Essentially Comparative Value does not Threaten Transitivity

Toby Handfield

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Abstract

The essentially comparative conception of value entails that the value of a state of affairs does not depend solely upon features intrinsic to the state of affairs, but also upon extrinsic features, such as the set of feasible alternatives. It has been argued that this conception of value gives us reason to abandon the transitivity of the better-than relation. This paper shows that the support for intransitivity derived from this conception of value is very limited. On its most plausible interpretations, it merely provides a necessary, but not sufficient, condition for intransitivity. It is further argued that the essentially comparative conception of value appears to support a disjunctive conclusion: there is incommensurability of value or betterness is not transitive. Of these two alternatives, incommensurability is preferable, because it is far less threatening to our other axiological commitments.

1 Introduction Contrast two ways of thinking about the value of a possible outcome of our actions. On the first approach, its value is entirely determined by the intrinsic properties of that state of affairs. A second hypothesis is that the value of the outcome can depend on extrinsic matters. Note that both hypotheses are about the value of an end – an outcome – rather than a means. So this is not the distinction between intrinsic and instrumental value. These are two hypotheses about the way in which an outcome can possess value, as an end. Consider the value that inheres in a gourmand's consuming and enjoying a fine meal. On the first hypothesis, this value might consist entirely in the various psychological, gustatory, and digestive states which are intrinsic to the state of affairs. On the second hypothesis, these factors may still be relevant, but other factors, not intrinsic to the state of affairs itself, may also contribute to its value. For instance, *what other meals were possible alternatives* might be one such extrinsic factor that is relevant to the value of the state of affairs.

If the second proposal were true, would this undermine the idea that the better-than relation is transitive? Larry Temkin [1] argues that it would. Temkin's argument is that we

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have good reason to think that value is 'essentially comparative'. The essentially comparative account of value is intended to contrast with an 'internal aspects' view – the idea that the value of an outcome depends only on its intrinsic properties. If value is essentially comparative, Temkin claims, we have good reason to think that the *better than* relation violates both transitivity and acyclicity.

In this paper, I show that the essentially comparative conception of value provides very limited support for intransitivity.[1] Temkin's characterisation of essentially comparative value admits of two readings: a stronger and a weaker. The weaker version provides a necessary, but not sufficient condition for intransitivity. The stronger version certainly fails to provide a sufficient condition, and depending on how the details are worked out, may not even be necessary. Moreover, instances of the stronger view which best support Temkin's ideas about transitivity will also threaten the asymmetry of *better-than*. This suggests that there are very general grounds for thinking that, even if we accept the essentially comparative nature of value, transitivity cannot be abandoned without unacceptable costs to our conceptual economy.

2 What is essentially comparative value? Temkin characterises the essentially comparative conception of value in a number of ways, not all of which appear to be identical. Here is one quotation, in which he 'roughly' characterises the view:

> [T]here is at least one outcome, O, such that there is no answer to the question of how good O is all things considered based solely on O's internal features; or, even if, for each outcome, O, there is an answer to the question of how good O is all things considered based solely on O's internal features, there are at least two outcomes, O_1 and O_2, such that how O_1 compares with O_2 all things considered is not simply a function of the extent to which O_1 is good, all things considered, based solely on O_1's internal features and the extent to which O_2 is good, all things considered, based solely on O_2's internal features. [1, 371]

In this passage, Temkin is entertaining whether betterness *is a function of* goodness, where goodness is *based solely on* the internal features of a state of affairs. Without undertaking extensive examination of the precise meaning of these phrases, I believe it is safe to assume "A is based solely on B" implies that A is a function of B. Moreover, the *is a function of* relation is transitive. So unpacking the quotation using these assumptions, Temkin is denying two distinct functional dependence claims.

1. The degree of goodness of a state of affairs A is a function of the intrinsic properties of A.

1. Here, *pace* authorities such as [2, 216], I follow what has become widespread usage, to use *intransitive* (rather than *nontransitive*) to mean not transitive, as opposed to never transitive.

2. The relative value of two states of affairs A and B is a function of the intrinsic properties of A and the intrinsic properties of B.

Despite Temkin's presentation, which implies that (1) could be true while (2) is false, this cannot be right. One thing is better than another if the first has a greater degree of goodness than the other. So if each of A and B have degrees of goodness determined by their respective intrinsic properties, then their relative value is determined also.[2]

For convenience, let us treat the denial of each of these claims as a distinct interpretation of what Temkin might mean by essentially comparative value. The denial of (1) is a relatively weak claim, which for mnemonic purposes we can call *no intrinsic monadic goodness*. The second view – the denial of (2) – is a stronger claim: *no intrinsic binary betterness*.

In addition to these relatively abstract characterisations of Temkin's view, he makes an ancillary claim about the way evaluative factors are sometimes significant and sometimes not, depending on the objects to be compared. We will consider this view also in what follows.

3 Implications for transitivity

3.1 The weak denial is necessary, but not sufficient, for intransitivity It would appear that, if an outcome's degree of goodness is a function of its intrinsic properties, and degree of goodness determines the overall ranking in terms of betterness, then transitivity follows [3, 293]. So if Temkin can establish *no intrinsic monadic goodness*, he will have established at least a necessary condition for the violation of transitivity.

Is this claim *sufficient* for transitivity to fail? It is not. Consider four objects, named for their heights: 11, 12, 13, and 15 inches tall. Define the relation *more fleepish* such that x is more fleepish than y if and only if x is two or more inches taller than y, otherwise x and y are unranked. 15 is more fleepish than 13. 13 is more fleepish than 11. In accordance with transitivity, 15 is more fleepish than 11. Indeed, the comparative fleepishness relation generates a strict partial order over any set of objects with determinate heights: it is a transitive and irreflexive (and therefore asymmetric) relation. Because various pairs of objects (e.g. 11 and 12) will be unranked with respect to their relative fleepishness, it is not the case that each object has a degree of fleepishness, determined by its intrinsic nature alone. It is only in comparing two objects that we can determine their relative fleepishness. Hence by analogy, transitivity of betterness can survive, even if (1) is false.

2. The claims are not equivalent, however. There may be partial orders of value in which facts about betterness depend on the intrinsic properties of the objects to be compared, while neither has a "degree" of goodness. Tolstoy's creative works may be a greater artistic achievement than Turgenev's, but both may be incomparable with Tchaikovsky's. These facts about relative value supervene on intrinsic properties of the artworks, so (2) is true; but there can be no degree of goodness associated with each, on pain of denying incomparability – for if they all had degrees of goodness, they would all be comparable: hence (1) is false.

So the weaker version of essentially comparative value is necessary, but not sufficient for the violation of transitivity. Given this limited result, we might expect that there is a tighter connection between the second interpretation and transitivity.

3.2 The strong denial is not necessary for intransitivity Temkin appeals to the idea that, in analysing value, different factors will be relevant, depending on what is being compared [1, 229]. So for instance, the following three claims about a hypothetical collection of objects is representative of the trouble we are supposed to face with moral evaluation.

3. In comparing x and y, we should appeal to factor F, and we will conclude that x is better than y.
4. In comparing y and z, we should appeal to factor F, and we will conclude that y is better than z.
5. But in comparing x and z, factor F is *not relevant*, so we may conclude that z is better than x. (Even though x would do better than z, if factor F was relevant in making our comparison.)

The above three claims generate a cycle of better-than relationships, so (assuming asymmetry) they would entail that better-than is not transitive.[3]

But what exactly do examples like this show with respect to claim (2)? If the relevance of factor F depends upon the intrinsic natures of the items to be compared, this example is consistent with (2). To illustrate, consider the following example.

> Suppose that x is *more ertnog* than y if and only if: (*i*) x and y differ in height by 2 or more inches and x is taller than y, or (*ii*) x and y do not differ by 2 or more inches in height and x has a greater mass than y.

One could recast this idea by saying that in comparing two objects for their degree of ertnog-ness, *mass is only relevant* if they differ by less than two inches in height. And indeed, for the objects in the table below, comparative mass is a relevant factor when comparing A with B, when comparing B with C, but not when comparing A with C. Consequently: A is more ertnog than B, which is more ertnog than C, which is more ertnog than A. Transitivity is violated, but in all cases, one's being more ertnog than the other depends entirely on the intrinsic nature of the objects compared.

Whether or not x is more ertnog than y is a function of the intrinsic natures of x and y, but the relation of being *more ertnog than* is not transitive. So by analogy, better-than may violate transitivity, even if (2) is true. In other words, the stronger version of Temkin's essentially comparative value – *no binary intrinsic goodness* – is not necessary for intransitivity.

3. The idea that different factors can be relevant or irrelevant, depending on the context of evaluation – has been defended elsewhere [4, 5, 6], though these arguments have tended to focus on *reasons* or *duties*, rather than upon goodness itself.

	Height (inches)	Mass (lbs)
A	11	5
B	12	4
C	13	3

3.3 Is the strong denial sufficient for intransitivity? If (2) is rejected, then whether A is better than B is not a function of the intrinsic properties of A and B. How then, should the betterness relation be understood? Until now, I have simply assumed that a relation can be identified with its extension: a set of ordered pairs, such that the first item in each pair is better than the second. This proposal will not work if it is not unequivocally the case that the pair (A, B) is or is not in the extension of the betterness relation, but that the betterness facts depend on some further factors.

In a number of places, Temkin suggests that what betterness relations obtain depends in part on the set of *feasible alternatives*.[4] Accordingly, instead of treating betterness as a binary relation we can treat it as a ternary relation holding between a set of alternatives, an outcome, and another outcome; or equivalently, as a function from the set of alternative-sets to binary relations between outcomes. We denote this binary relation, for a given alternative-set q, as $x >_q y$. This relation could be glossed as: "x is better than y, given that the set of feasible alternatives is q".[5] The question arises as to how to understand transitivity on either of these ways of conceptualising matters, and the natural option is to take it to amount to the transitivity of the binary relation $>_q$ for an arbitrary (but fixed) q, i.e., as the following:

> For all sets of feasible alternatives q, for all x, y, z, if $x >_q y$, and $y >_q z$, then $x >_q z$.

The proposal, in other words, is to hold fixed a set of feasible alternatives, and test for transitivity within the binary relation derived from that particular set. Betterness is "transitive" if and only if, for each possible set of feasible alternatives, the value of the betterness-function is a set of ordered pairs which is *itself* a transitive relation, as understood in the orthodox way. Understood this way, the strong version of essentially comparative value is consistent with transitivity.

Indeed, Temkin has no arguments to show that betterness will violate transitivity in

4. Melinda Roberts [7, 310] emphasises this interpretation.

5. Note, we can set aside the idea that, for all x, y, the uniquely appropriate set of alternatives is just $\{x, y\}$. This proposal would make betterness similar to the ertnog relation from the previous section: a case which is consistent with the truth of (2), so it is not an 'essentially comparative' view.

this sense. Rather, his examples involve something like the following:

$$A >_q B$$
$$B >_q C$$
$$\neg(A >_r C)$$

So the strong denial is not sufficient for intransitivity.[6]

For Temkin to build a case that we should reject the transitivity of betterness, he must convince us to understand transitivity differently, as a property of betterness *across different sets of feasible alternatives*. So for instance, we could say that betterness is "super-transitive" iff:

> For any two sets of feasible alternatives, q, r, for all x, y, z, if $x >_q y$, and $y >_q z$, then $x >_r z$.

Understood this way, Temkin's claim is that betterness is not super-transitive, and that this is the variety of transitivity we ordinarily ascribe to betterness. Temkin claims that we need something like super-transitivity to enable us to infer evaluative relations between different decision contexts, involving different sets of feasible alternatives [1, §13.1]. Consider an agent who has determined in one context that Apples are better than Bananas, and also that Bananas are better than Carrots. Such an agent, placed in a new context (where the feasible alternatives have changed), is asked whether Apples are better than Carrots. Admittedly, such an agent cannot infer from her earlier observations and the transitivity of betterness an answer to this question.

Following Voorhoeve, who makes a proposal (see note 6 above) that runs into an analogous complaint, I believe this inferential benefit is overrated by Temkin, and that the more modest transitivity property is still very important. The transitivity of *better-than* entails that our decisions can be *consistent* with betterness, in the sense that we are guaranteed that at least one option is *not worse* than any other option [8, 409–10]. If transitivity is violated, then we can have decision problems in which every option is worse than some other option. Consequently, it is unclear how our choices can be guided by betterness. This is reason enough to think the variety of transitivity described here is very important, even if it does not allow the sort of inferences Temkin has in mind.

6. Another proposal is canvassed by Alex Voorhoeve [8, 414–6], who suggests that we can individuate the states of affairs that stand in betterness relation more finely. So instead of seeking whether A and B stand in the better-than relation, we should more properly seek whether A-from-feasible-set-q is better than B-from-feasible-set-q. And Voorhoeve claims that all Temkin's alleged cases of transitivity-violation in fact involve equivocating on the relata. So for instance, Temkin might show that $A_q > B_q$; $B_q > C_q$; and $\neg(A_r > C_r)$. This does not suffice to violate transitivity.

   Voorhoeve's analysis is another attractive way to interpret Temkin's claims about value, but it is also *consistent* with the truth of (2). So although this proposal is similar in spirit to the analysis I offer, it begs the question against the essentially comparative view.

We can press the point further. If Temkin is correct that practical reason is dependent upon there being transitivity across different sets of feasible alternatives, then we should presumably also care about whether properties, such as asymmetry, hold across different sets of alternatives. That is, we usually assume that if A is better than B, B is not better than A. But do these properties of betterness hold across different sets of feasible alternatives? If $A >_q B$, does it follow that, for all r, $\neg(B >_r A)$? Call betterness super-asymmetric if it has this property.

To establish anything conclusively here is difficult, but considering one of Temkin's examples shows that there is at least prima facie support for the idea that super-asymmetry will be violated by betterness. Consider Temkin's discussion of the mere-addition paradox ([1, §12.1], originally in [9, §142]), in which we compare three populations, A, $A+$, and B (see figure 1). A is a population of uniformly high wellbeing. A and $A+$ differ by the mere addition of a population of individuals who have modest wellbeing, but lives worth living. B contains the same individuals as in $A+$, but they have equal wellbeing and although the individuals from A have lower wellbeing in B, the total wellbeing is significantly higher than in $A+$.

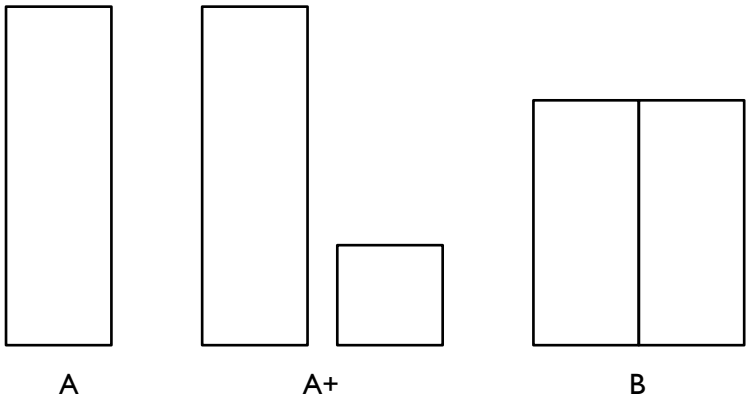

Figure 1: The mere addition paradox.

Temkin suggests that A is better than B, given these are the only two alternatives.[7] He also claims that B is better than $A+$, given these are the alternatives. Finally, he claims that $A+$ is at least as good as A, which violates the transitivity of betterness. He goes further, and suggests there could be variations on A whereby $A+$ is strictly better than A. In my

7. In the particular discussion I focus on here, Temkin addresses the relative value of these states of affairs with respect to utility; but it is clear that he thinks it possible that other circumstances could be such that utility is the overwhelmingly important factor for evaluation of these populations, so that the judgments we reach could be judgments about all-things-considered betterness. I will omit this complication in the following discussion.

preferred interpretation, these claims amount to:

$$A >_{\{A,B\}} B$$
$$B >_{\{A^+,B\}} A^+$$
$$A+ >_{\{A,A^+\}} A$$

Note that Temkin does not – as far as I am aware – commit to any evaluations of pairs of states, given *all three* are alternatives. One possibility is that, given all three, A is uniquely the best outcome. Therefore $A >_{\{A,A^+,B\}} A+$. Another possibility is that B is uniquely best, given all three. Hence $B >_{\{A,A^+,B\}} A$. This latter claim is especially plausible because B has the highest total utility. But if either of these is true, then betterness violates super-asymmetry. Betterness relations that obtain, given one set of feasible alternatives, are reversed, given a different set of alternatives.

While there are of course other possibilities that are consistent with super-asymmetry of betterness, my point is simply that, if we are open to the shiftiness of axiology that Temkin advocates, there does not appear to be any a priori reason to be confident that betterness will respect this property. I suggest the lesson to draw is that if Temkin's denial of (2) is correct, we must be careful not to inadvertently employ a plurality of betterness judgments that equivocate on the set of feasible alternatives. Doing so could radically undermine the coherence of our betterness judgments. This may be a serious limitation – and it is an important result to have established – but it is much less damaging to our understanding of practical reason than abandoning the transitivity of betterness, given a fixed set of feasible alternatives.

4 Must we accept incommensurability? In showing that the denial of (1) does not entail intransitivity, I used an example of a relation that generates only a partial order: some pairs are unranked or incommensurate with respect to fleepishness.[8] The question arises: is this a price that must be paid? If the weaker version of essentially comparative value is accepted, and we retain transitivity, must there be incommensurability of value?

Suppose, for reductio, that (1) is false and that a collection of 3 or more objects stands in a total, transitive ordering with respect to betterness. So we are to suppose that:

6. The degree of goodness of a state of affairs does not functionally depend on the intrinsic properties of that state of affairs. (The weak denial.)
7. A is better than B; B is better than C; A is better than C. And for any other objects in the domain of evaluation, betterness relations hold between any two objects, and the betterness relation is transitive. (There is no incommensurability of value.)

8. I do not here intend to draw any further distinction between the various species of incomparability, such as rough equality, being on a par, incommensurability, etc.

By familiar strategies for the construction of an ordinal utility scale, it will be possible to represent this ordering with a function that maps objects to natural numbers. So there exists a utility function u which maps states of affairs to numbers, such that x is better than y iff $u(x) > u(y)$. This puts our initial assumption under considerable pressure, because we have shown that it is at least possible to *represent* something that plays the same functional role as value, where this value is a function purely of the object of evaluation.

This argument may not be decisive, but it is highly suggestive. If the implication is correct, that given the failure of (1), we must choose either intransitivity or incommensurability, is there a reason to prefer that we preserve transitivity, even at the cost of incommensurability? That question is beyond my remit to answer conclusively, but I believe there is a strong prima facie case to retain transitivity:

First, on grounds of intellectual conservatism, there is already widespread acceptance of the existence of incommensurate value. Violation of transitivity, however, is highly unorthodox.

Second, there are money pump arguments to show that both intransitivity of preference and incompleteness of preference are liable to exploitation [10]. But to defend intransitivity against a money pump will almost certainly require a more radical proposal: that sometimes agents ought to make decisions *contrary* to their preferences. To defend incomplete preferences against a money pump merely requires that we have some additional criteria of choice when choosing between *unranked* options. An agent using a strategy such as status quo maintenance when choosing between unranked options, for instance, will be protected against a wide range of possible manipulations [11]. While status quo maintenance may not be classically rational, it would appear much easier to defend as rational than counter-preferential choice.

Third, some metaethical views appear to give us reason to think that our evaluations may not generate a complete ranking over all alternatives. Constructivist, or response-dependent accounts of value may well entail that limitations on our cognitive power will entail limitations on the range of the betterness relation. The strongest case for insisting that our evaluative relations must be complete, on the other hand, comes from implausible theories of revealed preference [12].

A final, indirect consideration is that some of Temkin's specific thought experiments – in particular his spectrum cases – have been interpreted as exemplifying incommensurability of value or indeterminacy, rather than intransitivity [13, 14, 15]. If these arguments are successful, they suggest that not only do we have the foregoing reasons to prefer incommensurability in general, but that in some of the particular cases that appear to threaten

transitivity, this preference has additional force.[9]

Monash University

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