

## Concepts of Knowability

### *Conceptos de cognoscibilidad*

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#### Abstract

Many philosophical discussions hinge on the concept of knowability. For example, there is a blooming literature on the so-called paradox of knowability. How to understand this notion, however? In this paper, we examine several approaches to the notion: the naive approach to take knowability as the possibility to know, the counterfactual approach endorsed by Edgington (1985) and Schlöder (2019) , approaches based on the notion of a capacity or ability to know (Fara 2010, Humphreys 2011), and finally, approaches that make use of the resources of dynamic epistemic logic (van Benthem 2004, Holliday 2017).

**Keywords:** knowability, counterfactual knowability, capacity to know, dynamic possibility to know.

#### Resumen

Muchas discusiones filosóficas dependen del concepto de cognoscibilidad. Por ejemplo, hay una floreciente literatura acerca de la así llamada paradoja de la cognoscibilidad. Sin embargo, ¿cómo hemos de entender la noción? En este paper, examinamos varios enfoques: el enfoque naive de tomar a la cognoscibilidad como la posibilidad de conocer, el enfoque contrafáctico defendido por Edgington (1985) y Schlöder (2019), enfoques basados en la noción de una



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capacidad o habilidad de saber (Fara 2010, Humphreys 2011), y finalmente, enfoques que emplean los recursos de la lógica epistémica dinámica (van Benthem 2004, Holliday 2017).

**Palabras clave:** cognoscibilidad; cognoscibilidad contrafáctica; capacidad de conocer; posibilidad dinámica de conocer.

## 1. Introduction

‘What can we know?’ is one of the main philosophical questions. It has two sides: first, it makes a question out of how we come to know whatever it is we know (in what ways we are able to know), and second, it makes an issue of the limits of knowledge (in what ways we are unable to know). Asking for the limits of knowledge entails asking what is knowable. The history of philosophy is rife with different positions on the question of what is knowable:

- Everything is knowable (epistemic optimism).
- Some things are knowable and some things are not (epistemic moderatism).
- Nothing is knowable (epistemic pessimism).

How, though, should we understand the notion of *knowability*? In this paper, we examine the problem of how to characterize the concept of knowability, and give an overview of various directions taken in the literature about the problem, to then raise some concerns.

The structure of the paper is as follows: in section 2 we examine the traditional concept of knowability in terms of there being a possibility to know, and raise the need for a factive concept of knowability. Next, we branch out into three different types of conceptualization that can handle this requirement: one that offers a counterfactual analysis (Edgington, Schlöder) (section 3), one that offers an analysis in terms of capacities (Fara, Humphreys) (section 4), and finally, one that makes use of the framework of dynamic logic (Van Benthem, Holliday) (section 5). After we discuss the conceptual issues that arise from these perspectives, in section 6 we will summarize and consider the similarities shared by these three perspectives, which point towards ways in which the discussion can be taken further.

## 2. Knowability as the possibility to know

The most common concept of knowability in the literature is that of there being a possible state of the world (potentially counterfactual) in which someone knows that something is the case.<sup>1</sup>

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<sup>1</sup> In this paper we are only concerned with ‘propositional’ knowability (‘it is knowable that  $\varphi$ ’), not with objectual knowability (‘ $x$  is knowable’).

## Naïve

$\phi$  is knowable iff it is possible to know  $\phi$

To present things more formally, we will make use of the framework of possible world semantics; we assume familiarity on part of the reader. Schematically, then, with  $k$  for the knowability operator,  $\diamond$  for possibility and  $K$  for knowledge:

$$k\phi \leftrightarrow \diamond K\phi$$

The meaning of  $\diamond$  (possibility) and  $K$  (knowledge) are supposed to be given by the ordinary semantic clauses:

- $K\phi$  is true at world  $w$  iff  $\phi$  is true at all worlds that are epistemically accessible from  $w$ .
- $\diamond\phi$  is true at world  $w$  iff  $\phi$  is true at least one world that is modally accessible from  $w$ .

Following this schema, the problem of characterizing the concept of knowability reduces to the problem of fully characterizing these operators. To illustrate: Kripke raises the problem in the context of a discussion of the concept of a priori knowability:

... And possible for whom? For God? For the Martians? Or just for people with minds like ours? To make this all clear might [involve] a host of problems all of its own about what sort of possibility is in question here. (Kripke, 1980, pp. 34-35)

The questions that arise naturally here are: How restrictive should we understand the sense of possibility here? What are the adequate logical/modal conditions for an account of knowledge?

An issue that came up fairly early (cf. Church, 2009; Fitch, 1963) with the Naive conception of knowability is that, in the context of theories committed to the thesis that all truths are knowable and under some fairly standard assumptions about the behaviour of the  $\diamond$  and  $K$  operators involved, it gives rise to what is now-called Fitch's paradox. The paradox states that, if all truths are possibly known, then every truth is in effect known. Epistemic optimism, namely the position that all truths are knowable, then implies omniscience. Many philosophers have tried to defend a version of epistemic optimism, but few would want to defend omniscience. This has motivated many to look for alternative conceptions of knowability that would avoid the paradox. Our goal here is to focus on the *concept* of knowability, not on the knowability *thesis* that all truths are knowable, so we won't dwell on this point. There are four strategies to deal with the paradox: biting the bullet, revising the underlying logic(s), restricting the thesis and reformulating the thesis with help of other concepts of knowability. In so far as the discussion that follows bears on the knowability thesis, it only does so in the context of the reformulation strategy.<sup>2</sup>

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<sup>2</sup> This is why we don't discuss any of the restriction strategies by Tenant (1997), Dummett (2001), Fischer (2013) or Artemov & Protopopescu (2013). For an overview of the discussion on the knowability thesis, see Brogaard & Salerno (2019).



There is a more general issue with the Naive concept of knowability: it is not factive, that is, there are (actual) falsehoods that are known in counterfactual states of the world, where they are not false. For example: one of the authors has only one sister, but he could have had two, so if he had counted his sisters, he would have known (in that scenario) that he has two sisters.<sup>3</sup> In many cases we worry about what is knowable about the actual state of the world, not about what is knowable in purely counterfactual scenarios. To address this point, we require a factive concept of knowability, so that in the relevant sense of ‘knowable’, only actual truths are knowable. For a factive concept of knowability, the following will hold:

**Factivity**

$$k\varphi \rightarrow \varphi$$

Brogaard & Salerno (2006) rehearse the following dialogue to stress the point that non-factive conceptions of knowability are problematic:

A: We could be discovered.

B: Discovered doing what?

A: Someone might discover that we are having an affair.

B: But we are not having an affair!

A: I didn’t say that we were.

Clearly, we don’t normally worry about what could merely happen, and by extension, about what could merely be possibly known.<sup>4</sup> Even if non-factive conceptions of knowability are admissible for some purposes, it would pay off to have a factive notion available.<sup>5</sup>

Nowadays there are alternative conceptualizations of knowability that restrict its range to (f)actual truths: having the counterfactual possibility to know that something is actually true (Edgington, 1985); actually having the capacity to know that something is actually true (Fara, 2010); having the potential to know (Fuhrmann, 2014); having the ability to know

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<sup>3</sup> There are other reasons to be careful about the factivity of possible knowledge. Heylen (2013, p. 96) notes the following consequence. First, suppose that theorems of arithmetic are possibly known. Second, assume a weak introspection principle: if one knows an arithmetical theorem, then it is possible that one knows that one knows that theorem. Third, assume some modal logic (i.e., the monotonicity rule for the diamond operator and modal axiom scheme 4 for the diamond operator). Fourth, suppose that possible knowledge is factive. Then it follows that theorems of arithmetic are known. Williamson (1992, p. 67) shows that the factivity of possible knowledge, in combination with the knowability of truth, and under the same modal assumptions as above entails a modal collapse: possibility entails truth. Heylen (2020b) mentions that the factivity of possible knowledge together with modal axiom scheme 5 entails that whatever is possibly known is also necessarily true.

<sup>4</sup> Cf. Sinhababu’s (2008) argument that perhaps we are allowed to worry about merely counterfactual affairs.

<sup>5</sup> It has been suggested that  $\varphi$  is knowable if and only if  $\blacklozenge K\varphi$ ,  $\blacklozenge$  ranges only over accessible worlds in which the non-epistemic facts are the same (Williamson, 1992; Tennant, 2009). For a critical discussion, see Heylen (2021).



(Spencer, 2017); becoming known to be previously true after a hypothetical future of updates (Holliday, 2017). We will examine several of these in the following sections.

### 3. The counterfactual approach

In this section we will discuss the counterfactual approach to knowability. This approach finds its contemporary roots in the work of Edgington (section 3.1) and it has recently been developed further by Schlöder (section 3.2).

#### 3.1 Edgington and counterfactual knowability

An early attempt to construe a factive conception of knowability is pursued by Edgington (1985). According to her, knowability could be characterized in terms of what could be known to be actually the case. Schematically:

**aK**

$\varphi$  is knowable iff it is possible to know that  $\varphi$  is actually the case.

$$k\varphi \leftrightarrow \diamond KA\varphi$$

We assume that the meaning of the A (actuality) operator obeys the following semantic clause, which is traditional:

- $A\varphi$  is true at  $w$  iff  $\varphi$  is true at the actual world,  $w_0$ .<sup>6</sup>

This effectively makes only actual truths knowable. Suppose that there is some  $\varphi$  that is not actually true, and which therefore cannot be actually true, and in turn, cannot be known to be actually true. Then,  $\varphi$  is unknowable, because it cannot be the case that it is known to be actually true in any world, because it isn't actually true in any world. It might be worth noticing that this also rules out knowability of contingently false statements about the actual past. For example: I didn't have breakfast this morning. So it is false at any world that I actually had breakfast this morning. It is unknowable in this sense, then, that I had breakfast this morning. Again, this is in contrast with non-factive conceptions of knowability like Naive. It could happen that I had breakfast; if I had, I could have known it.

Edgington points out that this concept of knowability can avoid Fitch's paradox. However, Rabinowicz & Segerberg (1994) show that the problem comes back with a vengeance: from the formulation of the knowability thesis using aK, we get  $A\varphi \rightarrow KA\varphi$ . Indeed, if  $\varphi$  is true at the actual world, then it is true at all worlds that  $A\varphi$ , and all worlds that are epistemically accessible worlds from a given world are among those, so  $KA\varphi$  is true at that given world. However, they also show that the issue can be circumvented using a different two-dimensional

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<sup>6</sup> In natural language there are some senses of 'actually' that do not follow these semantics. Edgington herself admits her use of 'actually' is a theoretical construct. Cf. Stephanou (2001).

semantics. The basic idea is that formulas have to be evaluated not with respect to a single world but to pairs of worlds:<sup>7</sup>

- $\diamond\varphi$  is true at the pair of worlds  $(w, v)$  iff there is at least one world  $w'$  that is modally accessible from  $w$  and  $\varphi$  is true at the pair  $(w', v)$ .
- $A\varphi$  is true at the pair of worlds  $(w, v)$  iff  $\varphi$  is true at the pair of worlds  $(v, v)$ .
- $K\varphi$  is true at the pair of worlds  $(w, v)$  iff  $\varphi$  is true at all the pairs of worlds  $(w', v')$  that are epistemically accessible from  $(w, v)$ .

We leave it to the reader to check that this indeed solves the problem.

A lingering problem is that this notion of knowability, by appealing to possible knowledge of actual truth, seems to require trans-world *de re* knowledge of actuality. Suppose, for example, that actually there is a truffle among the roots of an oak tree. In some possible world  $w$ , truffles are found among the roots with the help of a truffle hog. That there is a truffle among the roots of the tree is known at  $w$ , but this is not knowledge about what is the case at  $w_0$ . How could one know about the actual world in other worlds? The point is pressed by Williamson (1987, 2000) and Rabinowicz & Segerberg (1994).

Edgington's initial answer was to observe that the actuality operator works in relation to situations, not complete worlds. Schematically:<sup>8</sup>

**$\alpha K$**

Where  $\alpha$  characterizes the actual situation:

$$k\varphi \leftrightarrow \diamond K(\alpha \rightarrow \varphi)$$

The idea is as follows. Suppose that at some possible world  $w$  one knows that if  $\alpha$ , then  $\varphi$ . Then one knows that  $\varphi$  is true at all the  $\alpha$ -worlds, which includes the actual world.<sup>9</sup> But this also falls prey to triviality objections (Williamson; 1987, 2000; Fara 2010; Heylen 2020b). Williamson (1987) points out that, if  $\alpha$  is any correct description of the actual world and if  $\varphi$  is true at the actual world, then  $\alpha \wedge \varphi$  is also a correct description of the actual world. But  $K((\alpha \wedge \varphi) \rightarrow \varphi)$  is just knowledge of a logical truth. So, possibly knowing a trivial logical truth is then sufficient for the knowability of  $\varphi$ , even when  $\varphi$  supposedly is an empirical truth. A major drawback of  $\alpha K$  is that  $\alpha$  needs to be as complete a description of the actual world as is feasible, because that will shrink the set of worlds that satisfy the description to the smallest set that is feasible, with the singleton consisting of just the actual world as the ideal. A more promising approach consists in using a counterfactual, which selects the  $\alpha$ -worlds that are closest to the given world, where  $\alpha$  may be a very unspecific description (e.g., it rained last

<sup>7</sup> See also Heylen (2016).

<sup>8</sup> The schemes  $\alpha K$  and  $\alpha K'$  are based on Schlöder (2019).

<sup>9</sup> Note that since  $\alpha$  is an incomplete description, there are multiple ways to complete it.

night). In other words, the closeness relation compensates for the lack of specificity of the description. The proposed conceptualization of knowability is the following:

**$\alpha K$**

Where  $\alpha$  characterizes the actual situation:

$$k\varphi \leftrightarrow \diamond K(\alpha \Box \rightarrow \varphi)$$

The semantical clause for  $\Box \rightarrow$  is the following:

- $\varphi \Box \rightarrow \psi$  is true at a world  $w$  iff all  $\varphi$ -worlds that are closest to  $w$  are also  $\psi$ -worlds.

It turns out that the counterfactual-based version is not immune to trivialization worries (cf. Williamson, 1987; Fara, 2010; Schlöder, 2019; Heylen, 2020b). For any  $\psi$  that is true at the actual world and for any given world  $w$ , the closest  $\alpha$ -worlds also have to be the closest  $(\alpha \wedge \psi)$ -worlds, since otherwise a non-actual world in which  $(\alpha \wedge \neg\psi)$  is true is closer to  $w$ , which would undermine factivity. By assumption,  $\varphi$  is true at the actual world. Hence, the closest  $\alpha$ -worlds to any given world  $w$  are also the closest  $(\alpha \wedge \psi)$ -worlds. Therefore,  $\alpha \Box \rightarrow \varphi$  is necessarily equivalent to  $(\alpha \wedge \psi) \Box \rightarrow \varphi$ , which is just a logical truth.

### 3.2 The counterfactual approach revisited: Schlöder

Schlöder (2019) proposes a new analysis of counterfactual knowability that is meant to avoid the triviality objections:

**ARTK**

$\varphi$  is knowable either if it is known that  $\varphi$  or there is a way  $i$  to enquire whether  $\varphi$  that isn't actually successfully pursued but that, if it were to be successfully pursued, would impart the knowledge that  $\varphi$  would be true even if  $i$  wasn't successfully pursued. Using  $sp(i)$  to indicate that  $i$  is successfully pursued we get the schema:

$$k\varphi \leftrightarrow K\varphi \vee \exists i \in \text{Inq}(\neg sp(i) \ \& \ (sp(i) \Box \rightarrow K(\neg sp(i) \Box \rightarrow \varphi)))$$

This elaborates on Edgington's idea that we can have counterfactual knowledge of actual truths (in this case, that truths would have remained stable if we hadn't successfully pursued some lines of inquiry).

Schlöder's account is free from the triviality worries noted earlier because the knowledge of a counterfactual is not directly within the scope of a possibility operator but it is embedded within another counterfactual. The counterfactual  $\neg sp(i) \Box \rightarrow \varphi$  has to be true at all the closest  $sp(i)$ -worlds (relative to the actual world), but it is left open that it could be false (and, therefore, not logically true) outside the set of the closest  $sp(i)$ -worlds (relative to the actual world).

Factivity is enforced by stipulating that the following symmetry principle holds: if  $v$  is among the closest-to- $w$  worlds at which  $sp(i)$  is true, then  $w$  is among the closest-to- $v$  worlds

at which  $sp(i)$  is false. It is important to note that while this is a factive notion of knowability, it makes no use of the actuality operator, which dispenses with some of the worries that plagued Edgington's analysis. Yet, the symmetry problem that is appealed to is not without critics.

Heylen (2020b) points out that some lines of inquiries consist of multiple steps and that fact can be used to criticize the symmetry principle:

Suppose that in the actual world the line of inquiry was not successfully pursued because something went amiss before even the first step could be executed. Before you are walking into the kitchen, one of your children is crying and you go and check on the child instead. Now consider a possible world  $w$  in which you have successfully pursued the line of inquiry. Arguably, the worlds that are closest to  $w$  in which the line of inquiry has not been successfully pursued do not include the actual world but rather the worlds in which something went amiss before the line of inquiry could be completed. For instance, consider a world in which you did walk into the kitchen, you did open the cupboard, and you did look at the objects in front of you, but you did not yet rummage through the cupboard to look at the objects hidden behind other objects, because in this world you hear the music of an ice cream van and you decide to abort your search for cookies and buy an ice cream instead.

To solve this problem it was suggested by Heylen (2020b) to reformulate the symmetry principle to take multiple-steps procedures into account. The key idea is the following: if  $w$  is among the closest-to- $v$  worlds where the  $n$ -th step of the line of inquiry has been successfully executed, then  $v$  is among the closest-to- $w$  worlds where an earlier step  $m$  has not been successfully executed. We will not go into further details here, but do note that it is important to take the internal structure of lines of inquiries into account.

Another problem has been pointed out by Williamson (2000, 298) in a discussion of Edgington's work:

For example, let  $p$  be the proposition that there is a pebble at spatiotemporal location  $xyzt$ , and  $s$  be a situation in which  $p$  is true but unknown because the conditions for intelligent life emerge only long after  $t$  (the time of  $xyzt$ ). Let  $s^*$  be a situation as close as possible to  $s$  in which  $p$  is known. Cosmic history follows vastly divergent paths in  $s$  and  $s^*$ . In the closest possible situation to  $s^*$  in which  $p$  is unknown, it is unknown simply because no one chances to travel near  $xyzt$ ; such situations are far closer to  $s^*$  than to  $s$  in cosmic history.

Once more, it seems important to think about what possible lines of inquiries are. A minimal condition is that, for any possible line of inquiry, there is a possible world in which that line of inquiry is successfully pursued. (Otherwise it trivializes Schlöder's concept of knowability.) However, that leaves open whether it is successfully pursued by a possible agent or by an actual agent. If the latter, then there is a way to resist Williamson's counterexample.

Part of any putative line of inquiry into whether there is a pebble at coordinates  $xyz$  is for an agent to pursue this line of inquiry to travel to  $xyz$ , or to a point on a line of sight ending at  $xyz$ . But it is impossible for any agent in the actual world to travel to  $xyz$  or to a point on a line of sight ending at  $xyz$ , because any actual agent exists only at times much later than  $t$ . (In addition, it is assumed that time travel within the same world is not feasible for agents.) So, there is no possible world in which a line of inquiry into the presence of a pebble at  $xyz$  succeeds. Therefore, it is not a counterexample to the symmetry principle, which applies to worlds at which a line of inquiry has been successfully pursued. If possible lines of inquiries being successfully pursued is compatible with them being successfully pursued by possible agents, then Williamson's counterexample remains a serious challenge. To save factivity by appeal to symmetry from Williamson's counterexample, it suffices to restrict possible lines of inquiries to one that are possibly successfully pursued by actual agents. As we will see in the next section, in the analysis of knowability as the capacity to know, there is an explicit restriction to actual agents.

The counterfactual approach to knowability that was put on the contemporary research agenda by Edgington suffered first from a host of triviality problems. Recently, it was reinvigorated by Schlöder. It turns out that to ensure that Schlöder's counterfactual concept of knowability is factive it is important to think about the internal structure and the individuation of lines of inquiries.

#### 4. The capacity and the ability to know

A different line of thought about knowability makes a more direct use of the fact that knowability is a kind of ability. A somewhat natural approach towards abilities is to account for them in terms of items such as capacities, potentials or dispositions more generally. The idea is that one is able to do something if doing it is within one's capacities, or if there is a potential for one to do it, or if one is disposed to do it (given some triggering conditions). In turn, capacities, potentials and dispositions can be accounted for in various different ways (for an overview, see Choi & Fara, 2018). One theoretical possibility is that these items should be understood in terms of counterfactuals. In that case an analysis of knowability in terms of capacities would end up with some of the problems we have already raised for counterfactual approaches. Consequently, it is more interesting to consider if we can gain anything by rejecting the reduction of these items to counterfactuals. An approach of this kind is taken by Vetter (2015), who argues that dispositions are *sui generis* precisely in this way. Indeed, for Vetter objective modality is metaphysically dependent on what she calls *potentialities*. In a framework that takes dispositions as primitives, ability concepts are not analyzed in terms of counterfactuals, but rather in terms of the available dispositional primitives (cf. Vetter & Jaster, 2016) for a critical overview of some recent views of this sort).

This kind of approach to the concept of knowability is developed explicitly by Fara (2010), who proposes to use the notion of a *capacity* as his conceptually primitive notion. For knowability, he proposes the following analysis:

### Capacity to know

$\phi$  is knowable if actually there is an  $x$  who has the capacity to know that actually  $\phi$ .

$$k\phi \leftrightarrow A\exists x(CxKxA\phi)^{10}$$

The idea is that somebody has the capacity to know something if the agent possesses the appropriate epistemic faculties, exercised or not, that would yield knowledge in the appropriate circumstances. In Fara's words:

... one has the capacity to perform some feat provided one's internal constitution does not rule out the performance of that feat ... (2010, p. 68)

Interestingly, Fara distinguishes between the ability to know and the capacity to know: while the former typically is manifested in successes (in coming to know), the second can fail. According to him, for someone to have an ability, they must at least satisfy the condition that if they were to try, they would succeed. Consequently, the concept of capacity is (in Fara's reconstruction) relatively more fundamental than that of ability.

Fara takes the feature of capacities that they can fail to be manifested even further. In his conception, while typically abilities are possibilities to do something, capacities need not correspond to possibilities.<sup>11</sup> This gives a conceptual solution to Lewis (1976) grandfather paradox scenario: people can have the capacity to kill their own grandfather without there being a possibility where they indeed kill them (even if they tried). So Fara raises the possibility of having the capacity to do something impossible: the capacity to know something that is not possibly known, for example. Spencer (2017) also offers a notion of ability that is compatible with impossibility: according to him, one can be able to do the impossible.<sup>12</sup>

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<sup>10</sup> The  $Cx$ -operator (' $x$  has the capacity to ...') is an unary operator that can only meaningfully combine with an expression that expresses a verb in an infinitive clause (i.e. know). The  $Kx$ -operator (' $x$  knows that ...') is an unary operator that expresses a sentence (clause), not merely a verb. So, it would seem that both operators cannot be meaningfully combined. Fara (2010, p. 70, fn. 25) is aware of this issue and he proposes to read the formula  $CxKxA\phi$  as ' $x$  has the capacity to know that  $A\phi$ '.

<sup>11</sup> Similarly, Fuhrman (2014) talks about knowability as potential knowledge, and rejects that knowability entails possible knowledge and that possible knowledge entails knowability. In (Heylen, 2020a) it was shown that, for all models based on Fuhrmann's so-called 'hyperrelational' frames that make a certain the closure of potential knowledge under conjunction introduction frame-valid and that are not 'infinitary' in a certain sense, there are Kripke-style models based on bi-relational frames that are elementarily equivalent to the former. What this means conceptually is that, under those conditions, potential knowledge behaves as possible knowledge.

<sup>12</sup> Nguyen (2018) offers various arguments against Spencer; it is an open question if these arguments apply to Fara.



At this point it might be worth raising some more general issues about the individuation of capacities, abilities, and dispositions.

The first thing to notice is that capacities, abilities and dispositions can be context-sensitive: something or someone can have certain capacities or abilities restricted to certain circumstances only. Consider this scenario:

### **Wanderer**

Odysseus had been travelling at sea for many years, looking for a way back home. He was unable to find rest in Polyphemus island, where he lost many men. Neither could he in Circe's island. Even when he came back home he was restless, because of the pretenders that were trying to take over his household. Once he disposed of them with the help of his son, he was able to rest at last.

Odysseus' abilities seem to change as his circumstances change. Alternatively, we could say that they remain constant, but are *masked* as his circumstances change. Using Fara's distinction between capacities and abilities, we could say that Odysseus has the capacity to be at rest all throughout, even though at times it was impossible for him to be at rest, and thus, lacked the ability to be at rest. Capacities in Fara's sense are less sensitive to the circumstances than abilities; they are more about their subject (the individual who has them) than about the integrated system of the subject and its environment.

Transposing to the case of knowability, we should raise the question whether knowability is a property of (potential) epistemic agents, of the environment in which epistemic agents could be, or of the (potential) systems which integrate agents and environment. This gives three different ways in which the concept of knowability can be developed in a framework of this sort. We can thus pick apart different senses of 'capacity' in the characterization that we gave above. Whether any of these concepts would be more adequate, is an open question. For example, we could argue that the relation between the subject and the environment must always be taken into consideration.<sup>13</sup> We could ask what makes an individual able to know something across various changing environments. Or we could ask what in the environment makes it possible for individuals in the environment to know or ignore truths that are in some sense available in these environments. In principle, one could take a pluralist stance and argue that all three types of concept can be useful in different contexts.

Two types of factors involved in the constitution of the capacities and abilities of agents like us are worth some special attention: the technological and social circumstances. We are able to do certain things only because we have tools to do them. For example, measuring the pressure of a bike tire requires a special gauge. We are also able to do certain things only because we live alongside other people. These things often go hand in hand: the authors of

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<sup>13</sup> For example, somebody could argue that epistemic abilities are relational because epistemic concepts have anti-individualistic satisfaction conditions. Cf. Brandom (1995).

this paper were able to write this paper on their computers only because other people designed and manufactured those computers. This already puts some pressure on the distinction between capacities and abilities: we may be capable of certain things only because somebody has certain abilities, for example. Furthermore, abilities might require knowing how to do certain things. In this case, this know-how could also be relative to the use of technology or to social interaction. There is a sense in which merely *having* the tools to do something is not sufficient to be able to do it: one might also need to know how to use them. Can Odysseus sail his ship without a skilled crew? No, unless he also knows how to do the things the crew knows how to do. The same goes for the necessity of social skills (including communication skills): even if every member of Odysseus crew knew what to do and how to do it, could they sail the ship if they didn't also have the means to coordinate, or even evaluate the other crew members actions?<sup>14</sup> These points are obviously important in the context of knowability, where often methods of detection require specialized skill and coordination from various researchers (think, for example, of the experiments at CERN). An issue closely related to this has to do with the connection between having an ability, capacity or disposition to do something, and there being a counterfactual situation in which that thing is satisfied; in the cases where we deal with actions, we worry about the possibility of counterfactual successes. In Fara's conception, on the assumption that it is possible to try to know, the ability to know entails the possibility to know. Remember that for Fara, abilities to do something entail that if one tried to do it, one would succeed in doing it. Thus, if it is possible to try to do it, it is possible that one succeeds. This gives a necessary condition for having an ability, but also indirectly for having a capacity that depends on an ability (see above).

We can ask if that is also a sufficient condition. Suppose that there are possible worlds in which somebody lifts a 10 ton rock without mechanical assistance. Let's suppose that the person in question actually can barely lift 20 kg. It seems then that this person would have to be very different from the way they actually are in order to be like in the possible world where they lift 10 tons without assistance. So perhaps that world is too distant from the actual world for us to say that the person has the ability to lift it, even if it is possible. Maybe we would say that they have the ability if there is a good proportion of worlds where they lift it versus worlds where they don't. Some authors say that in those cases one has more than the ability: one has a *competence*.<sup>15</sup> The same can be said in the case of knowability: the possibility (for somebody) of counterfactual knowledge might not be sufficient for something to be knowable (for that person).

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<sup>14</sup> A crew can collectively navigate without any of the members of the crew being in a position to know all the required steps. For a description of the distributed cognitive processes involved in the navigation of a modern ship, see Hutchins (1995).

<sup>15</sup> Cf. Sosa (2010) for a discussion on the epistemological importance of the notion of competence, and Morales (2018, s. 4) for a critical overview of different accounts of competence.

The second point to raise has to do with the possibility of iterated capacities/dispositions. In his response to Fara (2010), Paul Humphreys (2011) elaborates on the point as follows:

There is a sense of ‘capability’ for which it is true that if an agent has, for example, the technological capability to construct an instrument that would provide evidence for a true sentence *p*, and the existence of that instrument would endow the agent with the epistemic capability to know that *p*, then the agent has the capability to know that *p*. There is another sense of ‘capability’ in which collapsing the successive capability claims is illegitimate. (p. 550)

Does, then, having the capacity to have the capacity to do something entail having the capacity to do it? Can we come from our capacities in distant possible worlds to the capacities we actually have in close worlds? How far is too far in this sense? Again, we will have a variety of concepts of knowability depending on the conditions we impose on iterated capacities. This goes back to our earlier point about the dependencies of capacities on technological and social circumstances. The features of circumstances make the capacity to have certain capacities available. For example, you might want to know the pressure of your bike’s tire. Typically, you will need some instrument to measure the pressure. So you might lack the capacity unless you had the required instrument, which in turn would make certain capacities available. Further, you might lack the capacity even if you had the instrument, if you didn’t know how to use it.<sup>16</sup> These problems of iteration put further pressure on the distinction between capacities and abilities.

The third point has to do with the fact we have just noticed that capacities and abilities can stand in various types of relations to each other. In the cases we just mentioned, some capacities might be *dependent* on abilities. There can be other hierarchies of capacities and abilities. For example, whoever has the capacity to chop down 20 trees has the capacity to chop down 19, and 18, and so on. Someone has the (more general) capacity to chop down trees and bring them to market might also have some of those capacities. In both cases the less general abilities compose the more general ones. Abilities can stand to each other in the relation that in order to be able to do something specific, you must have a more general ability, without it being possible to decompose the more general ability in more specific ones. For example, someone could argue that to understand a word in one circumstance it is necessary to be able to understand it in a vast array of circumstances.

The case of knowledge gives a different example of what we are discussing, more relevant to our concerns here: in order for one to know *p*, according to reliabilists, one’s method of forming the belief that *p* must be reliable, so that it would give the right result in a wide range

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<sup>16</sup> This is another point of contact with the literature on know-how. Indeed, the structure of the worry that having the capacity requires other capacities, which in turn might require other capacities, is at the base of a classic regress argument against intellectualism by Ryle (1949). Cf. also Carroll (1895).



of counterfactual cases. Methods of belief formation can also be more or less general. For example, one can form a belief based on (in order of decreasing generality):

1. directly looking at something,
2. directly looking at something in broad daylight,
3. directly looking at something in broad daylight and at a short distance,
4. directly looking at something in broad daylight and at a short distance from the front of my house,
5. on Friday 27 December at 12:07 directly looking at something in broad daylight and at a short distance from the front of my house.

Each of these methods of forming beliefs comes with different levels of reliability. The so-called Generality problem is to determine which of these belief forming methods is the relevant one. On the one hand what is to be avoided is that the individuation of belief forming methods is so fine-grained that they are tied to single beliefs, which are true or false, so that the reliability of a belief forming process reduces to the truth of the single belief that is formed. On the other hand it is also to be avoided that the individuation of belief forming methods is so coarse-grained that they are bound to result in false beliefs in a fairly wide range of circumstances. For example, directly looking at something in dim light or at something that is far away can easily result in false beliefs.

If knowability is analysed in terms of capacities or abilities and given that capacities and abilities can be more or less general,<sup>17</sup> then the question is what the relevant capacities and abilities are, where they can be neither too fine-grained nor too coarse-grained.<sup>18</sup>

The capacity/ability approach to knowability may seem *prima facie* promising: it gives a framework to make a series of distinctions and connections that are difficult to evaluate from other perspectives. However, this apparent strength is also its weakness, since in order to make good on the promise several significant difficulties concerning the individuation of capacities and abilities should be dealt with first. In this sense, Fara's proposal (and similar ones) is under-specified. We have seen this in that the distinction between capacities and abilities that it deploys is unstable. This general problem puts pressure on the purported autonomy of the capacity/ability framework from, for example, counterfactual approaches.

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<sup>17</sup> Cf. Mele (2003).

<sup>18</sup> The Generality problem is usually described in terms of processes, but this is not necessary. In the virtue reliabilism camp (like in Sosa, 2010), which emphasizes competences and abilities, some processes are also considered as competences or abilities. For a more extensive treatment of the Generality problem, see Conee & Feldman (1998), who raise it as a general problem against reliabilism, and Bishop (2010), who argues that the Generality problem affects all plausible epistemologies, both reliabilist and non-reliabilist.

## 5. The dynamic approach

A different conceptual framework for understanding knowability comes from the dynamic logic camp. Van Benthem (2004), Balbiani et al. (2008) and Van Ditmarsch et al. (2011) have proposed to analyze ‘ $\phi$  is knowable’ as ‘after the public announcement of some formula  $\psi$ ’ (with certain restrictions on  $\psi$  to avoid circularity), it is known that  $\phi$ . (We will not go into the formal details.) For reference, here is the proposal:

### Dk

$\phi$  is knowable iff after the public announcement of some formula  $\psi$  (with certain restrictions on  $\psi$  to avoid circularity), written  $\langle ! \rangle$ , it is known that  $\phi$ .

$$k\phi \leftrightarrow \langle ! \rangle K\phi$$

The semantical clause for public announcements is the following (Van Benthem & Liu, 2007; Balbiani et al. 2008):<sup>19</sup>

- $\langle !\psi \rangle \phi$  is true in the model  $M$  at the world  $w$  iff (1)  $\psi$  is true in the model  $M$  at the world  $w$  and (2)  $\phi$  is true at  $w$  in the model  $M| \psi$ , where  $M| \psi$  is identical to  $M$ , except that the epistemic accessibility relation  $R| \psi$  of  $M| \psi$  is defined as follows:  $\langle v, u \rangle$  belongs to  $R| \psi$  iff (i)  $\langle v, u \rangle$  belongs to  $R$  (the epistemic accessibility relation of  $M$ ) and (ii)  $\psi$  is true at  $v$  iff  $\psi$  is true at  $u$ .
- $\langle ! \rangle \phi$  is true in the model  $M$  at the world  $w$  iff there is a formula  $\psi$  (with certain restrictions to avoid circularity) such that  $\langle !\psi \rangle \phi$  is true in the model  $M$  at the world  $w$ .

However, this notion of knowability is not factive, since  $\phi$  is *interpreted* after the *public announcement has taken place*. In other words, after the announcement it is known that  $\phi$  is true or  $\phi$  is false, but not that  $\phi$  *was* true or  $\phi$  *was* false. Some formulas can be known to be true (false) after an announcement even though they were false (true) before the announcement. Take, for example, a situation in which it is unknown that there is an odd number of books in Williamson’s office. After the (true) announcement that there is an odd number of books in his office, it is known that there is an odd number of books in his office and, moreover, it is known that it is known that there is an odd number of books in his office.<sup>20</sup> So, what is known after the announcement is false before the announcement. It thus comes not as a surprise that Van Ditmarsch et al. (2011, p. 93) conclude that ‘[t]his restricts the philosophical relevance of interpreting ‘knowable’ as ‘known after an announcement’.

Holliday (2017)’s important contribution is to describe a framework in which something along the following lines can be expressed: after the public announcement of some formula  $\psi$  (again with certain restrictions on  $\psi$  to avoid circularity), it will in some hypothetical future

<sup>19</sup> This is the so-called ‘link-cutting’ version of public announcement logic.

<sup>20</sup> It is provable that  $\langle !\psi \rangle K\dots K\phi$  follows from  $\langle !\psi \rangle K\phi$ .

be known that  $\varphi$  was true *before that public announcement took place*. For reference, here is the core idea:

**FDk**

$\varphi$  is knowable iff after the public announcement of some formula  $\psi$  (again with certain restrictions on  $\psi$  to avoid circularity), it will in some hypothetical future be known that  $\varphi$  was true *before that public announcement took place*.

$$k\varphi \leftrightarrow \langle ! \rangle KY\varphi$$

We will not go into all the details of Holliday's proposal, but the basic idea is roughly as follows:

- $\langle ! \psi \rangle \varphi$  is true at a world  $w$  and a time  $t$  iff (1)  $\psi$  is true at world  $w$  and time  $t$ , (2) the epistemic accessibility relation at time  $t+1$  (which may be part of a hypothetical rather than actual future)<sup>21</sup> is defined by cutting links as before, and (3)  $\varphi$  is true at world  $w$  and time  $t+1$ .
- $Y \varphi$  is true at a world  $w$  and a time  $t$  iff  $\varphi$  is true at world  $w$  and time  $t-1$ .<sup>22</sup>

Clearly, this concept of knowability is factive: if at a world  $w$  and a time  $t+1$  it is known that  $\varphi$  was the case before the public announcement that resulted in the epistemic state at time  $t+1$ , then  $\varphi$  is true at a world  $w$  and time  $t$ .

What is striking about this concept of knowability is that it allows self-fulfilling announcements (Holliday, 2017, section 4.2). For example, it could be announced that next you will know that the number of books in Williamson's office on 19 December 2019 is odd. Suppose that it indeed (eternally) true that the number of books in Williamson's office on 19 December 2019 is odd. Then the announcement that next you will know that the number of books in Williamson's office on 19 December 2019 will (in some hypothetical future) lead to the knowledge that the number of books in Williamson's office on 19 December 2019 is odd. Let us also suppose that between the moment right before the announcement and the (hypothetical) moment right after the announcement no other epistemically relevant action pertaining to the subject matter is taken. It would then seem to be that the announcement itself is the source of the knowledge. This raises a vexing question:

[Y]ou could plausibly reason as follows: I have been told something that entails  $p$  by an authoritative source I trust, so  $p$  is true. Perhaps you could thereby acquire not only belief but also knowledge of  $p$ . Compare this with the case, used in science fiction stories, of an agent who encounters a machine that can predict the future, and

<sup>21</sup> In the actual future there may be a different public announcement resulting in a different epistemic accessibility relation at  $t+1$ .

<sup>22</sup> If there is an initial time  $t_0$ , then the right-hand side of the equivalence should be:  $\varphi$  is true at the initial time  $t_0$  or  $\varphi$  is true at time  $t-1$ .



the machine predicts for her that she will come to know some important proposition (tenselessly formulated, let us suppose). Does the agent thereby come to know the proposition? (Holliday, 2017, section 4.2)

To put the issue in sharper focus, let us consider another case. At  $xyz_t$  there is an earth worm, with  $xyz$  the spatial coordinates of a remote location in the Amazon rain forest and with  $t$  the time coordinate. That there is an earth worm at  $xyz_t$  is an eternal truth. Given the remoteness of  $xyz$ , no human beings were sufficiently close to  $xyz_t$  to discover that there is an earth worm at  $xyz_t$ . (Time  $t$  can be set before any human exploration of the Amazon rain forest took place or even before there were any humans.) Rodrigo lives in Chile in the early 21st century. If anybody has ever told Rodrigo that next he will know that  $p$  (e.g., a particular person is not to be trusted), it was always in cases in which Rodrigo discovered whether  $p$  via independent means (e.g., Rodrigo is put in a position in which that particular person can betray his trust). Let us use  $t'$  for the current date in the lifetime of Rodrigo.

In a hypothetical future Rodrigo hears the announcement that next he will know that there is an earth worm at  $xyz_t$ . “Next” here means: at time  $t'+1$ . Where is this announcement coming from? In the actual history (up to and including  $t'$ ) nobody has the means to determine that there is an earth worm at  $xyz_t$ . However, at the hypothetical future  $t'+1$  there is supposed to be an authoritative source regarding the presence of an earth worm at a remote location in the Amazon rainforest before it was explored by humans. Are we to imagine the sudden appearance of an oracle or a semi-divine message? On what basis could Rodrigo deem the announcement trustworthy? Even if he were to get into contact with an oracle and even if he starts to trust the oracle, his trust is presumably based on checking its proclamations or their implications by independent means and/or perhaps by studying the principles by which the oracle operates. In this scenario Rodrigo has no means to check independently of the oracle whether there is an earth worm at  $xyz_t$ . More importantly even, assuming that this authoritative source is indeed new, there is no time between  $t'$  and  $t'+1$  to start trusting the source on any basis that would lead to epistemic updates.

The dynamic approach to knowability in its original form, championed especially by van Benthem, faces the problem that it turns knowability into a non-factive notion. Holliday has shown that a dynamic approach can also yield a factive notion of knowability. Unfortunately, his version is confronted with the puzzling phenomenon of self-fulfilling announcements.

## 6. Conclusion

The scope of what is knowable is and has been a major point of discussion among philosophers. Epistemic optimists think that all truths are knowable, epistemic moderates think that some truths are knowable and some are not, epistemic pessimists think that no truths are knowable. All these positions presuppose that we have a good understanding of the concept of knowability.

As we have discussed throughout this paper, giving an analysis of the concept of knowability raises various technical and philosophical difficulties. Because of the importance of the concept, it is important to get things right, in the sense of capturing adequately the features that would allow the concept to serve its functions. For the reasons we mentioned in section 2, it is important to develop a factive concept of knowability.

The nowadays standard concept of knowability is the following: for something to be knowable is for that something to be possibly known. However, this concept of knowability is not factive. We have seen that, broadly speaking, there are three alternative approaches to the analysis of the concept of knowability:

- a) a *counterfactual* approach, first proposed by Edgington (1985) and recently revisited by Schlöder (2019),
- b) an approach making use of the notions of *capacity* and *ability*, exemplified by Fara (2010),
- c) an application of the framework of *dynamic logic*, exemplified by Holliday (2017).

We have seen how all these approaches can offer *prima facie* workable factive concepts of knowability. However, as we have also pointed out, each of these approaches faces difficulties.

The counterfactual approach to knowability has faced for a long time the difficulty of providing a counterfactual analysis of knowability that is factive on the one hand and non-trivial on the other hand. (A trivial concept of knowability is one that, for instances, makes only logical truths knowable.) Schlöder's recent contribution is a counterfactual analysis of knowability that is non-trivial. This analysis crucially involves lines of inquiries, which if they were successfully pursued, would result in the knowledge that, if they were not successfully pursued, that what is knowable would be true. However, we have seen that the factivity of his notion of knowability can be challenged. We have concluded that it is vital to reflect on the internal structure and the individuation of lines of inquiries.

The capacity approach to knowability says that for something to be knowable is to for an actual agent having the capacity to know that that something is actually true. According to Fara, a key difference with the counterfactual approach is that capacities are not analyzed in terms of counterfactuals but in terms of performances that are not ruled out by the internal constitution of the agents. Whether that is a real or imagined distinction is something we discussed. Capacities depend to some extent on circumstances, including social and technological contexts, where the agents need to be able to communicate and collaborate with other agents and where they need to be able to use the technological tools. If abilities are analyzed counterfactually, as Fara does (but Spencer does not), then there is again a counterfactual component in the analysis of knowability. Here as well we need to think further about what capacities and abilities are.

One difficulty that both the counterfactual approach and the capacity/ability approach to knowability face is the Generality problem. This problem was raised for the capacity/ability

approach but, as we will now argue, also applies to the counterfactual approach, at least in Schlöder's version. We can think of lines of inquiry as procedures that lead to knowledge, where these procedures can be described in more or less general terms. Each of those more or less generally described procedures may be more or less reliable. The difficulty consists then in picking the right grain of individuation of lines of inquiries.<sup>23</sup>

Finally, we have discussed the dynamic approach to knowability. Regrettably, the idea in its form due to van Benthem results in a non-factive concept of knowability. Holliday has recognized this problem and built on the work of van Benthem and others to develop a dynamic analysis of knowability that is factive. Roughly, the idea is that for something to be knowable, it has to be the case that, after some public announcement, it will in some hypothetical future be known that that something was true before that public announcement took place. However, Holliday's approach has as a mystifying consequence that self-fulfilling announcements can take place: if something is (eternally) true, then the announcement that next it will be known results in the knowledge that it is true.

Where the counterfactual approach and the capacities/abilities approach are confronted with questions about, for example, the grain of individuation of lines of inquiries and capacities/abilities, the dynamic approach has in a sense a deeper problem: the public announcements come out of nowhere. The update machinery has a black box at its core. The dynamic approach to the concept of knowability could profit from replacing the black box by, for instance, successfully pursued lines of inquiries.

Our work here was to offer a critical overview of the conceptual frameworks that we currently have in the literature. We hope this will offer some insight into how we can develop better solutions in the future. Ultimately, we hope that this will also lead to a better understanding of the limits of what is knowable.

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<sup>23</sup> From the perspective of our discussion of the structure of lines of inquiry, Lyons (2019) is also interesting. Lyons' proposed solution to the Generality problem is that cognitive processes should be individuated in terms of certain types algorithms and parameters for those algorithms; it is natural to see certain lines of inquiry as implementations of algorithms.



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