# Title:

Color experience is the phenomenal dual aspect of two-state quantum systems in a mixed state

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# Abstract

I show that the mathematical description of opponent-colors theory is identical to the mathematical description of two-state quantum systems in a mixed state. Following the principles of dual-aspect theory of phenomenal consciousness, which predicts an exact correspondence between a system’s phenomenal states and the objective states of its underlying physical substrate, I suggest that color sensations are phenomenal dual aspects of two-state quantum systems in a mixed state. Since nothing in this hypothesis suggests that what brings about the phenomenal experience that is dual to two-state quantum systems is the two-dimensionality of these systems, it is natural to generalize this hypothesis and suggest that other types of phenomenal experience (e.g., taste, odor, sound) are phenomenal dual aspects of quantum systems of higher dimensionalities. Finally, I propose that the two-state quantum systems that give rise to color in the brain are two-state ion-channels.

# Preface (or: Taking consciousness seriously)

Like many others, my views on phenomenal consciousness changed dramatically after reading David Chalmers’s attack on physicalism in ‘*The Conscious Mind’* (1996). Chalmers said nothing new, of course. Many thinkers before him, from Leibniz through Huxley to Nagel and Jackson, had noted the impossibility of physicalism. But Chalmers’s argumentation was so forceful and lucid (and came exactly at a time when people were beginning to realize that neuroscience could not possibly solve what, thanks to Chalmers, has become known as the ‘hard problem’) that it has led to a tectonic shift in the philosophy of mind: physicalism, which was the unchallenged prevailing metaphysical view of the second half of the 20th century, is now more and more commonly seen as a preposterous world view.

At the very beginning of this book, Chalmers urges us to ‘take consciousness seriously’. My understanding of this call for arms is that we should accept—really accept, not merely as lip service—that there is some process in the brain that gives rise to this wonder of wonders, phenomenal consciousness. Since now there doesn’t seem to be any physiological, biophysical, chemical, or physical process in the brain that is not known to modern science at some level of detail, the answer to the biggest mystery must be within our reach. On the other hand, this answer—as was so convincingly argued by Chalmers—is not going to come from scientific evidence because such evidence only deals with objective processes, relations, and functions, exactly the kind of stuff that is *not* phenomenal consciousness. Since the only way we have to understand the universe around us it through scientific evidence, the ‘hard problem of consciousness’ seems to be unsolvable (McGinn, 1989). There might be, however, a glimmer of hope. This glimmer comes from the stipulation of the dual-aspect theory of phenomenal consciousness that there exists an exact correspondence between phenomenal states and their underlying physical processes (Chalmers, 1996, chapters 6 and 8; Cortês et al., 2021; Lockwood, 1989, chapter 11; also see G. E. Müller’s famous psychophysical axioms (Boring, 1942, p. 89)). Here, for example, is how Lockwood (1989, p. 172) phrases this idea:

Take some range of phenomenal qualities. Assume that these qualities can be arranged according to some abstract $n$-dimensional space, in a way that is faithful to their perceived similarities and degrees of similarities... Then my… proposal is that there exists, within the brain, some physical system, the states of which can be arranged in some $n$-dimensional state space… And the two states are to be equated with each other: the phenomenal qualities are identical with the states of the corresponding physical system.

Therefore, if we find a physical system whose states can be shown to have the same structure as the states of some type of phenomenal experience, this physical system should be considered a good candidate to have a phenomenal dual aspect. This is precisely what I do in this paper. I show that there exists an *exact* correspondence between the mathematical structure of opponent-colors theory and the mathematical structure of two-state quantum systems in a mixed state. This leads to the conclusion stated in the title of this paper: color experience is the phenomenal dual aspect of two-state quantum systems in a mixed state. I will be happy to receive *any kind of response* regarding this hypothesis.

# 1. Introduction

As was first noticed by Leonardo da Vinci, there seem to exist six elementary color sensations: red, green, yellow, blue, white, and black (Valberg, 2001). These six sensations are elementary because they cannot be broken down to other color sensations (compare, for example, with the sensation of orange, which is perceived as some mixture of red and yellow). In his opponent-colors theory, Hering suggested that the six elementary colors are produced by three mechanisms, each containing a pair of *opponent* elementary color processes: a red–green mechanism, a yellow–blue mechanism, and a white–black mechanism (Hurvich & Jameson, 1957). The output of each mechanism results from the difference between the activities of its two constituent processes (which, of course, is why these processes are called opponent). Consequently, the color sensation that each mechanism produces is due to the process that is more active.

This is the standard formulation of opponent-colors theory. It faces two problems. First, while the colors produced by each of the hued mechanisms do not mix to yield a phenomenal intermediate (i.e., there are no greenish reds, reddish greens, yellowish blues, or bluish yellows), the colors produced by the white–black mechanism do have a phenomenal intermediate—gray (Boring, 1942, p. 209; Heggelund, 1974a; Ladd Franklin, 1899). This, of course, raises the question of why the white–black mechanism behaves differently from its hued counterparts, a question that opponent-colors theory doesn’t answer. The second problem facing Hering’s opponent-colors theory is that it predicts that when all three opponent-colors mechanisms are in equilibrium (i.e., when their activities are zero), we should perceive nothing at all, but observation shows that in such a case we perceive gray (Boring, 1942, p. 213; Hurvich & Jameson, 1957; Titchener, 1910, pp. 90–91). (See Hendel (2023) for a more detailed discussion of the problems in Hering’s theory.)

In a recent paper, based on earlier work by Heggelund (1974a, 1974b, 1991, 1992, 1993), Hendel (2023) provided a modification of Hering’s opponent-colors theory that doesn’t suffer from the above two problems. Here I show that the mathematical description of Hendel’s formulation of opponent-colors theory is identical to the mathematical description of two-state quantum systems in a mixed state. Although a mathematical identity between two theories in such disparate fields might be waved off as a meaningless coincidence, the dual-aspect theory of phenomenal consciousness, which suggests that one or more physical entities in our universe have phenomenal aspects that are dual to their physical aspects (see, e.g., Chalmers, 1996, chapter 8), predicts exactly such a correspondence between a system’s phenomenal states and the objective states of its underlying physical substrate (Chalmers, 1996, chapters 6 and 8; Cortês et al., 2021; Lockwood, 1989, chapter 11; also see G. E. Müller’s famous psychophysical axioms (Boring, 1942, p. 89)). Therefore, following the principles of dual-aspect theory, I suggest that color sensations are phenomenal dual aspects of two-state quantum systems in a mixed state.

# 2. The mathematical description of opponent-colors theory

The basic tenets of Hering’s formulation of opponent-colors theory and the two problems that this formulation of the theory faces were described in the *Introduction*. Heggelund (1974a, 1974b, 1991, 1992, 1993) modified Hering’s theory in a way that solved these two problems. Specifically, Heggelund suggested that—in contrast to the assumption that Hering (and the vast majority of other color scientists) had been making—the hueless colors do not lie along a one-dimensional continuum stretching between black and white, but rather require *two* dimensions for their full description: a luminous–black dimension and, orthogonal to it, a white dimension. Thus, Heggelund’s theory adds a seventh elementary color—luminous. This hueless color sensation, which appears only in colors that are perceived as emitting light (these are known as aperture colors), forms an opponent pair with black. The luminous–black opponent pair is analogous to the red–green and yellow–blue opponent pairs in that the colors luminous and black are mutually exclusive sensations. By contrast, on Heggelund’s theory, white has no opponent counterpart. Hence, this theory contends that white and black are not opponent colors, which explains why they mix to yield a phenomenal intermediate.

However, Hendel (2023) showed that although Heggelund’s theory successfully solves the two problems in Hering’s theory, it is not self-consistent. He then revised this theory and obtained a consistent opponent-colors theory. In Hendel’s version of opponent-colors theory, every color is described by the following four-dimensional vector:

$$\begin{array}{c}C=I\hat{x}\_{0}+\left(R-G\right)\hat{x}\_{1}+\left(Y-B\right)\hat{x}\_{2}+\left(Lu-Bk\right)\hat{x}\_{3},\#\left(1\right)\end{array}$$

where the vectors $\hat{x}\_{μ}$, $μ=0, 1, 2, 3$, constitute the standard basis of $R^{4}$, namely, $\hat{x}\_{0}=\left(1, 0, 0, 0\right)^{T}$, $\hat{x}\_{1}=\left(0, 1, 0, 0\right)^{T}$, and so on, where the superscript $T$ denotes the transpose operation; $I\geq 0$ is the luminance contrast signal (see details below); $R$, $G$, $Y$, $B$, $Lu$, and $Bk$ (which are all $\geq 0$) are, respectively, the values of the red, green, yellow, blue, luminous, and black processes, and therefore $\left(R-G\right)$, $\left(Y-B\right)$, and $\left(Lu-Bk\right)$ are, respectively, the red–green, yellow–blue, and luminous–black opponent-colors components of the color. The luminance contrast signal, $I$, i.e., the $\hat{x}\_{0}$-component of the color vector $C$, is the visual system’s representation of luminance contrast. For any light stimulus, its luminance contrast is the difference between the luminance of the stimulus itself and the luminance of its spatial surroundings (see more details in Hendel, 2023). Hendel (2023) showed that the value of $I$ is related to (a) the values of $R$, $G$, $Y$, $B$, $Lu$, and $Bk$ in the following manner:[[1]](#footnote-2)

$$\begin{array}{c}I=R+G=Y+B=Lu+Bk,\#\left(2\right)\end{array}$$

and to (b) the magnitudes of the three opponent-colors components as follows:

$$\begin{array}{c}\left(R-G\right)^{2}+\left(Y-B\right)^{2}+\left(Lu-Bk\right)^{2}\leq I^{2}.\#\left(3\right)\end{array}$$

Finally, the level of whiteness in the color, $W$, is determined by the value that completes the inequality in Eq. (3) to an equality, namely,

$$\begin{array}{c}W^{2}=I^{2}-\left(R-G\right)^{2}-\left(Y-B\right)^{2}-\left(Lu-Bk\right)^{2}.\#\left(4\right)\end{array}$$

# 3. The mathematical description of two-state quantum systems in a mixed state

The purpose of this section is to provide a brief review of the basic mathematical description of two-state quantum systems in a mixed state. This description can be found in many standard textbooks on quantum mechanics (e.g., Blum, 1981, chapter 1).

## 3.1 Two-state quantum systems

Two-state quantum systems are quantum systems that exhibit two physically distinguishable states relative to some measurement. Some common examples of two-state quantum systems are the spin state of spin-1/2 particles, the polarization state of photons, and atomic systems that can be approximated as effectively having only two electronic levels (Altepeter et al., 2004). The two physical states of a two-state quantum system will be represented by the two Hilbert-space vectors $\left.\left|+\right.\right⟩,\left.\left|-\right.\right⟩\in C^{2}$.[[2]](#footnote-3) These two vectors are taken to be orthogonal and therefore constitute a basis for the two-dimensional Hilbert space $C^{2}$. A two-state quantum system is then fully described by a *state vector*, denoted $\left.\left|ψ\right.\right⟩$, given by the following superposition (i.e., linear combination):

$$\begin{array}{c}\left.\left|ψ\right.\right⟩=a\left.\left|+\right.\right⟩+b\left.\left|-\right.\right⟩,\#\left(5\right)\end{array}$$

where $a,b\in C$ and $\left|a\right|^{2}+\left|b\right|^{2}=1$ (Blum, 1981, chapter 1). A measurement conducted to determine the state of the system described by Eq. (5) has a probability of $\left|a\right|^{2}$ of finding the system in the state $\left.\left|+\right.\right⟩$ and a probability of $\left|b\right|^{2}$ of finding it in the state $\left.\left|-\right.\right⟩$.

## 3.2 The Pauli operators

The Pauli operators, which are commonly denoted $\hat{σ}\_{i}$, $i=1, 2, 3$, are a set of three $2×2$ linear operators that obey a certain algebra (see Blum, 1981, chapter 1, for this algebra). In addition, the $2×2$ identity operator is often referred to as the zeroth Pauli operator, $\hat{σ}\_{0}$. In quantum theory, the Pauli operators are usually represented as matrices relative to the standard basis of $C^{2}$, $S=\left\{\left(1,0\right)^{T},\left(0,1\right)^{T}\right\}$. This so-called standard representation of the Pauli operators, which can be found in many textbooks on quantum mechanics, is given by

$$\begin{array}{c}\left[\hat{σ}\_{0}\right]\_{S}=\left[\begin{matrix}1&0\\0&1\end{matrix}\right], \left[\hat{σ}\_{1}\right]\_{S}=\left[\begin{matrix}0&1\\1&0\end{matrix}\right],\left[\hat{σ}\_{2}\right]\_{S}=\left[\begin{matrix}0&-i\\i&0\end{matrix}\right],\left[\hat{σ}\_{3}\right]\_{S}=\left[\begin{matrix}1&0\\0&-1\end{matrix}\right].\#\left(6\right)\end{array}$$

The brackets $\left[⋅\right]$ denote the matrix representation of a linear operator (see, e.g., Lipschutz & Lipson, 2009, chapter 6) and the subscript denotes the basis relative to which the matrices are given. The Pauli operators are Hermitian, i.e., $\hat{σ}\_{μ}=\hat{σ}\_{μ}^{†}≡\left(\hat{σ}\_{μ}^{\*}\right)^{T}$, $μ=0, 1, 2, 3$ (this can easily be verified from their matrix representation in Eq. (6)). This means that these operators are quantum observables operating on states of two-state quantum systems. I will therefore often refer to these operators as the Pauli observables. The set $\left\{\hat{σ}\_{0},\hat{σ}\_{1},\hat{σ}\_{2},\hat{σ}\_{3}\right\} $constitutes an orthogonalbasis for the vector space of $2×2$ operators (Aerts & Sassoli de Bianchi, 2017).[[3]](#footnote-4) Therefore, this basis, to which I will refer as the Pauli basis, also spans the space of all $2×2$ Hermitian operators, which is the space of all observables of two-state quantum systems.

It is easy to show that the eigenvalues of the three Pauli operators $\hat{σ}\_{i}$, $i=1, 2, 3$, are $\pm 1$. For each Pauli operator $\hat{σ}\_{i}$, $i\in \left\{1, 2, 3\right\}$, we will denote the eigenvectors associated with these eigenvalues by $\left.\left|\pm \hat{x}\_{i}\right.\right⟩\in C^{2}$, namely,

$$\begin{array}{c}\hat{σ}\_{i}\left.\left|\pm \hat{x}\_{i}\right.\right⟩=\pm \left.\left|\pm \hat{x}\_{i}\right.\right⟩.\#\left(7\right)\end{array}$$

Because the Pauli operators are Hermitian, the vectors in each of the pairs $\left.\left|\pm \hat{x}\_{i}\right.\right⟩$, $i\in \left\{1, 2, 3\right\}$, are orthogonal to each other and therefore constitute a basis of $C^{2}$.

We can employ the spectral decomposition theorem to obtain the following expression for each Pauli operator $\hat{σ}\_{i}$, $i\in \left\{1, 2, 3\right\}$:

$$\begin{array}{c}\hat{σ}\_{i}=\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.-\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.,\#\left(8\right)\end{array}$$

where the vectors $\left⟨\left.+\hat{x}\_{i}\right|\right.$ and $\left⟨\left.-\hat{x}\_{i}\right|\right.$ are, respectively, the transposed complex conjugate of the vectors $\left.\left|+\hat{x}\_{i}\right.\right⟩$ and $\left.\left|-\hat{x}\_{i}\right.\right⟩$. Therefore, Eq. (8) shows that each Pauli operator $\hat{σ}\_{i}$, $i\in \left\{1, 2, 3\right\}$, is composed of two underlying operators, $\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.$ and $\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.$, that operate in an *opponent* manner to each other. Since *any* two-dimensional vector is an eigenvector of the identity operator with an eigenvalue of $1$, we can use the eigenvalue decomposition theorem to express $\hat{σ}\_{0}$ (which, as will be recalled, is the identity operator) in the following manner:

$$\begin{array}{c}\hat{σ}\_{0}=\left.\left|+\hat{n}\right.\right⟩\left⟨\left.+\hat{n}\right|\right.+\left.\left|-\hat{n}\right.\right⟩\left⟨\left.-\hat{n}\right|\right.,\#\left(9\right)\end{array}$$

where $\left.\left|+\hat{n}\right.\right⟩$ and $\left.\left|-\hat{n}\right.\right⟩$ are any two orthogonal unit vectors in two-dimensional Hilbert space (for example, $\left.\left|+\hat{x}\_{i}\right.\right⟩$ and $\left.\left|-\hat{x}\_{i}\right.\right⟩$,$i\in \left\{1, 2, 3\right\}$). Equation (9) is known as the *completeness relation*.

## 3.3 Two-state quantum systems in a mixed state and the density matrix

A *mixture* of quantum systems is an ensemble of quantum systems, each described by a state vector, in which the quantum states of the systems do not coherently interfere with each other (Blum, 1981, chapter 1). The systems are then said to be in an incoherent state, or, as it is referred to more commonly, in a *mixed state*. Because the quantum systems that comprise a mixture do not interfere with each other, the mixture cannot be described by a state vector, but rather is described by a Hermitian operator (i.e., a quantum observable), referred to as the *density operator* (*ibid*.). This operator is customarily denoted by $\hat{ρ}$. In the case of two-state quantum systems, $\hat{ρ}$ will be a $2×2$ operator. It turns out that the $2×2$ density operator can be expanded in the Pauli basis, $\left\{\hat{σ}\_{0},\hat{σ}\_{1},\hat{σ}\_{2},\hat{σ}\_{3}\right\}$, in the following manner:

$$\begin{array}{c}\hat{ρ}=\frac{1}{2}\sum\_{μ=0}^{3}\left〈\hat{σ}\_{μ}\right〉\hat{σ}\_{μ},\#\left(10\right)\end{array}$$

where the brackets $\left〈⋅\right〉$ denote the expectation value of a quantum observable, $\left〈\hat{σ}\_{0}\right〉\in N$ is the number of two-state systems in the mixture, and $\left〈\hat{σ}\_{1}\right〉,\left〈\hat{σ}\_{2}\right〉, \left〈\hat{σ}\_{3}\right〉\in Z$ (*ibid*.).[[4]](#footnote-5) The expectation values of the Pauli observables appearing in Eq. (10) obey the following constraint:

$$\begin{array}{c}\left〈\hat{σ}\_{1}\right〉^{2}+\left〈\hat{σ}\_{2}\right〉^{2}+\left〈\hat{σ}\_{3}\right〉^{2}\leq \left〈\hat{σ}\_{0}\right〉^{2}\#\left(11\right)\end{array}$$

(*ibid*.). By applying linearity to Eqs. (8) and (9) it is easy to show that the expectation values of the Pauli observables $\hat{σ}\_{i}$, $i=1, 2, 3$, are given by

$$\begin{array}{c}\left〈\hat{σ}\_{i}\right〉=\left〈\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.\right〉-\left〈\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.\right〉\#\left(12\right)\end{array}$$

(Altepeter et al., 2004), and that the expectation value of the Pauli observable $\hat{σ}\_{0}$ is given by

$$\begin{array}{c}\left〈\hat{σ}\_{0}\right〉=\left〈\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.\right〉+\left〈\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.\right〉.\#\left(13\right)\end{array}$$

## 3.4 The Hamiltonian of two-state quantum systems

The time evolution of a quantum state is determined by the Hamiltonian operator $\hat{H}$. In the case of systems that are described by state vectors, the time-evolution equation is called the Schrödinger equation; in the case of systems that are described by density matrices (i.e., systems in a mixed state), the time-evolution equation is called the Liouville equation (Blum, 1981, chapter 2). Here we will only be interested in the Hamiltonian operators of two-state quantum systems. In this case, the Hamiltonian can be represented by a Hermitian $2×2$ operator. We will denote the eigenvectors of the Hamiltonian, which are commonly referred to as the energy eigenstates, by $\left.\left|\pm \right.\right⟩$. If the Hamiltonian is constant in time, it is easy to show that the eigenvector/eigenvalue equation for the Hamiltonian can always be written as

$$\begin{array}{c}\hat{H}\left.\left|\pm \right.\right⟩=\left.\left(E\_{0}\pm ε\right)\left|\pm \right.\right⟩,\#\left(14\right)\end{array}$$

where $E\_{0}, ε\in R$. For concreteness we will assume that $ε>0$. Hence, $\left.\left|-\right.\right⟩$ is the ground state, while $\left.\left|+\right.\right⟩$ is the excited state.

# 4. Color is the phenomenal dual aspect of two-state quantum systems in a mixed state

## 4.1 The identity between the mathematical descriptions of opponent-colors theory and two-state quantum systems

Let us postulate the following set of correspondences between the six expectation values $\left〈\left.\left|\pm \hat{x}\_{i}\right.\right⟩\left⟨\left.\pm \hat{x}\_{i}\right|\right.\right〉$, $i=1, 2, 3$, and the six variables $R$, $G$, $Y$, $B$, $Lu$, and $Bk$:

|  |  |  |
| --- | --- | --- |
| $\left〈\left.\left|+\hat{x}\_{1}\right.\right⟩\left⟨\left.+\hat{x}\_{1}\right|\right.\right〉⟷R$, | $$\begin{array}{c}\left〈\left.\left|-\hat{x}\_{1}\right.\right⟩\left⟨\left.-\hat{x}\_{1}\right|\right.\right〉⟷G,\end{array}$$ | $$(15a)$$ |
| $\left〈\left.\left|+\hat{x}\_{2}\right.\right⟩\left⟨\left.+\hat{x}\_{2}\right|\right.\right〉⟷Y$, | $$\begin{array}{c}\left〈\left.\left|-\hat{x}\_{2}\right.\right⟩\left⟨\left.-\hat{x}\_{2}\right|\right.\right〉⟷B,\end{array}$$ | $$(15b)$$ |
| $\left〈\left.\left|+\hat{x}\_{3}\right.\right⟩\left⟨\left.+\hat{x}\_{3}\right|\right.\right〉⟷Lu$, | $$\begin{array}{c}\left〈\left.\left|-\hat{x}\_{3}\right.\right⟩\left⟨\left.-\hat{x}\_{3}\right|\right.\right〉⟷Bk.\end{array}$$ | $$(15c)$$ |

Using this set of correspondences, we immediately see that the following correspondence exists between $\left〈\hat{σ}\_{0}\right〉$ in Eq. (13) and $I$ in Eq. (2):

$$\begin{array}{c}\left〈\hat{σ}\_{0}\right〉=\left〈\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.\right〉+\left〈\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.\right〉⟷I=R+G=Y+B=Lu+Bk,\#\left(16\right)\end{array}$$

$i=1, 2, 3$. Given Eq. (15), we also obtain that the following correspondences hold:

|  |  |
| --- | --- |
| $$\left〈\hat{σ}\_{1}\right〉=\left〈\left.\left|+\hat{x}\_{1}\right.\right⟩\left⟨\left.+\hat{x}\_{1}\right|\right.\right〉-\left〈\left.\left|-\hat{x}\_{1}\right.\right⟩\left⟨\left.-\hat{x}\_{1}\right|\right.\right〉⟷R-G,$$ | $$(17a)$$ |
| $$\left〈\hat{σ}\_{2}\right〉=\left〈\left.\left|+\hat{x}\_{2}\right.\right⟩\left⟨\left.+\hat{x}\_{2}\right|\right.\right〉-\left〈\left.\left|-\hat{x}\_{2}\right.\right⟩\left⟨\left.-\hat{x}\_{2}\right|\right.\right〉⟷Y-B,$$ | $$(17b)$$ |
| $$\left〈\hat{σ}\_{3}\right〉=\left〈\left.\left|+\hat{x}\_{3}\right.\right⟩\left⟨\left.+\hat{x}\_{3}\right|\right.\right〉-\left〈\left.\left|-\hat{x}\_{3}\right.\right⟩\left⟨\left.-\hat{x}\_{3}\right|\right.\right〉⟷Lu-Bk$$ | $$(17c)$$ |

(the equalities on the left-hand side of Eq. (17) are from Eq. (12), of course). Equation (17) matches pairs of opponent processes on the right-hand side with pairs of operators operating in an opponent manner to each other on the left-hand side. Now, from Eqs. (16) and (17) we notice that there exists an exact correspondence between $\hat{ρ}$ in Eq. (10) and $C$ in Eq. (11):

$$\begin{array}{c}\hat{ρ}=\frac{1}{2}\sum\_{μ=0}^{3}\left〈\hat{σ}\_{μ}\right〉\hat{σ}\_{μ}⟷C=I\hat{x}\_{0}+\left(R-G\right)\hat{x}\_{1}+\left(Y-B\right)\hat{x}\_{2}+\left(Lu-Bk\right)\hat{x}\_{3},\#\left(18\right)\end{array}$$

where, it will be noticed, the correspondence $\hat{σ}\_{μ}⟷\hat{x}\_{μ}$, $μ=0, 1, 2, 3$, is implicitly assumed. Another exact correspondence that follows from Eqs. (16) and (17) is one between Eq. (11) and Eq. (3):

$$\begin{array}{c}\left〈\hat{σ}\_{1}\right〉^{2}+\left〈\hat{σ}\_{2}\right〉^{2}+\left〈\hat{σ}\_{3}\right〉^{2}\leq \left〈\hat{σ}\_{0}\right〉^{2}⟷\left(R-G\right)^{2}+\left(Y-B\right)^{2}+\left(Lu-Bk\right)^{2}\leq I^{2}.\#\left(19\right)\end{array}$$

Overall, Eqs. (15)–(19) show that there exists an identity between the mathematical description of the proposed opponent-colors theory and the mathematical description of two-state quantum systems in a mixed state. (Notice that there is no need to find a parallel to Eq. (4) of the proposed theory (which is the equation for whiteness) in the quantum description of mixtures of two-state systems because this equation is merely definitional (that is, the level of whiteness in a color is simply *assigned* the value of the right-hand side of Eq. (4)).) The most important correspondence in the set of correspondences listed in Eqs. (15)–(19) is the one given in Eq. (18). This equation establishes that for every density operator representing the state of a two-state quantum system there corresponds a color experience represented by a vector in $R^{4}$, i.e.,

$$\hat{ρ}⟷C.$$

Interestingly, such a correspondence already exists in quantum theory itself; there, the $R^{4}$-vector is called the Bloch vector (Aerts & Sassoli de Bianchi, 2017; Bertlmann & Krammer, 2008).[[5]](#footnote-6) Hence, what Eq. (18) says is that the color vector $C$ is, in fact, the Bloch vector representing the state of a two-state quantum system.

## 4.2 A privileged basis for Hilbert space

In recent decades several notable researchers have advocated the hypothesis that the physical world is the result of a gigantic computation (Fredkin, 2003; Lloyd, 2007; ‘t Hooft, 2016; Wolfram, 2002; Zuse, 1982). A similar hypothesis, which elegantly explains the mysterious success that mathematics has in describing the physical world (Wigner, 1960), is that the universe is an abstract mathematical model (Carroll, 2022; Tegmark, 2008; also see Woit, 2015). The two hypotheses—namely, the world as computation and the world as a mathematical model—are, in fact, essentially identical. This is because mathematical models are inherently atemporal; hence, for a mathematical model to constitute an implementation of the physical universe it must be run in time. But this turns it into a computation.

In this paper I will adopt the general metaphysical framework suggested by the above hypotheses, namely, that the physical world is an implementation in time of a mathematical model. Moreover, I will assume that quantum mechanics provides a correct description of the physical world. Taking these two assumptions together, we arrive at the world picture suggested by Lloyd (2007), ‘t Hooft (2016), and Carroll (2022), in which the physical world is a computation that follows the rules of quantum mechanics.

In quantum theory it is often emphasized that a quantum state, i.e., a Hilbert-space vector, can be represented in any basis of Hilbert space (e.g., position space, momentum space, etc.). However, when one carries out *computations* with Hilbert-space vectors, one must commit to a definite representation of these vectors, namely, one must choose a particular basis for Hilbert space. Since in the metaphysical world view adopted here, the physical world results from a computation, we must stipulate that there exists a specific basis for Hilbert space in which all quantum states in the universe are represented. ‘t Hooft (2016), for example, refers to this privileged basis as the ‘ontological basis’. Carroll (2022) suggests that the privileged basis for Hilbert space is given by the energy eigenstates, namely, the set of eigenvectors of the Hamiltonian. This choice is based on the fact that the Hamiltonian is the most fundamental operator in quantum theory since it is what moves the quantum world in time.

Here I will follow Carroll (2022) in assuming that in the computation that gives rise to our universe quantum states are represented relative to the basis of the energy eigenstates. In the case of two-state quantum systems, these energy eigenstates are the vectors $\left.\left|\pm \right.\right⟩$ that satisfy Eq. (14). We will denote this privileged basis of two-dimensional Hilbert space by $P$, namely, $P=\left\{\left.\left|+\right.\right⟩ ,\left.\left|-\right.\right⟩\right\}$. All quantum states of two-state quantum systems are represented in the privileged basis $P$. This includes the vectors $\left.\left|\pm \right.\right⟩$ themselves. Clearly, this representation yields a coordinatization of in which the vectors $\left.\left|\pm \right.\right⟩$ are given by the vectors of the standard basis of $C^{2}$, namely, $\left[\left.\left|+\right.\right⟩\right]\_{P}=\left(1,0\right)^{T}$ and $\left[\left.\left|-\right.\right⟩\right]\_{P}=\left(0,1\right)^{T}$ (the subscripted brackets $\left[⋅\right]\_{P}$ denote a vector coordinatized relative to the basis $P$ (Lipschutz & Lipson, 2009)). This coordinatization of the privileged basis of $C^{2}$ will be denoted $PS$, namely, $PS=\left\{\left[\left.\left|+\right.\right⟩\right]\_{P}=\left(1,0\right)^{T} ,\left[\left.\left|-\right.\right⟩\right]\_{P}=\left(0,1\right)^{T}\right\}$. On our hypothesis, the universe coordinatizes the states of all two-state quantum system relative to $PS$. For example, the six vectors $\left.\left|\pm \hat{x}\_{i}\right.\right⟩$, $i=1, 2, 3$, are represented by $\left[\left.\left|\pm \hat{x}\_{i}\right.\right⟩\right]\_{PS}$.

Since quantum observables (i.e., Hermitian operators) operate on quantum states, consistency requires that if the latter are represented relative to some basis, the former must be represented relative to the same basis as well. Thus, in the quantum computational universe hypothesized here, quantum observables are represented as matrices relative to the privileged basis of Hilbert space, namely, the basis of energy eigenstates. A trivial example is the Hamiltonian, which in the basis of its own eigenvectors becomes the following diagonal matrix:

$$\left[\hat{H}\right]\_{PS}=\left[\begin{matrix}E\_{0}+ε&0\\0&E\_{0}-ε\end{matrix}\right].$$

The Pauli observables $\hat{σ}\_{μ}$, $μ=0, 1, 2, 3$, are represented by the matrices $\left[\hat{σ}\_{μ}\right]\_{PS}$. Since $PS$ is a standard basis, this matrix representation of the Pauli observables is given in Eq. (6). Finally, a density operator $\hat{ρ}$ of a mixture of two-state quantum systems is represented by a $2×2$ matrix $\left[\hat{ρ}\right]\_{PS}$. From Eq. (10) we see that this matrix can be expanded in the Pauli basis $\left[\hat{σ}\_{μ}\right]\_{PS}$, $μ=0, 1, 2, 3$, in the following manner:

$$\left[\hat{ρ}\right]\_{PS}=\frac{1}{2}\sum\_{μ=0}^{3}\left〈\left[\hat{σ}\_{μ}\right]\_{PS}\right〉\left[\hat{σ}\_{μ}\right]\_{PS}.$$

## 4.3 Color is the phenomenal dual aspect of two-state quantum systems in a mixed state

*Subsection 4.1* above established an identity between the mathematical description of two-state quantum systems in a mixed state and the mathematical description of opponent-colors theory (see Eqs. (15)–(19)). Such a correspondence between the states of a physical system and phenomenal states is exactly what the dual-aspect theory of phenomenal consciousness predicts. However, we are still falling short of the requirements of dual-aspect theory because the values on the left-hand sides of the correspondences in Eqs. (15)–(19) change with the choice of basis for Hilbert space. To clearly see the problem, consider, for example, Eq. (15). The expectation values $\left〈\left.\left|\pm \hat{x}\_{i}\right.\right⟩\left⟨\left.\pm \hat{x}\_{i}\right|\right.\right〉$, $i=1, 2, 3$, on the left-hand sides of the correspondences delineated by this equation depend on the specific basis that was chosen for $C^{2}$. Hence, different bases for $C^{2}$ will lead to different expectation values. But on Eq. (15), these expectation values correspond to the fundamental color sensations, $R$, $G$, $Y$, $B$, $Lu$, and $Bk$. Thus, as it stands, Eq. (15) makes the prediction that our color experiences should depend on the particular basis chosen for $C^{2}$. This, of course, is absurd. To solve this problem with Eqs. (15)–(19), we invoke the hypothesis of *Subsection 4.2* that the universe represents quantum states in a privileged basis. On this hypothesis, from nature’s point of view, the mathematical description of two-state quantum systems given in Eqs. (15)–(19) exists in *a specific form*, which is the representation relative to the privileged basis of $C^{2}$, $PS$. For concreteness, let us rewrite Eqs. (15)–(19) in this privileged representation. We begin with Eq. (15), which establishes the basic correspondence between the realm of two-state quantum systems and the realm of opponent-colors theory. This equation now becomes

|  |  |  |
| --- | --- | --- |
| $\left〈\left[\left.\left|+\hat{x}\_{1}\right.\right⟩\left⟨\left.+\hat{x}\_{1}\right|\right.\right]\_{PS}\right〉⟷R$, | $$\begin{array}{c}\left〈\left[\left.\left|-\hat{x}\_{1}\right.\right⟩\left⟨\left.-\hat{x}\_{1}\right|\right.\right]\_{PS}\right〉⟷G,\end{array}$$ | $$(20a)$$ |
| $\left〈\left[\left.\left|+\hat{x}\_{2}\right.\right⟩\left⟨\left.+\hat{x}\_{2}\right|\right.\right]\_{PS}\right〉⟷Y$, | $$\begin{array}{c}\left〈\left[\left.\left|-\hat{x}\_{2}\right.\right⟩\left⟨\left.-\hat{x}\_{2}\right|\right.\right]\_{PS}\right〉⟷B,\end{array}$$ | $$(20b)$$ |
| $\left〈\left[\left.\left|+\hat{x}\_{3}\right.\right⟩\left⟨\left.+\hat{x}\_{3}\right|\right.\right]\_{PS}\right〉⟷Lu$, | $$\begin{array}{c}\left〈\left[\left.\left|-\hat{x}\_{3}\right.\right⟩\left⟨\left.-\hat{x}\_{3}\right|\right.\right]\_{PS}\right〉⟷Bk.\end{array}$$ | $$(20c)$$ |

The next step is to convert the correspondences listed in Eqs. (16) and (17) into

$$\begin{array}{c}\left[\hat{σ}\_{0}\right]\_{PS}=\left〈\left[\left.\left|+\hat{x}\_{i}\right.\right⟩\left⟨\left.+\hat{x}\_{i}\right|\right.\right]\_{PS}\right〉+\left〈\left[\left.\left|-\hat{x}\_{i}\right.\right⟩\left⟨\left.-\hat{x}\_{i}\right|\right.\right]\_{PS}\right〉⟷I=R+G=Y+B=Lu+Bk\#\left(21\right)\end{array}$$

for $i=1, 2, 3$, and

|  |  |
| --- | --- |
| $$\left[\hat{σ}\_{1}\right]\_{PS}=\left〈\left[\left.\left|+\hat{x}\_{1}\right.\right⟩\left⟨\left.+\hat{x}\_{1}\right|\right.\right]\_{PS}\right〉-\left〈\left[\left.\left|-\hat{x}\_{1}\right.\right⟩\left⟨\left.-\hat{x}\_{1}\right|\right.\right]\_{PS}\right〉⟷R-G,$$ | $$(22a)$$ |
| $$\left[\hat{σ}\_{2}\right]\_{PS}=\left〈\left[\left.\left|+\hat{x}\_{2}\right.\right⟩\left⟨\left.+\hat{x}\_{2}\right|\right.\right]\_{PS}\right〉-\left〈\left[\left.\left|-\hat{x}\_{2}\right.\right⟩\left⟨\left.-\hat{x}\_{2}\right|\right.\right]\_{PS}\right〉⟷Y-B,$$ | $$(22b)$$ |
| $\left[\hat{σ}\_{3}\right]\_{PS}=\left〈\left[\left.\left|+\hat{x}\_{3}\right.\right⟩\left⟨\left.+\hat{x}\_{3}\right|\right.\right]\_{PS}\right〉-\left〈\left[\left.\left|-\hat{x}\_{3}\right.\right⟩\left⟨\left.-\hat{x}\_{3}\right|\right.\right]\_{PS}\right〉⟷Lu-Bk$. | $$(22c)$$ |
| Using Eqs. (21) and (22) we can represent the left-hand side of Eq. (18) relative to the basis $PS$ to obtain |  |

$$\begin{array}{c}\left[\hat{ρ}\right]\_{PS}=\frac{1}{2}\sum\_{μ=0}^{3}\left〈\left[\hat{σ}\_{μ}\right]\_{PS}\right〉\left[\hat{σ}\_{μ}\right]\_{PS}⟷C=I\hat{x}\_{0}+\left(R-G\right)\hat{x}\_{1}+\left(Y-B\right)\hat{x}\_{2}+\left(Lu-Bk\right)\hat{x}\_{3}.\left(23\right)\end{array}$$

Finally, when the left-hand side of Eq. (19) is represented relative to $PS$, we get

$$\begin{array}{c}\left〈\left[\hat{σ}\_{1}\right]\_{PS}\right〉^{2}+\left〈\left[\hat{σ}\_{2}\right]\_{PS}\right〉^{2}+\left〈\left[\hat{σ}\_{3}\right]\_{PS}\right〉^{2}\leq \left〈\left[\hat{σ}\_{0}\right]\_{PS}\right〉^{2}⟷\left(R-G\right)^{2}+\left(Y-B\right)^{2}+\left(Lu-Bk\right)^{2}\leq I^{2}.\#\left(24\right)\end{array}$$

Like their progenitors in Eqs. (15)–(19), Eqs. (20)–(24) establish an identity between the mathematical description of two-state quantum systems in a mixed state and the mathematical description of opponent-colors theory. However, in contrast to Eqs. (15)–(19), the mathematical description of two-state quantum systems delineated in Eqs. (20)–(24) is given relative to the privileged basis of Hilbert space. The most important correspondence in the set of correspondences listed in Eqs. (20)–(24) is the one given in Eq. (23). This equation establishes that for every state of a two-state quantum system, where the state is represented relative to the privileged basis of $C^{2}$, there corresponds a specific color experience represented by the color vector $C$. We saw in *Subsection 4.1* that the vector $C$ is, in fact, the Bloch vector representing a two-state quantum system in a mixed state. Thus, like its progenitor in Eq. (18), the content of Eq. (23) is that the color vector $C$ is the Bloch vector representing the state of a two-state quantum system. However, Eq. (23) goes beyond Eq. (18) in stating that this Bloch vector is given in a *specific coordinatization*.

Based on the exact correspondence between the mathematical description of two-state quantum systems in a mixed state and the mathematical description of opponent-colors theory that is given in Eqs. (20)–(24)—specifically, on the correspondence given in Eq. (23) between the density matrix in the privileged representation and the color $C$—and the principles of the dual-aspect theory of phenomenal consciousness, we reach the main hypothesis of this paper:

Color is the phenomenal dual aspect of two-state quantum systems in a mixed state.

# 5. Discussion

## 5.1 The differences between the hypothesis suggested here and previous suggestions relating consciousness and quantum mechanics

There have been many suggestions that consciousness and quantum mechanics might be related (e.g., Chalmers & McQueen, 2022; Hameroff & Penrose, 1996; Lockwood, 1989; Okon & Sebastián, 2020; Stapp, 1993; Wigner, 1961). The hypothesis made in this paper, i.e., that color sensations are phenomenal dual aspects of two-state quantum systems in a mixed state, is distinct from these past suggestions in three important respects. First, the motivation for the relationship between consciousness and quantum mechanics in the case of the current hypothesis stems from the identical mathematical description of opponent-colors theory and two-state quantum systems conjoined with the principles of the dual-aspect theory of phenomenal consciousness. By contrast, past suggestions that consciousness might be related to quantum mechanics were mostly motivated by ‘the fact that the problems in quantum mechanics seem to be deeply tied to the notion of observership, crucially involving the relation between a subject's experience and the rest of the world’ (Chalmers, 1996, p. 333). Namely, past suggestions have mostly attempted to relate the problem of consciousness with the problem of measurement in quantum mechanics. Second, whereas past suggestions relating consciousness and quantum mechanics were vague in what they meant by ‘consciousness’, the current hypothesis is very precise—it addresses a specific aspect of phenomenal consciousness, namely, color experience. Third, past suggestions of a relationship between consciousness and quantum mechanics all required that the brain be able to sustain a quantum *coherent* state. Given the physical properties of the brain, this is *prima facie* highly unlikely. Indeed, Tegmark (2000) has calculated that any coherent quantum state in the brain will decohere in a time that is orders of magnitude too short to be of any use for conscious processes. By contrast, quantum decoherence doesn’t pose a problem for the suggestion put forth here because on this suggestion color is a phenomenal dual aspect of a *mixture* of quantum systems, which is a population of quantum systems in an *incoherent* state.

## 5.2 Generalizing the argument to other types of phenomenal experience

On the suggested hypothesis, *any* two-state quantum system should have color experience as its phenomenal dual aspect. Since the world contains a vast number of such systems (e.g., the spin state of spin-1/2 particles or the polarization state of photons), the proposed hypothesis is clearly panpsychist in nature. Moreover, nothing in the hypothesis that color sensations are the phenomenal dual aspects of the states of two-state quantum systems suggests that what brings about the phenomenal experience is the two-dimensionality of these systems. It is therefore natural to generalize this hypothesis and suggest that other types of phenomenal experience result from quantum states of quantum systems with higher dimensionalities. On this suggestion, each type of phenomenal experience (taste, odor, sound, etc.) is ‘attached’ to a quantum system with a certain dimensionality. Thus, the various types of phenomenal experience that we humans (and presumably other creatures on earth) have are not generated *de novo* by the brain. Rather, these specific types of phenomenal experience are a fundamental part of our universe. Presumably, brains have evolved to tie these various types of phenomenal experience to various types of sensory information.

## 5.3 Two-state ion channels may be the two-state quantum systems that give rise to color

This paper proposes that color sensations are phenomenal dual aspects of two-state quantum systems in a mixed state. On this hypothesis, our color sensations arise from a mixture of two-state quantum systems that exists somewhere in the brain. What is the identity of these two-state quantum systems? Clearly, these systems must somehow be linked to color-related processes in the visual system. Further, since we are looking for a *quantum* system, it is likely to be found in the molecular level. Taking these two considerations together, I suggest that *two-state ion channels*—or, more likely, some component in them—are the two-state quantum systems whose phenomenal dual aspect is color. These channels, which lie at the very basis of neuronal activity, are large membrane-spanning proteins that transition between two discrete molecular conformations in response to the binding/release of a ligand or the presence/absence of voltage (for a review of ion channels, see Siegelbaum & Koester, 2000). The two molecular conformations of two-state ion channels correspond to two functional states: an open state, in which channel-specific ions are free to cross from the extracellular side to the intracellular side (or vice versa), and a closed state, in which no ions can pass through the channel. Since ion channels are huge protein molecules, and since the transition from one molecular conformation to the other entails many structural changes in the amino acids that comprise the channels (see details in DaCosta & Baenziger, 2013), it is unlikely that a two-state ion channel *as a whole* can be considered to be a two-state quantum system. Rather, it is more likely that the sought-for two-state quantum system is some *component* of the ion channel. Specifically, I suggest that the two-state quantum system that gives rise to color is the molecular component that resides at the orthosteric site, namely, the site where the channel’s agonist operates (Changeux & Christopoulos, 2016). The interaction with the channel’s agonist (e.g., the docking of a ligand) causes some conformational change in the orthosteric site which, in turn, cascades into the many structural changes that constitute the conformational change of the entire ion-channel molecule.

A well-established example of a conformational change in the orthosteric site of a receptor protein is given by rhodopsin, which is the photoreceptor molecule of rod cells in the retina (Tessier-Lavigne, 2000). The rhodopsin molecule is composed of a large protein, opsin, that is covalently bonded to a small, light-absorbing molecule—retinal. ‘In its nonactivated form rhodopsin contains the 11-*cis* isomer of retinal. Absorption of light by 11-*cis* retinal causes a rotation around the 11-*cis* double bond. As retinal returns to its more stable all-*trans* configuration, it brings about a conformational change in the opsin portion of rhodopsin, which triggers the other events of visual transduction’ (*ibid*., p. 511).[[6]](#footnote-7) If the *cis* and *trans* configurations of retinal constitute the only two physical configurations of this molecule, then the configuration state of the retinal molecule is a two-state quantum system. Therefore, on the hypothesis suggested here, the configuration of retinal has a phenomenal dual aspect of color experience. (Of course, I do not claim that our color sensations stem from retinal rods; rather, our color experiences arise from somewhere in the visual cortex (see below). That is, for some unknown reason, the color sensations presumably generated by retinal molecules in our retinas are not used by the brain for *our* conscious color experience.)

Where can we expect to find the hypothesized two-state ion channels that give rise to color? The recent work by Li et al. (2022) has shown that there exist cone-opponent functional domains in the primary visual cortex (V1). The hue preferences in these functional domains are geometrically organized into so-called ‘pinwheels’. It is natural to suggest that the first place to look for two-state ion channels that give rise to color experience is in the neurons within these cone-opponent functional domains.

# 6. References

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1. Hendel showed that this expression is one of two different, yet mathematically equivalent, expressions for $I$. For our purposes here, the expression in Eq. (2) will be more useful. [↑](#footnote-ref-2)
2. For our purposes here, Hilbert space simply means the vector space $C^{n}$, where $n=2, 3, …$ [↑](#footnote-ref-3)
3. The orthogonality of the Pauli operators is defined with respect to the inner product $\left〈\hat{σ}\_{μ},\hat{σ}\_{ν}\right〉=tr\left(\hat{σ}\_{μ}^{†}\hat{σ}\_{ν}\right)$, $μ, ν\in \left\{0, 1, 2, 3\right\}$, where $tr(⋅)$ is the trace operation. [↑](#footnote-ref-4)
4. Equation (10) gives the unnormalized version of the density operator. Usually, this operator is given with all values normalized by $\left〈\hat{σ}\_{0}\right〉$ (Altepeter et al., 2004; Blum, 1981, chapter 1). [↑](#footnote-ref-5)
5. Ordinarily, the Bloch vector is given as a vector in $R^{3}$. Here it is given by a vector $R^{4}$ in because we did not normalize the density operator by $\left〈\hat{σ}\_{0}\right〉$ (see footnote below Eq. (10)). [↑](#footnote-ref-6)
6. Notably, rhodopsin is not an ion channel, but rather a G-protein-coupled receptor. However, the biophysical principles behind its conformational changes are similar to those in ion channels (Changeux & Christopoulos, 2016). [↑](#footnote-ref-7)