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The philosophy of the concept and the specificity of mathematics

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Within narratives of twentieth-century French philosophy, Jean Cavaillès occupies the unusual position of being widely referenced whilst rarely being cited beyond a few stock phrases. He is frequently invoked alongside a litany of other founding figures of the so-called French 'epistemological tradition' (principally in conjunction with Gaston Bachelard, Georges Canguilhem and Alexandre Koyré), but the function of such indexing is generally only to outline, in highly abstract terms, a set of general methodological heuristics or conceptual orientations as background to the work of various luminaries of the 1960s' philosophical moment in France. In particular, Cavaillès is referenced for a single philosophical formula opposing the 'philosophy of the concept' to the 'philosophy of consciousness', a rather opaque disjunction which is imbued with rhetorical force via the mythologization of his double stature as a mathematician-philosopher and a resistant, executed by the Nazi occupying forces. Thus one finds that for all that Cavaillès is recognized as a precursor to more storied philosophical trajectories, he remains just that: a signpost towards later developments, a collection of key words.[1]

1. On Cavaillès's chequered reception, see Knox Peden, *Spinoza Contra Phenomenology: French Rationalism from Cavaillès to Deleuze*, Stanford University Press, Stanford CA,

This etiolated figure cast by Cavaillès in the reception history of twentieth-century French philosophy is not simply a contingent effect of the profoundly technical (on occasions borderline gnomic) nature of Cavaillès's writings. It also has deep roots in the philosophical programme that can be discerned therein, for there are intrinsic conceptual reasons that make it resistant to being exported into other domains. Indeed, I shall argue, the intrinsically restricted nature of Cavaillès's programme for mathematical philosophy is inseparable from its philosophical value. For Cavaillès's 'philosophy of the concept' is a theory of the *specificity of mathematics*, in a double sense: on the one hand, mathematics is defined as the domain of the production of *specific rational contents*; on the other, this very thesis, according to which mathematics is defined *by* its specificity, serves to erect an absolute demarcation of mathematics *from* every other domain of intellectual activity. In what follows I outline these two senses of specificity, each of which will be shown to bear upon respective incommensurate uses of the notion of the *singular* in Cavaillès's philosophical lexicon, and thus on the stakes of his much-vaunted 'Spinozism'.

The first sense: mathematics as rational specificity

In a letter to fellow radical Protestant Étienne Borne, written on 7 October 1930, Cavaillès returned to a polemic he had been developing against the Catholic philosopher Gabriel Marcel:

> I even wonder to what extent it is possible to attain the true naivety of the Saint without a prior submission to this necessity

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2014. In what follows I put aside a rich tradition of works in France that directly develop a 'Cavaillèsian' programme in mathematical epistemology *stricto sensu*: Jean-Toussaint Desanti, *Les Idéalités mathématiques*, Éditions du Seuil, Paris, 1968; Houyra Benis Sinaceur, *Corps et modèles: Essai sur l'histoire de l'algèbre réel*, VRIN, Paris, 1991; Alain Michel, *Constitution de la théorie moderne de l'intégration*, VRIN, Paris, 1992; Christian Houzel, Didier Nordon, Xavier-Francaire Renou, Henri Roudier and Jean-Jacques Szczeciniarz, *Pour Cavaillès*, Pont 9, Paris, 2021.

> which manifests the approach of God, immanent in mathematics, transcendent in love. And it is here that I locate my grievance against Marcel, his ignoring of the absolute value of the intelligible, of the rational: there is something divine even in the concept, *at least in the passage from one concept to another*. And it is here that we have the true Spinozist ontology, incomplete, but definitive in what it asserts.[2]

Doubtless, these are the words of the 'young' Cavaillès (aged 27) and cannot be taken to represent his mature philosophical perspective, not least because the theological context of this statement would not be explicitly endorsed by the later philosophico-mathematical essays. Nevertheless, these lines introduce a central theme of Cavaillès's philosophical project, namely the definition of mathematics in terms of its *movement*, a thesis that will be resumed in the closing lines of the posthumously published *On Logic and the Theory of Science*, where it will again be connected with Spinoza. I have in mind the sentence which immediately precedes Cavaillès's celebrated invocation of an opposition between a philosophy of consciousness and a philosophy of the concept, which concludes both that text and Cavaillès's extant writings. Given the central role played by these lines in Cavaillès's posterity, they bear citing in context:

> [O]ne of the essential problems for the doctrine of science is precisely that progress cannot be a mere increase in volume by juxtaposition, the prior subsisting with the new, but must be a perpetual revision of contents by way of deepening and erasure [*rature*] ... *Progress is material or between singular essences* [*essences singulières*], its motor the demand that each of them must be surpassed. It is not a philosophy of consciousness but a philosophy of the concept that can yield a doctrine of science. The generative necessity is not that of an activity, but of a dialectic.[3]

2. Jean Cavaillès, 'Lettres à Étienne Borne (1930–1931)', *Philosophie*, vol. 107, no. 4, 2010, pp. 13–45, p. 28; emphasis added. Translations from Cavaillès are my own unless otherwise noted.

3. Jean Cavaillès, *Œuvres complètes de philosophie des sciences*, Hermann, Paris, 1994, p. 560. Translation from *On Logic and the Theory of Science*, Urbanomic, Falmouth, 2021, pp. 135–6; emphasis added. Hereafter references to Cavaillès's main works will be cited from the *Œuvres complètes* as *OC*, and references to Mackay and Peden's translation of *On Logic and the Theory of Science* as *LTS*.

This passage is striking for the characteristic density of references implicitly invoked by Cavaillès, as it mobilizes his interpretation of results at the forefront of then-contemporary mathematics against Hegel ('juxtaposition'), Husserl and Kant ('philosophies of consciousness'), and Brunschvicg and Brouwer ('activity'). It is thus worth noting the positive valence given to Spinoza, the interpretation of whom in terms of a doctrine marked by the syntagm *essences singulières* is a cornerstone of French Spinozism.[4] In characterizing his notion of the progress of mathematics as being 'between singular essences', Cavaillès makes it clear that his theory of the becoming of mathematics belongs to the perspective of rationalist nominalism: the value of the rational – for which mathematics will serve not only as the paradigmatic but as the exclusive domain – lies not in its generality, but rather in the production of specific contents.

Cavaillès's understanding of mathematics in terms of specificity is best situated against the background of his primary philosophical interlocutors within post-Kantian reflections on mathematics and logic. Despite profound divergences in theoretical perspective, the elaboration of which would take us beyond our present purposes, Kant, Bolzano, Frege, Husserl and Carnap (each of whom plays an important role for the conjunctural intervention made by Cavaillès in *On Logic and the Theory of Science*) all in different ways praise mathematics for its universality or generality. As a heuristic, this conception of mathematics can be understood as being made up of two interrelated theses. On the *intra-mathematical* level, mathematical concepts are taken to be characterized by the fact that they intrinsically refer to all

4. An important reference here is to Léon Brunschvicg, *Les Étapes de la philosophie mathématique*, Librarie Félix Alcan, Paris, 1912, with which Cavaillès was familiar. In the course of Brunschvicg's defence of 'mathematism', he enters into polemic with Hyppolite Taine for having placed mechanistic philosophy 'under the patronage of Spinoza, that is to say, of the philosopher who saw most clearly the vanity of all classification into faculties as well as of all general ideas, who most insisted on the indefinite complexity of singular essences' (p. 563).

of the possible cases (or constructions) falling under a concept: naively, the concept 'triangle' refers not to any *particular* triangle, but immediately to all possible triangles. Further – whence the intrinsic nature of the reference – this concept does not point to some external, empirical set, with all the attendant problems of drawing a boundary to said reference, but rather exhibits each and every triangle, without remainder, owing to the fact that the concept contains its own rule of construction. This intra-mathematical generality founds Kant's focus on mathematics as paradigmatic for exhibiting the 'something = *X*', the Fregean project of mathematized logic as investigation into the domain of 'all that is thinkable', the Husserlian phenomenological focus on the 'object in general', and Bolzano's insistence that by calling the laws of formal mathematics '*general* [*allgemeine*], I mean it to be understood that mathematics never deals with a single thing as an *individual* but always with whole *genera* [*Gattungen*]'.[5] This intra-mathematical articulation of the generality of mathematics qua *indifference to content* can then be taken to ground a second level of generality, which we can call *generality of application*. The paradigm here is the application of mathematics to physics: it is precisely in so far as we take mathematical concepts to be 'pure' or 'empty' (devoid of any reference to a particular case) that we can understand the mathematization of a physical theory as bestowing on it an absolute generality, independently of any of the contingencies of the experimental situation. Hence, to mathematize is to de-particularize. The generality proper to mathematical concepts thus serves as a model for the more problematic ascription of generality to empirical concepts.

We shall return to the problem of the relation between these two levels of generality, but for now I will focus on Cavaillès's relation to the former, intra-mathematical thesis. The schema

5. Steve Russ, ed., *The Mathematical Works of Bernard Bolzano*, Oxford University Press, Oxford, 2004, p. 94.

just outlined is a simplification, but is fruitful for situating the way in which Cavaillès's epistemological investigations into modern mathematics were orientated by a fundamentally different problem. In short, Cavaillès's interest was in the ways in which mathematical objects are situated in fundamentally different ways within different formal settings, and in defining mathematics in terms of the mode of passage between these different settings. To choose an elementary example, when over the course of the nineteenth and twentieth centuries the operators of elementary arithmetic (such as the operation '+') were reconstituted within the framework of modern number theory, there is an obvious sense in which we can view this process as the extension of a particular gesture of 'generalization' central to classical algebra: in moving from the statement '1 + 2 = 3' to the general form '$a + b = c$', we transition from a statement bearing on particular objects (the intuitively understood whole numbers) to a statement bearing on generic objects (any arbitrary whole number). This is the 'moment of the variable', which Cavaillès takes to be one (but only one) of the fundamental operations of abstraction in mathematics, which he will variously name as 'idealization', 'generalization' or 'paradigmatic abstraction'.[6] Yet the identity between the elementary '+' and the abstract '+' is not something given, but rather is only constructed from the standpoint of the higher theory. We could pick numerous examples of such passages in the history of mathematics, but the conceptual point remains the same: mathematical history presents us with a sequence of reformalizations of its own basic notions, each of which can be seen in a certain sense as determinate complications of the 'same' operation, but none of which can strictly be identified with each other within a single unitary framework. Further, it is this very difference between different mathematical

6. *OC*, p. 511/*LTS*, p. 75.

theories that, for Cavaillès, constitutes the essence of mathematics itself, for 'each independent part of mathematics possesses its own modes of concatenation [*ses modes propres d'enchaînement*], which characterize it.'[7]

A fundamental (and unresolved) problematic of the Cavaillèsian programme thus becomes that of accounting for the identity within difference of mathematical operators and objects in these passages between different specific domains. Given that there is no meta-framework which can finally individuate the 'reality' of a particular operation, in what sense can we speak – as Cavaillès frequently does – of the re-situation of old notions within new frameworks as being an enrichment or 'extension' of the *same* notion? How are we to understand Cavaillès's recourse to formulations stating that new mathematical concepts contain 'more' content (or are 'deeper' or 'more profound') than prior concepts, given that his theory denies any possible field of comparison that could ground (in, say, quantitative terms) the idea of an 'increase' in intelligibility? These problems were acute for Cavaillès given that a large part of his theoretical work – essentially shaped by a sympathetic engagement with the Hilbertian formalist programme – was rigorously positioned against two programmes for constructing 'external' measures by which the identity of mathematical notions could be assured. On the one hand, there was the 'logicist' programme, exemplified by Frege and Russell, which Cavaillès stridently opposed as a reactivation of a Leibnizian ideal of a universal combinatory or 'theory of forms' seeking to enumerate (simultaneously) all of the possible forms of mathematical rationality. On the other, there were the various programmes for 'finitism', 'intuitionism' and 'arithmetism', which, starting from a basically Kantian inspiration, attempted to ground mathematics in a secure domain of

7. *OC*, p. 663.

intuitively graspable objects (be they 'whole numbers', 'marks', etc.) on which mathematical construction could be grounded. A large part of Cavaillès's first-order epistemological work can be read as a detailed engagement with the difficulties of the latter programme, with his judgement being ultimately negative, since 'the demand for possible arithmetization (Kronecker-Brouwer) is a misunderstanding of what is specifically mathematical: the unlimited procession of original intuitive modes.'[8]

In this notion of a procession between different intuitive modes, we find another prefiguration of the final doctrine of the passage between *singular essences* that closes *On Logic and the Theory of Science.*[9] The attempt to exhibit the specificity of mathematics so understood inaugurates a norm for reading its history, which orientates Cavaillès's epistemologico-historical writings. Cavaillès's commitment to the history of mathematics is thus downwind of his commitment to theorizing specificity: it is because the theoretical edifice is intended to show the progress (or production) of singular essences that it is necessary to investigate the genesis of these essences in particular historical documents, in the exact formulations made by existing historical mathematicians. In the remainder of this section I shall focus on two closely related themes that emerge in Cavaillès's elaboration of this theory of mathematics.

Operator–object duality and the necessary generation of new concepts

Among the texts that exerted a profound influence on Cavaillès's philosophical programme, a special place should be accorded to

8. *OC*, p. 579.

9. I am here gliding over significant developments that occur in Cavaillès's thought between the periods of the composition of his doctoral studies and *LTS*, in particular with respect to the concept of intuition. For an account of some of these, and their relation with the specific technical problem of 'effective calculability', see my article 'The Effective as the Actual and as the Calculable in Jean Cavaillès' (*Noesis*, 2022).

Dedekind's 1854 Habilitation address, which opens by stating Dedekind's intention to focus on 'the general manner in which, in the progressive development of this science, new functions, or, as one can equally well say, new *operations* [*Operationen*], are added to the chain [*Kette*] of previous ones'.[10] We find here a fundamental problem that will inform Cavaillès's work: the conjunction of *neccessity* and *creation*. The problem that Dedekind considers is the movement involved in the introduction of 'ideal' objects and operations in the development of mathematics. On Dedekind's view, whilst every science develops through the gradual introduction of new notions, the signal feature of mathematics is that such a process of introduction is necessary: the extension of the domain of objects and operations emerge from the kernel of the initial definitions in a regulated manner.

> [I]n this mathematics is distinguished from other sciences – these extensions of definitions no longer allow scope for arbitrariness but follow with an absolute necessity from the earlier primitive definitions, provided one applies the principle that the laws which flow from the initial definitions and which are characteristic for the concepts that they introduced have *universal validity* [*allgemeingültig*]. Then these laws conversely become the source of the generalized definitions if one asks: How must the general definition be conceived in order that the discovered characteristic laws be always satisfied?[11]

Dedekind's initial focus is on the reciprocal extension of the field of objects (i.e. the Z*ahlgebeit,* the number domain) and operations in the development of arithmetic and algebra. As Cavaillès states: 'Necessity intervenes here in a double movement.'[12] Taken from one side, the extended application of the basic arithmetical operations of addition, multiplication and exponentiation and

10. Richard Dedekind, 'On the Introduction of New Functions in Mathematics', first widely circulated in 1932 in *Gesammelte mathematische Werke III*, pp. 428–43. Translation from William Ewald, ed., *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, vol. 2, Oxford University Press, Oxford, 1996, p. 755.
11. Ewald, ed., *From Kant to Hilbert*, pp. 756–7, translation modified in line with Cavaillès's interpretation of the passage at *OC* 61.
12. *OC*, pp. 61–2.

their inverse operations immediately necessitates that we 'create the entire existing domain of numbers anew': the rigorous filling out of the space implied by these basic operations and objects (starting, that is, with an object domain restricted to the positive whole numbers), leads us to 'the negative, rational, irrational, and finally also the so-called imaginary numbers'.[13] Taken from the other side, a correlative modification of the domain of operations is now necessary, as they were not initially well defined for all of the objects (i.e. number classes) that have been occasioned by their application: for example, exponentiation initially has no meaning for the case of negative numbers or fractions. It must therefore be redefined in a more general setting by giving the general theorem for the addition of exponents, but this is to situate the operation of exponentiation on a higher plane, to give it a new meaning. Hence, the generation of a new definition such as, in our case, the replacement of concrete numbers by abstract variables in the law of exponentiation '$x^{a+b} = x^a \cdot x^b$', exemplifies a process in which 'every posited definition immediately generates a connecting thread with the existing system, but it is the whole bundle of them that is, in reality, to be understood as the new definition, which only condenses them to the highest degree'.[14]

Dedekind has here sketched a research programme into the introduction of new operations in mathematics. However, it contains a basic tension between two concepts that would traditionally be seen as opposed: necessity and creation (or 'generation' [*Erzeugung*]). Naively, if the 'new functions' introduced to account for the expanded application of operations were necessary, why were they not there already? Indeed, from a Kantian perspective the conjunction is nonsensical: given that, from the standpoint of the *Critique of Pure Reason* at least, necessity and universality are identified as co-constitutive characteristics of *a priori*

13. Dedekind, 'On the Introduction of New Functions in Mathematics', p. 257.
14. *OC*, pp. 61–2.

judgements, it makes little sense to speak of the necessary emergence of 'new functions', as per definition necessary and universal concepts should be *a priori*.[15] Cavaillès himself notes that Dedekind does little to resolve this tension concerning 'this necessary generation of new concepts'.[16]

Placing Cavaillès in the aftermath of these unresolved problems with the Dedekindian perspective on mathematics helps us to see how what I have called above the progress between 'specificities' in mathematics is of a piece with the *co-constitution* of *operations* and *objects*; that is, what Cavaillès's student Granger will call, in his various elaborations of the Cavaillèsian project, the perspective of *operation–object duality*.[17] In effect, this involves giving a primacy to the operation that is highly unusual in the history of philosophy: in so far as the domain of objects is seen as being *produced* by the development of operations, 'objects' no longer have any *a priori* status. This thesis is consistent with a broader attack that Cavaillès will mount on the Cartesian notion of grounding knowledge in simple ideas, which will be extended into a critique of attempts to ground mathematics in (discrete) intuitions or the notion of *evidence* (Husserl). Yet, by the same token, there is no possibility of according a fixed *a priori* status to 'operations', such that they would be conceived as a fundamental store of mental procedures which serve to produce the totality of mathematical objects. This latter point represents something that was a matter of fundamental theoretical struggle for Cavaillès, in so far as his doctoral dissertations are still orientated by the idea that the 'reality' of a mathematical theory can in some sense be individuated according to the presence of certain 'central

15. For a recent and profound elaboration on this 'hidden principle' of Kantian and post-Kantian philosophy, see Brice Halimi, *Le Nécessaire et l'universel*, VRIN, Paris, 2014, in particular ch. II.
16. *OC*, p. 62. Note that Cavaillès has turned Dedekind's assertion about the introduction of 'functions' and 'operations' into one about the generation of *concepts*.
17. The elaboration of this perspective is a central theme in Granger's work, but see in particular the analysis of Cavaillès's theory of abstraction in *Pour la connaissance philosophique*, Éditions Odile Jacob, Paris, 1988, ch. III, pp. 67–92.

intuitions' or 'gestures' that serve to unify it (so that a theory such as Cantor's initial invention of set theory could be seen as a progressive unfolding of the founding gestures of ordinal and cardinal 'counting').[18] It was in abandoning this perspective, and thus fully relativizing the notions of operation and object, that Cavaillès transitioned to the perspective of *On Logic and the Theory of Science*, which can be read as a relativization of the concept of the transcendental internal to mathematical work: different mathematical theories will be read as different operator–object domains, without recourse to an 'external' perspective that could explain their relations. The problematic of the 'philosophy of the concept' is thus the construction of a new concept of concept as the motor of this inter-transcendental variation.

The non-homogeneity of operations and the break with Kantianism

The thesis of the relativity of operations and objects just outlined must be connected to another central aspect of Cavaillès's project: the refusal of a 'Kantian' thesis concerning the homogeneity of operations in mathematics. In brief, this is another essential component of Cavaillès's rationalist nominalism: the thesis of object-operator duality prescribes the relativization of the transcendental, and this relativization will be specified each time by the singularity of different mathematical theories, in such a way that what is in principle denied is a general or *a priori* theory of the forms of mathematical reason. In this sense, Cavaillès represents a post-Kantian return to nominalism, one which results in a position that can seem paradoxical: the transcendental is each time particular. Unfolding this position requires a consideration of his tense relation to Kant, and in particular to French neo-Kantianism.

18. Cf. *OC*, p. 227.

Many of the difficulties of Cavaillès's position on this matter are on display when, responding to comments made by the mathematician Maurice Fréchet following the presentation of his doctoral works in 1939, Cavaillès states that 'I do not seek to define mathematics, but, by way of mathematics, to know what it means to know, to think; this is basically, very modestly reprised, the question that Kant posed. Mathematical knowledge is central for understanding what knowledge is.'[19] Two points should be made with respect to this. First, Cavaillès's statement is an axiom that prescribes an order of investigation: we ought to investigate mathematics in order to understand what thought is, and not the other way around; in other words, there is no *a priori* domain in which questions as to the essence of thought and knowledge can be posed in advance of (or conditioning) the progress of mathematics. We must accept this dogmatic aspect if we are to approach this philosophy in good faith. Second, if one takes this axiom seriously, one cannot stop there: if 'mathematics' – understood as the *effective* or *actual* realization of mathematical work, and not as some abstract definition – indeed holds the secrets of thought and knowledge, then this immediately prescribes a programme for wholesale reform of philosophy, which must now engage unreservedly with the entire body of mathematical production. Whence 'Cavaillèsianism' as a research programme. Yet it is surprising to find this position placed in Kant's own lineage, given that *On Logic and the Theory of Science* is in part structured by an extended critique of Kantianism, or 'the philosophy of consciousness', precisely on the grounds that it is a philosophy which attempts to delineate *a priori* conditions for thought prior to the actual development of mathematics.

Attention should thus be paid to the precise way that Cavaillès understood the Kantian programme, with respect to which it is

19. *OC*, p. 625. Jean Cavaillès and Albert Lautman, 'Mathematical Thought', trans. Robin Mackay, www.urbanomic.com/document/mathematical-thought, p. 20.

essential to refer to Brunschvicg's treatment of Kant in *Les Étapes de la philosophie mathématique*, a text that Cavaillès relies heavily on in his treatment of Kant in the doctoral works. On Brunschvicg's reading, mathematical philosophy is 'the cornerstone of the *Critique of Pure Reason*', and the core of the Kantian programme is to have posed to the problem of synthesis in such a way that, as Kant puts it in the introduction to the B edition of the *Critique of Pure Reason*, the questions 'How is pure mathematics possible?' and 'How is pure natural science possible?' are identified under the 'formula of a single problem': 'How are synthetic judgments *a priori* possible?'[20] As Brunschvicg summarizes: 'Kant realized that the solution of the problem with respect to the science of nature is the corollary of an analogous problem that, rather than solely concerning physics – that is, the application of mathematics to experience – is internal to mathematics itself.'[21]

The novelty of Kant thus lay in his having attempted to resolve the problems inherent in the application of thought to reality by positing a strict analogy with, so to speak, *the application of mathematics to mathematics*, such that, to use the classic example, the subsumption of the concepts '5' and '7' under the concept '12' exhibits synthesis in its pure form, with respect to which empirical cases of synthesis (say, subsuming the manifold of sensory data that is experienced when looking at the fingers of a normal human hand under the concept '5') is only a special case of this pure activity.[22] It is in this sense that the *Critique of Pure Reason* can first and foremost be read as a 'mathematical philosophy', with respect to which so-called 'sensory experience' is only a derived form. In turn, Brunschvicg locates Kant's central innovations with respect to the problem of pure mathematics as

20. Brunschvicg, *Les Étapes*, p. 257; Immanuel Kant, *Critique of Pure Reason*, Cambridge University Press, Cambridge, 1998, B19-20.
21. Brunschvicg, *Les Étapes,* p. 256.
22. I am borrowing the phrase 'the application of mathematics to mathematics' from Ian Hacking, *Why Is There Philosophy of Mathematics At All?*, Oxford University Press, Oxford, 2014.

a response to the problem of concatenation that d'Alembert takes up from Descartes: why is there – or, indeed, is there? – more in a 'chain' of deduction (2 + 2 = 4) than in (immediate) intuition (2 + 2 = 2 + 2)? Why was the whole content of a mathematical proof not there from the start? D'Alembert's answer is that on the level of the rational contents themselves, there is no novelty in this process, but merely the progressive unfolding of an initial definition which 'has not really been multiplied by this concatenation [*enchaînement*]', but 'has merely received different forms'.[23] What appears as progress for consciousness is only the gradual recognition of a rational arrangement that was latent in the relevant concepts.

For Brunschvicg the Kantian revolution lies in Kant's having taken the opposite stance: there is more in the conclusion of a demonstration than was present at the outset, and this supplement is added by *a priori* synthesis, such that in the progressive steps of a demonstrative chain we glimpse the activity of the mind or intelligence in its pure form (in its productivity), which will be seen to be identical with the activity (or progress) of science itself. Kant thus answers the problem of concatenation or deduction by transforming it into the problem of synthesis: 'The place of *a priori* synthesis does not lie in the connection between the terms of a judgment, or in the demonstration of such and such particular "numerical formula": it lies in the general process from which every particular number is derived, that is, in the creation of the notions themselves.'[24]

It is easy to see why on this reading Kant strongly prefigures Dedekind's analysis of mathematics as the necessary generation of concepts. And this is doubly so given that for Kant the primary moment of synthesis will be found in the basic

23. Jean Le Rond d'Alembert, 'Discours Préliminaire des Éditeurs' to the *Encyclopédie*, 1751, pp. ix. For Brunschvicg's citation of this passage cf. *Les Étapes,* p. 270.
24. Brunschvicg, *Les Étapes*, p. 270.

arithmetical operations, interpreted as *acts*, such that it will be 'in order to do justice to the sign +' that Kant will require the doctrine of the schematism: the synthetic unity of apperception in consciousness underwrites the entire synthetic process.[25] It is this ineliminable role given to the constructive activity of the mathematician which allows a reading of Kant as a philosopher of mathematical creativity, as Cavaillès notes in his thèse principale, *Méthode axiomatique et formalisme* (without yet taking the crucial step – which will define his later work – of rejecting Kant's theory for its very subjectivism): 'It is the synthetic activity of the *I think* that justifies the two characteristics of mathematical work: unpredictable becoming and absolute value. Absolute value because synthesis is required by the unity of apperception, unpredictable becoming because there is an effective constructive activity.'[26]

It should be clear here both why Kant is a central reference for the debate around the essence of mathematical thought, such as it will rage throughout the nineteenth century and on into the 1930s, and why it makes sense to ascribe to Kant the question that Cavaillès attributes to him: how does mathematics tell us what it is to think, what it is to know? But in all of this, we must note an essential assumption which Brunschvicg and Cavaillès took to underwrite the Kantian approach, namely that the treatment of the activity at stake in elementary mathematical examples is sufficient to give a philosophical basis for the treatment of higher mathematics, such that developments at a higher level of technical complexity will merely appear as special cases, and cannot be expected to threaten the theoretical edifice that has been developed with reference to simple cases. Whence Brunschvicg's rather qualified praise for the Kantian project as a whole. In one sense, Kant made an unsurpassable contribution to

25. Ibid., p. 271.
26. *OC*, pp. 34–5.

mathematical philosophy by producing an immanent philosophy of science:

> For the first time ... with Kant's doctrine concerning mathematics, the theory of science is, in relation to science itself, placed neither *above* science (as with Cartesian or Leibnizian metaphysics, which subordinate the principles of reason to theology), nor *below* science (as with English empiricism, which does not see mathematical notions as anything more than approximations of experience); the Kantian theory of science is exactly at the level of science itself.[27]

Yet, this immanence is achieved by insisting, by fiat, on the *operative homogeneity* of science, such that the concepts at work at its avant-garde will not be different in kind from those at stake at its most fundamental levels.[28] It is for this reason that, despite their internal divergences, the various attempts at the turn of the twentieth century to delimit a 'secure' domain of mathematical activity by referring all constructions back to a finite intuitive basis index themselves as belonging to Kant's lineage. As far as Brunschvicg was concerned, writing in 1912, it was not necessary to take a final stance on this debate: one could maintain a division which recognized Kant's essential contribution to foundational questions whilst leaving other avenues open when it came to developments at the forefront of modern mathematics. The Cavaillèsian research programme essentially begins once this *pax romana* is broken – that is, once it is no longer acceptable to separate the domains of 'technical' and 'foundational' mathematic work, which is precisely what he saw as the necessary result of the then-contemporary developments he approached in his epistemological studies.

On Cavaillès's view, answering 'the question that Kant posed', in the context of the foundational debates of the 1920s and 1930s, required a rejection of the Kantian programme at a quite

27. Brunschvicg, *Les Étapes*, p. 271.
28. Cf. ibid.

fundamental level. The ground on which this break is articulated is the refusal of the thesis of the homogeneity of operations in mathematics, which – qua the position of object–operation duality articulated above – equally entails a break with the thesis that mathematics deals with any particular or secure domain of 'objects'. Put otherwise, Cavaillès extracts from the Kantian lineage the thesis in *On Logic and the Theory of Science* that 'synthesis is coextensive with the engendering of the synthesized', but draws the conclusion from this that it is necessary to *stratify the concept of synthesis*, in a manner which entails a correlative stratification of the two 'unities' which were seen to underwrite the Kantian programme: on the 'subject' (or 'operator') side, there is a break with the thesis of the synthetic unity of apperception, whilst on the 'object' side there is an undermining of the supposedly 'general' form of the transcendental object = *X*.[29] Yet, it is precisely on these grounds that we rediscover the basic unresolved problem of the Cavaillèsian programme with respect to the inter-transcendental identity of mathematical operations, in so far as this strategy of *double stratification* is in fundamental tension with Cavaillès's commitment to what we could call the *continuity of mathematical becoming*. This tension is thrown into sharp relief when, in *On Logic and the Theory of Science*, he reworks his objection to Kant in the context of the challenge that Gödel's incompleteness results posed to the Husserlian theory of formal ontology:

> The body of a theory is a certain operatory homogeneity – as described by the axiomatic presentation – but when a theory is carried to the infinite, the iteration and the complications provide results and an intelligible system of contents that are ungovernable, and an internal necessity obliges it to surpass itself by way of an enlargement, which moreover is unforeseeable and only appears as an enlargement after the fact. There is no more juxtaposition than there

29. *OC*, p. 510/*LTS*, p. 74.

> is initial fixation; it is the entire body of mathematics that develops *in a single movement* across stages [*étapes*] and in diverse forms.[30]

If the mature Cavaillèsian programme is to be understood as working out the consequences of the rejection of any *a priori* or formal unity that could be seen to govern the process of synthesis, then in what sense is it possible to speak of the development of mathematics as the continuous unfolding of 'a single movement'? It is with respect to this problem that Cavaillès invokes an enigmatic notion of the 'polymorphy internal to a single rational concatenation'.[31] This notion of *internal polymorphy* is the closest thing we find to a 'definition' of mathematics in Cavaillès: mathematics just *is* the rational unfolding of a series of incommensurable theories in a movement which is continuous (each new theory resituates and transforms prior theories) but which cannot be unified under a single enumeration of forms. We find here again the first sense of the specificity of mathematics in Cavaillès's work, in so far as mathematics is understood as the domain of this rational polymorphy, and this polymorphy is in turn exhibited as the progress (and necessary relation) between different singular essences (or, in full Spinozist terms, between different 'ideas of ideas'). Cavaillès is thus an essential thinker of what Brice Halimi has called the 'problem of homogeneity': 'does there exist a homogeneous kind of entity encompassing all of which one can speak?'[32] Halimi's argument is that a *positive* answer to this question is the implicit assumption of the Kantian 'correlation' of the necessary and the universal. Cavaillès thus appears as a profound exponent of a *negative* answer to the homogeneity problem, all the more powerful because he claims to derive this negative consequence internal to the history of mathematics. This is one of the deepest senses of Cavaillès's

30. *OC*, p. 556/*LTS*,p. 131; emphasis mine.
31. OC, p. 510.
32. Halimi, *Le Nécessaire et l'universel*, p. 82.

‘anti-Kantianism’: contrary to what Halimi calls the correlation at the heart of the Kantian programme, Cavaillès delinks necessity from universality in order to attach it to specificity. By the same gesture we find what I will call the second sense of the specificity of mathematics, equally central to Cavaillès’s programme: mathematics is the *only* domain which possesses this character of rational polymorphy, in strict distinction from the other scientific disciplines. It is to the consequences of this second specificity that I will now turn.

The second sense: mathematics as distinct from the other sciences

By now the lineaments of Cavaillès’s theoretical perspective should be clear: mathematics has been designated as the domain of the production of specificities, and in turn the logical problem towards which mathematical philosophy is orientated is that of thinking the polymorphic relation between these specificities. It is with respect to this logical problem that a programme emerges of rereading the history (or ‘becoming’) of mathematics under a particular norm: that of revealing the identity of necessity and movement as the nature of the rational or the intelligible. However, this norm must be connected to another central aspect of this programme: the problem of the relation (or non-relation) between mathematics and physics, a problem which in turn stands for the profound gap between mathematics and the other sciences. What Cavaillès calls in *On Logic and the Theory of Science* ‘the fundamental problem of the epistemology of physics’ (*l’épistémologie physique*) is that mathematics and experimental science are characterized as two essentially irreconcilable domains of experience or of concatenation.

> the concatenation of physics [*l’enchaînement physique*] has no absolute beginning, any more than that of mathematics does … experimental

> acts engender yet more experimental acts by way of a *sui generis* concatenation which, at least in this regard, is independent – because it is of another essence – from the mathematical concatenation [*l'enchaînement mathématique*].[33]

On Cavaillès's view, the act of physical experimentation is essentially historically situated in a way that the mathematical act is not. This division may seem surprising given the theoretical importance of historical investigations for Cavaillès. It is important to note that although Cavaillès was by way of practice a historian of mathematics, he was ambivalent about the idea that mathematics has a history, properly called; hence his enigmatic reference to the investigation of 'this history, which is not a history'.[34] The ambivalence is as follows. On the one hand, mathematics must be understood as a progress or a becoming, and thus cannot be reduced to any universal or *a priori* formalism that would specify its development in advance, from which follows the central role of history for mathematical philosophy. On the other hand, this progressive character of mathematics is to be apprehended *post facto* through a reconstruction of the movement between its different rational contents, one which is of an entirely different order from the contingencies of the different formulations made by working historical mathematicians, with all of their attendant lacunae and misunderstandings, as much as their embeddedness in the facts of cultural history of intellectual biography. The archive of mathematical history is thus a kind of primary material through which the identification of necessity and progress can be exhibited, but the movement at stake is not itself identical with the development of historically produced works. It is in this sense that, rather than a historicity, mathematics possesses an *intrinsic logical temporality*, such that, in stark contrast to either classical rationalist theories of

33. *OC*, p. 522/*LTS*, p. 88.
34. *OC*, p. 664.

mathesis or to pure historicism, 'The fact that everything does not happen all at once [*d'un seul coup*] has nothing to do with history, but is the characteristic of the intelligible.'[35] It is quite otherwise for the case of physics, as well as all other experimental sciences, for in these cases there is an intrinsic link between experimental practice and the historically specific lived action of the experimenter. Thus in the transcription of the lecture course 'Causalité, nécessité, probabilité' given at the Sorbonne in the spring of 1941 we find the following stark opposition:

> What is physical – in opposition to mathematical – is the effective action of the physicist. Physical experience is situated in history whilst mathematical experience is not… Mathematical thought and physical thought mutually exclude each other (necessary concatenation [*enchaînement nécessaire*] on the one hand and historical concatenation [*enchaînement d'historique*] on the other).[36]

The designation of mathematical concatenation as necessary and physical concatenation as historical is founded in the different roles played by the subject in the two concatenations, connected to two competing notions of experience. In mathematical experience the mathematical subject performs an *experiment of pure thought* – that is, an experiment in which thought acts upon itself – whereas the experience/experiment in physics involves an essential aspect of *alterity*: thought experiments with something outside of itself. Thus Cavaillès states in a response to Ferdinand Gonseth in 1938: 'I do not believe it is possible to unify mathematical and physical experience under the same concept. There is an autonomous mathematical knowledge that is

35. *OC*, pp. 517–18/*LTS*, p. 83.
36. Cited in Paul Cortois, 'Cavaillès lecteur de Pascal', in Jean-Jacques Szczeciniarz and Baptiste Mélès, eds, *Hommage à Jean Cavaillès*, Hermann, Paris, 2018, pp. 37–62, p. 55. The telegraphic character of these lines is owed to the fact that they are cited from a transcript that Cortois made in 1988–89 of lecture notes on Cavaillès's course taken by Mme Marie-Louise Gouhier Dufour, and are thus not from Cavaillès's own hand (see p. 51). My reading in this section is influenced by Cortois's 'non-standard' interpretation of Cavaillès.

sufficient unto itself, and that therefore requires an idea of truth that is unrelated to physical truth.'[37] The severity of this position must be underlined, for it has the consequence that there is no possibility for a unified theory of science in the Cavaillèsian framework, and thus that Cavaillès's philosophy of the concept must be understood as being opposed to any project of general epistemology. Continuing his reply to Gonseth, Cavaillès makes one of his most startling enunciations to this effect:

> Whilst both experiences [i.e. in mathematics and physics, MH] stem from the same intuitive sensory activity, they thus each represent the culmination of two diametrically opposed evolutions. The description of these evolutions, and the study of the relations between them, seem to me to belong more to general anthropology than to epistemology.[38]

If we take Cavaillès's reference to anthropology here seriously, then on his account the relation between mathematics and physics fundamentally poses an essential problem for philosophy, but it is not, as traditional epistemology would have it, the problem of the rational ordering of the world or of the unity of scientific practice, but that of how to think the fact that in the contingent history of human societies we find points of contact between two incommensurable regimes: that of the production of rational knowledge and that of the organization of practical activity. The problem of the mathematics–physics differend thus in effect comes to stand as a surrogate for questions around the relation between reason and history. This point is all the more striking given that Cavaillès seems to have also thought this relation in terms of singularity, but now in a sense fundamentally different from the notion of 'singular essences' indexed above:

37. From Cavaillès's reply to Gonseth's presentation in Ferdinand Gonseth, ed., *Les Conceptions modernes de la raison. Entretiens d'été – Amersfoort (Septembre 1938)*, Volume I: *Raison et monde sensible*, Hermann, Paris, 1939, p. 41.
38. Ibid., p. 43.

> Is there an autonomous concatenation in physics? Appearance of the notion of existence which is to say of singularity... This notion of singularity = that which characterizes physical thought.[39]

In designating all physical thought qua 'thought of an existence' as the effective grasp of a singularity, Cavaillès advances a theory of the specificity of action which stands in opposition to any theoretical generality, but in a manner which in no way suggests a unification with the 'singular essences' found in the progress of mathematics.[40] The problem of the relation between reason and history, having been displaced onto that between mathematics and physics, is thus thought as the relationship between two incommensurable senses of the singular: the singular rational contents of mathematics and the historical singularity of the physical situation. But this comes with the consequence that the specificity of mathematics is characterized by its absolute difference from any applied discipline. From this, two final points follow.

First, it should be clear why Cavaillès's theory of the relationship between mathematics and physics is different from the position articulated above under the name of the *generality of application.* The mathematics which 'results' from the theorization of the physical situation is particular, just as much as the physical situation itself is, but they are two different *modes* of particularity (rational particularity and lived particularity). In turn, this makes it clear why Cavaillès is not a neo-Kantian. For Brunschvicg as much as for Cohen and the other authors of the Marburg school, physics and mathematics stood as joint paradigms of the transcendental. In contrast, with Cavaillès the form of synthesis is found to be fundamentally different for the intra-mathematical case and the case of the relation between thought and nature. Thus, starting from a novel theory of the applicability of mathematics to itself, we move from an analogy

39. From Dufour's course transcription, cited in Cortois, p. 58.
40. Ibid.

between the application of mathematics to mathematics and the application of mathematics to physics to a foundational disanalogy between these two forms of application. The consequence for the philosophy of the concept is that only in mathematics is it possible to make the modalities of transcendental variation precise; that is, to give a formalization of the way(s) in which a transcendental operator–object domain *shifts*. This is not possible for other sciences because of the different relation therein between theories and the determination of the objects on which they bear. It is thus in a very precise sense correct to read Cavaillès's overall theory as a contribution to the problem of the *historical a priori* or the *relativization of the transcendental*, but on the condition that we understand such a proposal as *strictly intra-mathematical*. The stridency with which Cavaillès takes pure mathematics as the exclusive paradigm of transcendental structuration is thus intimately bound up with Cavaillès's anti-Kantianism and his correlative rejection of the unity of science.

The second point concerns how focusing on this problem of the split between mathematics and physics provides a way to rethink the stakes of Cavaillès's resistance activity, and thus to resituate Cavaillès as a figure within the reception of French philosophy. Famously or infamously, the canonization of Cavaillès rests on an analogy constructed by his surviving collaborators between his roles as a resistant and as a mathematician-philosopher. Yet this analogy has been put to strikingly different ends. On the one hand, Raymond Aron's invocations of his last meeting with Cavaillès in London serve to ground this analogy in *a common concept of necessity*, 'which had command over practical imperatives as much as scientific propositions'.[41] It is in this spirit that Aron relayed in his obituary for Cavaillès the latter's statement to him on the occasion of

41. Cited from Aron's introduction to the 1962 *Philosophie mathématique* collection, reprinted in *OC*, p. 212.

their final meeting in London in 1943: 'I'm a Spinozist; I believe we submit to the necessary everywhere. The concatenations of the mathematicians are necessary, even the stages of mathematical science are necessary. This struggle that we carry out is necessary as well.'[42] On the other hand, Canguilhem sought to position the lesson of Cavaillès's work, life and death as exemplary of the combat between the universality of reason and fascism's negation of rationality, which in turn could be thought in terms of the resistance that rational necessity posed against the contingencies of history:

> [O]ne can understand that Cavaillès was a resistant according to logic. The deduction is simple. And for those who knew him, it is not imaginary. Nazism was unacceptable to the extent that it was the negation, savage rather than scientific, of universality, to the extent that it announced and sought the end of rational philosophy. The struggle against the *unacceptable* was thus *ineluctable*.[43]

Despite the differences in emphasis, both gestures served to lionize Cavaillès for the generation of the 1960s on the ground that there was an implicit deduction to be made from the aridity and rigour of his theoretical practice to the 'heroism' of his resistance activity.[44] In turn, this implied connection could serve as the rhetorical background to the politicization of the polemic between the 'philosophy of the concept' and the 'philosophers of consciousness', on the ground that Cavaillès's life was a kind of proof of the compact between scientific work and practical commitment, as opposed to the counter-proof of the inaction of the figure of the phenomenologist or philosopher of experience. It is thus striking that when one examines Cavaillès's texts, what one discovers is a philosophy that is in principle orientated in

42. Cited by Canguilhem in his 1967 inaugural address for the Amphithéâtre Jean-Cavaillès, reprinted in *OC*, p. 674. Translation from Peden's introduction to *LTS*, p. 19.
43. From Canguilhem's 1969 memorial radio lecture for Cavaillès, reprinted in *OC*, p. 677.
44. This thesis structures Peden's study *Spinoza Contra Phenomenology*; see in particular pp. 17–24.

the most profound ways against any possible synthesis between scientific thought and practical life.

What makes Cavaillès a philosophically generative figure through which to think this disjunction between thought and life is that it is exactly this point which is thematized in his comments on the mathematics–physics relation and his attendant critique of abstraction. For all that Cavaillès is known as a thinker of different forms of intra-mathematical abstraction, a red thread running through his work is that mathematical thought is not separate from the world, but rather is a qualitatively distinct manner through which the world is lived. Cavaillès thus outlines a *modal theory of abstraction*: to think the world mathematically is to think the same world as that of practical life, but to think under the image of necessity. Yet in no sense is this to be understood as an ontological split, as if mathematics represented the truth of world, with respect to which sensory and practical existence is a mere shadow. This point is expressed most stridently in the same response to Fréchet discussed above. Immediately following his invocation of the question posed by Kantian philosophy, Cavaillès continues by critiquing Fréchet's empiricist argument that mathematical concepts are produced by abstraction from an underlying sensory reality:

> Fréchet says: 'There are notions that are taken from the real world, and others that are added by the mathematician.' I respond that I do not understand what he means, since what is it to know the real world, if not to do mathematics on the real world?
>
> What do you call 'real world'? I am not an idealist, I believe in what is lived. To think a plane, do you live it? What do I think, when I say that I think this room? Either I speak of lived impressions, rigorously untranslatable, rigorously unusable by way of a rule, or else I do the geometry of this room, and I do mathematics.[45]

45. *OC*, p. 625/'Mathematical Thought', p. 20.

Cavaillès expresses here a position on the problem of abstraction that is notable for the equal distance it takes from empiricism and Platonism: the question of the 'relation' between mathematics and the real is ill-posed, for mathematics is to be thought as one immanent modality of the real. It is towards the articulation of this tender position that Cavaillès's whole theoretical work was directed:

> I spoke of a solidarity on the basis of sensible gestures. There is not, on the one hand, a sensible world that is given, and, on the other, the world of the mathematician, beyond it. … I believe that we never leave this starting point, in the sense that there is an internal solidarity and that each time we substitute for a less-well-thought mathematical object some more-thought-out objects, … all the same, we do not leave the sensible world.[46]

Read internally, the position here might seem to be constrained to the epistemology of mathematics. Yet read in terms of the ethical and political stakes of Cavaillès life and its mythologization, this position on the question of abstraction conjoins once more with Cavaillès's 'Spinozism', but this time on the terrain of the relation between what Étienne Balibar has called 'theoretical universalism' and 'practical universalism'. In his essay 'Sub specie universitatis', Balibar provides a suggestive heuristic which contrasts the Marxist–Hegelian tradition of thinking theory and practice in terms of a schema of ideal unification with what Balibar calls the 'Double Truth' strategy for thinking the universal, which he associates with the names of Spinoza and Wittgenstein. The latter holds that the demands of the theoretical and the practical are radically incommensurate, and thus must be thought together in a manner which preserves their independence whilst also accounting for their belonging to the same world, and making demands on the same actors. As Balibar summarizes: 'since in this conception there is nothing like an external (ideal,

46. *OC*, p. 626/'Mathematical Thought', p. 21.

or transcendental) point of view from which the difference could be reduced … philosophy becomes an exercise … in understanding why we always inhabit the same ("immanent") world in two contradictory manners which are *both universalistic*.'[47]

Given all that has been said above concerning the essential conjunction between the necessary and the specific in Cavaillès's work, it is evident that one cannot unproblematically inscribe Cavaillès in this tradition of a double strategy for thinking universality. Rather, what I suggest is that Cavaillès can be read as occupying a formally analogous position as a partisan of two incommensurable senses of necessity, the relation between which is philosophically fecund because it is theorized in terms of the contact between singular points. On the level of rational practice, he was led to a nominalist insistence that to do justice to the necessity of mathematical thought required locating the kernel of this necessity in the unsynthesizable passage between theories, and thus in harbouring the singular essence of each novel intelligible production. On the level of practical commitment, he indeed exemplified 'the logic of Resistance lived until death' eulogized by Canguilhem, but if one wishes to extract an ethics from this point it can only be of a paradoxical sort: to live life rationally is to bind oneself without remainder to the exigencies (or the singularities) of a particular situation.[48] What is in principle disbarred here is a unitary deduction between the two regimes. Two necessities, thus two specificities. Doubtless, the risk of hagiography abounds here; one which it is rare for writing on Cavaillès to avoid entirely. Nevertheless, it is precisely because both theoretically and practically Cavaillès's work and life suggest ways of thinking the difference between the necessary and the universal that he remains a point of departure for contemporary philosophy.

47. Étienne Balibar, 'Sub specie universitatis', *Topoi* 25, 2006, pp. 3–16, p. 7.
48. Cf. *OC*, p. 678 for Canguilhem's comment.