# Getting Back in Shape:

# Persistence, Shape, and Relativity

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#### Abstract

In this paper, we will introduce a novel argument (the "Region Argument") that objects do not have frame-independent shapes in special relativity. The Region Argument lacks vulnerabilities present in David Chalmers' argument for that conclusion based on length contraction. We then examine how views on persistence interact with the Region Argument. We argue that this argument and standard four-dimensionalist assumptions entail that nothing in a relativistic world has any shape, not even stages or the regions occupied by them. We also argue that endurantists have viable ways of preserving shape despite the Region Argument. The upshot of these arguments is that contrary to conventional wisdom, considerations about shape in relativity support endurantism rather than four-dimensionalism. We conclude by examining the implications of our discussion for the debate over Edenic shapes, noting that endurantists have a satisfying response to skeptical arguments about Edenic shapes similar to the one they have against the Region Argument.

Keywords: shape, four-dimensionalism, endurantism, special relativity,

#### Edenic properties

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## 1 Introduction

Although it is often said that special relativity favors four-dimensionalist views about material objects over endurantist ones, in this paper we will present a new argument that strongly suggests the opposite conclusion.<sup>1</sup> According to four-dimensionalists, material objects are divided into momentary parts in time ("temporal parts" or "stages"), just as they are divided into pointy parts in space. Endurantists deny this, claiming that material objects are "wholly present" at each moment of their existence.<sup>2</sup> Our argument can be summarized in the following manner: if standard four-dimensionalist assumptions about how objects possess shapes are correct, nothing has a shape in Minkowski spacetime—not even stages or the regions occupied by them. Since one of the main arguments for four-dimensionalism relies on the assumption that stages have shapes, our argument undermines an important motivation for this view. Furthermore, we will point out that the endurantist can easily respond to the argument in similar ways to how they respond to the problem of temporary intrinsics. In short, special relativity better motivates endurantism, which flies in the face of conventional wisdom.

Our paper will progress as follows. First, in §2.1, we will consider Chalmers' argument from length contraction that objects lack frame-independent shapes in special relativity. There, we will point out that Chalmers' argument has no force against four-dimensionalists. Then, in §2.2, we introduce a more formidable argument for the conclusion that objects lack frame-independent shapes: the Region Argument. In §2.3, we conclude the discussion of the Region Argument by examining an objection based on the invariance of the spacetime interval in Minkowski spacetime, and argue that it is lacking. Next, in §3, we explore the implications of views of persistence for the Region Argument. We explain how four-dimensionalism and endurantism should be understood in a rela-

<sup>&</sup>lt;sup>1</sup>See Gilmore, Costa, and Calosi (2016, §3) and Hawley (2023, §7) for a list of arguments often taken to suggest that special relativity favors four-dimensionalism over endurantism.

<sup>&</sup>lt;sup>2</sup>For some work on the debate, see Lewis (1988), Haslanger (1989), Lewis (2001), Sider (2003), Van Inwagen (2006), Miller and Braddon-Mitchell (2007), and Brower (2010).

tivistic setting. We then argue that the Region Argument, when combined with standard four-dimensionalist assumptions about temporary intrinsics, entails that nothing has a shape. We also argue that endurantists can exploit ambiguities in the Region Argument in a way that makes its conclusion unproblematic—ambiguities that are not available to four-dimensionalists. The main upshot of this section is that realism about shapes strongly favors endurantism over four-dimensionalism. Finally, we conclude the paper by discussing the ramifications of our observations for the recent debate over "Edenic" shapes, arguing that skeptical arguments against them based on special relativity fail from an endurantist perspective.

## 2 Relativity and Shape

#### 2.1 Chalmers on Length Contraction

David Chalmers has influentially argued that objects do not have shapes in a frame-independent manner on the basis of relativistic length contraction (e.g., see Chalmers (2012) and Chalmers (2021)). For example, a box might be rectangular in a certain frame, and square in a frame that is uniformly moving with respect to the first one due to the box's contraction along the direction of motion.<sup>3</sup> As Eddon (2010, 610) puts it in a slightly different context, "objects don't have their spatial shapes and sizes *simpliciter*; they have them *relative to a reference frame*."

A great deal of effort has been put into responding to Chalmers' argument (e.g., see

<sup>&</sup>lt;sup>3</sup>Chalmers is ultimately interested in arguing that in a relativistic world, objects do not have "Edenic" shapes, roughly, the shape properties that objects appear to have in perception (e.g., the seemingly square shape of a book). See Cutter (2017, 2301) for a clear presentation of the argument. We will briefly discuss Edenic shapes in §4.

Cutter (2017), Epstein (2018), and Saad (2021)). But despite the attention this argument has received, surprisingly, no one seems to have noticed that it contains a major flaw when examined from the perspective of the persistence debate. This argument by Chalmers does not put any pressure on a four-dimensionalist to believe that shapes are frame-dependent. To see why, first note that for four-dimensionalists, material entities are composed of "stages" or "temporal parts". Consider a basketball that has lost air and is being inflated. Four-dimensionalists believe that at each moment of time during the inflation process, there exists a momentary basketball "stage": an object that has all the properties that the basketball has at that time, but does not exist for any longer than an instant. None of these basketball stages share any parts in common. Observing the change in the basketball as its being inflated can be compared to looking at a flip book. Just as the pages in a flip book containing different drawings of a basketball can give the illusion of an enduring basketball being inflated if flipped through quickly enough, so the rapid succession of basketball stages with different levels of inflation generates the perception of an enduring basketball being inflated over time.

Chalmers' argument would not be compelling to a four-dimensionalist because, in the context of special relativity, they could say that different frames pick out different stages of the same material object. Each of these stages has its own shape *simpliciter*. So, for example, the shape of the basketball at the end of its inflation period might be spherical in one frame and ellipsoidal in another, but these descriptions of the basketball correspond to different stages. As it will later be evident through the case of the exploding generator and the book in §2.2, the stages one frame associates with the basketball inhabit different spacetime regions than the stages another frame associates with the basketball: it is not

<sup>&</sup>lt;sup>4</sup>Not at the fundamental level, anyhow.

as though individual stages are transitioning from spherical to ellipsoidal as we switch between different frames. We will explain how four-dimensionalism works in relativity in much more detail in §3.2, but for now, we just want to flag that this argument does not do enough to persuade four-dimensionalists that no object has a frame-independent shape.<sup>5</sup>

In the next subsection, we will present a novel argument (the "Region Argument") from special relativity to the conclusion that objects do not have shapes in a frame-independent manner. This argument, unlike Chalmers', does not rely on length contraction (i.e., it does not rely on the idea that an object is, say, spherical in one frame and ellipsoidal in the other). Instead, the Region Argument is based on the fact that, in special relativity, whether or not spacetime regions have shapes depends on the frame.

### 2.2 The Region Argument

The Region Argument can be put in this way:

- P1. The only shapes instantiated by spacetime regions of Minkowski spacetime involve relations to frames of reference.
- P2. If the only shapes instantiated by spacetime regions of Minkowski spacetime involve relations to frames of reference, then the only shapes instantiated by material objects (or stages) involve relations to frames of reference.
- C. Therefore, the only shapes instantiated by material objects (or stages) involve relations to frames of reference.

<sup>&</sup>lt;sup>5</sup>This point has been missed by other scholars as well. For example, Eddon (2010, 610) says that "none of the paradigmatic ephemera [e.g., color, shape, size] are had *simpliciter*, regardless of one's preferred theory of persistence." We not only think that four-dimensionalist have natural ways of responding to Chalmers' argument, but as we will see in §3.5, we also believe that endurantists have many tools to do the same.

The second premise can be motivated by adopting the rather intuitive principle according to which the shape of a material object at time t is the same as the shape of the region the object occupies at t. For instance, note how strange would it be to say both that the book cover has a square shape at noon and that the region it occupies at noon is circular or has no shape. But if the shape of a material object at a time is the same as the shape of the region it occupies at that time, and if the region an object occupies at a time has no shape that does not involve relations to a frame, then neither does the object. Put differently, if the shape property of a region involves a relation to a frame, and this shape is the same as the shape of the material object occupying that region at a certain time, then it follows that the shape of the object involves a relation to a frame. We find this premise hard to resist, and even Cutter (2017, 2304-5), who criticizes Chalmers' argument, endorses principles that support it. As for the first premise, the bulk of this section focuses precisely on defending it.

The strategy for motivating that spacetime regions of Minkowski instantiate shapes in a way that involve relations to frames can be summarized like this. If frames disagree about whether a region of Minkowski spacetime has a shape, then the region only has shapes in relation to frames. This is because all frames are equally valid with respect to objective reality: no frame is privileged with respect to describing the real features of a region, including its shape. In the following discussion, we will show that regions of spacetime that have shapes in a certain frame do not have a shape in many other frames because they are not made up of simultaneous points in the latter frames. Thus, regions in Minkowski spacetime only have shapes in ways that involve relations to frames, which is what we need to secure (P1) of the Region Argument.

We find it instructive to start with a much simpler case, namely, that of a Newtonian

universe with absolute space and absolute time. Think, for example, of the absolute locations in a Newtonian universe that the border of a book cover occupies at the time the power generator of a certain building explodes, and imagine connecting those absolute locations with straight lines. If one were to do that, one would get, for all frames in a Newtonian universe, a region with the same shape as that of the book cover, say a rectangle of 8 inches by 12 inches (let's leave depth aside to simplify the discussion). In particular, in a Newtonian universe, a frame attached to the bookshelf where the book stands, and a frame attached to a train in uniform motion would both agree about (1) the absolute points that the cover occupies at the time in question (i.e., they agree about the region occupied by the book at that time) and (2) the distance between any two points. Given (1) and (2), it follows that in a Newtonian universe, these frames (and any other one) agree about the shape of the region occupied by the book at the time the generator explodes. In this kind of universe, then, it seems natural to assume that regions of absolute space have shapes in a way that does not involve relations to frames.<sup>6</sup>

Consider now the case of Minkowski spacetime. In contrast to a Newtonian universe, in Minkowski spacetime time and space are not absolute, in the sense that there is no longer a unique way of decomposing regions of spacetime into spatial regions (which are three-dimensional) and temporal regions (which are one-dimensional); different frames will split spacetime into time and space in different ways, in a manner analogous to how different coordinate axes rotated with respect to one another split differently the components for a vector in the Cartesian plane. In other words, whereas in Newtonian spacetime all frames agree about what regions of spacetime are such that all the points in them are associated

<sup>&</sup>lt;sup>6</sup>A very similar reasoning applies to the case of Galilean spacetime, but for reasons of space we will omit it here (see also Cutter (2017)).

with the very same time, in Minkowski spacetime different frames disagree about what regions have this property (the regions that do have this property are called "simultaneity hyper-surfaces"). This is, of course, nothing else but the breaking of absolute simultaneity that special relativity is known for. But what does the breaking of absolute simultaneity have to do with the claim that regions in Minkowski spacetime have shapes in a way that involves relations to frames? This is what we will answer now.

Consider, once again, the book cover. As we did in the Newtonian case, we can ask: what is the shape of the region occupied by the cover at the time the generator explodes? In contrast to the Newtonian case, in special relativity, different frames will pick out different regions as the region occupied by the cover at the time of the explosion. If a frame associated with the bookshelf says that the cover occupies a region  $R_b$  at the time of the explosion, the frame associated with a train will pick out a different region  $R_t$  as the one occupied by the cover when the explosion happens. This is because the time of the event referred to as "the explosion" is associated with different simultaneity hypersurfaces depending on the frame, and so the book cover occupies sub-regions of different hyper-surfaces depending on the frame; it occupies sub-region  $R_b$  of the hyper-surface  $H_b$ picked out by the bookshelf frame, and sub-region  $R_t$  of the hyper-surface  $H_t$  selected by the train frame. It is worth stressing that the disagreement between the frames is not simply a matter of the time at which the cover occupies  $R_b$  or  $R_t$  or any other region but goes deeper than that. In particular, there is no moment whatsoever in the life of the cover at which it occupies region  $R_b$  according to the frame of the train, and there is no moment in the life of the cover at which it occupies region  $R_t$  according to the bookshelf frame. The same is true for any other region occupied by the book at any time in its life according to a given frame F. In any other frame F' in motion with respect to F, none of the regions selected by F as the regions occupied by the book at some point in its life is a region occupied by the book at some point in its life according to F'. Put in other words, were we to make a list of all the regions the book (exactly) occupies during its existence according to F, and were we to do the same for F', we would find that the two lists have no regions in common.<sup>7</sup>

Not only do different frames disagree about what region the book occupies at the time of the explosion and at any other time in its life, but as we will argue now, they also disagree about whether the regions in question have shapes because they disagree about whether the regions in question are *spatial* (i.e., whether they belong to simultaneity hyper-surface). This is important because, remember, if regions do not have shapes in some frames and do have shapes in others, then they only have shapes in a way that involves frames of reference. And that is all we need to secure (P1). In the bookshelf frame, all the points in the region  $R_b$  are simultaneous, and the shape of that region is simply the same as the shape of the book cover (square, say). But in the train frame, region  $R_b$  consists of spacetime points many of which are associated with different times, and likewise for region  $R_t$  when seen from the perspective of the bookshelf frame. That is, in the train frame, many points in  $R_b$  are associated with successive instants of times. So in that frame, no object (let alone the book) could occupy all the points in that region at a given instant, although an object could occupy a (proper) subset of points in  $R_b$  at a certain time provided that the subset belongs to a simultaneity hyper-surfaces in this frame. For any such subset of  $R_b$ , it makes sense to talk of its shape according to the train

<sup>&</sup>lt;sup>7</sup>This is not to say that the frames disagree about what spacetime points are part of the worldtube of the book (the worldtube is analogous to the worldline for an extended object), but just to say that they disagree about how those spacetime points are arranged into regions occupied by the object at every instant of time. See Balashov (1999) for an expansive discussion of this point.

frame because it is fully spatial (i.e., it is made of "simultaneous points") in that frame; just think of the shape of an object that was to occupy them at some instant. But asking what the shape of  $R_b$  is does not make sense in the train frame, for it would amount to asking what is the shape of a spacetime region that has some points associated with the past, some with the present and some with the future (e.g., the past, present and future of the explosion). In other words, it amounts to asking what is the shape of entities (such as spacetime regions) composed of other entities that belong to different times. As we will show in more detail now, asking about the shapes of these kinds of entities is conceptually confused, analogous to asking what (spatial) shape World War II had or about the shape of an earthquake.

We will give several examples to motivate why we think spacetime regions composed by non-simultaneous points do not have shapes. From the point of view of a given frame, it certainly makes sense to talk of the shape of the region  $R_{50\%}$  occupied by the basketball when it is halfway inflated (say it is ellipsoidal), or the shape of the region  $R_{100\%}$  the basketball occupies when it is fully inflated (say it is spherical). We take no issue with these innocent claims. Instead, what we take issue with is talking about the shape of regions that have points that belong to different times (in a given frame), such as the region  $R_U$  that is defined as the union of  $R_{50\%}$  and  $R_{100\%}$ . Our examples will show that it isn't clear what we could even mean when talking about the shape of a region such as  $R_U$  in the frame in question. The right thing to say, we will argue through our examples, is that the union lacks any shape in that frame. And what goes for  $R_U$  goes for any region larger than a spacetime point, for in some frame or other some of its points are associated with different times.

Before we present them, we want to note here that our examples will discuss problems

associated with the shapes of some material objects first. This may seem like a strange choice, given that we are trying to defend (P1) of the Region Argument, a premise that is about the shapes of regions: if we start with material objects, why not just argue for (C) directly? We proceed this way for two reasons. First, material objects are considerably easier to think about than spacetime regions. So it facilitates intuition to begin with material things. Second, there are some crucial examples involving material objects which can be used to reach conclusions about the shapes of regions. As we will make clear soon, if regions are occupied by certain types of objects, there are particular shape facts that must be true of the regions themselves. This second point can be used to show that the inability to attribute a shape to certain regions of spacetime is not a mere byproduct of what happens to occupy them, but it is rather a consequence of some properties of the region itself (e.g., the property of having points belonging to different times in some frames). If we had tried to sidestep the discussion of the shapes for regions and argued for (C) directly, one could reasonably worry that the problems with the attribution of shapes to the material objects that we will consider are generated by the particular examples, and do not universally apply to other material objects such as stages (more on this below).

For the first example, consider how we usually describe the shape of the fusion of two or more objects. We usually describe it in terms of how the objects are connected or overlapping each other. For instance, the shape of a fusion of a sphere and an ellipsoid that overlap along the ellipsoid's minor axis is different than the shape of a fusion of a sphere and an ellipsoid that overlap along the ellipsoid's major axis (see figure 1). If we were instead to imagine the sphere and ellipsoid as being far away from each other (and so not overlapping), it is unclear how we would describe the shape of their fusion. We wouldn't even know whether to say the shape the fusion had when they were far apart is

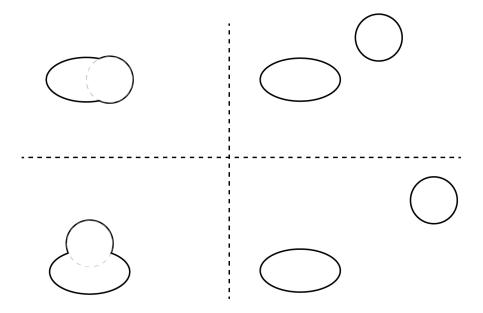


Figure 1: In the left column we find the resultant shape for the fusion of a circle and an ellipsis that are overlapping along the major axis of the ellipsis (top), and along the minor axis (bottom). Note that these are different shapes even though the two overlapping objects are the same. In the right column, we consider the case where the circle and the ellipsis are close (top) or far (bottom). In the absence of an overlap, it does not seem as if we had obtained a new shape for the fusion of the two objects in either case.

the same shape the fusion had when they were closer together (but still not overlapping). It seems more natural to say that in these cases the fusion has two parts, each with its own shape (spherical and ellipsoidal), but that the fusion does not itself have a shape properly speaking.

This problem is only compounded when we consider objects that do not exist simultaneously. Consider again the basketball occupying  $R_{50\%}$  and  $R_{100\%}$ . In the example from the previous paragraph, the spherical and ellipsoidal objects do not overlap, but they still stand in a spatial relationship to each other: they bear a certain distance relation at a given time. The basketball at  $R_{50\%}$  and the basketball at  $R_{100\%}$ , however, only have a

spatiotemporal distance from each other (in the frame in question, they are associated with different times). If it is hard to think about the shape of a fusion of non-overlapping objects when they all exist simultaneously, it is even more obscure how to think about the shape of a fusion of non-simultaneous objects (in this case they are not a spatial distance apart at any given time). It is more natural, in cases like these, to think that each object has a shape in the given frame (say ellipsoidal and spherical), but not their fusion.

A natural response to this case is to suggest that there is no such thing as the fusion of the basketball at  $R_{50\%}$  and the basketball at  $R_{100\%}$ . There's just, according to this response, a single basketball occupying both regions. Unless we assume that stages exist, there aren't two distinct objects here to fuse, there is just the basketball. Hence, it is just misleading, the response goes, to talk about the shape (or about the lack of shape) of the fusion because there is no such fusion to begin with.<sup>8</sup> Fortunately, we can set aside these kinds of difficulties by considering the situation if we assume that rather than there being a single basketball undergoing inflation, there exists a halfway inflated, ellipsoidal basketball at  $R_{50\%}$  and a different basketball at  $R_{100\%}$  that is fully inflated and spherical. We can suppose that both of these basketballs exist only for an instant, ensuring that they never co-exist.<sup>9</sup> Here we can certainly talk about two different objects that possess only a spatiotemporal distance from each other in the given frame, lacking a spatial distance (at a given time).<sup>10</sup> It is just as difficult to see how the fusion of these two objects could have

<sup>&</sup>lt;sup>8</sup>More precisely, there is no non-trivial fusion here. A basketball is, technically, the fusion of itself with itself.

<sup>&</sup>lt;sup>9</sup>Endurantists can admit momentary objects are possible, they just won't use them to analyze persistence.

<sup>&</sup>lt;sup>10</sup>One could propose that two different objects that do not co-exist in a certain frame are, in some sense, a spatial distance apart from one another. In particular, one might suggest that their spatial distance is simply the difference in their spatial coordinates in that frame (we disregard the difference in their time coordinate). But even though this can be done to define some sort of distance between non-coexisting things, it does not seem to be the right sort of distance that allows us to define a shape for the fusion of

a shape as it was to see how the fusion of the one basketball at distinct times could have a shape. So this kind of response does not ultimately dissolve the problem.

Now consider what happens when we modify the case and "remove" the basketballs from  $R_{50\%}$  and  $R_{100\%}$ , making them empty regions. Each still retains its spatial shape:  $R_{50\%}$  is an ellipsoidal region, and  $R_{100\%}$  is a spherical one. Each is spatiotemporally related to each other, but not spatially related to each other at a given time:  $R_{50\%}$  and  $R_{100\%}$  do not exist simultaneously. Nothing seems to have changed in this new case that would allow us to attribute a shape to their fusion,  $R_U$ . Removing the occupants from  $R_{50\%}$  and  $R_{100\%}$  does not somehow make them belong to a common time, nor does it alter the shape of either. Instead, thinking about the two momentarily-existing basketballs, one occupying  $R_{50\%}$  and the other one occupying  $R_{100\%}$ , help us reveal the spatial and temporal facts about the regions themselves (in the given frame). And an object composed of something that is ellipsoidal and exists for just a moment and something that is spherical and exists only during some other moment (in the same frame) doesn't have a shape, no matter what kind of thing it is. Whether it is a material object or a spacetime region seems irrelevant: what matters is that the parts of the object are separated by time.

As a further example, consider the case of length, which can be thought of as a onedimensional shape. Say that a pencil is 3 inches long at noon, and then someone cuts a

these objects (the pencil example, to be discussed later, also illustrates this point). Also, notice that if the object is moving, then the "distance" in question will grow bigger and bigger, but it would be very strange to say that this motion is making the fusion of the no-longer existing parts and the current parts grow (or change its shape). And note that this use of the term "distance" would be rather odd, as we surely would not (normally) say, for instance, that we are five meters away from a long-gone T-Rex even if the difference in the spatial coordinates is five meters. Of course, the spatial coordinates of parts of objects associated with different times do allow us to say what would be the actual distance between the relevant parts if they were to co-exist, and with that information one might then be able to say what would be the shape of an object that were to have those parts arranged in that way at a given time. But we should not confuse this way of talking about possible distances between two things with an actual distance between those two things.

third of it at 12:01 pm. In this case, we can talk of the length at noon (3 inches), the length at 12:01 (2 inches), and the length at any other time in the pencil's life. But what is the length of the pencil at noon and 12:01? Is it the sum of the two lengths? Their difference? We think that there is no such thing as the length of the pencil at noon and at 12:01, just as there is no such thing as the shape of the basketball when partially inflated and when fully inflated.<sup>11</sup> The length of an object, just like the two or three-dimensional shape, is a matter of the simultaneous arrangement of the parts of the object, that is, a matter of the arrangement of the parts of the object at this or that instant of time. Likewise, the fusion of the regions occupied by the pencil at noon and the pencil at 12:01 do not seem to have a length. One of those regions is 3 inches long, and the other is 2 inches long. But, being separated in time, their fusion lacks a length at any given time (one might also add that for an entity with only one spatial dimension, for it to have a well-defined shape it must have a well-defined perimeter, which is precisely what the entity in question seems to lack).

It should be clear that the lesson of these examples is not restricted to the regions  $R_{50\%}$ ,  $R_{100\%}$ , nor to the regions occupied by the pencil at the various times. The examples could have been altered in numerous arbitrary ways. We could have chosen different shapes, more than two regions, etc., and still have drawn the same conclusions. To put the point abstractly, if you want to think about some of the properties a spacetime region  $\mathcal{R}$  has in a certain frame, consider how  $\mathcal{R}$  would look when parts of it are occupied by a series of material objects with the following characteristics: each object in the series exists only

<sup>&</sup>lt;sup>11</sup>One can easily modify this example in the way we modified the basketball case to avoid worries about there not being two different objects to fuse. For example, one can consider two pencils that exist only for a moment. One exists at noon and is 3 inches long, the other exists at 12:01 and is 2 inches long (and looks as though it had been cut).

for a moment in the frame, and for each subregion of  $\mathcal{R}$  composed of simultaneous points in that frame, one and only one object in the series exactly occupies that subregion. This will tell you the shapes of the subregions of  $\mathcal{R}$  in the frame—that is, the shapes of all the subregions that have shapes in the frame. Our earlier discussion suggests that  $\mathcal{R}$  can have a shape in the frame only if all of the points composing  $\mathcal{R}$  are simultaneous in the frame. Otherwise, if  $\mathcal{R}$  has some points that are associated with different times in this frame, then it would be composed of a series of "simultaneous subregions" associated with different times (i.e., they will belong to different simultaneity hyper-surfaces in the given frame). Many of these subregions might have a spatial shape (for example, one of them might be a continuous subregion with a cubical boundary), but as our examples above make it clear, fusions of entities belonging to different times don't have shapes even though many of their parts might. And since every non-point spacetime region is made up of non-simultaneous points in some frame, the end result is that every non-point spacetime region lacks a shape in some frame.

It might be illustrative to offer one last example, this time one that does not center on material objects occupying regions (this will show that, in principle, material objects can be eliminated from our argument for (P1)). To be concrete, and keep things simple, say that we are working on a spacetime of two dimensions (one dimension in time and one spatial dimension). Then, choose a frame and some units, picking an arbitrary point to serve as the origin. Once we do that, let's pick out a region composed of a set of continuous spacetime points such that, at time t = 2s in the frame, the closest point to the origin in the set is one inch from the origin of the frame and the farthest point from the origin in the set is three inches from the origin of the frame (see Figure 2). In this frame, this region is just a straight line consisting of simultaneous points that go from

the coordinate (1,2) to the coordinate (3,2), where the first element corresponds to space (here one inch and three inches from the origin, respectively), and the second one to time (two seconds). Now consider a second region, this time composed of a set of continuous spacetime points such that, at time  $t = 5 \,\mathrm{s}$ , the closest point to the origin in the set is two inches from the origin of the frame and the further point from the origin in the set is three inches from the origin of the frame. Using coordinates, this region is just a straight line consisting of simultaneous points that go from the coordinate (2,5) to the coordinate (3,5). So in both cases, it seems clear that the two regions in question have a well-defined length at the corresponding time: one has a length of 2 inches, and the other one a length of 1 inch. But it also seems clear that if we consider the union of these two regions, that resulting region would not have a well-defined length. Would the length be the sum of the two lengths, say 3 inches? Would it be the difference, say one inch? Would it be something else? Being separated in time, their fusion seems to lack a length, just as it was noted with the case of the regions occupied by the pencil (note that we could have also considered an analogous case to the basketball example but using only a frame to pick out the various regions). In this instance, however, notice that nothing is occupying the relevant regions, which suggests that the problem in assigning shapes in cases like this one is not a byproduct of problems with the shapes of material objects that change shapes over time. Rather, the problem is associated with the fact that these regions belong to different times in the given frame.

To conclude, we do not think there is such a thing as the shape of a material object across time or such a thing as the shape of a spacetime region with points that belong to different times in a given frame. Hence, we do not think that  $R_b$ , the region the book occupies in the bookshelf frame at the time of the explosion, has a shape in the train

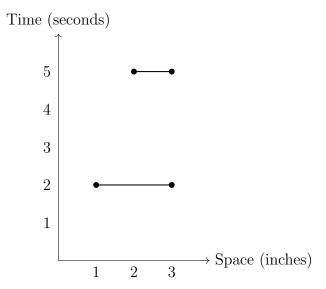


Figure 2: We use a frame of reference to pick out two different (empty) regions in a two dimensional spacetime, one composed of some spacetime points at a time of two seconds (the line from (1,2) to (3,2)), and the other one composed of some spacetime points at a time of five seconds (the line from (2,5) to (3,5)). Each of these regions has a well-defined length, but their fusion does not.

frame. But in the bookshelf frame,  $R_b$  certainly has a shape, namely, the shape of the book (say square). Thus, whether  $R_b$  has a shape depends on the frame, as we wanted to establish (this generalizes to any other spacetime regions, of course). This ends our argument for (P1).

Before considering an objection to the Region Argument, we want to remark on three points. First, even if someone pushed back on (P2)—perhaps by denying the principle that the shape of a region corresponds exactly to the shape of its occupant, which would be quite revisionary—they should still worry about (P1). Many have the intuition that shapes are necessarily possessed by things simpliciter, but regions of Minkowski spacetime do not possess them in this manner according to our arguments in this section. So, (P1) alone raises philosophical problems apart from (P2). Second, we pointed out in the

previous section that Chalmers' argument would not be compelling to a four-dimensionalist because the stages of a material object associated with each frame of reference are different. However, the discussion of the Region Argument makes clear that any stage associated with any frame of reference only has a frame-relative shape. Stages occupy regions, and those regions only have frame-relative shapes. Since any occupant of a region has exactly the same shape property as the region it occupies (as per the second premise), stages only have frame-relative shape properties as well. Thus, unlike Chalmers' argument, the four-dimensionalist cannot dismiss the Region Argument so easily (we will expand this point in §3.3). Third, and finally, the Region Argument is relevant even in the context of general relativity. After all, special relativity is locally valid in general relativity (see Linnemann, Read, and Teh (2023) for a recent discussion). At the scales we care about when talking of the shapes of ordinary objects such as books or trains or people or cats, general relativity reduces to special relativity, and the Region Argument kicks in.

### 2.3 The Interval Objection

Let us now consider a possible objection against (P1), that is, against the claim that the shapes of Minkowski spacetime regions involve relations to frames. The reader might be thinking that the spacetime interval (henceforth "Interval") allows us to define a shape for any spacetime region, regardless of the frame. How? Just compute the Interval, which is a frame-independent quantity, between all spacetime points in a region and use that information to construct a shape for that region. To be precise, there seem to be at least two main approaches for constructing shapes using the Interval. According to one, championed by Balashov (1999), the Interval allows us to construct (frame-independent)

4D-shapes for regions in Minkowski spacetime (see also Balashov (2014a)). According to the second approach, defended by Cutter (2017), the Interval is used to construct 3D (spatial) shapes of those regions that belong to simultaneity hyper-surfaces (see also Saad (2021)). In Cutter's words, "it is not a frame-relative matter what shape that [space-like] region has, at least in as much as the shape of a region is part of its objective physical structure (as described, e.g., by the Interval—an objective, frame-invariant magnitude)" (p. 2307).

As tempting as these approaches seem at first, we do not think that they help the defender of frame-independent spatial shapes. Consider, first, the idea that one can construct 4D-shapes using the Interval. There are two major problems with this approach. First, 4D-shapes in Minkowski spacetime will have sides extended in space combined with "sides" extended in time (after all, we are considering three spatial dimensions and one temporal one). But the notion of shape we are concerned about here cannot coherently be used in a way that combines space and time in this way. For example, using the notion of shape employed in the debate examined in this paper, it would be absurd to say that a region of spacetime has a shape made up of some seconds and other sides made up of some meters, just as it sounds absurd to say that my pencil is 3 seconds long and one centimeter wide. So we agree with Eddon (2010, 611-613) that in such a case, the term "shape" is simply being used in a non-standard way (when four-dimensionalists say that stages have shapes, for example, they clearly mean ordinary spatial shapes). <sup>12</sup> This last

<sup>&</sup>lt;sup>12</sup>Now, one might think that these spatio-temporal 4D-shapes are not that absurd given that many pictorial representations of spacetime regions do have shapes, or, similarly, given that the curves that we use to represent the worldline of objects in spacetime diagrams have well-defined lengths. But note that these pictorial representations, unlike the things they represent, are indeed fully *spatial*, so they surely have shapes and lengths in the ordinary sense of these terms. Our problem, rather, is with the idea that the things these figures represent have shapes in the same sense the figures doing the representation have.

point is related to our argument in the prior section that composites of objects at different times (or single objects extended in time) do not have shapes. Consider a worldsheet in a spacetime diagram. Whatever that sheet represents, whether a single rod moving with constant velocity for some time or successive distinct rods' spatial positions over time, it does not correspond to an object at a single time. Thus, for the reasons explained earlier, it would not make sense to say that there is such a thing as the length of whatever thing or things occupy the spacetime region represented by that sheet. And what goes for rods and spatially one-dimensional objects goes for objects with two or three spatial dimensions as well.

The second problem with the idea that one can construct 4D-shapes using the Inverval is that, in the general case, these 4D-shapes would be rather strange in other ways, as they might involve sides whose "length", as measured by the Interval, are positive and sides whose "lengths" are negative. This is a consequence of the fact that the Minkowski metric is not positive-definite, as standard metrics are (this is why it is often called a "pseudo-metric," after all). Indeed, the perimeter of some extended 4D-shapes of Minkowski spacetime might be zero or even negative precisely because the "lengths" might be negative. <sup>13</sup> But surely, the kinds of shapes that we ascribe to ordinary material objects never have a negative perimeter or a zero perimeter, just like they never have sides of temporal extension!

In short, then, even if one could somehow use the Interval to construct 4D-shapes for regions of spacetime, and even if those shapes were to be frame-independent, <sup>14</sup> they would

<sup>&</sup>lt;sup>13</sup>This can be understood as a consequence of the fact that the Interval does not satisfy the Triangle Inequality. See Saad (2021, 485) for a discussion of this point.

<sup>&</sup>lt;sup>14</sup>Notice that even if the "lengths" produced by the Interval are frame-independent, the *arrangement* of such "lengths" might still depend on the frame.

not have the right kinds of formal properties to play the role that shapes in the ordinary sense are supposed to play. That is, they would not be the same kind of shape as the shape that we can see in a picture of an object, or observe directly with our eyes, or the shapes that we learn about when studying Euclidean geometry, or the shape that temporal parts are said to instantiate, or the shape whose change in time led to the problem of temporary intrinsics, and so on. And as long as objects or regions of spacetime do not have this "ordinary" kind of shape in a way that does not involve relations to frames, the Region Argument stands, as this argument is ultimately about this kind of shape.

What about the approach that uses the Interval to construct 3D (spatial) shapes only for those regions that belong to simultaneity hyper-surfaces? Interestingly, all the issues afflicting the "4D-shapes approach" are avoided if the spacetime region in question is such that all of its points belong to a simultaneity hyper-surface. In particular, all the regions within a simultaneity hyper-surface are fully spatial, with no temporal extension whatsoever and only positive lengths and positive perimeters. Therefore, restricted to this kind of hyper-surface, one can indeed use the Interval to produce shapes as typically understood—the Interval recovers the standard Euclidean metric in this case. But while it may seem that this approach is able to resist (P1) in the Region Argument, it does not. For whether a space-like hyper-surface is a simultaneity hyper-surface is itself a frame-dependent matter. So, if the goal was to avoid having shape depend on frames of reference, we have not accomplished it. We have just moved the appeal to frames of reference "under the hood" since frames appear in the analysis of what it is to be a simultaneity hyper-surface.<sup>15</sup> This is not to say that whether H is a space-like hyper-

 $<sup>^{15}</sup>$ In slightly more formal terms, one can define simultaneity hyper-planes in terms of the Minkowski metric and the time-like field associated with an observer ("the frame"). Whether a hyper-plane H is simultaneous or not will depend on the orthogonality between the tangent vectors of the time-like field

surface itself depends on the frame. This is just to say that whether H is a space-like hyper-surface all the points of which are simultaneous is a frame-relative manner. Indeed, if H is a simultaneity slice in frame F, then, for any other frame in motion with respect to F, H is not a simultaneity slice (in that case, it is just a space-like hyper-surface with some spacetime points associated to different times), and so in those frames H will not have a shape, as argued in the prior section (at most it has a (spatiotemporal) 4D-shape in those frames). Contrary to what Cutter (2017, 2307) says, then, this approach still entails that regions have shapes in a frame relative manner. <sup>16</sup>

## 3 Persistence and the Region Argument

We have seen how shapes depend on reference frames in special relativity. We now turn to the question of what that means for shape's metaphysical status. We will argue that if the typical four-dimensionalist attitude toward shapes is correct, then the Region Argument entails shape nihilism: the position that nothing has shape. But if endurantist inclinations regarding shape are correct, there is no more difficulty in finding room for shapes in special relativity than there is in classical physics. Endurantists, we will show, can respect special relativity while either denying a premise of the Region Argument or defanging the Region

associated with the observer (at a point) and the tangent vectors in H. But this still means that whether H is a simultaneity hyper-plane or not depends on the frame, as it depends on what time-like vector field one considers (e.g., the one associated with a train, or the one associated with the bookshelf).

<sup>&</sup>lt;sup>16</sup>Saad (2021) seems to believe that it is possible to define shapes in space-like hyper-surfaces in general, regardless of the frame (2021, 486). According to Saad, a spacetime region is spatial if all the points in it satisfy the triangle inequality (computed via the Interval), and this is true of any region in a space-like hyper-plane whatsoever, regardless of the frame. However, we do not believe that satisfying the triangle inequality is sufficient for the points in a region to be counted as spatial. As we have argued in this section and the previous one, a necessary condition for a region to be spatial is that it belongs to a simultaneity hyper-plane, and this, unlike being a space-like hyper-surface, is a frame-relative matter (i.e., the very same space-like hyperplane will be counted as simultaneous in one frame and as non-simultaneous in another one).

Argument by exploiting certain ambiguities in it.

The plan for this section goes as follows. First, we begin with a brief description of theories of persistence in a classical setting, and then in a relativistic context. We then turn toward the "problem of temporary intrinsics", focusing on how four-dimensionalists and endurantists think about change in shape. Next, we examine the implications of the Region Argument for shape given four-dimensionalism. We conclude by examining how the Region Argument interacts with endurantism.

#### 3.1 Four-Dimensionalism and Endurantism

We already mentioned in §2.1 that according to four-dimensionalists, material reality is made up of momentary "stages" or "temporal parts". These stages compose 4D "worms" associated with material objects, though as we will see, four-dimensionalists disagree about the nature of that association. Endurantists take the opposite view and are defined by their rejection of stages. For endurantists, material objects are "wholly present" at each time at which they exist. For example, think again of the inflating basketball. Contrary to the stage theorist, the endurantist says the entire basketball exists at each moment during the inflation. Endurantists simply say that the basketball possesses different shapes at different times.

Both four-dimensionalism and endurantism require some care to be formulated relativistically, since in that context, they must account for frames of reference.<sup>17</sup> We start with relativistic four-dimensionalism, and once again use the basketball as our exemplar. As we observed in §2.1, different frames select different spacetime regions as simultaneity

<sup>&</sup>lt;sup>17</sup>For further information about what views of persistence look like in special relativity, see Gilmore (2014) and Balashov (2014b).

slices. Thus, the collection of stages that compose the 4D worm associated with the basketball is different according to different frames. For example, it might happen that in one frame, the stages of a given basketball that is fully inflated are spherical from the time the basketball is created until it is destroyed. In another frame in motion with respect to the first one, it might be that all the stages of the same basketball are ellipsoidal. However, the stages associated with the basketball that exist in different frames do have a special relationship. For any two frames  $f_1$  and  $f_2$ , W is the 4-dimensional worm composed by the stages associated with the basketball in  $f_1$  if and only if W is the 4-dimensional worm composed by the stages associated with the basketball in  $f_2$ . That is to say, all frames agree over what 4D worm the stages associated with the basketball compose (for a detailed discussion, see Balashov (1999) and Balashov (2014a)). To use a previous analogy, each frame is associated with its own, unique flip book for the basketball, but all flip books end up telling the story of the same 4D object.

Four-dimensionalists are divided into worm theorists and stage theorists. (See Sider (2003) Ch. 3 for an overview of both.) Worm theorists, also known as perdurantists, identify material objects with their associated worms. By contrast, stage theorists say that material objects are stages, with talk of the past or future of a given material object being analyzed in terms of past and future "counterpart" stages that count as material objects in their own right. In the context of relativity, perdurantists will say that our basketball is the 4D worm that all the frames agree about, with the stages particular to frames reflecting different ways of dividing the worm into momentary parts. Stage theorists will say that the basketball stages are basketballs themselves, linked together by counterpart relations that connect them across both times and reference frames. Spelling this out fully would require some care. For our purposes, however, it doesn't matter

whether a four-dimensionalist favors perdurantism or stage theory. Our arguments will undermine a purported benefit of *all* versions of four-dimensionalism, so we do not need to get into a detailed comparison. We prefer the term 'stage' to 'temporal part' in order to indicate our indifference between these views: 'temporal part' is suggestive of worm theory, but 'stage' is neutral (even if more commonly used by stage theorists).

Endurantists thinking about special relativity will not respond to the disagreement among frames over simultaneity slices by saying that there are different stages in different frames. After all, endurantists do not believe in stages. Instead, endurantists will say that what properties the basketball has varies not only by time, but also by frame of reference. Whether the basketball is 80% inflated depends not only on what time is specified but also on what frame of reference is specified as well (or put differently, it depends on what time in what frame). Material objects are "wholly located" at times in frames, rather than being "wholly located" at times alone. Still, all frames will agree about which 4D region is associated with the life or history of the enduring basketball—they just disagree about how that region is divided into times (i.e., they disagree about which points in that 4D region belong to simultaneity hyper-surfaces).<sup>18</sup>

It's natural to wonder why anyone would be a four-dimensionalist. Endurantism seems to capture our commonsense way thinking about material objects just fine. Why populate the universe with stages? While many arguments have been offered on behalf of four-dimensionalism, one in particular is relevant to us. It is said that only four-dimensionalism can solve the problem of intrinsic change, also known as the "problem of temporary intrinsics". In what follows, we will explore this argument in the wider context of the Region

<sup>&</sup>lt;sup>18</sup>Gilmore (2006) tends to talk about location in more absolute terms. We do not deny that the basic notion of occupation does not refer to frames, but the frame-relative notion (e.g., the notion appealed to when physicists say that a train is located inside a tunnel in the tunnel's frame) will have to be recovered.

Argument.

#### 3.2 Four-Dimensionalism and Shape

The problem of temporary intrinsics can be put as follows (we present it first in a classical way, then note how special relativity complicates matters.). Consider our basketball. When it is 100% inflated, it has the shape of a sphere. When it is 80% inflated, it has the shape of an ellipsoid (we may suppose). And yet, being spherical and being ellipsoidal are supposed to be contrary, intrinsic properties. How can one and the same object have two intrinsic properties that are incompatible with each other? If one points out that the basketball has those shapes at different times, this doesn't seem to help. For the worry is that this relational element, this indispensable reference to times, undermines the intrinsicality of the shapes. As it goes with shape, so it goes with intrinsic properties generally: size, density, and so on.

Four-dimensionalists have a simple answer to the previous question. When the basketball is spherical, there exists a basketball stage that is spherical. When the basketball is ellipsoidal, there exists a basketball stage that is ellipsoidal. These stages never possess any other shape. The worm theorist will then say that talk of the varying shapes of the basketball over the period of inflation is to be analyzed in terms of the shapes possessed by the basketball's temporal parts. The stage theorist will say that same talk is to be analyzed in terms of counterparts of the basketball stage we are looking at when we make the comment. But all four-dimensionalists agree that at rock bottom, shapes are possessed by objects simpliciter. Any reference to the shape of a malleable object at a time is analyzed in terms of the unchanging shapes of stages. In general, all attributions of

<sup>&</sup>lt;sup>19</sup>See Lewis (2001, 203-5) and Sider (2003, Ch. 4, §6.) for standard presentations.

intrinsic properties to ordinary objects are ultimately analyzable in terms of momentary things that possess those properties *simpliciter*.

Now suppose that in addition to our first basketball, a second basketball came into existence at a later time. We can suppose that this second basketball undergoes a perfectly parallel inflation to the first. Video footage of the two during their inflation would look exactly alike, instruments would yield all the same measurements, and so on. The four-dimensionalist will say that the stage of the first basketball lying at the beginning of the first's inflation has the exact same shape as the stage of the second basketball lying at the beginning of the second's inflation. Likewise, the two stages lying at the end of their respective basketball's inflation have the same shape. And in general, if a stage of the first basketball and a stage of the second basketball are both inflated to a certain percentage, the two have the exact same shape. This might seem obvious, but it must be pointed out: as we will see, some endurantists reject this claim. Regardless, while the example is specific to shape, four-dimensionalists would extend this temporal repeatability to intrinsic properties generally.<sup>20</sup>

Now to take special relativity into account. In relativity, the shape of that basketball will vary not only by time, but by frame of reference. To maintain their belief that shape is intrinsic and possessed *simpliciter*, four-dimensionalists say that the basketball has a stage at a time in a frame that is spherical *simpliciter*. It is no longer a stage at a time alone, but a stage at a time in a frame. Four-dimensionalists also think that shapes can be repeated across both times *and* frames. Suppose the video footage of the second basketball was taken not only at a different time, but in a different frame: the camera is

<sup>&</sup>lt;sup>20</sup>At least for qualitative intrinsic properties. There's nothing prohibiting four-dimensionalists from believing that stages have haecceities, but these sorts of exceptions have no relevance for our purposes.

set in uniform motion after the first video was taken. We can suppose that the second basketball is in motion as well but at rest with respect to the now-moving camera (i.e., the second basketball is at rest in the frame of the moving camera). Four-dimensionalists would still say that the beginning stage of the first basketball has the same shape as the beginning stage of the second basketball, provided that the inflation process recorded in the first frame parallels the inflation process recorded in the second frame.

We have already noted in §2.2 that there is a correspondence between the shapes of regions and the shapes of material objects occupying the regions. For four-dimensionalists, this correspondence also holds between regions and stages. So, given the four-dimensionalist paradigm, a persisting material object having a shape at a time in a frame can be reduced to it having a stage with that shape that exactly occupies a region all of the points of which exist at that time in that frame (i.e., the region belongs to a simultaneity hyper-surface in that frame). For the four-dimensionalist, the stage and the region it occupies have the same shape *simpliciter*.

These examples reveal that four-dimensionalists have several commitments. To help articulate them more precisely, it's useful to have a few new terms. Let us say that a spatiotemporal object is anything that is either a persisting material object, a spacetime region, or a stage. Let us say that  $F_0$  is an apparent intrinsic property just in case we would ordinarily consider  $F_0$  to be an intrinsic property, and let us say that  $F_1$  and  $F_2$  are apparently contrary just in case we would ordinarily consider  $F_1$  and  $F_2$  to be contrary. These notions admit of some vagueness, but they are workable: 'would ordinarily consider  $v_1, ..., v_n$  to be  $\Phi$ ' means something like 'would believe  $v_1, ..., v_n$  to be  $\Phi$  in the absence of exposure to philosophical or scientific arguments to the contrary'. With these concepts in hand, we can see how our examples illustrate the following four-dimensionalist theses:

Intrinsicality: Apparent intrinsic properties of material objects are intrinsic properties of spatiotemporal objects.<sup>21</sup>

**Conflict:** Apparently contrary intrinsic properties of material objects are contrary properties (i.e., cannot be possessed by a single thing *simpliciter* or at the same time in the same frame).

**Reproduction:** Intrinsic properties of material objects can be possessed by spatiotemporal objects at distinct times in distinct frames.

**Reduction:** For any spatiotemporal object o, intrinsic property of material objects F, time t, and frame f such that o instantiates F at t in f, o's instantiation of F at t in f can be reduced to some object that exists at t in f instantiating F simpliciter (e.g., o's stage at t in f instantiating F simpliciter).

Shapes are apparent intrinsic properties. Distinct determinate shapes are apparently contrary. So, the theses entail the four-dimensionalists' distinctive commitments about shape. In particular, they entail that shapes are intrinsic, can genuinely conflict with each other, are shared by distinct entities across different times and reference frames, and, in the final analysis, are possessed *simpliciter*.

### 3.3 Four-Dimensionalism and the Region Argument

So far, so good. But there is a problem for four-dimensionalism. When combined with the conclusion of the Region Argument, these theses entail that nothing has any shape. In

 $<sup>^{21}</sup>$ Note that not every apparent intrinsic property of material objects is an intrinsic property of every kind of spatiotemporal object: regions do not have mass, unless supersubstantivalism is true. The point is that if a spatiotemporal object has an apparent intrinsic property of material objects, it is intrinsic. For instance, shape is intrinsic for both stages and regions.

particular, three of the four theses entail the following conditional: if any spatiotemporal object has a shape at a time in a frame of reference, then some stage of that object has a shape that does not involve a relation to frames of reference.<sup>22</sup> The conclusion of the Region Argument, (C), entails that there is no such stage. By a simple *modus tollens* inference, it follows that no spatiotemporal object has a shape at any time in any frame of reference.

Here's how three of the theses entail the conditional. Intrinsicality entails that shapes are intrinsic properties of material objects. That fact and Reduction entail that any spatiotemporal object's having a shape at a time in a frame is reducible to some object existing at that time in that frame having that shape simpliciter (e.g., the stage of the object at some time in some frame). Now, by itself, that does not ensure shapes do not involve relations to frame of reference. It could be that the shapes are possessed simpliciter, but that frames are essential constituents of shape properties (e.g., a stage might instantiate being-square-in- $f_1$  simpliciter). However, Reproduction excludes that possibility. For example, it follows from Reproduction that there can be two different stages associated with two different times in two different frames that both have the very same shape property, say they are both cubical. Hence, Intrinsicality, Reproduction, and Reduction jointly entail that if any spatiotemporal object has a shape at a time in a frame of reference, then some stage has a shape that does not involve a relation to frames of reference (i.e., some stage has a shape simpliciter). Thus, if we combine this conclusion

 $<sup>^{22}</sup>$ Spacetime regions can be described as comprising stages. A stage of a region R (in a frame) is a subregion of R such that, for some point p in R, the subregion is composed of all and only the points in R that are simultaneous (in the frame) with p. Likewise, we can speak of a spacetime region having a shape at a time in a frame: for instance, there can be a square region at time t in a frame. So, if the conditional in question is true, any spacetime region that has a shape at a time in a frame has a subregion (a stage) with a shape that does not involve a relation to frames of reference. (P1) of the Region Argument, of course, states that there is no subregion with such a shape.

with (C), it then follows that no spatiotemporal object has a shape at any time in any frame of reference.

If they want to avoid shape nihilism, four-dimensionalists should find some way to resist (P1). They should look for some flaw in our rationale for it when presenting the Region Argument. But this is no easy task for four-dimensionalists. We will soon present various ways to resist (P1), but all of them require denying one of Intrinsicality, Conflict, Reproduction, or Reduction. It is unclear what grounds one could use to reject (P1) without compromising those four-dimensionalist principles or introducing problems equally as pressing as the Region Argument itself.

### 3.4 (Classical) Endurantism and Shape

Endurantists respond to the problem of temporary intrinsics in a variety of ways. As we will discuss in this section, in a relativistic context, all of them deny at least one of the four principles to which four-dimensionalists are committed (we do not intend to take any stance on which endurantist option is most plausible). Our ultimate goal is to study how these different endurantist views bear on the Region Argument of §2.2.

To begin, we start by considering endurantist options in a classical setting. This will allow us to understand how these views will look when relativity is taken into account.

Some endurantists maintain that apparently intrinsic properties are not intrinsic properties at all. Instead, they are relations to times (e.g., see Gilmore (2006, 227-8, fn. 7)). There is no such property as the property of being spherical, nor is there the property of being ellipsoidal; there are only the relations of being spherical-at and being ellipsoidal-at. Every apparently intrinsic property is analyzed as a dyadic relation. The problem of

temporary intrinsics is avoided because the supposedly intrinsic changes are not intrinsic changes after all.

Other endurantists deny that an indispensable reference to time in a property's attribution undermines its intrinsicality. These endurantists often hold one of two views, which we will label triadic instantiation and dyadic truth for convenience.<sup>23</sup>

Advocates of triadic instantiation maintain that the possession of a property is a triadic affair. Rather than there being a simple two-place relation between an object and a property, there is a three-place relation between an object, a property, and a time. (Lewis (1988, 65-6) introduces this option on behalf of the endurantist, and van Inwagen 2006 defends it.) Strictly speaking, there isn't really such a relation as having a property: there is only the relation of having a property at a time.<sup>24</sup> In the case of the first basketball, the shape of a sphere and the shape of an ellipsoid are both intrinsic, it's just that the "having at" relation is born by the basketball and each shape to different times. When both basketballs are considered, the segments in the history of each that are perfectly parallel to each other with respect to their degree of inflation share a common shape (say spherical). What varies are the objects (the two basketballs) and the times (when each basketball has the shape) that fill in the rest of the "having at" relation.

Advocates of dyadic truth say that propositions are not true or false *simpliciter*, but only true or false at certain times. (This is one of the options Haslanger (1989, 120-2) discusses under the adverbialist label.) The proposition that the basketball is spherical is

<sup>&</sup>lt;sup>23</sup>Both of these views, as well as several others, have been put under the umbrella of "adverbialism". We choose to be more specific and address them separately. Despite the common label, the differences are significant, and it is only by going into the details that we can later discuss how matters look when special relativity is on the table.

<sup>&</sup>lt;sup>24</sup>When it comes to temporary properties of material objects anyhow. Endurantists of this sort could posit a dyadic relation for permanent properties, or stick with a triadic relation in every case and say that permanent properties are instantiated at all times.

true at the time where it is 100% inflated and false at the time where it is 80% inflated. By replacing "true" with "true at", they keep instantiation a dyadic relation between an object and a property. Times are introduced as parameters for the evaluation of propositions rather than as relata of instantation.

There are several other endurantist views on temporary intrinsics that are worth mentioning in passing. Endurantists have proposed that apparently contrary properties are not necessarily contrary (e.g., see Miller and Braddon-Mitchell (2007)), have denied that intrinsic properties of material objects can be possessed at different times (Miller and Braddon-Mitchell (2007, 250) mention the view, citing van Inwagen 2006 as a defense), have argued that time indexes function like sentential operators (i.e., propositions are of the form "at t,  $\varphi$ "; this is the other option Haslanger (1989, 120-2) discusses under the adverbialist label), and have held that shapes are intrinsic properties of momentary property/substrata complexes rather than material objects (Brower (2010, 892-893)). To keep the discussion manageable, we will leave these particular endurantist views aside, but it is worth stressing that they could also be updated in light of relativity and put four-dimensionalist theses into question.  $^{25}$ 

So goes the endurantist options in a classical setting. Now for their relativistic variants, how they relate to the four-dimensionalist theses, and how they relate to the Region Argument.

<sup>&</sup>lt;sup>25</sup>For example, denying that the apparently contrary shapes an object possesses across times and frames are really contrary amounts to a rejection of Conflict, while denying that the intrinsic properties of material objects can be possessed at different times or frames amounts to a rejection of Reproduction.

#### 3.5 Endurantism and the Region Argument

Relativistic endurantist options can respond to the Region Argument by exploiting an ambiguity in the phrase "involve relations to frames of reference". They will admit that there is an element of frame-dependence in an object's instantiation of a property. In that minimal sense, the shapes instantiated by regions of Minkowski spacetime "involve relations to frames of reference." What they deny is that this frame-dependence in instantiation means that the instantiated property itself has a frame as a constituent. In other words, if "involve relations to frames of reference" is read in this minimal sense, they do not see any reason to believe that the frame-dependence evident in instantiation requires one to think that there is no such property as the property of being square simpliciter (i.e., they do not see any reason to believe that shape properties are of the form being square-in- $f_1$ . Though we speak here of properties, the same points apply to the view that shapes are relations). So, if "involve relations to frames of reference" is interpreted in this weak sense, the endurantist will say that the Region Argument is sound, but its conclusion is no more problematic than an object's shape being time-dependent in a pre-relativistic setting. They believe they already have the tools to explain time-dependence, and those tools extend straightforwardly to frame-dependence. And if "involve relations to frames of reference" were to be read in a stronger manner, as meaning that the only shape properties would be of the form, e.g., being square-in- $f_1$ , then the endurantist will say that (P1) is false. Either way, endurantist are unfazed by the Region Argument. To unpack these ideas, let's first consider endurantist views in the context of relativity.

In light of relativity, endurantists who said that apparent intrinsic properties of material objects are relations to times will now say they are relations to times and frames. In doing

so, they deny Intrinsicality. There is not the property of being spherical, but the relation of being spherical-at-in. Likewise with being ellipsoidal and the rest. This allows them to reject (P1) of the Region Argument if "involve relations to frames of reference" is interpreted in the more demanding sense. Shapes do not "involve" relations to frames in the sense of having specific frames as constituents—they are not properties like being circular-in- $f_1$ , being cubical-in- $f_2$ , etc. Rather, they are relations to times and frames. If the earlier endurantist view was satisfactory on the classical picture, it is hard to see why the updated relativistic view would not be satisfactory.

Endurantists who denied that an indispensable reference to time in a property's attribution undermines its intrinsicality will reject Reduction. They will now say that an indispensable reference to time and a frame in a property's attribution does not undermine its intrinsicality.<sup>26</sup> We can see this by seeing how triadic instantiation and dyadic truth will look once relativity is taken into account.

Triadic instantiation will be updated to quadratic instantiation. Instead of treating instantiation as a triadic relation between an object, a property, and a time, they can treat it as a quadratic relation between an object, a property, a time, and a frame. If the triadic analysis did not undermine the intrinsicality of a property, it is not clear why the quadratic analysis would. On such a view, there is a general property of being circular, it's just that it's only instantiated with respect to object, time, and frame triples.

Dyadic truth will become triadic truth. Instead of "true at", we have "true at ... and ...", where the last slot gets filled in by frames of reference. There is still instantiation

<sup>&</sup>lt;sup>26</sup>This is essentially the solution proposed by Epstein (2018, §§3-4), who draws a parallel between "adverbialist" solutions to the problem of temporary intrinsics and how he would propose we take reference frames into account when attributing shapes. However, Epstein does not remark on the fact that this option is not available to orthodox four-dimensionalists, given their commitments on how intrinsic properties are possessed.

simpliciter, but o's instantiating F at t and f is a matter of the proposition that o is F being true at t and f. No special difficulty is encountered when modifying the view. Here again we have a general property of being circular, but a particular claim that an object is circular is true at time and frame pairs rather than true simpliciter.

Thus, it is clear that the endurantist, unlike the four-dimensionalist, has various ways of defusing the Region Argument, and also Chalmers' length contraction argument examined in §2.1. A four-dimensionalist could respond by embracing one of the endurantist views on how spatiotemporal objects relate to properties, which are not logically incompatible with the existence of stages. Doing so, however, would put serious pressure on the four-dimensionalist to motivate their view. Given that accounting for intrinsic change is one of the primary motivations for accepting four-dimensionalism, taking on board an endurantist account of how things possess apparently intrinsic properties undercuts much of the reason for adopting the view. As we noted in §3.1, endurantism seems to offer a more intuitive conception of persistence: unless there is a compelling argument on behalf of four-dimensionalism, endurantism is the more plausible position.

### 4 Conclusion

In this paper, we have accomplished two major goals. The first was to introduce a new argument that there are not frame-independent shapes in special relativity. Unlike Chalmers' argument based on length contraction, the Region Argument does not overlook four-dimensionalism. The second was to show that, surprisingly, the Region Argument supports endurantism. Endurantists are able to exploit ambiguities in the Region Argument to disarm its implications, but four-dimensionalists are not in a position to

do so without abandoning their distinctive commitments on temporary intrinsics. Thus, against conventional wisdom, a careful consideration of shape in special relativity supports endurantism rather than four-dimensionalism.

As a final remark, we want to briefly connect our discussion of the Region Argument to the vibrant literature on "Edenic" shapes, roughly, the shapes apparent in perception (see Chalmers (2006), Chalmers (2019), and Chalmers (2021); Cutter (2017), Epstein (2018), and Saad (2021) are also entries in this debate). Chalmers' argument from length contraction against frame-independent shape is supposed to show that the shapes we seem to perceive are not present in objective reality. For example, imagine that Bud and Lou are each looking at their own box. Imagine further that in Bud's frame, Bud's box is square and Lou's box is rectangular, while in Lou's frame, Lou's box is square and Bud's box is rectangular. Presumably, Bud and Lou are both having the same shape perception: each perceives his box as square. But if that is the case, Bud can't be perceiving his box as square-in-Bud's-frame. Lou's perception of his box seems as valid as Bud's perception of Bud's, and were Lou perceiving his box as square-in-Bud's-frame, his perception would be wrong (as his box is not square in Bud's frame). By parallel reasoning, Lou can't be perceiving his box as square-in-Lou's-frame. So, the argument goes, Edenic shapes do not involve relations to frames, for supposing they do wrongly leads to massive attribution of error in perception.<sup>27</sup>

Endurantists have many ways of avoiding Chalmers' conclusion. Opponents of Intrinsicality will say that Bud and Lou are witnessing instantiations of the same shape relation. Bud is witnessing the instantiation of a square shape in his own frame at a

<sup>&</sup>lt;sup>27</sup>See Cutter (2017, 2301) for a condensed discussion of this argument, and Chalmers (2012, Ch. 7, §5) for an expanded discussion, as well as for other arguments.

certain time, and so is Lou in his own frame (and time). So some of the relata of the particular instantiation events they're beholding are different, but some are the same (i.e., it is the same square shape but the frame and time are different). In other words, in this view, Edenic shapes are really Edenic relations rather than Edenic monadic properties. Fans of quadratic instantiation and triadic truth will agree that Bud and Lou are seeing squareness, not squareness-in-Bob's-frame and squareness-in-Lou's-frame. It's just that the boxes' possessions of squareness involve relations to times and frames. The squareness itself, however, does not have any frame as a constituent. Hence, various endurantist views that can resist the Region Argument can also resist Chalmers' argument according to which there are no Edenic shapes in special relativity. In future work, it would be worth exploring in more detail the connection between different views on the persistence of material objects in a relativistic world, and the Edenic shape literature.

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