Chapter 6
Change, Event, and Temporal Points of View

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Abstract A “conceptual spaces” approach is used to formalize Aristotle’s main intuitions about time and change, and other ideas about temporal points of view. That approach has been used in earlier studies about points of view. Properties of entities are represented by locations in multidimensional conceptual spaces; and concepts of entities are identified with subsets or regions of conceptual spaces. The dimensions of the spaces, called “determinables”, are qualities in a very general sense. A temporal element is introduced by adding a time variable to state functions that map entities into conceptual spaces. That way, states may have some permanency or stability around time instances. Following Aristotle’s intuitions, changes and events will not be necessarily instant phenomena, instead they could be processual and interval dependent. Change is defined relatively to the interval during which the change is taking place. Time intervals themselves are taken to represent points of view. To have a point of view is to look at the world as it is in the selected interval. Many important concepts are relativized to intervals, for instance change, events, identity, ontology, potentiality, etc. The definition of points of view as intervals allows to compare points of view in relation to all these concepts. The conceptual space approach has an immediate semantic and structural character, but it is tempting to develop also logics to describe them. A formal language is introduced to show how this could be done.

1 Introduction

The interest of studying points of view has grown during the last decades. The idea of point of view is intuitively clear; it is a way to see the world. But when we want to explore the function and meaning of points of view in our cognition, we face a serious dilemma: To define exactly the concept of point of view, we have to select a

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specific field where points of view are relevant and applied. My personal observation is that the discussion about points of view is quite vain without formalizing basic concepts. Good formalism helps us to clarify our intuitions and in fact opens new research questions which are hard to see outside the formalism. This is especially evident in studying temporal aspects of points of view, the target of this article.

In my mind, the time aspect is underscrutinized in logic. A large majority of logical studies is related to a "static" aspect of logic; that is, temporal features of language are neglected and models are standard structures without a time component (cf. [16]). Another bias is that time is taken to be discrete leading to unnatural concepts like "the next moment" which are in contradiction with the intuition of the continuity of time. A related bias is to define the truth of formulas relative to single points in time (see [25, 27]), losing the process nature of change and motion.

My topic in this article is to study the concepts of change and event that are temporal and time related. The issue of time is philosophically sensitive. I consider it impossible to study temporal aspects of entities and changes without a philosophical framework. My strategy is to take Aristotle's philosophy of nature as a starting point. In his book *Physics*, Aristotle treated such important concepts like time, place, motion, change, and substance and tried to solve complex problems related to describing motion and change. What is interesting is that Aristotle adopted quite "modern" conceptions, such as continuity of time, space, and motion. Aristotle also developed a typology of changes, which is relevant also today. Although, Aristotle's conceptions are not defined in a clear, unambiguous way, there is enough consensus about its central points and suppositions, making it possible to discuss the Aristotelian conception of time and change.

To study temporal points of view, I use conceptual space approach. This approach permits me to formalize Aristotelian intuitions about time and change in a way that allows me to offer a new insight into them. I do not follow Aristotle's solutions in all aspects of my analysis. Although Aristotle's conceptions of continuity of time and the processual character of change are central for my presentation, I introduce modern analogues of the time related concepts of Aristotle that contain features not found in Aristotle's texts.

I have used the conceptual space approach in my earlier studies about points of view [12, 13, 15, 18, 19]. In that approach, properties of entities are represented by the location of entities in multidimensional conceptual spaces. The dimensions of spaces, called *determinables*, are qualities of entities in a very general sense. Concepts of entities are subsets or regions of conceptual spaces (see [10, 13]). Points of view could be defined in many ways with this approach. A clear way is to take a subset of determinables to represent points of view [13]. A more elaborated approach is to add theories or suppositions to points of view [15].

In this article, I adopt a slightly different strategy. First, I introduce a time element into the conceptual space approach by adding a time variable to state functions that map entities into conceptual spaces. In my notation, S(x, t) is the state of an entity x at a time instance t. In basic structure of temporal conceptual space I do not make any assumption about change. In this framework I explicate an Aristotelian concept of change according to which states have some permanency or

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stability around time instances. Thus changes and events are not instant phenomena, instead they are processual and interval dependent—change is defined relative to the interval during which the change is taking place.

What about points of view? My approach here is to take time intervals to represent points of view. To have a point of view is to look at the world as it is in the selected interval. Many concepts are relativized to intervals, for example, change, events, identity, ontology, and even potentiality. Therefore the definition of points of view as intervals allows me to compare points of view in relation to these concepts. For comparison I will apply also an interval algebra developed by Allen [1].

The conceptual space approach is semantic and structural in nature. It starts from sets of entities and from their aspects or qualities and considers their relations. But it is tempting to develop also logic to describe temporal conceptual spaces. I do that by introducing a language to talk about determinables and changes of entities. In this language, there is no direct reference to time instants. Instead, formulas are interpreted relative to conceptual spaces and time. The logic is not axiomatized in this article; its sole purpose is to show how a temporal logic could be developed in the framework of conceptual spaces.

2 Time and Change in Aristotle

In Aristotle's philosophy of nature, the concept of change (*kinêsis*) is a central one. Aristotle defines change as "the actuality of that which exists potentially, in so far as it is potentially this actuality (*Physica*. III, 1, 201a10–11, 201a27–29, 201b4–5)." That is, change rests in the potential of one thing to become another. In the change, the potentiality is in the process of becoming actual. In other words, change is the process of actualization of potentiality. When the change is complete, the potentiality has become actual.

In general, there are three kinds of change: *generation*, where something comes into being; *perishing*, where something is destroyed; and *transformation*, where some attribute of a thing is changed while the thing itself remains constant. The permanent form of a thing is called its *essence* and changing attributes are called *accidental properties* (*Physica*. V).

All change or process involves something coming to be from out of its opposite. Something becomes what it is by acquiring its distinctive form—for example, a baby becomes an adult. According to Aristotle there are two general kinds of change defined by logical concepts contradictory and contrary determinations (*Physica*. V, 1–2.)

Contradictory determinations hold between properties A and not-A, and contrary determinations hold between "different" properties, say A and B such that A and B cannot be attributed to same entities at the same time. In Book V of *Physics*, Aristotle makes a distinction between substantial change and change as *kinêsis*. Substantial changes are the generational (birth, genesis) and the perishing (phthora) of entities, that is, appearance and disappearance of entities. Substantial change is instant, happening at a certain moment of time.

Table 1 Types of changes in Aristotle (physics)

Categories	Type of change	Contradictory change	Contrary change
Substance	Birth (genesis)	Non-S $\rightarrow$ S (x starts to exist)	–
	Perishing (phthora)	S $\rightarrow$ non-S (x ceases to exist)	–
Kinêsis (real change) Quality Quantity Place	Qualitative change	–	F $\rightarrow$ F* (F and F* incompatible)
	Quantitative change	–	F is quantity with values n $\rightarrow$ m (n < m or n > m)
	Changing place	–	F is a place p $\rightarrow$ p* (p $\neq$ p*)

There are three kinds of *kinêsis* type change: qualitative change, quantitative change and change of place. A change in color of an entity (e.g., from green to yellow) is a qualitative change, and a change in size (growth) of an entity is a quantitative change. The change of place is easy to understand; in modern terms, the space coordinates of a moving entity change. *Kinêsis* is a processual change, taking an interval to complete.

Aristotle counts quantity and quality among the accidents that modify a subject or substance entirely and directly.[1] In *Metaphysics,* Aristotle defines quantity as follows: "We call a quantity that which is divided into constituent parts, each or every one of which is by nature something one and individual. Thus plurality, if it is numerically calculable, is a kind of quantity." (*Metaphysica*. 1020 a 7–10).

On the other hand, quality is anything apart from quantity that belongs to a substance (*Metaphysica*. 1020 b 7). According to Aristotle, in its primary sense, quality is that which distinguishes a thing in its essence. A second kind of quality are the properties of changing things insofar as its change is detectable, namely the properties in view of which changes are distinguished (*Metaphysica*. 1020 b 14). This distinction refers to the difference between essential and accidental qualities.

It is important to know that Aristotle used in *Physics* the concept of accidental change to refer to external changes of entities, exemplifying it with, a scholar walks (*Physica*. V, 1). If, on the other hand, something in a thing is changing, then the change is called *simple* by Aristotle. The simple change is often the change of some part of a thing, for example, a diseased eye becomes healthy. Aristotle says clearly in Book V of *Physics* that we can set aside accidental changes (*Physica*. V, 1, 224 b 25).

We can classify the concepts of change (Table 1) by the using the schema

"x which is S is F"

[1]http://peenef2.republika.pl/angielski/hasla/a/accident.html, accessible 6.10.2014.

where x is an individual, S is a substantial genus, and F is a quality, quantity or location. S is a form of x and it is unchangeable in Aristotle's system, whereas F is a changing accident.

Aristotle bound time and movement together; there is no time without motion and no motion without time (*Physica*. V). In the Book V, Aristotle made statements concerning the continuum of time and motion. Motion and time are continuous and they cannot be composed of indivisible atoms (of movement or time). An "atom" of time would be an instant of time with no duration and therefore it would be impossible to build an interval of time starting from these atoms, Aristotle argues. Similar arguments are given for the continuity of motion. According to Aristotle, changes in quantity and location are continuous. This means in modern terms that change in quantity is taking place as a process where the value of quantity is approaching the target values, the new state of entity.

Aristotle's conception of change of quality is not so clear. If a person is changing from illness to health, is that a discrete change: a person was ill and, after some moment of time, he is healthy. Aristotle seems to make an assumption, that there is a continuum of degrees of illness (or health), and similar for the change of color and of many other qualities. My interpretation of the Aristotelian conception of change is that quantities and qualities are both continuums and there is no jump or gap in change. Aristotle's theory of continuum is central for his theory of change. Change is a process concept. According to continuum theory, there is no way to say what is the "next state". Therefore it is not possible to define the change as referring to differences between one instant and the next. Also is not possible to define a change by referring to a single instant. Aristotle's solution is to accept the first moment of change (*Physica.* VI, 5). So the new state, after a change, begins at some moment and continues for some time (see Fig. 1). The first moment or state is the beginning of the actuality of some potentiality of an entity.

The background of this discussion about the "first moment" is the paradoxical character of change and motion when concentrating on single moments. If we say that x changed color from blue to yellow at the moment t, then it seems that x is blue and yellow at the same time. To avoid this logical contradiction, change must be analyzed in another way (see also [33]). Aristotle's solution is to split change into two parts: *Before t,* x is not yellow, and *from t onward,* x is yellow. This solution is based on the continuity of time. Zeno's paradox is related to the same issue: the motion is impossible because the moving object must be in rest and in motion at each moment (*Physica*. V).

I have described Aristotelian notion of change by pointing out several philosophical principles:

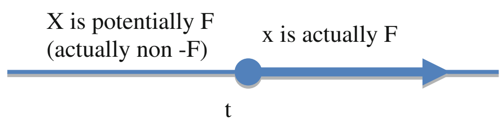

Fig. 1 Change identified by the starting point

- Time and motion are interdependent concepts.
- Time and motion are continuous, containing no jumps.
- There are changes related to substance (genesis and perishing) and changes related to accidents (*kinêsis*: quality, quantity and location).
- Substantial changes are instant (coming to be or perishing).
- Changes (*kinêsis*) in accidents are processual, continuous, and interval related phenomena.
- There is a first moment when change has taken place (the first moment of the actuality of the target state of change).
- Change is the actualization of potentiality.

I take these features of Aristotelian change theory as my starting point when developing temporal points of view. But I will make also some important deviations from them. Of course, each of these features has fueled extensive philosophical discussions [21, 22, 24], and my comments in this article are not intended to be scholarly.

3 Conceptual Spaces and Determinables

Conceptual space is an effective way to present properties of entities. Its idea is to map entities into quality space, where qualities are dimensions of the space. Johnson, in his book *Logic*, called these qualities *determinables*. Johnson's definition of determinables is the following:

> I propose to call such terms as colour and shape determinables in relations to such terms as red and circular which will be called determinates. (Johnson, Part I p. 174)

A determinable is something that could be further specified and this specification gives determinates (determinate values). Determinables could be seen from a linguistic viewpoint as general terms or from an ontological viewpoint as qualities, properties, or dimensions of things. Johnson's definition is linguistic, referring to *terms*. Johnson prefers to say that determinables are abstract names, which stand for adjectives [17]. According to A.N. Prior, "'red' is the proper name of an individual universal, if we may speak so; while 'colour' is the name of the class of universals to which this individual one belongs" ([28], 8).

My interpretation of determinables is that they are functors associating entities to their values. In schematic forms, determinables could be presented as follows: "the $\_\__1$ of $\_\__2$". For example, the phrase "the color of the table" presents the determination of "color" to the entity "table" and the value could be presented by the sentence:

> "The color of the table is brown."
> Using function notation the sentence could be coded
> [color](table) = brown.

I suppose that there is no absolute set of determinables describing the world as a whole. Determinables are first of all related to situations in which we live and act. We have to construct concepts to talk about our situations. Some determinables are physical and some mental, some are concrete and some abstracts, some are personal and some social, and so forth. In a traditional context (e.g. Aristotle), a distinction is made between qualities and quantities. In Johnson, determinables are intended to be qualities. But in a mathematical context, quantities are more natural and qualitative variables are transformed into quantities. So one can consider values of determinables to be just real numbers, but we do not assume that. So determinables could be qualities as well as quantities. It is interesting in this context that Aristotle seems to suppose that qualities are somehow continuous and have an infinite number of degrees; like degrees of colors or illness.

The number of determinables is a complex issue. In some earlier studies, like Hautamäki [13], it is supposed that the number of determinables for each set of entities is countable (see also [5]). Now it seems that it is better to assume that there are finite numbers of determinables needed to describe and identify entities in relevant applications. This is not a thesis concerning the absolute number of determinables. I suppose in this article that, in each context, a useful finite set of determinables is selected. This supposition reflects my conviction that the human (brain) capacity to identify entities is limited and so "ontology" in my sense is always finite in the sense of the fundamental variety of entities.

With the concept of determinable the concept of conceptual space is defined. Conceptual spaces are linked to entities by state functions, which "locate" entities in spaces.

For the definition, I use the notation B^A for the set of functions from the set A to the set B:

$$B^A := \{f \mid f\colon A \to B\}.$$

If $f \in B^A$ and $C \subseteq A$, then the *restriction of f into C*, denoted by f/C, is defined by

$$f/C := \{\langle x, y\rangle \in f \mid x \in C\}.$$

Let I be a finite set of determinables and let D be a set of (possible) values for determinables. We call the set

$$D^I = \{f \mid f\colon I \to D\}$$

a *conceptual space*. To talk about properties or qualities of entities, we just need a state function S from the set of entities E into the conceptual space D^I:

$$S\colon E \to D^I.$$

S(x) is the state of x. If S(x)(i) = a, we say that "i of x is a", for example

"the color of the pen is yellow"

when i is the determinable color, a is the color yellow and x is the pen. The state S (x) tells the "complete story" about x, because it specifies the values of all determinables for x.

As an example, let I = {color, form} and let S(x) = {⟨color, blue⟩, ⟨form, oval⟩} then the color of x is blue and the form of x is oval.

For clarity of terminology let us use the following terms to describe conceptual spaces:

Determinables, such as color, are aspects of entities and dimensions of space; we call them also functions of entities.
Determinates, such as red, are attributes of entities and values of determinables.

If S(x)(i) = a then a is a *property* of x and the value of the determinable i for x.

The term conceptual space comes from the fact that concepts could be presented as subsets of conceptual space. Let C be a subset of the conceptual space D^I then the concept C applies to all entities whose states belonging to C:

$$\text{“x is C” if and only if } S(x) \in C.$$

Note that in my presentation I “collect” all values to a single set D, which might be the set of real numbers. Although this is little bit artificial, in practice it does not set any restrictions on the applicability of determinables or conceptual spaces: it is a normal practice, say in statistics, to quantify qualities. In Hautamäki [13] all determinables have their own set of values.

The terms conceptual space or quality space are used in van Fraassen [31, 32], Stalnaker [30], Hautamäki [13, 14], Gärdenfors [10], and Clark [6], among others.

4 Time Aspect

The idea behind time is that entities change in time. Taking this as a starting point, we have to express change with a time variable. There are at least two options. Either we take time as one of the determinables or we connect time to the entities. I prefer the last one, because time is not considered a similar kind of aspect as determinables. Time is not a quality in the sense of Johnson’s logic, where the determinables are principles of “fundamentum divisionis”. Time is something to which changes of determinates are internally linked. On the other hand, both options make possible to express different aspects of changing entities. When time variable is a component of state function, as we suppose in this presentation, we could express continuity of entities as supposed in *endurantism*. It one take time to be one of determinables, then one could talk about entities with different temporal parts, like animals with relict parts (perdurantism).[2]

Let T be a set of times. For lucid presentation, we take T to be the set of real numbers $\mathbb{R}$. The elements of T are called (time) *moments* or *instants*.

[2] The concepts of enduratism and perdurantism was introduce to me by Manuel Liz. I would like to thank him for many valuable comments.

We extend the state function S to include the time beside the set of entities:

$$S: E \times T \to D^I, S(x,t) \in D^I, \quad x \in E,\ t \in T.$$

For convenience we shall use the expression i[x] for the phrases "the i of x" or "the value of determinable i for x". If S(x, t)(i) = a we write often "i[x] = a at t".

But further elaboration is needed, because if we add this concept of time to the state function without qualifications, changes of entities start to seem arbitrary and unnatural. For example, it might be that a determinable i has a value a at t, but not a before and after t. Intuitively entities have permanence over some period of time (cf. Aristotle). To express that aspect we use intervals to guaranty temporal unchangeability. The idea of our treatment of continuity is to suppose that, if an entity x has some property at t, it has the same property also near t, either before or after t.

Another crucial issue is the existence of entities. When time is under consideration, entities are changing in time but their existence might also be temporal. Entities appear or disappear. Between their beginning and ending, their existence is unbreakable, that is, there is no gap of existence between their appearance and disappearance. So intuition seems to propose that there must be an interval of existence for all entities. Before or after that interval the entity does not exist. There are no properties of entities outside their interval of existence, either. I take these ideas into my definition of temporal determination base.

For notation, the basic definitions for intervals of real numbers are

$[t,t^*] := \{x \in \mathbb{R} | t \le x \le t^*\}$, closed interval;
$(t,t^*) := \{x \in \mathbb{R} | t < x < t^*\}$, open interval.

Definition 1 A *(temporal) determination base* B is the structure

$$B := \langle I, F, D, E, T, S \rangle$$

where I, F, D and E are non-empty sets, I is finite and F is a subset of I, T is the set of real numbers $\mathbb{R}$, and S is a partial function from E × T into the set D^I such that

1. S is an injection for all time moments: if S(x, t) = S(y, t) then x = y;
2. if S(x , t)/F = S(y, t)/F for all t ∈ T, then x = y;
3. if S(x, t) is defined, then there is a closed interval π_x = [m, n] containing t such that

   if $t^* \in \pi_x$ then $S(x,t^*)$ is defined;
   if $t^* < m$ or $t^* > n$, then $S(x,t^*)$ is not defined;

The elements of I are called *determinables*, the elements of F are called *fundamental determinables*, the elements of D are called *determinates* or *determinate values*, the elements of E are called *entities*, the elements of T are call time *instants* or *moments*, and the function S is called a *state function.* Other determinables are called *supplementary.*

The set D^I of functions is called a *conceptual space* for entities in E. For all $x \in E$ and $t \in T$, $S(x,t) \in D^I$ is called the *state of x* at t. The notation S(x, t)(i) = a and S(x, t) = s implies that S(x, t) is defined. When we write S(x, t) = S(y, t) or S(x, t)(i) = S(y, t)(i) we mean that both are defined and identical or both are undefined.

The introduction of fundamental determinables is a novel feature in the theory of points of view as compared to Hautamäki [13, 15]. They are fundamental in the sense of providing a constraint for identity. In many applications of conceptual spaces, fundamental determinables include three space coordinates (x, y, and z axes).

Condition 1 above means, that whenever two entities are in the same state they are identical. It is a variant of the principle of Identity of Indiscernibles by Leibniz in his *Discourse on Metaphysics*, Sect. 9 [23]. But it is possible that two entities are in the same state but at a different time.

Condition 2 above is also a variant of Leibniz's principle: If two entities have the same values for all fundamental determinables at all moments then they are the same entity. Note that the condition permits the case that two different entities have same fundamental properties at some moment. Of course, then they have some differences in other determinables, otherwise they are identical by Condition 1. In this Condition 2, I make a deviation from Aristotelian conception of essence, because for him essence is stable and permanent (in form), whereas here fundamental determinables could change their values but still define identity over time. So my concept of "essence" is dynamic.

Condition 3 means that x has "properties" only in the interval $\pi_x = [t, t^*]$. We interpret that so that *x exists only in the interval* π_x. Before or after π_x, x is non-existent (or x has a virtual, empty existence). The interval π_x is called the *lifecycle* of x. At the instant *t, x appears* and, at *t*, x disappears.*

It is useful to generalize temporal state function to intervals, denoted by π, as follows:

$$S(x,\pi)(i) = a \text{ if and only if for all } t \in \pi:\ S(x,t)(i) = a,$$
$$S(x,\pi) = s \text{ if and only if for all } t \in \pi:\ S(x,t) = s.$$

Then I will write also that "i[x] = a in π" (Fig. 2).

Because a major interest for the time aspect comes from changes of entities, we have to define what change is in the context of conceptual space. As a basic case we take the change of the value of a determinable for an entity in some instant of time. It is quite complicated to define exactly the change point, because in a continuous

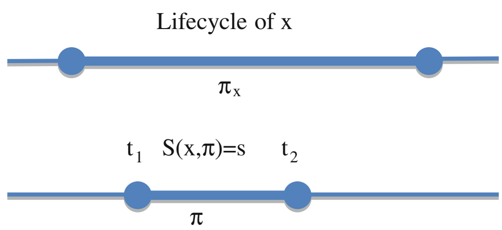

Fig. 2 The lifecycle of x and the interval condition for S(x, π) = s

time system there is no “next moment” of time. Of course, we could recognize the existence of change in the interval [t, t*] by observing that S(x, t) ≠ S(x, t*). But I want to study the process of change and therefore I use intervals to see where the change starts or ends in the Aristotelian sense. Note that my definition 1 of temporal determination base is neutral in relation to the nature of change.

Definition 2 Let B be a determination base B = ⟨I, F, D, E, T, S⟩ and let $i \in I$, $x \in E$, $t \in T$,

$$s \in D^I, a \in D, S(x,t) = s \text{ and } S(x,t)(i) = a.$$

x is *stabile in the interval* $\pi \subseteq \pi_x$ in respect of i if S(x, t)(i) is constant in π.

x is *stabile at t* in respect of i if there is an open interval π containing t in which x is stabile.
x A-changes at t in respect of i if there is an interval $\pi = (t^1, t^2)$ such that $t^1 < t < t^2$ and

1. $S(x, t^*)(i) = a$ for all t^* such that $t \le t^* < t^2$
   $S(x, t^*)(i) \neq a$ for all t^* such that $t^1 < t^* < t$

Or

2. $S(x, t^*)(i) = a$ for all t^* such that $t^1 < t^* \le t$
   $S(x, t^*)(i) \neq a$ for all t^* such that $t < t^* < t^2$

A-change is a nick name for Aristotelian change.

In the case 1 we say that i[x] *starts* (*begins*) to be a at t.
In the case 2 we say that i[x] *ceases* (*ends*) to be a at t.

If $i \in F$ is a fundamental determinable, the change of x in respect of i is called a *fundamental change*.[3]

The original Aristotelian model of change accepts only changes with Condition 1, that is, changes are always beginnings. I deviate here from Aristotle by accepting also endings as changes.

Aristotelian changes are “regular” in the sense that in them something is stabile either before or after of the change. But are all changes so nice? In theory not; entities can change all time following numerous different patterns. Instead of classifying them in any imaginable way, I just define a concept of “irregular change” as follows

[3]The concept of change is relevant when one studies the traditional doctrine of essentialism. There is the distinction between essential properties and accidental properties, where essential properties are those that survive change and accidental properties are those that do not (see Aristotle *De Int* 4a10, *Met.* 1028a31–33; [7]. In my system, fundamental determinables correspond to essential properties, whereas supplementary determinables might be called *accidental*.

An entity x *changes irregularly at t* in respect of the determinable i if for all intervals π containing t there is a moment of time t* such that

$$S(x, t^*)(i) \neq S(x, t)(i).$$

Note that if x changes irregularly at t in respect of i then x is not changing in Aristotelian sense nor it is stabile at t.

The other interesting case of change is the change without starting or ending point. In fact, even in the Aristotelian change one have accept this king of change. Namely, when i[x] ceases to be a at, it might be that i[x] = b in an open interval (t, t*) with b $\neq$ a. Then i[x] changes its value at t so that there is no starting point for the new value b.

Because the definition 1 of determination base is quite general and allows many patterns of change, it might be worthwhile to restrict possible patterns in very definition of determination base. For example, if we think that Aristotelian concept of change is a "natural one", we could adopt the following definition.

Definition 1A Let B = $\langle$I, F, D, E, T, S$\rangle$ be the determination base. Then it is *Aristotelian* if the state function S satisfies the following condition:

Whenever S(x, t) is defined and i $\in$ I there is a closed interval π containing t such that S(x, t)(i) is constant in the interval π.

This definition means, that entities keep their determinate values always during some interval. It is easy to define Aristotelian change in this setting: if S(x, t)(i) = a and t is the first point of the interval where i[x] = a and i[x] $\neq$ a before t in some interval then i[x] start to be a at t, and similarly for the ending case.

An important question is the nature of continuity in change. This issue could be studied by exploring "graphs" of states of entities. I use for that the concept of world-lines, which are functions from the set T of time moments into the conceptual space. This allows me also define time-related concepts.

If we think of conceptual space as multidimensional space and track how the state of an entity x changes in time, we will reach a "world-line" or curve of x in the space. So the life cycle of x could be represented by its world-line in conceptual space.

Definition 3 The elements of the set $(D^I)^T$ are called *world-lines* and they are functions from T into D^I. The subsets of $(D^I)^T$ are called (time-related) *concepts*.

The *world-line of an entity x* is the function w(x) defined by

$$w(x)(t) = S(x, t) \quad \text{for all } t \in \pi_x.$$

The *extension* of a concept C is the set

$$EXT(C) := \{x \in E|\ \text{there exist } f \in C \text{ such that } w(x) \subseteq f\}.$$

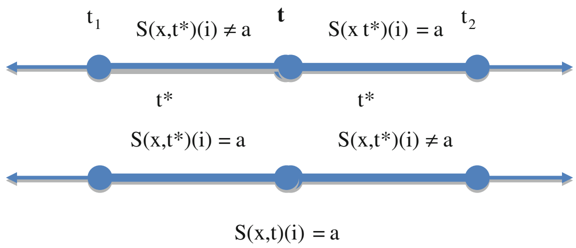

Fig. 3 There is an A-change: in the *upper line* i[x] starts to be a and in the *lower line* i[x] ceases to be a at t

w(x) is a function of the time mapping time moments in x's lifecycle into the state space reaching all states of x. In mathematical terms, a world-line is a *vector* of time. The extension of a concept is defined so that an entity satisfies the concept C, if its world-line is a part of some element of C. Note that we do not presuppose, that the world-line w(x) is member of C, because entities have lifecycles not covering the whole time span in T. The presented notion of concepts is interesting, because concepts do not only express the properties of entities but also they specify how properties of entities change in time.[4]

It is a difficult question, whether world-lines must be continuous. As we observed above, Aristotle seems to suppose that all changes are continuous. On the other hand, he hesitates to extend the continuity assumption to the case of qualitative change. I have defined determination base so that continuity over change moments is not supposed. So a jump is possible, as in the case A in Fig. 3. My intuition differs here from Aristotelian intuition. I see it as an empirical question whether a determinable is changing in a continuous way or not. A separate problem is that in genesis entities start to exist suddenly with a bundle of properties (qualities and quantities), and similarly they lose all properties when perishing. So genesis and perishing are not continuous changes; they are not *kinêsis* in Aristotle's terms.

In Fig. 4, the world-line A contains a jump or gap, the world-line B is a continuous curve, and the world-line C is a straight line, where the i1-coordinate is constant. Note that this kind of representation of world-lines does not tell how entities move in time in the conceptual space; for that a time dimension must be added to the coordinate system.

To reach a workable definition of the continuity of state function, one need to introduce a topology into conceptual spaces. Gärdenfors [10] has done this by supposing that there is a distance measure d defined between states of the space and using it to define a betweenness relation. Gärdenfors uses that relation to define the

[4]Many *scientific theories* could be presented as time-related concepts (e.g., [4]. Let Th be a time-related concept and x an entity. Then $Th \subseteq (D^I)^T$ and the proposition "$x \in EXT(Th)$" is an empirical claim stating that the entity x (a model of Th) "obeys the laws" of theory Th (see [15].

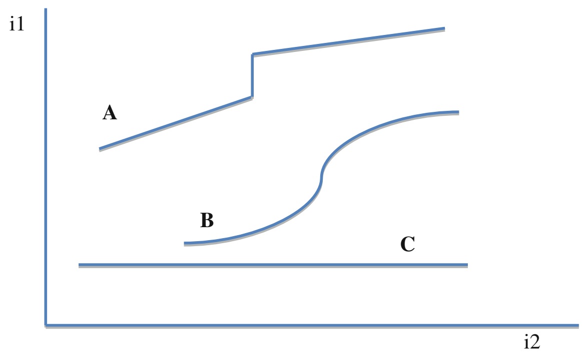

Fig. 4 A set of world-lines in a two dimensional space

convexity of subsets (regions) of the conceptual space. A subset C of the conceptual space is *convex* if for all states s and s* in C all states between s and s* are also in C [11]. I use convexity to define the continuity of state function S as follows:

The state function S: $E \times T \rightarrow D^I$ is *continuous* if sets S_x are convex for all $x \in E$ where

$$S_x := \{S(x,t) | t \in \pi_x\}.$$

This definition implies that if s and s* are states of an entity x then all states between s and s* are also x's states. Note that this definition says nothing about times when states in S_x are reached.

If we have an total order $\leq_D$ in the set D of values, then continuity could be define for determinables as follows:

The state function S is *continuous in respect of i* if for all entities x and for all intervals $\pi = [t_1, t_2]$ it holds that

if $S(x, t_1)(i) \leq_D a \leq_D S(x, t_2)(i)$ then there exists $t^* \in \pi$ such that $S(x, t^*)(i) = a$.

If S is continuous in respect of i then S(x, t)(i) reaches all values between any couple of values x reaches in any intervals during its lifecycle. Continuity implies that there is no gaps in world-lines of entities.

5 Temporal Points of View

A point of view is a special way to see the objects in cognition and in discourse. With point of view, some aspects of objects are selected as the focus of attention, leaving some other aspects out of consideration or awareness [26]. In conceptual space framework, several methods are used to approach points of view. In Hautamäki [13], points of view were defined to be a finite selection of determinables. In Hautamäki [15], another element was added to the definition of points of view: that of a theory. Thus a point of view is a structure V = ⟨B, K, T⟩, where B is a determination base, K is a (finite) set of determinables and T is a subset of subspace D^K called a theory (T is not time there).

The time aspect of objects leads naturally to consider time as an element of points of view. Instead of adding time to previous concepts of points of view, we take time as a sole element of points of view. Single moments of time are not relevant to define points of view, however. Instead, intervals of time are how we conceive time in cognition. A temporal point of view is an interval into which we concentrate our attention.

Definition 4 A *temporal point of view* over the determination base B = ⟨I, F, D, E, T, S⟩ is the structure TPV = ⟨π, B⟩, where $\pi \subseteq T$ is a closed interval.

When the determination base B is known, I identify a point of view ⟨π, B⟩ with the interval π. The interval π is closed, because it is natural to suppose that there are starting and ending instances in temporal points of view.

Identity of entities could be relativized to points of view. Of course, if entities are in the same state at some time they are identical. But if we consider the fundamental determinables of entities in some interval, we get an interesting concept of relative identity (or "incidence" as per [29]).

$$x \approx_\pi y \text{ if and only if for all } t \in \pi: \; S(x,t)/F = S(y,t)/F.$$

If $x \approx_\pi y$ I will say that x and y are *fundamentally identical in π*. This means that x and y identical in respect of all fundamental determinables in π. It might be that there is a moment t before or after π such that $S(x, t)/F \neq S(y, t)/F$ and therefore $x \approx_\pi y$ does not imply x = y.

Ontology is related to what exists and does not exist. *π-ontology* is the set of entities existing in π. Entities have their own lifecycles. Their appearance or disappearance could be relativized also to points of view. Entities could change in some respects and remain the same in other respects in π. It is interesting to recognize in which respects an entity is changing.

Definition 5 Let π be a point of view and $x \in E$. I will say that

x exists in π if $\pi \cap \pi_x \neq \varnothing$;
π-ontology is the set $O(\pi) := \{x \in E \mid \pi \cap \pi_x \neq \varnothing\}$;
x appears in π if $\pi_x = [t, t^*]$ and $t \in \pi$;
x *disappears* in π if $\pi_x = [t, t^*]$ and $t^* \in \pi$;
x A-*changes in π in respect of i* if i[x] A-changes at some $t \in \pi$;
x A-*changes in π* when x A-changes in π in respect of some $i \in I$;
x is *stabile in π in respect of i* if i[x] is constant in $\pi \cap \pi_x$;
The *extension of a concept C relative to π* is the set

$$\mathrm{EXT}(C, \pi) := \{x \in E | w(x)/\pi \subseteq f \text{ for some } f \in C\}.$$

6 Time Direction and Accessibility

So far all my definitions are in fact neutral to the direction of time, only the phrases "start" and "beginning" must be change to phrases "cease" and "ending" and vice verse. In fact all above concepts related to time and change are *symmetric*.

But intuitively and from the experience of every day life it seems plausible to suppose that time is "forward" directed. Our memory is building a picture of time flowing from past to present and now-moments turn to be passed away. One solution to this challenge is to bound the direction of time to temporal points of view by adopting the actual moment of time: the "now". A *temporal point of view with present time over the determination base B* is the structure VPT(n) = $\langle\pi, n, B\rangle$ where π is closed interval and $n \in T$ is the *now-moment*.

Then it is natural to call all moments $t < n$ *past time* and all moments $t > n$ *coming time*. Note that it is not supposed that n belongs to π. This means that the interval π under consideration in VPT(n) might be a passed interval or a future interval.

In the Aristotelian tradition the process of actualization of potentials is deeply bounded to the direction of time. We can even state that the process of actualization sets the direction to time. Behind the change there are causes, whether material, formal, efficient or final (*Physica* II, 3). These causes are answers to the question "why" something is moving or changing. According to Aristotle an event's **"final cause"** is the end toward which it directs; say the final cause of a seed is the full-grown plant. In summary, the whole theory of change and causes contains implicitly the direction of time, from causes to results.

One way to clarify the potentiality and the direction of time is to take into account of the possible transitions of states in conceptual space. I define the potentiality to be a relation between consecutive states on the basis of world-lines of entities.

Definition 6 The *accessibility* relation $A \subseteq D^I \times D^I$ is the set

$$\mathrm{ACC} := \{\langle s, s^* \rangle | S(x, t) = s, S(x, t^*) = s^* \text{ and } t < t^* \text{ for some x and } t, t^*\}.$$

If ACC(s, s*), we say that the state s* is *accessible from or possible after* the state s.

The *potentials of an entity x* at t is the set

$$\mathrm{POT}(x, t) := \{s \in D^I | \mathrm{ACC}(S(x, t), s)\}.$$

The *potentials of an entity x at t from the point of view* π is the set

$$\mathrm{POT}(x, t, \pi) := \{s \in D^I | s = S(x, t^*) \text{ and } \mathrm{ACC}(S(x, t), s)) \text{ for some } t^* \in \pi\}.$$

If s* is accessible from s then they are in the same world-line for some entity x, s* being a later state than s. Potentials of entities at t are their consecutive states after t. In POT(x, t, π) the set of potentials is restricted to states x owns in π.

Another interpretation of the accessibility relation ACC is that it expresses the *disposition of entities to change*. If $\langle s, s^*\rangle \in A$ and S(x, t) = s then x has the disposition to reach the state s*.

According to the definition of x's potentials at t, the state s is potential for x if it is accessible from it's state at t. We say that a potential s of x is *actualized* at t* if S (x, t*) = s. When potentiality is relativized to a point of view π, then only those states are considered to be potential which are actualized in π.

We can develop a numerical measurement or probability to express transitional potentiality:

$$P(s^*/s) = r \quad \text{and } r \in [0, 1]$$

P(s*/s) is the probability that an entity is in the state s* in the condition that it has been in the state s. To define this probability we take sets EV(X), $X \subseteq D^I$, to be *events* in the sense of probability theory:

$$\mathrm{EV}(X) := \{\langle x, t\rangle | S(x, t) \in X\}.$$

Note that EV({s}) = $\{\langle x, t\rangle\}$ when S(x, t) = s.

7 Comparison of Points of View

A temporal point of view is the structure TPV = $\langle \pi, B\rangle$, where $\pi \subseteq T$ is a closed interval. Let B be fixed. I will compare intervals of T. Relations of intervals could be classified by applying interval algebra developed by Allen [1].

Let $\pi = [t_1, t_2]$ and $\pi^* = [t_3, t_4]$, then there are basically seven different relations between π and π^* as shown in Table 2 and Fig. 5.

Table 2 Basic relations of intervals according to Allen [1]

Relation	Interpretation $\pi \rightarrow \pi^*$	Converse $\pi^* \rightarrow \pi$
(A) $t_2 < t_3$	(π) proceeds (π^*)	(π^* is) proceed by (π)
(B) $t_2 = t_3$	Meets	Met by
(C) $t_1 < t_3 < t_2 < t_4$	Overlaps	Overlapped by
(D) $t_1 < t_3 < t_2 = t_4$	Finished by	Finishes
(E) $t_1 < t_3 < t_4 < t_2$	Contains	During
(F) $t_1 = t_3 < t_2 < t_4$	Starts	Started by
$t_1 = t_3$ and $t_2 = t_4$	Equals	Equals

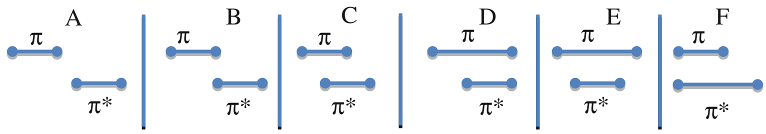

Fig. 5 Graphical presentation of relations between intervals (following [1]). *Note* The case of equality is a trivial identity

In Fig. 5 these concepts are presented by intervals.
Some observations about these cases:

In cases B, C, D, E and F, $\pi \cap \pi^* \neq ø$ and $\pi \cup \pi^*$ is also an interval.
In cases B, C and F, $\pi \cup \pi^*$ contains π, thus being an enlargement of π.
In cases D and E, π contains π^*.
In the case A, $\pi \cap \pi^* = ø$ and $\pi \cup \pi^*$ is not an interval.
In the case B, π and π^* meet or join each other.

The concepts of existence, ontology, change, permanence, and potentiality are all relativized to points of view. I point out only some interesting cases, leaving others to the reader.

If $\pi \subseteq \pi^*$ and $x \in E$ and $y \in E$ then:

$O(\pi) \subseteq O(\pi^*)$, more instances imply more existing entities;
$POT(x, t, \pi) \subseteq POT(x, t, \pi^*)$, more instances imply more potentiality;
$EXT(C, \pi^*) \subseteq EXT(C, \pi)$, because if $w(x)/\pi^* \subseteq f$ then $w(x)/\pi \subseteq f$;
if $x \approx_{\pi^*} y$ then $x \approx_{\pi} y$, if x and y are identical in an interval they are also identical in its subintervals.
The converses of these relationships do not hold in general.

8 Events

The nature of events is one of the most exiting topics in philosophy; discussed by such contemporary thinkers than Alain Badiou, Donald Davidson, Nelson Goodman, Jakerow Kim, David Lewis and W.V.O. Quine among others.[5] But the event is also important topic in Hellenistic philosophy, especially for Aristotle.

As an example we consider Kim's [20] account of events. According to him events are instantiations of properties. Events are composed of three things: Object (s) [x], a property [P] and time or a temporal interval [t].

Events are defined using the operator [x, P, t]. A unique event is defined by two principles:

[5] http://plato.stanford.edu/entries/events/, read on 9.9.2014.

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(a) the existence condition: [x, P, t] exists if and only if object x exemplifies the n-adic P at time t;
(b) the identity condition: [x, P, t] is [y, P*, t*] if and only if x = y, P = P* and t = t*.

Kim does not specify what kind of properties P's are, instead he states that only constructive properties create distinct events. Also events are related to time intervals, but not their spatial situations. A conceptual space approach opens new opportunities to handle events. I take Kim's proposal seriously and elaborate it in my framework.

Intuitively, events are associated to important changes in things. The "important" part is difficult to define without priorities of determinables and for that I use the concept of fundamental determinables. There are two kinds of events, the generation of a new thing or the appearance of new fundamental properties of existing things. Both changes can be treated by states of entities. Because changes are relative to points of view as time intervals, the concept of event is defined by referring to intervals. Roughly, a state s is an event relative to point of view π, if it is a new state inside π. My definition of the concept of event is in the spirit of Aristotle.

Definition 7 Let $B = \langle I, F, D, E, T, S\rangle$ be a determination base, let $\pi = [t_1, t_2]$ and let $TPV = \langle \pi, B\rangle$ be a temporal point of view. A triple $e = \langle x, s, t\rangle \in E \times D^I \times T$ is an *event from the point of view* π if

$x \in O(\pi)$, $s = S(x, t)$, $t \in (t_1, t_2)$, and there exists $i \in F$ such that
$S(x, t^*)(i) = a$ for all t^* such that $t \leq t^* \leq t_2$ and
$S(x, t^*)(i) \neq a$ for all t^* such that $t_1 \leq t^* < t$.

We say also that e is a *fundamental change of x in respect of i.* If there are several determinables changing at t, an event is said to be a *composite fundamental change*. The time moment t of the event is called *event time* and we can say that *e happens at t*.

According to the definition of event, the state s of an event $\langle x, s, t\rangle$ is new in the interval π. If the event e is a fundamental change of x in respect of i then i[x] will keep its value at t to the end of the interval π. A fundamental change is an appearance of a new value of a fundamental determinable. So the appearance of a new value of a supplementary determinable is not, as such, an event.

The definition allows there to be several changes taking place in the same event. For example the color and form could both change, thus being a composite fundamental change. So both changes are counted as parts of the same event (cf. [8].

To specify an event e at t one needs to pick up an entity x and a state s of x such that the combination $e = \langle x, s, t\rangle$ is an event in π. The entity x is going through a fundamental change at t and i[x] gets a new value for some fundamental determinable $i \in F$.

This definition for events is a clear advance in comparison to Kim's concept. Properties are specified to be actual values of determinables in events. The entity x exemplifies the properties presented by the state s. An event is also directed toward the future, that is, something new has happened and this will continue for some time. Identity condition for events is basically:

$$\text{if}\,\langle x, s, t\rangle = \langle y, s, t\rangle \quad \text{then } x = y.$$

The condition is satisfied. It follows from Condition 1 of the definition of determinable (Definition 1) that, if S(x, t) = s and S(y, t) = s then x = y (S is an injection). Of course, Kim's account is more complicated if one problematizes the identity conditions of time moments and states.

9 A Temporal Modal Logic of Viewpoints (TL)

There are many ways to elaborate a temporal modal logic over temporal determination bases. For example, formulas could be valuated in relation to instants of time or intervals of time. Here I use instants, but in truth relation I refer to intervals, as well. Time element could be added also to the language, but my choice is to treat time in semantics: Formulas are evaluated relative to conceptual spaces and time instances. For treating points of view, it is possible to introduce an interval operator. Scott [29] proposed to use *progressive tense* operator $[\leftrightarrow]$ defined by the stipulation (following my notation):

$[\leftrightarrow]\varphi$ is true at t iff there is an open interval $J \subseteq T$ with $t \in J$ such that φ is true at all $t^* \in J$.

According to Scott, McKinsey, and Tarski, the propositional logic of this operator is S4 (referred to in Scott [29], p. 160). I make some modifications to Scott's proposal.

9.1 Language of TL

The language TL is an extended first order language with two kinds of variables, positive and negative predicates for determinables and concepts, and with three modal operators for intervals. Variables for entities are x, y, x*, x**, and so forth and for determinates v, v*, v**, and so forth. Constants for entities are c, c*, c**, and so forth and for determinates a, b, a*, a**, and so forth. Variables and constants are terms, denoted by t, t*, t**, and so forth.

In the language TL there is an identity relation t = t* and a predicate EX(x) for existence of entities. To express determinables, I introduce a finite (non-empty) set Δ of determinable indexes and a two-place predicate P_δ for each $\delta \in \Delta$. Then I have also a set of one-place predicates C, C*, C**, and so forth for concepts.

Primitive formulas: $|\ t = t^* |\ EX(t)\ |[P_\delta](t, t^*)|\ [-P_\delta](t, t^*)|\ [C](t)|\ [-C](t)|$

t = t*	means "t is t*", t and t* are both constants for entities or for determinates;
EX(x)	means "x exists";
$[P_\delta](x,v)$	means "the value of the determinable δ for x is v";
$[-P_\delta](x,v)$	means "the value of the determinable δ for x is *not* v";
[C](x)	means "x is C";
[−C](x)	means "x is not-C"

Time operators:

$[\leftarrow\!\!\dashv]\varphi$	backwards φ holds;
$[\mapsto]\varphi$	forwards φ holds;
PoVφ	from the point of view φ holds.

Formulas: $|$ primitive formulas $|\ \neg\phi\ |\ \phi \wedge \psi\ |\ \forall x\phi\ |\ \forall v\phi\ |\ [\hookleftarrow]\phi\ |\ [\mapsto]\phi\ |\ \mathrm{PoV}\phi\ |$

A *sentence* is a formula without free variables.

9.2 Semantics of TL

The semantics of TL is a standard semantics for first order time logic except that modal operators are not interpreted by referring to single moments of time as are the standard operators now, past, and future (see [25]).

Definition 8 A *model* M of TL is a structure M = ⟨B, V, π⟩, where

1. B[6] = ⟨I, D, E, T, S⟩ is a determination base with a set I of determinables (I = Δ), a (non-empty) set D of determinate values, a (non-empty) set E of entities, a set T of time moments ($T = \mathbb{R}$) and a state function S from E into D^I (with condition 1 and 3 of the Definition 1);
2. $\pi \subseteq T$ is a closed (non-empty) interval;
3. V is a valuation function such that:
   - $V(c) \in E$, for all entity constants;
   - $V(a) \in D$, for all determinate constant;
   - $V(C) \subseteq D^I$, for all concept predicates.

[6] I leave the set of fundamental determinables away in this definition.

Let $M = \langle B, V, \pi \rangle$ be a model of TL, $t \in T$, and φ a sentence. The *truth relation* $M \vDash_t \varphi$, "φ is true in M at t", is defined as follows:

$M \vDash_t t = t^*$ iff $V(t) = V(t^*)$ (t and t* are constants);
$M \vDash_t [P_\delta](c, a)$ iff $S(V(c), t)(\delta) = V(a)$ and $S(V(c), t)(\delta)$ is defined;
$M \vDash_t [-P_\delta](c, a)$ iff $S(V(c), t)(\delta) \neq V(a)$ and $S(V(c), t)(\delta)$ is defined;
$M \vDash_t [C](c)$ iff $S(V(c), t) \in V(C)$ and $S(V(c), t)$ is defined;
$M \vDash_t [-C](c)$ iff $S(V(c), t) \notin V(C)$ and $S(V(c), t)$ is defined;
$M \vDash_t EX(c)$ iff $t \in \pi_x$, π_x is the closed interval where x exists;
$M \vDash_t \neg\varphi$ iff not $M \vDash_t \varphi$;
$M \vDash_t \varphi \wedge \psi$ iff $M \vDash_t \varphi$ and $M \vDash_t \psi$;
$M \vDash_t \forall x\varphi$ iff $M \vDash_t \varphi[\underline{e}/x]$ for all $e \in E$;
$M \vDash_t \forall v\varphi$ iff $M \vDash_t \varphi[\underline{a}/v]$ for all $a \in D$;
$M \vDash_t [\leftarrow\!\!\dashv]\varphi$ iff if there exists a moment $t^* < t$ such that $M \vDash_{t^{**}} \varphi$ for all t^{**} with $t^* < t^{**} < t$;
$M \vDash_t [\mapsto]\varphi$ iff if there exists a moment $t^* > t$ such that $M \vDash_{t^{**}} \varphi$ for all t^{**} with $t < t^{**} < t^*$;
$M \vDash_t PoV\varphi$ iff $M \vDash_{t^*} \varphi$ for some $t^* \in \pi$

$\varphi[\underline{e}/x]$ ($\varphi[\underline{a}/v]$) denotes the closed formula which we get after each free occurrence of x(v) in φ is replaced by the name $\underline{e}$ of the entity e in E (by the name $\underline{a}$ of the entity a in D); I stipulate also that $V(\underline{e}) = e$ ($V(\underline{a}) = a$). The interval π is the *point of view* of the model $M = \langle B, V, \pi \rangle$.

In this definition, I take the set I of determinables to be just Δ to simplify the treatment. A more accurate way would be to map Δ into I by a bijection. My definition of the truth condition for the point of view PoV operator differs from that of progressive tense operator in the sense of not demanding that the "actual state" is in the point of view.

Note that the state function S is partial causing truth-value caps. I solve this problem by taking negative predicates $[-P_\delta]$ or $[-C]$ as well as positive predicates $[P_\delta]$ or [C], thus avoiding introducing three-valued logic in which the third value is "undefined". This device is used in Hautamäki ([13]; see also [9]. For axiomatization of TL, one should include formulas to tell how $[P_\delta]$, $[-P_\delta]$, [C] and [−C] behave for all $\delta \in \Delta$:

$\forall x \forall v \neg([P_\delta](x, v) \wedge [-P_\delta](x, v))$;
$\forall x \forall v(EX(x) \rightarrow ([P_\delta](x, v) \vee [-P_\delta](x, v)))$;
$\forall x \forall v \forall v^*(([P_\delta](x, v) \wedge [P_\delta](x, v^*)) \rightarrow v = v^*)$ (functionality);
$\forall x \neg([C](x) \wedge [-C](x))$;
$\forall x(EX(x) \rightarrow ([C](x) \vee [-C](x)))$.

The concept "x starts to be a in respect of δ change in the respect of the determinable δ" is defined by the formula

$$Start(x,\delta,a) \leftrightarrow EX(x) \wedge [P\delta](x,a) \wedge [\mapsto][P\delta](x,a) \wedge \forall v([\leftarrow\!\!\dashv]([P\delta](x,v) \rightarrow \neg v{=}a)$$

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The predicate "entity x appears" is defines by the formula:
$App(x) \leftrightarrow EX(x) \wedge [\mapsto]EX(x) \wedge [\leftarrow\!\!\dashv]\neg EX(x)$.
The predicate "entity x disappears" is defined by the formula:

$Dis(x) \leftrightarrow EX(x) \wedge [\leftarrow\!\!\dashv]EX(x) \wedge [\mapsto]\neg EX(x)$.

Unfortunately, the larger development of this temporal logic is not possible in the context of this article. The fragments show, regardless, that a relevant modal logic for temporal points of view, based on conceptual space framework, could be built. I made similar proposal for non-temporal points of view in my article [15].

10 Conclusions

I have shown that interesting time related concepts like change, event, and potentiality could be represented and analyzed in the conceptual space framework. My elaboration was inspired by Aristotle's philosophy of nature. Aristotle's analysis of time and change in his *Physics* was shown to be interesting also from a "modern" viewpoint. Particularly his view that time and motion are continuous is still relevant and superior as compared to some recent attempts to develop time logic based on discrete treatment of time. The Aristotelian approach is to emphasize the processual character of motion and change. Therefore we have to consider intervals along with single moments of time. I used that insight in developing a temporal concept of conceptual space.

In my presentation of temporal conceptual space, time is not a determinable (or a dimension of entities). My device is to add time to state functions, mapping entities and moments of time into conceptual spaces. This works well, but the real challenge was to incorporate continuity. In Aristotelian model an entity x is in the state s at the moment t then it is also in some interval containing t as an end point. This means that entities are permanent in some period and it is not possible that they are in a state only in a single moment but not just before and after it. Still there are difficult problems in continuity. Aristotle thinks that states of entities are also continuous. This is clear in the case of quantities, but what about qualities? Aristotle was tempted to suppose that qualities are also continuous, for example a continuum of degrees of illness or redness. Some topology in conceptual spaces is needed before we can define the continuity of determinables. In this article, I just gave some hints how to define continuity of determinables and state function.

One aim of the article was to introduce a time-related concept of point of view in conceptual space framework. My choice is to take closed intervals of time to represent points of view. This means that to have a temporal point of view is to look at world from some period of time. So a point of view is a kind of "time window". Many time-related notions could be relativized to points of view, like existence, change, and event. This makes it possible to compare points of view.

I show also in this article how to build a temporal logic. In the logic TL three modal operators were introduced: PoV (point of view), [$\mapsto$] (forwards) and [$\leftarrow\!\dashv$] (backwards). These operators allow one to express, in TL, interesting time-related concepts like event, appearance, and disappearance. I also used positive and negative predicates for determinables: [P_δ](x, v) (the value of δ for x is v) and [$-P_\delta$] (x, v) (the value of δ for x is not v). By this device I could avoid introducing three-valued logic, the third value being that of undefined. There is a special predicate EX(x) to express the existence of entities.

The extension of the conceptual space approach to include time provided it with new dynamic features. It has been a common bias of logic that it has applied static conceptual models. It is perfect for describing ahistorical, synchronic structures, but if we want to apply logic to events and processes, new kinds of dynamic concepts have to use. Temporal conceptual spaces provide a promising framework to explore the world, changing in time.

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