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Source: *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, Vol. 1992, Volume One: Contributed Papers (1992), pp. 23-35
Published by: The University of Chicago Press on behalf of the Philosophy of Science Association
Stable URL: http://www.jstor.org/stable/192741
Accessed: 06/01/2010 15:33

Darcy's Law and Structural Explanation in Hydrology

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1. Introduction

According to a recent argument, models play two essential roles in the argumentative structure of solid state physics and chemistry (Hofmann 1990). On the one hand, models are the culmination of phenomenological description. That is, models are idealized representations of the molecular structures thought to be causally responsible for the processes experimentally monitored and measured. Secondly, theoretical physicists and chemists require that models ultimately be cast in a mathematical form appropriate for the application of the Schroedinger equation. In this respect models become the means through which the Schroedinger equation gives a theoretical unity to what would otherwise be a disparate set of empirical phenomenological laws and descriptions with limited scope. That is, it is an important theoretical goal to show that experimentally generated phenomenological laws can be approximately derived through an application of the Schroedinger equation to a necessarily idealized and simplified mathematical description of the relevant system. The two functions of models are not incompatible, but they do reflect two distinct theoretical orientations toward the interpretation of data.

The present paper extends these themes into the domain of hydrology. We consider a specific phenomenological relationship known as Darcy's law. Nancy Cartwright's discussion of phenomenological laws and fundamental laws provides some useful analytic vocabulary and initial insight, particularly when amplified by Bogen and Woodward's distinction between data and phenomena (Bogen and Woodward 1988, Woodward 1989, and Cartwright 1983). We thus begin with a summary of these topics and emphasize the potentially ambiguous nature of "explanation" due to the multiple functions of models. After a brief account of the origins and subsequent history of Darcy's law, we turn to a detailed analysis of the status of the law for both theoretical and applied hydrology. Two principal conclusions result. First, "structural explanation", in the sense specified by Ernan McMullin, is an accurate description of the derivations of Darcy's law carried out by theoretical hydrologists (McMullin 1978). Secondly, however, hydrologists themselves often refer to the derivation of phenomenological laws as a particularly extended exercise in the *description* of relevant phenomena rather than as an explanation of them. As a result, McMullin's realist in-

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terpretation of the implications of structural explanation must be modified in the case of hydrology. This is primarily due to the fact that, in many cases, the emphasis in hydrology is not on the hypothetical affirmation of new theoretical entities such as electrons or moving tectonic plates; hydrologists more often have reason to rely upon a multiplicity of conceptual models with idealized structures that are known to only imperfectly approximate actual materials. Finally, Darcy's law, when combined with the equation of continuity, also becomes the basis for a derivation of Laplace's equation applicable to hydraulic head. Laplace's equation and the relevant boundary conditions then serve as a mathematical model that is solved and applied in an instrumentalist fashion to predict and influence groundwater phenomena. A taxonomy of how models function with respect to Darcy's law thus offers insight into both the argumentative structure and the explanatory and descriptive goals of hydrology.

2. Models, Phenomena, and Structural Explanation

Discussion of theoretical explanation among philosophers of science has benefitted considerably from Bogen and Woodward's insistence that specific data are not the target of scientific explanation (Bogen and Woodward 1988, Woodward 1989, and Hofmann 1990). Data are too idiosyncratically dependent upon unique characteristics of specimens and instrumentation to warrant explicit explanatory attention. Rather, data provide evidence for the phenomenological relationships, conditions, or laws that are the potential subject matter for explanation. Theorists traditionally pay particularly close attention to those phenomena that are described by phenomenological laws.

In *How the Laws of Physics Lie*, Nancy Cartwright argued that the explanation of phenomenological laws in physics typically requires the application of fundamental laws such as Schroedinger's equation to an appropriately "prepared" model of the domain (Cartwright 1983). Starting from this combination of a fundamental law and an idealized model, physical and mathematical approximations generate the derivations that Cartwright originally referred to as theoretical explanations.

The role of models in this account has been misunderstood in a manner that requires some clarification. Kroes and Sarlemijn claim that the distinction between phenomenological laws and fundamental laws is not sufficiently precise (Kroes and Sarlemijn 1989). Using the example of the Van der Waals equation, they point out that the law is phenomenological in the sense that its two parameters must be specified experimentally. On the other hand, they also claim that the law could be considered to be fundamental because it is "derived from first principles" (Kroes and Sarlemijn 1989, p.324). But derivation from fundamental laws does not necessarily generate additional fundamental laws. More typically, the fundamental laws are applied to idealized models together with mathematical approximations. The results of these operations are precisely the phenomenological laws that Cartwright offers as our most reasonable candidates for laws of nature. Kroes and Sarlemijn also mistakenly disagree with Cartwright's provocative thesis that the fundamental laws of physics are false. Her point was that these laws say nothing specific about the real world until they are applied to suitably prepared models. Kroes and Sarlemijn claim that "Cartwright seems to confuse the validity of the boundary conditions with the validity of the fundamental laws" (Kroes and Sarlemijn 1989, p.326). A more correct rendering would emphasize that the approximate derivation of phenomenological laws by means of idealized models does not transform those phenomenological laws into fundamental laws; nor does an acknowledgment that models are highly idealized make fundamental laws, by contrast, "true" laws of nature.

J. Hofmann has clarified the functions of models in derivations of phenomenological laws similar to the Van der Waals equation (Hofmann 1990). On the one hand, models are a culminating stage in the description of phenomena; that is, models stipulate and emphasize selected structural aspects of the domain. Secondly, however, models provide this description in a mathematical form amenable to the application of a fundamental law. In this sense, models are a necessary requirement for what Cartwright called theoretical explanation in *How the Laws of Physics Lie*. In the terminology of that book, the two functions of models thus contribute to both phenomenological description and theoretical explanation.

There are good reasons to clarify these conclusions by emphasizing the more specific concept of structural explanation. Cartwright herself no longer considers the derivations she described in *How the Laws of Physics Lie* to constitute explanations. In a 1989 paper she decided to "reserve the word 'explanation' for scientific treatments that tell why phenomena occur" (Cartwright 1989, p.282). She attributes her revision to the fact that influential physicists such as Edwin Kemble hold that "the function of theoretical physics is to describe rather than to explain" (Cartwright 1989, p.275). From this point of view, the derivation of phenomenological laws from fundamental laws is a demonstration that there is an economical way to classify these laws as various applications of a few fundamental principles; explanatory causes are not addressed.

On the other hand, one of the points that emerged from a study of transition metal oxide models is that the wide variety of modeling techniques and motivations means that it is virtually impossible to give a general characterization that summarizes the relationship between phenomena and models (Hofmann 1990). But in some cases, surely, the structure stipulated by a model can be acknowledged to be part of an explanation of why the associated phenomena take place. This is in fact the interpretation Ernan McMullin refers to as structural explanation. His characterization is worth quoting (McMullin 1978, p.139):

> When the properties or behavior of a complex entity are explained by alluding to the structure of that entity, the resultant explanation may be called a structural one. The term "structure" here refers to a set of constituent entities or processes and the relationships between them. Such explanations are causal, since the structure invoked to explain can also be called the cause of the feature being explained.

Two points are in order here. First, the model alone seldom provides structural explanation. That is, phenomena typically are explained by applying fundamental laws to the structure stipulated by the model. In this sense, the combination of model and fundamental principles constitutes a theory of the phenomena. Secondly, McMullin takes the success of structural explanations to warrant an interpretation of the relevant model as a more or less accurate description of reality. In particular, when revisions of a model provide increasingly accurate derivations of phenomenological laws, McMullin argues that it is justifiable to conclude that the real structure responsible for the phenomena is approximately known. Before considering how these ideas apply to theoretical hydrology, note the following example McMullin cites (McMullin 1978, p.147):

> Geologists assume that a successful macrostructural explanation of such surface phenomena as sonar pulses can give reason to believe in the existence of sub-surface structures like pockets of water or oil. These structures play a role in the explanation of the phenomena similar to that played by molecular structures in the explanation of chemical phenomena. But in the geological case, the existence of the water or the oil can be directly ascertained. And the geologists' belief in the ontological reliability of the retroductive form has turned out to be amply justified.

Although the structure cited in this example is a macroscopic one, theoretical hydrology also makes use of conceptual models for the detailed and unobservable structure of soils and fluids. The resulting complications are best considered through a specific example.

3. Darcy's Law

In 1856 Henry-Philibert-Gaspard Darcy published a lengthy assessment of a proposed upgrading of the public water system for the French city of Dijon (Darcy 1856). His investigation of fountains called for information concerning the flow of water through sand filters; in an appendix to his report he included a description of his experimental work on this subject. His data analysis resulted in a relationship that has since come to be known as Darcy's law; the law is well known to hydrologists and Darcy's appendix has been partially translated into English (Freeze and Back 1983, pp.14-20).

Darcy included a diagram and a description of his apparatus. The sand representing the filter was contained in a vertical tube said by Darcy to be .35 meters in diameter and 2.5 meters in length (although the diagram labels the length at 3.5 meters). Darcy performed a sieve analysis on his sand and estimated the porosity at 38%. Care was taken in packing the columns to minimize entrained air. The height of sand could be varied above a screen and grillwork located .2 meters above the bottom of the column. Water entered the sand column from an adjacent hospital through a pipe near the top of the column and exited through a faucet in the chamber below the column. Both entry and output rates could be regulated. To record pressures, two mercury manometers were installed in the column above and below the sand. For the purposes of these measurements, the bottom of the sand was taken as the datum plane with elevation zero. This elevation coincided roughly with the bottom of the lower manometer arm.

In modern terminology, total hydraulic head is the sum of elevation head and pressure head. Elevation head is the distance between datum plane and the point of interest. Pressure head is the length of fluid registered by a manometer installed at the point of interest. Although Darcy did not explicitly explain all of his terminological conventions, his usage was consistent given the specific circumstances of his apparatus. In particular, since Darcy chose the bottom of the sand as his datum plane, the elevation head at the bottom of the filter was zero. The bottom of the column was open to the atmosphere; thus, using gage pressure conventions, the pressure head at the lower manometer position was zero. As a result, Darcy's calculations of hydraulic head amounted to adding the length of his sand column to the height of fluid in the upper manometer arm.

Darcy initially tabulated four series of data for sand columns with heights of .58, 1.14, 1.71 and 1.70 meters respectively. Water-hammer in the hospital plumbing forced him to use a mean value for the level of mercury in the upper manometer arm. This upper manometer value was reported as the mean pressure for each experiment after he converted his mercury pressure readings to equivalent heights of water.

Darcy performed as few as three and as many as ten different measurements of flow rates for four specific heights and types of sand; in each case he gradually increased the height of water in the upper manometer arm (the mean pressure) by adjusting his inflow and outflow faucets. In his first four sets of measurements, the lower end of the column was open to atmospheric pressure. Darcy observed that, for any given elevation head, the outflow volume invariably increased proportionally with the pressure head.

He then averaged the ratios of hydraulic head (Darcy's *charge*) to flow rate for each set of measurements, obtaining four proportionality constants. Darcy attributed the variation among the constants to differences in grain size and purity between the sands in different columns. He also claimed without argument that the data showed that the flow rates varied in inverse proportion to length of sand column. This conclusion was not obvious since the data provided did not include multiple measurements at fixed heads for different column lengths; however, it is substantiated by comparison of his data for differing column lengths with roughly equal mean pressure values. Darcy then performed a similar set of experiments differing mainly in that the pressure at the bottom of the column could be varied widely above or below atmospheric. He was satisfied that his earlier conclusions held in these cases as well.

One point that stands out in this analysis is the clarity of the distinction between data and phenomena. Darcy did describe the texture of his sand samples in some detail. However, aware of the unique nature of his apparatus and the variable effect of heterogeneities, he attributed little importance to the specific magnitudes of individual data readings. Rather, he emphasized two aspects of the general phenomenon he claimed his data supported: the proportionality of flow rate to total hydraulic head, and the inverse proportionality of flow rate to column length (Darcy 1856, p.593).

> Il parait donc que, pour un sable de même nature, on peut admettre que le volume débité est proportionnel à la charge et en raison inverse de l'épaisseur de la couche traversée.

Darcy assembled his conclusions in the following equation:

$$q = k(s/e)(h + e \pm h^*)$$

where:

q = rate of water flow (volume per time)

k = a coefficient dependent on the "permeability" of the sand

s = cross sectional area of the sand filter

e = length of sand filter

h = reading of the upper manometer arm

h^* = reading of the lower manometer arm

At this point, Darcy made use of the implications of his datum plane convention. Only under this convention, Darcy's law reduces to:

$$q = k\,(s/e)\,(h + e)$$

In modern format, using a particular sign convention, Darcy's law is usually written as:

$$Q = -KA\, dh/dl$$

where:

Q = rate of water flow (volume per time)

K = hydraulic conductivity

A = column cross sectional area

dh/dl = hydraulic gradient, that is, the change in head over the length of interest.

The law is often transformed by dividing through by the cross-sectional area and is then restated as:

$$q = Q/A = -K\ dh/dl$$

where q now has the dimensions of a velocity, and is referred to as the Darcy, or superficial, velocity.

Perhaps due to the ambiguous nature of some of Darcy's terminology and conventions, there was some initial confusion over the content of his law. This was cleared up largely due to the efforts of M. King Hubbert, who wrote several influential essays on Darcy's law beginning in 1940 (Hubbert 1969). Hubbert had been quite dismayed to find that widely used and respected texts in hydrology often stated Darcy's law as a proportionality between flow rate and pressure gradient alone, neglecting elevation head. His essays brought about a new consensus concerning the fact that hydraulic head functions as the potential in the law, and that total head is the sum of elevation head and pressure head.

On the other hand, there seems never to have been any doubt concerning the empirical basis of Darcy's law. For example, to quote an authoritative textbook by Freeze and Cherry, "Darcy's law is an empirical law. It rests only on experimental evidence" (Freeze and Cherry 1979, p.17). Darcy's law fully satisfies the requisite criteria to be considered a phenomenological law; it thus is a potential candidate for structural explanation.

In this respect it is somewhat surprising that in a recent discussion of Darcy's law, Shrader-Frechette repeatedly refers to it as either a "theoretical" or "fundamental" law (Shrader-Frechette 1989). She emphasizes the "idealized" nature of the law by claiming that it is "experimentally" verified only by applying the Bernoulli equation to an idealized model. She concludes that "this fundamental or theoretical 'verification' is highly idealized, since actual flow velocity is a function of the microstructure of the medium through which the water is flowing" (Shrader-Frechette 1989, p.335). Similarly, she claims that (Shrader-Frechette 1989, p.337):

> apart from the falsity of Darcy's Law on all three levels (micro, molecular, macro) what is significant is that 'corrections' to it do not come from the theory built into the law itself, but from phenomenological or observational factors not deducible from the theoretical or fundamental law.

There are several misleading aspects to these comments. First, Darcy's law is a phenomenological law rather than a fundamental law. Its limited accuracy and scope were recognized by Darcy in 1856 and have remained apparent to hydrologists ever since. The experimental discovery of the law, as carried out by Darcy himself, is a matter of quantitative measurements with specific soil samples and has no reliance on fundamental laws or theory. Furthermore, although hydrologists do distinguish velocities on several different orders of magnitude, Darcy himself was interested only in macroscopic discharge and its proportionality to directly measurable properties of his filters. The Darcy velocity is a macroscopic parameter influenced by quite possibly unmeasurable microscopic factors.

Although a full discussion of the historical development of Darcy's law is beyond the scope of this paper, one aspect that bears directly on present concerns is the recognition of both lower and upper bounds for the dependable use of the law's stated relationships. Briefly, some authors consider a lower limit below which there is no flow, positing the existence of a minimum threshold hydraulic gradient to motivate flow. The upper limit is of more practical significance; the law has been found, once again experimentally, to be inappropriate when the flow regime is not both laminar and dominated by viscous forces. In laminar flow the molecular velocity vectors are uniformly parallel to macroscopic flow. The determination of laminar flow is in turn dependent upon the magnitude of the Reynold's number, a dimensionless ratio of inertial forces to viscous forces. At low Reynold's numbers, viscous forces dominate, and Darcy's law is valid. There follows a transition zone in which inertial forces become more important; Darcy's law cannot be accurately applied to the nonlinear laminar flow in this zone. Flow in the turbulent zone is both nonlinear and nonlaminar, and deviations from Darcy's law can become very large. In many aquifer materials, the assumption of laminar water flow may not cause any inaccuracies. However, flow analysis predictions based upon Darcy's law in the presence of such matrix material as karstic limestones or highly fractured crystalline rocks can lead to large errors of great significance in problems such as contaminant transport. Flow in such cases cannot be described adequately by a linear relationship such as Darcy's law.

A purely practical difficulty is that virtually any matrix considered, even on the laboratory scale, will be heterogeneous to some extent. Relationships have been developed for estimating the composite conductivity for laminar flow through heterogeneous systems. However, field-scale investigations can fail to detect strata of significantly different conductivities. The presence of such heterogeneities does not theoretically prohibit the use of Darcy's law, but it lends uncertainty to generalizations on a large scale. It is for this reason that statistical models employing probabilistic parameter distributions are sometimes used to model the spatial variability of hydraulic conductivity. Of course, Darcy's own samples were heterogeneous to varying degrees. In this context, it can be seen that Darcy's law remains tied to its empirical roots, and that modern questions of its proper scope can be settled only through a similarly experimental approach.

Hydrologists also use Darcy's law to derive Laplace's equation as a governing equation for spatial variation of hydraulic head. Laplace's equation with its accompanying initial and boundary conditions then becomes a mathematical model applicable to a wide variety of specific sites. The solution of Laplace's equation relies upon either analytic methods or computer approximations, methods referred to as analytic and numerical, respectively (Wang and Anderson 1982). In this process, Darcy's law is taken as an experimentally verified relationship; the empirical foundation and limited scope of the law should always be borne in mind.

The difficulty and importance of delineating the valid application of Darcy's law at field scale were major issues in Shrader-Frechette's analysis of the Maxey Flats radioactive waste dump in Kentucky (Shrader-Frechette 1988). Nevertheless, it appears that in her subsequent more specific discussion of Darcy's law she overlooked the empirical basis of the law and then mistakenly interpreted as an "experimental verification" what was actually an example of structural explanation based upon a conceptual model. To clarify this distinction we must look more closely at structural explanation in hydrology.

4. Structural Explanations of Darcy's Law

Theoretical hydrologists draw several distinctions with respect to the models employed in their discipline. Physical models such as sandboxes with particular packing patterns are sometimes constructed in an attempt to replicate specific aspects of conditions encountered in the field. Electric analogue models have been wired to investigate conductivities and flowlines, using the parallel structures of Ohm's Law and Darcy's Law. The utility of these physical models is limited in that they tend to be highly site specific and therefore often do not generate results amenable to generalization. Theoreticians thus emphasize the use of tractable mathematical idealizations.

Mathematical models in their turn can be either deterministic or statistical. As noted earlier, statistical models include parameters that have probabilistic distributions rather than single values. In the case of deterministic models, a common source of motivation is a conceptual model that approximates the structure of the soil and fluids under study. In a survey of the subject, Faust and Mercer include the following description of how conceptual models are chosen to generate deterministic models (Mercer and Faust 1981, p.2).

> The first step is to understand the physical behavior of the system. Cause-effect relationships are determined and a conceptual model of how the system operates is formulated. For ground-water flow, these relationships are generally well known, and are expressed using concepts such as hydraulic gradient to indicate flow direction.

There is a clear resonance here with McMullin's insistence that descriptions of causal relationships can act as a starting point for what ultimately become structural explanations.

But before turning to more specific discussion of conceptual models, another set of distinctions should be noted. Hydrologists follow the conventions of physics and thermodynamics, generally classifying stipulations of structure as falling within one of three possible viewpoints: molecular, microscopic, or macroscopic. The molecular approach is the most detailed in that it stipulates the path of individual molecules within the fluid in motion. Great physical and mathematical precision is required by models on this scale in order to allow application of statistical techniques. The ultimate interests of hydrology are invariably on a larger scale, as in the case of Darcy himself. Therefore, hydrologists often move to a more coarse-grained approach in which the fluid within any particular pore of material is treated as a continuum rather than a collection of localized particles. The resulting microscopic models then represent various constraints placed upon the idealized continuous fluid in the porous medium.

For example, theoretical hydrologist Jacob Bear has collaborated with Bachmat in the invention and analysis of an elaborate microscopic model to represent fluid flow in a porous medium (Bear 1972, p.92). Bear describes the initial stage in this process as follows (Bear 1972, p.24).

> In the present text we shall adopt the continuum approach. Accordingly, the actual multiphase porous medium is replaced by a fictitious continuum: a structureless substance, to any point of which we can assign kinematic and dynamic variables and parameters that are continuous functions of the spatial coordinates of the point and of the time.

Similarly, Freeze and Cherry emphasize the far-reaching ramifications of structural stipulation, as they extend the discussion to macroscopic flow considerations (Freeze and Cherry 1980, p.17):

> This... may appear innocuous, but it announces a decision of fundamental importance. When we decide to analyze groundwater flow with the Darcian approach, it means, in effect, that we are going to replace the actual ensemble of sand

grains (or clay particles or rock fragments) that make up the porous medium by a representative continuum for which we can define macroscopic parameters, such as the hydraulic conductivity, and utilize macroscopic laws, such as Darcy's law, to provide macroscopically averaged descriptions of the microscopic behavior.

Bear's work provides an excellent illustrative example of how a conceptual model becomes the basis for both a structural explanation of Darcy's law and also for the construction of a much more general mathematical model for fluid flow. Before considering Bear's discussion of Darcy's law, his general conception of his own reasoning process should be noted. The following passage is important enough to be quoted at length; this is Bear's description of the two stages of his procedure that follow upon the introduction of a simplified conceptual model (Bear 1972, p.91).

> Once the model is chosen, the second step is to analyze the model by available theoretical tools, and to derive mathematical relationships that describe the investigated phenomenon. These relationships show how the various active variables (fluxes, forces, etc.) depend on each other. They also show which factors have, according to the chosen model, no influence on the investigated problem. The only way to test the validity of laws derived in this way is to perform controlled experiments in the laboratory (or to observe phenomena in nature). Such controlled experiments, which comprise the third step of this approach, will test the validity of the derived relationships among the variables. No theory developed by this approach can be accepted without first being verified by experiments.

Notice that Bear mentions that the goal of this procedure is to "describe the investigated phenomenon", a passage that calls to mind Cartwright's references to Kemble's conclusions about theoretical physics. We will see that Bear does not entirely avoid the term "explain", but he is reticent to use it because of the way models enter into his reasoning. Let us consider his derivations of Darcy's law.

In his most thorough treatment, he begins with the Bear-Bachmat conceptual model (Bear 1972, p.92). The fluid is idealized as an incompressible continuum and the medium is imagined to be a network of interconnected passages and junctions within a solid that is rigid and does not interact with the fluid. Additional idealizations include the assumption that "the fluid loses energy only during passage through the narrow channels and not while passing from one channel to the next through a junction" (Bear 1972, p.93). Bear then applies a complex averaging procedure in order to be able to assign values to dynamic variables within each representative elementary volume of the idealized continuous fluid. Finally, he applies an equation stating the conservation of linear momentum for a fluid system. The result is a general equation of motion which, when simplified for a homogeneous, incompressible fluid with small inertial forces, is an extension of Darcy's law to three dimensional flow in an anisotropic medium (Bear 1972, pp.104-106). Consequently, it is not surprising that Bear sometimes says that the law simply expresses conservation of momentum during fluid flow through a porous medium.

Although this derivation is Bear's most sophisticated analysis of Darcy's law, he also provides a review of several other derivations in which the mathematics is simplified by assuming at the outset that the fluid is homogeneous. He thus provides derivations from a wide variety of different conceptual models: capillary tube models analyzed by means of the Hagen-Poisseuille law, fissure models, hydraulic radius models, and resistance to flow models. Finally, one of his best known derivations uses a statistical model to take into account the disorder of actual porous media prior to averaging the Navier-Stokes equations over a representative elementary volume. In each case a conceptual model stipulates an idealized structure for the porous medium. Principles of conservation of energy or momentum then are applied to the model and Bear arrives at a version of Darcy's law through a series of approximations and idealizations.

It should be clear that each of these derivations provides an example of what McMullin calls structural explanations. Darcy's law is a phenomenological law generated by experimental data. That is, Darcy argued that the rate of water flow through samples similar to those he employed is proportional to hydraulic head gradient. To explain why this relationship holds, and to explore its limitations, idealized structures are postulated in order to carry out mathematical applications of fundamental physical principles such as conservation of linear momentum. Darcy's law follows only through a series of approximations that may include statistical analysis; it thus remains as much a phenomenological law as it was originally. Nevertheless, it has been brought under the explanatory umbrella of fundamental physical principles via analysis of structures depicted by conceptual models.

At this point we might ask what value these approximate derivations of Darcy's law hold for hydrologists. We have already seen that Bear sometimes writes that such a procedure provides an extended "description" of the phenomenon in question. However, in other passages he uses explanatory language. For example, in referring to his set of derivations of Darcy's law, he makes this comment (Bear 1972, p.92, emphasis added):

> In all these cases, the model is presented as an attempt to simulate, and *thus to explain*, phenomena observed in nature or in the laboratory. Sometimes several models are equally successful in explaining the relationship between observed excitations and responses. However, we must emphasize again that the proof of the validity of a model, and the only way to determine coefficients, is always the experiment.

Bear's ambivalence concerning description *versus* explanation is apparent in his account of the ultimate value of derivations based upon conceptual models (Bear 1972, p.92).

> With these thoughts in mind, a question sometimes arises as to why we bother with the model in the first place, since in any case we must eventually go back to the laboratory to determine the required coefficients. The answer is that in applying the conceptual model approach we gain an understanding of the investigated phenomenon and the role of the various factors that affect it. We also gain an insight into the internal structure of the various coefficients appearing in the equations that describe the investigated phenomenon. All this information is needed for planning the laboratory experiments.

These comments suggest a reassessment of McMullin's position that a sequence of increasingly accurate structural explanations provides an "approximately true" description of the causal components of the structure responsible for the phenomena explained (McMullin 1987, pp.59-60). In Bear's case, the major contrast to the scenarios emphasized by McMullin is that there is not necessarily a progressive modification of a single model with increasingly accurate results. Rather, a multitude of different models may be employed simultaneously or sequentially to explicate various aspects of the coefficients found in phenomenological laws. In the case of Darcy's law, for example, the hydraulic conductivity ultimately is not a "constant", but varies with both fluid and soil type. How the value of this coefficient varies with physical conditions is explored through a variety of models without claiming that any one of them provides a full account of the actual conduction process, even with future modifications in mind.

Analysis of the components of the conductivity "constant" in Darcy's law is an interesting example of this procedure. Darcy described the conductivity constant as primarily a function of grain size and sand purity. Modern versions of this coefficient include additional properties of both the fluid and the soil matrix, such as viscosity and tortuosity. These modern expressions do not always neglect interaction between the fluid and matrix. Conceptual models are chosen to reflect the significance assigned to various parameters, forces and relationships. Mathematical analysis of specific models generates experimental tests of these hypothetically dominant parameters.

For example, one of the structural explanations of Darcy's law provided by Jacob Bear is based on the capillary tube conceptual model. In this model, the void space within the solid matrix of the porous medium is imagined as a collection of uniform, parallel cylindrical tubes of diameter δ and length dimension s. The areal porosity, n, is the percentage of void space in a cross sectional area taken normal to the tubes. The fluid density is ρ, and the dynamic viscosity is μ.

The fundamental law to be applied to this model is the relevant version of the conservation of momentum principle, namely, the Hagen-Poisseuille law. Given a hydraulic head of ϕ and steady laminar flow of an incompressible fluid in a single, long cylindrical tube, this law states that:

$$Q = (\pi\delta^4\rho g/128\mu)\partial\phi/\partial s$$

where Q is the volume flow rate through the tube. Applying this equation to the capillary tube model and dividing through by the model's cross-sectional area gives:

$$q = (n\delta^2\rho g/32\mu)\partial\phi/\partial s = (k\rho g/\mu)\partial\phi/\partial s = K\partial\phi/\partial s$$

where $k = n\delta^2/32$.

This relationship is in fact Darcy's law where K is the hydraulic conductivity, and k is the intrinsic permeability but stipulated in terms of porosity and tube diameter. The capillary tube model thus provides insight into the dependency of hydraulic conductivity on two specific properties of the medium. Subsequent experimental measurements of permeability or conductivity provide information about the corresponding properties of the system. At the same time, the relevance of this insight is limited to media that can be approximated fairly accurately by the capillary tube model. The choice of an appropriate model thus is guided in part by decisions about what aspects of the medium are expected to have a major impact on permeability or conductivity. To cite Bear once again (Bear and Verruijt 1987, p.12):

> The real system is very complicated and there is no need to elaborate on the need to simplify it.... Because the model is a simplified version of the real system, *there exists no unique model* for a given groundwater system. Different sets of simplifying assumptions will result in different models, each approximating the investigated groundwater system in a different way.

5. Conclusion

In conclusion we should repeat that structural explanations not only provide an analysis of the factors relevant to the value of the coefficient in phenomenological laws, but also help specify the limitations within which the laws remain accurate. Fundamental laws, typically conservation of momentum, are applied to a wide variety of models to provide approximate derivations of phenomenological laws such as that of Darcy. However, in contrast to the examples emphasized by McMullin, hydrological models do not necessarily constitute a temporal sequence in which accuracy consistently increases in all respects. Furthermore, depending upon their objectives, hydrologists in the field are forced to consider a multitude of models as simultaneously applicable to a given system.

Analysis of the argumentative form of structural explanations also calls to our attention the mutually supportive roles of fundamental physical laws and models in an applied science such as hydrology. Fundamental laws can only be brought to bear upon models that are in an appropriate mathematical form. The idealized conditions incorporated into a model represent assumptions that permit the explanatory derivation to be carried out. Consequently, a statement of these conditions facilitates the empirical clarification of the domain in which phenomenological laws are applicable. Since hydrology has as its domain such a multitude of disparate individual groundwater systems, models function as an important scheme to classify these systems. Unless a system can be accurately represented by at least one model that functions in a structural explanation of Darcy's law, there is good reason to doubt that the law can be successfully applied to that system. As Wilfred Sellars pointed out long ago, an important characteristic of scientific explanation is the understanding it provides concerning why the phenomenological laws to be explained are in fact only approximately correct under limited circumstances.

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