Conversational Exculpature

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ABSTRACT: Conversational exculpature is a pragmatic process whereby information is subtracted from, rather than added to, what the speaker literally says. This pragmatic content subtraction explains why we can say “Rob is six feet tall” without implying that Rob is between 5’11.99” and 6’0.01” tall, and why we can say “Ellen has a hat like the one Sherlock Holmes always wears” without implying Holmes exists or has a hat. This paper presents a simple formalism for understanding this pragmatic mechanism, specifying how, in context, the result of such subtractions is determined. And it shows how the resulting theory of conversational exculpature accounts for a varied range of linguistic phenomena. A distinctive feature of the approach is the crucial role played by the question under discussion in determining the result of a given exculpature.

We are not always held to account for the full content of what we say. If Rob is 6’1.01” tall, he is taller than six foot one. And if he is 6’0.99”, well, then he is shorter than six foot one. So the claim

Rob is six foot one

strictly speaking entails that Rob’s height is in the tiny interval between 6’0.99” and 6’1.01”. But the use of (1) would not ordinarily commit a speaker to that unlikely consequence of what they said: one can assert (1) without being either dishonest or mistaken, even if it is strictly speaking false because Rob is really 6’1.13” or 6’0.86”. Likewise, if someone describes Ellen’s outfit by saying

Ellen wore the same type of hat as Sherlock Holmes

we are happy to ignore the implication that Holmes and his hat really exist. But why, exactly?

Here is a thought: perhaps some of our literal commitments can be waived for pragmatic reasons. Sure, taken literally, the speaker of (1) subscribes to the thesis that Rob is an exact integer number of inches tall. But a collaborative interlocutor recognises that the speaker is not serious about this, and waives that particular commitment. Thus, the proposition is pragmatically subtracted from (1)'s literal content, leaving a remainder to the effect that Rob is close to six foot one:

$p_1$: Rob is precisely six foot one

$q_1$: Rob is an exact integer number of inches tall

$(p_1 - q_1)$: Rob is six foot one to the nearest inch

Similarly, the speaker of (2), taken literally, assumes the basic tenets of the Holmes mythos, like what kind of hat he wears. But those are not serious commitments, and once subtracted, they leave a message that is just about Ellen’s hat:
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\[ p_2: \text{Ellen wore the same type of hat as Sherlock Holmes} \]

\[ q_2: \text{Sherlock Holmes wears a deerstalker} \]

\[ (p_2 - q_2): \text{Ellen wore a deerstalker} \]

It is a charming idea with some intuitive appeal. But it is unclear what to make of it unless we are told how this miraculous subtraction operation is supposed to work. The idea of logical subtraction is notoriously nebulous. Robert Jaeger (1976) and Lloyd Humberstone (2000, 2011) have recorded unsuccessful or inconclusive attempts at defining the notion, and until Stephen Yablo (2014) came in with a refreshing new approach, it was widely viewed as beyond repair. But even Yablo’s account of logical subtraction has not cleared the fog around the topic altogether, especially because of the way his approach is entangled with a revisionary and slightly obscure proposal for a non-compositional, ‘reductive’, truth-maker based semantics. This paper proposes a more straightforward understanding of pragmatic content subtraction, building on standard semantic notions. It then explores how pragmatic subtractions, understood this way, allow us to understand a range of seemingly unrelated linguistic phenomena, including loose talk and some forms of metaphorical speech.

We will call this mechanism of pragmatic content subtraction conversational exculpature. Conversational exculpature, the pragmatic subtraction of content, stands opposed to conversational implicature, if the latter is viewed as the pragmatic addition of content (as in Kent Bach’s influential taxonomy\(^1\)). The original meaning of “to exculpate” is to free from blame: the idea is that while an implicature embroils the speaker in a further commitment, an exculpature instead forgives a commitment.

On the present theory, conversational exculpature is driven by a version of Grice’s Maxim of Relation: essentially, it is a correction mechanism that comes into action when the speaker says something that, taken literally, is not wholly relevant to the conversational subject matter or question under discussion (QUĐ). For example, in a discussion about Ellen, (2)’s literal content \( p_2 \) is not wholly relevant because it brings in Sherlock Holmes. And in a context where we only want to know Rob’s height to the nearest inch, \( p_1 \) is not wholly relevant because it specifies his height to a greater degree of precision than the interests of the conversational participants require. Exculpature repairs such discrepancies.

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\(^1\) Bach 1994. As an example of addition, “Some students got an A” has a scalar implicature Not all students got an A, and its overall message Some but not all students got an A is the result of adding that to the statement’s literal content. Although Bach’s understanding of conversational implicature as content addition is common, not all authors draw the boundary between conversational implicature and other non-literal speech in this way. In particular, Paul Grice’s broad characterization of implicature appears to include the phenomena we are interested in, in that what is “implied, suggested, meant” in these cases is distinct from “what is said” (Grice 1967, p. 34). On a different taxonomy, exculpature can be classified as a type of relevance implicature.
Accordingly, it involves what Mandy Simons (2005, 2013) calls the contextual presupposition of an utterance: roughly, these are presuppositions that connect an utterance’s literal content to the question under discussion. It is these presuppositions, like \( q_1 \) and \( q_2 \) above, that are subtracted from the literal content in exculpature. Below we will see how an utterance’s literal content \( p \), the underlying contextual presupposition \( q \), and the question under discussion \( S \) jointly determine a unique, wholly relevant remainder \( r \). The prediction is that only this relevant remainder \( r \), and not \( p \) or \( q \), is seriously endorsed and added to the conversational common ground.

It is worth stressing that, following Yablo (2014, ch. 11-12), the notion of content subtraction presented here extends beyond cases where the equality \( p = (p - q) \land q \) holds: that is to say, content subtraction is not just an inverse of content addition or conjunction. For example, (2)’s literal content \( p_2 \) is not equivalent to \( (p_2 - q_2) \land q_2 \): since \( p_2 \) is true in worlds where Holmes and Ellen both wear a sombrero, it fails to entail both the subtracted proposition \( q_2 \) and the remainder \( (p_2 - q_2) \). The truth in the vicinity is that \( p \land q = (p - q) \land q \), wherever \( (p - q) \) is defined. Thus conversational exculpature is a retrenchment from the speaker’s overall commitments \( p \land q \) (literal content + contextual presupposition), but the resulting message \( p - q \) need not be entailed by \( p \) alone, as (2) illustrates. Consequently, pragmatic subtraction does not always lead to weakening of the literal content. In some cases, like (1), exculpature yields a message that is entailed by the literal content. But in cases like (2), the intended message is logically independent of the literal content, and in still other cases it is logically stronger. As we will shortly see, this feature of the account is central to its empirical success relative to competing accounts, which wrongly treat loose talk as a form of pragmatic weakening.

The introduction of the novel term “exculpature” may suggest an exaggerated claim to originality, so let me take a moment to cancel that implicature. This paper’s primary contribution is to synthesise and streamline existing ideas into a simple, formal pragmatic theory with reasonably clear empirical predictions. Many of those ideas, big and small, are taken from Yablo’s work, particularly his book *Aboutness* (Yablo 2014; see also Yablo 2005, 2006). Like Yablo, I build on Kendall Walton’s insights on the exploitation of make-believe in non-literal speech (Walton 1993, 2002). The idea that loose talk and metaphor both result from an effort to restore relevance echoes Sperber and Wilson (1986), although they have a different conception of relevance. The idea that presuppositions can make propositions relevant is explored in Simons 2005. Finally, it recently came to my attention that the linguist Manuel Križ (2015, 2016) exploits a very similar interaction between QUDs and presuppositions to account for non-maximality in plural definites, extending this idea in (Križ 2015, §3.A) to yield a treatment of the loose use of measurement expressions that closely parallels the one given below.
Section I describes the linguistic phenomena we will seek to capture in this paper, and raises some problems for existing attempts to account for them. Section II lays out the theory of exculpature in prose, and section III makes this formal and precise. Section IV applies the resulting theory to each of the examples from section I. Section V, on the logic of exculpature, shows how the theory accounts for some important general observations about the phenomena in question.

1. Loosening and Weakening

In the wake of Grice 1967, a vast amount of work has been done to understand the mechanisms behind pragmatic strengthening, so that we now possess sophisticated and predictive theories of how, for example, scalar implicatures are generated. While pragmatic weakening has been the object of a few studies, typically under the label “loose talk” (notably Sperber and Wilson 1986, Lasersohn 1999, Krifka 2002, Lauer 2011, Yablo 2014), I think it is fair to say that it remains a comparatively ill understood and understudied phenomenon. Consequently, there is no substantial, varied corpus of generally accepted examples of pragmatic weakening. For this reason, one of the most exciting aspects of the present theory is that it promises to provide a clear, systematic understanding of a widespread type of pragmatic weakening. This is of interest in itself, and also for the new light it throws on existing problems in semantics. At the same time, we will see that some of the theory’s key empirical strengths derive from the ways in which it extends beyond pragmatic weakening.

To get a general sense of the phenomena of interest, let’s begin by listing some putative examples of pragmatic weakening due to exculpature. Each statement below is accompanied by a proposition in italics that, at least on the most straightforward semantic treatment, is entailed or presupposed by the statement in question. However, in each case, it is easy to think of a setting where the statement is not naturally understood as committing the speaker to the italicised consequence:

- Rob is six foot one. [Rob is between 6’0.99” and 6’1.01”.] (1)
- Ellen wore the same type of hat as Sherlock Holmes. [Holmes really exists.] (2)
- The man over there drinking a martini is a notorious jewel thief. [Someone over there is drinking a martini.] (Donnellan 1966) (3)
- The dagger Macbeth saw in front of him was covered in blood stains. [There really was a physical dagger in front of Macbeth for him to see.] (Lewis 1983) (4)
- Hob believes a witch burned down his barn, and Nob believes she blighted his mare. [There are witches (one of them being the object of Nob’s belief).] (Geach 1967) (5)
- Crotone is in the arch of the Italian boot. [There is a city that is built on a piece of footwear.] (Walton 1993) (6)
- The number of Jupiter’s moons is four. [There are numbers.] (Frege 1884) (7)
Below, I hope to account for all these notorious examples pragmatically. Assuming a simple semantics, we can use the theory of exculpature to explain how their literal contents are transformed into messages that lack the problematic consequence. In each case, it has been suggested we instead need a semantics on which the statement in question does not entail the italicised proposition after all.\(^2\) But even if we can find an empirically adequate compositional semantics that does the job (which in many of these cases is questionable), it is clear such accounts will introduce significant complexities over a standard treatment. Thus Grice’s razor, which tells us not to complicate our semantics when there is a pragmatic explanation of the target phenomenon, rules in favour of a pragmatic strategy.\(^3\)

Another important attraction of the approach is its generality. Ostensibly, examples (1-7) illustrate unrelated phenomena, yet I hope to show they are all manifestations of the same pragmatic mechanism. Such a reduction is extremely theoretically satisfying, and no semantic strategy can hope to achieve a unification of comparable scope. The sweeping promise of a theory of conversational exculpature is to offer one elegant pragmatic solution to replace a hundred ugly semantic fixes.

Let me now touch on some general empirical observations that our theory should account for. The first is that, as I mentioned, loosening is not just a matter of weakening, contrary to the received wisdom. We can observe this in the examples given, but Carter (2016) showed how to make the point more crisply. If we consider the negations of standard examples of loose talk, it becomes evident that the weakening and strengthening due to loose talk are two sides of the same coin. Consider

Rob is not six foot one. \(\text{(1*)}\)

(1*) literally expresses a very weak proposition, namely that Rob’s height is not 6’1” — it could be any other height, for example 6’1.03”. But the loose reading of (1*), to the effect that Rob’s height is not in the neighbourhood of 6’1”, rules out that possibility. So the message (1*) sends is stronger than what it says literally. Similarly, (2)’s negation

\[^2\text{According to Sauerland and Stateva (2011), expressions like “six foot one” don’t denote specific heights, but intervals with context-dependent tolerances. Parsons (1979) and Crane (2012) argue statements like (2), (4) and (5) are literally true in virtue of the existence of fictional objects. Donnellan himself and Schiffer (2005) advocate semantic strategies w.r.t. (3). Black (1979) and Cohen (1993) advocate semantic treatments of metaphor (6). Semantic strategies for nominalising mathematical statements like (7) were developed by Putnam (1967), Hellman (1989) and Chihara (1990).}\]

\[^3\text{Given the state of knowledge on pragmatic weakening, semanticists who predict that a statement entails more than it intuitively communicates, currently have little choice but to adapt their semantics. Compare Strawson’s (1952) predicament when he observed English disjunctions and conjunctions imply more than the truth-functional account predicts. Because Strawson did not know about implicatures, he had to account for this semantically. In this manner, ignorance of implicatures led to a bias in the pre-Gricean era towards strong truth conditions, and it was one of Grice’s signal achievements to uncover that bias. In contemporary theorising, a lack of awareness of pragmatic weakening leads, I think, to an opposite bias towards weak truth conditions; the often fanciful semantic and metaphysical edifices that especially philosophers erected in response to examples like (1-7) illustrate the problem.}\]
Ellen did not wear the same type of hat as Sherlock Holmes. (2*)
is literally true no matter what Ellen wore: since Holmes does not exist, her hat cannot possibly be the same as his. However, (2*)’s message, Ellen did not wear a deerstalker, is not trivially satisfied in this way. And since there is no Italian boot, the statement
Crotone is not in the arch of the Italian boot. (6*)
may be literally true, but it still conveys misinformation about Crotone’s location.

In general, the negations of (1-7) above do not get to be assertable whenever the propositions in italics are false. The underlying reason seems to be that the loosened content of not- is always equal to the negation of the loosened content of . One might say loosening is “transparent” to negation. Clearly any theory of loose talk should account for this central datum. But as Carter points out, extant theories are doomed to failure here, because they assume loosening is a kind of weakening. The lesson typically drawn from examples like (1) is that such utterances only commit a speaker to their claim being “close enough to the truth for practical purposes” (Lasersohn 1999, p. 522). The job of an account of loose talk then becomes to articulate what it takes to be “close enough to the truth”. But since the literal truth is always close enough, any account of this sort predicts intended messages that are weaker than the literal content, and hence gets cases like (1*) wrong. Carter’s point about negation extends to other downward entailing environments. For example, the literal content of (1**) is quite weak, because pretty much no one is precisely six foot one:
Everyone who is six foot one will wear a size XL. (1**)
But (1**) is loosened to a stronger message, viz. Everyone who is around six foot one will wear a size XL.

It will be instructive to see in a bit more detail why the most popular accounts of loose talk fail on this count. Let’s begin with Lasersohn 1999, which handles examples like (1) and perhaps (3) above. According to Peter Lasersohn, loose talk arises because the semantic value of certain expressions is surrounded by a pragmatic halo of similar values. For instance, “six foot one” refers to the height 6’1”; but when used loosely, it has a range of similar heights as its halo, like the interval (6’0.5”, 6’1.5”). If an expression is not used loosely, its halo only contains its semantic meaning. The halo of a complex expression is the set of all values that can be formed by combining a value in α’s halo with a value in β’s halo.4 In this way, assuming “six foot one” is the only loosely used expression in (1), we obtain a set of propositions as the halo for (1): {Rob has height : ∈ (6’0.5”, 6’1.5”)}. In uttering (1), the speaker commits only to the truth’s being somewhere in that halo, which is the right prediction in the case of (1). But by the same token, the halo around (1*) is {Rob lacks height : ∈ (6’0.5”, 6’1.5”)}. Now, no

4 Lasersohn 1999, p. 527, 548-550. This is the default composition rule: expressions like “exactly” and “approximately”, which on Lasersohn’s account act directly on the halos, get special treatment.
matter how tall Rob is, some propositions in that halo are true: he always lacks the other heights. Thus Lasersohn’s account predicts, incorrectly, that (1*) effectively weakens to triviality. Similarly, (1**) is incorrectly predicted to weaken further. This defect is not due to incidental features of these examples: it is essential to the mechanics of Lasersohn’s theory that the literal meaning is always included in the halo, so the account is structurally incapable of predicting strengthening.

Lasersohn’s account also predicts unattested transparency failures of a different kind:

Emma and Jack both weigh five stone.  \( (8) \)

Assuming a halo \((65 \text{ lb, } 75 \text{ lb})\) around “five stone”, we get this halo for (8): \([\text{Emma and Jack both have weight } w : w \in (65 \text{ lb, } 75 \text{ lb})]\). But note that all propositions in that halo entail that Emma and Jack have exactly the same weight. Thus, Lasersohn’s account predicts that (8) is not even loosely assertable if, say, Emma weighs 71 pounds and Jack 68. Clearly that is the wrong prediction: the loose reading of (8) is \(\text{Emma weighs around five stone, and so does Jack}\). This is the conjunction of the loose readings of (8)’s conjuncts, and does not entail that they weigh exactly the same. We’ll see below that exculpature has the right logic to ensure correct predictions in (1*), (1**), (8) and a broad class of similar cases.

Another approach to loose talk, closer to the present one, holds that only relevant consequences of what we say are communicated. (Versions of this are explored in e.g. Sperber and Wilson 1986; Kao et al. 2014; Yablo 2014, §3.4 and ch. 5 — the proposal is distinct from Yablo’s account of pragmatic content subtraction in the same book.) Plausibly, in a typical context where (1) would be used, only consequences about Rob’s height to the nearest inch are relevant. All that (1) entails about that subject matter is that Rob is closer to 6’1” than to 6’2” or 6’0””. Thus this approach gets the right prediction for (1). Some of the other examples on our list are amenable to this kind of treatment, too. If only consequences about Ellen’s outfit are relevant, and we ignore the possibility that Holmes wears a different hat, we get (2)’s reading that Ellen wears a deerstalker. Similarly, (6) could be reduced in this way to a message about Italian geography. But again, this account predicts only weakenings, so that it gets the negations wrong. The literal truth of (1*) is compatible with Rob’s height to the nearest inch being anything, so it entails nothing on the matter. Likewise, since (2*) and (6*) are true just in virtue of non-existence of Holmes and the boot, they entail nothing about Ellen or geography. Thus this approach incorrectly predicts trivial readings for (1*), (2*) and (6*).

Another serious worry about this strategy is that it risks dramatically overgenerating loose readings (this criticism applies less to Sperber and Wilson than the others). Prima facie, the account assumes that irrelevant consequences of an assertion can in general be ignored. But that is simply not true. Suppose you ask “In which city did Louis Armstrong live in the sixties?” and I reply “He and his wife
Lucille got a nice place in Queens in the forties and stayed there the rest of their lives.” Clearly this cannot be heard as saying only Armstrong lived in New York in the sixties. Yet that is all it entails on the topic you raised. Similarly, if I ask “Is Emma over twenty-one?”, and you answer “She’s twenty-seven,” you are not just claiming that Emma is over twenty-one. Such examples can be multiplied effortlessly. If reduction to relevant consequences is an occurrent phenomenon at all, we are owed an explanation for why it is so rarely observed. A satisfactory pragmatic theory does more than recover the alternative readings we find. To be genuinely explanatory and predictive, it must also fail to produce alternative readings we do not find. One of the key virtues of the account presented in the next two sections is that it fails to generate alternative readings in most contexts: exculpature is only defined given a suitable configuration of literal content, contextual presupposition and subject matter.

II. Speaking as If

Not everyone knows what a derby is. So you may not know what I mean if I say “Ellen wore a derby.” But I can overcome the expressive limitations this would seem to pose with a simple trick. Everyone does know what kind of hat Charlie Chaplin used to wear; exploiting this, I can say

Ellen wore the kind of hat Charlie Chaplin used to wear.  (9)

The literal content of (9) is not that Ellen wore a derby: it is about how Ellen’s and Chaplin’s hats compare. But (9) still gets that information across, thanks to the fact that we both know what kind of hat Chaplin wears, even if you don’t know what it’s called. In this way my remark appeals to our shared information that Chaplin used to wear a derby in order to connect its literal content to the topic at hand, that is Ellen’s outfit.

Mandy Simons calls these kinds of appeals contextual presuppositions (Simons 2005, 2013; see also Thomason 1990). The notion is best introduced by contrasting it with the more traditional concept of a sentence presupposition. The sentence “Charlie is playing the tramp again” carries the presupposition Charlie played the tramp before: it makes reference to that piece of background information, which is assumed whenever the sentence is used. By contrast, a contextual presupposition attaches to an utterance, not to a sentence. Simons gives the example of a professor who starts a meeting by saying “Listen up everyone, it’s three o’clock”. Intuitively, the professor’s remark presupposes that the meeting is supposed to start at three. It is from this assumption that the claim derives its relevance. Of course the sentence “It’s three o’clock” makes no reference to meetings. This is a contextual presupposition of the utterance: it is what the participants in the exchange must assume to make the utterance relevant (see Simons 2005, p. 5). Similarly, it is not the sentence (9) that makes reference to the fact that Chaplin used to wear a derby, but rather my particular use of it in this situation.
Now suppose that Nina, in the middle of another story about our hat-loving friend Ellen, uses (2), “Ellen wore the same type of hat as Sherlock Holmes”. At first blush, Nina looks to be pulling off the same hat trick we just saw: not everyone knows what a “deerstalker” is, so she exploits our shared knowledge about Holmes to get information about Ellen’s hat across. Except that in this case, it is is not shared knowledge exactly to which Nina appeals, or even shared belief: she is only speaking as if the Holmes myth were true. Speaker and audience are fully aware that the body of information referred to is a fictional one. But since it is understood that she is not trying to say anything about Holmes, and instead addresses the question what Ellen wore, it does not make a practical difference.

Thus, cases like (2) show that contextual presuppositions can be made “not because we really believe them, but in pursuit of some expressive goal. (We may believe them; but that is not why we are at the moment treating them as true)” (Yablo 2014, p. 174). There is nothing especially radical about the notion of presupposition without belief: it has long been recognised that in cases involving deception or fiction speakers presuppose things they do not really believe (see e.g. Stalnaker 1970, p. 39-40). But cases of “speaking as if” do raise a special set of questions. Nina has perfectly sincere communicative intentions in spite of her fictitious presuppositions: she is telling us that Ellen wore a deerstalker. How did we disentangle that message from the fiction she presented it in? How is it possible to process an assertion without taking on board the assumptions on which it is based? Those are the questions the theory of exculpature seeks to answer.

In the case at hand, three contingent propositions seem to be principally involved in the transition:

\[ p_2: \text{Ellen wore the same type of hat as Sherlock Holmes} \]
\[ q_2: \text{The Sherlock Holmes myth} \]
\[ r_2: \text{Ellen wore a deerstalker} \]

There is \( p_2 \), the literal content of the statement. Then there is the shared background to which the speaker appeals; here, that contextual presupposition \( q_2 \) is a compendium of well-known aspects of the Holmes story with which Nina can reasonably assume familiarity. It does not really matter how this is spelt out exactly, as long as \( q_2 \) entails that Holmes wears a deerstalker. Then there is the message \( r_2 \) Nina got across. How are the three related?

5 Contrary to Kripkean orthodoxy, we will assume there are possible worlds containing Holmes (see Bacon 2013, Partee 1989). Issues specific to empty proper names are inessential to the phenomenon of interest, which still arises if we substitute the description “the famous sleuth from Baker Street” for the name “Sherlock Holmes”.

6 In fact (2) is a little ambiguous because it is not clear what “the same type of hat” means. To keep it simple, I assume here and throughout that any two deerstalkers (and any two derbies) count as the same type of hat. Given that literal reading, (2) produces the message Ellen wore a deerstalker. As a reviewer pointed out, (2) can also convey the more specific information that Ellen wore a plaid, flapped deerstalker. That is because, on a reading of “type” that individuates hat types more finely, (2)’s literal content is stronger, affecting its communicated content as well.
First of all, note that \( p_2 \) and \( r_2 \) are equivalent conditional on \( q_2 \): \( q_2 \) and \( p_2 \) jointly entail \( r_2 \), and \( q_2 \) and \( r_2 \) jointly entail \( p_2 \). This means that in any conversation where \( q_2 \) is part of the common ground, adding the proposition \( p_2 \) to that common ground has precisely the same effect as adding \( r_2 \). Imagine a conversation between people who honestly believe all the Holmes novels report, and who really take \( q_2 \) for granted. For these Holmes believers, (2) and “Ellen wore a deerstalker” express the same incremental information. Given their shared assumptions, the statements have the same upshot, the way (9) and “Ellen wore a derby” have the same upshot to us, Chaplin believers. This suggestive observation naturally leads us to a first hypothesis about what is going on here: plausibly, by speaking as if the Holmes stories were true, and thus in a sense pretending to believe them, Nina invites us to join the pretence, and interpret what she said as a Holmes believer would (see Walton 2002, §10.2-3).

But that cannot be the whole story. Arguably, the equivalence between \( p_2 \) and \( r_2 \) given \( q_2 \) explains why, to Holmes believers, (2) amounts to another way of saying that Ellen wears a deerstalker. But it is no complete explanation of why (2) conveys that information to us, Holmes skeptics. That is because, aside from \( r_2 \), a host of other propositions are also equivalent to \( p_2 \) given \( q_2 \). For example:

- \( r_2^* \): Ellen wore the same hat as Holmes, and Holmes is cleverer than Watson.
- \( r_2^{**} \): Either Ellen wore a deerstalker, or Mrs. Hudson does not live on Baker Street.
- \( r_2^{***} \): Either Ellen wore the same hat as Holmes, or Holmes is no detective and whales sleep standing up.

Even once we recognise Nina’s presupposition \( q_2 \), it is unclear why we should understand her as communicating the information \( r_2 \) specifically, rather than some other proposition conditionally equivalent to \( p_2 \), like \( r_2^* \), \( r_2^{**} \) or \( r_2^{***} \) (or indeed \( p_2 \) itself). To Holmes believers, all of these express the same incremental information. That is to say, the truth-conditions of \( p_2 \), \( r_2 \), \( r_2^* \), \( r_2^{**} \) and \( r_2^{***} \) diverge only in possible worlds where \( q_2 \) is false. For the purpose of updating the beliefs or common ground of a Holmes believer, or any information state containing \( q_2 \), the truth-conditional profile of a proposition with respect to those worlds is irrelevant. So to Holmes believers, distinctions that surface only in Holmes-free worlds do not matter very much.

But things are rather different for us Holmes skeptics, who take ourselves to inhabit one of those Holmes-free worlds. At the end of the day, the purpose of Nina’s assertion is that we add her intended message to our own seriously held beliefs and our own serious common ground. Here, the question whether her message is \( p_2 \), \( r_2 \), \( r_2^* \) or one of the others makes all the difference in the world. On the one hand, \( p_2 \) and \( r_2^* \) cannot be added to our beliefs and common ground without inconsistency. On the other side of the spectrum, adding \( r_2^{**} \) changes nothing: our beliefs already entail \( r_2^{**} \) in virtue of the second disjunct. And anything in between is also possible. For instance, \( r_2^{***} \) effectively adds the
information that whales sleep standing up. The truth-conditional distinctions that matter least to Holmes believers are precisely those that matter most to us Holmes skeptics. So in a sense, the fact that Nina’s intended message shares a truth-conditional profile with $p_2$ in Holmes worlds does not narrow down our interpretative options at all.

That is not to say there is nothing to the observation. But it does show that to extract the message $r_2$ from (2)’s literal content $p_2$, we must have discerned some additional feature of $r_2$ that renders $r_2$ an especially plausible candidate to be Nina’s intended message, and sets it apart in this regard from all the other propositions sharing $p_2$’s truth-conditional profile in Holmes worlds. That is where relevance enters the picture. Recall that the utterance of (2) occurs in the context of a story about Ellen, so that we have good reason to assume that Nina is trying to say something about her.

Now there we have something that distinguishes $r_2$ from $p_2$, $r_2^*$ and the rest. The proposition $r_2$ is about Ellen’s outfit and nothing else: it does not raise orthogonal issues about detectives or whales. All one ever needs to know about in order to check whether $r_2$ is true is Ellen’s outfit. Adding relevance to the picture, we can reconstruct the pragmatic reasoning taking us from Nina’s literal assertion to the message $r_2$: “We know that Nina does not seriously believe what she said. She does not believe in Holmes and besides, she is not talking about Victorian detectives. She must in fact be telling us something relevant, i.e. something about Ellen. It is clear enough how what she said connects to that topic: Nina is talking as if Holmes were a real detective, who really wore one of those funny hats. Given this assumption, what she said is another way of saying Ellen wore a hat like that. Accordingly, that must be the information about Ellen she intends for us to pick up on.”

Thus, $r_2$ is singled out on the basis of two features: (A) it is about Ellen’s outfit and (B) it is conditionally equivalent to $p_2$ given $q_2$. Generalising from this example, the contours of an account of exculpature begin to emerge. In cases of exculpature, the speaker’s intended message $r$ is determined on the basis of the literal content $p$ of their statement, and two contextual clues: the contextual presupposition $q$ to which the speaker appeals, and the subject matter $S$ they address. The speaker’s message $r$ is the unique proposition that is (A) just about $S$ and (B) equivalent to $p$ given $q$. (In the next section, we will define talk of subject matters and relevance more rigorously, establishing the exact conditions under which (A) and (B) do indeed single out a unique message.)

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7 Of course one need not perform this monologue to understand (2). Pragmatic and semantic processing rarely involve conscious thought. This type of pragmatic argument serves a heuristic purpose, clarifying how the assertion interacts with the conversational maxims and background knowledge to produce a certain communicative upshot.
A sign that this is on the right track is that this account looks to display the right behaviour under negation to capture the observations from section I. Note that the propositions
\[ \neg p_2: \text{Ellen did not wear the same type of hat as Sherlock Holmes} \]
\[ \neg r_2: \text{Ellen did not wear a deerstalker} \]
are also equivalent given \( q_2 \), and that \( \neg r_2 \) is also wholly relevant to Ellen’s outfit. Thus, in the same contexts where (2) conveys the message \( r_2 \) due to exculpature, its negation (2*) is predicted to send the message \( \neg r_2 \). This captures the fact that (2*) is not trivially assertable in virtue of Holmes’ non-existence. More generally, \( r \) is equivalent to \( p \) given \( q \) just in case \( \neg r \) is equivalent to \( \neg p \) given \( q \). So assuming that a proposition \( r \) and its negation \( \neg r \) are always about the same topics, in any context where exculpature takes us from the literal claim \( p \) to the message \( r \), it should be expected that its negation \( \neg p \) is transformed into the message \( \neg r \).

To clarify the dual dependence of the message on the contextual presupposition and the topic of conversation, it will be helpful to consider a different example whose non-literal meaning, I expect, is opaque to the reader:

\( \text{Amy travelled to Alexandria and back before Nut swallowed the sun.} \) \hspace{1cm} (10)

Supposing it is clear from the context that no true sun-swallowing took place, we know we need some alternative interpretation of (10). But unless you are an Egyptologist you are probably left guessing as to what it is, because you do not know the mythology to which (10) appeals. Is Nut the personification of thunder, and did Amy get back before thunderclouds floated in? Or is Nut the goddess of harvest, and did she return before the wheat fields turned golden? Or is Nut the harbinger of the apocalypse and did Amy return before the end of the world? Or is Nut like Rahu in hinduism, who swallows the sun to cause a solar eclipse? Clearly the interpretation of (10) turns on this matter. As it happens, Nut is the Egyptian sky goddess, and here is the relevant myth:

\[ q_{10}: \text{In the morning, Nut gives birth to the sun god Ra in the East. He spends the day sailing his bark along her watery body, which is arched over the world: we observe his journey in the motion of the sun. At night he arrives exhausted at her mouth on the western horizon, where he dies. Nut then swallows his body, causing the world to fall into darkness. She gives birth to him again the next day.} \]

That helps: according to \( q_{10} \), Nut swallows the sun whenever it sets, so (10) must be a colourful way of saying that Amy arrived before sunset. But which sunset? If \( q_{10} \) were really true, the sun would set at the same time everywhere in the world. Thus, conditional on \( q_{10} \), (10)’s literal content \( p_{10} \) is equivalent to, for instance, the following:
$r_{10}$: Amy travelled to Alexandria and back before the sun set on Alexandria.

$r_{10}^*$: Amy travelled to Alexandria and back before the sun set on Tripoli.

$r_{10}^{**}$: Amy travelled to Alexandria and back before the sun set on Amy.

These all have different truth conditions: the sun sets about an hour later in Tripoli than in Alexandria. Depending on whether (10) is used in the context of a story whose main action takes place in Alexandria or Tripoli, reading $r_{10}$ or $r_{10}^*$ will be more plausible; in a story where Amy is the clear protagonist, $r_{10}^{**}$ might be the most likely reading. The present account explains that variation by noting that in these different contexts, different subject matters would be addressed:

$S_{10}$: Where did Amy go and what was the time of day in Alexandria when she got back?

$S_{10}^*$: Where did Amy go and what was the time of day in Tripoli when she got back?

$S_{10}^{**}$: Where did Amy go and what was the time of day for Amy when she got back?

Of all the propositions that match $p_{10}$ in $q_{10}$-worlds, only $r_{10}$ is wholly relevant to $S_{10}$, only $r_{10}^*$ is wholly relevant to $S_{10}^*$, and only $r_{10}^{**}$ is wholly relevant to $S_{10}^{**}$, and thus the discourse question settles the intended reading. Where the context leaves it unclear which question is addressed, there is a corresponding ambiguity about the intended reading of (10).

Here is another way to think of it: to get the reading $r_{10}^*$, one must recognise first that (10)'s literal content makes reference to the global sunset of Egyptian myth, and second that this mythical sunset represents a particular local sunset, namely the one in Tripoli. More generally, on the present account, exculpature requires knowledge of both the speaker’s point of departure (the myth presupposed) and of their target (the topic aimed for). By contrast, Yablo’s pragmatic content subtraction, “pivoting on a presupposition”, tries to get by with just this point of departure. Where the present account exploits the conversation’s subject matter to pick out the intended message $r$ from a range of alternatives, Yablo tries to make hyperintensional features of the literal message $p$ and the presupposition $q$ do this job. The limitations of that approach come out in a case like (10), where the target dependency comes through clearly: to know which of the messages $r_{10}$, $r_{10}^*$, $r_{10}^{**}$ etcetera is intended, it does not suffice to know the myth to which the speaker appeals, and knowing it in hyperintensional detail will not help.\footnote{It is not plausible that the three readings result from ‘localised’ variants of $q_{10}$. To get the reading $r_{10}^*$, for instance, we would need a myth $q_{10}^*$ entailing that Nut’s sun-swallowing causes the sun to set only on Tripoli, and nowhere else. How is that even supposed to work? Does Tripoli have its own sun? No doubt one can concoct some bizarre story $q_{10}^*$ that produces the right remainder, but that cannot explain how speakers and hearers who never heard that story arrive at the reading $r_{10}^*$. The reading $r_{10}^*$ also cannot be obtained indirectly, as a relevant consequence of a ‘global sunset’ reading: the proposition *Amy travelled to Alexandria and back before the sun set everywhere in the world*, much like *Amy travelled to Alexandria and back before hell froze over*, fails to entail $r_{10}$, $r_{10}^*$ or $r_{10}^{**}$.}

We really need to know what timezone we’re talking about.


III. The Formalism

In this section, we will sharpen the account informally expressed in the last section by formulating it within the possible worlds framework. Propositions, like the literal content \( p \), the contextual presupposition \( q \) and the speaker’s message \( r \), will be modelled in the standard way as sets of worlds:

A full proposition (or simply proposition) is a subset of logical space \( \Omega \), that is a set of possible worlds. A proposition \( p \) is true at \( w \) just in case \( w \in p \) and false at \( w \) otherwise. The complement \( \Omega \setminus p \) of \( p \) is denoted “\( \neg p \)”. (a)

To compare propositions within a limited area of logical space, it will be helpful to define partial propositions and restrictions:

A partial proposition is an ordered pair of disjoint sets of worlds. \( \langle t, f \rangle \) is true at \( w \) just in case \( w \in t \) and false at \( w \) just in case \( w \in f \). It has no truth value at worlds outside of \( t \cup f \). (We will treat the partial proposition \( \langle p, \neg p \rangle \) as identical to the full proposition \( p \)). (b)

The restriction of the proposition \( p \) to the proposition \( q \), written \( ptq \), is the partial proposition \( \langle p \cap q, \neg p \cap q \rangle \). (c)

Using this notation, we can compactly express the thought that \( p \) and \( r \) have the same truth values in all \( q \)-worlds by writing \( ptq = rtq \) — the restrictions of \( p \) and \( r \) to \( q \) are the same.

We also need a way to make sense of all the talk of relevance and subject matters. For this, we employ the notion of a subject matter as a partition of logical space (see Hamblin 1958, Lewis 1988):

A subject matter or question is a partition of logical space \( \Omega \). Two worlds \( w \) and \( v \) agree on \( S \), written \( w \sim_S v \), just in case \( w \) and \( v \) are contained in the same partition cell of \( S \); thus \( \sim_S \) is an equivalence relation on \( \Omega \). (d)

A proposition \( p \) is wholly about (or simply about) \( S \) just in case \( p \) is a union of \( S \)-cells. (Equivalently, \( p \) is about \( S \) iff \( p \) is closed under the relation \( \sim_S \)). A partial proposition is (wholly) about \( S \) just in case it is a restriction of some full proposition about \( S \). (e)

A proposition \( p \) has no bearing on \( S \) just in case \( p \) intersects every \( S \)-cell. (f)

For instance, the subject matter how many cows there are groups possible worlds together according to the number of cows they contain; it contains a partition cell for all the cowless worlds, a cell for the worlds inhabited by one very lonely cow, a cell for the worlds with exactly two cows, and so on. The proposition There are between a million and a billion cows is about that subject matter: it is the union of the one-million-cow cell, the one-million-and-one-cow cell, and so on. However, the proposition There are more cows than squirrels is not (wholly) about this subject matter, since its truth value depends on the number of squirrels as well as the number of cows. It does have some slight bearing on the question, because it rules out the possibility that there are no cows. The proposition There are more than a hundred hippos has no bearing at all on the question how many cows there are.
Figure I: A proposition (wholly) about S and a proposition with no bearing on S.

It can help to think about these things visually. Figure I displays two maps of logical space. Each depicts a different full proposition: the region where the proposition is true is coloured green, the region where it is false red. Meanwhile, the thick black lines represent the boundary lines between six cells of some subject matter $S$. The diagram on the left represents an arbitrary proposition about $S$: i.e. a union of cells of $S$. On the other hand, the diagram on the right represents an arbitrary proposition without any bearing on $S$, compatible with every $S$-cell.

We can compare subject matters or questions by how finely they individuate the possibilities:

A subject matter $S$ is at least as fine-grained as, or at least as big as, or contains, or entails, another subject matter $T$ just in case every cell in $T$ is a union of $S$-cells. (g)

The subject matter how many cows and how many bulls there are contains the subject matter how many cows there are because it is based on a stronger equivalence relation, and thus makes more distinctions between worlds. If a proposition is wholly about one subject matter, it is automatically wholly about every bigger subject matter too. All propositions are about the biggest subject matter, everything, that puts each world in its own cell.

So much for subject matters in the abstract. What is it about a conversation that makes a particular subject matter the subject matter of that conversation? Or putting it in terms of questions, what is it that makes a particular question the question under discussion in a particular context? In line with the trend set by Roberts 1996, Groenendijk 1999, Ciardelli et al. 2013 and others, I take the subject matter of a conversation to be part of the conversational context, or what is often called “the conversational scoreboard” (Lewis 1979). And I want to think of a conversation’s subject matter as modelling the evolving interests of the conversational participants, similar to the way a conversation’s context set is standardly taken to model the participants’ evolving presuppositions (Stalnaker 1974).

Roughly speaking, the subject matter of a conversation is the partition $S$ such that $w \sim_{S} v$ if and only if the differences between $w$ and $v$ are ignored for the purpose of the conversation. For example, in a conversation about what happened yesterday, the only differences between worlds that matter are differences in yesterday’s events. Worlds that agree on those events, and differ with regard to, say, what will happen tomorrow, or how tall Napoleon was, occupy the same cell in the conversation’s
subject matter: such distinctions between worlds are momentarily ignored. The proposition \( p \) is *wholly relevant* in a context just in case \( p \) is *wholly about* the conversational subject matter \( S \). (As with “about”, I will often drop the “wholly” and simply say “relevant” to mean “wholly relevant”.) Like the context set, the conversational subject matter evolves over time, as participants in the conversation resolve or abandon old questions, and raise new ones to salience that were previously ignored.

Let’s apply some of these notions in the context of our old hat example (2). Recall that we took the sentence to be uttered in a context where we are interested in the question *what Ellen wore*. The cells of that subject matter each correspond to a possible outfit, containing precisely the worlds in which Ellen wore that outfit. (2)’s literal content \( p_2 \) has a bearing on this topic: it rules out cells where Ellen wore no hat at all. Still, within any cell where Ellen did wear a hat, we find both \( p_2 \)-worlds where Holmes wears that same hat, and \( \neg p_2 \)-worlds where he does not. In terms of our diagrams, \( p_2 \) ‘colours outside the lines’ so that \( p_2 \) is not wholly relevant in the context. By contrast, (2)’s intended reading \( r_2 \), *Ellen wore a deerstalker*, is relevant: it is true throughout all cells in which Ellen wore an outfit that includes a deerstalker, and false throughout all other cells. Finally, consider the the presupposition \( q_2 \), the Holmes story that links \( p_2 \) to \( r_2 \). For all that story says, Ellen might have worn any outfit, so \( q_2 \) has no bearing at all on the topic. Recall that since \( q_2 \) entails that *Holmes wears a deerstalker*, \( p_2 \) and \( r_2 \) are equivalent conditional on \( q_2 \), or in our new notation, \( p_2 \triangleright q_2 = r_2 \triangleright q_2 \).

Putting all this together, we can discern a way to compute the the relevant message \( r_2 \) from irrelevant literal content \( p_2 \) with the aid of \( q_2 \) and \( S_2 \). To see that, consider figure II below, where the general situation is depicted visually. The middle diagram represents a partial proposition: worlds where it is true are coloured green, worlds where it is false are in red; worlds where it lacks a truth value are left blank. More specifically, it represents the restriction of the non-relevant \( p \) (top left) to \( q \) (top right): it has a truth value matching \( p \) just where \( q \) is true. Since \( p \triangleright q = r \triangleright q \), this same partial proposition is also a restriction of \( r \), which is about \( S \). Consequently, unlike \( p \), \( p \triangleright q \) takes on *at most* one truth value per \( S \)-cell, so that \( p \triangleright q \) is about \( S \) (df. (e)).

Furthermore, since \( q \) has no bearing on \( S \) and thus intersects every \( S \)-cell, \( p \triangleright q \) also takes on *at least* one truth value per cell. In sum, \( p \triangleright q \) associates a unique truth value with each cell, and since \( p \triangleright q = r \triangleright q \), that is in each case the truth value \( r \) takes on in that cell as well. Consequently, there is a simple way to single out \( r \) in terms of \( p \triangleright q \) and \( S \): \( r \) is the unique full proposition about \( S \) that is true where \( p \triangleright q \) is true, and false where \( p \triangleright q \) is false; the bottom diagram is the unique way to complete the colouring of the middle diagram without drawing outside the lines. Or as we put it in the last section: \( r \) is the unique full proposition wholly about \( S \) that matches \( p \)'s truth-conditional profile in \( q \)-worlds.
To make this easier to express, we define the completion of a partial proposition by a subject matter:

The completion of a partial proposition \( \langle t, f \rangle \) by the subject matter \( S \), written \( S(\langle t, f \rangle) \), is well-defined just in case \( \langle t, f \rangle \) is about \( S \). Then \( S(\langle t, f \rangle) \) is this, possibly partial, proposition:

\[
S(\langle t, f \rangle) = \{ w : w \not\sim_S v \text{ for some } v \in t \}, \{ w : w \not\sim_S v \text{ for some } v \in f \}
\]  

(h)

If \( \langle t, f \rangle \) is not wholly about \( S \), or equivalently if there is an \( S \)-cell containing both \( t \)- and \( f \)-worlds, \( \langle t, f \rangle \) cannot be consistently completed and so \( S(\langle t, f \rangle) \) is undefined. If every \( S \)-cell contains \( t \)- or \( f \)-worlds but not both, we are in the situation of figure II, and the completion \( S(\langle t, f \rangle) \) is a full proposition. If \( \langle t, f \rangle \) is about \( S \), but some \( S \)-cells contain neither \( t \)- nor \( f \)-worlds, we can still complete \( \langle t, f \rangle \), but the resulting completion will not cover all of logical space. In that case, \( S(\langle t, f \rangle) \) is a strictly partial proposition, lacking a truth value in the \( S \)-cells that do not intersect \( t \cup f \).

With that final definition in place, we now have a precise and elegant way to express the core of the present account. In conversational exculpature, the utterance’s literal content \( p \), the underlying contextual presupposition \( q \) and the topic of conversation \( S \) jointly determine the intended message \( r \) as follows: \( r = S(\langle p, t, q \rangle) \). The central predictive claim of the account is simply that this reading \( S(\langle p, t, q \rangle) \) is available in any context where it is well-defined. With that formal characterisation of the account in hand, we can investigate its ramifications more systematically.
To begin with, the fact that exculpature exhibits the kind of transparency to negation observed in section I can now be seen to be due to the fact that the treatment of truth and falsehood by the operation ‘completion by $S$’ are perfectly symmetric (that is, the operation is self-dual). Figure III below nicely illustrates why, as a consequence, completing the contraditories $pq$ and $\neg pq$ by $S$ produces another pair of contraditories $S(pq)$ and $S(\neg pq)$, so that $S(\neg pq) = \neg S(pq)$.

For related reasons, exculpature is also transparent to the other Boolean operators: $S((a \land b)q) = S(aq) \land S(bq)$ and $S((a \lor b)q) = S(aq) \lor S(bq)$. Thus we get transparency for all truth functions, in line with the observations from section I (see section V for details).

The formalism also gives us a more instructive way of thinking about the role of the subject matter in exculpature. In section II, we introduced relevance as a constraint to whittle down the class of candidate messages. Now we have a more evocative image. Through completion, the subject matter projects the truth conditions beyond the initial domain $q$ onto other areas of logical space. Speakers explicitly specify the boundary line between true and false within the region $q$, then expect listeners to work out how to “go on in the same way”, to use Yablo’s phrase (2014, pp. 142-3). The conversation’s subject matter provides the structure needed for this.

How far beyond the boundary a subject matter takes us depends on the boundary and the subject matter. From df. (c) and (h), we see that $S(pq)$ has a truth value throughout the following region:

$$\{w : w \sim s v \text{ for some } v \in q\} = \{w : w \sim s v \text{ for some } v \in p \cap q\} \cup \{w : w \sim s v \text{ for some } v \in \neg p \cap q\}$$

The QUD reduction operator $p \mapsto ps$ used by Yablo (2014, §3.4) and others lacks that symmetry. Since the dual of ‘strongest relevant proposition $p$ entails’ is ‘weakest relevant proposition entailing $p'$, $(\neg ps) = \neg (ps)$ for non-relevant $p$. 

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\[\text{Figure III: Completion and negation}\]
This region is the strongest proposition about \( S \) that \( q \) entails. Thus, in general, the less \( q \) says about \( S \), the farther we can project, and the closer the completion is to a full proposition. \( S(p \triangledown q) \) is a full proposition, and \( \{ w : w \sim_S v \text{ for some } v \in q \} = \Omega \), precisely in case \( q \) has no bearing on \( S \).

In the examples from the last section, we do indeed find that the \( q \)s in question have no bearing on the corresponding \( S \)s, which is to say they are compatible with every \( S \)-cell. The Sherlock Holmes story \( q_2 \) says nothing about what Ellen wore, and the Egyptian myth \( q_{10} \) tells us nothing about Amy’s travels or their timing: what is subtracted in exculpature is the irrelevant bit. The fact that contextual presuppositions are often irrelevant in our sense is essentially connected to their role as a bridge from the speaker’s claim \( p \) to the question under discussion \( S \) (Simons 2005). Where \( q \) is compatible with a certain answer to \( S \), there is a question as to whether adding \( p \) to \( q \) still leaves the answer open. But if \( q \) rules out a certain answer to \( S \) already, there is nothing to test. Thus contextual presuppositions \( q \) can play their bridging role only because they have little or no bearing on \( S \). They owe their ability to make the speaker’s utterance relevant in part to their own irrelevance.

Next, let me state a useful result, which we will appeal to time and again:

Let \( p, q \) and \( r \) be full propositions, and let \( S \) be a subject matter. Then \( r = S(p \triangledown q) \) if and only if the following three conditions are met:

- \( r \) is about \( S \). \( (Aboutness) \)
- \( p \triangledown q = r \triangledown q \). \( (Equivalence) \)
- \( q \) has no bearing on \( S \). \( (Independence) \)

In case only the final condition fails, \( S(p \triangledown q) = r \triangledown s \), where \( s \) is the strongest proposition about \( S \) entailed by \( q \).\(^{10}\)

Below, whenever we need to check that the present account appropriately connects a given literal meaning to the message actually communicated, we can fall back on this easy checklist. Its three conditions can be rephrased as follows: to satisfy \( Aboutness \), \( S \) has to be at least as big as the binary subject matter \( \{ r, \neg r \} \). To satisfy \( Equivalence \), \( q \) has to entail \( (p \equiv r) \), where “\( \equiv \)” denotes material equivalence. Finally, \( Independence \) puts upper limits on the size of \( S \) and the strength of \( q \), demanding that \( q \) remain compatible with every \( S \)-cell.\(^{12}\)

One important observation to draw from result (i) is that the function \( S(p \triangledown q) \) is highly robust with respect to \( S \) and \( q \) — within generous bounds, we can vary the subject matter \( S \) and the presupposition

\(^{10}\) Proof: \( Aboutness \) holds iff \( r \) has one truth value per \( S \)-cell (df. (e)). Given \( Aboutness \), \( Equivalence \) holds iff \( p \triangledown q \) matches that one truth value \( within \) each \( q \)-compatible cell (df. (c)) and \( S(p \triangledown q) \) matches \( q \) \( through \)out each \( q \)-compatible cell (df. (h)), i.e. throughout the region \( s = \{ w : w \sim_S v \text{ for some } v \in q \} \). Thus \( Aboutness \) and \( Equivalence \) hold iff \( S(p \triangledown q) = r \triangledown s \). Finally, \( s \) is equal to \( \Omega \) iff \( q \) is compatible with every \( S \)-cell, that is iff \( Independence \) holds (df. (f)). \( \blacksquare \)
without affecting the value of \( S(p\land q) \) at all. This robustness is important for two related reasons. First, it makes the non-literal messages easier to find. If a very particular presupposition and a very particular subject matter were needed to get to the speaker’s intended meaning, that would leave a lot of room for miscommunication. But as it is, the listener only needs to be in the right ballpark to understand the speaker. Second, this robustness accounts for the fact that in most of our examples, the same non-literal meaning is accessible in a varied range of contexts, in conversations about different topics, and to listeners with different backgrounds.

For instance, in (2), to get the message \( r_2 \), *Ellen wore a deerstalker*, the subject matter of the conversation needn’t be exactly *what Ellen wore*. Some smaller subject matters, and just about any bigger subject matter will do just as well, since the message \( r_2 \) is wholly relevant to those bigger subject matters too: Aboutness will still be satisfied. This means that pretty much any conversation where Ellen’s outfit is amongst the interests of the conversational participants should be conducive to deriving this message \( r_2 \) — it need not be the sole conversational focus. Furthermore, if we take the possibility into account that listeners may accommodate by expanding the subject matter, the reading is also accessible in conversations where this is easily added to the subject matter of the conversation. The only constraint on expanding \( S \) is that it should not invade on \( q \)’s territory, in order to avoid conflict with Independence. For instance, if we are narrating the adventures of Ellen and Holmes, \( q \) would actually be relevant, which is why (2) can only be read literally in that context.

Similarly, the listener does not have to guess precisely what fiction \( q_2 \) the speaker has in mind in order to get to the right message: any \( q \) that entails Holmes wears a deerstalker puts us in good shape as far as Equivalence is concerned. Independence does put limits on \( q \): if \( q \) entails answers to questions at issue in the conversation, that limits the region \( s \) of logical space where \( S(p\land q) \) has a truth value.

### IV. Applications

Now that the account of conversational exculpature is on the table, we can enjoy the fruits of our labour and investigate potential applications. Of the seven puzzle sentences listed on p. 4-5, we have so far only dealt with number (2). This section examines each remaining example in turn. Without

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11 It follows immediately that whenever \( S(p\land q) = r \) for some \( S \) and \( q \), it is also the case that \( B_r(p\equiv q) = r \), where \( B_r \) is the polar question \([r, \neg r]\). For contingent \( r \), \( B_r(p\equiv q) = r \) iff \((p\equiv r) \) overlaps both with \( r \) and with \( \neg r \), that is whenever there are \( pr \)-worlds as well as \( \neg p\neg r \)-worlds. Thus we get the following result: for given \( p \) and \( r \), there exist \( S \) and \( q \) such that \( S(p\land q) = r \) iff (A) \( p \) and \( r \) are compatible, and (B) \( \neg p \) and \( \neg r \) are also compatible. (In the special cases \( r = \bot \) and \( r = \top \), drop (A) or (B) respectively.) That certainly does not imply that through exculpature, a given utterance can convey just about any message compatible with its literal content! Exculpature only takes place in a context where the topic under discussion and the utterance’s contextual presuppositions really do satisfy the (i)-conditions.
exception, these discussions raise complex questions that will need to be side-stepped. Attending to all the issues and nuances attaching to any one of these applications would require a dedicated study. Let’s remain superficial for now, and, in a spirit of exploration, map out a few beginnings.

Each application falls into roughly the same pattern. We start out with a problematic statement that, in context, has a reading \( r \) distinct from its apparent semantic content \( p \). The aim is to explain the discrepancy using the present account of conversational exculpature. Such an explanation has two components. First, identify a contextual supposition \( q \) and subject matter \( S \) that are plausibly representative of the contexts where the statement has the reading of interest. Second, establish that \( S(p \triangleq q) = r \) by running through the three conditions from result (i) above: Aboutness, Equivalence and Independence. In some cases, Independence won’t be fully satisfied, so that \( S(p \triangleq q) \) is a strictly partial proposition \( r \triangleq s \). But as long as \( s \) covers all salient possibilities, such limitations are harmless. I will usually write down something like the minimal natural choice of subject matter \( S \) and presupposition \( q \): bigger subject matters and stronger presuppositions do the job, too.

Example (1): Scales, Numbers, and Measurements

Measurement expressions are often used loosely. Consequently, statements like “John arrived at 6 o’clock”, “There were two dozen people at the party” and “The universe is 14 billion years old” ordinarily convey weaker messages than they literally express, while their negations convey stronger messages. Looseness is different from vagueness. In a scientific context, the statement “The molar mass of water is 18.015 grams.” conveys a very precise and determinate piece of information, namely that the molar mass of H\(_2\)O is at least 18.0145 and less than 18.0155 grams. But this is loose talk all the same, because the statement literally expresses something stronger.

Round numbers generally get looser readings. It is fine to say “It’s four o’clock” when you know that it is in fact 3:58. But saying “It is three fifty-seven” would be misleading, even though that’s strictly speaking closer to the actual time. Likewise, “It is thirty degrees outside” admits a looser reading than “It is twenty-four and a half degrees outside”. Such observations have convinced linguists that numerals and measurement terms are grouped together in conventionally determined scales or expression-choice spaces of varying granularity (Krifka 2002, Sauerland and Stateva 2011, Solt 2014).

For instance, heights can be measured on the feet-and-inch scale: \{ … , “four foot eleven”, “five feet”, “five foot one”, “five foot two”, … \}, which is finer than the feet scale \{ … , “four feet”, “five feet”, “six feet”, … \}, but coarser than the quarter-inch scale \{ … , “five foot one”, “five foot one and a quarter”, “five foot one and a half”, … \}. In general, a scale is coarser or finer depending on whether fewer or
more quantities are represented on it. The use of a coarser scale leads to more loosening; hence the round numbers that appear on coarse scales admit looser readings. However those same numbers typically occur on fine scales as well, which is why they can also be interpreted more strictly.\(^\text{12}\)

The theory of exculpature can explain why it is that scales produce loosening, and also why this effect depends on the granularity of the scale, without needing to posit any semantic ambiguities. First note that a typical scale omits most values: for example, the height 6’1.3” is nowhere represented on the feet-and-inch scale. This poses a puzzle about why scales are useful at all. Most people’s heights, for instance, are not on the feet-and-inch scale, since most of us are not an exact integer number of inches tall. If you want to point out Rob’s height on the scale, you only have so many options: “Rob is five foot eleven”, “Rob is six feet tall”, “Rob is six foot one”, and so on. You are ignoring all the intermediate possibilities here. Thus the use of a scale to specify Rob’s height presupposes that his height is on the scale, which at first blush looks like a strangely implausible assumption to be making. But as it happens those scale presuppositions can be easily exculpated.

Our standards of precision about personal height differ from context to context, and this is reflected in the conversational subject matter: What is Rob’s height to the nearest inch? and What is Rob’s height to the nearest foot? are distinct questions: the former makes more distinctions than the latter. The higher our standards of precision, the more distinctions are relevant, and the finer-grained the question under discussion. The granularity of the question under discussion in turn affects the scale the speaker employs to address it. In particular, a speaker using the feet-and-inch scale is not interested in answering the question What is Rob’s height? to arbitrary levels of precision. The scale is just too coarse for that. But it is eminently well-adapted for addressing the coarser question Which height on the feet-and-inch scale is closest to Rob’s? In this way, finer scales are naturally associated with more fine-grained questions, and thus the speaker’s choice of scale reveals which distinctions between heights are considered relevant.

Putting all this together, it becomes clear why a sentence like (1), “Rob is six foot one”, offers fertile ground for exculpature. The expression “six foot one” also occurs on the quarter-inch scale, but since personal height is more commonly measured on a feet-and-inch scale, (1) is most naturally interpreted relative to that scale. On that interpretation, (1) is associated with a scale presupposition \(q_1\) and a question \(S_1\). With these parameters, the account predicts the loose reading \(r_1\) after exculpature.

\(^{12}\) Modifiers like “exactly” and “approximately” can be used to address such scale ambiguities. In their absence, looser readings are often preferred, since the coarser scales tend to be used more frequently (see Krifka 2009).
\( p_1 \): Rob is six foot one.

\( q_1 \): Rob is some integer number of inches tall.

\( S_1 \): Rob’s height to the nearest inch

\( r_1 \): Rob is six foot one to the nearest inch (that is, he’s between 6’0.5” and 6’1.5”).

To check this, we establish that \( S_1(p_1 q_1) = r_1 \) by running through our three conditions. The cells of \( S_1 \) are propositions of the form \( \text{Rob’s height in inch is within the interval } [n-\frac{1}{2}, n+\frac{1}{2}] \), for positive integers \( n \). Since \( r_1 \) is one of those propositions, it is about \( S_1 \), so \( \text{Aboutness} \) is satisfied. The only way to be an integer number of inches between 6’0.5” and 6’1.5” tall, is to be 73 inches tall, or 6’1”. Thus \( p_1 \) and \( r_1 \) are equivalent given \( q_1 \), giving us \( \text{Equivalence} \). Finally, being an integer number of inches tall is compatible with every cell of \( S_1 \), establishing the final condition \( \text{Independence} \).

Since an inch is defined as 2.54cm, statement (11) has precisely the same literal truth conditions as (1).

\[
\text{Rob is 185.42 cm tall. (11)}
\]

But (11) uses a much finer scale than (1), ignoring fewer possibilities: \( \{ \ldots , "180.00 \text{ cm}" , "180.01 \text{ cm}" , "180.02 \text{ cm}" , \ldots \} \). Thus (11) carries a weaker scale presupposition and addresses a more fine-grained question than a normal utterance of (1). That is why (11) does not have a loose reading anywhere near as weak as \( r_1 \). (11)’s literal content \( p_1 \) can only weaken a little bit to

\[
\text{r}_{11}: \text{Rob is between 185.415 cm and 185.425 cm tall.}
\]

If we interpret (1) as employing a half- or a quarter-inch scale, we get readings that are intermediate in strength between \( r_1 \) and \( r_{11} \).

**Example (3): Donnellan descriptions**

Keith Donnellan (1966) observed that definite descriptions appear to have what he calls a “referential use”. He argued, \textit{contra} Russell, that definite descriptions used in this way pick out their intended referent, irrespective of whether or not that referent satisfies the descriptive content of the definite description. Thus sentence (3) can be true on the relevant reading even if no one is drinking a martini.

\[
\text{The man over there drinking a martini is a notorious jewel thief. (3)}
\]

According to Donnellan, what matters for the truth of this reading of (3) is that the man the speaker intends to draw attention to (\textit{that guy}) is a notorious jewel thief; whether that man is actually drinking a martini is irrelevant. Donnellan concluded that the definite article “the” is lexically ambiguous between a referential and a Russellian or attributive sense.

Kripke (1977) counters that, even in languages where definite descriptions only have a Russellian/attributive meaning, the referential uses Donnellan points to should be expected to arise for pragmatic reasons. Wielding Grice’s razor, Kripke concludes that English is such a language, and that the
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semantic ambiguity Donnellan posits does not exist. But Kripke only gives the barest sketch of the pragmatic mechanism that is meant to generate this alternative reading. The present account can fill the lacuna. Assuming, for simplicity, the Russelian analysis,\footnote{If you think (3)'s literal content is a Strawsonian partial proposition, that works too. To accept partial inputs into the exculpature mechanism, simply extend the definition of a restriction in the obvious way: $(t, f) q =_{df} (t \cap q, f \cap q)$. On this version of the account, the Independence condition is that $(t \cup f) \cap q$ has no bearing on $S$.} (3) expresses the proposition $p_3$ below. The background supposition $q_3$ and the subject matter $S_3$ take us via the familiar paths from $p_3$ to $r_3$:

$p_3$: There is one man there who is drinking a martini and he is a notorious jewel thief.
$q_3$: There is one man there who is drinking a martini and it is that guy.
$S_3$: What is that guy’s job?
$r_3$: That guy is a notorious jewel thief.

Aboutness and Equivalence are easily seen to be satisfied. $q_3$ entails that that guy exists, but provided we take it that worlds in which he does not exist share an $S_3$-cell with worlds where he exists but has no job, this does not interfere with Independence. Thus we have all three conditions, and $S_3(p_3q_3) = r_3$.

One might ask why (3*) cannot convey $r_3$, although it is equivalent to (3) on the present treatment. The reason is that (3*) explicitly asserts that there is a man with a martini there, in a separate conjunct. Thus (3*) does not, in any sense, presuppose $q_3$, and the conditions for exculpature are not met. By contrast, (3) can be heard as contextually presupposing $q_3$ because it discreetly stashes its martini in the de finite description, the traditional home of presuppositions since Frege (1889).

Example (4): Veracity exculpatures

“Is this a dagger which I see before me, … or art thou but a dagger of the mind, a false creation, proceeding from the heat-oppressed brain?” Macbeth is unsure, but when describing the content of his vision he certainly speaks as if there were a concrete object floating in front of him, “handle toward [his] hand”, covered in “gouts of blood”. He speaks as if the vision were veridical, although he does not believe it. Given his tense state of mind, the poor Thane may be forgiven his ontological laxity. But we display it too, for instance when describing the episode as follows:

The dagger Macbeth saw in front of him was covered in blood stains. (4)

Our insouciance about referring to non-existent objects in describing beliefs and experiences is all too well known to philosophers and linguists for the many thorny puzzles it poses about attitude reports. One of those puzzles is Peter Geach’s famous Hob-Nob problem, which is up next. In this subsection, I outline a general approach that exculpature allows us to take to this class of problems. The core idea is
that in (4), we presuppose that Macbeth’s dagger is real to make it easier to describe the content of his apparition, in much the same way that in (2), the presupposition that Holmes was real made it easier to describe Ellen’s hat. More generally, we tend to speak as if any representation we are describing is accurate, whether or not we believe this, and we can do so with impunity because this contextual supposition is easily exculpated. We will call such exculpatures veracity exculpatures.

The habit of speaking as if the representation under discussion is accurate crops up whether we are describing a statement, a book, a statue, an image, a story, or a belief. Its most blatant manifestation is in a very common kind of indirect discourse. As an example, here is a description of Rafael’s portrait Girl with a Unicorn: “The painting shows Maddalena Strozzi shortly before her wedding. She has long blonde hair and is sitting in front of the open window wearing a red-and-gold dress. On her lap she holds a curious-looking, woolly-haired baby unicorn.” After the first sentence, this description does not mention the painting at all, but describes the sitting in the indicative mood, as if it really happened that way. (Which of course it didn’t. The sitter was probably holding a less exotic beast: in 1959, an X-ray revealed a small dog painted underneath the unicorn; see Coliva 2016.)

How is it that a falsehood like (12) conveys accurate information about this portrait, even though, taken literally, it says nothing about a painting?

Maddalena has a woolly-haired baby unicorn on her lap. (12)

To answer that question using veracity exculpatures, we must interpret (12) as literally concerned with what happened during the sitting. We then get:

\[ p_{12}: \text{During the sitting, Maddalena had a woolly-haired baby unicorn on her lap.} \]
\[ q_{12}: \text{The Girl with a Unicorn is a completely accurate depiction of the sitting.} \]
\[ S_{12}: \text{What is depicted in Rafael’s Girl with a Unicorn} \]
\[ r_{12}: \text{The Girl with a Unicorn depicts a woolly-haired baby unicorn on Maddalena’s lap.} \]

Evidently, \( r_{12} \) is about what happens in the painting (Aboutness). And assuming the painting is a completely accurate depiction of the sitting, it depicts a woolly-haired baby unicorn on Maddalena’s lap if and only if there really was one (Equivalence). What about Independence? What happens according to an image is typically independent of what happens in the situation it depicts, so there tends to be the possibility that content and reality match. Thus, \( q_{12} \) is compatible with most \( S_{12} \)-cells: to a close enough approximation, \( S_{12}(p_{12} ! q_{12}) = r_{12} \). Still, there are some exceptions to Independence in this case. In particular, the \( S_{12} \)-cell where the Girl with a Unicorn has no content at all is incompatible with \( q_{12} \), and hence \( S_{12}(p_{12} ! q_{12}) \) will not have a truth value there.\(^{14} \)
A similar veracity exculpature accounts for (4)'s apparent failure to entail that there was really a dagger Macbeth saw:

\[ p_4: \text{The dagger Macbeth saw in front of him was covered in blood stains.} \]
\[ q_4: \text{Macbeth's vision was accurate.} \]
\[ S_4: \text{The content of Macbeth's visual experience} \]

In much the same way as in example (12), we get a message \( S_4(p_4 \land q_4) \) which concerns only the content of Macbeth's visual experience, and therefore fails to entail that there was a dagger in front of Macbeth. Just as with (12), \( S_4(p_4 \land q_4) \) is partial, having a truth value only in worlds where the content of Macbeth's visual experience is compatible with its own accuracy.

Just what is that message \( S_4(p_4 \land q_4) \)? It is tricky to say it explicitly. It is something to the effect that

\[ r_4: \text{Macbeth's visual experience was as it would be if he were to see a bloody dagger.} \]

where the “as if” is understood to connote sameness of content. While it is a helpful approximation, this may not quite be the right way to put it — counterfactuals are treacherous! The analogy with (12) suggests an alternative paraphrase:

\[ r_4^*: \text{Macbeth's visual phenomenology depicts a bloody dagger.} \]

But that sounds unduly esoteric, and I am not sure it is really better. We noted before that exculpature can expand our expressive range. Perhaps there is no natural literal form of expression that precisely captures (4)'s message \( S_4(p_4 \land q_4) \). Thankfully it is easily expressed non-literally, using (4).

Example (5): Hob, Nob and the witch that wasn’t

Hob believes a witch burned down his barn, and Nob believes she blighted his mare. (5)

The pronoun “she” in the second conjunct appears to lack a suitable antecedent. Presumably, it has to refer back to some witch. But which witch? The witch that burned down Hob’s barn? That cannot be right, because for all (5)'s first conjunct says, there may be no such witch. Indeed, for all it says, there may be no witches at all, not even a witch whom Hob holds accountable for burning down his barn. Maybe “she” picks out an imaginary witch, existing only in Nob’s belief worlds. But in that case,
which of Nob’s doxastic witches is it? After all, Nob may believe in multiple witches. Is it the witch Nob thinks burned down Hob’s barn? That cannot be right either. For all (5) says, Nob may not know about Hob or his barn. Intuitively, (5) correctly characterises a scenario where Hob and Nob read the same made-up newspaper story about a witch, after which Hob blames her for his burnt barn and Nob for his blighted mare, while neither knows of the other’s misfortune (Edelberg 1986). At this stage, it becomes tricky to say what it is (5) is trying to tell us about Nob’s beliefs at all.

To begin with, note that the following variants of (5) are more easily accounted for:

Hob knows/discovered/realised a witch burned down his barn, and Nob believes she blighted his mare.  

(5*)

Why does the substitution of a factive verb make a difference? Well, thanks to the factivity, the first conjunct of (5*) does deliver a witch for the pronoun to pick up on. It is widely accepted that statements of the form “a knows that p” presuppose the truth of p, and may be treated as equivalent to “p, and a knows it” (e.g. Karttunen 1974, Schlenker 2009, §2.3). Thus (5*) becomes

A witch burned down Hob’s barn, Hob knows/discovered/realised it, and Nob believes she blighted his mare.  

(5**)

(5**), in turn, is relatively unproblematic. It is a run-of-the-mill case of an anaphoric pronoun with an indefinite antecedent: “she” picks up on the antecedent “a witch” just as it might in “A woman walked in; I think she got a raspberry milkshake.”

In the context of (5*)’s second conjunct, the speaker is seriously committed to the content of the first,

q5: Hob knows a witch x burnt down his barn.

The content of the second conjunct, i.e.

p5: Nob believes she x blighted his mare,

picks up on the witch from q5 (the subscripted x’s track the anaphoric relations). Thus q5 makes a witch available for the pronoun in (5*). Now in the context of (5)’s second conjunct, the speaker emphatically fails to endorse q5: they only said Hob believes that a witch burnt down Hob’s barn, and that belief might be false. But while the speaker may not believe that Hob’s beliefs are accurate, I submit that the use of the pronoun shows the speaker is speaking as if these beliefs were accurate, that is as if Hob’s belief amounted to knowledge. So while their serious commitments may differ in the context of the second conjunct of (5) and (5*), the same contextual presupposition q5 is made in both

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15 There is a technical question about how these anaphoric connections should be treated formally. A full discussion of the difficulty would take us too far afield, but I will mention that my own preferred solution involves treating q5 and p5 as sets of world/assignment pairs rather than just sets of worlds, along the lines of Heim 1983. Such a framework also enables an elegant treatment of example (12*) (p. 26n): we say (12*) presupposes only that St. George accurately depicts some situation or other, and let the quantifier in this presupposition bind a situation variable in (12*)’s literal content.
cases. Furthermore, the presence of this contextual presupposition is sufficient to make the indefinite available as an antecedent for “she_x” in (5) as well as (5*).

Thus the upshot of (5)’s second conjunct is the result of a veracity exculpature that subtracts from its literal content \( p_5 \) the unendorsed contextual supposition \( q_5 \). This leads to a message \( S_5(p_5 \triangleleft q_5) \approx r_5 \) that is innocent of \( p_5 \)’s commitments to witches: we pragmatically purge the witch from (5) in much the same way that we pull the dagger out of (4):

\[
S_5: \text{What happened according to Hob and Nob} \\
r_5: \text{Nob’s beliefs are as they would be if Hob knew a witch}_x \text{ burned his barn, and Nob believed she}_x \text{ blighted his mare.}
\]

As everyone writing about Hob and Nob observes, the interesting reading of (5) tells us something significant about the relationship between Hob and Nob’s beliefs. Intuitively, we would like to say the beliefs it attributes to Hob and Nob are about the same witch. Provided it is understood that this could be a merely possible witch, like the merely possible witch the newspaper story is about, that may well be the right way of putting it. Alternatively one might say that (5) tells us that Hob and Nob’s beliefs are coordinated in the manner of Fine 2007 (ch. 4), where this is emphatically not understood as coordination on some particular witch. Either way, to incorporate the relational aspect of (5) into \( S_5(p_5 \triangleleft q_5) \), \( S_5 \) should be taken to ask about the joint content of Hob and Nob’s beliefs, where this incorporates at least (A) the content of Hob’s beliefs, (B) the content of Nob’s beliefs and (C) any interesting relations between them, like coordination.

**Example (6): Waltonian metaphors**

Anyone who has read Walton’s famous 1993 paper on prop-oriented make-believe knows roughly where the Italian coastal town Crotone is. Yet the paper does not include an itinerary, a map, or the town’s coordinates. Rather, Walton included this false but memorable claim:

\[
\text{Crotone is in the arch of the Italian boot. (6)}
\]

There is no such thing as the Italian boot. Italy itself is certainly not footwear. Even if you had the shoe size, you could not use it that way: like so many non-boots, the country lacks such essential prerequisites as a sole, a heel and, indeed, an arch.

Walton is under no misapprehensions in this regard. He does not really believe that

\( q_6: \text{The region separating the Adriatic and the Tyrrhenian Sea, which is in actual fact the locus of the landmass of Italy, is instead occupied by a boot of vast proportions.} \)

But he is talking as if this story, or something like it, were true. It is the contextual presupposition that
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is exculpated from $p_6$ to extract a piece of information about $S_6$.

$p_6$: Crotone is in the arch of the Italian boot.

$S_6$: The locations of Italian cities and regions

To express the geographical information (6) conveys literally, I must first dub the region of the globe that the boot’s arch occupies according to $q_6$ “Arcatania”. There goes:

$r_6$: Crotone is in Arcatania.

Let’s check that $S_6(p_6|q_6) = r_6$. The message $r_6$ answers the question where on the globe Crotone is. This question is part of $S_6$, which gives us Aboutness. If $q_6$ were true, the arch of the Italian boot would cover Arcatania. So given $q_6$, any town built on that arch is in Arcatania and vice versa, making $p_6$ and $r_6$ equivalent (Equivalence). Finally, $q_6$ is compatible with any distribution of cities and regions on the Italian landmass, so we have Independence. (Or enough Independence, anyway: $q_6$ does entail Italy is not, for instance, a guitar-shaped island in the Pacific.)

Walton observed that (6) is an instance of a more general phenomenon, which he calls prop-oriented make-believe. In make-believe games, children can turn into fearsome knights, sticks into swords, and scooters into horses. The sticks, the scooters and also the children are props in the game. Developments in the real world (the world of the props) correspond in more or less systematic ways to developments in the fictional situation these props represent: for Ben’s knightly alter ego to win the horse race, Ben has to win the scooter race. And when Kelly and her followers evicted Harvey from the treehouse and obtained the cowboy hat, her fictional alter ego usurped the throne and became Queen of the Realm.

The props in a game like this effectively constitute an image of a make-believe world. With (12) and (12*), we saw that it is possible to describe a real painting by talking about the (partly) made up situation it depicts. Walton observed that similarly, we can describe the props of a game by talking about the make-believe they represent. Thus we can tell Kelly her steed has been stabled as a way of informing her that her scooter is in the shed. In such cases we are mainly interested in the props, and our participation in the game is said to be prop-oriented rather than content-oriented. (6) talks about a fictional boot in order to describe the prop depicting it, in this case Italy itself.

A Waltonian metaphor has two essential ingredients: (A) a collection of props, and (B) a game that conjures up a make-believe situation by associating each possible state of the props with a state of that make-believe situation. In the context of such a game, any claim $p$ about the make-believe situation acquires a metaphorical meaning $r$ about the props to the effect that the props are in one of the states that the game associates with the truth of $p$. 
As the analogy with (12) suggests, there is a neat general way of accounting for Waltonian metaphors using the exculpature account. All we have to do is to slot its two ingredients (game and Props) into the two contextual parameters the exculpature account requires:

\( g \): The make-believe state matches the state of the props in thus-and-such a way.
\( P \): The props

If we think of the props as constituting an image of the make-believe situation, then the subtraction of \( g \) is a veracity exculpature. As long as the state of the props is independent of the make-believe situation they represent, the mapping \( p \mapsto P(p|g) \) then takes us from claims about the make-believe situation to their metaphorical meaning as specified above.

Admittedly the Independence condition puts limitations on this version of Walton’s account that are not inherent in the original. Consider the mirror game: in this game, we pretend the contents of the room are a mirror image of what they actually are. Everything in the room is both a prop and part of the make-believe situation. In this game, we say “Alice goes left” in case she actually goes right. The metaphorical meaning of this statement is Alice goes right. But this is incompatible with its literal meaning. Thus the exculpature version of Walton is structurally incapable of getting this prediction: as long as \( P(p|g) \) is contingent it must be compatible with \( p \) (see p. 20; see also Yablo 2014, §12.3).

If we think of the props as constituting an image, the props in the mirror game depict themselves. In fact, they structurally misrepresent themselves, and that is where the trouble lies: in virtue of going left, Alice represents herself as going right. Thus, any asymmetry in the room is incompatible with the mirror game’s veracity assumption \( g_M \) and \( P_M(p|g_M) \) only has a truth value in worlds with a perfectly symmetric room. (Incidentally, the mirror game is perfectly boring in those worlds, as it involves no make-believe.) Can we work around this flaw? Perhaps. But for now we must skip over the rabbit-holes of self-reference, on to the next and final application.

**Example (7): Applied mathematics**

Of all the insipid morality tales we foist on our children in the name of education, none outrank the Fable of Arithmetic on the primary school syllabi. A brief reminder of its central plot points:

*The Myth of the Natural Numbers (mn)*: Beyond the outer reaches of our physical universe, there is the Platonic Realm of Mathematics. Amongst the denizens of this land are the unchanging Natural Numbers, arranged on the Natural Number Line. All the way on the left sits the number Zero. Immediately to Zero’s right sits One. To the right of One sits Two, and so on. To the immediate right of every natural number sits another natural number.
Every natural number numbers the class of natural numbers seated to its left and all and only classes equinumerous to that class. The End.

The key to mn’s lasting success lies not in its literary qualities or its uplifting morals, but in its usefulness as a conversational exculpature. As our Holmes example (2) illustrates, exculpature allows us to exploit just about any well-known story for the purpose of describing the world. But mn and its many sequels and variants are especially notable in this regard, enabling the elegant expression of complex generalisations for which no straightforward non-mathematical paraphrase is available.

In order to explain how this works, we need to treat mn as contingent. So we will assume there are worlds with a platonic realm and worlds without one, just as we took there to be worlds with and without Sherlock Holmes before, and worlds with and without unicorns. Some will complain that mn is a necessary falsehood. Others that it is a necessary truth. As before, this is not the place to address such metaphysical quibbles in detail. But I will remark that many of them can be assuaged by considering a zoological variant of mn: just replace the numbers in the above passage with squirrels, arranged on an infinite Squumber Line. The resulting tale of squumbers, while implausible, is surely a contingent hypothesis! As it happens, that story can do all the same work as mn.

One reason mn is effective as an exculpature is that it concerns a parallel universe that is in every way isolated from our own cosmos. In our applications so far, we have had to be careful that the contextual suppositions we wanted to exculpate did not interfere too much with the subject matter under discussion, since that interference limits the area of logical space on which the resultant message has a truth value. Well, in this subsection we can throw caution to the wind, because mn has no bearing on the whole vast subject matter that is

C: our concrete universe

So if we are talking about any topic that is part of C, we can appeal freely to mn in describing it. That is because any exculpature of mn to C satisfies Independence. In this way, mathematical exculpature always delivers full propositions about the concrete world.

To see how this goes, consider Frege’s example (7).

\( p_7: \) The number of Jupiter’s moons is four.

\( p_7 \) is true only in worlds containing the number Four. By exculpating mn from \( p_7 \) we get a message \( r_7: \)

\( r_7: \) Jupiter has a moon and another and another and another and those are all its moons.

The truth value of (7)’s literal content \( p_7 \) is affected by changes in the platonic realm: remove number Four (or squirrel Four) to make it false. But there’s no way to change \( r_7 \)’s truth value without moving around some very large rocks in our own universe, so \( r_7 \) is purely concrete (Aboutness).
That leaves Equivalence. Suppose $mn$ and $p_7$ are both true. Then the class of numbers to the left of Four and the class of Jovian moons are equinumerous, so that is there is a 1-1 function $f$ between them. Hence there is a moon $f(Zero)$, a moon $f(One)$, a moon $f(Two)$ and a moon $f(Three)$, all distinct, and those are all the Jovian moons. That gives us $r_7$. Conversely, suppose $mn$ and $r_7$ are both true. Call one moon $M_{Zero}$, another $M_{One}$, another $M_{Two}$ and the final one $M_{Three}$. By $r_7$ we have named all of them, and thus the function $M_i \mapsto i$ is a bijection, whence we gather from $mn$ that Four is the number of Jovian moons, so that $p_7$ is true. Thus Equivalence holds and hence $C(p_7 \land mn) = r_7$.

Let’s consider some other inputs to the map $p \mapsto C(p \land mn)$. Clearly if $p$ is already concrete, $C(p \land mn) = p$ since $p$ is about $C$. What if $p$ is a purely mathematical claim? (That is, one about the denizens of the platonic realm.) Well, for those statements $C(p \land mn) = \top$ when $p$ is entailed by $mn$, $C(p \land mn) = \bot$ when $\neg p$ is entailed by $mn$, and $C(p \land mn)$ is undefined otherwise. So in particular, the concrete upshot of any mathematical theorem is simply the necessary truth. From the perspective of a fictionalist, this is a welcome result because it reconciles the fictionalist thesis that many mathematical theorems literally express falsehoods with the universally held intuition that they express necessary truths. Both may well be true: the necessary truth just happens not to be the semantically determined content of the theorem, but rather the message it expresses through mathematical exculpature.

Speaking as a fictionalist, Yablo likes to suggest that mathematical exculpature serves to recover nuggets of truth from the larger falsehoods we wrap them in (e.g. Yablo 2014, p. 205). It is wonderful imagery, but a bit misleading. Mathematical exculpature does not always, or even typically, have this effect. By fictionalist lights, unwrapping ‘No kiwi has an even number of seeds’ reveals a falsehood hidden in a larger truth. Replace the kiwi with an avocado, and you get a truth wrapped in a truth. Statement (7) wraps a falsehood in a falsehood (writing in 1884, Frege couldn’t have known better, but Jupiter has at least sixty-seven moons).

So it is better to say that mathematical exculpature reveals the concrete nugget wrapped inside a larger abstraction $p$. Emphasising topic change over truth value change also clarifies why exculpature should be of interest to platonists and nominalists alike. For platonists, mathematical exculpature preserves truth value, because $p$ and $C(p \land mn)$ always have the same truth value in $mn$-worlds. But platonists can agree that $p$ and $C(p \land mn)$ differ in subject matter: while the former has a bearing on the platonic realm, the latter is solely concerned with concreta.

Here is a question both platonists and nominalists must face: whether they exist or not, why should causally inert, non-physical entities like numbers be of interest to natural scientists who study our
physical universe? The present account suggests a simple answer: they are not. Natural scientists are interested in concrete matters, and the talk of mathematical objects just serves as an efficient way to express complex hypotheses about those concrete matters. The concrete information conveyed by a particular mixed mathematical expression is the same whether the abstract entities it refers to exist or not, so working scientists can ignore that ontological question.

Or can they? Mathematics is not just used to make claims about the world, but also to reason about it. Physics is full of extended derivations in which mathematical and physical talk interact seamlessly. In checking the validity of such derivations, a working physicist seems to treat the applicable mathematical theorems as literally true. How can such reasoning be reliable if it is actually based on falsehoods? For an answer to that question, we turn to our final section.

V. The Loose Logic of Exculpature

When contextually presupposing something, even if it is not seriously endorsed, we act in many ways as if it were true. In particular, we often reason as if what we said was literally true. For instance, the following piece of reasoning is perfectly persuasive:

\[ p_A: \text{Lazio is in the knee of the Italian boot, and Calabria is in the toe.} \]
\[ p_B: \text{The knee of the Italian boot is north of its toe.} \]
\[ \therefore c: \text{Lazio is north of Calabria.} \]

Why should arguments like this one be compelling? Sure, the argument is valid, construed literally — it is impossible for the premises \( p_A \) and \( p_B \) to be true and the conclusion \( c \) to be false. But since, for lack of a boot, both premises are literally false, that does not by itself justify the conclusion. Yet this type of reasoning ‘within’ a metaphor, or with premises that are only loosely true, occurs all the time. How can it be reliable?

To discuss this matter, it will help to introduce some notation. Fixing a particular contextual supposition \( q \) and a particular subject matter \( S \), we can represent the map \( p \mapsto S(p \mid q) \) with a partial unary operator “\( \bigcirc \)” (to be pronounced “nug”) that unwraps literal contents to reveal the relevant ‘nuggets’ inside. The symbol “\( \bigcirc \)” is meant to suggest a loosening gesture. Depending on the application, \( \bigcirc \) takes you from literal to metaphorical, from strict to loose, from attributive to referential, from platonist to concrete. Or, interpreting “\( \bigcirc \)” as the operator \( p \mapsto S_{\delta}(p \mid \delta \theta) \), from boot-talk to geography: for instance, \( \bigcirc p_A \) is a message about the location of Lazio and Calabria.

Our puzzle can now be put as follows: in making the above argument, we are implicitly making the argument that \( \bigcirc p_A, \bigcirc p_B \therefore \bigcirc c \). But why should this other argument cut any ice? As it turns out, the
present account has an answer: arguments that are valid on a literal construal are guaranteed to be valid on a loose or metaphorical construal, too. In our case, the validity of $p_A, p_B \vdash c$ entails the validity of $\mathcal{O} p_A, \mathcal{O} p_B \vdash \mathcal{O} c$. Thus the fact that $\mathcal{O} p_A$ and $\mathcal{O} p_B$ are true guarantees that $\mathcal{O} c$ is true. And since $c$ is already about $S_6$, $\mathcal{O} c = c$, so $c$ is also literally true. In general, we may conclude from the metaphorical or loose truth of the premises of a valid argument that its conclusion is metaphorically or loosely true as well. In the special case where the conclusion is already wholly relevant without exculpature, its loose and literal readings are identical, so that the literal truth of the conclusion is also guaranteed. This answers the question on which we ended the last section: whenever a partly mathematical derivation leads to a conclusion about the physical world, that conclusion is already supported by just the concrete, non-mathematical content of the premises (see also Dorr 2010). Thus, such derivations are reliable even if they are based on literally false premises.

Before proving the result formally, we can try to understand intuitively why it holds. As $p_A, p_B \vdash c$ is valid, the intersection of the premises, thought of as an area of logical space, is included in the conclusion. So this inclusion must also hold as restricted to boot worlds: the area where $p_A \models q_6$ and $p_B \models q_6$ are both true is included in the area where $c \models q_6$ is true. The completion by $S_6$ essentially inflates each partial proposition to fit the region $\{w : w \sim_{S_6} v \text{ for some } v \in q_6\}$, while retaining their relative logical ‘shapes’. Thus it preserves the inclusion, and the region where $S_6(p_A \models q_6)$ and $S_6(p_B \models q_6)$ are both true is included in the $S_6(c \models q_6)$-region. So $\mathcal{O} p_A, \mathcal{O} p_B \vdash \mathcal{O} c$ is valid.

Below the general result (j) is stated in terms of multiple-conclusion sequents “$X \models Y$”. Recall that such a sequent is true just in case the disjunction of $Y$ follows from the conjunction of $X$. Thus, if $X$ and $Y$ are sets of (partial) propositions, all of which have truth values on the same region of logical space, $X \models Y$ if and only if in every world where all $x \in X$ are true, some $y \in Y$ is also true.

For any propositions $p_i, i \in I$ and $c_j, j \in J$ such that $\mathcal{O} p_i$ and $\mathcal{O} c_j$ are well-defined,

$$\begin{align*}
\text{If } \{p_i\}_{i \in I} & \models \{c_j\}_{j \in J} \text{ then } \{\mathcal{O} p_i\}_{i \in I} & \models & \{\mathcal{O} c_j\}_{j \in J} \quad (j)
\end{align*}$$

Proof. Without loss of generality, we may take the set of all worlds to be $\{w : w \sim_{S_6} v \text{ for some } v \in q\}$. Then $\mathcal{O} p_i = S(p_i \models q)$ and $\mathcal{O} c_j = S(c_j \models q)$ are full propositions. We need to show that $\{S(p_i \models q) : i \in I\} \models \{S(c_j \models q) : j \in J\}$, i.e. that $\cap S(p_i \models q) \subseteq \cup S(c_j \models q)$. First note that this inclusion holds as restricted to $q$-worlds:

A. $\cap p_i \subseteq \cup c_j$ (given: this is the assumption that $\{p_i\}_{i \in I} \models \{c_j\}_{j \in J}$)
B. $\cap (p_i \cap q) \subseteq (\cup c_j \cap q)$ (from A, intersecting both sides with $q$)
C. $\cap (p_i \cap q) \subseteq \cup (c_j \cap q)$ (from B)
D. $\cap (S(p_i \models q) \cap q) \subseteq \cup (S(c_j \models q) \cap q)$ (from C, using the fact that $S(a \models q)$ and $a$ match in $q$-worlds)
E. $\cap (S(p_i \models q)) \cap q \subseteq (\cup (S(c_j \models q)) \cap q$ (from D)

Now, let $w$ be any world in $\cap S(p_i \models q)$. Then for any $i$, $w$ is in $S(p_i \models q)$. Pick a $v \in q$ so that $w \sim_{S_6} v$ (thanks to our simplifying assumption, we can always do this). Since $S(p_i \models q)$ is about $S$ and $w \in S(p_i \models q)$, we have $v \in S(p_i \models q)$.
Hence $v \in S(p, q) \cap q$. Thus $v \in \cap_i S(p, i, q) \cap q$. So by (E), $v \in (\cup_j S(c, j, q)) \cap q$. Therefore $v \in S(c, i, q)$ for some specific $j \in J$, whence also $w \in S(c, i, q) \subseteq \cup_j S(c, j, q)$. So $\cap_j S(p, i, q) \subseteq \cup_j S(c, j, q)$, which is what we set out to show.

The transparency observed in section I can be viewed as a consequence of (j). Take the case of conjunction: since $(a \land b)$ entails its conjuncts $a$ and $b$, $\cup(a \land b)$ entails $\cup a$ and $\cup b$. Conversely, $a$ and $b$ jointly entail $(a \land b)$, so that $\cup a$ and $\cup b$ jointly entail $\cup(a \land b)$. Thus $\cup(a \land b)$ is equivalent to $(\cup a \land \cup b)$. That’s why the loose reading of “Emma and Jack both weigh five stone” is the conjunction of the loose reading of “Emma weighs five stone” and the loose reading of “Jack weighs five stone”. To establish the general result, we first need a lemma:

The space of propositions $p$ such that $\cup p$ is well-defined is a complete Boolean Algebra.

(That is, it’s closed under negation, arbitrary conjunction and arbitrary disjunction). (k)

Proof. Assume that $\cup p_i$ is well defined for all $i \in I$. Recall that by the definition of a completion (df. (h)), $\cup p$ is well-defined if and only if $p \cup q$ is about $S$, that is if and only if $p \cup q$ is the restriction to $q$ of some full proposition $s$ about $S$ (df. (e)). So there’s a proposition $s_i$ about $S$ for each $i \in I$ such that $p \cup q = s \cup q$. It follows that $(\neg p) \cup q = (\neg s) \cup q$, $(\bigcap_{i \in I} p_i) \cup q = (\bigcap_{i \in I} s_i) \cup q$ and $(\bigcup_{i \in I} p_i) \cup q = (\bigcup_{i \in I} s_i) \cup q$. Now since the $s_i$ are unions of $S$-cells, $\neg s_i \bigcap_{i \in I} s_i$ and $\bigcup_{i \in I} s_i$ must also be unions of $S$-cells, which is to say that they’re also about $S$. Thus $(\neg p) \cup q$, $(\bigcap_{i \in I} p_i) \cup q$ and $(\bigcup_{i \in I} p_i) \cup q$ are all about $S$, so that $\neg p$, $\bigcap_{i \in I} p_i$ and $\bigcup_{i \in I} p_i$ are well-defined. ■

Putting (j) and (k) together, we get:

‘$\cup$’ is transparent to Boolean operators:

A. $\neg \cup p = \cup \neg p$
B. $\bigcap_{i \in I} \cup p_i = \cup \bigcap_{i \in I} p_i$
C. $\bigcup_{i \in I} \cup p_i = \cup \bigcup_{i \in I} p_i$

for any propositions $p$ and $p_i$ such that $\cup p$ and $\cup p_i$ are well-defined. (l)

Proof. (A): $p$ and $\neg p$ are inconsistent, i.e. $\{p, \neg p\} = \emptyset$. Since $\cup p$ is well-defined, so is $\cup \neg p$ by result (k). So by (j), $\{\cup p, \cup \neg p\} = \emptyset$, whence $\cup \neg p = \neg \cup p$. On the other hand, $\emptyset = \{p, \neg p\}$. By (j), $\emptyset = \{\cup p, \cup \neg p\}$, and so $\neg \cup p = \cup \neg p$.

(B): $\bigcap_{i \in I} p_i = p_i$ for every $j$. Now (k) tells us $\bigcap_{i \in I} p_i$ must be well-defined since the $\cup p_i$ are. Hence by (j) we get $\bigcap_{i \in I} p_i = \cup p_i$ for every $j$. Thus $\bigcup_{i \in I} p_i = \bigcap_{j \in J} \bigcup_{i \in I} p_i$. Conversely, note that $p_i \bigcup_{i \in I} p_i = \bigcap_{j \in J} \bigcup_{i \in I} p_i$. Again by (j), it follows that $p_i \bigcup_{i \in I} p_i = \bigcap_{j \in J} \bigcup_{i \in I} p_i$. Hence we have $\bigcap_{i \in I} \cup p_i = \cup \bigcup_{i \in I} p_i$.

(C) can be established in the same way as (B), and it also follows from (A) and (B) by De Morgan’s. ■

Sometimes, we allow our descriptions of the world around us to be visited by creatures from another place, whether they be Egyptian goddesses, titanic boots, Baker Street detectives, squumbers, baby unicorns or men with martinis. Often when we do this, we barely notice that we are not speaking literally. The simple logic of $\cup$ helps explain the relaxed attitude. Result (j) tells us that logically
speaking, the relevant messages behind our words are a perfect mirror image of the fantasies we wrap them in.

(Thanks to Simona Aimar, Sam Carter, Hartry Field, Jeremy Goodman, Brian King, Mandy Simons, Philippe Schlenker, Benjamin Spector and Steve Yablo for their questions and stimulating conversation. Thanks to two anonymous reviewers, Chris Barker, Justin Bledin, Kyle Blumberg, Paul Boghossian, Ben Holguin, Stephen Schiffer, and especially Martín Abreu Zavaleta and Matt Moss for their insightful comments on earlier drafts. For their tireless assistance and encouragement special thanks are due to Cian Dorr and also to Jim Pryor, who patiently spurred on the development of these ideas long before they were any good.)

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