The Accident of Logical Constants

Tristan Grøtvedt Haze

tristanhaze@gmail.com

University of Sydney

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ABSTRACT: Work on the nature and scope of formal logic has focused unduly on the distinction between logical and extra-logical vocabulary; which argument forms a logical theory countenances depends not only on its stock of logical terms, but also on its range of grammatical categories and modes of composition. Furthermore, there is a sense in which logical terms are unnecessary. Alexandra Zinke has recently pointed out that propositional logic can be done without logical terms. By defining a logical-term-free language with the full expressive power of first-order logic with identity, I show that this is true of logic more generally. Furthermore, having, in a logical theory, non-trivial valid forms that do not involve logical terms is not merely a technical possibility. As the case of adverbs shows, issues about the range of argument forms logic should countenance can quite naturally arise in such a way that they do not turn on whether we countenance certain terms as logical.

Keywords: nature of logic, formal validity, logical constants, logical terms.

Introduction

First-order logic has greater scope than propositional logic - it recognizes all the valid forms that propositional logic recognises, and more besides. But what determines the scope of (a) logic? Some have said that the scope of (a) logic is determined by which terms are (counted as) logical constants.[1] Some examples:

> The choice of logical constants determines which sentences are counted as logical truths. (Warmbrōd (1999), p. 504.)

> It is clear that an answer to the question "What is a logical constant?" would provide us with the means to answer the question "Where are the limits of logic?" (Došen (1989), p. 362.)

> Philosophically, my goal has been to find out what distinguishes logical from nonlogical terms, and, on this basis, determine the scope of (core) logic. (Sher (1991), p. 4.)

> At least under most views, logic is concerned only with certain kinds of correct arguments, and in fact only with arguments whose correctness is due to the peculiar properties of the expressions in a certain set. (...) In the most general, least theory-laden conception of it that seems possible, the problem of logical constants is the problem of demarcating in some principle-based, non-arbitrary-looking way the set of expressions that logic should deal with as directly responsible for the logical correctness of arguments (…). (Gómez-Torrente (2002), p. 2.)

[1] The parentheses are there because I want to stay neutral on, to use the terminology of Sagi (2014), the issue of ‘principled’ vs. ‘relativistic’ accounts of the scope of logic.

Some have even suggested that we need logical terms in order to do logic at all:

> For if no terms are designated as logical constants, logic will lose the generality that comes with the ability to isolate and study argument forms. (Hanson (1997), p. 376.)

> All parties agree that permitting all expressions to have variable interpretations makes formal treatments of validity impossible, since no argument form would preserve truth under all interpretations of the logical "constants", and all agree that permitting no expressions to have variable interpretations does not allow for multiple instances of a given logical form, thus precluding the study of *formal* logic. Hence, all agree on the need for logically relevant constants. (Bueno & Shalkowski (2013), p. 2.)

But there are logically correct - i.e. formally valid - arguments whose logical correctness does not depend on the meanings of logical terms. Which argument forms are countenanced in a logical theory does not depend only on which (if any) logical terms are present in the language of the theory. And the problem of the scope of (a) logic does not reduce to the problem of demarcating the logical terms.

These points have been obscured by certain historical developments. For instance, the popular Bolzano-Tarski[2] explication of logical consequence, which in effect takes a language as given and then raises the issue of which expressions in that language count as logical, tends to obscure

[2] See Tarski (1936/1956).

the fact that decisions affecting the scope of a logic may have already been made by the time the language has been determined. Also, it happens to be the case that in the most common presentations of the most common types of logic - classical propositional and first-order logic - all valid forms, except the identity inference (from A infer A)[3], involve logical terms.

There are deeper, but still contingent, reasons for logical terms playing such a prominent role in the study of logic, having to do with the way human beings are trained or constituted. As Ramsey noted long ago:

> We might, for instance, express negation not by inserting a word "not", but by writing what we negate upside down. Such a symbolism is only inconvenient because we are not trained to perceive complicated symmetry about a horizontal axis [...]. (Ramsey (1927), pp. 161 - 162.)

Gil Sagi and Alexandra Zinke are two recent authors who have argued that the scope of (a) logic is not always a matter of logical terms. Sagi (2014) develops a formal theory of ‘semantic constraints’ on a language, designed to offer a way of theorizing about logical form more general and flexible than focusing only on the issue of logical vs. extra-logical terms. Zinke (2018) notes - in passing, really - that propositional logic can be done without logical terms, and that therefore the logical/extra-logical distinction outruns the distinction between logical and extra-logical *terms*. (However, we will see that Zinke treats this as little more than a “strictly speaking” sort of point, and - in my view at least - pulls back from appreciating its full significance.)

[3] Thanks to an anonymous referee for another journal for pointing out to me the need for this exception.

It is the purpose of this paper to further develop these insights. First, I extend Zinke's technique for avoiding logical terms in propositional logic to first-order logic with identity. Second, I consider the case of adverbs, which highlights the importance of not letting the issue of logical termhood hog the limelight by making it clear that it is not even *contingently* the case that issues about (a) logic's scope generally appear as issues about logical termhood.

Zinke (2018, p. 133 - 134) notes that we can do propositional logic without having any terms for truth-functions. It is well known that there are two binary connectives, the Sheffer stroke and the Peirce dagger, both of which are functionally complete all by themselves. Zinke points out that if we only have one connective, we might as well omit it: to form a truth-functional compound, we could just write two sentences next to each other, using brackets as usual for scope distinctions. She writes:

> In such a language, there are no logical constants in the narrow sense, i.e., there are no elements of the alphabet which are classified as logical. The only logical notion there is, is not represented by a sign of the alphabet, but by the particular way the terms are arranged. (Zinke (2018), p. 134.)

Lest it be thought that this is a trick which stops working in a less basic setting, it is worth noting that we do not need to stop at propositional logic.[4] We can without much trouble devise a

[4] The ideas which follow were arrived at independently; thanks to an anonymous referee for another journal for making me aware that Zinke (2018), after quoting Ramsey's remark about negation quoted above, anticipated the propositional part.

language with the full expressive power of standard first-order languages which is free of logical terms. Contemporary presentations of first-order logic typically involve two or three kinds of logical terms: propositional connectives, one or two quantifier symbols, and perhaps an identity predicate. As we just saw, we don't need propositional connectives. Secondly, we don't need a special quantifier symbol either: we may take universal quantification as basic and use the old-fashioned '(x)' style of notation (e.g. instead of '$\forall x\mathrm{F}x$' write '$(x)\mathrm{F}x$'). Thirdly, we may express identity by simply putting two terms together and adding brackets. (See the Appendix for a formal specification of the language just sketched.) Finally, in case anyone is tempted to count brackets as logical terms on the grounds that they are symbols whose significance is in some sense the same in all interpretations of the logical language, note that brackets may be avoided in a two-dimensional notation: instead of writing a left bracket, move up, and instead of writing a right bracket, move down. (The strategy of Polish notation, another well-known means of avoiding brackets, is unavailable in the present setting since we don't have a connective to signal the beginning of our truth-functional compounds.) Similar two-dimensional techniques may be employed to permit more truth-functions, and the existential quantifier, to be taken as basic if desired.

The eliminability of a symbol for identity is, I think, particularly instructive, since it suggests that at least some issues about logic's scope could quite easily have arisen in a guise other than that of an issue about logical termhood. People have differed over whether the theory of identity is part of logic proper, with some maintaining that while first-order logic without identity is pure logic, first-order logic *with* identity is a first-order logical theory about a particular subject-matter, namely identity.[5] Since we typically represent identity claims in formal languages using a special symbol, '=', this issue typically appears as an issue about logical termhood. But this could quite easily have been different. In English, we often express identity claims using 'is'. And just as the 'is' of predication disappears upon logical regimentation, so too would the 'is' of identity if we did things slightly differently as described above. The issue of whether identity falls under logic's scope then appears as an issue about whether to countenance a particular form of wff.

Now, what does all this show? About the propositional logic case, Zinke draws the following moral:

> We are unable to classify a *term* as logical in a case like this, but this does not mean that there are no logical constants in a wider sense in such a language. In the language just described, a grammatical construction rule has to be characterized as logical. Strictly speaking, the problem of logical constants should therefore not be understood as restricted to drawing a line between the logical and the non-logical terms of the alphabet. (Zinke (2018), p. 134.)

I take the point that if by 'logical constants' we just mean something like 'logical features of the language', then none of what we have seen above shows that we don't need *these*. But I think we should demur from extending the meaning of 'logical constants' in this way. In this connection, it is interesting to note that although in the phrase 'logical constants', the emphasis for present

[5] See Quine (1986, pp. 61 - 64) for discussion.

purposes is really on the 'logical' - *logical* constants as opposed to extra-logical ones - the connotation of the phrase easily shifts so that the emphasis falls on 'constants': logical *constants* as opposed to expressions in logical languages whose meanings vary. This shift also fits with the fact that 'constant' tends to be used in logic nowadays only for individual constants, and not in a way that would naturally cover connectives or quantifier symbols. Indeed, this terminology of 'logical constants', euphonious and entrenched as it may be, has arguably been outdated ever since the *Tractatus*, where, while so influentially rethinking the nature of formal logic, Wittgenstein declares that the so called 'logical constants' are not representatives.[6]

Terminology aside, there is a deeper point that needs to be made here. Above, we eliminated symbols which, in the development of logical theories, normally and naturally get used - although in the case of identity, it seems as though it could have easily gone either way - in favour of an alternative way of accomplishing the same thing. This invites the thought that logic, while *technically* it could be done without logical terms, is for us human beings a matter of studying how our logical terms work. Something like this thought may be behind what Zinke goes on to say after the above-quoted passage:

> While it is important to keep this in mind, I will nevertheless continue to speak of the problem of logical constants and say that it focuses on drawing and justifying the line of

[6] See Wittgenstein (1922, §4.0312). From the point of view of our present concerns it is quite important that 'logical constants' appears in scare-quotes in this remark. While the ancestor of this remark in Wittgenstein's early *Notebooks* (Wittgenstein (1969)) does not have the scare-quotes, the scare-quotes appear even in the notoriously buggy first edition, in German, of the *Tractatus*, and persist in both the German and English texts of the Ogden-Richards edition (Wittgenstein (1922)), as well as the Pears-McGuiness translation (Wittgenstein (1961)). They do however tend to go missing when the remark is quoted - for instance in Biletzki & Matar (2018, §2.2) (the main *SEP* entry on Wittgenstein), Armstrong (2004, p. 54, f.n. 1), and Sellars (1962, p. 39). (Ironically, the title of Sellars' paper is 'Truth and "Correspondence"' - another significant use of scare-quotes!)

> demarcation between the logical and the non-logical terms. This not only helps keep the discussion simple, it is also legitimate as far as one is solely concerned with natural language where we indeed have terms representing the logical notions. (Zinke (2018), pp. 134 - 135.)

In my view this is an unfortunate retreat, and the final thought above about how things are in natural language is importantly misguided. A good way to see this is to consider the case of adverbs. Consider the following natural language arguments:

> *Deletion:*
>
> Socrates is running quickly.
>
> Therefore, Socrates is running.
>
> *Addition:*
>
> Socrates is not running.
>
> Therefore, Socrates is not running quickly.

Both arguments are necessarily truth-preserving. Furthermore, there seems to be a clear sense in which they are both necessarily truth-preserving in virtue of their forms. Their being necessarily truth-preserving does not turn on the specific meanings of ‘running’ and ‘quickly’, but rather has to do with the fact that ‘quickly’ belongs to a certain category of expressions - factive, or intersective, adverbs - all members of which can be deleted from atomic statements and added to negated atomic statements while preserving truth (among other inferential behaviours, such as compounding and permutation). Indeed, such arguments are naturally formalised by augmenting first-order logic with an extra category of expressions[7] so that they get translated as follows:

Rs^q

$\therefore Rs$

$\sim Rs$

$\therefore \sim Rs^q$

These arguments may then be determined to be necessarily truth-preserving in virtue of their forms. Some may think that logic should countenance such adverb-involving valid forms, while others may maintain either that the natural language arguments which motivate such a view are not formally valid after all, or that such arguments should be analyzed or regimented into non-adverb-involving valid forms. I am not taking a stand on this issue; the point is that here we have an issue about the scope of logic - about the range of valid forms it should countenance - which does not turn on which expressions are counted as logical constants. Rather, the issue in this case naturally occurs at the level of which grammatical categories and modes of composition should be included in (a) logic.[8]

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[7] See my (ms.) for a development of this approach to the logic of adverbs, including a model-theoretic semantics and a sound and complete proof system. For an approach to handling the logic of adverbs with specially augmented languages see Thomason & Stalnaker (1973) and Pörn (1983). For an approach in which adverbs disappear on analysis or regimentation see Davidson (1967).

[8] Gareth Evans highlights the case of adverbs (and adjectives) in a related context:

> The validity of some inferences is said to be explained by reference to the meanings of the particular expressions occurring in them, while that of other inferences is due, rather, to the way in which the sentences are constructed out of their parts. The inference from 'John knows that snow is white' to 'Snow is white' is given as an example of the first type (...). The inferences from 'John ran breathlessly' to 'John

We are now in a position to see what might be wrong with Zinke's thought that, while strictly speaking it isn't generally correct, to speak nevertheless as if the bounds of logic were a matter of the distinction between logical and extra-logical terms is 'legitimate as far as one is solely

> ran' and from 'John is a large man' to 'John is a man' may, tentatively, be taken to be examples of the second type. (Evans (1976), p. 49.)

We might expect Evans to put the inference from 'Snow is white and grass is green' to 'Snow is white' in the former category, on the grounds that its validity may be explained by reference to the meaning of 'and'. But Evans seeks to assimilate this with the second type. He continues:

> The distinction I have gestured towards is not without its intuitive appeal, and for many years philosophers have been trying to provide a basis for it in harmony with what they took to be its importance. The debate centred upon, and eventually ran aground upon, the problem of identifying a set of expressions as the logical constants. For if we are determined to say that the inference from ⌜P and Q⌝ to P is valid in virtue of structure, then the distinction between it and the detachment inference with 'knows' must reside in some difference between 'knows' and 'and'. (Evans (1976), p. 49.)

This sends Evans in search of a notion of 'structurally valid inference' under which fall the adverbial and 'and'-involving inferences, leaving the 'knows'-involving inference on the non-structural side. Having such a notion may be all to the good, but from the present point of view, on which we are happy to say that many formally valid inferences turn on the meanings of particular expressions (albeit logical ones), Evans's way of motivating his search runs together two distinctions which cross to yield four categories of deductive inferences:

(i) Logical inferences which turn on the meanings of particular terms (e.g. from 'Snow is white and grass is green' to 'Snow is white').
(ii) Logical inferences which do not so turn (e.g. from 'Socrates ran quickly' to 'Socrates ran' (perhaps), or from 'AB' to 'A' in a logical notation where conjunctions are formed by putting conjuncts side-by-side).
(iii) Extra-logical inferences which turn on the meanings of particular terms (e.g. from 'John knows that snow is white' to 'Snow is white').
(iv) Extra-logical inferences which do not so turn (e.g. from 'John *Snow is white*' to 'Snow is white' in a hypothetical language where knowledge attributions are made by writing a statement in italics next to someone's name).

Why did Evans write as if he wanted to find a principled way to avoid saying that the validity of instances of 'and'-elimination and the like 'is to be explained by reference to the meanings of the particular expressions occurring in them'? We find a clue in an idea, derived from Davidson (1973, p. 81), which Evans pursues but finds difficulties with: that in a (broadly Davidsonian) theory of meaning, a word like 'and' is treated via a recursive clause rather than a base clause. We might say that 'and', so treated, is syncategorematic - i.e. that it does not have a meaning by itself. But that doesn't mean it doesn't have a meaning in a broad sense.

(Thanks to an anonymous referee for making me aware of Evans (1976) and for a suggestion about how to think about it.)

concerned with natural language where we indeed have terms representing the logical notions'. The talk of 'logical notions' here rather stacks the deck against a case like that of adverbs, resulting in a subtle non sequitur. For aspects of logic that *are* naturally dealt with using logical terms, it is natural enough to talk of 'notions': the notion of identity, the notion of *all*, the notion of *or*, and so on. But with arguments like Addition and Deletion above, there is no analogous use of 'notions' which fits very well as a way of talking about what is responsible for their apparent validity.

To sum up, the ubiquity of logical terms in the valid forms of standard logical theories is not an essential feature of formal logic. The work done by logical terms in some logical languages may in others be taken over by various aspects of syntax - categories of symbol, modes of composition of symbols, or even modes of appearance of symbols (italicization, writing upside down, or in different colours, etc.). In the case of identity it seems reasonable to think that this could quite easily have gone either way. Furthermore, the issue of how or whether (a) logic should accommodate a case like that of adverbs naturally occurs in such a way that no question arises about whether certain logical terms should be countenanced. Accordingly, philosophers of logic should beware of attaching too much (of the wrong kind of) importance to the notion of logical terms.

Appendix: FOL without logical terms[9]

Vocabulary:

[9] The style of presentation used here is modelled on that of Smith (2012, p. 280 and preceding), which uses a standard language involving logical terms.

Names:

$a, b, c, \ldots t$

If we need more, we use subscripts (i.e. $a_2, a_3, \ldots, b_2, b_3, \ldots$).

Variables:

x, y, z, u, v, w

As with names, we use subscripts if we need more.

Predicates:

$A^1, B^1, C^1, \ldots, A^2, B^2, C^2, \ldots$

Superscripts indicate the number of argument places, and may be omitted for convenience. As with names and variables, we use subscripts if we need more.

Brackets:

(,)

Definition of *term*:

(i) Names are terms.

(ii) Variables are terms,

(iii) Nothing else is a term.

Wffs:

(i) Where $\underline{P}^{\underline{n}}$ is an *n*-place predicate and $\underline{t}_1 \ldots \underline{t}_n$ are terms, the following is a wff:

$\underline{P}^{\underline{n}}\underline{t}_1 \ldots \underline{t}_n$

A wff of this kind is called an *atomic* wff.

(ii) Where α and β are wffs, $\underline{x}$ is a variable, and $\underline{t}$ and $\underline{u}$ are terms, the following are wffs:

(αβ)

($\underline{tu}$)

($\underline{x}$)α

(iii) Nothing else is a wff.

A model may be thought of as consisting of a domain D, a mapping from names to members of D (their *referents*), and a mapping from *n*-place predicates to (possibly empty) sets of *n*-tuples (the predicates' *extensions*).

Truth in a model M may then be defined as follows (for the quantification clause, we use α[$\underline{x}$] to stand for an arbitrary wff with no free occurrence of any variable other than $\underline{x}$, and α[$\underline{a}$/$\underline{x}$] to stand for the wff resulting from α[$\underline{x}$] by replacing all free occurrences of $\underline{x}$ in α[$\underline{x}$] with the name $\underline{a}$):

1. A wff $\underline{P}^{\underline{n}}\,\underline{a}_1 \ldots \underline{a}_n$, where $\underline{P}^{\underline{n}}$ is an *n*-place predicate and $\underline{a}_1 \ldots \underline{a}_n$ are names, is true in M iff the ordered *n*-tuple consisting of the referents in M of $\underline{a}_1$ through $\underline{a}_n$ in that order is in the extension of $\underline{P}^{\underline{n}}$.

2. A wff ($\underline{no}$), where $\underline{n}$ and $\underline{o}$ are names, is true in M iff $\underline{n}$ and $\underline{o}$ have the same referent.

3. A wff (αβ), where α and β are wffs, is true in M iff α and β are both false in M.

4. A wff (x)α[x] is true in M iff for every object *o* in the domain D of M, α[a/x] is true in M^{a}_{o}, where a is some name not assigned a referent in M, and M^{a}_{o} is a model just like M except that in it the name a is assigned the referent *o*.

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