Let 'Let n be such an x' Be: A Reply to Meléndez Gutiérrez[1]

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Abstract: I defend Haze's argument against the Breckenridge-Magidor theory of instantial reasoning from an objection by Meléndez Gutiérrez. According to Breckenridge and Magidor, in reasoning like 'Some *x* is mortal. Let *n* be such an *x…*', the '*n*' refers to a particular object but we cannot know which. This surprisingly defensible view poses an obvious threat to widespread notions in the philosophy of language. Haze argues that the theory leads to absurdity in cases like 'Some *x* is unreferred-to by any expression. Let *n* be such an *x…*' and should therefore be rejected. Meléndez Gutiérrez counters that Haze's argument is just a case of Berry-like paradox and thus fails to refute the Breckenridge-Magidor theory. I argue that the analogy breaks down: unlike the intuitively compelling and widely believed well-ordering principle about positive integers, the principle drawn from Breckenridge and Magidor that plays a supposedly analogous role enjoys no such status, and is instead simply shown to be false by Haze's reductio. The possibility of such a response is obscured when Meléndez Gutiérrez portrays Haze's argument as involving a stipulation that '*n*' is to refer to an unreferred-to object. On the contrary, Haze's argument does not assume that expressions like '*n*' work by means of referring at all, and simply lets stipulations like 'Let *n* be such an *x*' be themselves, without imposing a theory on them. Once this is clarified, we can see that Haze's argument is unaffected by Meléndez Gutiérrez's objection.

Keywords: instantial reasoning, arbitrary reference, paradox, quantification.

1. The Breckenridge-Magidor Thesis and Haze's Reductio

Consider the following argument:

(1) There is someone x such that for every person y, x loves y [Premise]
(2) Let John be such a person
(3) For every person y, John loves y [Existential Instantiation on 1]
(4) Let Jane be an arbitrary person
(5) John loves Jane [Universal Instantiation on 3]
(6) There is some person x such that x loves Jane [Existential Generalisation on 5]
(7) But since Jane was an arbitrary person, for every person y there is some person x such that x loves y [Universal Generalisation on 6]

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[1] For anonymous review purposes I wrote this paper as though I was a different person. (This footnote has been added after acceptance.)

In their (2012), Breckenridge and Magidor argue that 'John' and 'Jane' in this argument refer to particular people in the world, although we cannot know which. While this thesis may be counterintuitive, Breckenridge and Magidor argue that it lets them account for bits of instantial reasoning like the above in a way which beats alternative theories.

In this way, what may seem at first like an arcane topic in logic—how instantial reasoning works—has, as well as its intrinsic interest, broader interest in the philosophy of language. If Breckenridge and Magidor are right that 'John' and 'Jane' above refer to particular people, although we cannot know which, many widespread ideas about how the reference of a name is determined are under threat. Take for example the descriptivist idea that the reference of a name is determined by an associated description, or a cluster of descriptions.[2] Or take the causal picture made famous by Kripke, according to which a name refers to its bearer in virtue of standing in a particular causal relation to that bearer.[3] It is hard to make out how either of these rival proposals could be made to apply in this case, if Breckenridge and Magidor are right. In this way, the ascendance of the Breckenridge-Magidor thesis threatens to send philosophers with a story to tell about reference-determination back to the drawing board.

In his (2016), Haze argues that this account cannot be right, since it leads to absurdity when applied to an example like the following (in which 'unreferred-to object' means an object that never has been, and never will be, referred to):

(1) All unreferred-to objects are white and there is some unreferred-to object. [Premise]
(2) All unreferred-to objects are white. [Conjunction Elimination on 1]
(3) There is some unreferred-to object. [Conjunction Elimination on 1]
(4) Let O be such an object.
(5) O is white. [Universal Instantiation on 2]
(6) There is some white object. [Existential Generalization on 5]

The Breckenridge-Magidor account leads to absurdity, Haze argues, because it leads to the absurd conclusion that 'O' in the above reasoning refers to an object that never has been, and never will be, referred to.

2. Meléndez Gutiérrez's Objection to Haze's Reductio

In her (2023), Meléndez Gutiérrez makes a case that Haze's reductio fails. She first rehearses Berry's paradox about 'The least positive integer not specifiable by any expression of fourteen words or fewer' (call this definite description 'L'). Given that there are finitely many words and hence finitely many strings of fourteen words or fewer, and given that the positive integers are well-ordered (i.e. each nonempty subset of them contains a least element), L

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[2] See Frege (1892) and Russell (1911), and—for the cluster version—Searle (1983).
[3] See Kripke (1980).

must specify an integer not specifiable in fourteen words or fewer. But L is itself fourteen words long. Thus we have a paradox.

Meléndez Gutiérrez argues that the absurdity that Haze charges Breckenridge and Magidor with is just another manifestation of Berry-like paradox, and hence should not count against the Breckenridge-Magidor account. She writes:

> A definition is *impredicative* if it refers to, or quantifies over a set containing its definiendum. Haze's argument involves what might be called an *impredicative stipulation*: the term 'c' is to refer to one of the objects that are never referred to by anything. The stipulation whereby 'c' is introduced, that is to say, invokes a set that includes 'c' itself—namely, the set containing everything.[4]

And later, summing up:

> Both Haze and Berry invoke sets whose members are characterised by not being the referents of any one of the expressions of a certain class. Let us call these sets B and H, respectively. [...] In Berry's paradox, it is [the well-ordering principle] that allows us to single out one of the members of B. Analogously, by the theory of arbitrary reference, it is possible to refer arbitrarily to an individual member of any non-empty set. In Haze's argument, this theory allows us to refer arbitrarily to one of the elements of H. The parallelism is patent. To my mind, then, blaming Haze's paradoxical result on the theory of arbitrary reference, as Haze does, is as confused as blaming the well-ordering principle for the genesis of Berry's paradox.[5]

3. Why Meléndez Gutiérrez's Objection Fails

The chief disanalogy between Berry's paradox and Haze's reductio is that, while the well-ordering principle about positive integers is both intuitively extremely compelling and widely agreed upon by mathematicians, the principle that it is possible to refer arbitrarily to an individual member of any non-empty set enjoys no such status. On the contrary, the latter principle is eyebrow-raising to begin with, and what's more, the mere consideration of sets of unreferred-to objects immediately throws it into serious doubt.

At this point, someone might try to reply on behalf of Meléndez Gutiérrez as follows. The metasemantic terrain involved in the Haze case is less familiar and, perhaps *because* it is less familiar, currently less well understood than the arithmetical terrain involved in the Berry case. But once you begin to consider the metasemantic terrain seriously, the principle that it is possible to refer arbitrarily to an individual member of any non-empty set becomes highly compelling, perhaps even non-negotiable, much like the well-ordering principle about positive integers.

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[4] Meléndez Gutiérrez (2023), p. 1447.
[5] Ibid., pp. 1447-1448.

However, no such case has been made. Indeed, we have no good reason to think that an account of instantial reasoning must be committed to the idea that the superficially name-like terms introduced in such reasoning literally *refer* to objects at all. When, as quoted above, Meléndez Gutiérrez writes that 'Haze's argument involves what might be called an *impredicative stipulation*: the term "c" is to refer to one of the objects that are never referred to by anything', this is potentially misleading on the present point. In the sample argument at the core of Haze's reductio argument, the stipulative part runs 'Let O be such an object'. To insist that this is *really* a stipulation to the effect that some expression is to *refer* to something is to beg the question in favour of accounts like that of Breckenridge and Magidor.

It bears emphasizing that the sample argument at the core of Haze's reductio argument does not *itself* involve anything patently absurd. On the contrary, part of Haze's point is that it uses familiar rules of deductive reasoning and is extremely hard to fault. The dialectic runs as follows: look, here's a bit of deductive reasoning that is surely in order. But if we assume the Breckenridge-Magidor account of instantial reasoning, we get the absurd result that 'O' in this reasoning refers to an unreferred-to object. Hence, we should reject the Breckenridge-Magidor account of instantial reasoning.

4. A Possible Response and Why it Fails

There remains a possible line of response which could be made on Meléndez Gutiérrez's behalf.[6] In the previous section, the central point against Meléndez Gutiérrez's objection is that the well-ordering principle enjoys much better support than the principle that it is possible to refer arbitrarily to an individual member of any non-empty set, and hence it is much more plausible to reject this latter principle in light of Haze's reductio—and with it, the Breckenridge-Magidor account of instantial reasoning—than it would be to reject the well-ordering principle in light of Berry's paradox. But if we are forced by Berry's paradox and the like to stratify language anyway, preventing us from having quantifiers which can range over (among other things) the expressions making up the language to which they belong, then Haze's reductio can no longer be given and hence we need not reject the Breckenridge-Magidor account. In this way, perhaps Berry's paradox and the like force us to take a measure which blocks Haze's reductio as a side-effect.

I grant that if we had to stratify all language in this way, then Haze's reductio could no longer be given. In that case, we could no longer express the idea of an unreferred-to object, since these are supposed to be objects which aren't referred to by anything whatsoever, including the expressions we're using to express the idea of an unreferred-to object! The main problem with this line of response is that it's highly dubious that we *do* have any obligation to stratify all language in this way. It's a commonplace in discussions of the semantic paradoxes that, while something like the Liar sentence ('This sentence is false') cannot be constructed if we ban self-reference, in natural languages like English, other self-referential sentences seem

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[6] Thanks to an anonymous referee for pushing me in the direction of considering this line of response.

fine and even clearly true (e.g. 'This sentence contains five words').[7] And note that banning self-*reference* alone would not block Haze's reductio—at least, not in any obvious way. It seems we would need to lay down that a language cannot even *quantify* over its own expressions.

Meléndez Gutiérrez, in the following passage, appears to grant that the language stratification response to semantic paradox may not ultimately be the right response, but nevertheless seems to pursue something like the line of thought I am concerned with in this section:

> Language stratification has well-known disadvantages as a strategy to solve semantic paradoxes. Nevertheless, my aim is not to propose a superior alternative, but merely to show that, like any one of the standard paradoxes of self-reference, Haze's argument can be dismantled if self-reference is banned for the relevant language; and, consequently, to contend that its paradoxical result is best blamed not on the Breckenridge-Magidor theory, but on self-reference.[8]

The crucial transition here (signified by 'consequently') seems to me unjustified. Just because there is *a* way of blocking semantic paradoxes like Berry's paradox—namely, insisting on language stratification—which also blocks Haze's reductio, that doesn't at all mean that the contradiction at the heart of Haze's reductio is best blamed on failing to adhere to language stratification (and again, the kind of stratification needed to block Haze's reductio would need to ban a language quantifying over its own expressions). Compare: enforcing a ban on sport would prevent sporting injuries, but that doesn't mean that sporting injuries are best blamed on failing to adhere to a ban on sport. Finally, the above use of 'paradoxical result' in relation to Haze's argument is unjustified. Haze's argument involves a contradiction—that is by design; it is a reductio style argument. That does not mean the argument has a paradoxical result; there is nothing paradoxical about an intriguing philosophical theory turning out to be false. This is important because calling the result of the argument 'paradoxical' might make it look more plausible than it should that, whatever the right solution is to Berry's paradox and the like—even if it isn't language stratification—that solution would also apply to the result of Haze's argument and neutralise it somehow.

5. A Moral

One lesson to draw from this debate is that, in some regions, it is crucial to distinguish quite sharply between quantification and reference.[9] This distinction threatens to collapse under the Breckenridge-Magidor account, and Meléndez Gutiérrez's response to Haze is marred by a failure to keep it clearly in view. The notion of referring to an unreferred-to object is admittedly absurd, but we can nonetheless express the idea of an unreferred-to object, and

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[7] See also the remarks in Section II of Kripke (1975) on Tarski's hierarchy and its apparent unfaithfulness to the realities of natural language.

[8] Meléndez Gutiérrez (2023), p. 1447. Footnote omitted.

[9] Thanks to an anonymous referee for emphasising this point and pushing me to bring it out more explicitly.

there is nothing obviously absurd about quantifying over such objects. After all, it seems true to say that there are uncountably many real numbers that no one will ever refer to.

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