# Title:

A self-consistent opponent-colors theory

# Abstract

Hering’s opponent-colors theory suggests that our color sensations are produced by three opponent-colors mechanisms: a red–green mechanism, a yellow–blue mechanism, and a white–black mechanism. Noticeably, whereas the pair of colors produced by each of the hued mechanisms don’t mix to yield phenomenal intermediates, the colors produced by the hueless (‘achromatic’) mechanism, white and black, do have a phenomenal intermediate—gray. Hering’s theory doesn’t provide an explanation for this different behavior of the white–black mechanism. It is therefore unclear how Hering’s theory accounts for the existence of gray. This paper focuses on an elegant solution to this problem suggested by Paul Heggelund in the 1970’s. Combining Heggelund’s theory with Hering’s opponent-colors theory leads to a beautiful description of phenomenal color space. Here I point out, however, that the combined theory is not self-consistent. I go on to revise Heggelund’s theory to obtain a self-consistent opponent-colors theory.

# Keywords:

color, phenomenology, opponent-colors theory, opponent-process theory, Hering

# 1. Introduction

It is widely accepted that our color experience contains six *elementary* color sensations: white, black, red, yellow, green, and blue (Valberg, 2001). What makes these sensations elementary is that none of them is perceived as being composed of any other color sensation. For example, the elementary version of red (often referred to as *unique* red) is perceived as a purely red sensation that cannot be broken down to more basic color sensations. This should be contrasted with, say, orangish red, which is perceived as a mixture of red and yellow. All colors can be described as some combination of two, three, or four of the six elementary color sensations (Hård et al., 1996). The six elementary colors fall into two phenomenally distinct groups: one group contains two hueless (or ‘achromatic’) colors (white and black); the other group contains four hued (or ‘chromatic’) colors. The gamut of all hueless colors can be arranged in a one-dimensional phenomenal continuum that begins in black, continues to dark grays and then light grays, and ends in white. The gamut of all hues can also be arranged in a one-dimensional phenomenal continuum. However, in contrast to the gamut of the hueless colors, this one-dimensional continuum is *closed* (that is, if we start at an arbitrary hue and move continuously along the hue dimension, eventually we will return to the hue that we started with). This closed continuum is often portrayed as a circle known as the hue circle.

In the last quarter of the 19th century, the German physiologist Ewald Hering noticed that there are certain combinations of the four elementary hues that don’t appear along the hue circle: red and green do not mix to yield intermediate hues (i.e., there are no greenish reds or reddish greens) and neither do yellow and blue (i.e., there are no bluish yellows or yellowish blues). By contrast, any hue from the red–green pair freely combines with any hue from the yellow–blue pair to yield phenomenal intermediates (reddish yellows, bluish greens, and so on). Based on these phenomenological observations, Hering proposed that our sensations of hue are produced by two opponent-colors (or opponent-processes) mechanisms: a red–green mechanism and a yellow–blue mechanism (Hering, 1878, pp. 118–119; for modern presentations of this theory, see Hurvich & Jameson, 1957; Palmer, 1999, pp. 108–114; Shevell & Martin, 2017). Each such mechanism consists of two *elementary-color processes* that operate in an opponent (or antagonistic) manner to each other. Thus, the output of each mechanism results from the difference between the activities of its two constituent processes. As its name implies, each elementary-color process is assumed to give rise to an elementary (i.e., unique) hue. For example, the red–green mechanism consists of one process that gives rise to unique red and another process that gives rise to unique green. The hue sensation that is produced by each mechanism is due to the elementary-color process whose activity is in excess relative to its opponent. Consequently, opponent hues are never perceived together in one color. In other words, opponent hues are mutually exclusive sensations. Thus, Hering’s theory indeed explains the missing intermediate hues along the hue circle.

What about the pair of hueless elementary colors, white and black? Do they also form an opponent pair? The situation here is more complicated than for the hued elementary colors. On the one hand, because white and black—similarly to red and yellow or green and blue—combine to produce a phenomenal intermediate (gray), they don’t seem to form an opponent pair. On the other hand, in the phenomena of afterimages and simultaneous color contrast, white and black behave analogously to the hued opponent pairs (Ladd Franklin, 1899; Titchener, 1910, p. 75). Thus, there is conflicting evidence as to whether white and black form an opponent pair. It is clear, however, that one cannot have the cake and eat it too: either white and black are opponent to each other, in which case they must be mutually exclusive sensations (namely, gray is *not* due to their mixture), or gray is taken to be a mixture of white and black, in which case white and black cannot be opponent to each other. Confusingly, however, Hering’s approach to this dilemma was to hold on to both its horns (Heggelund, 1974a): he suggested that white and black are due to a third pair of opponent elementary-color processes (Hering, 1878, pp. 118–119), yet also contended that gray results from the mixture of white and black (Hering, 1878, pp. 58–62), which of course means that they are not mutually exclusive and hence not opponent.

The inconsistent treatment of the hueless colors in Hering’s theory did not go unnoticed by his contemporaries or by the phenomenologists of the generation after him (Boring, 1942, p. 209; Ladd Franklin, 1899).[[1]](#footnote-2) Here, for example, is Ernst Mach (1897, p. 35) (whose ideas about color greatly influenced Hering):

The only point that still dissatisfies me in Hering’s theory is that it is difficult to perceive why the two opposed processes of black and white may be simultaneously produced and simultaneously felt, while such is not the case with red–green and blue–yellow.

And here is Christine Ladd Franklin (1899, p. 78; italics in the original):

A chief objection to the view of Hering, for those who have been interested in its theoretical aspect, is the inconsistency which meets us at the very beginning; why should black and white be regarded as an *antagonistic* sensation-pair, when they do not destroy each other, but give us, on the contrary, the whole series of grays?

At least two early attempts were made to fix this problem in Hering’s theory. The earliest attempt, which was very influential at the time, was made by the prominent experimental psychologist G. E. Müller (Boring, 1942, p. 213; Ladd Franklin, 1899). There was also a later (and less-known) attempt by F. L. Dimmick (1929, 1948, 1962).[[2]](#footnote-3) Both Müller and Dimmick solved the problem of the hueless colors in Hering’s theory by positing that (a) white and black and opponents and are therefore mutually exclusive and (b) grayness is produced by a non-opponent mechanism that is separate from the white–black mechanism (see Boring, 1949, for a review of the Müller and Dimmick theories). The Müller theory suggested that the non-opponent mechanism generated a constant level of grayness. The Dimmick theory suggested that the non-opponent mechanism generated a level of grayness that complemented the outputs of the opponent-colors mechanisms to the level of the color’s ‘visual intensity’.

Although the Müller and Dimmick theories ostensibly solve the problem of the hueless colors in Hering’s theory, both theories make claims that have been contradicted experimentally or that counter simple observation. Therefore, both theories are untenable. The problem with the Müller theory is that it suggests that all colors should have *a constant level of gray* in them. This is not consistent with what observers actually report: when observers are asked to estimate the level of grayness in colors, they give *variable* estimates for different colors (Evans, 1959; Quinn et al., 1985). Moreover, the fact that colors patches viewed in isolation (‘unrelated colors’) don’t have *any* grayness in them (Fairchild, 2005, p. 89; Shevell, 2003) clearly contradicts Müller’s theory. (Another major difficulty in the Müller theory is that because it doesn’t assume that gray is an elementary color, it must make the odd—and ultimately inconsistent—assumption that white and black are sometimes mutually exclusive sensations and sometimes are not (Ladd Franklin, 1899).) The problem with the Dimmick theory is that it assumes that gray is an elementary color (this is because white and black are taken to be mutually exclusive). This assumption is clearly opposed to the strong intuition that gray is the phenomenal intermediate of white and black and therefore not elementary. And indeed, experiments aimed at directly testing this assumption refuted it unequivocally (Quinn et al., 1985; also see Logvinenko & Beattie, 2011).

In a series of papers starting in the 1970’s, Paul Heggelund (1974a, 1974b, 1991, 1992, 1993) proposed an elegant solution to the problem of the hueless colors in Hering’s theory. Based on systematic observations on the properties of hueless colors, Heggelund proposed that, in addition to black and white, there exists a third elementary hueless color—*luminous* (Heggelund, 1974a). This hueless sensation exists in colors that are perceived as emitting light. For example, this sensation is present in the color of stars in the night sky or in the color produced by light bulbs. On Heggelund’s suggestion, the gamut of the hueless colors should be extended to end in this luminous sensation. That is, the hueless colors stretch from black, through grays, to white, and then continue through luminous whites all the way to a color that is purely luminous. Thus, the color positioned opposite to black on the continuum of hueless colors is luminous, not white. This, in turn, suggests that luminous, not white, is the opponent color to black (Evans, 1974, p. 100; Heggelund, 1974a, b; but see Vladusich et al. (2007), who claim that luminous and black are not mutually exclusive sensations). Hence, according to Heggelund’s theory, white and black are not opponent to each other, which explains why they unproblematically combine to yield gray as an intermediate. Heggelund’s theory of hueless colors therefore neatly solves the most serious flaw in Hering’s theory.

The addition of the luminous sensation to the cadre of elementary color sensations means that there are seven elementary colors rather than six. (And since the luminous sensation is taken to be produced by an elementary-color process., there is now a total of seven of those as well.) Importantly, there is independent evidence to support Heggelund’s proposal that luminous is a third elementary hueless color. First, Evans (1959), based on experimental work somewhat resembles that of Heggelund but preceded it, emphasized the existence of a luminous attribute in hueless colors (he used the terms ‘*fluorent*’ and ‘*fluorescence*’). (Notably, however, Evans’s overall model of the hueless color was different from Heggelund’s (Heggelund, 1974a).) Second, Izmailov and Sokolov (1991) conducted experiments where observers were asked to rank the perceptual distances between pairs of hueless colors. Multidimensional scaling analysis of the results showed that they could be best accounted for by adding a luminous attribute to the hueless colors (Izmailov and Sokolov used the term *bright*). Results consistent with those of Izmailov and Sokolov were later also obtained by Bimler et al. (2009).

When we combine Heggelund’s model of hueless colors with Hering’s model of hued colors we obtain a model wherein any color results from a mixture of one, two, three, or four of the following phenomenal components: a component of red *or* green, a component of yellow *or* blue, a component of luminous *or* black, a white component (Heggelund, 1991, 1993). On Heggelund’s extension of Hering’s opponent-colors theory, we can represent every color by the following four-dimensional vector:

where the unit vectors , , are the standard basis of , namely, , , and so on (the superscript stands for the transpose operation); is the level of activity of the white elementary-color process; , , , , , and (which are all ) are, respectively, the levels of activity of the red, green, yellow, blue, luminous, and black elementary-color processes, and therefore , , and are, respectively, the outputs of the red–green, yellow–blue, and luminous–black mechanisms. It is noteworthy that the vector model of Eq. (1) is reminiscent of other vector models of color (Guth, 1991; Guth & Lodge, 1973; Guth et al., 1980; Ingling & Tsou, 1977), but differs from these models in that it contains the component.[[3]](#footnote-4)

Heggelund (1974a, 1991) completed his theory by suggesting that the *perceived intensity* of a color, which we will denote by , is given by the Euclidean norm of ,[[4]](#footnote-5) namely,

Heggelund used the term ‘color strength’ rather than the term ‘color intensity’ that I use here.[[5]](#footnote-6) Neither term, however, is commonly used. Nevertheless, there are two good reasons to prefer the term ‘color intensity’ (or ‘color strength’) over the much more commonly used term ‘brightness’, which refers to the perceived sensation of the amount of light emitted or reflected from a colored area (Kuehni, 2003, p. 367). First, the sensation of brightness is usually taken to stretch from dim to bright (or dazzling) (Evans, 1974, p. 97; Kuehni, 2003, p. 367; Shevell, 2003). However, from the definition of color intensity in Eq. (2) it is clear that a patch of black color is perceived as having some intensity (for example, a deep black color is perceived as an *intense* black), yet a black patch is neither dim nor bright (the distinction between dark and dim was emphasized by Evans (1974, p. 86)). Second, different authors define brightness differently (e.g., compare Shevell’s (2003) definition with that of Gilchrist (2007)) thus leading to much confusion (Evans, 1974, pp. 7–8; Heggelund, 1974b). Therefore, I will follow Heggelund in the decision to use a straightforward, non-ambiguous term for the sensation of overall magnitude of color strength.[[6]](#footnote-7)

Figure 1 provides a scheme of phenomenal color space according to Heggelund’s theory (Heggelund, 1991).



**Figure 1.** Phenomenal color space according to Heggelund’s extension of Hering’s opponent-colors theory. The outputs of the three opponent-colors mechanisms are projected into the three orthogonal opponent-colors axes shown in the figure. White is located at the intersection of these three axes. In contrast to Hering’s theory, Heggelund’s theory has no problem explaining the existence of gray because in this theory white and black are not opponent colors; rather, the opponent sensation to black is luminous. Another advantage of the Heggelund theory over the Hering theory is that it unifies surface and self-luminous colors (the latter more often called aperture colors) into a single phenomenal space. Even though the Heggelund theory requires four independent phenomenal dimensions for the description of a color (see Eq. (1)), the figure depicts phenomenal color space in three dimensions. This was achieved by requiring that all the colors in this space have the same intensity (color intensity is given by in Eq. (2)). Consequently, all colors in the figure are contained inside (or are on the surface of) a ball whose radius is the fixed value of (see text for details). In the figure itself, to prevent clutter, the borders of this ball are shown only partially: one octant and the hue circle.

Notice that even though the Heggelund theory requires four independent phenomenal attributes for the description of a color (see Eq. (1)), Fig. 1 shows phenomenal color space as only three-dimensional. This was achieved by requiring that all colors in the figure have the samecolor intensity, i.e., the value of is fixed. To see how this works, notice that if color intensity is fixed at a value of , then Eq. (2) becomes

This equation restricts all colors to lie on the surface of a *four-dimensional* sphere with a radius of . However, we can also express Eq. (3) as

Since , Eq. (4) shows that for a fixed value of color intensity , all colors are contained within, or on the surface of, *a three-dimensional ball* with a radius of . This three-dimensional ball is depicted in Fig. 1.[[7]](#footnote-8) To prevent clutter, the figure shows the borders of the ball only partially: one octant (between luminous, unique red, and unique yellow) and the hue circle.[[8]](#footnote-9) A convenient way of thinking of the colors in the phenomenal color space of Fig. 1 is through their level of whiteness. From Eq. (3) we see that this level is given by

That is, the amount of whiteness in a color is given by the difference between the fixed level of color intensity and the overall magnitude of the three opponent-colors components. Hence, when all three opponent-colors pairs are in equilibrium, the perceived color is purely white. In Fig. 1 this is indicated by having pure white located at the intersection of the three opponent-colors axes, namely, at the origin of phenomenal color space. As we move away from the origin, the color contains less and less white, i.e., it becomes more and more saturated[[9]](#footnote-10). At the borders of the color ball, the colors have zero whiteness in them and are therefore fully saturated.[[10]](#footnote-11)

The scheme of phenomenal color space in Fig. 1 highlights another advantage in Heggelund’s extended gamut of hueless colors (Heggelund, 1991). To understand this advantage, we need to acquaint ourselves with the two ‘modes’ of color perception:

[C]olors may be perceived in two different modes often called the light and the object mode... In the light mode the colors are perceived as a property of the light emitted from a field. These colors are called light colors. In the object mode the colors are perceived as a constant property of object surfaces. These colors are called surface colors… (Heggelund, 1993, p. 1709).

Here I refer to what Heggelund called light colors (which are more often referred to as aperture colors (e.g., Heggelund, 1974a; Judd, 1940)) as *self-luminous* colors. Evidently, then, the addition of luminous as an elementary color sensation seamlessly unifies surface and self-luminous colors into a single space.

In summary, combining Heggelund’s theory of the hueless colors with Hering’s opponent-colors theory suggests that every color is determined by the values of four *independent* variables—, , , and . By ‘independent’ we mean that neither of these variables is a function of the others. Hence, the value of any one of these variables can change without affecting the others. Notice that if this weren’t the case, less than four dimensions would be needed for the unequivocal description of any color. This, of course, is what makes it reasonable to assign each of the variables , , , and to a component of the color vector in Eq. (1). In addition to four independent variables, there is one *dependent* variable in Heggelund’s theory—. That this variable is dependent is immediately evident from its definition in Eq. (2), which shows that the value of is a function of the four variables , , , and .

The description of Heggelund’s theory so far is the one given in the three papers he published on the subject from 1974 to 1991. However, Heggelund later realized that to account for his observations, could not be assumed to be independent; rather, its value is dependent on , , , and (Heggelund, 1992).[[11]](#footnote-12) More specifically, the observations suggest that the amount of white in a color is correlated to the difference between color intensity and the overall magnitude of the opponent-colors components. Crucially, the change of from an independent variable to a dependent one is destructive to the theory of Eqs. (1) and (2). The reason is that with being a dependent variable, Eq. (1) for the color vector is no longer valid (see previous paragraph). This, in turn, means that Eq. (2) for , which simply gives as the magnitude of , must be invalid as well. Thus, once is taken to be a dependent variable, Heggelund’s theory collapses.[[12]](#footnote-13)

In summary, Hering’s opponent-colors theory contains a major problem: it does not explain how it is that white and black, which in this theory are taken to form an opponent pair, give rise to a phenomenal intermediate (gray). Heggelund extended Hering’s theory in a way that elegantly solved this problem (and, in addition, neatly combined surface and self-luminous colors into a single framework). However, as was just shown above, Heggelund’s extension of Hering’s opponent-colors theory is untenable because it cannot consistently account for the dependencies between its variables. Since opponent-colors theory is the consensual model of color phenomenology (e.g., Abramov & Gordon, 2005; De Valois & De Valois, 1993; De Valois et al., 1997; Gordon et al., 1994; Hård et al., 1996; Hurvich & Jameson, 1957; Ingling et al., 1996; Li et al., 2022; Rezeanu et al., 2023; Shevell & Martin, 2017; Schmidt et al., 2014; Stockman & Brainard, 2010; Werner & Wooten, 1979; for an outline of an alternative theory, see Conway et al., 2023), the fact that we do not have a version of this theory that is self-consistent is, of course, problematic. The goal of this paper is to rectify this situation by revising Heggelund’s theory to obtain a self-consistent opponent-colors theory.

# 2. Obtaining a self-consistent opponent-colors theory

## 2.1 Revising Heggelund’s theory

As explained in the last part of the *Introduction*, Heggelund’s observations indicated that must be taken as a dependent variable. This *necessarily* means that should now be taken as an independent variable. That is, in contrast to what Heggelund originally suggested in Eq. (2), cannot in fact be a function of , , , and . We know that this has to be the case (i.e., must be an independent variable), because otherwise color would only need three phenomenal dimensions for its full characterization, but we know that four are needed. Now, since it is that is now taken to be an independent variable, whereas is taken to a dependent one, we clearly need to replace the color vector of Eq. (1) with a color vector in which the variable takes the place of the variable as the -component of the vector, namely,

Notice that we denote the new color vector by to emphasize its distinctness from the original color vector . Thus, the first step in our revision of Heggelund’s theory is the replacement of Eq. (1) by Eq. (6).

What we are still missing are equations for and . We start with the latter. Since is a dependent variable, we need to find its functional dependence on the variables , , , and We are aided by two guidelines. First, as mentioned above, Heggelund’s results indicated that the amount of white in a color is correlated with the difference between color intensity and the overall magnitude of the opponent-colors components. Second, even though the Heggelund theory of Eqs. (1) and (2) is now invalid, the phenomenal color space that it gives rise to (i.e., Fig. 1) remains the most sensible option for a color space because it is *maximally symmetric* (the *Discussion* elaborates on why symmetry constitutes a powerful argument in our context). Why? Firstly, because this space locates white, which is the only elementary color that doesn’t have an opponent counterpart, at the intersection of the three opponent-colors axes. Any other choice would violate this symmetry. Secondly, because all colors in this space are contained within *a ball*, the most symmetric shape in three dimensions. Hence, we would like to retain the phenomenal color space of Fig. 1. With the above two guidelines, we immediately conclude that the functional dependence of on , , , and must be given by

This expression for gives rise to the phenomenal color space depicted in Fig. 1, exactly as desired. This is because the behavior of whiteness in this space is described by Eq. (5), which is a specific case of Eq. (7). It is also evident from Eq. (7), again as desired, that white ‘fills the gap’ between color intensity and the magnitude of the opponent-colors components.

Taking stock, we see that our revision of Heggelund’s theory currently consists of Eqs. (6) and (7). The missing ingredient in obtaining a self-consistent opponent-colors theory is an equation for , which will replace Eq. (2) in Heggelund’s theory. This is what we do next. We begin by noticing that an implication of Eq. (7) is that the following inequality necessarily holds:

For if this inequality didn’t hold, Eq. (7) would assign an imaginary value to , which is impossible. Now, it is clear that for Eq. (8) to always be obeyed, the value of must be a function of the activities of the six elementary-color processes , , , , , and . Otherwise, how would the visual system ensure that Eq. (8) *always* holds? In addition to the constraint of Eq. (8), we must also ensure that is an independent variable (see above), namely, we must demand that is not a function of the three opponent-colors components, , , and . The expression for is therefore tightly constrained. I argue that the *simplest possible* expression for that obeys the above two constraints is the following:

This is a very sensible expression for color intensity. It can easily be shown that this expression obeys the inequality of Eq. (8).[[13]](#footnote-14) And, obviously, in this expression is not a function of the values of the three opponent-colors components. Thus, the expression for in Eq. (9) obeys the two constraints stated above. What makes this expression the simplest one possible are the following two features. First, it is perfectly *symmetric* with respect to the six elementary-color processes since all these processes contribute to in the same way. Second, all elementary-color processes contribute *linearly* to the value of (i.e., they appear in the first power in the expression for ). Notice that this is identical to the way in which the elementary-color processes contribute to the values of the opponent-colors components (i.e., the elementary-color processes appear in the first power in the expressions for these components). In conclusion, Eq. (9) provides the simplest possible expression for and we therefore adopt it as the expression for color intensity in the proposed theory (the *Discussion* elaborates on why adopting the simplest possible expression for is the most sensible approach). By doing that we have attained what we set out to do, namely, to find an expression for that replaces Eq. (2) in Heggelund’s original theory.

Equations (6)–(9), when taken together, form a self-consistent opponent-colors theory that replaces Heggelund’s theory. Here is a brief summary of the proposed theory. The theory suggests that every color is determined by the values of four independent variables—, , , and . Therefore, every color can be described by the color vector  of Eq. (6). The intensity of a color, , is given by the summed activity of all six elementary-color processes (Eq. (9)). The level of whiteness in a color is determined by the process , which is dependent on the values of the four components of the color vector . More specifically, as Eq. (7) shows, the level of whiteness in a color is determined by the difference between the color’s intensity, , and the overall magnitude of the three opponent-colors components. Finally, from Eq. (7) is easy to see that for a fixed value of , phenomenal color space is described by Fig. 1. All colors in this space are contained within (or are on the surface of) a ball whose radius is the fixed value of .

## 2.2 An alternative formulation of the proposed theory

The previous subsection argued that the expression for in Eq. (9) is the simplest one possible. This claim was based on two features of this expression: first, it is symmetric, i.e., all six elementary-color processes contribute to in the same way; second, it is linear, i.e., the elementary-color processes in this expression appear in the first power. An expression for that is different from the one in Eq. (9) but shares these two features is the following:

Although the expression for in Eq. (10) seems to be totally different from the one in Eq. (9), this subsection will demonstrate that if the variables , , , , , and are assigned a slightly different meaning from their original one as elementary-color processes, then the two expressions for become mathematically equivalent. The conclusion will be that Eqs. (9) and (10) give rise to two different, yet mathematically equivalent, formulations of the proposed opponent-colors theory. The advantages of formulating the proposed theory in two different ways will also be outlined.

As was just noted, in the subsequent development, the variables , , , , , and will be assigned a different meaning than their original one as elementary-color processes. To keep things clear and prevent confusion, in this subsection we will therefore denote the elementary-color processes by , , , , , and . For example, in the new notation, Eq. (9) for color intensity will read:

Similarly, in terms of the newly named elementary-color processes, Eq. (6) for the color vector will be written as

where the explicit expression for from Eq. (11) was used for the -component.

We now define six *auxiliary variables*, , , , , , and , in terms of the elementary-color processes (in their new notation) in the following way:

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where is given in Eq. (11). The first thing that can be easily verified from Eq. (12) is that

That is, the expressions for the opponent-colors components in terms of the auxiliary variables are equal to the expressions for these components in terms of the elementary-color processes. Next, notice that summing the terms for and in Eq. (12a) yields , as required by Eq. (10). The same result is obtained for and in Eq. (12b) and for and in Eq. (12c). We thus see that the expression for in Eq. (10) holds true for the auxiliary variables , , , , , and defined in Eq. (12).

We now notice that because Eqs. (10) and (13) hold, Eqs. (6)–(8) are valid even when the variables , , , , , and appearing in them are taken as the auxiliary variables defined in Eq. (12). This means that we have obtained an alternative formulation for the proposed theory. The two formulations share Eqs. (6)–(8), but differ in their expressions for color intensity—Eq. (9) in the original formulation, Eq. (10) in the new formulation. It should also be remembered that the meaning of the variables , , , , , and is different in the two formulations.

There are two advantages of having two formulations of the proposed theory, one quite obvious, the other more subtle. The obvious advantage is that each of the two formulations provides a different view of opponent-colors theory, thus giving us more insight into the theory. Both formulations of the theory suggest that there exist six color processes, , , , , , , that are organized into three opponent pairs: , , and . But here the differences between the two formulations begin to emerge. In the original formulation, the color processes are taken to be *independent* of each other. Namely, the activity of each of the six color processes is determined independently of the others. Consequently, the overall activity of the six processes is given by their sum (Eq. (9)). By contrast, the activities in the six processes of the alternative formulation are not independent of each other. Rather, as is evident from Eq. (10), these activities are constrained such that the summed activity of each pair of opponent processes is identical. The second, more subtle, advantage of having two mathematical formulations for the proposed theory becomes apparent if one wishes to identify the actual processes in the brain that give rise to color (see the *Discussion* for an elaboration on this issue). In such a case, one of the formulations might turn out to be more easily associated with those processes.

# 3. Discussion

When stripped to the bone, color exhibits only four fundamental phenomenal properties:

1. There are seven elementary color sensations: red, green, yellow, blue, luminous, black, and white. All colors are due to some mixture of these seven elementary colors (though not all of them simultaneously).
2. The first six of the seven elementary colors listed above are arranged into three opponent pairs: red–green, yellow–blue, luminous–black. The defining feature of each opponent pair is that the two colors in it are phenomenally mutually exclusive, namely, they never appear together in one color.
3. Every color has some level of intensity.
4. The seventh elementary color, white, does not have an opponent color. The amount of white in a color is correlated with the difference between colors intensity and the overall magnitude of the three opponent-colors components.

The goal of this paper was to obtain a self-consistent theory that can account for these properties. This was achieved by the opponent-colors theory of Eqs. (6)–(9) (or, alternatively, Eqs. (6)–(8) and (10)). The resulting theory accounts for properties (i)–(iv) in the following manner:

1. There exist six elementary-color processes: , , , , , and . Each of these processes gives rise to an elementary color sensation. We thus get red, green, yellow, blue, luminous, and black.
2. The elementary-color processes are organized into three opponent pairs: , , and . Consequently, in any one color we can only perceive red *or* green, yellow *or* blue, luminous *or* black.
3. The intensity of a color, , is given by the summed activity of all six elementary-color processes (Eq. (9)). (Alternatively, the intensity of a color is given by the sum of the two processes in any of the three opponent pairs (Eq. (10)).)
4. There is a seventh elementary-color process, , which gives rise to a seventh elementary color—white. The value of is given by Eq. (7). As this equation shows, the amount of whiteness in a color is given by the difference between color intensity and the overall magnitude of the three opponent-colors components.

For a fixed level of color intensity, the theory gives rise to the phenomenal color space shown in Fig. 1. All colors in this space are located inside, or on the surface of, a three-dimensional ball whose radius is the fixed level of color intensity.

The guiding principles in the development of the proposed theory were *simplicity* and *symmetry*. In particular: (a) the motivation for adopting Eq. (7) for was the fact that it leads to the most symmetrically possible phenomenal color space, which is the one depicted in Fig. 1; (b) in the color vector (Eq. (6)), the elementary-color processes in each of the three opponent pairs appear in the first power, which, of course, is the simplest possibility; (c) the motivation for adopting the expressions for given in Eqs. (9) and (10) was their simplicity (specifically, these expressions are linear) and symmetry. The end result, the theory of Eqs. (6)–(9) (or, alternatively, of Eqs. (6)–(8) and (10)), is therefore *the simplest and most symmetrical theory* that can self-consistently account for the fundamental phenomenal properties of color (points (i)–(iv)).

Undoubtedly, however, the vast majority of color researchers today would view the proposed theory as, at best, an extremely simplified description of the *true* theory of color, and the phenomenal color space of Fig. 1 as, at best, an idealized caricature of *true* color space. After all, we know that the relationship between the properties of the visual stimulus (i.e., its power distribution and spectral composition) and the properties of the perceived color is horrendously complicated (Fairchild, 2005; Kuehni, 2003). And we know that the neurophysiological processes that are color-related are not easy to interpret and in no way neatly align themselves with color phenomenology (e.g., Derrington et al., 1984; Lennie et al., 1990; Mollon, 2009; Valberg, 2001; but see the recent work by Li et al. (2022)). And we also know that trying to model how the phenomenology of color comes about from the physiological properties of color-sensitive cells is a complex business (De Valois & De Valois, 1993; Guth, 1991; Guth & Lodge, 1973; Guth et al., 1980; Ingling & Tsou, 1977; Rezeanu et al., 2023; Schmidt et al., 2014; Schmidt et al., 2016; Stockman & Brainard, 2010). Given all that, how could the phenomenology-based, ridiculously simple model of Eqs. (6)–(9) and the unrealistically symmetric phenomenal color space that corresponds to it (Fig. 1) possibly have any merit?! Surely they couldn’t.

The sentiment that phenomenology cannot be trusted to give us anything (or almost anything) of scientific value, and that we should instead turn to psychophysics and neurophysiology, is deeply entrenched. Already in 1942, Boring, while discussing Hermann Ebbinghaus’s color pyramid from 1902, which is a description of phenomenal color space reminiscent of the space of Fig. 1, had the following to say about the declining scientific status of phenomenology:

For a while Ebbinghaus’ double pyramid represented the last stand of the phenomenologists against the encroachments of the nervous system upon psychology: here in the color pyramid, it was argued, there is at least one fact that is independent of both the stimulus and of physiology. That there are but few psychologists any longer to cherish such a last leaf on the tree of mentalism goes to show how phenomenology perpetually, in the development of psychology, loses the battle to experimentalism. (Boring, 1942, p. 149)

In this (almost poignant) paragraph, Boring describes the 20th-century process in which the objective sciences of the mind, psychophysics and neurophysiology, have gained superiority over the subjective science of mind—phenomenology. However, since there has now been a growing realization that physicalism *cannot* be true (Chalmers, 1995, 1996; Goff, 2017; Jackson, 1982; Nagel, 1974), I argue that it is perhaps time to reconsider the reflexive rejection of the veridicality of phenomenological data. I base my argument on a non-physicalist view of phenomenal experience according to which all types of phenomenal experience (i.e., sound, odor, taste, color, etc., and presumably also types that are not experienced by humans) are *fundamental* ingredients of our universe. Because they are fundamental, namely, not composed of more basic precursors, it is necessarily the case that they were created *fully formed* at the birth of the universe. I’ll refer to this view as the *FFF view*, for *Fundamental and thus Fully Formed*. The FFF view has several significant advantages over the competing non-physicalist views, which argue that the types of phenomenal experience that exist in biological creatures came about either from combinations of more basic types of phenomenal experience or from combinations of proto-experiences, which are themselves nonconscious precursors to phenomenal experience (Chalmers (2015, 2017) gives a systematic analysis of all these views).[[14]](#footnote-15) First, the FFF view doesn’t have to invoke an unknown and metaphysically dubious ‘mental chemistry’, to use Coleman’s (2012) term,[[15]](#footnote-16) to explain how basic types of phenomenal experiences or proto-experiences combine to give the types of phenomenal experience that we know (this is the ‘quality combination problem’ (Chalmers, 2017)). Second, it doesn’t have to explain how proto-experiences somehow combine to become ‘fully-blown’ conscious. Third, it provides a solution to the ‘palette problem’ (Chalmers, 2017).[[16]](#footnote-17). One possible objection to the FFF view is the seemingly adaptive associations between some types of phenomenal experience and their corresponding behavioral responses. In defense, we can invoke the hypothesis suggested by Zietsch (2024), according to which these associations are only *seemingly* adaptive.

With the FFF view established, we can now return to the question of the veridicality of phenomenological data, and specifically the veridicality of this data in the case of color phenomenology. My argument is that subscribers to the FFF view should expect the phenomenal properties of color to exhibit simplicity, symmetry, and beauty. This argument is based on the fact that the mathematical description of the structure of the fundamental *physical* ingredients of our universe exhibits simplicity, symmetry, and beauty (e.g., consider the mathematical description of spin-1/2 in quantum mechanics). Therefore, by analogy, there is every reason to believe that the mathematical description of the structure of the fundamental *nonphysical* ingredients of our universe will exhibit similar features. Consequently, subscribers to the FFF view should give high credence to the opponent-colors theory proposed here (along with its corresponding phenomenal color space (Fig. 1), in all its glory) since this theory provides the simplest and most symmetric mathematical description of the phenomenal properties of color summarized in points (i)–(iv) above.

To summarize, adherents of a non-physicalist worldview who are convinced by the claims made by the FFF metaphysical view should consider the proposed theory to be the most plausible mathematical description of color phenomenology. This theory proposes that our color sensations arise from the visual system’s ‘sampling’ of the independently-existing phenomenal color space depicted in Fig. 1. More specifically, after brains (or more primitive types of nervous systems) learned to convert electromagnetic radiation into electrochemical signals, they figured out a way to use these signals to tap into a *preexisting* phenomenal color space. The myriad irregularities, asymmetries, and nonuniformities that appear when psychophysicists try to organize our color percepts into three-dimensional solids (see Kuehni, 2003) are therefore to be understood the visual system’s partial, imperfect, and nonuniform ‘sampling’ of the symmetric and uniform[[17]](#footnote-18) phenomenal color space of Fig. 1.

But how does the visual system supposedly ‘tap into’ or ‘sample’ phenomenal color space? One attractive option is provided by the *dual-aspect theory* (or double-aspect theory) of phenomenal consciousness (see, e.g., Benovsky, 2016; Chalmers, 1995, 1996, chapter 8; Taylor, 1963, chapter 2). This theory suggests that conscious sensations are phenomenal duals of the physical states of some physical entity or physical process.[[18]](#footnote-19) On this theory, there should exist an exact correspondence between the structure of the phenomenal states of any type of conscious experience and the structure of the physical states of the underlying physical system (Chalmers, 1995, 1996, chapter 8; Lockwood, 1989, chapter 11; also see G. E. Müller’s famous psychophysical axioms (translated in Boring, 1942, p. 89)).Therefore, if one takes the dual-aspect theory seriously (and see Benovsky (2016) for why we all should), then there should exist in the brain a physical system whose mathematical description is identical to the mathematical description of color phenomenology that is suggested here (i.e., Eqs. (6)–(9) and Fig. 1).[[19]](#footnote-20) We can exploit this logic to obtain a sanity check for the suggestion that color sensations are the phenomenal duals of the physical states of some physical system: there should exist a physical system whose mathematical description is given by Eqs. (6)–(9). That a well-known physical system with this exact mathematical structure indeed exists will be shown in a follow-up paper.

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1. Quite perplexingly, however, this acute problem in Hering’s theory is almost completely ignored in modern accounts of opponent-colors theory. For example, Palmer (1999), in his well-known textbook about vision, comments on the problematic status of the white–black mechanism in Hering’s theory, but simply asserts that ‘There is thus something qualitatively different about the achromatic dimension.’ (p. 110), without any attempt to explain why or how this difference comes about. [↑](#footnote-ref-2)
2. It is noteworthy that relatively recently, Nayatani (2001, 2002) proposed a modification to Hering’s theory, which—to the best of this author’s judgement—is virtually identical to the modification suggested by Dimmick. [↑](#footnote-ref-3)
3. Another difference between Heggelund’s model and these past models is that the opponency in these models was *cone* opponency rather than true *color* opponency (Guth’s 1991 model is an exception in this regard). [↑](#footnote-ref-4)
4. Heggelund (1974a, 1992, 1993) provided an alternative possibility wherein color intensity is given by the L1 norm of the color vector $F$ rather than by the Euclidean (i.e., the L2) norm of Eq. (2), namely,

$$I≡\left‖F\right‖\_{1}=\left|W\right|+\left|R-G\right|+\left|Y-B\right|+\left|L-Bk\right|.$$

However, in order to only follow a single thread of equations, I will use the definition of $I$ given in Eq. (2). This choice has no appreciable effect on the forthcoming development and conclusions. [↑](#footnote-ref-5)
5. I prefer the term ‘color intensity’ over Heggelund’s ‘color strength’ because it generalizes more naturally to other sensory modalities, e.g., sound intensity, odor intensity, and so on. [↑](#footnote-ref-6)
6. Heggelund’s decision to use the term color strength rather than brightness was indeed motivated by an attempt to prevent confusion: ‘the term color strength is preferred to brightness, since to most people brightness is closely associated with aperture colors, and therefore unsuitable to denote the intensity aspect of the surface colors.’ (1974a, p. 1075). (The definitions of aperture and surface colors are given below.) [↑](#footnote-ref-7)
7. Heggelund (1991) did not realize that his equations mean that for a fixed level of color intensity all colors are contained within a three-dimensional ball. Instead, he *arbitrarily* chose a double-cone shape, stating that ‘The actual structure probably depends to a large extent on the selected value for the constant color strength.’ (Heggelund, 1991, p. 317). [↑](#footnote-ref-8)
8. Had we chosen the L1 norm form of color intensity instead of the Euclidean norm of Eq. (2) (see Footnote (4) above), all colors in the phenomenal color space of Fig. 1 would have been contained within *a double pyramid* and the hue circle would have replaced with a hue square. [↑](#footnote-ref-9)
9. The term saturation is ordinarily used only with respect to hued colors. Here I am generalizing it to include the hueless colors luminous and black as well. Thus, a pure luminous color and a pure black color are taken to be fully saturated. [↑](#footnote-ref-10)
10. This is idealized—full saturation is never perceived for hued colors. Namely, even the most saturated hues (which are the spectral hues) have some whiteness in them (Gordon & Abramov, 1988; Gordon et al., 1994; Jacobs, 1967). [↑](#footnote-ref-11)
11. His actual conclusion was that $W$ was dependent on the value of the $\left(L-Bk\right)$ component and on the stimulus’s luminance (Heggelund, 1992, Eqs. (5) and (6)). That he confined his conclusion to only the $\left(L-Bk\right)$ component is readily understood as stemming from the fact that his 1992 paper only dealt with hueless colors. Thus, the suggestion made in the text that $W$ is also dependent on the other two opponent-colors components is a natural generalization of Heggelund’s original conclusion. In addition, in the text I replaced Heggelund’s luminance, which is a psychophysical magnitude, with color intensity, $I$, which is a perceptual attribute. This replacement has two justifications. First, in the case of whiteness, color intensity can be equated with brightness, which is the perceptual representation of luminance (Gilchrist, 2007). (The luminance of a visual stimulus is a psychophysical magnitude that measures the amount of light in the stimulus that is available for usage by the visual system. For a more formal definition, see, e.g., Lennie et al. (1993).) Second, if the level of whiteness did not depend on color intensity, holding color intensity fixed would not reduce the dimensionality of color from four to three, as was done when we obtained Fig. 1. But Heggelund never claimed that the fact that $W$ is a dependent process made the phenomenal color space of Fig. 1 untenable. (We also know independently of Heggelund that if color intensity is held constant, color requires only three dimensions for its unequivocal description (Boring, 1949).) [↑](#footnote-ref-12)
12. Heggelund himself did not seem to have noticed that the fact that $W$ is a dependent variable is inconsistent with his earlier model. One possible reason for that might be that in his 1992 paper, which attempted to quantitatively model his observations on the hueless colors, he did not model color intensity, $I$. Had he done so, he probably would have realized that there is a circular dependency between $W$ and $I$. [↑](#footnote-ref-13)
13. To show this we begin with the following trivially true inequality:

$$\left(R-G\right)^{2}+\left(Y-B\right)^{2}+\left(L-Bk\right)^{2}\leq \left(R+G\right)^{2}+\left(Y+B\right)^{2}+\left(L+Bk\right)^{2}.$$

Next, notice that the following inequality is also trivially true:

$$\left(R+G\right)^{2}+\left(Y+B\right)^{2}+\left(L+Bk\right)^{2}\leq \left(R+G+Y+B+L+Bk\right)^{2}=I^{2},$$

where the expression for $I$ in Eq. (9) was used for the equality on the right-hand side. Since the right-hand side of the first inequality equals the left-hand side of the second inequality, we can concatenate the two inequalities above and immediately arrive at the following inequality, which is the desired result, namely, Eq. (8):

$$\left(R-G\right)^{2}+\left(Y-B\right)^{2}+\left(L-Bk\right)^{2}\leq I^{2}.$$

 [↑](#footnote-ref-14)
14. All the views I discuss here are panpsychist or panprotopsychist. [↑](#footnote-ref-15)
15. Coleman himself credits Mill with the term. [↑](#footnote-ref-16)
16. The palette problem was formulated by Chalmers (2017, p. 183) as follows:

There is a vast array of macroqualities, including many different phenomenal colors, shapes, sounds, smells, and tastes. There is presumably only a limited palette of microqualities… How can this limited palette of microqualities combine to yield the vast array of macroqualities?

One possible set of solutions to the palette problem is what Chalmers dubbed large-palette solutions. Like the FFF view advocated in the text, these solutions suggest that all types of phenomenal experience are fundamental (i.e., are not composed of any precursor) and have therefore existed in their current form since the creation of the universe. [↑](#footnote-ref-17)
17. By ‘uniform’ I mean that this space has a Euclidean metric. By contrast, color order systems, for example the Munsell color system, are notoriously nonuniform, namely, they have a non-Euclidean metric (Judd, 1969; Kuehni, 2003, chapters 7 and 8). [↑](#footnote-ref-18)
18. Dual-aspect theory can be interpreted as a variant of property dualism, as a variant of neutral monism, or as a theory in its own right (Benovsky, 2016; Stubenberg, 2014, section 9.4; Van Gulick, 2022, section 8.1). The way I present the theory here is closer to the property dualism interpretation, but this has little, if any, significance in our context. [↑](#footnote-ref-19)
19. The term ‘physical system’ should be construed here in a biological context. Thus, given our knowledge of how neurons work, this hypothesized physical system might be a small component of a much larger biophysical system, like an ion channel. [↑](#footnote-ref-20)