![Springer logo]

# <span>ANTTI</span> The Logic of Viewpoints*

HAUTAMÄKI

**Abstract.** In this paper a propositional logic of viewpoints is presented. The language of this logic consists of the usual modal operators $L$ (of necessity) and $M$ (of possibility) as well as of two new operators $A$ and $R$. The intuitive interpretations of $A$ and $R$ are "from all viewpoints" and "from some viewpoint", respectively. Semantically the language is interpreted by using Kripke models augmented with sets of "viewpoints" and with a new alternativeness relation for the operator $A$. Truth values of formulas are evaluated with respect to a world and a viewpoint. Various axiomatizations of the logic of viewpoints are presented and proved complete. Finally, some applications are given.

In ordinary logic, the truth-value of a sentence depends only on the world considered. The modern discussion about scientific change, paradigms conceptual schemes, and so on, shows that it is interesting to try to create logics in which the truth-value of a proposition depends also on ways to conceptualize the world. In this paper 'viewpoint' means 'a way to conceptualize the world'. We can think that phrases like 'conceptual scheme', 'linguistic scheme', 'conceptual framework', 'theoretical perspective' are synonyms of 'viewpoint'.

In this paper we study the logic of viewpoints on propositional level by using modern intensional logic à la Kripke (for references see Chellas 1980). The basic idea is to interpret formulas in respect of worlds and viewpoints. We leave the inner structure of viewpoints unspecified. The formal language to be considered contains, besides the usual operators $L$ for necessity and $M$ for possibility, two new operators $A$ for absoluteness and $R$ for relativity. The interpretation of these new operators will be that $AP$ is true at $w$ from the viewpoint $i$ if and only if $P$ is true at $w$ from all (relevant) viewpoints $i'$, and $RP$ is defined as the formula $\neg A \neg P$. This account shows that the operators $A$ and $R$ behave very much like the modal operators $L$ and $M$.

We think that the transition from ordinary logic to the logic of viewpoints is a logical counterpart to the philosophical transition from "metaphysical realism" to "internal realism"[1].

---

[1] The terms "metaphysical realism" and "internal realism" are from Putnam 1978. My interpretation of these realisms is that in metaphysical realism we are correlating our language to reality "an sich" but in internal realism we are correlating our knowledge to the reality as it is seen from our viewpoint.

## §1. Language of the logic of viewpoints

The language $\mathscr{L}(L, A)$ is defined as follows. The *vocabulary* of $\mathscr{L}(L, A)$ is: $X$, $\neg$, $\&$, $L$, $A$, and brackets $(,)$, where $X$ is a denumerable set of propositional variables. The set of *formulas* of $\mathscr{L}(L, A)$ is the smallest set $F$ such that

1. $X \subseteq F$,
2. If $P \in F$, then $\neg P$, $LP$ and $AP \in F$,
3. If $P, Q \in F$, then $(P \& Q) \in F$.

Other connectives $(\vee, \rightarrow, \leftrightarrow)$ are defined in the usual way. Operators $M$ and $R$ are defined as follows:

$$MP \quad \text{iff} \quad \neg L \neg P \quad \text{(possibility)}$$
$$RP \quad \text{iff} \quad \neg A \neg P \quad \text{(relativity)}$$

The formulas $AP$ and $RP$ can be read "absolutely $P$" (or perhaps "invariably $P$") and "relatively $P$", respectively.

If we combine two different operators together, we get 16 "double operators": $LA$, $AL$, $LR$, $RL$, $MA$, $AM$, $MR$, $RM$, $LM$, $ML$, $LL$, $MM$, $AR$, $RA$, $AA$, $RR$, of which the first eight are quite interesting. We suggest that these operators can be read as follows. $LAP$ is read "formula $P$ is necessarily absolute", $ALP$ is read "formula $P$ is absolutely necessary", $LRP$ is read "formula $P$ is necessarily relative", $RLP$ is read "$P$ is relatively necessary", and so on.

## §2. Semantics of the logic of viewpoints

We define now a Kripke style model for the language $\mathscr{L}(L, A)$[2].

The structure $\mathfrak{M} = \langle W, I, R, S, V \rangle$ is a *model* for the language $\mathscr{L}(L, A)$ if and only if $W$ is a non-empty set (a set of possible worlds), $I$ is a non-empty set (a set of viewpoints), $R$ and $S$ are relations in $W \times I$, that is $R$ and $S$ are subsets of $(W \times I) \times (W \times I)$, and $V$ is a function (a valuation) from $F \times W \times I$ to $\{0, 1\}$ such that

---

[2] This account shows that Krister Segerberg's "two-dimensional modal logic" resembles our logic of viewpoints. But in his system the universum $U$ is essentially a two dimensional space $W \times W$. We get the same kind of model, if we take the set $I$ to be also $W$. Then our operators $L$ and $A$ correspond to Segerberg's operators $\boxminus$ and $\boxminus$. Besides these, there are four other operators in Segerberg's system, to which there are no counterparts in our system. But it can be added that the semantics of these other operators is very unnatural if $I \neq W$. Note also, that $\boxminus$ and $\boxminus$ are both *S5*-modalities in Segerberg's system. (See Segerberg [7])

I got the idea of using a special set of viewpoints in semantics from P. Needham's tense logic, see Needham [3].

(i)    $V(\neg P, w, i) = 1$   iff   $V(P, w, i) = 0$,

(ii)   $V(P\&Q, w, i) = 1$ iff   $V(P, w, i) = V(Q, w, i) = 1$,

(iii) $V(LP, w, i) = 1$   iff   $V(P, w', i) = 1$ for all $w'$ such that $\langle w, i \rangle R \langle w', i \rangle$, and

(iv) $V(AP, w, i) = 1$   iff   $V(P, w, i') = 1$ for all $i'$ such that $\langle w, i \rangle S \langle w, i' \rangle$.

We can read $V(P, w, i)$ "the truth-value of $P$ at $w$ from the viewpoint $i$". From this reading we see that it is natural to let the accessibility relations $R$ and $S$ depend on both "coordinates" $w$ and $i$. Technically this idea is realized by taking the relations $R$ and $S$ to be relations in $W \times I$. This definition yields to great generality; for example it is possible that $\langle w, i \rangle R \langle w', i \rangle$ but not $\langle w, i' \rangle R \langle w', i' \rangle$. So $w'$ can be an alternative to $w$ from one viewpoint but not from another. Similar remarks hold in the case of $S$.

## §3. Different systems

Let the language be $\mathscr{L}(L, A)$ and let the schemas $A_0, \ldots, A_5$ and $A'_1, \ldots, A'_5$ be

$A_0$    $P$, if $P \in F$ is a tautology.

| | | | | |
|---|---|---|---|---|
| $A_1$ | $L(P \to Q) \to (LP \to LQ)$ | | $A'_1$ | $A(P \to Q) \to (AP \to AQ)$ |
| $A_2$ | $LP \to P$ | | $A'_2$ | $AP \to P$ |
| $A_3$ | $P \to LMP$ | | $A'_3$ | $P \to ARP$ |
| $A_4$ | $LP \to LLP$ | | $A'_4$ | $AP \to AAP$ |
| $A_5$ | $MP \to LMP$ | | $A'_5$ | $RP \to ARP$ |

The rules MP, RL, and RA are

MP:    $P, P \to Q / Q$

RL:    $P / LP$

RA:    $P / AP$.

We can take different combinations from these schemas and rules. As is well known, the schemas $A_0 - A_1$, $A_0 - A_2$, $A_0 - A_3$, $A_0 - A_2$ and $A_4$, and $A_0 - A_2$ and $A_5$ with rules MP and RL constitute the modal systems $K, T, B, S4$, and $S5$, respectively. Let $x$ and $y$ be any symbols from the set $\{K, T, B, S4, S5\}$. By a $V(x, y)$-*system* we mean a system, whose rules are MP, RL, and RA, and which is $x$-system with respect to $L$ and $y$-system with respect to $A$. For example, the axiom-schemas of the system $V(T, B)$ are $A_0 - A_2$ and $A'_1 - A'_3$. So $V(T, B)$ is a $T$-system with respect to operator $L$ and a $B$-system with respect to operator $A$. Analogically, $V(K, T)$ is a system, whose axiom-schemas are $A_0$, $A_1$ and $A'_1$, $A'_2$. There are 25 different $V$-systems.

If $\Sigma$ is a $V$-system, then the set of $\Sigma$-*theorems* is the smallest set of formulas, which contains all instances of the axiom-schemas of $\Sigma$ and is closed

under its rules. If $P$ is a $\Sigma$-theorem, we write $\vdash_\Sigma P$ or simply $\vdash P$. A sentence $P$ is *deducible* from a set of formulas $\Gamma$ in a system $\Sigma$ if and only if there are formulas $P_1, \ldots, P_n$ in $\Gamma$ such that a formula $(P_1 \& \ldots \& P_n) \to P$ is a $\Sigma$-theorem; if so, we write $\Gamma \vdash_\Sigma P$.

## §4. Models of $V$-systems

We shall define the concept of a $\Sigma$-model, where $\Sigma$ is a $V$-system. If $\Sigma$ is the system $V(x, y)$, then a model $\mathfrak{M} = \langle W, I, R, S, V \rangle$ for $\mathscr{L}(L, A)$ is a $\Sigma$-*model* iff the relation $R$ is any relation, a reflexive relation, a reflexive and symmetric relation, a reflexive and transitive relation, or an equivalence relation in $W \times I$, when $x$ is $K, T, B, S5$, respectively, and similarly for $S$ and $y$.

We shall say that a formula $P$ is *true* in the model $\mathfrak{M} = \langle W, I, R, S, V \rangle$ if and only if $V(P, w, i) = 1$ for all $w \in W$ and $i \in I$. We shall say that a formula $P$ is $\Sigma$-*valid* if and only if $P$ is true in every $\Sigma$-models. We write $\vDash_\Sigma P$ to mean that $P$ is $\Sigma$-valid.

## §5. The choice of a $V$-system

It is well known that $S5$ is the strongest of the modal systems $K, T, B, S4$, and $S5$, and that $B$ and $S4$ are stronger than $T$. $K$ is the weakest of these systems. It has also been argued that a modal system for necessity must be at least as strong as $T$. But what about the operator $A$? The intuitive interpretation of the accessibility relation $S$ is not so evident as the interpretation of $R$. It is better to call it the relation of *alternativeness*. In what sense can a viewpoint be an alternative to another viewpoint? One sense is that these viewpoints are comparable or *commensurable*. We can define commensurability in several ways. For example, $i$ and $i'$ are commensurable if they give at least to one propositional variable the same "meaning", that is: there is a variable $p$ in $X$ such that $V(p, w, i) = V(p, w, i')$ for all $w \in W$. This relation is a reflexive and symmetric relation, that is, an analogy or similarity relation. But now the alternatives of a viewpoint $i$ are independent of worlds, and the above condition is perhaps too strong. Let us try the following definition: $\langle w, i \rangle$ $S \langle w, i' \rangle$ iff $V(p, w, i) = V(p, w, i')$ for some $p$ in $X$. In this case, too, the relation $S$ is a similarity relation, but the alternatives to $i$ depend on the world considered.

If we think that the relation $S$ is a commensurability relation it must be at least reflexive and symmetric (if not an equivalence relation). So we can think that the weakest "right" system is $V(T, B)$.

## §6. Completeness

Let $\Sigma$ be a $V$-system. A set of formulas is *$\Sigma$-maximal* if it is consistent and contains as many formulas as it can without becoming inconsistent. We give first some lemmas without proofs.[3]

> LEMMA 1. *If $\Gamma$ is a $\Sigma$-maximal set of formulas, then*
> (i) $P \in \Gamma$ *iff* $\Gamma \vdash_\Sigma P$ *($\Gamma$ is closed under MP),*
> (ii) $\neg P \in \Gamma$ *iff* $P \notin \Gamma$,
> (iii) $P \& Q \in \Gamma$ *iff both* $P \in \Gamma$ *and* $Q \in \Gamma$.

> LEMMA 2. (i) $\Gamma \vdash_\Sigma P$ *iff* $P \in \Delta$ *for every $\Sigma$-maximal set $\Delta$ such that $\Gamma \subseteq \Delta$.*
> (ii) $\vdash_\Sigma P$ *iff* $P \in \Delta$ *for every $\Sigma$-maximal set $\Delta$.*

> LEMMA 3. *The following rules of inference are valid in any $V$-system*:
> $$\frac{(P_1 \& \ldots \& P_n) \to P}{(LP_1 \& \ldots \& LP_n) \to LP} \qquad \frac{(P_1 \& \ldots \& P_n) \to P}{(AP_1 \& \ldots \& AP_n) \to AP} .$$

The proof of Lemma 2. is based on *Lindenbaum's Lemma*: every consistent set of formulas has a maximal extension.

SOUNDNESS THEOREM. *Let $\Sigma$ be a $V$-system. Then every theorem of $\Sigma$ is true in every $\Sigma$-model: if $\vdash_\Sigma P$, then $\vDash_\Sigma P$.*

The proof is a standard one.[4] We show only that the rule RA preserves validity. We assume that $\vDash_\Sigma P$. Let $\mathfrak{M}$ be a structure $\langle W, I, R, S, V \rangle$ and suppose that $w \in W$ and $i \in I$. Let $i'$ be an arbitrary viewpoint such that $\langle w, i \rangle S \langle w, i' \rangle$. Because $P$ is valid, $V(P, w, i') = 1$ and so $V(AP, w, i) = 1$.

COMPLETENESS THEOREM. *Let $\Sigma$ be a $V$-system. Then every $\Sigma$-valid formula is a $\Sigma$-theorem: if $\vDash_\Sigma P$, then $\vdash_\Sigma P$.*

PROOF. If we can find a canonical model for $\Sigma$ such that $V(P, w, i) = 1$ if and only if $P$ belongs to some maximal set, then proof of the theorem is clear.

Let $\Sigma$ be a $V$-system. A structure $\mathfrak{M} = \langle W, I, R, S, V \rangle$ is a *canonical $\Sigma$-model* if and only if
- $W = \{w: w \text{ is a } \Sigma\text{-maximal set of formulas}\}$;
- $I = \{i: i \text{ is a bijection from } W \text{ onto } W\}$;
- $V$ is a valuation such that $V(p, w, i) = 1$ iff $p \in i(w)$, $p \in X$;
- $\langle w, i \rangle R \langle w', i' \rangle$ iff $i = i'$ and $i(w)^+ \subseteq i(w')$, where $i(w)^+ = \{P: LP \in i(w)\}$ for all $i \in I$ and $w \in W$;

---

[3] Proofs of lemmas can be founded in Chellas [1].
[4] See Chellas [1].

    —   $\langle w, i \rangle S \langle w', i' \rangle$   iff   $w = w'$ and $i(w)^- \subseteq i'(w)$, where
      $i(w)^- = \{P: \ AP \in i(w)\}$ for all $i \in I$ and $w \in W$.

It is possible to show that a canonical $\Sigma$-model is a $\Sigma$-model, that is, if $\Sigma$ is a $V$-system then relations $R$ and $S$ have required properties. For example, if $\Sigma$ is $V(\boldsymbol{B}, \boldsymbol{B})$-system then $R$ is a reflexive and symmetric relation and so is $S$ too. Let us prove that $R$ is a symmetric relation in this case. We have to show that if $i(w)^+ \subseteq i(w')$ then $i(w')^+ \subseteq i(w)$. Suppose that $i(w)^+ \subseteq (w')$ and $P \in i(w')^+$. If $P \notin i(w)$, then $\neg P \in i(w)$ (Lemma 1. (ii)). Because $i(w)$ is maximal and $\neg P \rightarrow LM \neg P \in i(w)$ (axiom $A_3$), then $LM \neg P \in i(w)$. But so $M \neg P \in i(w)^+$ and by supposition $M \neg P \in i(w')$. Equivalently $\neg LP \in i(w')$ and this contradicts the supposition $P \in i(w')^+$ because this means that $LP \in i(w')$. Hence $P \in i(w)$. So $R$ is symmetric.

    We prove now the main thing about canonical models:

(∗)     $V(P, w, i) = 1$   iff   $P \in i(w)$.

The proof is by induction on the complexity of $P$.

  i. If $P \in X$, then (∗) is true by definition of $V$.
 ii. Let $P$  be $\neg Q$:

        $V(\neg Q, w, i) = 1$    iff   $V(Q, w, i) = 0$
                       iff   $Q \notin i(w)$ (inductive hypothesis)
                       iff   $\neg Q \in i(w)$   (Lemma 1 (ii)).

iii. Let $P$ be $Q \& T$:

        $V(Q \& T, w, i) = 1$   iff   $V(Q, w, i) = 1$ and $V(T, w, i) = 1$
                       iff   $Q \in i(w)$ and $T \in i(w)$ (ind. hyp.)
                       iff   $Q \& T \in i(w)$ (Lemma 1 (iii)).

iv. Let $P$ be $LQ$:

        $V(LQ, w, i) = 1$     iff   $V(Q, w', i) = 1$ for all $w'$ such that
                            $\langle w, i \rangle R \langle w', i \rangle$
                      iff   $Q \in i(w')$   —"—    (ind. hyp.)

    We have to show that

    $Q \in i(w')$ *for all $w'$ such that* $\langle w, i \rangle R \langle w', i \rangle$   iff   $LQ \in i(w)$.

    Suppose that $Q \in i(w')$, for all $w'$ such that $\langle w, i \rangle R \langle w', i \rangle$. Let $w'' \in W$ be such that $i(w)^+ \subseteq w''$. Because $i$ is a bijection, there is a $w' \in W$ such that $i(w') = w''$. From the definition of $R$ it follows that $\langle w, i \rangle R \langle w', i \rangle$ and $Q \in i(w') = w''$. So $Q \in w''$ for all $w''$ such that $i(w)^+ \subseteq w''$. But so $i(w)^+ \vdash_{\Sigma} Q$ (Lemma 2 (i)) that is there are formulas $P_1, \ldots \ldots, P_n$ in $i(w)^+$ such that $\vdash_{\Sigma}(P_1 \& \ldots \& P_n) \rightarrow Q$. By lemma 3 it holds that $\vdash_{\Sigma}(LP_1 \& \ldots \& LP_n) \rightarrow LQ$. So $i(w) \vdash_{\Sigma} LQ$ because $LP_1, \ldots, LP_n \in i(w)$ by definition of $i(w)^+$. Finally $LQ \in i(w)$ (Lemma 1 (i)).

    Now suppose that $LQ \in i(w)$ and $w'$ is a world such that $\langle w, i \rangle R \langle w', i \rangle$.

It follows from the definition of $R$ that $i(w)^+ \subseteq i(w')$. So $Q \in i(w)^+$ and consequently $Q \in i(w')$.

v. Let $P$ be $AQ$:

$$V(AQ, w, i) = 1 \quad \text{iff} \quad V(Q, w, i') = 1 \text{ for all } i' \text{ such that } \langle w, i\rangle\, S\langle w, i'\rangle$$
$$\text{iff} \quad Q \in i'(w) \qquad -,,- \qquad \text{(ind.hyp.)}$$

We show that $Q \in i'(w)$ for all $i'$ such that $\langle w, i\rangle S\langle w, i'\rangle$ iff $AQ \in i(w)$. Suppose that $Q \in i'(w)$ for all $i'$ such that $\langle w, i\rangle S\langle w, i'\rangle$. Let $w''$ be a world such that $i(w)^- \subseteq w''$. Because $I$ is a set of *all* bijections, there is an $i'$ such that $i'(w) = w''$. It follows from the definition of $S$ that $\langle w, i\rangle S\langle w, i'\rangle$. By supposition $Q \in i'(w) = w''$. Hence $i(w)^- \vdash_\Sigma Q$ (Lemma 2 (i)). So there are formulas $P_1, \ldots, P_n$ in $i(w)^-$ such that $\vdash_\Sigma (P_1 \& \ldots \ldots \& P_n) \to Q$. By lemma 3 it holds that $\vdash_\Sigma (AP_1 \& \ldots \& AP_n) \to AQ$. Because $AP_i \in i(w)$, $i(w) \vdash_\Sigma AQ$. So $AQ \in i(w)$ (Lemma 1 (i)). Suppose that $AQ \in i(w)$. Let $i'$ be a viewpoint such that $\langle w, i\rangle S\langle w, i'\rangle$. It follows from this that $i(w)^- \subseteq i'(w)$. Because $AQ \in i(w)$, $Q \in i(w)^-$ by definition of $i(w)^-$, and finally $Q \in i'(w)$.

Now let us suppose that a formula $P$ is not a $\Sigma$-theorem. Then there is a $\Sigma$-maximal set $w \in W$ such that $P \notin w$ (Lemma 2. (ii)). But then $\neg P \in w$ (Lemma 1. (ii)). Because $I$ is the set of all bijections, $Id \in I$ ($Id$ is the identity function on $W$). So $w = Id(w)$ and $\neg P \in Id(w)$. Hence $V(\neg P, w, Id) = 1$ by (*), that is $V(P, w, Id) = 0$. So $P$ is not valid. This concludes the proof. $\square$

## §7. Modalities

A *modality* is any sequence of the operators $\neg, L, M, A$, and $R$, including the empty sequence. Within a $V$-system two modalities $m$ and $m'$ are equivalent if and only if for every formula $P$ the formula $mP \leftrightarrow m'P$ is a theorem. It is quite easy to show that in every $V$-system all of the modalities $LA, AL, LR, RL, MA, AM, MR, RM$ are distinct, that is, any two of these are not equivalent. There are no *reduction laws* for these eight modalities. So it seems to me that every $V$-system has infinitely many distinct modalities.

## §8. Applications of the logic of viewpoints

### 8.1. Overdetermined modal logic [5]

Let $\mathfrak{M} = \langle W, I, R, S, V\rangle$ be a model for $\mathscr{L}(L, A)$ where $S$ is a reflexive relation. Let $\mathscr{L}(L)$ be a language with necessity operator $L$ but without

---

[5] Our overdetermined and underdetermined modal logics are analogous to corresponding non-standard logics of Rescher and Brandom [6].

the operator $A$. We give the semantics to $\mathscr{L}(L)$ by the following definitions:

$$|P|_{w,i} = t \quad \text{iff} \quad V(RP, w, i) = 1$$
$$|P|_{w,i} = f \quad \text{iff} \quad V(R\neg P, w, i) = 1.$$

It is clear from these definitions that

$$|P|_{w,i} = f \quad \text{iff} \quad |\neg P|_{w,i} = t.$$

But it does not hold that, if $|P|_{w,i} = t$, then $|\neg P|_{w,i} = f$. Despite of this, the law of contradiction holds in the form $|P \& \neg P|_{w,i} = f$.

We say that the world $w$ is *normal* from the viewpoint $i$, if $\langle w, i \rangle$ $S \langle w, i' \rangle$ implies $i = i'$. Otherwise the world $w$ is said to be *non-normal* from the viewpoint $i$. It is possible, if $w$ is non-normal, that

$$|P|_{w,i} = t \quad \text{and} \quad |\neg P|_{w,i} = t$$

or

$$|LP|_{w,i} = t \quad \text{and} \quad |\neg MP|_{w,i} = t.$$

In this sense it is reasonable to call this logic overdetermined. Let $\Sigma$ be a $V(x, T)$-system, where $x$ is $T$, $B$, $S4$, or $S5$, and let $\mathfrak{M}$ be a $V(x, T)$-model. Now it is evident that for all formulas of $\mathscr{L}(L)$ it holds:

if $\vdash_x P$, then $|P|_{w,i} = t$, for all $w \in W$ and $i \in I$

if $P \vdash_x Q$ and $|P|_{w,i} = t$, then $|Q|_{w,i} = t$, for all $w$ and $i$.

In this overdetermined modal logic, the following general principle does not hold:

if $|P_1|_{w,i} = \ldots = |P_n|_{w,i} = t$ and $P_1, \ldots, P_n \vdash_x Q$, then $|Q|_{w,i} = t$.

But if we read this principle "collectively", we get a valid principle:

(*)      if $|P_1 \& \ldots \& P_n|_{w,i} = t$ and $P_1, \ldots, P_n \vdash_x Q$,   then   $|Q|_{w,i} = t$

### 8.2. Underdetermined or schematic modal logic

Let $\mathscr{L}(L)$ and $\mathfrak{M}$ be as in an overdetermined logic. We state a truth-definition as follows:

$$|P|_{w,i} = t \quad \text{iff} \quad V(AP, w, i) = 1$$
$$|P|_{w,i} = f \quad \text{iff} \quad |\neg P|_{w,i} = t.$$

In a non-normal world $w$ from the viewpoint $i$ it can be that

$$|P|_{w,i} \neq t \quad \text{and} \quad |\neg P|_{w,i} \neq t$$

or

$$|LP|_{w,i} \neq t \quad \text{and} \quad |\neg MP|_{w,i} \neq t.$$

In this sense this logic is underdetermined. Also in an underdetermined logic all theorems and all one-premise inferences of $x$ ($x$ is $\mathbf{T}, \mathbf{B}, \mathbf{S4}$, or $\mathbf{S5}$) hold in $\mathfrak{M}$. The above general principle (∗) does hold.

### 8.3. Tense logic

In this case the set $I$ in a model structure $\mathfrak{M} = \langle W, I, R, S, V \rangle$ can be the set of real numbers and $S$ can be

      a.   universal relation,
      b.   the relation $\{\langle\!\langle w, i\rangle, \langle w, i'\rangle\!\rangle: i > i' \text{ and } w \in W\}$,   or
      c.   the relation $\{\langle\!\langle w, i\rangle, \langle w, i'\rangle\!\rangle: i < i' \text{ and } w \in W\}$.

The meanings of operators $A$ and $R$ are in the case of

      a.   'always' and 'sometimes',
      b   'it has been till now the case that' and 'it was the case that',
      c.   'it will be from now on the case that' and 'it will be the case that'.

### 8.4. Dialectical contradictions and complementarity

My account of dialectical contradictions is that they are not contradictions in reality but some kind of epistemological antinomies[6]. Typically, in a dialectical contradiction it is the case that *two mutually exclusive* propositions $P$ and $Q$ do apply to the same situation. For example, a micro-object can be corpuscular ($P$) and also wavelike ($Q$), but when an object is conceived to be corpuscular it can not be conceived to be wavelike, and conversely. My idea is to interpret this situation such that an object is corpuscular and wavelike from different viewpoints. In addition these two properties *complete each other*: if an object is corpuscular, it must be also wavelike, and conversely. Therefore, I want to call dialectical contradictions *complementary oppositions*.

In the logic of viewpoints we can express dialectical contradictions as follows: formulas $P$ and $Q$ form a *dialectical contradiction* with respect to a model $\mathfrak{M}$ if and only if

    (i)   $\mathfrak{M} \vDash \neg(P \& Q)$   (exclusiveness)
    (ii)  $\mathfrak{M} \vDash P \to RQ$  ja  $\mathfrak{M} \vdash Q \to RP$  (completeness).

Observe that if $\mathfrak{M} \vDash P$, then $\mathfrak{M} \vDash \neg Q$, and if $\mathfrak{M} \vDash Q$, then $\mathfrak{M} \vDash \neg P$. If, for example, $V(P, w, i) = 1$, there is a viewpoint $i'$ such that $\langle w, i\rangle S\langle w, i'\rangle$ and $V(Q, w, i') = 1$. In this case $P$ and $Q$ are both true at $w$ but, of course,

---

    [6] See Hautmäki [2].

from different viewpoints $i$ and $i'$. My definition shows that dialectical contradictions are not logical contradictions. The paradoxical character of dialectical contradictions disappears, when we take viewpoints (or respects) into consideration[7].

## Bibliography

[1] BRIAN F. CHELLAS, *Modal Logic, An Introduction*, Cambridge University Press, Cambridge 1980.
[2] ANTTI HAUTAMÄKI, *Dialectics and points of view, Ajatus* 39 (1983) (forthcoming)
[3] PAUL NEEDHAM, *Temporal Perspective, A Logical Analysis of Temporal Reference in English*, Uppsala 1975.
[4] HILARY PUTNAM, *Realism and reason*, in H. PUTNAM, *Meaning and the Moral Sciences*, Routledge and Kegan Paul, London 1978.
[5] HILARY PUTNAM, *Quantum mechanics and the observer, Erkenntnis* 16 (1981), pp. 193-219.
[6] NICHOLAS RESCHER, and ROBERT BRANDOM, *The Logic of Inconsistency, A Study in Non-Standard Possible-World Semantics and Ontology*, Basil Blackwell, Oxford 1980.
[7] KRISTER SEGERBERG, *Two-dimensional modal logic, Journal of Philosophical Logic* 2 (1973), pp. 77-96.

DEPARTMENT OF PHILOSOPHY
UNIVERSITY OF HELSINKI

---

[7] It seems to me that we can apply my account of dialectical contradictions to the interpretation of quantum mechanics. Compare with Putnam 1981, where he writes that if $p$ and $q$ are two incompatible propositions, "(w)hat turns out to be the case is that one can know that $p$ and one can know that $q$ ... but one is not allowed to have *a single text* in which one says both $p$ and $q$" (p. 210). Instead of texts Putnam speaks also about perspectives (p. 199), points of view (p. 209), and frames (p. 212). I am indebted to Professor Ilkka Niiniluoto for drawing my attention to Putnam's article.