum of action frustrates their ability to appear. If the quantum of action is postulated to have genuine reality, however, there is simply no such thing as such a classical manifold of continuously and deterministically evolved properties of position and momentum. I suggest, then, that the above obscurities in Bohr's response vis-à-vis the quantum postulate and "disturbance by measurement" were in part consequences of the fact that Bohr was himself still struggling to break free of the worldview of classical physics too much to offer a clear statement of precisely why and how developments in quantum theory force a break from it, especially given his desultory writing style.[49] Nonetheless, the quantum postulate does indeed arguably imply that "a more detailed analysis of [EPR] phenomena" than their representation by wavefunction (1) is "*in principle* excluded" [Bohr, 1949/1996, p. 375].

[47]See also [Bohr, 1948, p. 317]. What remains to be specified in arriving at this view is why and in what sense Bohr was concerned here exclusively with "experimental conditions."

[48]This suggestion finds support in the historical work of Makoto Katsumori [Katsumori, 2011, pp. 28-31, 50-53, 69-77], who shows that this change accompanied Bohr's rejection of the language of "disturbance." See also [Folse, 1989, pp. 263-265] and [Faye, 1991, pp. 135-137].

[49]Indeed, we can see Bohr oscillating between these two ways of thinking about the quantum of action within the same articles as early as 1929 and as late as 1939. In 1929, Bohr writes that "the discovery of the quantum of action" shows us that "...any observation necessitates an interference with the course of phenomena, which is of such a nature that it deprives us of the foundation underlying the causal mode of description" indicating the former point of view, but then one sentence later writes that "...this should not be regarded as a hindrance to further advance; we must only be prepared for the necessity of an ever extending abstraction from our customary demands for a directly visualizable description of nature," indicating that our conceptualization of "the course of phenomena" as "interfered with" is only a demand of the classical worldview, in accordance with the latter point of view [Bohr, 1929/1985a, p. 249]. In 1939, in the very article in which Bohr first introduces the above novel definition of "phenomenon," Bohr still voices that measurements of quantum systems have an "essential influence on the phenomenon itself," in contradistinction to the status of measurements vis-à-vis physical systems "within the scope of classical physics" [Bohr, 1939/1996, p. 311]. Thank you to Thomas Ryckman for drawing my attention to such passages.

Conclusion

The quantum postulate—the postulated reality of the Einstein-Planck-de Broglie relations—implies the falsity of Pre-Existing Properties for measurements of position and momentum in all contexts, and so also for such measurements on EPR states. The quantum postulate implies that Pre-Existing Properties is false in such contexts because it implies that any localized atomic entity must have a spread in the momentum it actually has, and so measurements of position and momentum are not deterministically settled by the pre-measurement state of any such entity. This has clarified that remarkably instrumentalist turns of phrase in Bohr's response to EPR are actually expressions of consequences of a physical postulate, namely that different measurements of position and momentum on identically prepared states are qualitatively different physical phenomena in which such states discontinuously and indeterministically evolve into the states they do upon measurement. Because Bohr also took this response to consist in his offering his famous or infamous "complementarity" as a viewpoint that resolved the difficulties posed by EPR [Bohr, 1949/1996, p. 372], the above reconstruction of this response shows this viewpoint itself to be an attempt at the task Anton Zeilinger urges, viz. the conceptualization of the content of quantum theory through an uncontroversial and universally-acceptable "foundational conceptual [physical] principle" [Zeilinger, 1999, pp. 631-2].[50] In closing, I would like to briefly sketch this viewpoint and its merits further.

The above reconstruction shows that complementarity, insofar as it concerns itself with atomic physics, includes a conceptualization of the nature of properties of position and momentum of atomic entities. It is thus in a non-technical sense committed to the reality of atomic entities that possess such properties. The view also has a notable point of convergence with Heisenberg, namely in holding that atomic entities possess objective propensities to give a certain necessarily statistically predicted distribution of outcomes in measurements of position and momentum [Heisenberg, 1958, pp. 41, 52-3, 180-1]. But where Heisenberg advanced or at least suggested that such propensities were mere potentia, Bohr advances that atomic entities posses such propensities given the way in which they actually posses properties of position and momentum prior to measurement, namely such that their degree of spread in momentum is inversely proportional to the degree to which they are localized in space.

This latter point is worth emphasizing and contrasting with what I will call for present purposes the "textbook" view of quantum mechanics in its account of measurements of position and momentum, which will bring out that complementarity is entirely compatible with the modern account of quantum measurements that uses POVMs to model simultaneous "unsharp" measurements of observable quantities. The textbook view is characterized by the following commitments:[51]

[50] Thus, adducing the quantum postulate in response to EPR is not the assumption of the completeness of quantum mechanics per se, as Folse claims [Folse, 1989, p. 261]; rather, the postulate shows EPR's conclusion to be false on general physical grounds and informs Bohr's view of what quantum theory represents in the first place, the latter of which Folse urges in a later article [Folse, 2002].

[51] All four commitments can be found in e.g. the most recent textbook from Griffiths and Schroeter [Griffiths and Schroeter, 2018, pp. 17-19, 125-126, 133-136]. Dirac and von Neumann were the first to codify them in the quantum formalism [von Neumann, 1955, pp. 217, 351-357, 418], [Dirac, 1958, pp. 34-36].

1. Wavefunctions are superpositions of eigenstates of operators acting on a space of those wavefunctions.
2. Eigenstates of operators are entirely determinate states that give a corresponding measurement outcome with $p = 1$.
3. Measurement outcomes are given by a real numbers associated with eigenstates called "eigenvalues."
4. The process of measurement "collapses" superpositions to the eigenstate corresponding to the measurement outcome obtained.

The textbook account of position measurements in particular further holds that the amplitude of the wavefunction $|\psi(x)|^2$, integrated over some region of space, gives the probability that the particle will be found somewhere within that region of space; according to commitments (2)-(4), the results of position measurements are localizations of the atomic entity to a single point a, as they must collapse the state to a delta function centered on a, such that immediately subsequent measurements of position will once more give a with $p = 1$.[52]

The textbook account of position measurements is certainly not forced by any evidence from any natural phenomenon or experiment, in that we never observe any atomic entity being localized at single point, but rather only within some finite region. At the very least, it is more epistemically modest to model measurements as localizations of some entity within such regions. Bohr's account of position measurements is in this respect superior to the textbook one: his description of position measurements as a "proper reduction of the spatial extension of the fields" is not committed to (2)-(4), but only holds that the degree of localization of a wavefield changes upon position measurement [Bohr, 1927/1985b, p. 94]. The form of the momentum operator rules out commitments (2)-(4) for momentum measurements straightaway: the (three-dimensional) eigenstates of the momentum operator are elementary plane waves of infinite extent; measurements of momentum are measurements of entities localized in (three-dimensional) space; therefore no measurement of momentum ever results in an eigenstate of the momentum operator.

That the eigenstates of the momentum operator are elementary plane waves is a codification of the Einstein-Planck-de Broglie relations, that is, a codification of the quantum postulate. Central to Bohr's reasoning as above reconstructed is the observation that this postulate implies that no atomic entity can be localized to any degree in (three-dimensional) space and have a fully determinate momentum, and thus no actual atomic entity whatsoever can have a fully determinate momentum. Bohr's view of momentum measurements is thus emphatically not that of the textbook account, in that Bohr correctly holds that an atomic entity in a momentum-measuring device absolutely cannot be in the eigenstate of the momentum operator. This observation clarifies that momentum measurements—displacements of parts of the measuring device in interacting with the atomic entity—must be the net result of momentum exchange between atomic entities and the measuring device over multiple modes (each of which is associated with a corresponding wavelength), and that the momentum any actual (spatially localized) atomic entity can have at

[52]von Neumann as well as Dirac codified this part of the textbook view [von Neumann, 1955], [Dirac, 1958]; it appears that Pauli was the first to suggest it in print [Pauli, p. 83n1].

all must be associated with changes in how that entity is localized (say, a mass times the group velocity of a wavepacket representing the motion of an approximately free particle).

It was, then, a mistake ever to have thought that eigenstates of position and momentum operators must be the bearers of observable determinacy. Eigenstates of the momentum operator are not observable determinate states of any atomic entity because they are of infinite spatial extent; eigenstates of the position operator are not observable determinate states of any atomic entity because they are of no spatial extent whatsoever. That Bohr was sensitive to both points clarifies what he must have meant when calling the classical pictures of space-time localization and momentum-exchange "complementary but exclusive" features of the quantum-theoretic description of atomic entities [Bohr, 1928/1985c, p. 148]. He cannot have meant that exclusively momentum or position of atomic entities are ascertainable by different experimental set-ups, and that this ascertainment is of a completely determinate, quasi-classical property. For a measurement of momentum of an atomic entity involves the exchange of momentum of a measured and *thus localized* atomic entity with a measuring device. This localization (i) amounts to the only thing a position measurement could be in the first place, namely an atomic entity being localized to some degree with respect to some physical system we call a measuring device, and (ii) implies that there is an objective spread in the momentum the entity has when and after exchanging momentum with the measuring device, for the reasons rehearsed in Section 5. Momentum measurements are thus ipso facto position measurements, and the atomic entities upon having their momentum measured possess a degree of spread in their momentum as all real such entities must according to the quantum postulate; it is thus possible to measure position and momentum simultaneously—the HUR, on Bohr's view, are not a pronouncement of the impossibility of simultaneous such measurements, but a representation of the reciprocal relation in which the spatial extent and degree of spread in momentum stand for all atomic entities during all stages of measurement. The classical pictures of position and momentum thus appear to be "exclusive" for Bohr in that they are two ideal, mutually opposed poles, viz. complete localization on the one side and perfectly sharp momentum on the other, neither of which are ever themselves actually instantiated per se [Bohr, 1928/1985c, p. 148], but both of which are brought to bear on our understanding of any actual atomic entity through consideration of how localized the entity is and how sharply its momentum is defined, both of which are thus necessary for our description of atomic phenomena [Bohr, 1929/1985b, p. 288].[53,54]

[53]On the ideality of elementary plane waves in Bohr, see e.g. [Bohr, 1927/1985a, p. 97], [Bohr, 1927/1985b, p. 92], [Bohr, 1927/1985c, pp. 116, 129, 132], [Bohr, 1928/1985c, pp. 149, 156], [Bohr, 1939/1996, p. 304]. Bohr continually counterposes the ideality of such plane waves with "the idea of material particles" which are as such localized in space [Bohr, 1927/1985a, pp. 76, 79], [Bohr, 1927/1985b, p. 92], [Bohr, 1927/1985c, p. 116], [Bohr, 1928/1985c, p. 149], [Bohr, 1939/1996, p. 304].

[54]One of Bohr's observations that invites confusion in this connection is that in certain measurements of position we "cut ourselves off from" any knowledge of the momentum transfer between the particle and the rigidly bolted diaphragm through which it passes, which can appear to suggest that no simultaneous measurements of position and momentum are possible at all [Bohr, 1935, p. 697]; this observation is correct as far as it goes, but, as I have just argued, does not imply that measurements of momentum in general cannot also serve as measurements of position. With such examples, I take it that Bohr means to illustrate what approaching the classical ideal of any (known) complete localization of an atomic entity demands, namely rigidly bolting the diaphragm to a laboratory frame (otherwise, the measuring slit is liable to move during measurement, which means that the region in which the atomic entity is interacting with the diaphragm is larger than it would be if the same diaphragm were bolted, and thus further from the classical ideal), and pointing out that in such situations, there is, owing to the quantum postulate, a spread in how the atomic entity exchanges momentum with the rigidly bolted diaphragm in proportion to how small the slit is. For some reason (plausibly simply as a conse-

Complementarity is thus entirely compatible with the modern account of quantum measurements that uses POVMs to model simultaneous "unsharp" measurements of observable quantities, in that the former holds that all atomic entities are never completely localized and always have a spread in their momentum.[55] It is worth emphasizing, however, that the spread in the fully determinate momentum all atomic entities have does not in the first instance represent a probability measure over what classically observable momentum characteristic of a given atomic entity we will observe in certain experimental conditions, for the components of the spread are eigenstates of the momentum operator, none of which are themselves determinately observable. Rather, the only momentum we can observe is associated with the way the spatial localization of an entity changes, where only non-eigenstates of momentum can ever be so localized. Likewise, the only position we ever observe is an atomic entity being localized in a finite region of space, and so the simple elements (points) in terms of which this localization is represented (with $|\psi(x)|^2$) do not represent the possible states in which an entity could be observed in a position measurement.

This raises the question: of what occurrence, if not the ascertainment of a fully determinate momentum or fully determinate position, do probabilities vis-à-vis position and momentum in quantum theory represent? In broad outline, the answer must be that of finding the atomic entity of interest in a site of possible localization in an experimental arrangement in a non-eigenstate of both the momentum and position operators; this is because the only observable states of any atomic entity are non-eigenstates of both the momentum and the position operators, as these states and only these states are at all localized to any finite degree. Apparently, such a state obtains after a transition from a pre-measurement state to one of many states possible upon measurement, and the state immediately pre-measurement is what specifies the probabilities of such transitions. This speaks for understanding measurements to be discontinuous, fundamentally probabilistic transitions from ψ_i to ψ_j, where ψ_i and ψ_j are non-eigenstates both of position and of momentum, the probabilities of which are presumably given by $|(\psi_i, \psi_j)|^2$, in accordance with Born's rule.[56] In this respect, Bohr's view is like the textbook view in holding measurement events to be discontinuous changes of state, but where the textbook version of "collapse" is a transition from an indeterminate potentiality to a quasi-classical "sharp" observed property, for Bohr, measurement events of position and momentum are apparently discontinuous changes of states that are essentially quantum-mechanical through and through, namely non-eigenstates of both momentum and position. In short: Bohr apparently conceived of measurements of position and momentum as "quantum jumps" in roughly the sense he himself originated.[57]

quence of additional epistemic sensibilities), Bohr is also concerned to observe that such momentum transfer is itself unknown in such cases.

[55] See [Busch and Lahti] for details on POVMs vis-à-vis simultaneous unsharp measurements.

[56] It is worth noting in this connection that not all historical instances of Born's rule were advanced with the eigenstate-eigenvalue link. See for example [Born, 1926, pp. 865-6].

[57] To be clear, there is no reason to think that subjective acts of ascertainment (such as visually attending to an experimental arrangement) "cause" said quantum jumps, in the way that such acts are thought to cause "collapse" of the wavefunction on the orthodox or pop-science view of quantum measurement: the most natural thing to say here is that we ascertain the measurement record, which is itself a perceivable amplification of an *instance* of such a jump. The only thing we as experimenting subjects can do to make a causal difference to the quantum phenomena is change the experimental arrangement and thus consider a different experimental context—

So far, we thus see that complementarity is a view of quantum theory according to which (i) states of atomic entities at all stages of all atomic phenomena have a spread in their determinate momentum, consonant with the modern, POVM account of such measurement and (ii) measurements of position and momentum are discontinuous changes of non-eigenstates of both position and momentum, i.e. quantum jumps in the given experimental context. The last point worth considering for present purposes is the significance of experimental contexts for the scope and nature of quantum-mechanical representation. Call this the question of Bohr's "experimental contextualism."

Clearly, Bohr takes his experimental contextualism to follow from the quantum postulate (see Section 4). The above reconstruction of Bohr's response to EPR identifies a sense of this postulate—the postulated reality of the Einstein-Planck-de Broglie relations—that implies the falsity of EPR's conclusion (Pre-Existing Properties for position and momentum of both particles in an EPR state) on general physical grounds. Recall that the implied falsity of EPR's conclusion proceeds *via* an ontological conception of the HUR that does not itself rest on any appeal to experimental arrangements—rather, it is an immediate consequence of the Einstein-Planck-de Broglie relations (see Section 5). Clearly, the HUR also play a role in the quantum-mechanical formalism, conceived of as "adequate tool ... for deriving predictions, of definite or statistical character, as regards information obtainable under experimental conditions described in classical terms..." [Bohr, 1948, p. 314]. (Cf. [Bohr, 1928/1985c, p. 154].) But the above reading finds the qualitative outline of Bohr's understanding of measurements of position and momentum to be expressing further consequences of this ontological conception of the HUR, and "predictions ... as regards information obtainable under experimental conditions" certainly include predictions of outcomes of measurements of position and momentum; thus whatever role the HUR play in the quantum-mechanical formalism must be articulated through articulating further consequences of the ontological conception of the HUR in so many different experimental conditions suited to measure either position or momentum. More generally, this reading suggests that the entirety of the logic and content of the quantum-mechanical formalism so-conceived, insofar as it aims at predictions of quantities of position and momentum, must be worked out as so many consequences of the Einstein-Planck-de Broglie relations for how to conceive of said representation.[58]

In short, the above reading suggests that Bohr's experimental contextualism vis-à-vis position and

itself a different physical situation—in which different jumps can occur and be amplified such that we can perceive them with our senses.

[58] That this was Bohr's general point of view finds strong textual support in "On the Notions of Causality and Complementarity":

> Incidentally, it may be remarked that the construction and the functioning of all apparatus like diaphragms and shutters, serving to define geometry and timing of the experimental arrangements, or photographic plates used for recording the localization of atomic objects, will depend on properties of materials which are themselves essentially determined by the quantum of action. Still, this circumstance is irrelevant for the study of simple atomic phenomena where, in the specification of the experimental conditions, we may to a very high degree of approximation disregard the molecular constitution of the measuring instruments. If only the instruments are sufficiently heavy compared with the atomic objects under investigation, we can in particular neglect the requirements of [the HUR] as regards the control of the localization in space and time of the single pieces of apparatus relative to each other [Bohr, 1948, pp. 315-6].

Here, the quantum of action is presented as necessary for experimental arrangements even to have the properties they have that make them suitable as experimental arrangements in the first place, and the representation of these arrangements with classical concepts is presented as some pragmatically justifiable coarse-grained representation of configurations of essentially quantum-mechanical constituents.

momentum is a consequence of the postulated reality of a bona fide physical principle, viz. the Einstein-Planck-de Broglie relations. This is somewhat puzzling, in that these relations should also have consequences for how to think of atomic processes that do not occur in any possible experimental context narrowly construed, e.g. those that occur in the interior of the sun. But Bohr insists that the quantum-mechanical formalism is a tool for deriving predictions under well-defined experimental conditions.[59] Charity of interpretation strongly favors the above reconstruction of Bohr's thought as far as it goes, and so this interpretative puzzle calls for resolution. There may well be other charitable ways of proceeding, but two non-exclusive possibilities suggest themselves to this author: (i) Bohr had additional reasons for restricting the scope of the quantum-mechanical formalism as then constituted to the representation of occurrences in experimental contexts narrowly construed; (ii) Bohr thought the notions of "experimental context" and "classical concepts" he was invoking could be taken sufficiently loosely to make intelligible any possible truth-apt representation of quantum-mechanical states whose nature and context-sensitive propensities to change discontinuously are conceived in accordance with the Einstein-Planck-de Broglie relations.

The latter option is easiest to square with Bohr's apparent readiness to accept the reality of atomic entities outside of experimental contexts narrowly construed, for example electrons in "air showers" initiated by cosmic rays [Bohr, 1939/1996, p. 321]. It also seems ad-hoc to restrict the reality of discontinuous state transitions to experimental contexts narrowly construed; further, later Bohr indeed advances that quantum state transition is fundamentally probabilistic, and when he does, there is no indication of any such restriction–e.g. after remarking that

> ...a wholly new situation in physical science was created through the discovery of the universal quantum of action... ,

he declares that

> ...the specification of the state of a physical system evidently cannot determine the choice between different individual processes of transition to other states, and an account of quantum effects must thus basically operate with the notion of the probabilities of occurrence of different possible transition processes [Bohr, 1948, p. 313].

Note that the quantum of action is called 'universal' here and the indeterminism of state transitions that is supposed to follow from it is not qualified in any way. And if Heisenberg's recollection of Bohr's response to Schrödinger regarding whether one could hope to do away with quantum jumps during the latter's first visit to Copenhagen in Autumn of 1926 is to be believed, Bohr at one point thought that such jumps occurred *in the sun*:

> No, one can't hope for [an alternative explanation of Plank's law]. For [its] meaning has already been clear for 25 years. And besides, we see discontinuities; we see jumping [*das Sprunghafte*] in atomic phenomena immediately, for example on a scintillator or in a cloud chamber. We see that a flash of light suddenly appears on a screen or that an electron suddenly passes through

[59] Thank you to an anonymous reviewer for pushing me on this point.

> a cloud chamber. You can't simply ignore such jumping events and pretend they don't exist [Heisenberg, 1986, Ch. 6 – own translation].

The topic of discussion at this moment was apparently the meaning of Planck's formula for blackbody radiation, which famously made (at least mathematical) sense of the fact that the sun shines, and Bohr was apparently emphatic that understanding it requires admitting the reality of quantum jumps. Moreover, Bohr advances that one *sees* such jumps in the laboratory, i.e. that so-many quantum-mechanical measurements just are jumping events [*das Sprunghafte*] that we observe in a laboratory setting. Though this issue requires much more thorough study, this all strongly suggests two points. First, if and to the extent that this is possible, we should favor a liberalized reading of what Bohr means by the necessity of "experimental conditions" represented by "classical concepts" for representing atomic phenomena. Second, Bohr conceived of measurements of position and momentum as quantum jumps just like any other in nature, which simply happen to occur in a laboratory. If this is right, then despite its reputation, Bohr's view of quantum theory is arguably a more thoroughgoingly universally quantum conception of nature than both the textbook view as well as neo-Bohrian attempts to generalize experimental contexts to classical contexts broadly construed.[60]

The work of further interpreting Bohr's view as it has emerged here is considerable. Can we find an understanding of the supposed necessity of "experimental contexts" and "classical concepts" for quantum-mechanical representation that is consonant with the view as so far reconstructed? Precisely how does continuous Schrödinger evolution fit into the dynamics of quantum states, given that it is apparently not universal according to Bohr? When and why do quantum states "jump" into other ones? How do the time-energy uncertainty relations figure in the kinematics and dynamics of quantum systems? Are quantum properties of position and momentum to be conceived relationally? And so on. Regardless of how these questions are to be answered, however, the above reconstruction undoubtedly shows that behind certain instrumentalist trappings of early Bohr's language, one finds an earnest attempt to conceive of the objective content and workings of quantum theory. Because this attempt worked out consequences of a physical principle unquestionably standing at quantum theory's foundations, it is one worth taking seriously for those who take such foundations seriously.

[60] For one example of such a neo-Bohrian attempt, see [Auffèves and Grangier, 2016]. For an objection that such an approach makes quantum theory non-universal, see [Brukner, 2017].

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