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On luck and modality

Luck plays an important role in debates in political philosophy, ethics, free will, and epistemology. As such, work that focuses on the nature of luck is also important.¹ One influential view is the modal account, which holds that the chanciness element in lucky cases should be thought of in terms of modal fragility. The thought is that we can give a rough ordering of possible worlds in terms of how closely they resemble the actual world. Hence a nearby possible world is quite similar to the actual world, whereas a distant possible world is quite unlike the actual world. According to the modal theorist, a significant event is lucky if and only if: “it obtains in the actual world […] but] there are—keeping the [relevant] initial conditions for that event fixed—close possible worlds in which this event does not obtain” (Pritchard 2014, p. 599). This way of defining luck has a number of putative advantages over control-based or probability-based views. First, it is able to handle potential counterexamples to these theories.² Second, the modal account of luck has been used by Duncan Pritchard (2005) and Rik Peels (2017) to explicate instances of moral luck and Gettier cases.

However, there are many different ways of capturing the above relationship between luck, the actual world, and possible worlds. As such, there are three extant modal theories: proportional, distance, and density-based views. I argue that each of these accounts is subject to counterexample. I then argue that whether an event is lucky is often determined by how one describes the relevant initial conditions of an event, and it is an open question what initial conditions are relevant. Thus,

¹ For an alternate view on the importance of conceptual analysis on luck and issues in epistemology and ethics see Ballantyne (2014) and Anderson (2019).
² For example, whether the sun rises tomorrow is completely outside of anyone’s control and significant but is either non-lucky or only involves a small degree of luck. This event, however, is modally robust. On the probabilistic side, it is unclear what notion of probability is relevant to luck, and deterministic events (for example, a dice roll or roulette spin), which have a probability of 1, may still be lucky. However, one’s win at such “games of chance” is modally fragile.
modal accounts of luck are subject to a kind of reference class problem similar to the reference class problem that is a seemingly unavoidable and serious objection to frequentist accounts of probability.³

1. The proportional view

The proportional view holds that the degree of luck for an event should be cashed out in terms “of the proportion of close possible worlds in which it would fail to occur—the larger the proportion of such close possible worlds is, the luckier the event” (Broncano-Berrocal 2016). Such a view is held by Pritchard (2004, 2005), Neil Levy (2011), and E. J. Coffman (2015). Witness Levy:

Event $E$ is chancy if it occurs in the actual world at $t^*$, but fails to occur in a large enough proportion of possible worlds obtainable by making no more than a small change to the actual world at $t$, where $t$ is a temporal interval just prior to $t^*$ (2011, p. 17, emphasis original)

There are differences, however, in how each of these theorists demarcate what amounts to a large enough proportion of worlds. Pritchard (2005) and Coffman (2007) argue that the target event is a matter of luck only if it fails to occur in at least half of the nearby possible worlds where the relevant initial conditions for the event are held fixed. But this threshold is arbitrary and false. Samuel’s victory at Russian roulette is lucky for him even though this event is probable, that is, in five out of the six nearby, relevant worlds Samuel survives (Rescher 1995, pp. 24-25). Levy argues that what amounts to a large enough number of worlds is going to depend inversely on the significance of the event. Coffman (2015, p. 40) discusses this view and calls it the inverse proportionality thesis. According to Coffman, the inverse proportionality thesis allows for too many instances of luck as it entails that nearly any highly significant event is lucky as long as it involves some element of chance. Levy could respond that although it might sound odd or be

³ For more on the reference class problem for frequentist accounts of probability see Hájek (2009) and La Caze (2016).
inappropriate to call a significant but highly probable event lucky, it is nonetheless true since such events still involve some level of chance. Luck is a scalar notion. But perhaps what Coffman has in mind is that the inverse proportionality thesis needs to be supplemented by a threshold condition such that an event is lucky only if it fails to occur in enough of the relevant worlds. Regardless of these differences, each of these accounts claim that an event in the actual world is lucky to the extent that there are a number of close possible worlds in which this same event fails to occur.

The putative strength of the proportional view lies in its ability to explain a multitude of paradigmatically lucky cases. Consider *Fair lottery*, which is similar to a case given by Pritchard (2005, 2014):

Suppose Smith buys a Powerball lottery ticket. This involves Smith selecting five unique numbers from one to sixty-nine and one number from one to twenty-six. On the day of the Powerball drawing, five white balls, which correspond with the numbers picked from one through sixty-nine, and one red ball, which corresponds with the number picked from one through twenty-six, are drawn randomly from two machines. The lottery drawing is fair. No agent knows which numbers are going to be picked, and no agent engages in a nefarious plot to alter the outcome of the drawing or to ensure that any one person wins or loses. It so happens that all of Smith’s white balls hit, and he wins a million dollars as a result.

Smith’s winning the lottery is a lucky event, and Pritchard argues that the modal account of luck best explains this fact. Smith’s winning, while it obtains in the actual world, is modally fragile. According to Pritchard, in most if not all nearby worlds Smith will be tearing up his lottery ticket in disgust as it is a loser. Holding the relevant initial conditions for the event fixed—that is, the lottery is fair, Smith purchases a ticket, and the rules for Powerball are the same across worlds—, all that has to change for Smith to lose the lottery is for a few numbered balls to fall slightly differently from the machine (Pritchard 2005, p. 128). Such worlds are not only numerous but remarkably similar to our own, hence, according to the proportional theorist, Smith’s win is
extremely lucky. There are a high proportion of nearby worlds in which Smith’s ticket is a loser, thus his win in the actual world is very lucky.

However, while the proportional view might seem to explain many lucky cases such as *Fair lottery*, it is, in fact, nonsensical. This is because proportions involve finite numbers. But as Carter and Peterson (2017, p. 2177) note, the set of nearby possible worlds is an infinite set. Consider, as Pritchard himself admits, that there is at least one nearby, possible world, P₁, in which Smith wins the lottery and one nearby, possible world, P₂, in which Smith loses the lottery. For example, P₁ and P₂ could be identical to the actual world except for how a few of the lottery balls fall.⁴ Consider further that there are other possible worlds P₃ and P₄ that are identical to P₁ and P₂, respectively, except for one minor detail, say the position of Smith’s car in his driveway. But P₃ and P₄ are also very similar to P₁ and P₂. Relocating Smith’s car in his driveway is a small change. But, in such a case, P₃ and P₄ are also nearby the actual world, though they are slightly farther away than P₁ and P₂. But it should now be clear that there are an infinite number of possible worlds like P₃ and P₄ that are identical to either P₁ or P₂ except for a small, trivial change. Thus, there are an infinite number of nearby worlds in which Smith wins the lottery, and there are also an infinite number of nearby worlds in which he loses. But if this is right, then it does not make mathematical sense to say, as each version of the proportional view does, that there are *more* nearby worlds, a *wide enough set* of nearby worlds, or a *higher proportion* of nearby worlds in which Smith’s winning occurs or fails to occur. Such a claim is analogous to saying that there are more odd numbers than there are prime numbers. In reality, both sets of numbers are countably infinite, and as such we should not spell out the relationship between luck and possible worlds via proportions.

⁴ P₁ could still involve the repositioning of a few balls; they could simply fall in a different order. Thus, Smith still wins by hitting on all five of the white balls.
2. The distance view

Pritchard’s (2014) account differs from his (2005) proportional view:

[T]he degree of luck involved varies in line with the modal closeness of the world in which the target event doesn’t obtain (but where the [relevant] initial conditions for the event are kept fixed). We would thus have a *continuum* picture of the luckiness of an event, from very lucky to not (or hardly) lucky at all. (2014, p. 600, emphasis original)

Broncano-Berrocal (2016) labels such a position the distance view because it says that “the degree of luck of an event varies as a function of the distance to the actual world of possible worlds in which it would fail to occur.” But Pritchard’s account, while it paints a continuum picture of lucky events, also contains a threshold condition. According to Pritchard, if there are no nearby worlds where the event does not obtain, then we no longer consider the event to be a matter of luck. For example, winning a coin toss is very lucky because there is a nearby world in which one loses. A small change concerning the initial position of the coin or on the forces that act on it could result in the coin landing on its opposite side. Such a world is remarkably similar to our own. In contrast, consider another event: the sun’s rising tomorrow (Latus 2000). The distance view correctly judges this event to be non-lucky. This is because there is no nearby world in which the sun fails to rise. Following Lewis (1979), a world in which the sun does not rise would be quite distant from our own both spatiotemporally and in terms of the physical laws governing nuclear fusion. Blowing up the sun is, hopefully, not a small change.

Furthermore, the distance view, unlike the proportional view, is able to correctly describe the degree of luck in *Fair lottery*. This is because the closest world in which Smith does not win the lottery is very similar to the actual world. The only difference between the two worlds could be how a few of the lottery balls fell. However, consider the following case *Fair lottery 2:*
Suppose Donald buys a Powerball lottery ticket. This involves Donald selecting five unique numbers from one to sixty-nine and one number from one to twenty-six. On the day of the Powerball drawing, five white balls, which correspond with the numbers picked from one through sixty-nine, and one red ball, which corresponds with the number picked from one through twenty-six, are drawn randomly from two machines. The lottery drawing is fair. No agent knows which numbers are going to be picked, and no agent engages in a nefarious plot to alter the outcome of the drawing or to ensure that any one person wins or loses. It so happens that Donald is a loser. Only two of his numbers, corresponding with the white balls, hit, which is not enough for a payout.

Intuitively, Donald’s not winning the Powerball is either slightly unlucky or not a matter of bad luck at all. According to the distance view, however, Donald is extremely unlucky that he lost. Remember that in *Fair lottery* Smith is very lucky to win the Powerball because there is a nearby world in which he loses. But the same kind of point holds in *Fair lottery 2*. Donald is very unlucky because there is a nearby world in which he wins. In terms of distance, the two cases are symmetrical as they both involve the repositioning of three lottery balls. As Pritchard, himself, tells us:

[T]he possible world in which one wins a lottery, while probabilistically far-fetched, is in fact modally close […] all that needs to change is a few coloured balls fall in a slightly different configuration (2014, p. 596)

Given the rules of Powerball, if two white balls or one white ball and one red ball had fallen slightly differently, then Donald would have won a hundred dollars. If three of the white balls had fallen slightly differently he would have won a million dollars, and if four of the balls had fallen differently he would have hit the jackpot and won hundreds of millions of dollars. Each of these scenarios involves just a small change to the actual world. To the distance view’s credit, it does get the order of these possibilities correct. It is a bigger change to reposition four balls as opposed to two. However, it incorrectly attributes a great deal of chancy luck to these cases. The world where Donald wins a million dollars because three white balls fall slightly differently is close to our own (similarity relations are not the same as real-world probabilities), but, contrary to the
distance view, this does not entail that the Donald in the actual world is the victim of terrible bad luck. This event, that is Donald’s not winning at least a million dollars, is simply too likely to occur (consider that the odds of correctly guessing all five of the white balls are 1 in 11,688,053⁵) for its occurrence to be considered very unlucky.⁶

A distance theorist could object that Donald’s loss is not very unlucky because it is of little significance. Powerball tickets only cost two dollars. First, this reply is not open to Pritchard (2014, p. 604) as he holds that significance is unnecessary for luck. Instead, Pritchard aims to give a metaphysical account of luck purely in terms of possible worlds. Second, this reply misses the mark as luck theorists hold that degrees of luck can vary for two reasons: significance and chanciness. But what Fair lottery 2 shows is that the distance view gets the degree of chanciness in Donald’s case wrong.

Pritchard would likely respond to Fair lottery 2 by arguing that his view does capture our intuitions about near (or not so close) lottery misses. Pritchard might point out that Donald would not feel very unlucky if all of his numbers were way off, but if Donald correctly guessed his first two numbers and was then only off by one number on all if his other guesses, he would feel very unlucky. But this is because he, unlike the first Donald, was really close to a huge payout, and Pritchard’s distance view has a way of substantiating the second Donald’s claim.

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⁵ Note that the odds of guessing the other three white balls correctly after correctly guessing the first two numbers is 1 in 47,905 or (67 x 66 x 65) / (3 x 2 x 1). Again, there is something to be said about the verdict implied by the distance view that the Donald who loses after correctly guessing the first two balls is unluckier than the Donald who incorrectly guesses all of the balls. However, my objection is that the distance view gets the degree of luck involved in such cases wrong. I would like to thank an anonymous reviewer for helping me emphasize this point.

⁶ Coffman (2015, p. 64) and Hawthorne (2004, pp. 4-5) discuss a similar problem with ease of mistake approaches to knowledge that make use of Pritchard’s modal account. Consider that there might be one nearby world where one’s justified belief is false due to a highly improbable physical anomaly—say a quantum particle flashing in and out of existence at a particular time. But the fact that there is one such nearby world, does not entail that one’s justified, true belief in the actual world—say that there is a coffee cup on the desk—has been “Gettiered” or is particularly lucky.
First, given that Pritchard wishes to give a purely objective account of luck in terms of possible worlds, it is, at the least, highly questionable of him to appeal to the subjective responses of individuals about whether or not they feel that they are lucky, unlucky, or non-lucky. Of course, I am putting words in Pritchard’s mouth in the above reply to *Fair Lottery 2*, but Pritchard does cite empirical support for his account via subject’s judgments about lottery cases. However, subjects are often wrong about whether or not an event is lucky. Consider Ballantyne’s (2012, p. 329) example of an anorexic man who unbeknownst to him and through sheer accident receives adequate nutritional supplement. This man is the beneficiary of good luck regardless of his own or another agent’s assessments. Furthermore, people are frequently mistaken about the chanciness at issue in lucky or non-lucky cases. Consider the mud punter who is convinced that he was not lucky but really knew that his horse was going to win. The point here is that we should not blindly accept an agent’s own judgments about luck but instead assess to what degree an agent, such as Donald, is objectively (un)lucky.

Returning to the above response, it is irrelevant to the case whether Donald is one number off in each of his guesses. This is because if the correct number was 1, then a guess of 2 is just as incorrect as a guess of 62. A guess of 2 is no closer to the correct answer than any other wrong guess. Such a case only has a superficial gloss of closeness. However, I agree that a Donald, say D₁, who hits on his first two guesses but misses on the rest is in a certain sense unlucky than a Donald, say D₂, who incorrectly guesses all of the balls. This is because there is a time t₁ (right after his first two balls hit) in which D₁ does have a chance of winning a million dollars, whereas

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7 This is would not be questionable regarding Pritchard’s earlier accounts. Pritchard’s (2004, p. 23) view is that there is a subjective component to luck, and he defines significance subjectively: “The type of luck, and its very existence from that agent’s point of view thus depends upon the significance that the agent attaches to the event in question” (2004, p. 19). Pritchard’s (2005, p. 132) view also has a significance condition, but here he defines significance in terms of what an informed agent would find significant.
D₂ at that same time has no chance of winning such a prize. That the distance view can make sense of such a case is an advantage of the account, and this way of reading Donald’s response is the best way to interpret the above reply to *Fair lottery 2*. However, a probabilistic view of luck such as Steglich-Petersen’s (2010, 2018) or Gregory Stoutenburg’s (2015, 2018) can also capture this intuition. After all, the odds of success for D₁ and D₂ are different at t₁. But regardless of one’s intuitions about the luckiness of D₁ and D₂, this response misses the point of *Fair lottery 2*. The point of *Fair lottery 2* is that there is at least one nearby world where Donald wins the lottery simply by purchasing a ticket. Pritchard, himself, admits that it is a small change to reposition a few lottery balls. But this does not mean that the Donald who fails to win the Powerball in the actual world is the victim of terrible bad luck. Most people think that Donald is only slightly unlucky or that there is no bad luck in the case, and we would not nor should we countenance a Donald in the actual world who bemoaned his awful luck at not winning a million dollars at the Powerball despite his first two balls hitting. This is because his loss is expected to occur and is not very chancy.

3. The density view

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8 I would like to thank an anonymous reviewer for this point.
9 Of course, a modal theorist could claim that chanciness thought of in terms of probability is not the same as the chanciness element in lucky cases, hence we should not be surprised that a distance-based modal account and a probabilistic account will diverge in certain cases—such as in *Fair lottery 2*. However, the point of this counterexample is that Pritchard’s distance-based modal account sometimes gets the chanciness condition wrong. A small change in a nearby world could cause an event to fail to occur but if that change, itself, is extremely unlikely, the occurrence of the event in the actual world will—contrary to the distance view—not involve a high degree of luck. Thus, the relationship between luck and possible worlds cannot be captured via distance alone.
Carter and Peterson (2017) argue for a density-based account of luck that contains two novel conditions: modal weight and density. Carter and Peterson’s explanation of modal weight is as follows:

If an event is lucky, then it is an event that occurs in the actual world but whose modal weighted likelihood is above some appropriate threshold. The term “modal weighted likelihood” refers to a measure that considers which worlds E occurs in and the distance of those worlds from the actual world, such that (i) the weight assigned to the occurrence of E in a world decreases as the distance from the actual world increases, and (ii) an event is less lucky the more worlds it occurs in, everything else being equal. (2017, p. 2181, emphasis original)

As previously argued in response to the proportional view, condition (ii) is problematic since it invokes the notion of there being “more worlds”, yet there are an infinite number of possible worlds, and within an infinite set words like ‘more’ and ‘most’ have no well-defined mathematical meaning. To solve this problem, Carter and Peterson appeal to the concept of mathematical density. They argue that the fact that both E and not-E occur in an infinite number of worlds does not entail that E and not-E are equally lucky. This is because the density of E worlds and not-E worlds may differ. Carter and Peterson argue this point via an analogy from mathematics:

We know that the countably infinite set of positive integers is no larger than the countably infinite set of perfect squares. Despite this, the perfect squares become increasingly scarcer as we move upwards from 1 towards infinity. There is, therefore, a sense in which one is luckier if one by random happens to pick a perfect square than a non-perfect square. Mathematicians use the notion of density for articulating the observation that the perfect squares are scarcer than the positive integers. For our present purposes an intuitive understanding of density will be sufficient. (2017, p. 2181)

With this intuitive understanding of density in mind, Carter and Peterson’s definition of luck is as follows:

Let \( d(E, x) \) denote the density of E at distance x from the actual world. For each distance x from the actual world, \( d(E, x) \) assigns a value \([0,1]\) that represents the

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10 Ian Church’s (2010) modal account is similar to the proportional view but involves the notion of modal weight. Perhaps a charitable reading of Pritchard also involves something similar to modal weighted likelihood. However, such a reading pushes his view very close to Carter and Peterson’s.
density of E-worlds at distance x from the actual world, such that d(E, x) = 0 if and only if E occurs in no world at distance x, and d(E, x) = 1 if and only if E occurs in all worlds at distance x. Let w(x) denote the weight assigned to events that occur at distance x from the actual world. It is plausible to assume that w(x) approaches 0 as x approaches ∞ and that w(x) approaches 1 as x approaches 0. The expected modal likelihood ml of E can then be defined as follows:

$$ml(E) = \int_{x=0}^{\infty} w(x) \cdot (1 - d(E, x))$$

(2017, p. 2181)

This account has a number of advantages. For one, it can make sense of the fact that luck admits of degrees. According to the above formula, if an event E occurs in the actual world but no other worlds, it will be maximally lucky, and if E occurs in all possible worlds it will be non-lucky. Furthermore, the above account allows for lucky events to lie on a continuum based on how they are distributed amongst all possible worlds, and this view could be supplemented by a threshold T such that E counts as lucky if and only if ml(E) > T (2017, p. 2182).

The density view also gives us the right result in Powerball cases. The worlds in which one loses the lottery are much denser than the worlds in which one is a winner. This way of explicating the relationship between a lucky event and possible worlds can make sense of the fact that one only wins the Powerball if one’s own, specific numbers are drawn, whereas there are a prodigious number of lottery combinations in which one loses. As such, were one to select from a class of worlds at random it is tremendously unlikely that in this picked world one is a lottery winner. Thus, winning the lottery involves a great deal of luck, while losing the lottery is only slightly unlucky.

Despite these advantages, Carter and Peterson’s density view cannot explain why cases such as Jennifer Lackey’s (2008) *Buried treasure* are lucky. Lackey’s example is as follows:

Sophie, knowing that she had very little time left to live, wanted to bury a chest filled with all of her earthly treasures on the island she inhabited. As she walked around trying to determine the best site for proper burial, her central criteria were, first, that a suitable location must be on the northwest corner of the island—where she had spent many of her fondest moments in life—and, second, that it had to be a spot where rose bushes could flourish—since these were her favourite flowers. As it happens, there was only one particular patch of land on the northwest corner of the island where the soil was rich enough for roses to thrive. Sophie, being
excellent at detecting such soil, immediately located this patch of land and buried her treasure, along with seeds for future roses to bloom, in the one and only spot that fulfilled her two criteria. One month later, Vincent, a distant neighbor of Sophie’s, was driving in the northwest corner of the island—which was also his most beloved place to visit—and was looking for a place to plant a rose bush in memory of his mother who had died ten years earlier—since these were her favourite flowers. Being excellent at detecting the proper soil for rose bushes to thrive, he immediately located the same patch of land that Sophie had found one month earlier. As he began digging a hole for the bush, he was astonished to discover a buried treasure in the ground. (2008, p. 261)

Lackey’s argument from this case is as follows:

1. Vincent’s discovery is paradigmatically lucky as he was not looking for and had no knowledge of the valuable treasure (2008, p. 262).
2. Although “circumstances just happen to fortuitously combine in such a way” so that Vincent finds the treasure, his discovery is not modally fragile—it is instead modally robust. (2008, p. 263, emphasis original). It occurs not only in the actual world but in all of the nearest possible worlds where the relevant initial conditions for the event are held fixed.

Therefore, modal fragility is not a necessary condition for an event’s being a matter of luck.

Carter and Peterson argue that their view can handle Buried treasure type cases because it takes into account not just nearby worlds but all possible worlds:

Vincent’s modal weighted likelihood of finding Sophie’s treasure was low. By acknowledging that events in all possible worlds count [we …] can explain why Vincent’s discovery of the buried treasure was a lucky event […] as] there are a vast number of worlds in which either Sophie or Vincent would not have been digging at all, or would not have visited the island, or would have been dead, and so on (2017, pp. 2182-2183, emphasis original)

Carter and Peterson argue—contrary to premise 2—that Vincent’s discovery of the treasure is modally fragile. When a proper sampling of all possible worlds is taken, Vincent’s discovery is scarce, and the worlds in which he does not discover the treasure are near enough to the actual world for the event to be considered very lucky.
I agree with Carter and Peterson that there are a “vast number of worlds” (in fact an infinite number of worlds) in which Vincent does not discover the treasure. However, consider the reasons that Carter and Peterson cite for this fact, that is, that Vincent may never have visited the island, may never have attempted to plant the rose bushes, or that he might be dead. It strikes me as highly implausible that these are the reasons that explain why Vincent’s discovery is lucky. This is because it is trivially true that Vincent does not find the treasure in a world in which he does not exist or never encounters the treasure. That is, it is not a matter of luck that Vincent fails to find the treasure in such a world; it is logically certain that this will be the case. As such, it is a mistake to compare worlds in which Vincent does not exist to the worlds where he finds the treasure. Doing so tells us nothing about whether the targeted event is a matter of luck.

Furthermore, suppose that Carter and Peterson are correct, and Vincent is very lucky for the reasons that they cite. If this is true, then nearly any non-necessary event will also be very lucky. This is because most events would fail to obtain for similar reasons, for example if a key agent died before the event or was prevented from acting. But this proliferation of very lucky events is absurd. Perhaps all events are somewhat lucky in this sense. For example, it may be a matter of luck that I exist at all, call this event E*, but if it were not for E* then I could not have performed some later action E. However, even if we think that some of the luck from E* transfers to E, it seems implausible that E* and E are necessarily, equally lucky.\(^1\) As Coffman (2009, p. 503) argues, there is a difference between being positioned to do something and once positioned being able to perform that action. From the fact that I am very lucky to have been born, it does not follow that I am also very lucky to be able to raise my right arm when I wish to answer a question.

\(^1\) Coffman (2009) takes this objection a step further and argues that it is false that the luck from E* transfers over or infects E. Peels (2017, p. 206) also makes a similar point. Coffman’s (2015) view, however, holds that luck can transfer in this way when E* is the primary contributor to E.
Raising my right arm is something that I do intentionally and have control over, it is not a matter of chance. But Carter and Peterson’s view entails that nearly any event will be very lucky due to the fact that in a dense selection of worlds a relevant agent will not be positioned such that he or she will be able to carry out the targeted event. But this is an absurd rendering of our ordinary conception of luck. Furthermore, it would make the concept of little philosophical interest. If nearly every event is in this sense very lucky, then it is not modal luck that separates knowledge from justified, true belief as both our known and Gettiered beliefs will be very lucky.

4. An objection to possible modal accounts

Of course, even if I am correct that the proportional, distance, and density views are flawed, this does not show that there is not some other way of properly defining luck in modal terms. One obvious next step is to retain Carter and Peterson’s conditions of modal weighted likelihood and density so as to explain Powerball cases but to add to their account Pritchard’s restriction that we can only consider possible worlds in which the relevant initial conditions of the event are the same as in the actual world. However, the fact that any plausible version of the modal account must select a set of fixed initial conditions to meaningfully compare what actually is the case with alternative scenarios opens up the theory to a serious objection, that is, that the extent to which an event is a matter of luck will then depend on how one sets these initial conditions. But since it is an open question how these conditions are set, the modal account is vulnerable to a reference class problem similar to the reference class problem that is troublesome for frequentist accounts of probability. As such, the modal account does not actually give us an analysis of the sense of chance involved in lucky cases. One could know all of the modal facts and still be unable to determine

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12 I would like to thank an anonymous reviewer for the wording of this point.
whether an event in the actual world is a matter of luck. For the remainder of this paper, I fill in the details of the above argument.

Since I hold that the above reference class problem is a serious objection to any plausible version of the modal account, I first need to show why an initial conditions clause is necessary. For one, as shown in section 3, initial conditions are necessary in order to get the right verdict in many cases as there will often be a dense selection of possible worlds in which the relevant agent either does not exist or is not involved in the targeted event. However, what happens in these worlds, regardless of their distance to the actual world, will usually be irrelevant to the luckiness of the event in the actual world. For example, there may be a dense selection of nearby worlds in which I do not exist or there is no state-run lottery, but this does not affect the degree of luck involved in my lottery win in the actual world. Relatedly, an initial conditions clause is also necessary so that we can pick out across worlds the “particular kind of event that we want to assess for luckiness” (Pritchard 2014, p. 599). Consider, again, Lackey’s Buried treasure. In this example, we need some description of the event to meaningfully compare with alternative scenarios. Without such a description, we would not even be able to say whether this event occurs in other possible worlds.

However, the necessity of this initial conditions clause means that the luckiness of an event in the actual world, say Vincent’s discovery of the treasure, will depend on how the relevant initial conditions of the case are set. In order to see that this is the case, consider Pritchard’s point that in Lackey’s original presentation of Buried treasure important details concerning the initial conditions of the case are left under-described (2014, p. 610). In order for Lackey’s example to work, certain facts about the case have to remain fixed across worlds, that is, that this one treasure-shaped patch of land is the only place on the island where roses grow, that Sophie and Vincent’s
mother have identical flower and island preferences, and that the treasure is buried at a specific depth. A small change to any of these conditions and Vincent no longer finds the treasure. For example, Vincent does not find the treasure in a nearby world where the treasure is buried six feet underground as this is far deeper than he would dig to plant the rose bushes. Thus, contrary to premise 2, Vincent’s discovery is modally fragile, at least in Lackey’s original presentation of the case.

Pritchard then argues that for Lackey’s case to work as intended we need an example where the initial conditions for the event are described such that Vincent’s discovery is no longer modally fragile—call such a revised scenario *Stipulated buried treasure*. In *Stipulated buried treasure*, all of the circumstances that combine such that Vincent finds the treasure must, themselves, be robust or held constant across nearby worlds. For example, it would have to be stipulated in the case that only one precise spot on the island is suitable for planting both the rose bushes and the treasure. But Pritchard argues that in such a case we will all recognize that “the target event was bound to happen [and that …] Vincent is guaranteed to find the treasure” and that once it is clear that Vincent discovers the treasure in all nearby worlds we will no longer think that he is lucky, which is exactly what the modal account predicts (2014, p. 611).\(^\text{13}\)

Given this analysis, the extent to which Vincent’s discovery in the actual world is lucky depends on how the initial conditions of the case are set. If one holds, similarly to *Stipulated buried treasure*, that a relevant initial condition of the case is that Vincent plants a rose bush in a certain spot near where the treasure is already located, then his discovery is not a matter of luck as it is

\(^{13}\) I suspect that many will not have this intuition and hold instead that an event may be modally robust and lucky. For example, Steglich-Peterson (2010) discusses eight different variations of *Buried treasure* cases in regard to Vincent’s epistemic position and modal fragility and argues that lucky cases are restricted—not to cases where the Vincent’s discovery is modally fragile—but to cases in which Vincent is not in a position to know that he will discover the treasure.
modally robust. But if the description of the case is more akin to Pritchard’s reading of *Buried treasure*, then Vincent’s discovery is modally fragile and lucky. In this way, the modal account of luck is vulnerable to a reference class problem that is similar to the reference class problem that is troublesome for frequentist accounts of probability. Consider a case in which a particular person contracts a disease. If we take our reference class to be the entire population within a certain time-frame or region, then this event (that is, said person’s having a certain disease) may be probabilistically unlikely. However, if we take the relevant reference class to be people who are born with certain genes or who are exposed to the contagion, then the odds of this event’s occurrence vary wildly and may even be quite likely. The same can be said for the modal account of luck and how one fixes relevant initial conditions. Is one lucky, unlucky, or non-lucky to win the lottery, lose the lottery, find buried treasure, or be struck by lightning? It depends on how the relevant initial conditions of these events are set.

Because matters of luck can shift in this way, modal theorists owe us either some guidelines or preferably a principled, independent account of how to set initial conditions. Without such an account, the modal theory does not actually give us an analysis of the sense of chance involved in lucky cases. This is because we cannot say whether Vincent’s discovery is lucky or non-lucky until we pick a set of conditions to keep constant across worlds, and if there is no reason for selecting one set of initial conditions over another, then the modal account is no better than an intuitionist or purely subjectivist account of luck. But before considering how initial conditions could be set, a few notes of clarification are in order.
First, this problem is a metaphysical worry and not an epistemological objection. One could know all of the facts concerning all possible worlds (for example, the distance of all possible worlds to the actual world and on which worlds Vincent finds the treasure) and still not know anything about the luckiness of the event in the actual world. For even if Vincent does not find the treasure in a dense selection of nearby worlds, we also need to know if on these worlds the relevant initial conditions are the same as in the actual world. Thus, this problem is a metaphysical objection in that it is a matter of what the relevant initial conditions of Vincent’s discovery (or any event) actually are. It is a worry about what kind of property or properties make a condition in the actual world a relevant initial condition to be held constant across all possible worlds.

Second, Stipulated buried treasure and Buried treasure both pick out the same event in the actual world. The difference between the two cases is a matter of how the initial conditions of the event are described, and it is an open question which description should be preferred; both interpretations of the case are reasonable. Lackey obviously meant, similarly to Stipulated buried treasure, for Vincent’s discovery to be modally robust based on the details of the case. Levy (2011, pp. 20-21) also views the example in this way. He argues that given the initial conditions of the case Vincent is guaranteed to find the treasure in all or nearly all nearby worlds. He even compares the case to one in which Vincent’s discovery is over-determined by a nefarious neuroscientist with God-like powers. As such, he views the case as non-lucky, whereas Pritchard argues that the original description of the case is modally fragile and lucky. Which description is correct? Should the depth at which the treasure is buried, the conditions of the soil on the island, or Sophie’s flower preferences be kept constant? There are no prima facie answers to these questions.

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14 I would like to thank an anonymous reviewer for this objection and Thomas Mulligan for helping me with this response.
With these clarifications in mind, let us now consider the ways in which modal theorists have gone about setting initial conditions. Pritchard argues that only the salient features of the event should be held fixed so that we are able to pick out the relevant event in question across worlds without guaranteeing that the event obtains and that the best we can do is to pick out such features on a case-by-case basis. He acknowledges that this is highly theoretical and admits of vagueness, but he argues that luck, itself, is also vague and that we will usually be able to pick out the relevant initial features of an event (2014, p. 599). First, as shown in our analysis of *Buried treasure*, it is not at all obvious what the salient features of many events actually are. This is the very question that the modal theorist owes us an answer to. Second, Pritchard’s non-guaranteeing clause is odd. Some events or states of affairs—say, that a triangle has three sides or that a particular person is born with a genetic disease—will be guaranteed to occur in nearby worlds even given minimal conditions. Lastly, Pritchard’s response that luck is vague is unhelpful. A good theory of luck should be able to explain why Smith’s winning the Powerball is very lucky, why Donald’s loss is only a little unlucky, and it cannot be the case that Vincent’s discovery is both lucky and non-lucky in the same way. However, due to this reference class problem, Prichard’s account fails to give a non-vague analysis of the sense chance at play for any of these events.

Levy (2011) builds on Pritchard’s account by holding that initial conditions are contextually sensitive. According to Levy, “claims about […] luck […] exhibit the same kind of context sensitivity and consequent instability that claims about knowledge are held to exhibit by epistemological contextualists (e.g. DeRose 1999)” (2011, p. 34). Unfortunately, an appeal to context does not shed any light on how initial conditions should be set in *Buried treasure*. In order to see this, let us compare *Buried treasure* with DeRose’s (2002) *Bank Cases* here summarized by William Larkin:
In these cases a subject S claims to know that the bank is open on Saturday morning. This proposition is in fact true and S’s belief is based on the ‘quite solid grounds’ that S was at the bank two weeks ago and found that it was open until noon on Saturdays. In the first case—Low Bank—S and his wife are deciding whether to deposit their paychecks on Friday or wait until Saturday morning, “where no disaster will ensue if (they) waste a trip to the bank on Saturday only to find it closed” (170). In the second case—High Bank—“disaster, not just disappointment, would ensue if (they) waited until Saturday only to find (they) were too late” (170).

Intuitively, in Low Bank S knows that “the bank is open on Saturday mornings” (call this proposition P), whereas in High Bank S does not know that P. If this is right, then the knowledge attribution “S knows that P” can vary across contexts without a change in the subject’s epistemic relation to this proposition. But notice that this is because other relevant features are built-in to these two, different cases. For example, in Low Bank the stakes are low. No disaster will occur for S if he and his wife do not cash their paychecks on Friday night. The opposite is the case in High Bank and this is why we are willing to assert that S has knowledge in the first case but not the latter. However, the difference between Lackey’s original Buried treasure and Stipulated buried treasure is not a difference about contextual features or stakes. In both versions of Buried treasure, the event that occurs in the actual world—that is, Vincent’s discovery—is the same. Both sides agree, for example, that, in the actual world, Sophie’s favourite flowers are rose bushes and that rose bushes only grow on one spot on the island. Both sides could also agree on all of the modal facts about whether Vincent discovers the treasure on other possible worlds. The only difference between the two cases is about what possible worlds are thought of as suitable for comparison. In this way, Buried treasure is disanalogous with DeRose’s Bank Cases and, as such, it is unclear how an appeal to context could resolve a dispute about, for example, whether Sophie’s flower preferences should count as relevant initial conditions.

5. Conclusion
I have argued that extant modal accounts are subject to counterexample. Furthermore, I have argued that modal accounts need to incorporate an initial conditions clause. However, while this initial conditions clause is necessary, it poses a problem. That is, how one describes the initial conditions of an event changes the extent to which the event in question is a matter of luck, and it is an open question what the relevant initial conditions of an event are—appeals to modal distance, intuition, and context do not help resolve this issue. Perhaps there is some principled way of setting initial conditions that is immune to counterexample. I have not argued against this possibility. However, our analysis of luck and modality has shown that the modal account of luck is not as modal as it seems as the question of how initial conditions should be set cannot be answered via modal facts alone.
References


