A GEOMETRIC UNIVERSE WITH TIME FLOW

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Note on 3rd draft.

This is the third main draft presentation of this theory. I am adding details incrementally, as there are extensive ramifications. The purpose of these draft versions is to make it freely available to anyone else interested in developing it or publishing it. The present model is well defined, and has not changed significantly in the past year. This draft adds more details of some derivations, etc. The core theory is some 25 years old, with three major evolutions of the model (1989-1997; 1999-2004; 2012-2015). The variable dynamics defined in the first sections fix the present model in a specific form. Variations on this specific dynamics are possible, and would represent alternative models, from the more general class of 6-dimensional hyper-spherical models. Other plausible models of this class exist. The present model is the simplest version I can conceive, and has been stable for a number of years.

The main new feature of the present model, added in 2014, is the proposal of 'strings', i.e. the internal structure of the hyper-sphere. This is in the final section, and does not affect other key predictions. It may be considered the most speculative and dramatic part of the model. However to me it is the key unifying structure missing from earlier developments. Pre-2012 versions of the theory were called 'time flow physics'. From around 2012, with a modified set of transformations and then the addition of the 'string' structure, I have called this the 'geometric model'.

There are ample results to justify a full development as a project in theoretical physics. If it is taken on as a funded research project, the empirical evidence should rapidly become decisive. It requires a number of specific studies to decide empirically, primarily in cosmology and astrophysics, where it makes strong and novel predictions. However it is a new type of unified foundational theory of physics, and as such lies outside the boundaries of orthodox research in physics. Multi-dimensional theories of space are dominated by orthodox string theory, which contradicts the approach here in its foundational principle. The approach here will remain invisible to physics for the foreseeable future. The only viable development is probably through mathematics, not physics proper, as mathematicians have freedom from the metaphysical paradigm that dominates modern physics, which rules out the geometric model from consideration as a physical theory before it can even start.

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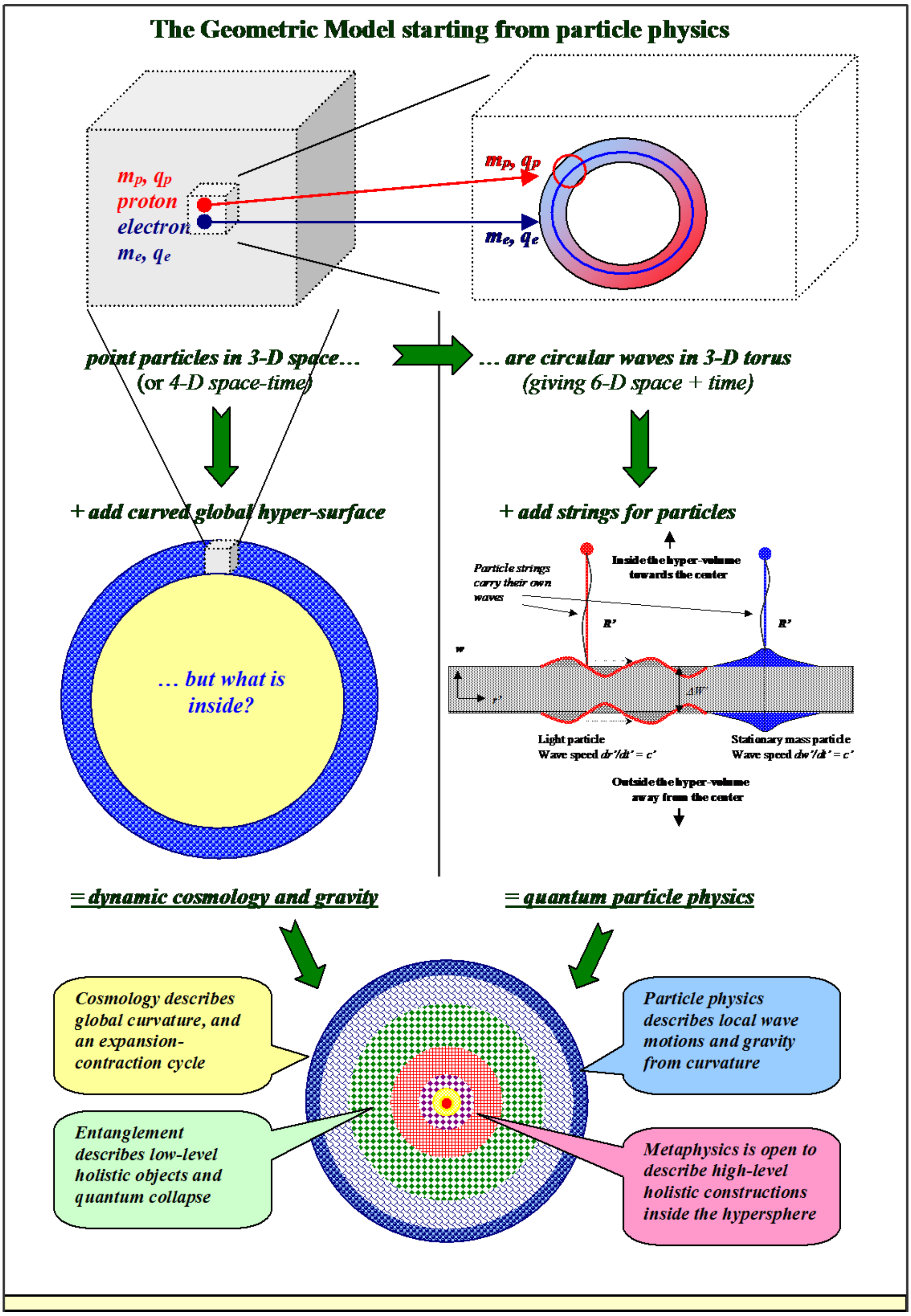

Figure 1. *The geometric model is a holistic theory of the universe, providing new foundations for particle physics, quantum mechanics, gravitational theory and cosmology. It opens a new space for metaphysics.*

INTRODUCTION.

This study presents a new type of foundational model unifying quantum theory, relativity theory and gravitational physics, with a novel cosmology. It proposes a six-dimensional geometric manifold as the foundational ontology for our universe. The use of lower-dimensioned manifolds (less than 10 dimensions) has been overlooked since the advent of string theory, and this general kind of model (with an extrinsically curved multi-dimensional space replacing the intrinsic curvature of space-time) has been overlooked since the advent of General Theory of Relativity. The theoretical unification is simple and powerful, and there are a number of novel empirical predictions and theoretical reductions that are strikingly accurate. It subsequently addresses a variety of anomalies in current physics. It shows how incomplete modern physics is by giving an example of a theory that is genuinely unified. It dramatically alters the interpretation of the nature of time, space and matter drawn from modern physics. On a broader plane, it profoundly challenges expectations about a naturalistic account our own existence, including the explanation of personal identity. It opens a new space in the universe beyond the familiar realm we see in three dimensions, and if the theory is verified, this must be the prime candidate for the location of our subjective existence, rather than the world of matter and atomic interactions.

I contend here that there is sufficient evidence to support this theory as *the leading proposal for a unified foundational theory* at the present time. It might not last for long in this status, once it receives serious attention, because it is subject to strong empirical verification or rejection. But that is what we should *expect* from a real theory. I know of no other proposal today that approaches it for comprehensiveness and simplicity as a foundational model. It puts the fragmented jig-saw puzzle of modern physics back together in a way that works realistically. I contend that it provides a more promising research program than the popular alternatives of our day: string theory, or the conventional dual paradigm of GTR and quantum field theory, or the few other well-known attempts to resolve foundational problems, such as super-symmetry, the holographic universe, many worlds quantum theories, deterministic quantum theories, etc. There are immediate empirical tests available, it is conceptually transparent and relatively easy to solve mathematically, and its consequences for both physics proper and naturalistic metaphysics are spectacular.

The theory makes strong and (so far) accurate empirical predictions, supported by powerful theoretical reductions. It drastically reduces the number of independent parameters (or universal constants) required in the Standard Model and cosmology. It determines a much simpler and stronger cosmological model than the conventional Big Bang cosmology, forcing a fundamental re-evaluation of the kludge of dark substances needed to keep the present theory of our cosmos working. It shows the host of anomalies and puzzles that now beset modern physics in a new light. It describes a new kind of theoretical and physical interconnectivity at the most fundamental level of nature, repeatedly glimpsed in both quantum phenomenon and cosmological coincidences, but not yet comprehended or explained. The result is a *post-modern physics,* moving on from the set of theoretical paradigms that has effectively ruled since the 1930's. I have elsewhere called this *time flow physics*, and the reason will be clear by the end of this introduction.

To illustrate the radical nature of the theory immediately with an example, it predicts the following as a natural law-like relationship[1]:

$$T = 2\hbar^2/m_e m_p^2 Gc = 13.823 \text{ billion years}$$

On the left, T is the measured age of the universe. The empirically estimated value of T today is between about *13.80 – 13.84 billion years,* on the two best independent measures. The prediction above is almost exactly at the mid-point of these two estimates, and accurate to about 0.1%.

What does this prediction mean? On the right, $2\hbar^2/m_e m_p^2 Gc$ involves only locally measured universal constants.[2] The constants involved have wide-ranging values,

[1] I note the uncertainty in this prediction is getting the factors of 2 and π correct. Given the exact match with the empirical value, it seems clear that this must be the correct version. Note also that the model predicts a direct relationship for the radius, R, not the age, T. The *measured age, T,* in the perspective of the theory really represents a disguised measurement of radial expansion, which has been interpreted in conventional theory as measurement of time. All conventional interpretations of cosmological measurements are theory-dependant, and have to be reinterpreted.

from 10^{-34} to 10^{8}. This is an accurate numerical relationship, as you can easily verify with a calculator. It is dimensionally correct: the quantities on both sides are *times*. So it makes no difference what system of units are used. It is a striking coincidence that it so accurate, given it combines such a range of astronomically large and small numbers. Writing out the terms numerically we have:

$$T = \frac{2 \times 1.05457 \times 10^{-34} \times 1.05457 \times 10^{-34}}{9.1094 \times 10^{-31} \times 1.6726 \times 10^{-27} \times 1.6726 \times 10^{-27} \times 6.6738 \times 10^{-11} \times 2.9979 \times 10^{8}}$$

What is the chance that this comes by coincidence to exactly: 4.3621×10^{17}secs = 13.823 $\times 10^{9}$ light years – within about 0.1% of the best measured values? Analysis shows the chance is about 1 in 40,000.

Yet it will still be unbelievable to most physicists that this could be a *law-like relationship*. In the conventional paradigm, it can only be a coincidence, because the local constants on the right are *static constants* and never change, while the age of the universe is increasing. The geometric model directly contradicts this paradigm: it entails that some universal constants *change with the expansion of the universe (rather than with time directly),* and it entails there is such a *law-like relationship* with precisely this functional form. It implies that the universal constants, which are measured locally, reflect global properties of space. This challenges a fundamental paradigm: the notion that the constants are independent, arbitrary, static numbers, that were mysteriously set at the Big Bang, characterising our universe but not carried by any mechanism or substance or properties.

The vast majority of orthodox physicists and philosophers of physics will reject any such theory that contradicts their paradigmatic *metaphysical* expectations – and tell everyone else that it is nonsense. But this is to reject it without evidence – to reject it as not deserving to have any evidence considered. If everyone takes this view, then no new theory that challenges paradigmatic assumptions could ever be examined on evidence. Of course, orthodox physicists do not *want* their assumptions challenged. They do not *want* new theories. The development of new ideas is not for orthodox

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[2] m_e is the mass of the electron, m_p is the mass of the proton, h is Planck's constant, G is the gravitational constant, c is the speed of light.

physicists, it is for heterodox scientists. We may also ask: what is *the orthodox explanation* of this coincidence? Their answer is: *it is a chance coincidence.* The chance of this is ostensibly about 1 in 40,000. That is a mighty big coincidence to write off as chance. And there are at least two more such ‘coincidences’ to explain in this area alone – which is only a fragment of the full evidence for the full theory.

Getting past the gatekeepers of orthodoxy to be allowed to present or publish an idea is by far the biggest challenge in developing a new theory – more difficult than any technical problems. Perhaps it will add some credibility to know that one of the greatest theoretical physicists of all time, Paul Dirac, proposed a theory of cosmology with similar relationships, first a very simple theory in 1939, and when that was rejected empirically, a more sophisticated theory in 1969; and he maintained his faith in it to the end of his life. In fact Dirac and also Eddington, another great C20th physicist, with profound intuition, both independently recognised in the 1930’s that there is not just one but at least *three* ‘cosmological coincidences’ of this kind.

The first is that above, involving the *gravitational constant*; the second involves the value of the *electromagnetic constant*; the third involves the *number of fundamental particles.* Eddington and Dirac had only approximate and somewhat flawed versions of the relationships. Approximate because they had no way to determine the correct combination of *electron and proton masses* required, which are determined precisely in the form above in the geometric model. And flawed, because they proposed the relationship as directly between *time* (or *age* of the universe) and the values of the constants, whereas the true relationship is with *space*. The physical constants reflect properties of space. The fact they are changing is because space is stretching.

But even in approximate form, the so-called ‘large number coincidences’ are very striking, and they have never been explained in conventional physics. The theory here predicts these in a far more precise form than Dirac was able to, and they provide striking evidence. This is also more convincing because the theory starts with a foundational model that delivers the ordinary *laws of physics* first, and subsequently determines these striking cosmological relationships in a precise way. Dirac attempted to construct such a theory in reverse, starting with the large number coincidences and reconstructing a fundamental theory to make the relationships law-like. This was a brilliant piece of theory construction, and he came remarkably close to succeeding.

The geometric model may be wrong of course – but what makes it *scientific* is that if it is wrong this should show up with closer investigation of the empirical evidence. This is in contrast to other popular research programs, notoriously string theory, but also super-symmetry and others, that struggle to make *any* empirical predictions, and struggle for any prospect of confirmation or disconfirmation - but nonetheless continue to monopolise huge scientific resources[3]. The geometric model makes *multiple strong predictions* – of phenomenon and relationships for which there are no reasons or explanations in conventional theories. It will not take billions of dollars of high-tech experiments or decades of work by thousands of specialists to reach a conclusion about it. The development of this theory should be able to be completed to a point where the evidence is objectively decisive, by a small handful of expert mathematical physicists, in a short period, on the scale of a year rather than a decade, using known experimental data.

A new theory of this kind also does not have to be *true* to be worth investigating of course. The main claim that makes it worth investigating is that it is a *realistic theory.* I do not for certain if it is true - that is what I would like to find out. However I am quite sure that at this point it is a *realistic theory.* It is realistic in two senses.

First, in the philosophical sense of *ontological realism:* the model postulates *real entities* that provide *realistic explanations* for the ordinary laws of physics. It explains *why* relativity theory and quantum theory work as well as they do as predictive empirical theories, by deriving their laws from a deeper and simpler causal

[3] "In theoretical physics, string theory has absorbed a lot of people and funds, as well as marginalizing and deprecating other approaches to the same problems... Lee Smolin thinks that string theory is not only speculative but the conclusions are circular, the concepts are arbitrary and the hierarchical structure of this scientific community is quite outlandish. The Nobel Prize winner in Physics, Sheldon Glashow, wonders whether string theory is not more appropriate for an Institute of Mathematics or even a Faculty of Theology rather than to an Institute of Physics. Unzicker considers physicists working in that theory as being like a sect or mafia. Another case is the search for supersymmetric particles in dark matter, which occupies more than thousand people at CERN. And what happens when, after a long period of search, when huge amounts of money have been consumed, the experiments or observations do not find any evidence in favour of these theories? Then the groups claim that we must carry out exploration at higher energies and they ask more money." [Corredoira, 2013, p. 86.]

mechanism. Various aspects of those conventional theories are simply postulated as 'fundamental principles' with no explanation; they now receive explicit explanations, relating them precisely to a deeper reality.

Second, in the pragmatic sense, it is *empirically realistic* in matching accurately with known data and phenomena. In this sense, it is a realistic competitor with conventional theories. As far as the empirical evidence goes at present, it explains a number of key phenomenon, key relationships, more comprehensively than conventional theories, it predicts certain striking new phenomenon accurately, and no strong disconfirming evidence has appeared so far.

There is no avoiding the fact that the geometric model involves a *real paradigm shift,* and it is as dramatic and shocking as anything encountered in physics before. It changes the very fundamental conception of the universe. It proposes we live in a vast cathedral of space, an incredible gothic construction, interconnected behind the façade of familiar three-dimensional space by a world normally referred to as the realms of magic. The geometric model postulates a six-dimensional geometric manifold, with a simple topology and simple laws, as the foundational reality of physics. In terms of interpreting ordinary sensory perception, the proposal is that we normally perceive only a surface of this space, a three dimensional *hyper-surface*. But within the larger manifold of space are complex structures, that hold the surface structures in order. There is no scientific recognition of this vast interconnectedness – but its existence is pointed to by multiple phenomenon of experimental and theoretical physics, and many will say by phenomenon of consciousness and subjectivity and spirituality also.

The proposal of a higher-dimensional space for physics is not new, being proposed in modern physics in the 1920's in the Kaluza-Klein theory. It became accepted and subsequently dominant in the 1980's with the development of string theory. The geometric model however does not use the puny 'strings' and crepuscular space of string theory: it introduces a full-fledged hyper-dimension, with strings stretching across a vast inner theatre of hyper-space, enclosed like a balloon by the physical surface of the 3-dimensional space we see and feel around us[4]. It supports a cosmic orchestra of harmonic vibrations that maintains the material world in order. It asserts a *cosmic center to the universe* – a center that we are all directly connected to, almost instantaneously. It confirms a profound, almost instantaneous connectivity across the whole of the universe. It maintains that the three-dimensional space through which we see the stars shinning is just a surface, supporting what we know as the world of matter, but only the surface of something far more complex and interconnected. It maintains an absolute law of local causality – and the speed of light still rules on the material surface – but the internal connectivity is a more profound dimension of causality, and not constrained by the surface speed of light.

In metaphysical terms, it makes the world 'one substance', a monism, leaving behind the dualistic distinction between *particles of matter* and the *space they are embedded in.* Particles are now wave-modes *of space itself*, not additional entities – just as water waves are not separate entities from the water itself - for which the C19th model of light as a wave-vibration of the luminiferous aether is a perfect analogy. This breaks down any prospect of *metaphysical materialism.* For space may be called an 'aether' but is not itself a 'material substance', nor even a 'substance' in the normal connotation of that term, and material particles are not fundamental things. We are not reducing one form of materialism to yet another. The material world emerges as a veil of illusion – a real aspect of the world, but, like a TV screen, illusory in what, to us, it *appears to contain*. The 3-dimensional mechanical world of ordinary physics is a surface projection over a deeper reality.

The most pointed existential question of philosophy, *what we ourselves are in this world,* becomes entirely open again. Our physical bodies are part of the material world, but that physical world – the *three-dimensional space* – is no longer all that exists, and the construction of our *minds, consciousness, souls, spirits,* however we conceptualise our self-identity, can only be thought to pertain to the 'inner

[4] It is profoundly different to string theory in conception, proposing a six dimensional extrinsically curved spatial manifold, with a specific simple topology, instead of the 10 or 11 dimensional *space-time* of string theory. This will be a reason for string theorists to reject it, for they think they have proven that any space-time of less than 10 dimensions is out of the question. But the geometric model does not make the same foundational assumptions, and the string-theoretic 'proof' is irrelevant to it. From this point of view, the paradigm that has built up in string theory is a major reason that alternative *realist multi-dimensional theories* like the geometric model have been overlooked.

dimensions'. For there are internal constructions of strings, carrying information, mediating causality, that could represent any number of entities that are commonly experienced and hypothesised as non-physical or transcendent aspects of self. To take this possibility seriously is a mind-altering experience. But it is contemplated not as a metaphysical speculation, nor as an item of religious faith, but as a scientific implication of a complete theory of physics. This is what gives its power to shock.

This is the ultimate paradigm shift involved, an overtly *metaphysical* one, and something the materialist philosophers and scientistic positivists will fervently hope will be consigned to their Bonfire. For it threatens the whole bureaucratic edifice they have constructed on the back of Scientific Authority. But this is not the subject here. As to the more strictly *scientific* conceptual shift, it seemed very extreme when I first began to think of this theory; but over the past 25 years many of the seemingly radical ideas have become familiar and almost commonplace.

The notion that the universe is *multidimensional* is now a commonplace. The notion that the constants are dynamic is unpopular, but seriously considered by some, and there are respectable experiments testing it. There are more wildly metaphysical ideas in currency, such as the many worlds interpretation of quantum mechanics that holds that the world itself bifurcates continuously; or the notion that time-lines can be circular and that worm-holes through space-time generated by black holes can connect different space-times; or that the universe is a hologram; or that the universe is a fractal pattern.

The geometric model does not involve anything this extreme: it is a model of a simple *realist mechanical ontology,* albeit a non-materialist one. But it involves some theoretical paradigm shifts that most physicists will find more alarming and absurd. One is that *time flow is real and space is real.* Another is that *the universe is in a cyclic process of expansion and contraction.* Another is that *neither General Relativity nor quantum mechanics is fundamental, and principles from these are not the starting point for the theory, they are predictions from it.* The laws and phenomenon of both GTR and QM are explained from a more fundamental basis.

Why do orthodox physicists find such possibilities as these so threatening, when they no longer raise an eyebrow at time travel, worm-holes, dark matter and dark energy, or a universe appearing from a singularity in nothing? That question is beyond the scope here, but see [Corredoira 2013] for a detailed view of the psycho-social interactions between ‘orthodox’ and ‘heterodox’ beliefs in science.

Scope of the study.

The main purpose of this introduction is to explain the core concepts required to understand the theory. I also present some case for the empirical success of the theory, although this is not the main purpose. The main case for it is really just the devastating simplicity with which the model can reproduce and explain the heart of modern physics, and produce powerful novel predictions that are inexplicable in the conventional framework. If these claims are established, there will be no real argument about it from scientific or philosophical realists.

The simplicity is both ontological (simplifying the substances and laws of physics) and theoretical (simplifying the formalism and principles). This is reflected by the fact that the mathematics is relatively simple, and should be highly accessible to any theoretical physicist or general mathematician with a good understanding of geometric manifolds and the partial differential calculus. Continuum mechanics is as good a starting point as anything. It does not require the intensely specialised formalisms of obscure applications of quantum field theory or general relativity or string theory that modern physicists are preconditioned to expect as the *starting point* of any new physics. The mathematics starts from first principles and *reconstructs* the key features of quantum, relativity and cosmological theories.

I emphasise this *simplicity* partly because it is the sign of a successful unification, and partly to encourage general students of physics or mathematics to examine it without fear that they will be out of their depth in obscure technicalities. The mathematical development presents the model postulates directly, and derives basic solutions, showing how the essential laws of conventional physics arise as limiting approximations, unifying relativistic quantum mechanics, special and general relativity, electrodynamics and cosmology. A variety of predictions are obtained along the way, supporting the case that the theory is theoretically and empirically realistic. The mathematics requires fluency with partial differential equations. It is presented in the style of applied mathematical modelling as usual in applied physics, and should be accessible to generalists. I have included quite a lot of relatively simple mathematical workings in places, to make derivations as transparent as possible, and illustrate how to work with the formalism, which is novel in one important respect.

The key novelty in formalism lies in the treatment of properties of an expanding differentiable manifold, which requires *scale transformations of space, time, mass and charge variables,* to adequately represent the evolution equations for fundamental constants. This was foreseen by Dirac in his pioneering attempts to develop a related cosmological theory, and is not mathematically novel, being comprehended within a partial differential calculus. An analogous treatment of variables is evident in the transformations between Eulerian and Lagrangian variable systems for classical continuum mechanics, which deals with the similar problem of transforming from a 'material' system of variables carried along with the flow of a substance, to a 'geometric' system fixed to a spatial frame. We essentially apply a similar insight to the universe itself and its fundamental 'material properties', evident as the universal constants: $c, h, G, \varepsilon_0, m_e, m_p, q$.[5]

The prospect of such a theory will appeal to a special kind of personality, found occasionally in academic mathematics, physics and philosophy, and in every other walk of life: intellectuals motivated by an independent spirit, who perceive present theoretical paradigms as excessively complex to represent the true simplicity of nature, and who are open to a fundamental conceptual revision in order to find the truth. Some philosophical implications are emphasised in this introduction, but a detailed discussion of philosophy is left for another place.

[5] Most applied mathematics and physics required to understand this paper is found in basic texts such as [2, 14, 28, 29] (maths) and [4, 7, 11, 20, 22, 23,19, 32] (physics). More advanced topics in differential geometry, GTR and cosmology can be found in [16, 18, 24, 30, 31, 35, 36]. Dirac's later cosmological theory is referred to in [5, 6]. Results used from cosmology are widely known and referenced here to the Wikipedia cosmology and astrophysics sites, which provides updated guides to the literature and measurements. Reference to the Pioneer spacecraft anomalies are given in (25, 26). A few other papers of special interest are referenced, but no attempt is made to survey the literature on the various topics.

The main body of this study presents the mathematical model for the *simplest model of universe of this kind*. This model makes simplifying assumptions, similar to the standard *cosmological hypothesis*, that matter in the universe is evenly distributed, the universe is essentially similar in every direction we look, there is a stationary frame determined by the isotropic frame of the cosmic background microwave radiation (CBMR). This is reflected by making the manifold topology spherically symmetric. This simple model needs refinement, but it corresponds to our observed universe closely, predicting fundamental features that are otherwise inexplicable. The application of the theory is open to much further development, but this treatment is intended to formalise the concepts required to conceptualise this kind of theory.

This presentation is a staging post in the development, and whatever its successes it is bound to be incomplete, by necessity in an attempt to develop such a wide-ranging theory from scratch. There are multiple empirical tests and theoretical ramifications that remain to be studied. But the number of successful applications of the theory is already substantial, and it has passed multiple tests that might easily have forced its rejection. No evidence has yet come to light so far to significantly disconfirm it. The present study is provided as a research document for any other researchers interested in following a similar line of investigation, because it represents a definitive way forward, especially in constructing the critical formalism required to express this kind of theory. I now go on to a detailed explanation of this.

The formalism for model variables.

From the mathematical point of view, the most important thing is to have a formal theory of the *variable systems,* with transformations of the variables and physical constants, which is required to map from the ‘static’ system of conventional physics to the ‘dynamic’ system here. This is the main point of novelty in the mathematical formalism. The conceptualisation of this was the most difficult point in the development – and is probably one main reason such a theory has not been developed before. It is not essentially novel however, as Dirac proposed this in principle in his later cosmological theory, but without having an adequate model of a real cosmology to illustrate it in real detail.

Dirac's late cosmological theory has fallen on deaf ears, despite being the most revolutionary idea he ever developed. It was his favourite and most profound idea. Dirac was the prime creator of modern relativistic quantum theory (quantum electrodynamics: QED) along with a host of other theoretical discoveries, and celebrated as one of the greatest theoretical physicists of all time, with a powerful physical intuition combined with the mathematical facility to substantiate his ideas. It is perplexing that subsequent generations of physicists have discarded his favourite idea without a backwards glance. I will spend a substantial part of this introduction explaining the formalism and the physical concepts behind it, because it is really the breakthrough concept required to develop the model. If it were not for Dirac's efforts, I would likely have been stuck at this point in the development, so I am very grateful for his work, and I hope this study may help someone else in turn in the same way.

Because this is the real point of novelty in the theory, I will illustrate at this point what I mean by this formalism. The cosmological model models the universe as an expanding manifold, like the ordinary Big Bang cosmology up to a point, although the mechanics are very different. The geometric manifold is six dimensional, but the point of the formal treatment of variables and constants is a general point independent of this.

with Time Flow

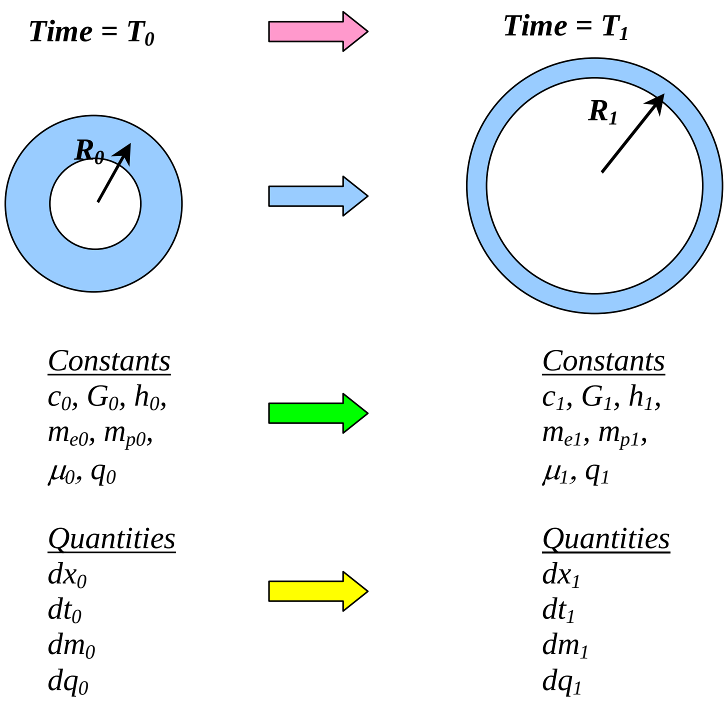

Figure 2. *The universe expands from a smaller radius R_0 at an initial present time, T_0, to a larger radius R_1 at a later time, T_1.*

In the geometric model, the seven local *fundamental constants* required for the ordinary physics of gravity, quantum mechanics and electromagnetism actually characterise properties of *space*, and they change with the expansion of space. In conventional physics they are *static constants*, mysteriously set at arbitrary values at the creation of the universe. In the geometric model they evolve, and are treated as *dynamic constants*, and they are related to each other formally through the expansion parameter, *R*.

Because the seven constants are so fundamental to physics – they essentially *contain all known physics* - it is useful to illustrate their occurrence in some primary laws of conventional physics.

$F_{Gravity} = md^2r/dt^2 = -mMG/r^2$	*Newton's gravitational force:*	G, m
$F_{Electric} = m_1d^2r/dt^2 = q_1q_2/4\pi\varepsilon_0r^2$	*Coulomb's electrical force:*	ε_0, q
$E = hf = hc/\lambda$	*Energy of a quanta of light:*	h, c
$E = mc^2$	*Energy of a mass:*	c, m
$c^2d\tau^2 = c^2dt^2 - dr^2$	*Special relativity metric:*	c

The *constants* are fundamental to these laws. The other terms, r, t, τ, are space, time and proper time variables (or parameters) along trajectories of particles. They are quantified variables (or parameters). All the complex apparatus of ordinary physics lies in calculating their differentials in three-dimensional geometry. But the very capacity to even *express* fundamental laws lies in the existence of the universal constants. They are often treated and thought of simply as numbers. But they are not numbers: they are *complex quantities.* They are constructed from underlying quantities of *space*, *time*, *mass* and *electric charge.*

Conventional physics has developed no formalism to represent how the constants *could change.* This is partly why models like the geometric model have remained invisible: dynamic constants cannot even be represented from the perspective of conventional physics. It would upset all the intricate formalism of tensor calculus. The primary difficulty, which Dirac recognised, is that if the fundamental constants change, *the physical quantities (or units) of space, time, mass and charge* upon which the constants are based must also change. We cannot possibly have a theory in which constants change without a consistent scheme for a *change of physical quantities* at the same time.

We can think of this in instrumentalist terms of measurement units. Suppose we define physical standards for *length* and *time* at the present time, and call them dx and dt. These can be thought of as small 'differential amounts' of length and time. But these physical standards depend upon the properties of matter and light for their definitions. And those properties – e.g. the time for light to travel a certain distance, the radius of a hydrogen atom, the mass of a proton, the intrinsic frequency of a quantised particle, or the charge on an electron – themselves depend on the fundamental constants. This interdependency of *the properties of matter that we use to define measurable quantities* and *the fundamental constants that determine those properties* creates a difficult problem when we try to formalise any theory of dynamic constants.

To illustrate, suppose we define *the mass of the proton, m_p*, as our fundamental unit of mass, *dm*. But then we wish to propose a theory in which *the mass of the proton decreases with the expansion of the universe.* Then we surely have to account for this in our definition of the unit of mass. For at a later time, with a larger radius for the universe, by hypothesis the mass of one proton has decreased. So now we have to count one proton as representing *less mass* than we did originally. Suppose the *evolution of the proton mass* decreases like this:

$$m_{p1} = m_{p0}\, R_0/R_1$$

So the mass decreases linearly as R increases. Now how do we define the 'measurement unit' of mass, *dm?* If we stick with our proton mass definition, so that $dm = m_p$ at all times, we have to change it to:

$$dm_1 = dm_0\, R_0/R_1$$

Thus our measurement unit becomes dynamic. Now we really have *two measurement units* going on: (i) the ratio of a given mass to the proton mass, which is constant through time (assuming all particles evolve their masses identically), and (ii) the ratio of a given mass to the proton mass *times the real mass of the proton at the time of measurement* - which now decreases with time, as the proton mass decreases.

We will call the first mass unit the *conventional mass* – for it is how mass is really defined in conventional physics. We use the symbol: *dm* for this, and the symbol m_p for the conventional mass of the proton. This definition ensures that the conventional measurement of the mass of a given object does not change over time. It cannot ensure that the *true mass* does not change over time though – we cannot force nature to follow our *definition of measurement*. We will call the second mass unit the *true mass,* and use the symbol *dm'* for the true mass unit, and m_p' for the true mass of the proton. To express our dynamic mass law we then have, for arbitrary radius R:

$dm \equiv m_p$	*Definition of conventional mass unit*
$m_p = m_{p0}$	*Evolution of conventional proton mass is constant*
$dm_0' = dm_0$	*True mass unit at present defined as conventional mass*

$m_p' = m_{p0}'R_0/R$	*Evolution of true mass – the postulate of dynamic mass*

$m_{p0}' = m_{p0}$	*True proton mass equals conventional mass at present*
$dm' = dm\ R_0/R$	*Transformation from conventional to true mass unit*
$m_p' = m_p\ R_0/R$	*Transformation from conventional to true proton mass*

(The second three relationships follow from the first four.)

Thus we see that we are immediately involved in a rather complicated looking group of relationships to represent the simple dynamic mass law: $m'_{p1} = m'_{p0}\ R_0/R_1$. There are four kinds of relationships here:

- *instrumental definition of conventional units*
- *boundary conditions matching conventional to true units at the present time*
- *transformations between the two systems of units*
- *laws of dynamics for the evolution of mass*

This is only for *mass,* which is the simplest constant – what about the other constants, which involve complex combination of quantities of time, space, mass or charge? How can we ensure a consistent *system* of transformations and relationships? And then, what about the *laws of physics* – like the laws for gravity, electromagnetic force, relativity theory and quantum mechanics? Won't *these* start to become terribly complicated if we allow complex evolutions of constants and quantities? Indeed, the difficulty is even greater when we realise that *R* and *T*, and all our spatial and temporal variables, are also subject to such transformations.

If we started trying to look for a set of such transformations to *start* doing physics, without any clues, it would be very confusing, for there are a considerable number of possibilities. It is possible to develop a theory from this point of view, evaluating all the feasible possibilities, and perhaps that is also a worthwhile project. But the geometric model actually developed from the opposite direction. I started with a novel physical model for the laws of micro-physics, and was then left perplexed when I realised it required a scheme of evolving constants. However, once I had the essential concept of how to formulate such a scheme in a set of differential transformations, which came from Dirac, I found that the geometric model more or less *forces a set of variable transformations and dynamic laws.* When I worked it out carefully, it led to the astonishing predictions explained below. And it leads to a version of the micro-theory that is simpler when it is unified with the cosmological theory. Indeed, the gravitational theory cannot be formulated properly without it.

But in the beginning there is this new complexity, and before going on, I will note the initial reaction is to look for a way out of having to introduce this new complexity into the variables. There are three main kinds of arguments to avoid it. One is just that "there is no evidence that the constants change at all" – but that is simply not adequate. We cannot see if there is evidence if we cannot even *conceptualise* a theory that allows us to test for it. And when we do conceptualise such a theory, we find that certain features that appeared merely as 'cosmological coincidences' in the conventional theory are real and fundamental physical relationships, reflecting the dynamic nature of the constants.

The second and more popular objection is that "there is no such thing as a 'true systems of quantities'. For suppose we do make up an alternative system of quantities: well then, we can simply transform back into our conventional system, and write the laws in that system, as they are already written in conventional physics. What if mass does not change in our conventional system, but it does change in the alternative system? Well then, the conventional system is simpler and preferable. But there is no sense in which one is *true* and the other *false:* for we can define measurement systems as we please. They are conventional."

This is a familiar sentiment but it is mistaken, and a fallacy we need to get over. First it would mean that stasis of physical constants could be achieved simply by *adjusting measurement conventions* or *coordinate system conventions.* But that is not true. There are serious experiments done to test empirically whether constants such as the gravitational constant or the fine structure constant change over time. This is an empirical matter. More important, the laws of physics are proposed as universal relationships that maintain the same form through time – they have *time translation* *symmetry.* This must determine an intrinsic metric for time, up to a scale transformation – for if we make a non-linear transformation of time, the laws of physics will *no longer be time translation symmetric.* And the same goes for other physical quantities. It is true we can *transform coordinate systems to represent quantities with numbers* as we like. But physical relationships represented by equations of physics are between *real physical quantities,* not just between numbers.

A third objection is that "such a scheme would contradict the principle of the Special Theory of Relativity, which states that the laws of physics appear identical to all observers, independent of their motion. If the constants appear to change with *time,* and simultaneity itself is conventional and depends on the frame of the observer, different moving observers would have to see dynamic constants as having *different values across space at the same time.* Thus no scheme of dynamic constants can be consistent with STR".

The answer to this is firstly that the dynamic laws for the constants in the present theory are parametised by *the radius of space, R, not by the absolute time, T.* Observers at different points in *space-time* will judge the *radius* of space differently, and there is no immediate contradiction of this with transforming to arbitrary frames of simultaneity – at least, no contradiction that is not an equivalent problem for *all cosmology generally*. More generally however the Special Theory of Relativity is simply inapplicable in this context: *it does not apply to space on a cosmological scale in an expanding universe at all.* It is a local theory, applying to electromagnetic theory. STR postulates a *flat space-time,* but the General Theory of Relativity postulates a *curved space-time.* They explicitly *contradict* each other.

Extrapolating from the 'bloc universe metaphysics' of STR to rule out a theory of dynamic constants is really absurd. Such arguments are only taken seriously by philosophers with an agenda to prove a 'bloc universe' metaphysics of time, a view that was initially thought to be implied by STR (as famously proposed by Minkowski), until STR itself was found to be *the wrong theory of space-time*. It is as well to be explicit about the failure of any such arguments based on STR, because these kinds of arguments are the popular source of attack against a realistic view of space, and likely to be repeated *ad infinitum* by philosophers.

The fact is that if we want to conceptualise dynamic constants in an expanding universe, we *have to introduce a system of variable transformations*, like that illustrated above. This was Dirac's central point. Precisely such a system is introduced here. The formalism for this is the backbone of the geometric model. This is why the equations in the following pages are filled with *dashed and subscripted symbols for constants and variables.*

- Dashed variables indicate we are working in true variables rather than conventional variables.
- The zero subscript indicates variables *at a specially defined present time, when we set values of conventional and true variables equal.*

Central concepts of the geometric model.

We now turn to other central concepts of the theory. The *geometric model* supposes that the three dimensional physical space we see is part of a *(smooth, differentiable) geometric manifold of six dimensions*, defined as an extrinsically curved manifold, supporting wave motions on a finite 5-dimensional hyper-surface. All 'material particles' of ordinary physics are instances of these wave motions, and they all obey exactly the same fundamental laws. They obey very simple laws of wave motion, determined by a few very fundamental symmetries. These motions give rise to what we see as different kinds of particles, with forces mediating their interactions, through electrodynamics and gravitation.

The 6 dimensional manifold volume is defined with a very specific topology, forming a 'hyper-sphere-torus'. On the macro-scale (~ *R = 14 billion light years)* it is a *three-dimensional hyper-sphere;* on the micro-scale (~ *$W = 10^{-13}$ meters)* it is a *three-dimensional torus*. The hyper-sphere determines cosmology, the world of stars and galaxies, while the micro-torus determines the particle world. The topology is dynamic, and the universe is presently expanding. The equation for the expansion is precisely solvable on simplifying assumptions, and determines the cosmological evolution of our model universe on the large scale. The solution derived here is cyclic: our universe is in a process of expansion and collapse. The large-scale expansion is time symmetric, but on the local scale it drives irreversible processes, giving rise to the irreversible thermodynamics and irreversible micro-physics that we readily observe in physical processes.

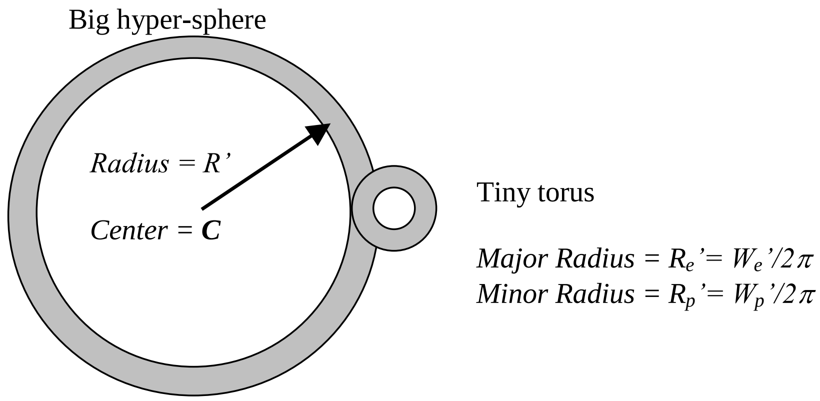

Figure 3. *The geometric manifold is a six dimensional spatial manifold with a five dimensional surface. Three surface dimensions form a symmetric hyper-sphere, which is visible to us as our three-dimensional space. Vibrational modes entirely in this dimension appear to us as light. Two other surface dimensions form the surface of a torus, which vibrates with energetic waves we see as mass particles.*

The initially disconcerting feature of this model development for the modern physicist will be that *no foundational assumptions from relativity or quantum theory are included in the fundamental model*. Laws of STR, GTR and QM are *derived from* the model, not imposed in its definition. Practically every conventional attempt to develop a unified theory *starts with* assumptions from relativity theory (e.g. a space-time metric) or from quantum mechanics (e.g. uncertainty relations). For it is widely assumed that these provide the foundational starting point for any theory. String theory, for instance, introduces a multi-dimensional space with ‘strings’, but already assumes that the strings are *intrinsically quantised*, and that the space is metricised as a *space-time manifold.*

This paradigm preconditions the expectations of modern physicists very deeply, and there is great difficulty seeing past it. Most physicists believe that *space-time* and *quantisation* are two fundamentally real features that modern physics has conclusively proved – and they must be put into any model at a foundational level. They are right that these are real features of physics, in the normal energy scales and time and space of our physical environment. But the view here is that they are not *foundational.*

In the same sense, C19th physics effectively demonstrated the *atomic hypothesis* – that matter is composed from discrete atoms. This is true in one sense, but it turned out not to be the *foundational* theory of matter. Atoms themselves are composed of something else – subtle 'quantum particles', with their mysterious ghostly 'wave functions'. Analogously, relativistic and quantum phenomenon are real, but the present theories of these are not regarded here as *foundational.* They are generated from a more fundamental mechanics. This shift is difficult – perhaps psychologically impossible - for most physicists to contemplate, but it is essential to conceive the present model. But rather than labouring the issue here, I just want to side-step it as a preliminary source of confusion, and go on to present the geometric model in its own terms.

All conventional physics is generated from the 6-D geometric manifold. I emphasise again that this manifold *is not a space-time manifold,* it is a purely *spatial manifold.* It supports energetic waves, representing particles. These wave motions are *not intrinsically quantised* – 'quantisation' is generated from boundary conditions, the simply fact that wave motions are *bounded by the surfaces of space in the micro-dimensions.* The basic equations of STR and quantum mechanics are determined immediately as the wave solutions from the boundary conditions. The ordinary laws of physics emerge precisely in ordinary limits, and describe how particles behave very accurately in the present epoch. But there are divergences from the standard theories in extreme domains – at very high energies, and at very small or very large scales. The most striking effect is at the cosmological scale, with a radically different cosmological model to the present one.

Perhaps what may seem initially confusing from this point of view is that the fundamental constants of ordinary physics – c, h, G, ε_0, m_e, m_p, q – appear almost immediately in the statement of the model. Don't these characterise ordinary physics you may ask – and indicate that we have smuggled in assumptions from ordinary physics instead of providing an ontological *reduction* of it, as claimed? Doesn't the appearance of Plank's constant, *h*, for instance, mean that we have smuggled in an assumption from quantum mechanics? Doesn't *G* mean we have smuggled in gravitation theory, and *c* mean we have smuggled in relativity theory? But this is not so: we do not smuggle in hidden assumptions. Rather, these constants appear because they *connect the model with ordinary physics.* And they are not fundamental: they are all *reduced to properties of space.*

To illustrate, the model postulates that *all waves (in the manifold surface) travel at a universal speed*. What is this speed? We symbolise it: c'. It has to match what physicists have observed empirically as the speed of light, *c*. Hence we set: $c' = c$. Or more exactly, we set: $c_0' = c_0$, because the model and conventional values only match at the *present time.* Similarly, in the model, all wave motions carry energy proportional to their frequency, *f*. What is the constant of proportionality? We symbolise it h'. It has to match what physicists measure as Plank's constant, *h,* so that: $E' = h'f'$ just as $E = hf$. Similarly, all waves distort the manifold (stretch space), according to their mass. What is the constant relating the distortion to mass? We call it G' and it must match what physicists measure as the gravitational constant, *G*.

We do not smuggle the *laws* from relativity theory or quantum theory into the model. The model has its own laws, with a tiny number of parameters characterising the *state and topology of space.* But we find the values of these parameters have been discovered already by conventional physics – as the 'universal constants'. Hence the appearance of these constants in the model is simply the *connection* with conventional physics. To emphasise this, the *model constants* are symbolised c', h', G', etc, i.e. with dashed variables, corresponding to the conventional constants *c, h, G,* etc. There is a very specific reason for emphasising this difference: the model parameters do not behave exactly like the conventional constants. Their magnitudes change as the universe expands. The model is connected to the content of ordinary physics by the transformations relating them to the conventional constants and variables.

The fundamental parameters of the model are not the physical constants at all, but simply the parameters to specify the state of the geometric manifold. There are only *three* spatial parameters at a moment of time, illustrated in the previous diagram:

W_e	*The major circumference of the miro-torus*
W_p	*The minor radius of the miro-torus*
R	*The radius of the large hyper-sphere*

There is one more parameter to specify the physical state of the manifold. This is represented as the *speed of expansion*[6]*:*

dR/dt	*The speed of expansion of R*

There is one further parameter, which must be derived in a more complete theory, but is treated here as an independent parameter, the *total mean particle number:*

N	*The number of fundamental masses in the universe*

This last parameter is not precisely related to the other parameters or constants in the present version of the theory, and we will ignore it here.[7] The three spatial parameters and one velocity parameter are all the parameters required in the model to *determine conventional physics, including the constants*. Conversely, the conventional constants *determine all these parameters.*

Model Parameters		*Conventional Constants*
W_e, W_p, R, dR/dt	$\rightleftarrows$	c_0, G_0, h_0, m_{e0}, m_{p0}, μ_0, q_0

[6] Although it can equally be represented as the *present age, T_0,* meaning the time since the start of the present expansion cycle, or the Big Bang.

[7] It is observed in the treatment of gravity that this should be determined by the other parameters, with the relationship: $N \sim (R/W)^2$, but fixed shortly after the Big Bang, when *mass-energy* became trapped in the *W-dimensions*, rather than being dynamic.

Figure. 4. *Four model parameters determine the constants. The constants over-determine the model parameters.*

Something of profound importance appears when we consider *that the number of parameters required in the model is less than the number of universal constants* – and yet all the constants are determined in the theory by the model parameters. This means that *some of the universal constants are redundant, and the model must show how to reduce some of the universal constants to others.*

This is a characteristic expected from a unifying model. In fact, the model *reduces the number of fundamental constants by two.* This is reflected by two simple empirical predictions, seen in the next section.

At the same time, we observe that *the model parameters only involve space and time and particle (or wave) counts* – what has happened to *mass* and *electric charge?* These are independent physical quantities in conventional physics, and intrinsic to the physical dimensions of all the universal constants except the speed of light. How can we have a theory that predicts such quantities if it does not contain them at a fundamental level?

This can happen when we have an *ontological reduction.* In classical thermodynamics, the macroscopic quantity of *temperature* is reduced to the microscopic quantity of *average energy per particle.* The macroscopic quantity of *pressure* is reduced to the microscopic quantity of *average force per surface area.* In relativity theory we have an equivalence of *mass and energy.*

The four fundamental quantities of *time, space, mass and charge* have been the ontological bedrock of physics since Coulomb formally introduced the electric force in 1785 to the present. No accepted modern theory has reduced any of these from the framework. However the geometric model dispenses with both *mass and charge* as fundamental quantities, replacing them with properties inherent in *spatial waves in the manifold.* This is sign of how fundamental the revision of physical ontology is.

We now illustrate two reductive relationships predicted among conventional constants in more detail. This demonstrates two strong empirical predictions. It also introduces *dimensional analysis* as a fundamental tool. The importance of dimensional analysis can hardly be overemphasised. It the touchstone of sanity in any attempt to construct a physical theory, a logical or *semantic* tool that transcends any particular theory. For an equation is logically incoherent as a *proposition* if it is not dimensionally balanced.

This is because equations are *propositions* that state *identities between physical quantities:* $A = B$, and we can only identify quantities of the same type. We cannot identify an apple with an orange. We cannot have an equation that identifies a certain amount of time with a certain amount of space, for instance: $x = t$. We always need a function to *convert quantities of time to quantities of space,* like: $x = ct$. This is an insight into the fundamental constants too: they are not numbers, they are *functional entities* that convert one kind of quantity into another kind of quantity. By working through the two following examples we see into the logic of the theory, and verify its logical coherence.

Two predictions of reductive relationships.

We now review two empirical predictions that are the most striking and easiest to verify numerically – you just need a calculator and list of values of the constants, along with the best estimates for the age of the universe. We start with a list of the quantities from conventional physics, and then list the fundamental model parameters of the model that replace them. There are nine relevant quantities from conventional physics: seven fundamental local constants, and two cosmological variables, viz. the present age and present expansion rate (Hubble parameter) of the universe.

Table 1. Conventional constants.

Local Constants						
	Domain	**Variable**	**Name**	**Value**	**Units**	**Dimensions**
1	Particle mechanics	m_e	mass of electron	*9.1094E-31*	*kg*	M
2	Particle mechanics	m_p	mass of proton	*1.6726E-27*	*kg*	M
3	STR	c	speed of light	*2.9979E+08*	*m/s*	X/T
4	QM	h	Planks constant	*6.6261E-34*	$m^2 kg/s$	XXM/T
5	Gravity	G	gravitation constant	*6.6738E-11*	$m^3/s^2 kg$	XXX/TTM
6	EM	ε_0	electric constant	*8.8542E-12*	$s^2 C^2 m^{-3} kg^{-1}$	TTQQ/XXXM
7	EM	q	elementary electric charge	*1.6020E-19*	*C*	Q
	EM	q_e	electron charge	*1.6020E-19*	*C*	Q
	EM	q_p	proton charge	*1.6020E-19*	*C*	Q
Cosmological Variables						
8	Cosmology	$R = T/c$	age-radius* of the universe	*1.3819E+10*	*l.y.*	X
	Cosmology	T_1	time since BB - min	*1.3798E+10*	*y*	T
	Cosmology	T_2	time since BB - max	*1.3840E+10*	*y*	T
	Cosmology	T	time since BB - avg	*1.3819E+10*	*y*	T
9	Cosmology	H	Hubble parameter - aprox.	*7.1E-11*	*1/y*	1/T

The fundamental constants are known with high accuracy. The age is measured by two different methods that now agree to within about 0.3%. The Hubble parameter, representing the present normalised rate of spatial expansion, is much less accurate, with a measurement error on the scale of at least 10%. These are the only quantities we need to consider in the primary development. The constants are the *independent fundamental constants* required for long-range forces, i.e. gravity and the EM force.

The 'standard model' of quantum mechanics, of course, has dozens of additional independent parameters, but these are to deal with the short-range strong and weak forces, and to model the menagerie of subatomic particles, including quarks. But we do not deal with that yet: the new model is established first for the real and indisputable realm of long-range forces and stable observable particles (electrons, protons, photons, neutrons) central to the major phenomenon of physics. Of course there are other real particles – their tracks can be seen in particle accelerators (unlike quarks[8]) – but we construct these few primary fundamental particles first (electrons, protons, photons), and leave the construction of other particles from waves modes for further elaboration, after the central theory is established.

There are also many other 'constants of physics' of course, appearing in thermodynamics and physical chemistry, that are not fundamental. E.g. Avogadro's number and Boltzmann's constant are macroscopic constants, but do not appear in the fundamental laws. Rather, they connect properties of bulk matter to micro-physics. The new theory remodels the *micro-physical laws themselves,* not the subsequent reductions of macro-theories. The exception is that the new theory does give a new law-like explanation of why irreversible thermodynamics rules the macroscopic world of processes in principle – something long unexplained in conventional physics.

In any case, the theory recognises the seven constants and two cosmological quantities in Table 1 as characterising the *fundamental properties and state of the spatial manifold itself.* But according to the model, *seven + two = nine* is more parameters than needed: the manifold is completely characterised by only *four parameters* (or possibly five, if we include *the total particle number, N*, but this is not seen as fundamental in the present model). The model also entails that the seven local constants determine some of the *cosmological quantities.*

Is this even possible? Surely physicists would have noticed if there was a simple reduction of this kind – that we could derive the values of some constants from

[8] The theory of quarks – sub-nuclear particles - is quite different to the well-established domain of EM and gravity, and it threatens to dissolve into a *mathematical* construction in the context of the new model. Quarks have never been directly observed in physics – they are inferred from a *mathematical model* for the internal construction of nuclear particles – and in the geometric model, this theory of quarks must be re-evaluated as a kind of calculus for wave harmonics in the finest level of micro-structure of the manifold. On this point, and the detailed treatment of the full menagerie of fundamental particles, the theory remains open.

others? Well, you don't notice things if you don't look at them. To show this is possible we now write the two main reductions, expressed in the two following empirical predictions.

Prediction 1. Measured age T of the universe from: $\{\hbar, m_e, m_p, G, c\}$

$$T = 2\hbar^2/m_e m_p^2 Gc = 13.823\ b.y.$$

Alternative version: the measured 'age-radius', $Z = Tc$ from: $\{\hbar, m_e, m_p, G\}$.

$$Z = 2\hbar^2/m_e m_p^2 G = 13.823\ b.l.y.$$

This numerical relation is accurate to within about 0.1% of the empirically measured age. It is certainly a true numerical relationship – put the numbers for the local constants into a calculator and see for yourself – but it is astonishing to claim that it is a *law-like relation.* In conventional physics it can only be a coincidence. But that is precisely what the geometric model entails: *this relationship is law-like*.

For clarity, it should be emphasised that the *age of the universe*, T, is what physicists claim to measure as 13.8 billion years. The quantity defined as Z is simply a distance defined by Tc, which on the conventional theory would be the distance a photon of light would travel in the period from the Big Bang to the present. The real 'radius of the universe' on the conventional theory is unknown, and much more difficult to estimate. Best estimates are presently anywhere from about 5 to 10 times this distance – but these estimates are highly theoretical, and they depend on assumptions about dark matter, cosmological constants, etc. The *age*, T, and the *physical radius*, R, must be independently measured, and are not simply related.

On the geometric model, this interpretation is reversed: it is the *model radius, R', that is precisely and simply determined by the fundamental constants,* whereas the *model* *age, T',* is more difficult. The geometric theory determines a precise relationship between the conventional quantity *Z* and the model radius, *R'*.[9]

In any case, the coincidence that such a simple combination of constants gives exactly the measured age: $2\hbar^2/m_e m_p^2 G = 13.823$ *b.l.y.* is *prima facie* improbable, as observed next. This prediction is almost precisely the mid-point of the two best estimates. The fact that this relationship is *empirical* is obvious: the quantities on either side are determined by very different measurement procedures.

Before examining this, there is a second prediction, determining the electric charge, or equivalently, the electric fine-structure constant, from the proton-electron mass ratio. In its first approximation it is:

Prediction 2. Electric charge from {m_p, m_e, ε_0, h, c}.

$$q = (m_p/m_e)^{1/3}(2\varepsilon_0{}'hc)^{1/2} = 1.5316E\text{-}19 \text{ Coulombs.}$$

Correct value is: $q = 1.6020E\text{-}19$. This is accurate to within 4.6%. This is equivalent to predicting the electric fine structure constant:

$$\alpha \approx (m_e{}'/m_p{}')^{2/3} = 1/149.95 \text{ (dimensionless).}$$

Correct value is: $\alpha = 1/137.0665$. This is accurate to within 9.4%. This prediction is modified by a second-order term in the full theory, but the simple version is already sufficiently accurate to be a surprising coincidence. Both relationships are remarkably

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[9] From the point of view of the model, what is conventionally measured as the *age, T,* is really a measurement of a *distance, Z,* in disguise. The model changes the interpretation of conventional cosmological measurements, which are theory-dependant. This is relatively easy to comprehend, because the measurement of *T* is measured through spatial quantities (wave-length of light that has travelled to us from the Big Bang event), and the *age* is inferred from this through a complex chain of reasoning. This depends on the conventional cosmological theory. When this chain of reasoning is reworked in the alternative geometric theory, we find that we are really measuring a radius, not an age. Moreover, the conventional age and radius, *T* and *R,* do not correspond directly to the model age and radius, *T'* and *R':* we have to transform the physical units from one system to the other. This set of transformations is at the heart of the geometric model.

accurate when we consider the magnitudes of the quantities being combined. E.g. if we write out the terms for the first prediction numerically we have:

$$T = \frac{2 \; X \; 1.05457 \; X \; 10^{-34} \; X \; 1.05457 \; X \; 10^{-34}}{9.1094 \; X \; 10^{-31} X \; 1.6726 \; X \; 10^{-27} X \; 1.6726 \; X \; 10^{-27} \; X \; 6.6738 \; X \; 10^{-11} \; X \; 2.9979 \; X \; 10^{8}}$$

What is the chance that this comes to exactly: $4.3621 \; X \; 10^{17}$secs = 13.823 $X \; 10^{9}$ light years – within 0.1% of the empirical value? A probability analysis shows this first prediction has about a 1/40,000 chance of being this accurate by coincidence. The second prediction has about a 0.04 chance of being this accurate by coincidence. Given there are two predictions, the chance of both being this accurate by coincidence is about 1 in 1,000,000. A maximal chance, supposing we have made some error in deriving combinations of small constants (like *2* and π), so the prediction are miscalculated by a factor of 10, is still about: 0.04 x 0.05, or about 0.002.

These relationships are not coincidences. Either they are real law-like physical relationships, or they have been manufactured in the theory construction. The question for the sceptic is whether they have been *manufactured* – discovered numerically and then ‘reversed engineered’ to look like predictions of an independent theory. Did I just notice them and make up a theory that *appears* to predict them? Or are they really objectively predicted by a theory that has good independent motivation? It will become evident that the theory does genuinely determine such relationships, and it is so tightly determined that it is impossible to force such results out of it. To show why we first need to review a dimensional analysis.

Dimensional Analysis.

Dimensional analysis shows that there is strictly limited scope for such relationships to exist. It might first be thought that since there are a number of constants (seven), plus small logical constants such as 2 and π, one might just randomly combine them in various powers, until a number matching the 'age of the universe' pops out, and then claim this as a theoretical relationship. There are lots of combinations using just small powers, and it might appear you could construct a close approximation to any *number* you wanted.

That is true if we are talking about *numbers*, but any such relationship must be *dimensionally correct* to make any physical sense. E.g. only combinations of the seven constants *that give the dimension of time* can be used as a possible relation predicting that age of the universe. How many such combinations are there? And how are they *spaced*? This is what determines the chance of an apparently accurate relationship appearing by coincidence.

This is easy enough to analyse. The key lies in the number of *dimensionless combinations* that can be formed from the constants. We can start by concocting the simplest combination giving a *length*, viz. $W_e = h/m_e c = 2.4263E\text{-}12$, where m_e is the electron mass. (This is the length unit intrinsically related to the electron). Now suppose we concoct a second combination of constants, call it W_1, also giving a length. Then the ratio of: W_e/W_1 is a *dimensionless combination of constants.*

Conversely, starting with W_e, we can construct any other combination that gives a length by multiplying it by some *dimensionless combination of constants.* In fact this is the only way we can construct such combinations. Hence we see that any such relations like Predictions 1 and 2 are intrinsically constrained by the *dimensionless ratios* available.

How many *independent dimensionless ratios* can we construct from our seven constants? The answer is exactly *three*. The reason is that there are exactly four physical quantities involved (from the dimensional analysis: X, T, M *and* Q), representing four degrees of freedom. Three degrees of freedom remain (7-4 = 3). Each allows us to construct an independent dimensionless combination. Three natural ratios can be defined. Their simplest forms are as follows.

The three local dimensionless ratios.

Table 2. *Three dimensionless constants.*

Local dimensionless constants					
	Domain	**Variable**	**Name**	**Value**	**Dimensions**
1	Cosmology	$D_{ep} = hc/Gm_e m_p$	Dirac's ep constant	*1.9535E+42*	1
2	EM	$\alpha = q^2/2hc\varepsilon_0$	Fine structure constant	*1/137.0665*	1
3	Particles	$\rho = m_p/m_e$	mass ratio (proton/electron)	*1836.1527*	1

Now any powers or products of these are also dimensionless. Conversely, all dimensionless combinations that can be formed from the seven constants can be formed as powers and products of these. For instance, there are four alternative variations of what I have called the *Dirac constant* that we can form by substituting different combinations for the *mass* term*:*

Table 3. *Variations of the Dirac Constant.*

Variations of the Dirac constant					
	Domain	**Variable**	**Name**	**Value**	**Dimensions**
$\mathbf{D_e}$	Cosmology	$D_e = hc/Gm_e^2$	Dirac's e constant	*3.5869E+45*	1
$\mathbf{D_p}$	Cosmology	$D_p = hc/Gm_p^2$	Dirac's p constant	*1.0639E+39*	1
$\mathbf{D_{ep}}$	Cosmology	$D_{ep} = hc/Gm_e m_p$	Dirac's ep constant	*1.9535E+42*	1
$\mathbf{D_m}$	Cosmology	$D_m = hc/G(m_e^{1/3}m_p^{2/3})^2$	Dirac's m constant	*1.5953E+41*	1

These are all related by powers of the *mass ratio*, $\rho = m_p/m_e$. E.g. $D_p = D_e/\rho$, $D_{ep} = D_e/\rho^2$, $D_m = D_e/\rho^{2/3}$. We will see that the geometric model determines that the last one, D_m, is actually the appropriate one to use in forming relationships. To see the reason, note there are similar variations of the *mass ratio.*

Table 4. *Variations of the Mass Ratio.*

Variations of the mass ratio					
	Domain	**Variable**	**Name**	**Value**	**Dimensions**
ρ	Particles	$\rho = m_p/m_e$	p-e mass ratio	*1836.1527*	1
$1/\rho$	Particles	$\rho^{-1} = m_e/m_p$	Inverse p-e mass ratio	*5.4462E-04*	1
γ	Particles	$\gamma = \rho^{2/3} = (m_e m_p^2)^{1/3}/m_e$	normalised mass ratio	*149.9475*	1

From the point of view of the geometric model, the last form of the ratio is the critical one to use in forming relationships. This is because the model determines that the meaningful quantity of an *average particle mass* is the numerator of the last quantity:

Average particle mass in the model.

Table 5. *Particle mass in the model.*

Average particle mass in the Geometric Model					
	Domain	**Variable**	**Name**	**Value**	**Dimensions**
m	Particles	$m = (m_e m_p^2)^{1/3}$	Average mass	*1.3659E-28*	M

Why not use another combination to define an average mass, e.g. $(m_e m_p)^{1/2}$, or: $(m_e + m_p)/2$? These alternatives might seem sensible at first, since there are equal numbers of electrons and protons in the universe. But the choice of *m* is determined by the *geometric model,* not by what might seem 'sensible' from ordinary physics. It is not an average in terms of a *mass average,* but rather, in terms of a '*volume average'.* I explain this next because the appearance of this mass combination is critical in the model. (And the correct determination of this is exactly what is missing in earlier attempts to explain the 'large-number coincidences' by Dirac and Eddington.)

The fundamental postulate of the model is that the universe is a *hyper-sphere-torus.* It has a 6 dimensional hyper-volume equal to: $V_6' = (2\pi^2 R'^3)(2\pi^2 R_p'^2 R_e')$, where the (dashed) variables are the (large) hyper-sphere radius, R', and two (tiny) torus dimensions, R_p' and R_e'. The functional combination of: $R_p'^2 R_e'$ is because R_p' is the *minor radius of the torus,* and R_e' is the *major radius of the torus,* and: $2\pi^2 R_p'^2 R_e'$ is consequently the volume of the torus.

Now the fundamental model postulate is that:

- *The 6-dimensional manifold volume is invariant*

The manifold stretches as the universe expands, so that R' increases, and $R_p'^2 R_e'$ consequently decreases in direct proportion to $R_p'^3$ to maintain the total volume – just as a rubber balloon becomes thinner in its surface as it is inflated, keeping the total volume of its material constant.

The length R' is therefore inversely proportional to the product: $(R_p'^2 R_e')^{1/3}$. Now the torus dimensions, R_p' and R_e', are determined by the *model connection with QM,* so that: $R_e' = \hbar/2m_e c$, and: $R_p' = \hbar/2m_p c$. Thus the quantity: $(R_p'^2 R_e')^{1/3} = \hbar/2(m_p^2 m_e)^{1/3}c$, which appears in the *conservation of volume equation*, has the meaningful relationship to R'. This is the fundamental reason the meaningful combination of masses in the model is: $(m_p^2 m_e)^{1/3}$. It is not an arbitrary choice: it is determined by the *geometry of the model.*

Parameters of the geometric model.

Let us now specify the fundamental parameters of the geometric model. There are four parameters needed to fully define the present state of space. These are the spatial parameters: R', R_p' and R_e', which define the relative dimensions of the 6-D hyper-sphere-torus, and the present *rate of expansion,* which may be expressed (like the Hubble parameter) as a normalised rate: $(dR'/dt')R'$.

(Alternatively we can take the present *time, T',* which is the time from the start of the expansion cycle, which we think of as the *age of the universe* in Big Bang cosmology. The solution to the expansion cycle makes $\{R', R_p', R_e', (dR'/dt')\}$ and $\{R', R_p', R_e', T'\}$ inter-definable. The former is preferred.)

The time (i.e. present age) and expansion rate are more complex, and we need to know the equation of motion to see how these relate, but the spatial parameters are simple, and we can see the relationships of these with the physical constants directly.

Table 6. *The Geometric Model Spatial Parameters.*

Geometric Model Spatial Parameters						
	Domain	**Variable**	**Name**	**Value**	**Units**	**Dimensions**
1	Model	$W_e = h/2m_e c = 2\pi R_e$	e-circumference	$1.2132E-12$	m	X
2	Model	$W_p = h/2m_p c = 2\pi R_p$	p-circumference	$6.6070E-16$	m	X
	Model	$W = h/2mc = (W_e W_p^2)^{1/3}$	avg circumference	$8.0905E-15$	m	X
3	Model	R'	universe radius	?	m	X

R' is the *universe hyper-sphere radius* in the model. We normally write the model in the *circumference* variables: $\{2\pi R', W_e, W_p\}$, rather than the radius variables: $\{R', R_e', R_p'\}$, to help confusing the symbols. Note the *average circumference, W,* defined above, is determined by the *e-circumference* and *p-circumference,* and is not an independent parameter. Most critical relationships can be written in terms of *W.*

The first two variables, W_e and W_p, are determined by the model connection with conventional physics, and may be taken as fundamental assumptions for the present. The third, *R',* needs to be determined by the model. These three quantities give two independent dimensionless ratios *(spatial ratios of the model)*. (Three quantities with one dimension: 3-1 = 2 degrees of freedom).

Table 7. *Geometric Model Dimensionless Spatial Ratios.*

Geometric Model Dimensionless Spatial Ratios						
	Domain	**Variable**	**Name**	**Value**	**Units**	**Dimensions**
1	Model	$W_e/W = \gamma = (m_p/m_e)^{2/3}$	fine ratio	149.9475	1	1
2	Model	$D' = 2\pi R'/W = 2\pi R'h/2mc$	large ratio	?	1	1

Note that these follow directly from the general model assumptions. These are the *only two dimensionless quantities in the model that can be constructed from the spatial parameters.*

Now the seven constants of conventional physics gave us three dimensionless quantities: the *mass ratio, Dirac consant,* and *fine structure constant*. The first model ratio is logically equivalent to the *mass ratio*. So this is not an independent empirical identity – it is part of the theoretical definition of the model. This leaves two dimensionless quantities in each theory. If the geometric model does represent a *reduction* of conventional physics, then *there must be an empirical identity between the two pairs of ratios*. The relationships are postulated as:

$D' \approx D_m$	*Large-ratio prediction*
$1/\gamma \approx \alpha$	*Fine-ratio prediction*

There are no other choices for the general forms of these relationships within the model. The only uncertainty in specifying the relationships exactly is the choice of small numeric constants, 2 and π, that appear when we expand these identities. But these two simplest forms are the correct ones, determined by the full model!

The second relation is actually the simplest, it is precisely *Prediction 2* given above. It means that the *electric fine structure constant is determined by the fine ratio in the model – and is hence identified with the appropriate mass ratio.* This is a reduction of the *electric force constants* to the ratio of the *electron-proton masses*.

The first relation is slightly more complicated. It expands into:

$$2\pi R'/W = (2\pi R')(2mc/h) = hc/Gm^2.$$

Hence:

$$R' = h^2/4\pi G m^3 = h^2/4\pi G m_e m_p^2 = \pi\hbar^2/G m_e m_p^2$$

This is very similar to Prediction 1: $Z = 2\hbar^2/m_e m_p^2 G$. The difference is in the small constants on the hand side. It means we must identify: $R'/Z = \pi/2$. I.e. to assert this as a model prediction, we must identify the model-theoretic quantity R' as identical to $Z\pi/2$. This requires more detail and is left for the main development. But it is readily seen by now that such a relationship must be forced by the model, and must match empirical observations accurately if the model is correct. The surprising fact that both relationships observed above match accurately gives strong evidence for the model.

This is only one aspect of the evidence. To work, the model also needs to predict the ordinary 'laws of nature', including the properties of particles, their law-like interactions through forces, their relativistic and quantum behaviour. This is the bulk of the model development, summarised below. Before that we address another logical issue in the reduction of the constants.

It may seem that since there are *eight* conventional quantities (including T *or* $Z = Tc$), and only three model parameters determining them, that there should really be five reductive relationships, not just the two we have seen. But this does not take into account that the model parameters considered so far have only one dimensional quantity: *space (or length),* whereas the conventional quantities involve four dimensional quantities: *space, time, mass, charge.* The best way to view this is through so-called *natural units.*

Model reductions of the constants in natural units.

To assign numerical quantities to the conventional variables requires a system of measurement units. Because there are four dimensional quantities, we are free to assign numerical units of *1* to four of these. So let us define units so that:

Natural Units. $\underline{m_e} = 1$
$\underline{q} = 1$
$\underline{c} = 1$
$\underline{h} = 1$.

Underlined symbols indicate these are in natural units. Four quantities remain that then need to be measured empirically to determine their numeric values in these units, i.e. $\underline{m}_p$, $\underline{G}$, $\underline{\varepsilon}_0$, and $\underline{Z}$. In conventional physics, these are empirically independent – they are not predicted by any theory, and must be measured independently.

Note that we can calculate these quantities in natural units quite easily by using the *dimensionless ratios*, D_e, ρ *and* γ. This is an important practical use for dimensionless quantities, based on the fact that:

- Dimensionless quantities do not change their value when the systems of units is changed – for they have no dependence on the dimensional quantities or measurement units.

Thus for the dimensionless quantities: $\underline{D}_e$, $= D_e$, $\underline{\rho} = \rho$ and: $\underline{\gamma} = \gamma$.

Since: $\underline{D}_e = \underline{hc}/\underline{Gm}_e^2 = 1/\underline{G} = D_e$, we simply have in natural units:

$$\underline{G} = 1/D_e = 1/3.5869E+45.$$

Similarly: $\underline{\rho} = \underline{m}_p/\underline{m}_e = \rho$, so:

$$\underline{m}_p = \rho\underline{m}_e = \rho = 1836.1527.$$

And: $\underline{\alpha} = \underline{q}^2/2\underline{hc\varepsilon_0} = \alpha$, so:

$$\underline{\varepsilon}_0 = 1/2\alpha = ½\ 1/137.0665.$$

Note these are *numerical relationships*, but we should be aware that they do not represent the dimensional constructions, because we have removed the dimensional quantities that have numerical values of 1 in natural units. Such representations do not appear dimensionally balanced. Strictly we should add symbols '1' for the unit quantities, because they still represent physical quantities, but this is just a little cumbersome. It is dangerous to write equations in a form that does not show their physical dimensions. For we can easily make the mistake of writing numerical equations that are not physically meaningful. A proper notation must reflect the dimensions, and should render meaningless equations as ill-formed constructions. This is sometimes ignored in physics for convenience of simplifying notation.

Similarly, in the model, let us set $\underline{W}_e = 1$ to define the natural unit for space. There remain two quantities to be measured empirically, $\underline{W}_p$ and $\underline{R}'$. These two model-theoretic quantities, $\underline{W}_p$ and $\underline{R}'$, must then determine the four conventional quantities, $\underline{m}_p$, $\underline{G}$, $\underline{\varepsilon}_0$, and $\underline{Z}$, resulting in *two theoretical and two empirical reductions.* In these units, the reductions are:

Theoretical reductions: $\underline{m}_p = 1/\underline{W}_p$

$\underline{Z} = \underline{R}'/\pi$

Empirical reductions: $\underline{G} = \underline{W}^2/2\pi\underline{R}'$

$\underline{\varepsilon}_0 = \underline{W}_p^{2/3}/2$

The first two may be considered 'theoretical definitions' of the model quantities from the conventional measurements – for we have no way of *empirically measuring* the quantities $\underline{R}'$ and $\underline{W}_p$ except by measuring $\underline{m}_p$ and $\underline{Z}$, and using these relationships. The second two are *independent empirical relationships forced onto the conventional quantities by the model.* Reducing the model-theoretical quantities $\underline{R}'$ and $\underline{W}_p$ out of the equations, these give the predictions:

$$\underline{G} = 1/2\underline{m}_p^2\underline{Z}$$

$$\underline{\varepsilon}_0 = 1/2\underline{m}_p^{2/3}$$

These are simply *Prediction 1* and *Prediction 2* that we started with, written in natural units. As we saw, the first is empirically accurate within about 0.1%, the second is accurate within about 4.6%.

Note there is another reduction when we take the fourth model parameter, *time,* into account. The corresponding conventional variables are the *expansion rate* (the Hubble parameter) and the *present true age.* These two quantities are reduced to just one in the model. This reduction gives another empirical relation, but it is more complex to explain, and the empirical quantities are not known very accurately, we pass over this here. I note that there is yet another empirical quantity not mentioned here, the total particle numbers in the universe – the numbers of electrons, protons and neutrons. This gives another empirical model relationship, but again this is more complex, and only discussed briefly in the section on gravity. What should be evident by now is that the model has very strong, precisely defined, and simple empirical predictions.

Summary of reductive relationships.

Let us summarise what the argument up to this point means. First it shows that *if a theory of this kind is possible* – a theory where a spatial manifold characterised by just three spatial dimensions can be used to reduce ordinary physics with its seven constants and the cosmological variable $Z = Tc$ – then *there must be reductive relationships with this form*. To show what these are, we drew on just two basic principles of the theory – the model definitions of the small dimensions, and the fact that it requires conservation of the hyper-space volume. These by themselves are almost enough to fully determine the relationships! There is no *a priori* reason in physics why such a theory should be possible. At first it seems extremely unlikely – few conventional physicists would consider it a possibility. The fact that the two relationships shown above are empirically accurate shows that it is *prima facie* possible. This is very surprising in itself. It is evidence that a realistic theory exists.

One might also expect that the geometric model would characterise space with some further parameters – more than just the spatial dimensions. In the geometric model, space appears somewhat like a material continuum, a *substance* - the substance that Lorentz and other thought of as the *aether* - and we are used to parameters in continuum mechanics characterising mechanical properties of substance. But this is different: *there are no further 'mechanical' properties of space*. The geometric manifold is a 'logical continuum', not a material substance – much as Euclidean space is a 'logical space', not a material substance. All its fundamental properties seem to be symmetry or conservation properties – such as scale symmetry, spatial isometry, conservation of volume, universal wave speed. Its empirical properties are its shape or topology. It is this very logical simplicity that makes it such a powerful model – and quite irreducible to yet another level of *substance*.

The geometry does carry properties – the ordinary constants like c, h, G characterise properties evident to us when we experiment with particles – and in describing the physics it supports we must connect the model to these. But they are not additional fundamental properties. They emerge reductively from the geometry. This may be at first hard to believe, but it is exactly what the model provides.

We now move on to the other central aspect of the model, the derivation of the ordinary *laws of physics* from the mechanics of the geometric manifold. This gives us a set of precise theoretical predictions, and shows the *unification* of the laws. They all derive from one source of fundamental structure. What we have seen so far shows that a reductive theory is possible: what we see next shows that it reflects the real micro-physics of our universe, the laws so painstakingly developed in the classical, relativistic and quantum theories.

Five predictions of theoretical laws.

If a simple model can account for the *laws* of ordinary physics - QM, relativity theory, particle physics, gravity, electromagnetism, cosmology, thermodynamics - then it has a powerful case to be considered as the foundational theory of physics. This is verified by looking at the general theoretical predictions of the model (as opposed to specific empirical predictions). Ordinary physics recognises two macroscopic forces (gravity and the electromagnetic force), along with a number of fundamental particle types (photons, electrons, protons, neutrons), particle properties (mass, electric charge, spin, energy, momentum), and equations for forces governing the motion of particles. These are primarily represented in three theories: relativistic quantum mechanics (QED), and the Special and General Theories of Relativity (STR and GTR). If the model can duplicate *these* laws, and explain why they hold from a simple basis, then it has a serious claim as a unified theory. In fact this was the starting point of the model, not the empirical predictions like those given above, that subsequently appeared as if by magic. I now survey some of the key theoretical predictions.

The model generates these ordinary laws of theoretical physics in the appropriate limits. We could include conservation of energy and momentum, but they are principles used to determine the construction of the model, so although they are entailed by the model, they are not independent predictions. The model is deliberately constructed to ensure them, so it is no coincidence they hold. They are evidence for the plausibility theory, but not the kind of distinguishing evidence that we look for to decisively *confirm* the theory. The decisive evidence is the coincidence that specific laws of conventional dynamics come out of the model.

In the following I state the predicted laws in conventional form without using transformed variables (i.e. dashed variables) except where necessary. They can be taken as predictions of the laws at the present moment (without allowing for the evolution of constants with time.) They are stated more precisely in the main derivations. The cosmological predictions however must be stated precisely, with explicit distinctions of true and conventional variables.

Prediction 1. Special Relativity metric.

The geometric model predicts the STR metric in the local limit of flat space (Section 4). The model starts with the following *universal speed postulate*:

Total speed on the manifold surface (particle or wave).

Local speed of light

$$\underline{du/dt} \;=\; \underline{sqrt(dr^2 + dw^2)}/\underline{dt} \;=\; \underline{c}$$

Euclidean distance on the manifold surface.

Real time

The variable *u* here is used for distance on the manifold surface. This simply means that *any wave-front or particle* in the manifold surface travels at a universal local speed, *c*. The spatial distance is simple Euclidean distance on the 5-dimensional surface of the manifold. The speed is the local speed of light. The law applies to *all energetic particles,* mass particles and photons alike.

The STR metric equation is:

$$\underline{cd\tau} = \underline{sqrt(\, c^2dt^2 - dr^2)}$$

Proper-time interval

Space-time interval

These are equivalent through the model definition of proper time:

$$\underline{dw} = \underline{cd\tau}$$

Manifold distance

Proper-time interval

The STR metric is just the model *speed postulate* rearranged, using the model definition of *proper time.* The two equations are thus mathematically equivalent, but they represent different things. The essence of the geometric model is that *what we see as proper-time ('process time'* or *'clock time'), $d\tau$',* in ordinary physics, is really generated by motion in space around the curled-up dimension we call *W*.

The *STR metric equation* is the foundation of STR and all the usual mechanics of Special Relativity – time dilation, space dilation, mass dilation, the Lorentz transformations, $E = mc^2$, etc. The geometric model entails it, and consequently predicts all the usual mechanics of Special Relativity.

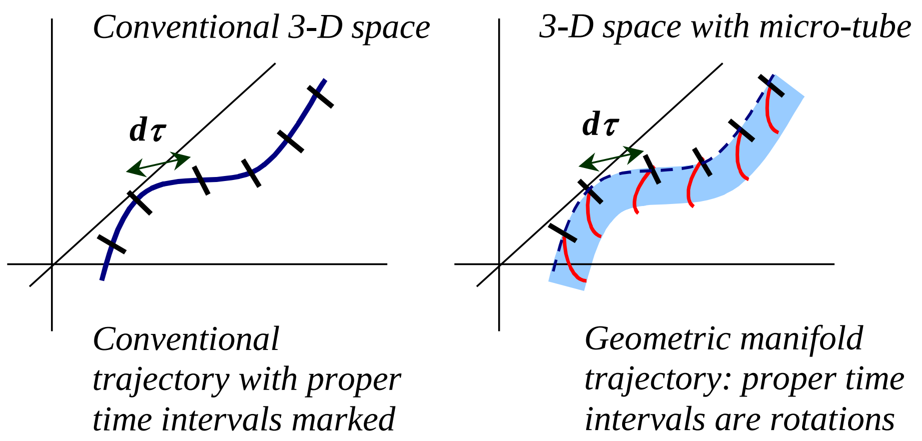


Figure 5. *Conventional and geometric manifold trajectories.*

In conventional physics, *periodic processes* occur as a particle or physical system moves along a trajectory in 3-D space. These processes measure *proper time.* Space, time and proper time are related by the STR metric equation. In the geometric model, particles really move along the surface of a 'tube' rolled up in extra microscopic dimensions, the *W-dimensions,* as they move through ordinary space. Their total speed is *c*. Periodic processes correspond to *rotations around W.* (This is what *quantum wave processes* correspond to in the model). The motion is described by the *speed postulate:* particles (or wave disturbances) move at speed *c* on what is now a 5-dimensional hyper-surface.

There is no analogue for the extra *W* dimension in conventional physics, and *proper time* is introduced there as a fundamental quantity, in addition to space and real time. Conversely, in the geometric model, proper time is reduced to a form of spatial motion. From the latter perspective, STR has rearranged what is a genuine Euclidean metric into the pseudo-metric of space-time.

The *universal speed equation* is the foundation of the geometric model. It represents the underlying mechanical model that explains the STR metric. Particles are really *wave-like disturbances in the manifold surface,* and *waves at the same point in space will travel at the same speed.* This postulate means that the manifold is *non-dispersive.* High-energy (high-frequency) waves travel at the same speed as low-energy waves. Light waves travel at the same speed as mass waves. Light waves travel purely in the three visible dimensions. Mass waves have components in the curled-up (torus) *W* directions. The components in the *W* directions represent *rest mass.*

From the point of view of the geometric model, STR mechanics is *derived* from a more fundamental construction. The construction explains why STR appears to be law-like, and also shows why it is only approximately true: when the manifold is stretched or curved the simple STR metric fails. By contrast, the STR metric is *postulated* as fundamental in STR (just as the Euclidean metric is *postulated* as the defining feature of Euclidean space). It is conceived as reflecting the existence of a *space-time manifold* as the fundamental basis of physical reality. It is encoded in the tensor calculus for STR. This inference to the 'space-time manifold' is a metaphysical leap, from the Lorentz symmetry of the metric, to an ontological interpretation - and via an unfortunate fallacious epistemological interpretation ("if we can't measure something it is not real"). Taking this metaphysical speculation as *conclusive,* and then using it as a reason to rule out any other possible alternative explanation, is the fundamental dogma that plagues modern physics. Most physicists today cannot distinguish this metaphysical interpretation from the empirical theory. They take the *space-time philosophy* to be the essential and irrevocable meaning of relativity.

Lorentz tried to find a physical, mechanical explanation for STR, explaining it from something more fundamental. He sought a mechanical compression of the aether, affecting particles embedded in it. His attempts failed, and he is often ridiculed for continuing to pursue such an explanation long after STR became popular. This ridicule is usually to the effect that Lorentz did not appreciate the metaphysics of space-time, as interpreted by Minkowski and the early Einstein, to the effect that *simultaneity relations are unreal,* being merely subjective to the motion of an observer.[10]

But the geometric model provides a reductive explanation that does exactly what Lorentz's intuitions told him. The physical mechanism Lorentz missed is the postulate of extra *microscopic dimensions* of space - with the reduction of particles themselves to wave-motions of space. This overthrows the dualism of having both *space* and *embedded particles.* If Lorentz had recognised this possibility, he could have found the mechanism he sought, and might have predicted the features of quantum mechanics twenty years before de Broglie, Schrodinger, Heisenberg and Dirac, which follow immediately from the geometric manifold, as we see next.

[10] The most average of physics teachers will shake their heads in perplexity that one of the greatest physicists – indeed *the chief inventor of relativistic physics itself* - could not appreciate what they so clearly and simply comprehend as its obvious conclusion. The argument is crystal clear, they say: *STR means that no absolute space-time frame is empirically measurable, so it is not physically real.* They forget that the CMBR determines an absolute frame of for the entire universe. They forget that this reasoning is conditional on the *absolute truth of STR*, but that it was subsequently found to be the *wrong theory of space-time,* being specifically contradicted by GTR. They do not know that it is a faulty inference in any case: there is no valid inference from *knowability to existence,* or as philosophers would say, from *espistemology to ontology.*

Prediction 2. Quantum particle-wave.

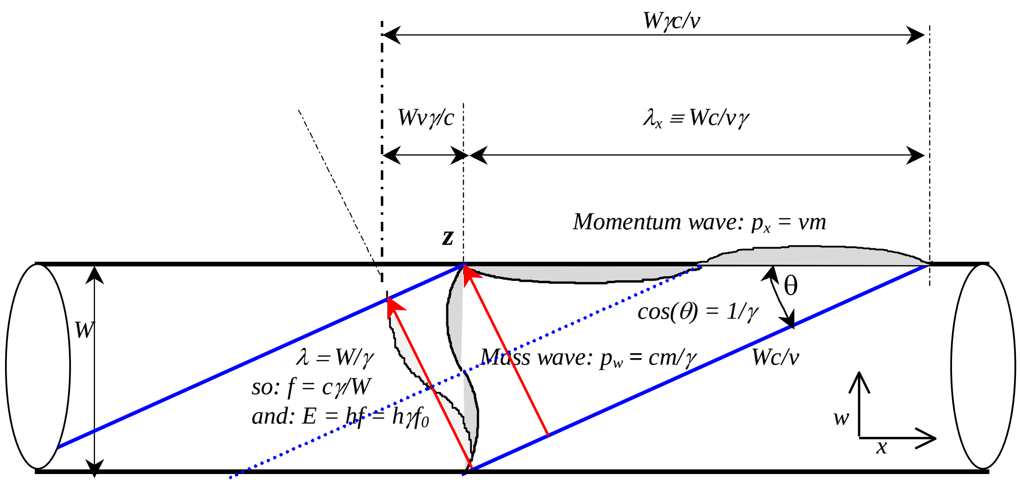


Figure 6. *Geometry of a moving wave in a pipe corresponds precisely to relativistic quantum mechanics.*

The fundamental model represents all particles as *waves*, and postulate conservation of energy. We can work out all the simple properties of a *relativistic quantum particle*, including photons and mass particles, from the geometry illustrated in the diagram above. Here I just summarise the most basic predictions that let us turn from the classical picture of particles as point masses to the quantum picture of particles as waves. Energy of a wave is defined by:

$$E = hf$$

f is the wave frequency, the fundamental property of a wave, with the dimension of $1/T$. The constant h is required as a universal constant of proportionality: $h = E/f$ for *all waves*. This is to say that there is only one *form* of energy, just as all waves travel at a universal speed. Because it converts *frequency (1/T)* to *energy* (MX^2/T^2), h must have the dimension of angular momentum (MX^2/T). Physicists identified h as Plank's constant at the end of the C19th. The connection with *mass* arises because energy is also identified with *mass energy* or *kinetic energy* by:

$$E = mc^2$$

Since mass is not fundamental but a defined concept, we can take this as the reductive definition of mass for the model. Combining these for a stationary mass particle gives:

$$hf = mc^2$$

The stationary mass is a wave mode around a W dimension. For a wave with one full cycle in W, this has wavelength: $\lambda = W$ (circumference of the W dimension). It has speed c. Hence the frequency is:

$$f = c/\lambda = c/W$$

Combining with the previous equation gives:

$$W = h/mc$$

This lets us determine the model parameter W from conventional constants, h, m and c. In fact this is for a full wave mode in W. However the lowest-energy wave mode possible is a half-wave, where: $f = c/2\lambda = c/2W$, for which we instead get:

$$W = h/2mc$$

Note that the possibility of different wave modes is forced logically, by the boundary condition on the wave. It is not an extra postulate. The boundary condition is simply that once we have completed a full cycle around W, we are back where we started from, so the wave amplitude must be identical to the point we started at. A half-wave mode satisfies this, and has the lowest possible energy. Higher wave modes are possible, but assumed to be relatively unstable, as their energy can decay into waves of lower energy. We infer that energy trapped in the W dimensions take up half-wave modes as stable states.

We now interpret the two *fundamental mass particles,* in the stable lowest energy states, as the *electron* and the *proton.* This determine the two model parameters, called W_e and W_p, representing the major and minor circumference of the micro-torus.

$$W_e = h/2m_ec \qquad \text{Major (electron) circumference}$$

$W_p = h/2m_pc$ Minor (proton) circumference

These mass-waves have angular momentum, since they are equivalent to *rotating masses* in the extra dimensions. Their angular momenta are given by their *mass times radius times speed*, which is $m(W/2\pi)c$, giving respectively:

$L_e = h/4\pi = \hbar/2$ Electron angular momentum

$L_p = h/4\pi = \hbar/2$ Proton angular momentum

Thus we predict there must be an intrinsic angular momentum, $\hbar/2$, associated with the fundamental particles. Other wave-modes can have multiples of this.

The electron magnetic moment is also accurately predicted from the simplest model, taking the electric charge as circulating W_e at speed c, as:

[9.3] $\mu_e = qcR_e = qcW_e/4\pi = q\hbar/2m_e = -9.2730 \times 10^{-24}$

The experimental value is: $\mu_e' = -9.2848 \times 10^{-24}$. The ratio to the prediction is 1.0013. For the proton, the prediction on this simple model is wrong by a factor of about 2.8, and the proton model is clearly more complex. This is not surprising: what is surprising is that model continues to get these extremely small values so close at all. This is more strong evidence it is realistic.

I note here that the true relationships to determine W_e and W_p should be as above, i.e. $h/2mc$, because they should be the lowest-energy wave modes, and this gets the intrinsic angular momentum correct. In previous drafts in certain places I have sometimes used: $W = h/mc$ as a general relationship for simplicity, but it should be: $W = Nh/mc$, where N is the wave-mode for the particle in question. This generally makes no difference to the logic, except when we need to calculate results. When the theory is applied to cosmology and to fundamental particle physics, the factor $N = ½$ is generally required, and usually noted at that point.

Relationships for moving waves are easily calculated from the wave geometry. The result is precisely that for a free quantum mechanical particle, like the de Broglie mass wave, or the Schrodinger quantum wave, but with exact *relativistic properties.* This is represented more comprehensively by observing that the waves are precisely the solutions to the relativistic *Klein-Gordon equation,* next.

Prediction 3. Klein-Gordon equation of relativistic QM.

It is easily shown that the basic complex-valued solution for a wave in the space is:

$$\begin{aligned} \Psi(x,w;t) &= A\, Exp(i/\hbar)(p_x x + p_w w - (E_x + E_w)t) \\ &= A Exp(i/\hbar)(p_x x + p_w w - (E_x + m_0 c^2)t) \end{aligned}$$

This solution is determined *purely from the boundary conditions on the space.* There are no *assumptions* from quantum theory smuggled in. The time differential satisfies the equation:

$$\frac{\partial \Psi}{\partial t} = \frac{-i}{\hbar}\left(\frac{m_0 c^2}{\gamma} + m_0 {v_x}^2 \gamma\right)\Psi = \frac{-i}{\hbar} mc^2 \Psi = \frac{-i}{\hbar} E_{Total} \Psi$$

(Using η for $\hbar$ because MS Equation does not have the symbol $\hbar$.) This is the model prediction of the Klein-Gordon equation - the relativistic version of the time-dependant Schrodinger equation - for a free particle without spin components. ([18, 39]). When spin components are added, we get the Dirac equation. However, the spin is not something added as an additional postulate in our model: it is already intrinsic to the physical model. This solution satisfies the usual momentum eigenvalue equation and similar standard results of quantum mechanics, e.g.:

$$p_x \Psi = -i\hbar \frac{\partial \Psi}{\partial x}$$

This shows that the particle model matches perfectly with relativistic quantum wave functions for mass particles and photons alike, used as the basis for quantum electrodynamics. This conformity with QED is an extremely powerful theoretical prediction of the model. It took physicists many years to discover this, after relativity theory and non-relativistic quantum mechanics were discovered. It appears immediately and almost effortlessly from the geometric model.

Prediction 4. GTR and the Schwarzschild metric for weak gravity.

The geometric model is very similar to GTR in that the force of gravity is a direct result of the distortion of space by mass-energy, and particle trajectories are determined by a general *geodesic principle*. In weak gravity, the predictions are indistinguishable. The main empirical predictions of GTR are represented by the distortion of the flat-space-time STR metric into the curved-space-time Schwarzschild metric by a central mass.

$$d\tau^2 = dt^2\, c^2/k^2 - dr^2k^2/c^2 + r^2d\theta^2/c^2 - (r\,\sin(\theta)\,d\phi)^2/c^2 \qquad \textbf{GTR Line Metric}$$

Where k is defined:

$$k(r) = (1\text{-}2MG/c^2r)^{-1/2} \qquad \textbf{Definition of } \boldsymbol{k}$$

The geometric model gives the same form, but with k replaced by K (“big K”):

$$K(r) = exp((MG/c^2)(1/r + 1/\pi R)) \qquad \textbf{Definition of } \boldsymbol{K}$$

The first terms in the series expansion are identical to k, and it differs only in higher-order terms, which are usually very small. In ordinary gravity, the theory is almost indistinguishable from GTR predictions, but there are a number of areas of difference.

The geometric model equation is better seen rearranged into the form of a *speed metric* however:

$$(K^2dr^2 + dy^2 + dz^2 + dw^2)/dt^2 = c^2/K^2 \qquad \textbf{Speed Metric}$$

This is mathematically equivalent to the line metric form, but like the earlier *speed function*, it corresponds more accurately to the interpretation of the underlying model. This is not a fundamental postulate however: it is derived from the underlying mechanics of the manifold, which is governed by the fundamental *strain function:*

$$W(r) = W_0 \exp((MG/c^2)(1/r - 1/\pi R)) \qquad \textbf{Strain Function}$$

This tells us how the torus dimensions, *W*, are stretched by the energy of the mass *M*. There is no analogue of this in GTR, for there is no underlying mechanism for gravity - only mathematical solutions to the abstract mathematical GTR field equation.

The geometric theory of gravity is very close to GTR, and this shows it is an empirically realistic model. But it differs from GTR in special applications. These provide direct empirical tests.

Prediction 5. The electromagnetic force and Maxwell's equations.

Three essential features of EM are represented here, but Maxwell's equations are not derived in this version. This is to be subsequently added.

One feature already mentioned is the prediction of the fundamental electric charge and fine structure constant. The second is the STR metric and Lorentz transformations, which are the basis of EM theory. Third is the implicit solution for the photon wave function contained in the relativistic QM solution above. The explicit model for Maxwell's equations and the photon wave function will be given in a future version. This is the solution if we simply embed the EM fields in the manifold, without reduction to manifold properties. But the full reduction of the EM field in terms of strain functions, similar to the gravitational theory, has not been completed.

Prediction 6. Quantum wave collapse, entanglement and coherence.

The *wave-particle duality* demonstrated by Einstein, Bohr, de Broglie and others in the early development of quantum mechanics has remained the source of its fundamental mystery ever since. It is evident in the phenomenon of *wave function* *collapse.* A particle wave function is spatially extended, and can disperse as a wave, but when it interacts in a 'measurable event', it abruptly *localises.* Particles act like discrete entities in this way: their energy is transferred in *quanta,* and does not disperse into multiple parts, like water waves or sound waves. In the photo-electric effect, all the energy of a single photon, *hf,* is transferred to an electron at once, in a single absorption event.

This is a *non-local phenomenon:* the quantum wave function may extend over large reaches of space, and yet acts holistically, as if the parts are instantaneously connected. Quantum *entanglement* refers this feature when a multiple-particle system is prepared in a state represented by a single quantum wave function. The particles act like a single system, and display correlated properties, even when they are separated by large distances. The classical example is the spin-correlated state of a pair of particles like two electrons in the singlet state. When one is measured, this has an apparently *instantaneous effect* on the other at a different location.

The wave function collapse is probabilistic and irreversible. There is still no formal specification of the physical conditions that cause collapse. As a result, some infer that it is sparked by the consciousness of an observer. Others propose it is caused by unknown physical conditions. The many worlds interpretation holds that it reflects a bifurcation of worlds or realities. The positivists disavow the problem and hold that the theory is merely calculating device for predicting measurements with no realist interpretation. There are various other interpretations.

This has been the deep source of controversy about quantum mechanics since the 1930s. There is no accepted *realist* interpretation. This especially upset Einstein, Schrodinger and Dirac, pioneers of the original theory. The mathematical possibility of a deterministic mechanism was shown by de Broglie and Bohm, but still involves non-local effects. It gives particles distinct identities, with distinct positions and trajectories; but particles are now guided by 'pilot waves', which are themselves non-local, and without a realist interpretation in their turn. It is an important insight, but not yet developed as a realistic mechanism.

A related problem for the conventional paradigm is that *if there is a realist interpretation of collapse, then it must define an absolute frame of simultaneity.* In any event, the non-local correlations prove that *information* is transmitted across space non-locally, faster than light, without any known causal mechanism. It is still unknown whether this phenomena permits controlled super-luminal signalling, despite various attempted proofs to the contrary.

Another related problem with little recognition is raised by Kus'menko [17], who argues that the *coherence* of quantum waves has never been explained or justified. Quantum particle waves are modelled as coherent wave packets, but there appears to be no physical principle for this explicit in the theory. It appears to be a boundary value assumption imposed in practical applications, but not based on any explicit physical principle. No physical explanation of it is known. This is evident also because the probability theory governing collapse is time asymmetric or irreversible, but most quantum theorists claim the theory is intrinsically reversible in principle. Irreversibility is claimed to derive merely from imposing time asymmetric boundary conditions. Yet it is intrinsic to the probability theory and the assumption of coherence, for waves are always assumed to collapse to new coherent states following measurement, and subsequently disperse, never the reverse.

Although quantum physicists officially denied for decades that such foundational problems are meaningful or important, no one who has seriously studied the subject is any doubt that there are severe problems, and many believe there is something radically *missing* from quantum theory. The theory has proved itself a good predictor, but it is incomplete. Its failure to deal with gravity – the quantisation of gravity being inconsistent – is another sign of this.

The geometric model addresses this whole cluster of problems through what are here called *strings.* The *string structure* of the geometric model is predicted by the gravitational equation. (A strong symmetry in the solution is further determined by conservation of momentum). It serves to explain *wave-particle duality, wave collapse, entanglement and coherence* of quantum particles. It provides a physical structure, viz the string at the center of mass of each particle, *for which no analogue exists in ordinary quantum theory.* This structure appears independently in the present theory from the treatment of gravity, so it is not an *ad hoc* addition. But it explains the problematic features noted above. The fact that it is explicitly missing in the conventional theory emphasises the lack of a realistic explanation for these major features of quantum reality.

Strings arise naturally in the geometric model, because at the center of the mass-wave of any particle, the strain function becomes extremely large, eventually extending across the radius of the hyper-sphere inside the universe itself. The mass-wave turns into a very thin ‘string’ of manifold when we get close to its center. This is similar in a sense to black holes in GTR. If particles had point-like masses, then in GTR they would form black holes at their centres. The GTR solution has an event horizon singularity at a small radius (at $r = 2MG/c^2$), and a naked singularity at the center ($r = 0$). This is physically impossible for a real particle. A key motivation of conventional string theory is to remove the central singularity. The geometric model strain function also has a mathematical singularity at the center ($r = 0$), but this is not physical, because the function does not extend right to the center. Instead, the function gives rise to ‘strings’, or thin threads of the manifold, close to the center. These are postulated to stretch right across the inside of the universe, indeed joining one side to the other. Strings of *entangled particles* interact by literally becoming entangled with each other in the higher dimension. The entanglement of strings is a physical analogue for the normal entanglement of wave functions described in Hilbert spaces.

This general structure explains how particles are individuated: every particle is identified with a distinct string. The string exists at a specific point, and the wave function is the manifold wave dispersed around it. The string follows a path with the wave function, and is guided by it, while at the same time holding it together. The wave function is interpreted realistically, as the manifold wave amplitude, with its periodic motion. The imaginary-valued amplitudes represent phases for the rotations of waves in the extra dimensions. It is proposed but not proved here that the de Broglie-Bohm hidden variable theory is a formal analogue of this.

The particle wave is *coherent* because it is held together by the string perturbation. At larger distances, it generates the gravitational strain (as its average displacement). Entangled quantum particles are literally connected by their strings. The speed of wave propagation along the strings is very fast, by an order of magnitude some 10^{40} times greater than the speed of light in the surface space. This is because strings are so thin, and wave speed is inverse to the strain. Information is transferred super-luminally through the strings, connecting points or particles in the surface space together.

This does not yet give formal predictions different to standard quantum probability theory. Rather, it provides the realistic mechanism to explain the mysteriously missing elements of that theory. It leads to a formal derivation of the probability mechanics. It must ultimately predict certain divergences from the standard theory. It cannot lead back to deterministic materialistic laws in the classical sense, because the material particles under-determine the string states. It is unknown whether it leads to determinism or probabilism or something else in the larger picture.

It should be mentioned also that this is where the treatment of *mind* enters the picture. Our minds are connected to our physical brain states, without being reducible to them. The brain states are complex quantum entanglements. It is thus natural to suppose that mental states are connected directly to the entanglements of strings, rather than to the simple mechanical state of the particles as in the materialist view.

Prediction 7. Cosmological predictions.

In stating any results from the cosmological model, we must explicitly distinguish true variables from conventional variables. There are a lot of different predictions and implications for cosmology. The key result in this model is a solution for $R'(T')$.

$$R'(t) = (V_0'^2 R_0'/c_0'^2) \sin^2((c_0'/2R_0')\, t')$$

This is a cyclic function, giving a symmetric expansion and contraction of the universe. It has to be transformed into conventional variables, *R* and *T*, to be related to conventional cosmology. We must subsequently interpret the point in the cycle that we are presently at. This is still somewhat uncertain. But there are numerous predictions that result from this. I list some.

- The conventional gravitational constant, *G,* is decreasing, at a normalised rate of about 10^{-11} parts per year. This depends on exactly what point in the expansion we have reached.
- Other conventional constants should appear to be essentially static, even though in true variables they are dynamic.
- The predicted Hubble constant conforms fairly closely to that measured from galactic red shifts. The uncertainty in this is that the measurement has be derived from the red shift of galaxies outside the local cluster. The required sample is some ½ to 1 billion light years distant, so it relates to the expansion in the past.
- The expansion of the universe appears to be accelerating in conventional measurements, but is actually started decelerating in real variables. The appearance of acceleration is predicted as an illusion, stemming from a failure to transform variables.
- The conventional measured age of the universe really reflects spatial expansion, not age. The age in conventional variables for time is roughly twice the measured age. This should be evident in the anomalous formation of galaxies, very old long-lived stars, etc. Precisely such anomalies have troubled cosmology for decades.
- We are substantially through the expansion cycle, and the true time left to maximum expansion and re-collapse is only 10% or so of the present age. It appears larger in conventional variables, but still on the scale of roughly the age of our solar system, not the extreme time span assumed in conventional cosmology.
- Dark matter and dark energy do not exist as currently thought. The whole of galactic dynamics has to be revisited and recalculated. Exactly what the geometric model entails is not clear. There may be analogues of dark matter and dark energy. The geometric model means that radiation (i.e. light) exerts a pressure for expansion, just as matter exerts a pressure for contraction. The model makes forms of ‘exotic matter’ a possibility. In the first instance, however, galactic dynamics needs to be recalculated *without* such assumptions, and depending on the results, the possibility of exotic matter needs to be recalculated.

Synopsis of model assumptions and predictions.

The following summary may help interpret the model development.

- The approach is to define the geometric manifold ***U*** and its essential properties as a self contained system, using a system of dashed variables, and then to specify the physical interpretation of its elements in terms of correspondences to ordinary physics, based on explicit transformations to undashed (conventional) variables.
- The geometric manifold, ***U***, is first defined and its properties developed. The guiding principles are very general: conservation of energy and momentum, scale invariance, time translation invariance, and invariance of the manifold volume.
- ***U*** is a 6-dimensional hyper-volume bounded by a 5 dimensional hyper-surface. Its global shape is the (geometric) product of a 3-D hyper-sphere with a 3-D torus.
- The *3-D hyper-sphere* is the large dimension, parametised by R', the radius of the universe.
- The torus is very small and gives the fine structure, parametised by the two torus circumferences: W_p' and W_e'.
- The rate of expansion: dR'/dt', is required to determine the dynamic state, including the age of the universe.
- These three lengths and the expansion rate define the mechanical state of the manifold at a moment in time.
- The three lengths in the model are precisely related to the seven conventional constants of physics: $c, h, G, m_{electron}, m_{proton}, q, \varepsilon_0$, through their three independent dimensionless ratios. The dimensionless ratios are: $\rho' = m_p/m_e$ (the mass ratio), $D' = hc/m^2G$ (the Dirac constant) and: $\alpha = q^2/2\varepsilon_0 hc$ (the fine structure constant).
- The model variables for space, time, mass and charge (x', t', m', q') are different to the conventional variables (x, t, m, q), and we must provide a system of transformations to map from the model variables to their counterparts. Only when this is done can we derive conventional physical laws and interpret conventional measurements from the model.
- This is closely related to the fact that in the model, the 7 fundamental physical constants ($c', h', G', m_e', m_p,' q', \varepsilon_0$'), are dynamic and change values with the

expansion of the manifold. But we cannot tell how their conventional counterparts (c, h, G, m_e, m_p, q, ε_0) may change until we have established the variable transformations: $(x', t', m', \mathrm{q}') \rightarrow (x, t, m, q)$.

- These are represented as differential transformations of the form: $dt = f'dt'$, where f' is a function that must be determined from the geometric model and the interpretation of the conventional measurement of t and other variables.
- The key feature is that the transformations are all *simple functions of the normalised radius,* $\hat{R}' = R'/R_0'$.
- $\hat{R}'$ provides the best fundamental *parameter* to write evolution functions for the fundamental 'constants', *c', h', G', etc.* This helps ensure *scale symmetry,* which is a fundamental symmetry of the geometric manifold.
- We can only derive functions of time, t', after we solve R' as a function of time: $R' = R'(T')$.
- The physical intuition is that the physical constants really characterise properties of the *space manifold itself,* and as space expands with R', these properties change in response – exactly as the wave speed in an elastic medium will increase when it is stretched.
- Hence we write all the physical constants as functions: *$c'(\hat{R}')$, $h'(\hat{R}')$, etc.* The variable R' is itself a function of time: i.e. $R'(T')$. To obtain the time variation of the constants we have to solve the function for $R'(T')$.
- There are consequently three kinds of primary equations required to set up the model variables and constants for the general model.
- First, transformations between model and conventional variables, like this:
    - $dx = \hat{R}'dx'$ and $dt = \hat{R}'^2\boldsymbol{T}'$
- Second, evolution equations in the model variables, like:
    - $c' = c_0\hat{R}'$
- And subsequently, *evolution equations in our ordinary variables*, like:
    - $c = c'/\hat{R}' = c_0$
- The latter equation follows from the first two using dimensional analysis.
- This example means that the speed of light, c, in our conventional variables is constant even though the true speed, c', is changing in true variables.
- We can state laws in ordinary variables but they will not appear time translation invariant. The invariance of the laws fixes the correct metric for time.

- The model also provides natural mechanisms to model:
  - *Quantum wave function collapse*
  - *Non-local connectivity and quantum entanglement*
  - *Large-scale gravitational anomalies*
- The model makes some direct empirical predictions, including:
  - *Predicts the measured current age:* Z_0/c *from* $\{2, \pi, c, h, G, m_e, m_p\}$.
  - $Z_0/c = h^2/2\pi^2 m_e m_p^2 Gc = 13.823$ *billion years.*
  - *Predicts the fundamental electric charge q from* $\{2, \pi, c, h, m_e, m_p, \varepsilon_0\}$.
  - $q = (m_p/m_e)^{1/3}(2\varepsilon_0 hc)^{½}$
  - *Predicts the rate of change of G from the radial expansion rate: dR'/dt'.*
  - $dG/dt = 9.8\ 10^{-13}$ *parts per year.*
  - *This gives the normalised rate:*
  - $(dG/dt)/G = 1.4\ 10^{-11}$ *parts per year.*
  - *(Subject to more precise determination of the expansion rate: dR'/dt').*
  - *Predicts the true age of the universe is 32.0 billion years.*
  - *Predicts structures much older than 13.8 billion years should have formed.*
  - *Predicts structures have had much longer to form than conventionally thought, with stronger gravity through the early stages.*
  - *Predicts the present radius of the universe is 21.7 b.l.y.*
  - *Predicts the present circumference of the universe is 136.5 b.l.y.*
  - *Predicts the Hubble parameter over the expansion of the universe.*
  - *Predicts that the recent past expansion of the universe radius, R, will appear to be accelerating in conventional variables.*
  - *Predicts small differences from GTR for solar-system scale phenomena, including 14 +/-3 seconds for the anomaly in the Pioneer space craft trajectories in 2003.*
  - *Predicts conventional cosmology will generate multiple anomalies.*
  - *Predicts the hypothetical substances of dark matter, dark energy, and the cosmological constant, are not real substances. They have been inferred from incomplete theories of gravity and cosmology.*
  - *Predicts GTR black holes are not real.*

- The manifold is a *smooth continuum* and is not intrinsically 'quantised'. It is simply a *space manifold,* not a 'relativistic space-time manifold'. Space has a Euclidean metric. The manifold is a curved surface in 6-D space. No underlying discrete or atomic structure is proposed for the manifold, which is treated as a pure continuum. The fundamental structures of STR, GTR and QM are genuine emergent features of the model.
- The two most direct empirical predictions are as follows.
- The simplest model predicts the conventional electric fine structure constant as the model fine structure ratio:
    - $\alpha = (W_p'/W_e')^{2/3} = (m_e'/m_p')^{2/3}$
    - This is equivalent to predicting the electric charge:
    - $q' = (m_p'/m_e')^{1/3}(2\varepsilon_0'h'c')^{½}$
- The model predicts that: $2\pi R'/W' = D'$, where the *Dirac constant,* $D' = h'c'/m'^2G'$ is the second dimensionless constant. This gives a prediction of the measured age of our universe using the following relations.
    - The model predicts a universe 'radius':
    - ***Predict R₀'.*** $R' = h'^2/4\pi m_e'm_p'^2G'$
    - Conventional cosmology measures age, *T*, corresponding to a distance, Z.
    - $Z = Tc$.
    - The model postulates the relations:
    - **Interpret Z_0.** $Z = R/\pi = R'2/\pi$
    - Hence this predicts:
    - **Predict Z_0.** $Z = h^2/2\pi^2 m_e m_p^2 G = 13.823$ *b.l.y.*

The paradigm shift of the geometric model and *time flow physics*.

I conclude with some remarks about the main difficulty in introducing such a theory, which is the conceptual paradigm shift involved. The biggest problem in front of any such theory is obtaining an evaluation of it at all, because it confronts paradigmatic assumptions that most experts are unwilling – or psychologically unable – to contemplate. And yet the development of any new theory of physics that is going to see us out of the present impasse in the subject has to introduce a more or less radical paradigm shift. The present theories (STR, GTR, QED, etc) are impressive as empirical theories in their specific domains, but they simply don't fit together properly as a whole. There has to be something fundamentally wrong with them and missing from them. The geometric model very clearly and specifically identifies the prime candidate for what is wrong. It reflects a deeply flawed metaphysical paradigm at the heart of modern physics. I have treated this more extensively in other places, as a more general proposal to restore a paradigm of *time flow physics.*[11] The geometric model is the direct outcome of this. I summarise some these themes here.

Physicists are notoriously defensive about their theoretical paradigms. The mainstream 'philosophy of physics' consists of bland popularist accounts of philosophy by physicists, and bland popularist interpretations of physics by philosophers. Its academic formulation has become a verbose and labyrinthine dogmatisation of conventional paradigms: the entropy state that doctrinal philosophy approaches when no external work is applied to it. While lacking originality itself, it is sustained by a powerful defence mechanism, attacking challenges to its authority. To make a case for the theory presented here, it is necessary to side-step false attacks from this quarter, and focus attention instead on the real scientific case for the theory. I would argue that the 'paradigm shift' involved is not really problematic at all from a scientific point of view: it simply challenges certain flimsy points of an academic metaphysics that has been dogmatised in the interpretation of modern physics.

I should emphasise that I am not at all opposed to metaphysical interpretations or speculations, or to real philosophical analysis. On the contrary, our various scientific

[11] See Holster, *The Time Flow Manifesto.* Web References 40.

theories are loaded with metaphysical assumptions just below the surface, and it is the proper task of philosophy to analyse these and to propose new metaphysical ideas. This is unavoidable and inevitable. We see this readily in historical scientific theories, and it is equally true of our modern theories. Our capacity for metaphysical ideas is not some unfortunate non-scientific irrationality: it is our primary intellectual gift, and it is the real driving force behind scientific theorising. Theories like the holographic universe, the many worlds interpretation of quantum mechanics, fractal universes, super-symmetry, string theory – as well as quantum mechanics and relativity theory themselves - all involve metaphysical invention and speculation. I think these are all brilliant ideas, and illustrate very important concepts. It is important to consider the meaning and possibility of these concepts, whether or not they turn out to be true. For they are all made to address real issues, and we learn by contrasting different possibilities, not by having some privileged access to truth.

Philosophy is primarily about meaning where science is primarily about truth: but none of us has any privileged knowledge of either. I am a pluralist when it comes to metaphysics and philosophy, just like art and culture. What I am deeply opposed to is the adoption of a single totalitarian metaphysics imposed on all science and philosophy as the official view of reality – and especially the projection of a doctrinal metaphysics to destroy other ideas. This is what the mainstream C20th scientific philosophy became under the domination of positivism, and what we live with today: a program for a totalitarian philosophy, reflecting the agendas of small circles of privileged academics.

The major difficulty for anyone trying to approach a new theory like the geometric model is of course that it *does* involve a significant conceptual shift. We all find it painful to revise our fundamental concepts – they become part of our psychological architecture. Yet it is impossible to propose a unified theory of physics without altering the conceptual paradigm. The geometric model implies a paradigmatic shift in essential concepts – and in the end, a startling change in our vision of the physical world we inhabit.

I want to distinguish two kinds of paradigms in this respect: academic ones, and real ones. Academic paradigm shifts relate to revisions of conventional philosophical concepts. These are commonly met with great academic sound and fury, but are seen in retrospect as revisions of confusions and misguided dogmas, and we often wonder what all the fuss was about when we review them historically. Real paradigm shifts bring positive changes in our vision of what the world is really like. They are positive discoveries. This is what we really want to achieve in science and philosophy. But in the course of science and philosophy we always need to confront academic paradigms first, to open a space to the imagination before real paradigm shifts are possible.

Academic paradigms.

The geometric model theory contradicts a central academic paradigm in the modern philosophy of time. To me, this is a group of metaphysical ideas that has been cobbled up behind physics proper as a rationalisation of the scientific theory. The dominant paradigm is a version of 'space-time metaphysics' called the 'bloc universe'. This tells us that time is just a form of space and that the passage of time is an illusion. It is an attempt to materialise time as an object – giving us the 'space-time manifold' as a concrete spatial object that forms the eternal fixed theatre of all reality. I have nothing against people who want to develop such ideas. What I object to is the claim that this idea has any scientific proof, the claim that it been proven to rule out any alternative scientific theory that does not conform to its metaphysical prescription.

The geometric model contradicts this metaphysical prescription. In physical terms, it requires that the unique isotropic frame for the CMBR has to be adopted as the unique frame for the proper description of the universe as a whole. But this is something cosmologists and astrophysicists already assume, usually without acknowledging the contradiction with the bloc universe metaphysics . The geometric model returns us more dramatically to a world with a real frame of simultaneity, and thus a real division between past, present and future. We describe this as time flow. This is the first academic paradigm shift of the geometric model. I will explain why it is merely academic, and not a real scientific discovery shortly.

For a second and closely related academic paradigm shift, a conventional dogma for many decades has been that the laws of physics are reversible or time symmetric (because time itself is just a symmetric spatial dimension). The geometric model contradicts this, and returns us to a physical universe that is intrinsically directional in time. In the geometric model, the laws apply in the future direction of time. We can cause what happens in the future, while we can merely infer what happens in the past. This is empirically evident in the time-directed probabilistic laws of quantum mechanics. This also contradicts the first paradigm, because time cannot be asymmetric in the bloc universe view - where it is just a direction in space.

For a third academic paradigm shift, the conventional doctrine among quantum physicists is that the quantum wave function is unreal, and wave function collapse has no physical reality, the wave function being merely a formal calculating device. The geometric model makes quantum wave function collapse real, and as such it has a definite frame of reference – a definite order of non-local collapse events - whether or not we can directly measure the order locally. This implies that wave function collapse involves faster-than-light causality, and faster-than-light transmission of information – whether or not we can use it to communicate. This also contradicts the first paradigm – the primary reason this anti-realism was adopted in quantum theory in the first place being to maintain the myth of Lorentz symmetry as universal.

The contradiction of these three modern metaphysical dogmas is the reason the theory is subtitled 'A Geometric Universe *With Time Flow'*. For those who find this scandalous from their exposure to the pop-philosophy of physics, it may be added that these conventional doctrines have been subject to strong criticism for decades, by a small but astute minority of philosophers and physicists. They are recognised by realist philosophers in the field to be controversial arguments that invoke metaphysical assumptions, and most certainly not incontrovertible scientifically-proven results of the kind real physics provides. The problem is that this conventional metaphysics has been so widely propagandised that most physicists (at least those who take any notice of the philosophy of their subject at all) have come to believe that these metaphysical myths are a foundational part of physics. In fact they have little relation to the reality of physics.

These three doctrines concern very abstract ideas: philosophical doctrines that have become ideologies. They are used to prescribe the possibilities of physics by dogmatising present theories as metaphysical doctrines. If physics was finished, and would never be required to change its theories in the future, dogmatising it might be less harmful. The problem is that physics is radically unfinished. Modern physics is radically unfinished, just as much as classical physics was. Yet the conventional philosophers of physics are intent on supporting a materialist scientistic philosophy, and their main claim in this respect is that the laws of physics tell us everything we can really know about the world. They want to interpret the laws of physics for us in this perspective. Materialising time itself is intended as the last triumphant step in the vision of positivist-materialists. But we cannot interpret what 'the laws of physics' mean for metaphysics when we still only have a radically incomplete theory of the laws. The real problem at this stage is that of trying to complete physics. By locking physics into these metaphysical assumptions, the philosophers have tried to prescribe against any conceptual change.

The three dogmas mentioned above are academic philosophical paradigms, because they make no genuine contact with reality. No one really believes psychologically that 'time is just space', that 'there is no difference between the past and the future', that 'there is no present moment'. In fact I don't think anyone can actually conceive this idea. We all assume from the very nature of our experience that we exist in the present moment, that we no longer exist in the past, and we do not yet exist in the future. This common sense view is affirmed by the geometric model. But this is not a real discovery of the geometric theory – not a real paradigm change – for every child already knows it. It merely redresses an academic confusion.

Any suggestion of realism about time flow has been condemned by leading philosophers of physics as nonsensical for decades now, and provides the excuse to reject alternative theories before reading a single equation; the excuse to deny realist theories consideration. It will seem bizarre to the general public that a proposed theory of physics would be condemned because it affirms the passage of time – something everyone experiences - but this is precisely the situation in the modern philosophy of physics.

Equally, 'time reversal symmetry' applies to certain laws of physics taken in isolation, but only fragmentally. It is a property widely misidentified by specialists (the technical subject is in a state of confusion for decades), and it does not hold for the physical world as a whole. There is no evidence that the universe as a whole could possibly ‘run backwards in time’, that thermodynamic processes could be reversed, or that our physical world does not have an intrinsically directional process. These are arm-chair speculations that have bizarrely taken on a status as received wisdom. Again, it might seem bizarre that a theory would be condemned as unrealistic because it implies a temporal directionality in physical processes that we see all around us. But again, this is precisely the situation in the modern philosophy of physics. The academics in this area typically express their certainty that the laws of physics must be time symmetric – despite the fact that after decades of work they cannot even give a coherent explanation of what time symmetry means.

Similarly, physicists are well aware that there is something very real about quantum waves, since wave interference and wave function collapse have very real effects. But it is typical to maintain an anti-realist dogma that “quantum mechanics is merely a calculus for predicting the outcomes of measurements”. This is a hang-over from early C20th positivism, an excuse to ignore the tricky problem of trying to explain what the wave function and its collapse really is. Again, this was a view propagandised and adopted by physicists around the mid-C20th, for temporary convenience. It subsequently became dogmatised as philosophical wisdom due to the vacuum of a realistic philosophy of physics. After decades of official denial, many physicists increasingly recognise that a realist theory of quantum mechanics is not a ridiculous possibility – or that it is not ridiculous at least to question the anti-realist assumptions.

If the geometric model is rejected from consideration because it contradicts such metaphysical dogmas, then of course it will not be evaluated at all. But serious physicists are too realistic to hold onto the academic dogmas foisted by philosophy for long, once they see real evidence that contradicts them. To take the example of time flow, everyone in astrophysics assumes that the stationary (isotropic) frame for the MWBR (microwave background radiation) is a special frame, and uses it to define the ‘present moment’ across cosmological eras. This contradicts the ‘bloc universe’ theory that no such global frame can possibly be defined . Working physicists will just shrug this contradiction off and say: “I don’t really care. I go along with your philosophy if everyone else does, but it makes no difference to me in the end, I am doing experiments, not philosophy. The MWBR is real. I've been measuring it." The existence of the MWBR does nothing to contradict the local relativistic phenomena – time, space and mass dilation – it merely contradicts the rationalising interpretation that there can be no physical definition of a universal frame of reference because there is no such thing as the past, present and future.

To illustrate the overthrown of an academic paradigm with another relevant example, we have Stephen Hawking's famous argument that black holes can radiate energy. Initially, by his own account, his claim sparked outrage and mockery – for physicists knew at that time that it is impossible for anything to escape from a black hole. This had become a paradigm of academic theory for all of about 10 years. Hawking was only fortunate he was in a position to get his proof objectively reviewed. Once the experts heard that Hawking's proof had been mathematically verified, and that there is even a plausible mechanism for 'Hawking radiation', they quickly switched view-point. Soon enough they were teaching it as new scientific fact, as if they had verified it themselves.

The reason they could switch 'paradigms' so easily in this case is that they had no really deep convictions about black holes in the first place: their convictions were based on second-hand academic opinion. For black holes are entirely theoretical entities: no one has ever observed a black hole or experimented on one, and Hawking radiation does not contradict any direct or indirect experience, or any known experiments at all. Physicists were not outraged initially because the idea was shocking to their real world view – black holes are too abstract to hold deep psychological convictions about - only because it contradicted a belief in academic authority.

Real paradigm shifts.

Many so-called 'paradigm shifts' are of this sort: academic controversies over mistaken theoretical dogmas. But some discoveries in physics really are shocking and disturbing to our world view: these are real paradigm shifts. The Copernican revolution, changing from an Earth-centred to a heliocentric cosmology, induced a very real psychological shock in people. For the visualisation that the Earth is the 'center of the world' is a real belief, it connects to a mental picture around which we intuitively organise our conceptual model of the world. It is not just an abstract academic idea. Similarly, the discovery that the Milky Way is just one of numerous nebulae, and the universe has a vast collection of such galaxies, was a decisive change of view. It suddenly made us very small indeed in the scheme of things. The universe opened out from a cosy neighbourhood of local stars, into a vast cosmology of bewildering size.

The discovery of the expanding universe, with the Big Bang at the start of physical time, and the likelihood of an eternal 'heat-death' of the universe in the future, was probably the biggest metaphysical shock of C20th physics. It is somewhat abstract, to be sure, but it still makes real contact with our conception of reality. Time itself has a start; our universe is not eternal; our very world appeared out of an explosion of fantastic abruptness, through a state of indescribably chaos in which nothing like our physical environment even existed. And everything that exists and has ever existed is doomed to extinction; inevitably, unavoidably and eternally, for we cannot stop the march of time. This is something most scientists now believe to be true, and it makes contact psychologically.

Two other revolutionary shocks occurred early in C20th physics, both central to the theory here. The fundamental discovery of relativity theory is that the intrinsic speed of physical processes slows down for moving systems, and this was a real shock for physicists – it contradicts the intuitive visualisation that simple motion through space has no causal effect. The fundamental discovery of quantum physics is that particles are not solid lumps, like billiard balls, but aethereal waves, that can interfere with each other at a distance, jump around randomly, spontaneously 'tunnel' out of boxes, and have mysteriously entangled properties and non-local correlations in their behaviour, and this was the second real paradigm shift of early C20th physics. Although again somewhat abstract, it is has a profound metaphysical consequence that we can visualise. It changes our psychological landscape in a very real way to recognise that we are living in such a soft and ghostly world, that we thought was hard and definite. It also led some to entertain consciousness as a fundamental reality.

I make this distinction of academic and real paradigm shifts, because I see the philosophical controversy over the fact that the geometric model supports time flow – i.e. a real distinction between past, present and future - as essentially academic and superficial, and I think physicists should not waste too much time on it. The 'bloc universe' metaphysics, proposing the unreality of time, has never had any real meaning as a view of existence. There are thousands of ponderous books and articles and documentaries in recent decades trying to prove it and explain it – but we should remember that no one any longer reads the scholastic pulp of the C19th positivists either, who were intent on proving the unreality of atoms.[12]

However the geometric model also represents a real paradigm shift, not just an academic one, and this is dramatic and shocking. For it changes the very fundamental nature of the universe, turning it into a theatre of hyper-space. However, it is the aim here to present the physics, not to try to address these metaphysical issues, and after pointing out the radical transformation of the world that this theory really involves, I leave the reader here to draw their own conclusions about the metaphysics.

Back on Planet Physics, there are *real paradigm shifts*. The fundamental constants change with the expansion of the universe; the cosmological scale is directly connected to the micro-scale; black holes do not exist; there are gravitational holes of a different kind; the universe is in a cyclic process of expansion and contraction; it did not start from a singularity but bounced from a previous cycle; this cycle drives the one-way thermodynamic process; substances like dark matter and dark energy and cosmological constants (and all dark things invented to make astronomy consistent) are speculative; the coherence of quantum particles reflects a hidden mechanism in nature; wave-function collapse and particle-like behaviour also reflects this hidden mechanism; the list can go on.

[12] The larger problem is that foundational and philosophical questions about physics have become sidelined as a specialised domain of *academic philosophy,* organised as *philosophical programs.* Philosophers defend their beliefs polemically and block competing programs: it is about *winning arguments,* not establishing knowledge. Genuine philosophers of physics and science consequently have nowhere to work; being repelled by the culture of academic philosophy; and being expelled by the positivism within physics itself, which leaves no space for conceptual enquiry. Until physics takes back responsibility for its own philosophy, and treats it seriously again, it will fail to be more than a shadow of its former self, in the glorious days when it was called *natural philosophy,* times still within the generational memory of the last era of great modern physics, that produced Plank, Lorentz, Bohr, Einstein, Schrodinger.

There are many points, because a radical redress of the foundational theory means that all the present anomalous phenomenon of physics are up for reappraisal. And there are an awful lot of anomalies in modern physics.

But perhaps the most stunning implication for theorists will be simply this: only four parameters are required to characterise the global state of the universe, determining all the universal constants of physics, along with the radius, age, and cyclic life-span of the universe. The radical implications are evident because the theory takes away numerous degrees of freedom in the type of universe we inhabit, represented in conventional physics by the number of independent empirical constants (like G, c, h, etc). In the present model, we are left with just one empirical degree of freedom determining the specific laws of nature characterising our particular universe: the mass ratio of the electron to proton mass. And even this may disappear in a more complete version, leaving us with a universe that is logically unique. As a result, the 'laws of nature' are almost reduced to a purely logical construction.

In the present model, the mass ratio is regarded as empirical, fixed shortly after the Big Bang event and constant for the current era. But there is a second degree of freedom, *the present time in the universe*, which is the chief empirical variable relative to our experience. The latter reflects the fact that we are doing *time flow physics.*

Materialism.

From the point of view that the geometric model is a realistic theory, it already shows that certain philosophical possibilities are real probabilities, and in this, it radically undermines the materialist philosophy that presently dominates the world of science. Materialist assumptions are often claimed as definitive conclusions from present theories of physics. They are claimed to provide conclusive 'scientific answers' about the ultimate fundamental nature of reality. But the real possibility of the geometric model undermines any claims of materialism to have strong *scientific evidence* behind it. Scientific evidence is only strong when it *rules out other possibilities* as realistic explanations for things. Given there is a realistic alternative theory with such radically different *metaphysical consequences*, it is clear the modern scientific materialists have made the fatal mistake of claiming a universal metaphysical scheme from a defective interpretation of our temporary and inadequate scientific theories.

Philosophically, the geometric model represents something of enormous personal significance to us. It removes the case that our personal identity and subjective reality is reducible to purely physical atomic processes, as the materialists claim. Instead, there is a larger space of reality than the three-dimensional material world. There is another *dimension to reality.* On the assumption of the theory, this must be filled with complex information-carrying structures, and it becomes the prime candidate for the causal foundation of our personal identities. Our personal existence must be intimately connected to the structures and processes within this higher dimension.

All questions about personal identity, including survival after physical death, that the materialists believe are closed, are reopened to scientific enquiry. The fact that this is *scientifically realistic* shows that the materialist metaphysics, proclaimed with such sound and fury by the scientific philosophers of our day, is merely speculation. The fact that there is such a realistic prospect of a comprehensive *causal ontology connecting the physical to the meta-physical dimensions,* with precise mathematical laws binding them together, shows that a meta-physical conception of personal identity and consciousness is entirely realistic within the naturalistic scientific paradigm.

The result I believe must be the reintegration of the scientific world view with traditional metaphysical, spiritualist and religious, world views. Both are about genuine aspects of reality: neither is complete by itself. From this point of view, the materialists are wrong. For spiritualist and religious and alternative metaphysical world views do not deny the features of the material world, they only claim that it is not *all the world,* and the appearance it presents is in some degree illusory. If the geometric model (or anything similar) is correct, they are right. They are right at very least that it is an open possibility. The materialists deny the reality of anything beyond the material dimension, they deny any possibility of a larger or hidden aspect to the world. If the geometric model (or anything similar) is correct, they are wrong. They are wrong to claim to know the truth of materialism.

SECTION 1. BASIC GEOMETRIC MODEL EQUATIONS.

0. The global spatial model and variables.

The model postulates a six-dimensional hyper-volume or *space manifold*, bounded by a five-dimensional hyper-surface. The global topology is a "*torus X hyper-sphere"*.

Note that rather than adopting a 5-D Riemannian geometry for the curved surface, we analyse this in a Euclidean space of 6 dimensions. Treating it as an extrinsically curved spatial volume rather than an intrinsically curved Riemannian space is essential to the model. Note Whitney's theorems [31,35,36] that show any intrinsically curved N-dimensional Riemannian space (e.g. 3-D) can be modelled as an extrinsically curved hyper-surface in 2N Euclidean space (e.g. 6-D).

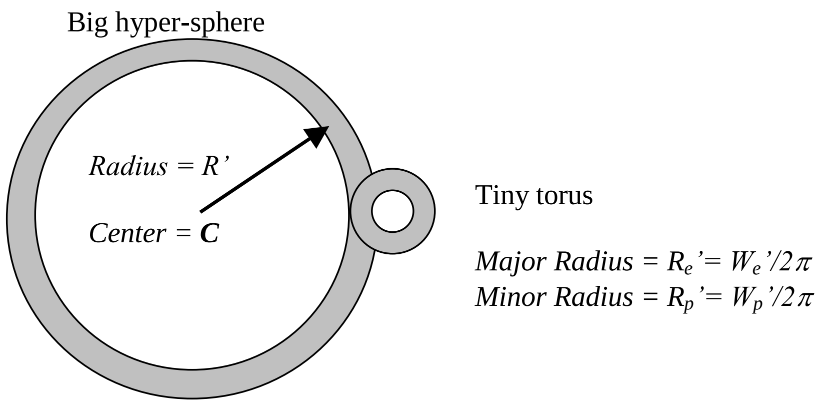


Figure 3 (Repeated).

The *big hyper-sphere* has a 3 dimensional surface, which is a hyper-sphere in 4 dimensions. Its surface directions correspond to the three ordinary dimensions of our 'visible universe', and the large radius is on the scale of $R' = 10^{27}$m. The shaded areas represent the manifold of 'ordinary space'. The center, ***C***, is external to the manifold.

The *tiny torus* has a two dimensional surface, and is on the scale of 10^{-13}m. It will contain 'energetic waves' as tiny perturbations, representing two fundamental mass particles, the electron and proton.

The manifold is dynamic: *R'* increases with the expansion of the universe (like a balloon being inflated and thinning out). The global geometry at any moment of time is specified by the three radii – or better, by any two independent *(dimensionless) ratios of the radii.* The model is *scale invariant,* so there is no absolute scale for the Euclidean space itself. This will mean that all the constants in the 'ordinary laws of physics' are essentially determined by just two fundamental dimensionless parameters.

To begin with we assume a smooth, locally flat manifold. Later we introduce perturbations, which are wave motions that strain the manifold locally. To define the laws we first need to specify some spatial variables, as follows.

Spatial variables.

At any point in ordinary space, $r' = (x', y', z')$, on the hyper-sphere surface, the micro-torus appears as an ordinary 3-D torus, i.e. a 3-D volume bounded by a 2-D surface (a sub-space). We consider variations in this w.r.t. *r'* and *t'* later, but for the moment we define the general global dimensions.

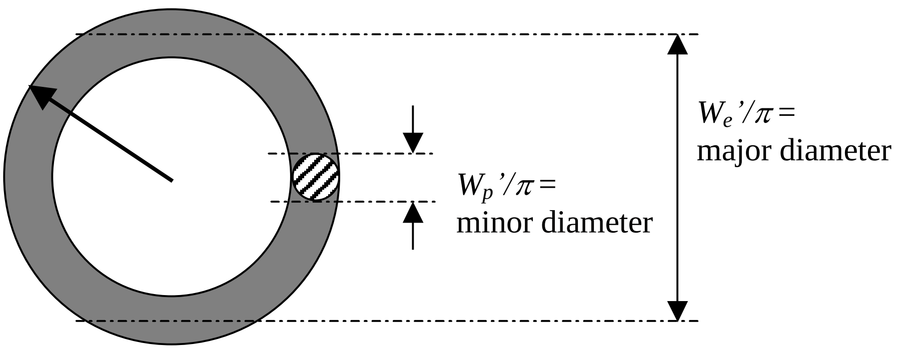


Figure 7. *Geometry of the micro-torus.*

The major circumference of the (the *electron circumference)* is W_e'.

The major radius (the *electron radius*) is: $R_e' = W_e'/2\pi$.

The minor circumference (the *proton circumference)* is W_p'.

The minor radius (the *proton radius)* is: $R_p' = W_p'/2\pi$.

The Universe circumference is W_u'.

The Universe radius is: $R' = R_u' = W_p'/2\pi$.

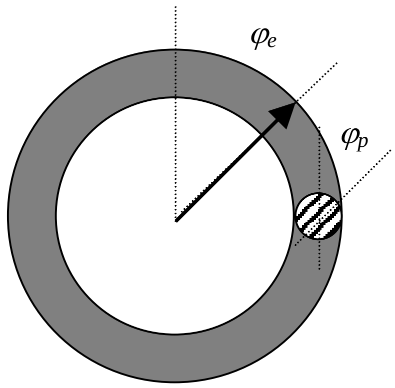


Figure 8. *Angular coordinates for the torus.*

Positions around the micro-torus surface can be specified using angular coordinates: (φ_e, φ_p). These are useful for representing periodic motions or wave functions. Infinitesimals on the torus surface are represented using:

$dw_e' = Re'd\varphi_e'$ and: $dw_p' = Re'd\varphi_p'$

Orientation of the Torus.

We postulate that the central axis of the torus points towards the center of the hyper-sphere in 6 dimensions. This gives rotations around the major circumference an orientation: clockwise or anti-clockwise relative to the direction to the center. This corresponds to positive and negative electric charges. This determines the QM time reversal operator and CPT theorems, an issue that remains unresolved in conventional physics, see (3, 13, 14).

1. Postulate of Euclidean Metric.

Distance in both sub-spaces is Euclidean. In the torus surface:

$$dw'^2 = dw_e'^2 + dw_p'^2$$

In ordinary space:

$$dr'^2 = dx'^2 + dy'^2 + dz'^2$$

So total surface distance between two surface points is determined by:

[1] $du'^2 = dr'^2 + dw'^2 = dx'^2 + dy'^2 + dz'^2 + dw_e'^2 + dw_p'^2$

2. Postulate of Invariant Volume.

A fundamental postulate of the model is *invariance of the six dimensional hyper-volume, denoted* $\boldsymbol{V_6'}$. (Incompressible manifold). We start with the preliminary exercise of determining this volume.

Exercise. Determine the 6-D volume.

The simplest way is simply to multiply the *3-D hyper-volume of the hyper-sphere by the 3-D volume of the torus.* The total volume is found by integrating the torus volume over each point in the hyper-surface, but since the torus volume is constant for the locally 'flat' space, this is just the product of the *hyper-surface times the torus volume.* Later when we consider gravitational curvature, the micro-dimension *W* is strained outwards by the embedded energy of a mass-energy wave, and we must integrate the function for *W(r),* but for the 'background space' modelled as a smooth flat manifold we simply have the global 6-dimensional volume as above.

The volume for the *hyper-sphere* is $2\pi^2R'^3$ (not $4/3\pi R'^3$ as for the ordinary sphere). This is found by integrating around a sphere of radius *R'*:

Hyper-Sphere Volume.

$$Vol_{R'} = \int_{R'=0\ to\ \pi R0'} 4\pi R_0'^2 \sin^2(R'/R'_0)\, dR'$$

$$= 2\pi R_0'^2[R'-\sin(R')\cos(R')]_{[0\ to\ \pi R0']}$$

$$= 2\pi^2R'^3\ (meters^3) \qquad \text{[In universe radius variable]}$$

$$= W_u'^3/4\pi\ (meters^3) \qquad \text{[In universe circumference variable]}$$

Note the radius, R', is not the same as the apparent 'surface distance', measured across normal 3-D space, e.g. as a distance travelled by light during expansion from $R'=0$ *to* R_0'. The volume integrals are derived in more detail in Section 3 on gravity.

The torus volume is just that for an ordinary torus:

Torus Volume.

$$\begin{aligned} Vol_W' &= (\pi R_p'^2)(2\pi R_e') \\ &= 2\pi^2 R_p'^2 R_e' && \text{[In radius variables]} \\ &= W_p'^2 W_e'/4\pi && \text{[In circumference variables]} \end{aligned}$$

We define: $W = (W_p'^2 W_e')^{1/3}$ as the 'average circumference', so that:

$$Vol_W' = W'^3/4\pi$$

The total volume is then:

$$V_6 = (\pi/2)\, R'^3 W'^3$$

This is equivalent to:

$$V_6 = (\pi/2) L_0^6$$

where L_0 is defined as: $\sqrt{(R'W')}$. This will be an invariant quantity of *length* in the model.

We can do this more formally by full integration. The 6-D volume V_6 of the manifold is given by the integral of dV_6' over the hyper-sphere, where dV_6' is the 6-D differential volume element, and equal to the *spherical shell element times the micro-torus volume* at each point *r:*

$$dV_6'(r') = (4\pi R'^2 \sin^2(r/\pi R'))\; 2\pi^2 (W'(r')/2\pi)^3\, dr'$$

[**spherical shell]** **[micro-torus]**

Note that r moves over the 3-D *hyper-surface*, not into the 6-D radius R'.
Rearranging, all the numerical constants cancel:

$$dV_6'(r') = R'^2 \sin^2(r/\pi R')W'(r')^3\, dr'$$

To cover the whole manifold this should be integrated over: $r = 0 \text{ to } \pi R'$:

$$V_6' = \int_{r=0 \text{ to } \pi R'} R'^2 \sin^2(r/\pi R')W'(r')^3\, dr'$$

Assuming W is uniform:

$$\begin{aligned} V_6' &= R'^2 W^3 \int_{r=0 \text{ to } \pi R'} \sin^2(r/R')\, dr' \\ &= ½\, R'^2 W'^3\, [r - \pi R' \sin(r/\pi R')\cos(r/\pi R')]_{0 \text{ to } \pi R'} \\ &= (\pi/2)R'^3 W'^3 \\ &= (\pi/2)L_0^6 \end{aligned}$$

We now state this as a fundamental postulate of the model, with alternative representations in different radius and circumference variables for reference.

[2] The total hyper-volume, V_6', of the spatial manifold is constant.

V_6'	$= (2\pi^2 R'^3)(2\pi^2 R_p'^2 R_e')$	[hypersphere X torus]
	$= 4\pi^4 R'^3 R_p'^2 R_e'$	[Rearrange]
	$= (\pi/2) R'^3 W_p'^2 W_e'$	[Definition of W_p', W_e']
	$= W_u'^3 W_p'^2 W_e'/16\pi^2$	[Definition of W_u]
	$= (\pi/2) R'^3 W'^3$	[Definition of W]
	$= (\pi/2)L_0^6$	[Definition of L_0]
	$=$ ***Constant***^{6}*(Meters*^{6}*)*	

Note that this conservation of volume is an idealisation for the smooth manifold. There is a small change in volume on expansion, as the strain due to embedded masses changes. This is shown in detail in Section 3 on gravity. For the main development of the cosmological model in the next two sections however, conservation of volume is assumed, and this assumption is valid for prediction of the main cosmological effects.

This immediately gives a simple equation relating *R'* to the torus dimensions:

[2.1] $R'^3 = \boldsymbol{Constant}^6/2\pi W_p'^2 W_e'$ [rearrange (2)]

Note that the (length) ***Constant*** is a *fixed length* throughout time – an invariant for the expanding universe. In contrast, W_p and W_e change with time. Because this combination: $W_p'^2 W_e'$ appears whenever the torus *volume* is used, we define an 'average torus radius', *W'* for convenience:

[2.2] $W' = (W_p'^2 W_e')^{1/3}$ [Definition of *W'*]

This lets us write [2] or [2.1] more simply as:

[2.3] $R' = \boldsymbol{Constant}^2/2\pi W' = \boldsymbol{Constant}^2/2\pi(W_p'^2 W_e')^{1/3}$

[2.4] $\boldsymbol{Constant} = (2\pi\, W'R')^{½}$

The ***Constant*** and W_p' and W_e' will be determined as functions of local constants, and we can then give a prediction of the universe radius, *R'*. We then have to transform *R'* → *R*, i.e. into ordinary variables, and relate it instrumentally to cosmological age and distance measurements, before we can interpret it physically in terms of the measured age, *Zc*.

3. Postulate of Universal Speed.

The second postulate is that *there is a universal speed of (wave) propagation on the 5-D hyper-surface of the manifold, denoted c'*. (More precisely: *at any two points on the* *manifold with identical spatial strain, perturbations have identical speed of wave propagation).*

[3] $du'/dt' = \sqrt{((dr'/dt')^2 + (dw'/dt')^2)} = c'$ or equivalently:

$du'^2 = dr'^2 + dw'^2 = c'^2 dt'^2$

Note that *c'* will more generally be a function of the *strain tensor* of the manifold at each point. The strain changes with the expansion, *R',* and with local perturbations in *W'*. But for the moment we assume a *locally flat* manifold as the background space.

Rearranging this we see it has the same form as the Lorentz metric in STR:

[3.1] $dw'^2 = c'^2 dt'^2 - dr'^2$ [Rearrange (3)]

4. Interpretation of Proper Time and STR Metric.

To interpret (3.1) as a physical model for STR, we take *dr'* as *dr* (ordinary space), *dt'* as *dt* (ordinary time), and we are forced to interpret *dw'* as proper time:

[4] $dw'^2 = c'^2 d\tau'^2$ [Interpret proper time]

Substituting into [3.1] this gives:

[4.1] $c'^2 d\tau'^2 = c'^2 dt'^2 - dr'^2$ [Substitute (4) into (3.1)]

If we take a (local) coordinate transformation representing a *velocity boost in r',* we must use the Lorentz transformation to relate the two coordinate systems, exactly as in STR, since the metric is identical in form. However, in the present model, this is really obtained as a *Galilean* velocity transformation applied to a system with a *constant speed of light.*

Note also that the *Lorentz transformation* is only valid locally – when we look at the global curvature of the hyper-sphere, there is a unique stationary frame. A velocity boost in one direction, say *x'*, corresponds globally to a rotating frame, which is accelerating. This means that there is unique global *time frame* with unique simultaneity relations. In practise, this is the frame in which the MWBR (microwave background radiation) is isometric.

4.1 Visualising the local geometry.

It is worth trying to visualise the physical interpretation of this. In the 'flat' space, on a local scale, the velocity-boosted coordinate system will give the same form of mechanical laws as the original. But this is a velocity added *in the r' sub-space only*. The coordinates for *w'* cannot be given a velocity boost, because the motion is circular, and *the number of revolutions between two events is invariant.* I.e. the distance $dw'^2 = c'^2 d\tau'^2$ is invariant under any physically valid transformation. This corresponds to the *invariance of* $c'^2 d\tau'^2$. We can visualise this by 'unrolling' one of the torus dimensions. (Epstein, 1983 [7] uses this as a method for visualising the STR relations.)

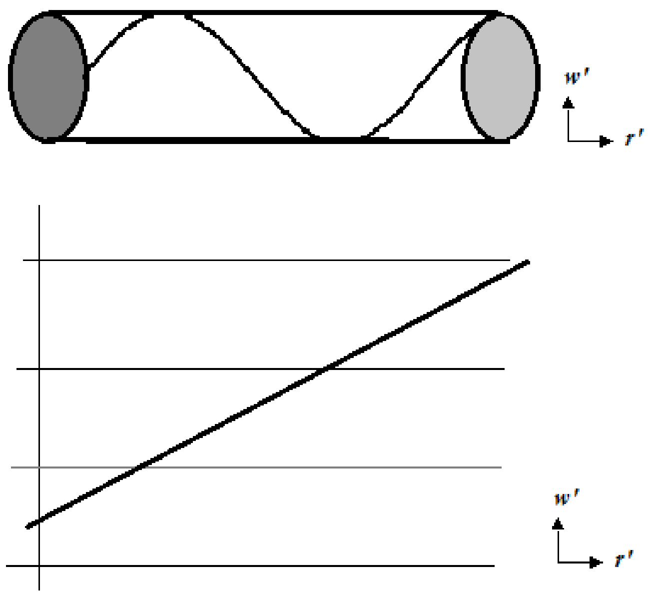


Figure 9. *A line on a cylinder developed into a plane.*

A 'straight line' trajectory on the surface of a cylinder is really curved (like a spring) when viewed in 3-D space, but if we unroll the cylinder onto a flat plane, it appears as a straight line. The *w'* dimension is periodic however – the horizontal lines shown are spaced at ½ the periodic distance. We can transform the coordinates for *r'* (e.g. take: $r'' = r' + Vt'$), and retain a non-accelerating trajectory and the same metric. But we cannot transform the circular coordinate in the same way, because this implies a rotating coordinate system, which would be accelerating – and directly contradicted by the fact that it would change the number of waves observed passing a given point (which implies the invariance of proper time). Refer to Figure 4 for a diagram.

More generally we can represent a (real or imaginary sinusoidal) *wave* on the surface, and write an equation for this. Solving for the boundary conditions, we will find the solution is precisely the simplest type of relativistic quantum ‘particle wave’, described by the Klein-Gordon equation ([18, 39]). But first we have to introduce the model interpretation of *energy* and *mass* and *fundamental particles*.

4.2 Interpretation of Fundamental Particle Waves.

We introduce three basic particles in this model, the *electron, proton and photon.* These correspond to wave modes in the manifold. Photons are surface waves in the (x,y,z) or *r* hyper-sphere. An electron is the lowest wave-mode, $N=1/2$, in the large torus dimension. A proton is the first wave-mode, $N=1/2$, in the small torus dimension. Other fundamental particles must be constructed in the same way, using higher wave-modes. Note that this model takes quarks as orthogonal wave components of the fundamental heavy particle, the proton.

We set up the basic *model of the continuum* here, describing the *continuous evolution* of the manifold, and interpret particles as localised waves. The mechanism for *wave function collapse* and explanation of why particles act as *particles* is given subsequently in the theory of strings. The basic model determines the cosmology and local (gravitational and EM) forces quite precisely. What should also appear are models for *neutrinos, mesons, etc* which should be fundamental particles, but the extension to a full theory of particle physics is not presented in this version.

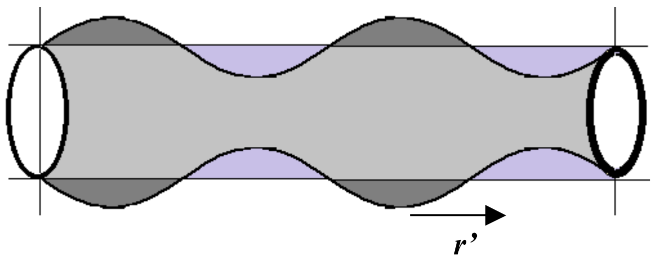


Figure 10. *Plane waves travelling in r’ direction with amplitudes in W.*

A photon is a wave-like distortion, travelling in an *r’* direction with no speed in other directions. Hence: *dr’/dt’ = c’.* A stationary electron is the lowest mode of wave-like distortion of the W_e ring, travelling around W_e with no speed in other directions. Hence: $dw_e'/dt' = c'$. The electric charge and electric force is generated by the ‘screw-like’ shape of the wave disturbance that this generates. The intrinsic angular momentum is from real rotation in the higher dimensional space. A stationary proton is taken as the lowest mode of wave-like distortion of the W_p ring, travelling around W_p. We have to add a circular wave in the W_e ring to give it an electric charge. But we ignore the electric mass component of the proton here. The energies add by their squares and this modifies the energy by only about $\sqrt{(1\text{-}1/1836^2)} = 1.00000015$, which is negligible here.

5. Interpretation of Rest Mass Frequency.

Wave *frequency, f’,* is defined as *the number of wave-nodes passing a fixed point in r’ per unit of time.* Generally, harmonic solutions with wave number *N* per revolution are possible, starting with a *half-wave: N = 1/2*. The rest mass frequency is for a wave stationary in *r’*. This means that it travels at *c’* around one or other torus dimensions. We count *N waves* for revolution of a circular dimension, *W*. Since the wave travels at *c’,* generally:

[5] $f_0' = Nc'/W'$ [Rest mass frequency for wave mode *N* around *W*]

In the simplest model the proton and electron can be taken as first full wave solutions in their torus dimensions.

[5.1] $f_{p0}' = c'/W_p'$ [Proton rest mass frequency]

[5.2] $f_{e0}' = c'/W_e$ [Electron rest mass frequency]

In the more precise model, they are the half-wave solutions in their torus dimensions.

[5.1*] $f_{p0}' = c'/2W_p'$ [Proton rest mass frequency]

[5.2*] $f_{e0}' = c'/2W_e$ [Electron rest mass frequency]

We might expect that there should also be a possible particle with wave-mode in the large circumference of the hyper-sphere:

[5.3] $f_{u0}' = N_u c'/W_u$ [Rest mass frequency a 'Universe Particle']

I have not tried to interpret this third particle here. It will have extremely small mass.

---

6. Postulate of Wave Energy.

The fundamental energy equation must be the same for mass particles as for photons, for the wave theory to be unified, and also to be continuous with STR, and is postulated to be universal for all waves. *h'* corresponds to *h*, Planck's constant.

[6] $E' = h'f'$ [Energy equation for waves]

7. Interpretation of Mass Energy.

The relativistic mass-energy equation and momentum equation must also be continuous with STR. Since *E'* has already been defined above, this can be taken as the definition of *mass* in the model. (This is the first time the mass variable, *m'* appears).

[7] $E' = m'c'^2$ [Energy equation for waves in terms of mass]

Setting these equal we have:

[7.1] $E' = h'f' = m'c'^2$ [(6) and (7)]

[7.2] $f' = m'c'^2/h'$ [Rearrange]

Note that these are quite general, for particles in motion and light, but we interpret them next for the special case of *rest mass waves* (where: $dr'/dt' = 0$) first. For: $dr'/dt' <> 0$, the relativistic mass dilation, etc, must be derived first.

I should note that [7] is not the most general kinetic energy. This is more generally defined in a non-accelerating frame of reference in the six dimensional Euclidean space, subject to Galilean transformations. Thus it includes *the motion of the manifold itself*. The general kinetic energy is: mV^2, with V the total speed. This is important when we solve the energy equation, where we have to take the *radial velocity of the expansion of R'* into account, as well as the local velocity (which is always c'). Later we write for a free particle:

[7.3]
$$\begin{aligned} E' &= m'c'^2 + m'(dR'/dt')^2 \\ &= m'(c'^2 + (dR'/dt')^2) \\ &= m'V^2 \end{aligned}$$

8. Torus Dimensions from Particle Rest Masses.

Substituting (5) into (7.2) we obtain the basic relation:

[8] $m' = Nh'/c'W'$ [General rest mass for wave mode N around W']

Using (5.1) and (5.2) we obtain the basic relations:

[8.1] $m_{p0}' = h'/c'W_p'$ [Rest mass of proton]

[8.2] $m_{e0}' = h'/c'W_e'$ [Rest mass of electron]

Using (5.1*) and (5.2*) these are corrected by a factor of ½.

[8.3] $m_{p0}' = h'/2c'W_p'$ [Rest mass of proton]

$m_{e0}' = h'/2c'W_e'$ [Rest mass of electron]

The quantities m_{p0}' and m_{e0}' correspond to the *particle rest masses.* Since we can measure masses directly rather than *W's*, we normally use this in the rearranged form:

[8.4] $W_p' = h'/c'm_{p0}'$ or $R_p' = \hbar'/c'm_{p0}'$ [Rearrange (8.1)]

$W_e' = h'/c'm_{e0}'$ or $R_e' = \hbar'/c'm_{e0}'$ [Rearrange (8.2)]

Note $\hbar$ enters naturally when converting from circumference to radius variable.
With the correction of ½:

[8.5] $W_p' = h'/2c'm_{p0}'$ or $R_p' = \hbar'/2c'm_{p0}'$ [Rearrange (8.3)]

$W_e' = h'/2c'm_{e0}'$ or $R_e' = \hbar'/2c'm_{e0}'$

Note these relationships between the *model parameters* on the left and *locally measured constants* on the right are determined by the model, not an additional postulate. This lets us determine the size of the torus from empirical measurements of particle rest masses, *h'* and *c'*. The fact that we *measure* m_{p0}' and m_{e0}' directly and *infer* W_p' and W_e' does not mean that the former are fundamental. In the *realist* view taken here, the *model* determines the fundamental entities, and measurements are secondary.

As with *W, w*e define the following combination of masses, *m*:

[8.7] $m' = (m_p'^2 m_e')^{1/3}$ [Definition of *m'*]

$W' = (W_p'^2 W_e')^{1/3}$ [c.f. Definition of *W'*]

Substituting from 8.1, 8.2 we get:

[8.8] $m' = h'/c'W'$ [8.7, 8.1, 8.2]

$W' = h'/c'm'$

Alternatively substituting from 8.3 we get:

[8.9] $m' = h'/2c'W'$ [8.7, 8.3]

$W' = h'/2c'm'$

9. Intrinsic Angular Momentum and Magnetic Moment.

Particles at rest in *r'* are idealised as circular wave motions around a *W* dimension to visualise their frequency behaviour, and if there was just one dimension for spin, this would be equivalent to their mass spinning in a circle. This would give them an 'intrinsic' angular momentum, with magnitude defined by: $L = mvr$.

[9] $L = m'R'c' = m'W'c'/2\pi$

Hence for *half-wave* particles with: $W'=h'/2m'c'$, we predict the *angular momentum:*

[9.1] $L_e = h/4\pi = \hbar/2$ Electron angular momentum

[9.2] $L_p = h/4\pi = \hbar/2$ Proton angular momentum

We identify this *orbital angular momentum* as the *spin angular momentum*. Full treatment of spin is more complex, as noted in Appendix 4, but this is a fundamental prediction of the model. Intrinsic angular momentum is considered a mysterious quantum property, without a realistic interpretation. This shows it has a realistic interpretation as a *physical angular momentum.* This is strong evidence for the model.

Similarly intrinsic magnetic moments are also a quantum effect, predicted when we consider the electron and proton simply as charged particles spinning in a circle:

[9.3] $\mu_e' = Nq'c'R_e' = q'c'W_e'/4\pi = q'\hbar'/2m_e' = -9.2730 \times 10^{-24}$

The experimental value is: $\mu_e' = -9.2848 \times 10^{-24}$. The ratio to the prediction is 1.0013.

For the proton:

[9.4] $\mu_p' = Nq'c'R_p' = q'c'W_p'/4\pi = q'\hbar'/2m_p' = 5.0502 \times 10^{-27}$

The experimental value is: $\mu_e' = 1.4106 \times 10^{-26}$. The ratio to the prediction is 2.7932.

The model is precisely accurate for the electron, but only approximately accurate for the proton. The latter is clearly more complex than this simple model allows. That is not surprising when we consider the complex geometry of rotation in the torus that generates the ‘current’. What is surprising is that the result is so realistic. We are combining quantities with extremely tiny magnitudes (10^{-34}, 10^{-27}, 10^{-19}), and we get an answer within a factor of 2.8. The almost perfect accuracy for the electron prediction is astounding. These simple predictions are further strong evidence the model is realistic.

SECTION 2. THE RELATIVISTIC QUANTUM WAVE.

The model immediately predicts the basic *relativistic quantum wave,* in the form of the Klein-Gordon equation, as the complex wave function satisfying the boundary conditions of the manifold. (The non-relativistic Schrodinger equation is an approximation to this).

10. Relativistic Quantum Wave Solution.

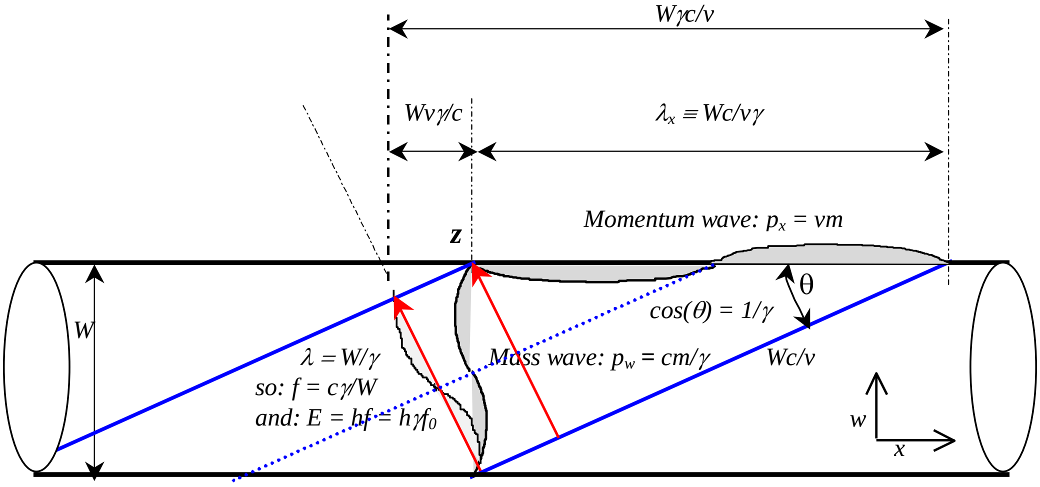


Figure 4 *(Repeated). Geometry for a simple mass-momentum plane wave.*

This illustrates a plane wave, with wave nodes (constant amplitude) shown in blue. The central node (dotted blue line) is at half a wave length. Wave vectors (direction of the wave motion) are shown in red. The wave fronts move in the direction of the wave vectors (red) at speed c. The 'true wave-length' corresponds to the length of the (red) wave vector arrows, and simple geometry gives the length as: $\lambda = W/\gamma$. Wave fronts (or nodes) arrive successively at the fixed point **z** every time a node travels half the true wave-length, and since it is moving at the speed c, the full period is: $T = c/\lambda = c\gamma/W$. The wave moves from right to left in ordinary space, x, at speed v. It appears to have a wave-length in *ordinary space* of: $\lambda_x = Wc/v\gamma$.

The basic complex solution for wave motion in one dimension, x, is:

[10.1] $\Psi(x,w;t) = A\ Exp(i/\hbar)(p_x x+p_w w-(E_x+E_w)t)$

$= AExp(i/\hbar)(p_x x+p_w w-(E_x+m_0c^2)t)$

Defining: $\gamma^2 = 1/(1-v^2/c^2)$ and: $\hbar = h/2\pi$. (Note we use: $\eta = \hbar = h/2\pi$ in equations below as the MS Equation editor does not have the symbol for $\hbar$.) It is shown how this is derived below. It represents a Klein-Gordon wave function for a free particle, i.e. the simplest variety of *relativistic Schrodinger wave*, as shown by differentiating.

[10.2] $$\frac{\partial\Psi}{\partial x} = \frac{iv_x\gamma m_0}{\hbar}\ \Psi = \frac{i}{\hbar}v_x m\Psi = \frac{i}{\hbar}p_x\Psi$$

Hence this solution satisfies the usual momentum eigenvalue equation:

[10.3] $$p_x\Psi = -i\hbar\frac{\partial\Psi}{\partial x}$$

For the second spatial differential:

[10.4] $$\frac{\partial^2\Psi}{\partial x^2} = \frac{-v_x{}^2 m}{\hbar^2}\ \Psi \quad \approx \frac{-2m}{\hbar^2}E_x\Psi \quad \textit{for low velocities.}$$

Similarly for the spatial differentials w.r.t. *w:*

[10.5] $$\frac{\partial\Psi}{\partial w} = \frac{icm_0}{\hbar}\Psi$$

[10.6] $$\frac{\partial^2\Psi}{\partial w^2} = \frac{-m_0{}^2c^2}{\hbar^2}\ \Psi$$

So the total second spatial derivative is:

[10.7] $$\nabla^2\Psi = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial w^2} = \frac{-m^2c^2}{\hbar^2}\ \Psi = \frac{-mE_{Total}}{\hbar^2}\ \Psi$$

And the time differential is:

[10.8] $$\frac{\partial\Psi}{\partial t} = \frac{-i}{\hbar}\left(\frac{m_0 c^2}{\gamma} + m_0 v_x{}^2\gamma\right)\Psi = \frac{-i}{\hbar} mc^2\Psi = \frac{-i}{\hbar} E_{Total}\Psi$$

This is the Klein-Gordon equation - the relativistic version of the time-dependant Schrodinger equation - for a free particle (without spin components). ([18, 39]). This property shows that the particle model is *prima facie* ideal for modeling the quantum wave functions for particles.

10.1 Deriving the QM Solution.

We specify the boundary conditions for the waves in our manifold. e consider a free particle, as depicted in Fig.9, with ordinary motion only in the *x-direction,* and for simplicity we can ignore the two other (*y* and *z*) directions (partial differentials w.r.t. *y* and *z* are all zero). Space-time points can be represented by: (*x,w; t*).

[10.1.1] The periodic nature of *w* means that the points: $\mathbf{P_0}$ = (0,0;0), and $\mathbf{P_1}$ = (0,W;0) are identical space-time points.

We assume there is a wave function: $\Psi = \Psi(x,w;t)$. The values of this wave function are complex, but we ignore what they represent here. All we really need are boundary conditions, which apply whatever the wave amplitudes represent, along with previous energy and momentum relationships, which relate the energy and momentum to the frequency and wavelengths of simple complex sinusoidal solutions. 38.1 determines the first boundary condition:

[10.1.2] $$\Psi(x,w;t) = \Psi(x,w+W;t)$$

The derivation of general solutions of this kind of wave function are well known, and I just specify the simplest *complex sinusoidal wave function solutions,* and point out the equivalence with *QM* solutions. The simple solutions are separable as the products of four wave functions, labeled as follows:

[10.1.3] $$\Psi(x,w;t) = \Psi_x(x;t)\Psi_w(w;t) = \psi_{px}(x)\psi_{pw}(w)\psi_{tx}(t)\psi_{tw}(t)$$

ψ_{px} and ψ_{pw} have respective wavelengths: $\lambda_x = W(c/v\gamma)$, and $\lambda_w = W$ (for spin-1 particle). $\psi_{tx}(t)$ and $\psi_{tw}(t)$ have respective wave speeds: $v_x = V$, and $v_w = c/\gamma$, and periods: $T_x = \lambda_x/v_x = Wc/V\gamma$ and $T_w = W\gamma/c$. The period of the full wave is: $T = W/c\gamma$, with speed c.

Boundary conditions pertaining to the space-time *origin*: $(x,w;t) = (0,0;0)$, for the full periodic wave function $\Psi(x,w;t)$, are:

[10.1.4] $$\Psi(0,0;0) = \Psi(\lambda_x,0;0) = \Psi(0,\lambda_w;0) = \Psi(0,0;T)$$

And more generally at an arbitrary point $(x,w;t)$:

[10.1.5] $$\Psi(x,w;t) = \Psi(x+n_x\lambda_x, w+n_w\lambda_w; t+n_tT), \quad \text{where } n_x, n_w, n_t \textit{ are integers}.$$

These conditions are satisfied by a complex plane wave with crests traveling at velocity $v_x = V$, $v_w = c/\gamma$, *and* $v_{total} = c$. A simple complex plane wave solution is:

[10.1.6] $$\Psi(x,w;t) = A\,Exp((2\pi i/W)(xv_x\gamma/c + w - t(c/\gamma + v_x^2\gamma/c))$$

Substituting for W using (13), this is:

[10.1.7] $$\begin{aligned}\Psi(x,w;t) &= A\,Exp(i/\hbar)(p_xx+p_ww-(E_x+E_w)t)\\ &= AExp(i/\hbar)(p_xx+p_ww-(E_x+m_0c^2)t)\end{aligned}$$

The complex conjugate is also a solution of the boundary conditions, but is a 'time-reversed' solution, as obtained by taking the ordinary time reversal, *T*, of the quantum (Schrodinger) equation. It naturally represents the anti-particle of (38.7). Note that the combined operation of time reversal and complex conjugation, *T**, which is normally adopted as the 'time reversal' operator in QM, leaves the Schrodinger equation and the solution (32) invariant. The general wave function (for a finite volume) can be expanded as a sum of such plane waves. E.g. (4, Ch. 10).

SECTION 3. THE COSMOLOGICAL MODEL VARIABLES.

The next sections give the dynamics and transformations for variables and constants in the cosmological model, relating to dynamic expansion of the radius *R*. This is more general than a specific solution to the expansion cycle, which is obtained in the following section.

11. Dimensionless ratios and natural units.

The model gives us two independent dimensionless ratios, defined by:

[11.1] Large Ratio:

$$W_{universe}'/W' = 2\pi R'/W'$$
$$= R_{universe}'/R_{partticles}'$$

[11.2] Small Ratio:

$$W_p'/W_e' = R_p'/R_e'$$
$$= (m_p'/m_e')$$
$$= (m'/m_e')^{3/2}$$

The second generally appears in the 'normalised' form:

[11.3] Small Normalised Ratio:

$$\gamma = W_e'/W' = m'/m_e'$$
$$= (m_p'^2 m_e')^{1/3}/m_e'$$
$$= (m_p'/m_e')^{2/3}$$

Note: $W_p'/W' = m'/m_p' = (m_p'^2 m_e')^{1/3}/m_p' = (m_e'/m_p')^{1/3}$.

There are seven fundamental constants in the ordinary physical theory we wish to model (including gravitational and electric constants): $c, h, G, m_p, m_e, q_e = -q_p, 4\pi\varepsilon_0$. (Note there are really eight constants, but we assume that the elementary electric charge of the proton and electron are equal and opposite: $q_e = -q_p$. Strictly this is empirical too and should be predicted by the model, but it is interpreted as a definition here.)

There are four physical dimensions in the ordinary theory: *space, X, time, T, mass, M, charge, Q*. This mean that exactly three independent *dimensionless* quantities can be defined:

[11.4]	$D' = h'c'/m'^2G'$	[Definition: Large dimensionless constant]
[11.5]	$\rho' = m_p'/m_e'$	[Definition: Fundamental mass ratio]
or:	$\gamma' = (m_p'/m_e')^{2/3}$	[Definition: Small normalised ratio]
[11.6]	$\alpha' = q'^2/2\varepsilon_0'h'c'$	[Definition: Fine structure constant]

Any other dimensionless quantities are defined as functions of these. It is instructive to define 'natural units' for the dimensions to view the relationships in a simplified form (although we continue to work in real units in the rest of the theory). We can set four independent constants equal to 1, and take this as the definition of the *units*. The most natural choice is:

[11.7] NATURAL UNITS A DEFINED

$c' = 1 \qquad h' = 1 \qquad m' = 1 \qquad 2\varepsilon_0' = 1$

This entails the following values for dimensionless ratios:

[11.8] NATURAL UNITS A: DIMENSIONLESS CONSTANTS

$D' = 1/G'$

$\rho' = m_p'/m_e'$

$\alpha' = q'^2$

This illustrates that there are only 3 degrees of freedom, or three empirical relationships that can be added.

12. The fundamental ratio postulates.

We can now state the three fundamental relationships of the model, that relate the *three fundamental ratios that characterise the global state of space* on one side, to the *magnitudes of the seven local constants* on the other side.

[12.1] *Small Ratio equals ρ' (defined by: m_p'/m_e')*

$$W_e'/W_p' = \rho' = m_p'/m_e'$$

[12.2] *Large Ratio equals D' (defined by: $h'c'/m'^2G'$)*

$$(2\pi R')/W' = D' = h'c'/m'^2G'$$

[12.3] *Small Normalised Ratio equals 1/α'*

$$(m_p'/m_e')^{2/3} = 2\varepsilon_0'h'c'/q'^2$$

Note it is speculated that the last relationship is corrected with a fine factor:

[12.3*] *Small Normalised Ratio equals 1/α'* **+ *(dR'/dt')*^{2}**

$$(m_p'/m_e')^{2/3} = 2\varepsilon_0'h'c'/q'^2 + (dR'/dt')^2$$

However this is not proved. When we come to analyse *R',* this assumption lets us fully determine the cosmological model, and the solution is realistic. Otherwise we must use the Hubble parameter as an extra measurement, to determine the present speed of expansion. It is not known if this extra postulate is correct. However there must be *some postulate of this kind,* correcting the 5% error in the prediction of the find structure constant from [12.3], and relating the final parameter dR'/dT' to the constants, and asserting this serves to highlight this.

12.1 The fundamental mass ratio.

The first relationship (12.1) is determined by the model from: $W' = h'/m'c'$ (8.8) (or equally from: $W' = h'/2'm'c'$, (8.9). We use this to fix the size of W' to start with. This is an empirical prediction and subject to evidence, since the size of W' might have turned out to be unrealistic, and because it is part of the larger theory of wave functions, that predicts the de Broglie wave-length, spin, etc. But it is equivalent to fixing a metric scale for the model space (in meters) that compares to our conventional metric scale for space (in meters). So it is not directly testable by independent measurement of W'. After adopting this, the first and third relationships provide direct independent empirical tests.

Note that the variable dynamics given later, and: *14.4-14.5,* means m_p/m_e is constant. This fixed constant (*1836*) fixes the kind of particle universe we are in. The origin of this number is unexplained by the theory. But it is supposed that, in the early universe, when matter dissociated from radiation, this ratio was 'frozen out' into the energy balances of the universe. It is treated as fixed here. The fine dynamics are later related to small changes in the *fine structure constant,* relative to the fixed *normalised mass ratio.*

12.2 The radius of the universe.

The second relationship (12.2) determines the function for R', the model radius of the universe:

[12.4] $R' = W'h'c'/2\pi m'^2G'$ [True Radius of the Universe]

Assuming the simplistic version: $W' = h'/m'c'$ (8.8) gives:

$R' = h'^2/2\pi m'^3G'$ [substitute W', 8.8]

But we need to use the accurate version: $W' = h/2mc$ (8.9) which gives:

$R' = h'^2/4\pi m'^3G'$ [substitute W', 8.9]

Note we cannot relate R' (in the model) to Z/c (the empirical measurement age) directly: we must establishing the variable transformations from: $R' \rightarrow R$, and the dynamics and measurement process for Z/c so we can relate it to: $R \rightarrow Z/c$, remembering that R does not appear in the conventional theory. See later sections on light trajectories, co-moving distance measurements. In any case, this model must posit the measurement relation:

[12.4*] $Z_0 = R_0/\pi = R_0'/2\pi$ [Interpret Z, match measurement to model]

This relation needs to be analysed in detail, but it then entails:

Predict *Z*. $Z_0 = h^2/2\pi^2 m_e m_p^2 G = 13.823$ *billion l.y. (distance)*
Predict *Z/c*. $Z_0/c = h^2/2\pi^2 m_e m_p^2 Gc = 13.823$ *billion years. (time)*

Note that this means that this conventional *age* measurement is really a *distance measurement* in disguise. Z_0/c converts to a time T^*_0, which reflects the conventional *age* of the universe as measured experimentally by cosmologists.

12.2 The empirical age of the universe.*

Conventional cosmology measures the age, T^*_0, corresponding to a distance, $Z = T^*_0\, c$. I have labelled this 'age': T^*_0 because it is not the real current age, T_0, in the model, it is really a distance in the model, $R'2/\pi$. The two best measurements of the *conventional age of the universe* are currently[13] 13.798 and 13.84 billion years, determined experimentally by two different methods. The true value is expected to fall between these two values. Note the *empirical relationship* is given by:

- T^*_0 is in: [*13.798, 13.84*] b.y. [Time measurement]
- $Z_0 = c\,T^*_0$ is in: [*13.798, 13.84*] b.l.y. [Distance measurement equiv.]
- $R_0/\pi = h^2/2\pi^2 m_e m_p^2 G = 13.823$ *b.l.y.* [Distance - model prediction]
- $(R_0/c\pi = h^2/2\pi^2 m_e m_p^2 Gc = 13.823$ *b.y.* [Time - model prediction]
- Prediction: $T^*_0 = h^2/2\pi^2 m_e m_p^2 Gc$ is accurate to 0.1%.

[13] In mid-2013.

12.3 The static electric charge and fine structure hypothesis.

The third relationship (12.3) means we can predict the fine structure constant from the fundamental mass ratios, or reverse this to deal with the directly measurable quantities, and predict the (positive) magnitude of the fundamental electric charge from the other five other constants. From (12.3), the *static electric postulate:*

[12.5] $q' = (m_e'/m_p')^{1/3}(2\varepsilon_0'h'c')^{½}$ [12.3, rearrange]

$q'^2/2\varepsilon_0' = (m_e'/m_p')^{2/3}\, h'c'$ [Equivalent]

This shows the electric properties are reducible to the properties in the mass ratio, universal speed *c'*, and quanta of angular momentum *h'*.

Table 9.

Elementary electric charge	Value	Units	Dimension	Meaning
q-predicted = $(2hc\,\mu)^{1/2}\,(m_e/m_p)^{1/3}$	1.5316E-19	Coulombs	Q	elementary electric charge predicted
q-measured	1.6020E-19	Coulombs	Q	elementary electric charge
Ratio	**0.9561**			

Calculation shows the predicted value is about 4.5% less than the empirical value.

This indicates the model is strong, but the discrepancy is still highly significant. It indicates a mechanism that has not been represented.

12.3 The dynamic electric charge and expansion rate hypothesis.*

To explain the discrepancy of 4.5% in *q*, I propose an extension in the form of (12.3*), the dynamic hypothesis. This can be considered as a provisional hypothesis to make the present model fully definite, but there are other possibilities.

The dynamic version of the fine structure ratio postulate, (12.3*) can be rearranged to:

[12.6] $q'^2/2\varepsilon_0' = (m_e'/m_p')^{2/3}(h'c')\mathbf{(1 + (dR'/dt')^2/c'^2)}$ [12.3*, rearrange]

$q' = (m_e'/m_p')^{1/3}\sqrt{(2\varepsilon_0'h'c')}\mathbf{(V'/c')}$

Given we are interested in the time variation, $d(q'^2/2\varepsilon_0')/dt'$, of the left hand side, this predicts a fine dependence on the radial acceleration, $d((dR'/dt')^2/c'^2)/dt$. This is tiny and not included in the transformations given below, 13 - 17.

The manifold property we use to reconcile the empirical discrepancy in the electric charge prediction is the universe expansion rate, dR'/dt'. This adds orthogonally to the surface speed c' to give the total speed: $V' = (c'^2 + (dR'/dt')^2)^{1/2}$. Defining $1/\beta$ as:

[12.7] $1/\alpha + 1/\beta = \gamma$ [Definition of $1/\beta$]

$137 + 13 = 150$

$\alpha = 1/137,\ \beta = 1/13, \gamma = 150$

- If $c^2 = 1$, then $V'^2 = 150/137$, and $(dR'/dt')^2 = 13/137$ and $c'^2 = 137/137$

The hypothesis is that *two electric components* add to give the true invariant γ, from two velocity-squared related components, $1/\alpha$ arising from speed in the manifold: c'^2, and $1/\beta$ from the speed of the (orthogonal) manifold: $(dR'/dt')^2$. (12.3*) is equivalent to postulating that: $c'^2/\alpha + c'^2/\beta = c'^2\gamma = V'^2$. Or equivalently:

[12.8] $c'^2\alpha\gamma = V'^2$ [*12.3**. Postulate: Fine Structure Hypothesis]

$c'^2\alpha/\beta = (dR'/dt')^2$ [Equivalent, using definition 12.7]

$dR'/dt' = c'\sqrt{(\alpha/\beta)}$ [Rearrange]

The empirical values are very close to:

[12.9] $dR'/dt' = c'\sqrt{(13/137)}$

$= c'\sqrt{0.0949}$

$= c'\ 0.308$

But note that unlike other mechanisms (gravity, QM particle-waves, dynamics of constants, etc) derived here, this fine dynamic dependence of the electric constants has not yet been shown to derive directly from the model. Nonetheless some relation of this kind should exist in the model. This assumption gives us a direct way to determine the expansion speed. After completing the time-dependant solution in Section 2, we will use this to estimate the age of the universe.

13. The variable transformations.

The dynamics of the constants and variable transformations are now defined. The following variable transformations are required for consistency with the evolution equations, given in the next section.

[13.1]	$\hat{R}' = R'/R_0'$ and: $\hat{R} = R/R_0$	*Definition of normalised radius universe*
[13.2]	$\check{T}' = T'/T_0'$ and: $\check{T} = T/T_0$	*Definition of normalised age*
[13.3]	$\hat{R}'dx' = dx$	*Space metric transformation*
[13.4]	$\hat{R}'^2 dt' = dt$	*Time metric transformation*
[13.5]	$\hat{R}'dm' = dm$	*Mass metric transformation*
[13.6]	$\hat{R}'dq' = dq$	*Electric charge metric transformation*

14. The evolution equations for the model constants.

The following equations are the key to the model. They are justified from first principles when we examine the physical mechanism carefully, but it is sufficient here to simply state them and demonstrate their consistency.

[14.1]	$c' = c_0\hat{R}'$	*Evolution of speed of light constant*
[14.2]	$h' = h_0/\hat{R}'$	*Evolution of Planck's constant*
[14.3]	$G' = G_0$	*Evolution of gravitational constant*
[14.4]	$m_e' = m_{e0}/\hat{R}'$	*Evolution of electron mass*
[14.5]	$m_p' = m_{p0}/\hat{R}'$	*Evolution of proton mass*
[14.6]	$q_e' = q_{e0}/\hat{R}'$	*Evolution of elementary electron charge*
[14.7]	$\varepsilon' = \varepsilon_0/\hat{R}'^2$	*Evolution of electric force constant*
[14.8]	$\mu' = \mu_0$	*Evolution of magnetic force constant*

Note that the electric constant dynamics here determines the *static* fine structure constant, which remains fixed. For the dynamic version, (14.6-14.8) have small second-order terms added, but we do not consider this further here.

Terms with subscript *0* represent *present values.* These equations make the true variables continuous with the conventional variables at the present moment, seen by substituting the value: $\hat{R}' = 1$ (which is its present value). For clarity, I will state these explicitly as boundary conditions.

15. The boundary conditions at the present time and origin

[15.1] $R = 0$ if $R' = 0$ *Zero radius of the universe*

[15.2] $T = 0$ if $T' = 0$ *Zero age of the universe*

[15.3] $dx_0' = dx_0$ *Current space metric*

[15.4] $dt_0' = dt_0$ *Current time metric*

[15.5] $dm_0' = dm_0$ *Current mass metric*

[15.6] $dq_0' = dq_0$ *Current electric charge metric*

[15.7] $c_0' = c_0$ *Current speed of light constant*

[15.8] $h_0' = h_0$ *Current Planck's constant*

[15.9] $G_0' = G_0$ *Current gravitational constant*

[15.10] $m_{e0}' = m_{e0}$ *Current electron mass*

[15.11] $m_{p0}' = m_{p0}$ *Current proton mass*

[15.12] $q_{e0}' = q_{e0}$ *Current elementary electron charge*

[15.13] $\varepsilon_0' = \varepsilon_0$ *Current electric force constant*

[15.14] $\mu_0' = \mu_0$ *Current magnetic force constant*

I list the present values of the physical constants for convenience. Five decimal places are sufficient here.

[15.15] $c_0 = 2.99793 \times 10^8$ m/s

$h_0 = 6.62607 \times 10^{-34}$ Js

$G_0 = 6.67384 \times 10^{-11}$ Nm^2/kg^2

$m_{e0} = 9.10938291 \times 10^{-31}$ kg

$m_{p0} = 1.672621777 \times 10^{-27}$ kg

$m_0 = 1.3695 \times 10^{-28}$ kg

$\varepsilon_o = 8.85418782 \times 10^{-12}$ F/m

$q = 1.6020 \times 10^{-19}$ C

16. The dimensional relations.

We use *dimensional analysis* next to obtain the evolution equations for the constants in conventional variables. We can think of dimensional quantities as *basis vectors* for the system of physical units, using ***X, T, M, Q*** and ***X', T', M', Q'*** respectively for the two systems. These are related exactly like the differential transformations above.

[16.1] $dx \equiv \mathbf{X} = \hat{R}'\mathbf{X'}$ *Space dimension changes*

[16.2] $dt \equiv \mathbf{T} = \hat{R}'^2\mathbf{T'}$ *Time dimension changes*

[16.3] $dm \equiv \mathbf{M} = \hat{R}'\mathbf{M'}$ *Mass dimension changes*

[16.4] $dq \equiv \mathbf{Q} = \hat{R}'\mathbf{Q'}$ *Electric dimension changes*

17. The evolution equations for the conventional constants.

In the conventional theory, the constants are static, so that in conventional units, $c_0 = c$, $h_0 = h$, etc, at all cosmological times. We now verify that our model has this consequence for all conventional constants *except G*. We can obtain the evolution equations for the constants in our conventional variables from *dimensional analysis* of the quantities, and the previous transformations.

[17.1] $c \equiv \mathbf{X/T} = X'/\hat{\mathbf{R}}'\mathbf{T'} \equiv c'/\hat{R}' = c_0$ *c is constant*

[17.2] $h \equiv \mathbf{MX^2/T} = \hat{R}'\mathbf{M'X'^2/T'} \equiv \hat{R}'h' = h_0$ *h is constant*

[17.3] $G \equiv \mathbf{X^3/MT^2} = \mathbf{X'^3/M'T'^2}\hat{R}'^2 \equiv G'/\hat{R}'^2 = G_0/\hat{R}'^2$ ***G is decreasing***

[17.4] $m_e \equiv \mathbf{M} = \hat{R}'\mathbf{M'} \equiv \hat{R}'m_e' = m_{e0}$ *m_e is constant*

[17.5] $m_p \equiv \mathbf{M} = \hat{R}'\mathbf{M'} \equiv \hat{R}'m_p' = m_{p0}$ *m_p is constant*

[17.6] $q_e \equiv \mathbf{Q} = \hat{R}'\mathbf{Q'} \equiv \hat{R}'q_e' = q_{e0}$ *q_e is constant*

[17.7] $\varepsilon \equiv \boldsymbol{Q^2T^2/MX^3} = \boldsymbol{Q'^2T'^2/M'X'^3} \equiv \hat{R}'^2\varepsilon' = \varepsilon_0$ $\quad$ *ε_0 is constant*

[17.8] $\mu \equiv \boldsymbol{MX/Q^2} = \boldsymbol{M'X'/Q'^2} = \mu' = \mu_0$ $\quad$ *μ_0 is constant*

The model predicts that *all the constants except G* appear invariant in conventional variables, even though they really change in true variables. *G* is decreasing with $\hat{R}'^2$ in conventional variables, but is the only invariant in true variables.

However we still have to calculate how *G* is changing with $\hat{R}$, i.e. radius variable in conventional units, as opposed to $\hat{R}'$. I should emphasise that this reflects the essential point that Dirac realised in his later work on cosmology (from 1969), except his theory is parametised by time rather than space. His initial (1939) theory of evolving constants, which did not take any change of variables into account, predicted that *G* was decreasing by $1/T^2$, but this was empirically disproved, and this rate of change is too fast to be physical. Dirac subsequently realised that incorporating a variable transformation is necessary and the decrease of *G* may appear different in conventional variables to its real decrease in true variables.

18. The relation between spatial variables in the two systems.

We use the differential transformations above to relate *R* to *R'*, i.e. the expansion of the universe in the two variable systems. We must integrate using 16.1: $dR/dR' = dx/dx' = \hat{R}'$, and use the boundary condition that: $R = 0$ *when* $R' = 0$.

[18.1] $R_1 = \int_{0,R1} dR$

$$= \int_{0,R1'} dR/dR'\,dR'$$
$$= \int_{0,R1'} \hat{R}'\,dR'$$
$$= \int_{0,R1'} R'/R_0'\,dR'$$
$$= [R'^2/2R_0']_{0,R1'}$$
$$= R_1'^2/2R_0'$$

Giving:

[18.2] $R_1 = R_1'^2/2R_0' = R_1'\hat{R}_1'/2$

[18.3] $R_0 = R_0'/2$

[18.4] $\hat{R}_1 = R_1/R_0 = \hat{R}_1{}'^2$

And the inverse relations:

[18.5] $R_1' = 2R_1/\sqrt{\hat{R}_1}$

[18.6] $R_0' = 2R_0$

[18.7] $\hat{R}_1' = \sqrt{\hat{R}_1}$

The relation between R and R' is quadratic, not linear, but at the *present moment* the relationship is the simple linear one (18.3), (18.6). This depends only on the transformation 16.1 and the boundary conditions at the origin. But note that we will *not* assume a singularity in R' or R at the temporal origin: we find a minimum value of R' and R close to the origin, when the universe 'bounces' in a cyclic expansion and collapse process. Note this result is general and does not assume any evolution equation for R' in terms of T'.

19. The relation between time variables in the two systems.

We use the variables T and T' specifically for *the age of the universe*. They are identical to the general variables: t and t' respectively when we set the origins as: $t = t' = T = T' = 0$ at the 'start' of the universe. However t or t' as general time variables are normally assumed to have a conventional origin, whereas T and T' have a specified origin.

The 'start' of the universe in the expanding or cyclic models here is taken as the time when the radius theoretically goes to 0. However, in the cyclic model developed here, the time variable in a cycle only has a valid range from a value T_{MIN}' (or T_{MIN}) slightly larger than 0, when the universe is at its minimum radius, through T_{MAX}' (or T_{MAX}) at its maximum radius, and on through to $2T_{Max}' - T_{MIN}'$ (or $2T_{MAX} - T_{MIN}$) when it has contracted back to its minimum radius again.

To relate the time variables T and T' to each other (the age of the universe in the two systems), we must integrate using 16.2: $dT/dT' = dt/dt' = \hat{R}'^2$, and boundary condition: $T = 0$ *when* $T' = 0$.

[19.1] $\quad T = \int_{0,\,T1} dt \quad = \int_{0,T'} dt/dt'\, dt'$

$$= \int_{0,T'} \hat{R}'^2\, dt'$$

$$= \int_{0,T'} R'^2/R_0'^2\, dt'$$

However (unlike the spatial integration) we cannot perform this integral until we know the evolution equation for R' in terms of T', i.e. the function for $R'(T')$. A solution for this is determined in the following section, but we continue here with results that are independent of the solution.

(The solution is: $R'(T') = R'_{MAX}\, sin^2(\pi\, t'/2T'_{MAX})$, where: R'_{MAX} is a maximum radius of expansion and T'_{MAX} is the (first) time this radius is reached (after $T'=0$). I note the result of the integral here as: $T = (R'_{MAX}/R_0')^2(3T'/8 - sin(AT')cos(AT')/2A + sin(2AT')cos(2AT')/16A)$, where: $A = (\pi/2T_{MAX}')$ and: $T_{MAX}' = \pi R_0'/c_0'$, so that: $A = (c_0'/2R_0')$.

20. The evolution of G in conventional variables.

We can now relate the evolution of G directly to R. Substituting 18.7 into 17.3 gives:

[20.1] $\quad G(R) = G_0/\hat{R}_1 = G_0R_0/R_1 \quad$ [18.7 in 17.3]

The model therefore predicts that *the conventional measurement of G will appear proportional to* $1/R_1$ in conventional variables, whereas it will *appear proportional to* $1/R_1'^2$ in true variables. Note that although the true G' is invariant, the *force of gravity* (for two masses at a constant distance) still weakens in true variables – but this is due to the reduction of *mass* in true variables, rather than a reduction in the gravitational constant. I reiterate that the two variable systems are mathematically consistent with each other as coordinate descriptions, but we identify one as the ‘true system’, because it gets the correct time metric to ensure the time translation invariance of the laws of physics.

21. The evolution of the Dirac constant.

The *dimensionless constant* that we can generate from the fundamental constants *c, h, G,* and a fundamental mass *m,* is here called the *Dirac constant, D.* This played the central role in Dirac's development of a theory of evolving constants. For an arbitrary mass *m* we repeat the definition above for convenience:

[21.1]	$D_m = hc/m^2G$	Definition of the D_m for mass *m*
	$D_m' = h'c'/m'^2G'$	and mass *m'*

Without a model, Dirac did not know whether to use the mass of the electron, m_e, or proton, m_p, or a combination, but we have seen that our geometric model requires the combination: $m = (m_e m_p m_p)^{1/3}$ to define key relationships. We now define some properties of *D*. Since the constants are time dependant, *D* and *D'* are time dependant, and we write its fundamental equations as a function: *D'(R'),* or *D(R).* Using the evolution equations for the constants we see that:

[21.2]	$D_{m0} = h_0\, c_0\, /m_0\, {}^2G_0$	D_m at the present time

[21.3] $\quad D_m'(R') \quad = h'c'/m'^2G'$

$$= \hat{R}_1{}'^2 h_0 c_0/m_0{}^2 G_0$$

$$= \hat{R}_1{}'^2 D_0$$

[21.4] $\quad D_m(R') \quad = hc/m^2G$

$$= \hat{R}_1{}'^2 h_0 c_0/m_0{}^2 G_0$$

$$= \hat{R}_1{}'^2 D_0$$

We can then determine the corresponding function in *R*:

[21.5] $\quad D_m(R) \quad = \hat{R}_1 D_m(R')$

Hence $D = D'$ and it is invariant w.r.t the *system of variables.*

[21.6] $D_m'(R') = D_m(R') = D_m(R)$

This identity is required because D is *dimensionless.* The difference between *c', h', G', m'* and *c, h, G, m* is that they use different coordinate scales and physical units to represent the same physical quantities. For a dimensionless quantity, the physical quantities cancel out, so there is no dependence on the coordinate scales or variable system. This is a verification of self-consistency.

22. The evolution of the fine structure constant.

The second dimensionless constant is the *fine structure constant*, defined as:

[22.1] $\alpha' = q'^2/2\varepsilon_0'h'c' \approx 1/137.035999074$ [Definition of FSC]

$\alpha = q^2/2_0hc$ [In conventional variables]

Using the transformations (14.1-14.8) and (17.1-17.8) these are found invariant w.r.t. R, with the same value in either variable system.

[22.2] $\alpha' = q'^2/2\varepsilon_0'h'c' = (q_{e0}/\hat{R}')^2/2(\varepsilon_0/\hat{R}'^2)(h_0/\hat{R}')c_0\hat{R}' = q_{e0}^2/2\varepsilon_0h_0c_0 = constant$

[22.3] $\alpha = q^2/2\varepsilon_0hc = = q_{e0}^2/2\varepsilon_0h_0c_0 = \alpha' = constant$

But note that the *invariance* of this quantity is considered an approximation of the simplest model. In the dynamic model, $1/\alpha$ is proposed to increase to become equal γ at maximal expansion. This gives $1/\alpha$ a secondary dependence, on $(dR'/dT')^2$. Recent empirical studies suggest that α may have small variations in time or space.

The postulate that: $(m_p'/m_e')^{2/3} = q'^2/2\varepsilon_0'h'c'$ is therefore invariant on both sides and equivalent in both systems:

[22.4] $(m_p'/m_e')^{2/3} = (m_{p0}'/m_{e0}')^{2/3} = (m_{p0}/m_{e0})^{2/3}$

$$= q'^2/2\varepsilon_0'h' = q_0'^2/2\varepsilon_0'h_0' = q_0^2/2\varepsilon_0 h_0$$

The equivalent relationship for the electric charge is similarly consistent:

[22.5] $q' \quad = q_0/\hat{R}' \quad = (m_p'/m_e')^{1/3}(2\varepsilon_0'h'c')^{½}$

$$= (m_{p0}'/m_{e0}')^{1/3}(2\varepsilon_0/\hat{R}'^2)^{½}(h_0'/\hat{R}')^{½}(c_0'\,\hat{R}')^{½}$$

$$= (m_{p0}'/m_{e0}')^{1/3}(2\varepsilon_0'h_0'c_0')^{½}/\hat{R}'$$

23. The spatial variables for expansion.

To maintain some physical intuition, we now begin to relate the model expansion more carefully to the conventional measurement of cosmological variables, before solving the expansion function.

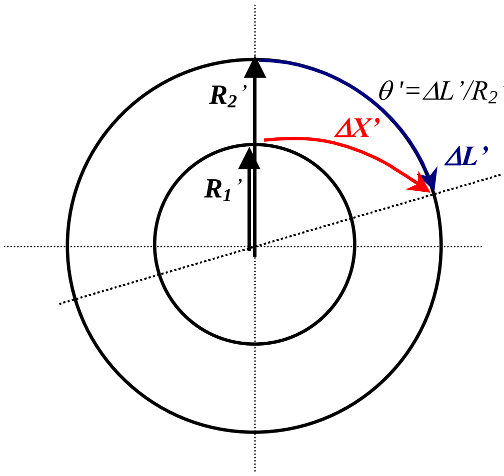


Figure 11. *Variables to describe the expansion of the universe and photon trajectory.*

The universe expands from radius R_1' at time T_1' to radius R_2' at T_2'. A light-beam is emitted at T_1' and travels along the *XYZ*-surface (of the spatial hyper-sphere), and is detected at T_2'. Its path is shown in red. It moves radially due the expansion of the universe, and tangentially through space at the local speed of light, c', due to its velocity in the *XYZ*-surface. The *co-moving distance* between the point of emission and detection is $\Delta L'$. This is the distance between the two points of space *at the time it is detected*. The real distance it has travelled in ordinary space (i.e. on the *XYZ*-

surface) is $\Delta X'$. The full distance it has travelled (in 6-dimensional space) is larger than $\Delta X'$, because it also has a radial component, in R', which is not included in $\Delta X'$. The radial angle travelled is $\Delta\theta'$. This is not the change of angle of the light trajectory, but simply the angle between the emission and detection points relative to the center, given by:

[23.1] $\Delta\theta' = \Delta L'/R_2'$ [Definition of $\Delta\theta'$]

When we consider measurements in ordinary cosmology, of time or age, radius and circumference of the universe, co-moving distance, and distance travelled by a light-beam, we have to translate these into the concepts of the new model. This is done in part by transforming from conventional variables *(T, R, etc)* into the 'true variables' (dashed variables) of the model *(T', R', etc)*. But it also requires reinterpretation of the concepts, because the new model has a different geometric and dynamic structure to the conventional model. There is no higher-dimension of space (R') in the conventional model, which uses only *intrinsic curvature of ordinary space*, and does not represent curvature as extrinsic curvature of a hyper-surface in a higher-dimensional space, as in the new model.

24. The 5-D hyper-surface of the universe.

The *5-D hyper-surface of the universe* is given by the total hyper-volume times the (torus surface/torus volume), the latter being:

[24.1] $W_{surf}'/W_{vol}' = W_e'W_p'/(W_e'W_p'^2/4\pi)$ *(meters^{-1})*

$= 4\pi/W_p'$ *Hyper-surf/hyper-vol*

$= 4\pi\hat{R}'/W_{p0}'$

This increases linearly with R'. From this we have the total hyper-surface:

[24.2] $Surface_{RW}' = (R'^3.W'^3\pi/2)(4\pi/W_p')$ *(meters^5)*

$= 2\pi^2 R'^3 W'^3/W_p'$

SECTION 4. COSMOLOGICAL MODEL SOLUTION.

We can now solve the expansion function, *R'(T')*. So far the model has been general but obtaining a specific solution for *R'(T')* narrows it down to a unique temporal process. To obtain this solution, we use a postulate that *the total energy and total momentum is fully represented by the state of the manifold, i.e. sum of particle-wave energies and momenta.* Conventional cosmology adds a number of 'hypothetical substances' to 'fine tune' their theories against observation – dark matter, dark energy and cosmological constants are required to reconcile the conventional theory to observation. The approach here ignores these 'hypothetical substances' of current cosmology and takes *the energy and momentum sources provided explicitly by the model as complete.* The hypothetical dark matter, dark energy and cosmological constant must be taken to refer to the *phenomena* that these are meant to explain, and the phenomena must explained by mechanisms in the model instead. Dark matter, dark energy and cosmological constants are not admitted as *energy sources* unless their substances are found in the model, and they are experimentally confirmed.

The solution is simple, being the Cardiod curve. This solution is not fundamental to the model though, since perturbations from our perfect boundary conditions, or additional structures, fine-structure evolution, etc, may lead to alternative solutions, which may have the same essential cyclic behaviour, or may have distinct long-term behaviours. The cyclic solution described here is the paradigm solution however, and gives realistic predictions.

25. Postulate of total energy of a free particle.

The ideal result we want, to fully specify the cosmological model, is the evolution equation for *R'* in terms of *T'*, the global age of the universe. This taken as the time *T'* since the Big Bang. We will now derive a solution for this by determining an energy equation for a free mass, and assuming *conservation of total kinetic energy*. The total kinetic energy of a free point-mass *m* in the global frame is defined by:

[25.1] $E' = m'V'^2$ [Define Energy]

$= m'c'^2 + m'(dR'/dt')^2$

$= constant$ [Postulate Energy Conservation]

This is on the assumption that the speed *c* is *orthogonal to the expansion, dR'/dt.*

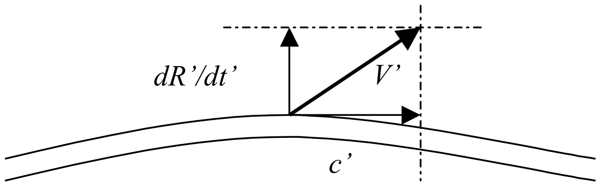


Figure 12. *Total resultant velocity, V', for a free particle.*

This takes the kinetic energy from both the motion *in* the space manifold and the motion *of* the space manifold into account. We define the total speed of the particle:

[25.2] $V'^2 = c'^2 + (dR'/dt')^2$ [Definition of *V'*]

The energy equation is then equivalent to:

[25.3] $m'V'^2 = E_0' = constant'$ [sub 25.2 in 25.1]

The present value, E_0', must be constant. The present speed and energy is then:

[25.4] $m_0'c_0'^2 + m_0'(dR_0'/dt')^2 = m_0'V_0'^2 = E_0'$ [present values in 25.3]

From (25.2) and the evolution of *m'* (14.4) we have:

[25.5] $V'^2 = E_0'/(m_0'/\hat{R}') = V_0'^2\hat{R}'$ [14.4, 25.2]

From the evolution of *c'* we have:

[25.6] $(dR'(t')/dt')^2 = V_0'^2\hat{R}' - c_0'^2\hat{R}'^2$

$$= (V_0'^2/R_0')R'(t') - (c_0'^2/R_0'^2)\, R'(t')^2$$

26. The Solution for the Expansion Cycle: *R'(T')*

This differential equation in $R(t)$ is solved by:

[26.1] $R'(t) = (V_0'^2 R_0'/c_0'^2) \sin^2((c_0'/2R_0')\, t')$ [Solve 25.6]

This means the maximum expansion is found when $\sin((c_0'/2R_0')\, t') = 1$, giving:

[26.2] $R'_{MAX} = (V_0'^2/c_0'^2)\, R_0'$

And the time of maximum expansion is found when $(c_0'/2R_0')\, t' = \pi/2$, giving:

[26.3] $T'_{MAX} = \pi R_0'/c_0'$

This is a remarkable result of the model: by determining the current radius and current speed of light we can determine the time of maximum expansion. Since R_0' is determined from the local constants h_0, G_0, m_{e0}, and m_{p0} alone, this means T'_{MAX} is determined by these constants and c_0.

We can write the evolution equation as the cyclic solution:

[26.4] $R'(t) = R'_{MAX} \sin^2(\pi\, T'/2T'_{MAX})$

However we will not extend the solution all the way to $R' \rightarrow 0$. Instead it will 'bounce' when R' takes its minimum value when: $\hat{R}' \rightarrow \hat{R}_0'/\sqrt{D_0}$. (Which is only about 10^{-20} of the present radius.)

At the time of maximum expansion, we have: $R'(t) = R'(T'_{MAX}) = R'_{MAX}$, and all the velocity is in c', giving: $V'(T'_{MAX}) = V'_{MAX} = c'_{MAX}$, and the overall average speed of expansion is: $R'_{MAX}/T'_{MAX} = c'_{MAX}/\pi$.

27. The Cardiod function for *R'*.

Note that the evolution function for *R'* given in (26.4) is a *Cardiod function* ('heart function'). The Cardiod is normally written in the form: $r = 2a(1+cos(\theta))$. This is seen to be equivalent to the form of 26.4: $r = bsin^2(\psi)$ by using the identity: $\cos(\theta) = cos(\theta/2+\theta/2) = cos^2(\theta/2)-sin^2(\theta/2)$, so that: $r = 2a(1+cos(\theta)) = 2a(1+ cos^2(\theta/2)-sin^2(\theta/2)) = 2a(2cos^2(\theta/2)) = 4a(cos^2(\theta/2)) = 4a(sin^2(\theta/2+\pi/2))$. Setting: $b = 4a$ and: $\psi = \theta/2+\pi/2$ gives the function in the form: $r = bsin^2(\psi)$. Or inversely, setting: $a = b/4$ and: $\theta = 2\psi-\pi$, the equation: $r = bsin^2(\psi)$ converts to the form: $r = 2a(1+cos(\theta))$, giving the equivalent form of 26.4:

[27.1] $$R'(t) = (R'_{MAX}/2)(1-cos(\pi T'/T'_{MAX}))$$

The latter is simpler to integrate and has a single cycle from: $\theta' = 0$ *to* 2π. For a convenient graph of the radial expansion as a Cardiod function, we define the time variable cyclically as an *angle*, θ', by:

[27.2] $$\theta' = T'\pi/T'_{MAX} \qquad or: \qquad T' = \theta' T'_{MAX}/\pi$$

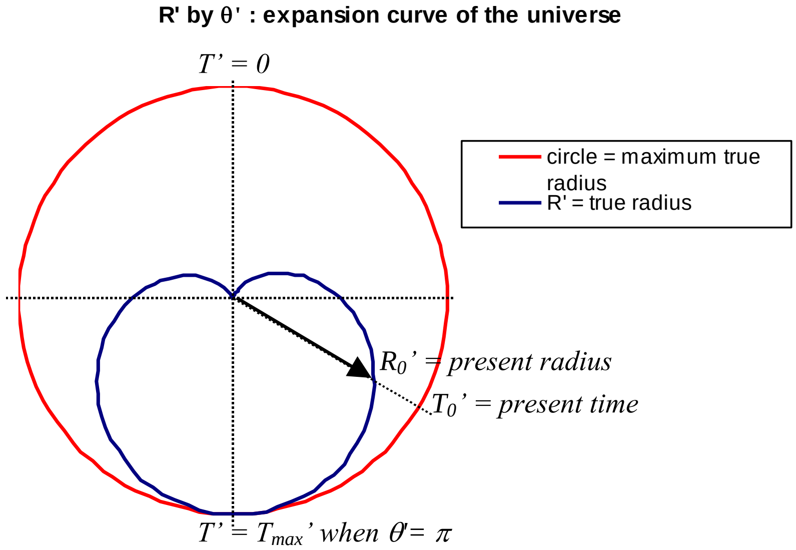


Figure 13. *The expansion curve of the universe, shown as a cyclic function of time, represented by the angle: $\theta' = \pi\ T'/T'_{MAX}$. The radius R' is the radial distance from the center of the circle. For the moment we ignore the tiny divergence at the origin, caused by the fact that we do not take the radius all the way back to 0 at $T' = 0$.*

Note that the time defined as: $T'_{MAX} = \pi R_0'/c_0'$ is half-way through the full cycle. We show next that this cardiod function is also precisely the trajectory of a photon on the *XYZ*-surface.

28. The cardiod function for light trajectory.

Consider a photon of light emitted at $T' = 0$ *(or slightly later),* that travels continually around the universe. This moves parallel to the *XYZ*-surface, or orthogonally to the radial vector of expansion, at a speed $c' = c_0\hat{R}'$ at each moment. We assume it moves in the *x-direction* (using a rotating basis vector for ***x*** that remains parallel to the surface) and define its radial angle as:

$T' = T_{max}'$ when $\theta' = \pi$

[28.1] $d\theta' = dx'/R'$

The angular velocity is then:

[28.2] $d\theta'/dT' = dx'/dT'\ (1/R')$

By the definition of the model, the speed of light is:

[28.3] $dx'/dT' = c' = c_0\hat{R}' = c_0R'/R_0'$

Substituting into 6.41 gives:

[28.4] $d\theta'/dT' = c_0/R_0' = constant$

I.e. light rotates at a constant angular speed, c_0/R_0'. Integrating and taking the angle as: $\theta' = 0$ at $T' = 0$ gives:

[28.5] $\theta' = T'(c_0/R_0')$ or: $\Delta\theta' = \Delta T'(c_0/R_0')$

where: $\Delta\theta'$ is the change in angle in a period of $\Delta T'$. We can determine the angular rotation at the point of maximum expansion, half-way around the expansion-contraction cycle, using 26.3: $T'_{MAX} = \pi R_0'/c_0$, giving:

[28.6] $\theta_{MAX}' = T_{MAX}'(c_0/R_0') = \pi$

This means that the trajectory of light follows exactly the same cardiod function as the expansion function for R' in θ, as given above. We can use the geometric properties of the cardiod to solve light trajectories, etc.

29. Co-moving distance of a photon.

The co-moving distance of a photon moving between two moments of time is easily defined in terms of the radial angle θ', as defined above. A change in the radial angle of $\Delta\theta'$ in a period $\Delta T'$ corresponds to a co-moving distance $\Delta L'$:

[29.1] $\Delta L' = \Delta\theta' R'$

where *R'* is the radius at the final time (time of detection). Using 6.44 gives:

[29.2] $\Delta L' = \Delta T' c_0 R'/R_0' = \Delta T' c_0 \hat{R}' = \Delta T' c'$

where *c'* is the speed of light at end of the period (time of detection). In the special case where we detect light at the present time from the origin of the universe (approximately the last scattering surface), we define $\Delta L' = L_0'$, and $\Delta T' = T_0'$, and $c' = c_0'$, giving:

[29.3] $L_0' = T_0' c_0'$

Hence the present *co-moving distance of light from the origin* (the Big Bang, or shortly thereafter) is simply the *present age of the universe times the present speed of light.*

In the special case of light from the origin of the universe detected at the time of maximum expansion, L_{MAX}' is half the maximum circumference:

[29.4]
$$\begin{aligned} L_{MAX}' &= \pi R_{MAX}' \\ &= T_{MAX}' c_{MAX}' \\ &= T_{MAX}' c_0 \hat{R}_{MAX}' \\ &= T_{MAX}' c_0 R_{MAX}'/R_0' \end{aligned}$$

This reconfirms that $T_{MAX}' = \pi R_0'/c_0$.

30. Light distance from emission to detection.

The distance a photon *travels* in the *XYZ-surface* between emission and detection (or through a change in radial angle $\Delta\theta'$) is denoted by $\Delta X'$. This is different to the co-moving distance, $\Delta L'$. $\Delta X'$ is defined through the differential:

[30.1] $d(\Delta X') = c' dT' = c_0 (R'/R_0') dT'$

Note that: $d(\Delta X') \equiv dx' = R'd\theta'$, and: $dx'/dT' = c'$ and: $dx'/dT' = Rd\theta/dt = c_0R/R_0' = c'$. This simply reflects the fact that at any moment of time, the photon is travelling at local speed c' in the XYZ-surface. To get the distance travelled in an interval $\Delta T'$, we integrate (30.1) over the time period in question, using: $T_{MAX}'/R_0' = \pi /c_0$ to simplify:

$$[30.2] \qquad \Delta X' = \int_{\Delta T'} c_0 \,(R'/R_0')\, dT'$$

$$= (c_0R_{MAX}'/2R_0')\int_{\Delta T'} (1-\cos(\pi T'/T_{MAX}'))\, dT'$$

$$= (c_0R_{MAX}'/2R_0')\, [T' - (T_{MAX}'/\pi)\sin(\pi T'/T_{MAX}')]_{\Delta T'}$$

$$= (c_0R_{MAX}'/2R_0')\Delta T' - (c_0R_{MAX}'T_{MAX}'/2\pi R_0')\, [\sin(\pi T'/T_{MAX}')]_{\Delta T'}$$

$$= (c_0R_{MAX}'/2R_0')\Delta T' - (R_{MAX}'/2)\, [\sin(\pi T'/T_{MAX}')]_{\Delta T'}$$

$$= \tfrac{1}{2}\, (c_{MAX}'\Delta T' - R_{MAX}'[\sin(\pi T'/T_{MAX}')]_{\Delta T'})$$

Taking the interval $\Delta T'$ to go from $T' = 0$ to T' we write:

$$[30.3] \qquad X' = \tfrac{1}{2}\, (c_{MAX}'T' - R_{MAX}' \sin(\pi T'/T_{MAX}')$$

For the special value: $T' = T_{MAX}'$, we have:

$$[30.4] \qquad X_{MAX}' = \tfrac{1}{2}\, c_{MAX}'T_{MAX}' = \tfrac{1}{2}\, L_{MAX}'$$

I.e. light travels half the maximum circumference of the universe in a full orbit.
For the special value: $T' = T_0'$, we have:

$$[30.5] \qquad X_0' = \tfrac{1}{2}(c_{MAX}T_0' - R_{MAX}' \sin(\pi T_0'/T_{MAX}')$$

$$= \tfrac{1}{2}\, R_{MAX}'(c_0T_0'/R_0' - \sin(\pi T_0'/T_{MAX}')$$

31. Maximum radius and present time.

To relate R_{MAX}' to T_0' through 26.2, i.e. $R'_{MAX} = (V_0'^2/c_0'^2)R_0'$, we can write $V_0'^2$ as function of T_0':

[31.1] $V_0'^2 = c_0'^2 + (dR'/dT')_{|0}^2$

To find $dR'/dT'_{|0}$ we first find: dR'/dT' from the function for R':

[31.2] $dR'/dT' = d/dt((R_{MAX}'/2)(1-\cos(\pi T'/T_{MAX}'))$
$= (\pi R'_{MAX}/2T_{MAX}') \sin(\pi T'/T'_{MAX})$
$= (c_0 R'_{MAX}/2R_0') \sin(\pi T'/T'_{MAX})$

And take its value at $T' = T_0'$:

[31.3] $dR'/dT'_{|0} = (c_0 R'_{MAX}/2R_0') \sin(\pi T_0'/T'_{MAX})$

Substituting into (31.1):

[31.4] $V_0'^2 = c_0'^2 + (c_0 R'_{MAX}/2R_0')^2 \sin^2(\pi T_0'/T'_{MAX})$
$= c_0'^2 (1 + (c_0 R'_{MAX}/2R_0')^2 \sin^2(\pi T_0'/T'_{MAX}))$

Then:

[31.5] $R'_{MAX} = (V_0'^2/c_0'^2)R_0' = (1 + (c_0 R'_{MAX}/2R_0')^2 \sin^2(\pi T_0'/T'_{MAX}))\, R_0'$

32. Time to maximum expansion.

From 26.3 and 12.4 we can obtain the time of maximum expansion as a function of the local constants alone, as:

[32.1] $T'_{MAX} = \pi R_0'/c_0'$
$= h_0^2/2\pi m_0^3 G_0 c_0$
$= h'^2/2\pi m'^3 G' c'$
$= \pi Z_0/c_0$

This is an invariant: it does not depend on R' or T'. The value will appear to be the same value at any point in history (once the units for the variables are set at one point in history). T'_{MAX} is predicted to be:

[32.2] $T'_{MAX} = \pi\, 13.823$ *billion years*

$= 43.43$ *billion years'*

But note that this is in *true variables.* To convert back to conventional variables, T or T_{MAX}, we will need to determine how far through the expansion process we are. For this we need to independently determine either the current T', or the current V' or the maximum R'.

33. Expansion Rate and Hubble Parameter.

We now define the *spatial expansion rate at a time T or T',* equivalent to the Hubble parameter, *H or H'.*

[33.1] $H'(T') = (dR'/dt')/R'$ Definition of Hubble parameter (true variables)

$H(T) = (dR/dt)/R$ Hubble parameter (conventional variables)

Note that since: $R = R'^2/2R_0'$ and $dt'/dt = R_0'^2/R'^2$

[33.2]

$$
\begin{aligned}
H &= (dR/dt)/R \\
&= (d(R'^2/2R_0')/dt')(dt'/dt)/(R'^2/2R_0') \\
&= (d(R'^2)/dt')(R_0'^2/R'^2)/(R'^2) \\
&= 2R'((dR'/dt'))(R_0'^2/R'^4) \\
&= 2((dR'/dt')/R')(R_0'^2/R'^2) \\
&= 2H'/\hat{R}'^2
\end{aligned}
$$

The present values are related by:

[33.3] $H_0 = 2H_0'$, *or*

$H_0' = H_0/2$

And since $R_0 = R_0'/2$, we have:

[33.4] $H_0R_0 = H_0'R_0' = dR_0'/dt' = dR_0/dt$

The Hubble parameter is currently estimated as:

[33.5] $H_0 \approx 1/(14 \text{ billion years})$

Hence the empirical value in the model is:

[33.6] $H_0' \approx 1/(28 \text{ billion years})$

Some simple useful relationships are:

[33.7] $dR'/dt' = H'R'$

[33.8] $V'^2 = (H'R' + c'^2)^{½}$

Using the equation for V_0':

[33.9] $V_0'^2 = (H_0'R_0' + c_0'^2)^{½}$

I do not try to evaluate the Hubble parameter predictions here.

34. Present Expansion Speed, Present Age, Maximum Radius.

We now need the *present age* or *present position in the cycle* to know all the parameters of the model. We can determine either *the present age, T_0', the maximum radius, R_{MAX}', or the present expansion rate, dR_0'/dt',* as they are inter-dependant. Their relationships are summarised below.

We assume that we have a method of determining dR_0'/dt', or V_0', and summarise the relationships for the other two variables. The problem of inferring the age empirically can be done in independent ways, by using the estimate from the dynamic fine structure postulate, Hubble parameter observations, expansion rate observations (accelerating expansion), and cosmological observations of various kinds.

The prediction of dR_0'/dt' from assuming the dynamic fine structure hypothesis is shown below the general equations.

[34.1] $R_{MAX}' = (V_0'^2/c_0'^2)R_0'$ [31.1, Cardioid model]

[34.2] $R_{MAX}' = \gamma\alpha R_0'$ [34.1, 12.8, dynamic fine structure]

$= 150/137\, R_0'$ [Empirical value]

$= 1.095\ R_0'$ [Our universe is 95% inflated]

Use (26.4): $R' = R'_{MAX}\ \sin^2(\pi T'/2T'_{MAX})$, applying the present time:

$R_0' = (V_0'^2/c_0'^2)R_0'\ \sin^2(\pi T_0'/2T'_{MAX})$ [26.4 at present time]

$c_0'/V_0' = \sin(T_0'c_0'/2R_0')$ [rearrange]

[34.2] $T_0' = (2R_0'/c_0')\ \arcsin(c_0'/V_0')$ [rearrange, substitute 32.1]

Using the dynamic fine structure postulate:

[34.3] $T_0' = (2R_0'/c_0')\ \arcsin(1/\sqrt{(\gamma\alpha_0)})$ [rearrange, substitute 12.8]

$\theta' = T'(c_0/R_0')$ is the angle in the cardioid function.

[34.4] $\theta'/2 = T'(c_0/2R_0') = \arcsin(1/\gamma\alpha_0)$ [34.3, rearrange]

$= \arcsin(\sqrt{(137/150)}) = \arcsin(0.956)$

$= 1.272$ *radians* $=145$ *degrees*

$= 0.8098$ *of universe half-cycle*

This is required to resolve the rate of change of G. The solution for T is obtained by integrating the Cardiod solution directly:

[34.5] $T = (R'_{MAX}/R'^2)\ (3/8\ T' - \sin(AT')\cos(AT')\ T'_{MAX}/\pi$

$+ \sin(2AT')\ \sin(2AT')T'max/8\pi)$

Note that in the model solution, the true conventional age, T_0, is larger than the 'measured age', Z/c.

[34.6] $T_0 = 32.04\ b.y.$ [Modelled true conventional age]

This is closest to the measure of *proper time,* and means that the universe is more than twice as old as the standard measured age Z/c allows. This should be evident in old planets or long slow processes that would otherwise not have time to complete.

35. Rate of change of G.

The rate of change of G is predicted from the model by differentiating: $G = G_0R_0/R$.

[35.1] $dG/dT = d(G_0R_0/R)/dT$
$= -G_0R_0/R^2\ (dR/dT)$

Hence the rate per unit of G is predicted as:

[35.2] $(dG/dT)/G_0 = -R_0/R^2\ (dR/dT)$

At the present time, $R = R_0$, predicting the present rate of change of G as:

[35.3] $(dG/dT)/G_0 = -1/R_0\ (dR/dT)$

To solve this requires the rate of expansion or age in the model.

35. Prediction of rate of change of G.

The best-fit model at present has: $R_0' = \pi 13.8\ b.l.y.$ and $dR'/dt' = 0.307\ c_0$, and consequently predicts:

[35.4] $(dG/dT)/G_0 = -1.4 \times 10^{-11}$ *per year*

This indicates the scale for the prediction of the normalised rate of change of *G* we should expect to observe. The magnitude decreases as we get close to the mid-point of the expansion cycle, and eventually becomes zero, but for the late-mid-range in the expansion, where we appear to be, it is in the range of 10^{-11}.

35.1 Empirical Observation of Change of G.

Is the rate of change of about -1.4×10^{-11} parts per year in *G* consistent with observation? At time of writing, a number of studies claim to fix the lower bound for the evolution of *G* at about: $-(dG/dt)/G < 10^{-11}/yr$ over intermediate cosmological periods. The firm results are *prima facie* only just consistent with the model. But a challenge is evident in stronger results claimed recently using other methods, claiming bounds of about $-(dG/dt)/G < 3 \times 10^{-13}$/yr for local variations in *G*. This would be out of plausible range of the present model predictions, unless we are within a few degrees of the top of the cycle. Careful modelling of the observations should answer this question. However this question is not answered yet. Some general points are made below.

G is the most difficult of the fundamental constants to measure directly, estimates of its value are the most theory-dependant, and its value has the weakest precision of all the fundamental constants. The best estimates of variations in *G* appear to come from four main methods: *lunar laser ranging (giving local and current variations in G only), Big Bang Nucleosynthesis, Hubble diagrams of Type 1a supernovae,* and analysis of *stellar evolution of white dwarfs.*

Cosmology and astronomy has been a dynamic empirical science over the last 50 years, and it has done an extraordinary job observationally and theoretically, uncovering a range of anomalies in the theories, and revising assumptions in patch-works of applied physics (dark matter, dark energy, cosmological constant, etc). But a fundamental problem is the complexity and theory-dependence of interpretation of the observations. Even within the assumptions of the standard theory, researchers have trouble giving robust analysis, and research is in a state of revision of previous results. Experiments in this area make idealising assumptions, and these often need to be revised by later researchers. These highly theoretical measurements depend on chains of reasoning, and it can only take one bad link to make results misleading.

When we reinterpret the results of the conventional *measurement processes* we cannot put in its 'hypothetical substances' or depend on its ontological assumptions. We only have to reinterpret the chain of measurement assumptions and process in terms of the model, and work out what the *transformations are from our predictions in our model variables to conventional theoretical variables and then to the conventional measurement variables: R'→R→Z→T.*

Physicists generally assume that *G* is constant, and experiments *to detect changes in G* are probably expected to have null results and may be likely to become *experiments to improve the experimental bound showing lack of any change in G.* To match strong theoretical expectations, we unconsciously look for consistent results and discard anomalous results, and keep recalculating measurements until they fit expectations. It may need dedicated experiments by researchers expecting to find that *G* does change on this scale (10^{-11}/yr). I could not evaluate the model prediction without more detailed analysis. Conversely, the measured rate of change of gravity, as it is improved experimentally, may be used to determine a best-fit model. If *G* is found to be stationary to a high precision, then we can only infer from the model that we are at the maximum expansion state, or in a stationary expansion solution, and compare with other measurements, of the Hubble parameter, etc.

SECTION 5. PREDICTION OF GRAVITY.

36. The Geometric Theory of Gravity.

The theory of gravity in the geometric model is determined by the effect on space of placing a mass-energy wave in the manifold. The primary effect is to *strain* space locally around the embedded mass-energy. This curves space by expanding the micro-dimensions, *W'.* The energy is stored by the strain, like stretching a rubber band. The effect on the motion of a neighbouring particle is generated by two effects of the strain. First, there is a change of *shape* of the 5-dimensional spatial surface, which becomes convex around the mass. Second, there is a reduction in the *wave speed, c*. Waves move on geodesics on the surface, conserving energy. Curving the space and creating a differential speed field changes the paths they follow.

The strategy for deriving the geometric theory of gravity therefore starts with determining the *strain function* for a single particle. We then need to specify the strain function for multiple particles – the *superposition principle* for the strain function. We can subsequently calculate the effect of the strain on the motion of neighbouring particles, by determining effects on *geodesic paths*, and determining effects of the strain on spatial properties, primarily the wave-speed, c. We will see that the natural strain function generates a central acceleration, which in the first order gives an inverse-square law identical to Newtonian gravity, and in the second order gives the same effects as the General Theory of Relativity, with gravitational red-shifts and curved light paths. In the third order, the local effects of gravity are slightly different from GTR. However the present section deals with developing the theory, not its detailed applications[14].

[14] In the solar system, gravity should appear be very slightly different to the predictions of GTR. The effects should be evident in free-fall trajectories of spacecraft or comets over long periods of time. A separate study indicates this accounts for the anomalies in GTR predictions of the Pioneer spacecraft trajectories, which showed up conclusively in data after about 20 years. There are differences predicted in the orbital distances of planets, but these are difficult to detect, because the planetary center of mass is difficult to determine precisely. There should be small differences detectable in the precession of the perihelion of Mercury. Conclusive evidence could be established by sending a spacecraft in free-fall from Earth to the orbit of Jupiter, at a suitable velocity, taking about 3 years to complete. In the theoretical realm, GTR black holes are rejected, and a simpler kind of gravitational hole is predicted.

The main point we will arrive at in this section is written as a *speed metric:*

[36.1] $(K^2 dr'^2 + dy'^2 + dz'^2 + dw'^2)/dt'^2 = c'_\infty{}^2/K^2$ **Speed Metric**

with the geometric model factor *K* ('big K') defined by:

[36.2] $K(r') = exp((M'_\infty G'_\infty/c'_\infty{}^2)(1/r' + 1/\pi R'))$ **Definition of *K***

Although for most practical calculations we can just use:

[36.3] $K(r')^* = exp((M'_\infty G'_\infty/c'_\infty{}^2 r')$ **Definition of *K****

This is the metric equation for a stationary central mass. The terms M'_∞, G'_∞, c'_∞ are the mass, gravitational constant and speed of light for the background space, at a large distance from the mass. They are explicitly distinguished from the values of these constants in the gravitational field. We call [36.1] a 'speed metric' rather than 'line metric', although it is formally equivalent, because it explicitly gives the *speed in the 6-dimensional manifold* at a point on a trajectory.

The solution is perfectly analogous to the Schwarzschild solution, and very similar quantitatively, but with our term *K* ('big K') replaced by the term *k* ('little *k*'):

[36.4] $k = (1-2M'_\infty G'_\infty/c'_\infty{}^2 r')^{-1/2}$ **Definition of *k***

This gives the equation usually written in spherical coordinates as:

[36.5] $d\tau^2 = dt^2/k^2 - dr^2k^2/c^2 + r^2d\theta^2/c^2 - (r\sin(\theta)\, d\phi)^2/c^2$ **GTR Line Metric**

or equivalently in the local Cartesian coordinates at the field point (r, θ, ϕ) as:

$d\tau^2 = dt^2/k^2 - dr^2k^2/c^2 + dy^2/c^2 - dz^2/c^2$ **GTR Line Metric**

In the context of the geometric model, this usual Schwarzschild metric is rearranged as a speed metric, and model constants and variables (dashed) substituted for ordinary constants and variables:

[36.6] $(k^2dr'^2 + dy'^2 + dz'^2 + dw'^2)/dt'^2 = c'_\infty{}^2/k^2$ **GTR Speed Metric**

To give a multi-particle theory, we subsequently give principles for *mass superposition* and *covariant transformations,* to obtain equations for multiple masses and moving masses. The resulting theory in this form is very similar to GTR, and empirical predictions can be calculated using similar techniques to GTR.

We could take [36.1] as the essential postulate of the geometric theory of gravity. However it is not a fundamental postulate: it needs to be derived from the underlying mechanics of the geometric model. In this section we work through the derivation and justification of this from first principles, in stages that illustrate the motivation and fundamental principles. This is an antidote to the approach in GTR, where the field equation is simply postulated, and the Schwarzschild metric derived from it mathematically, on the assumption that the energy-momentum tensor for a central point mass is adequately defined. In the geometric model we have a clear underlying mechanical derivation and causal explanation for the effects. In GTR the fundamental equation is simply postulated on the grounds that it represents a universal symmetry, and the theory is an exercise in mathematical formalism, with little prospect to visualise a physical interpretation.

The geometric model must correspond to Newtonian gravity and GTR in appropriate limits for a realistic theory. Hence we can start with Newtonian gravity and GTR, and derive a strain function to match known physics. We can then show it is justified from the fundamental model. Thus we begin with the effect of curvature of the manifold surface, determine the solution for the strain function required in the Newtonian limit, then generalise this as the line-metric equivalent of the Schwarzschild solution to GTR. We then justify this from the model perspective. We begin here with general features of the geometry, before obtaining solutions.

36.1 Lines on the curved surface.

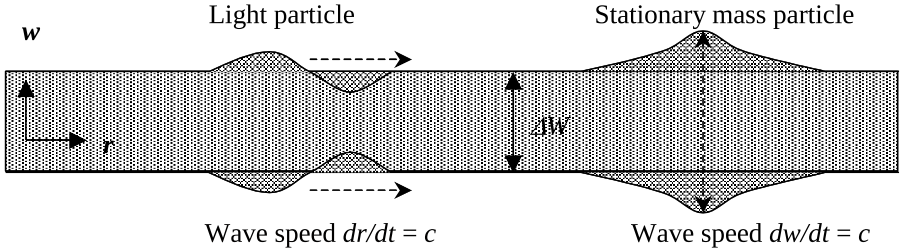


Figure 14. *Mass-energy wave strains space. Cross-section through space. Energetic waves create distortions of the surface. This has the effect of 'funneling' other waves towards them, and the acceleration can be calculated if we know the strain equation and the wave speed. This tells us the perturbation in W.*

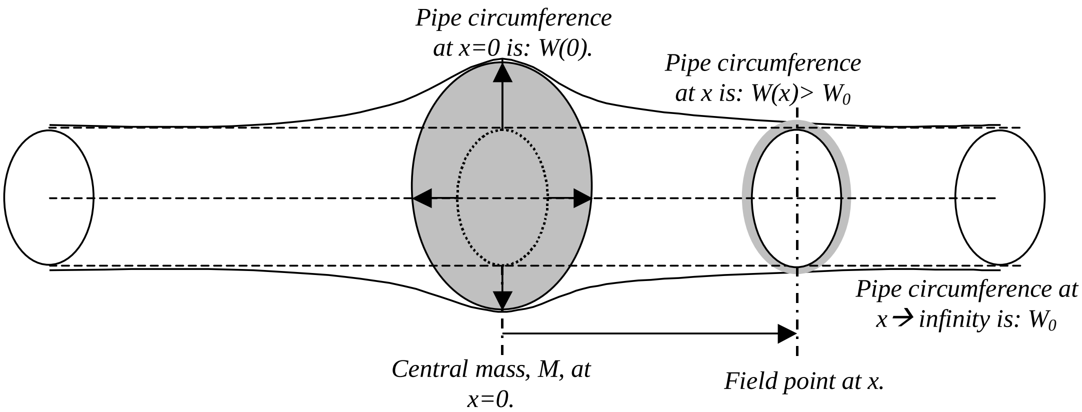


Figure 15. *Gravitational curvature in a simple pipe. The geometric theory of gravity is specified by giving the functional relationship between a given mass-energy and the curvature it generates, along with a superposition principle specifying how curvatures of multiple masses add together, and the effect of this strain on the wave-speed.*

As the first step to visualise the mechanism, first consider the surface of a cylinder (pipe) unrolled, with straight-line trajectories drawn on it, and then rolled up again. A trajectory with no sideways velocity (in the x direction) forms a circle, while an oblique trajectory forms a *spiral* or *screw*. In either case, given the trajectory maintains a constant total speed c, and there is no acceleration in the x-direction. Straight lines are geodesics in this case.

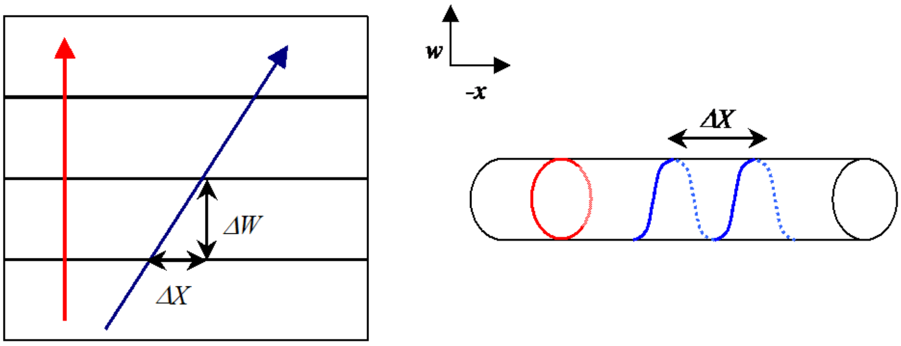


Figure 16. *Geodesic point-motion on a pipe. The increment Δx in the x-direction is constant with each rotation. Neither trajectory is accelerating. Both trajectories are geodesics. Note we take the negative x-direction to be to the right.*

Now consider instead a section of a cone (trumpet-like shape), rather than a straight cylinder. Unroll it, draw a straight line trajectory on it, and roll it up again. This trajectory – which is really a straight line - will be accelerating towards the larger end of the cone. Conversely, if you draw a circle around the cone, and unroll it, it no longer forms a straight line – it is an arc of a circle on the developed surface. It maintains a constant *x-position*, but it is actually accelerating away from the larger end of the cone – its centripetal acceleration.

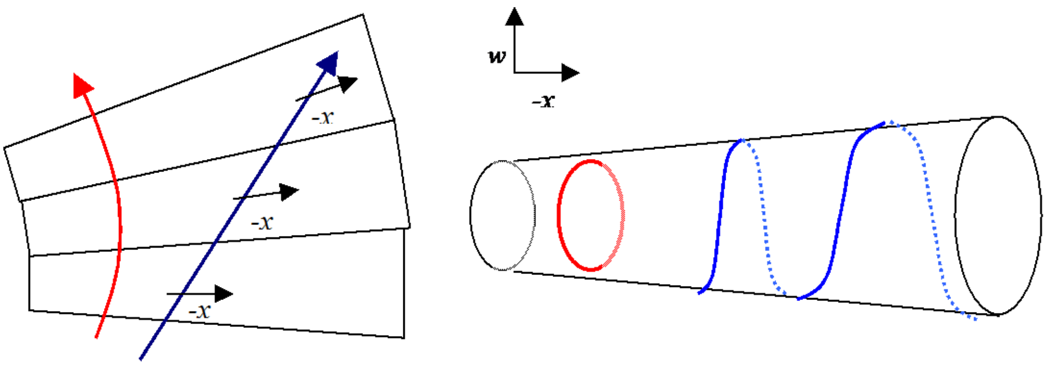


Figure 17. *Straight-line -motion on a cone. Rotation at constant x (red line) is an accelerating path. For a straight-line path, the increment Δx in the x-direction is not constant with each rotation. The geodesic path is accelerating in the (negative) x-direction. Note the wide end of the cone corresponds to the direction of a source mass, so we take this to be the negative x-direction, so we have x ≡ r in normal radial coordinates for a central mass.*

Straight-line motions on the surface of the cone accelerate in the (negative) *x-direction*, towards the open end of the cone. The *x*-acceleration at a point can be derived geometrically. It increases with the internal angle of the cone, increases with the speed of the trajectory, and decreases with the circumference of the cone.

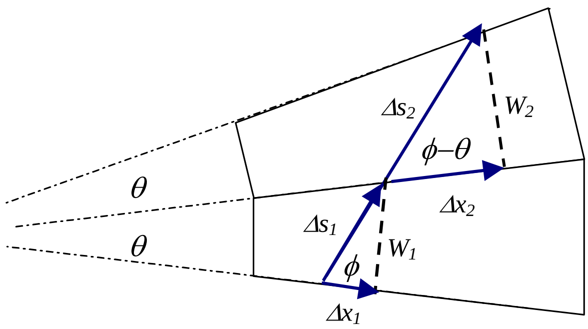


Figure 18. *Point-motion on a cone. We can determine the x-acceleration of a straight-line motion geometrically, from first principles.*

Note if we define a time increment: $\Delta t \rightarrow dt$ as the time to travel: $\Delta s \rightarrow ds$, then: $\Delta s/\Delta t \rightarrow ds/dt = c$, the speed of light at the field point. Then: $\Delta x/\Delta t \rightarrow dx/dt = v_x$, and: $dx = cos(\phi)ds$. Thus: 3

$$cos(\phi) = dx/ds = (v_x dt)/(cdt) = v_x/c$$

$$sin(\phi) = \sqrt{(1- cos^2(\phi))} = \sqrt{(1-v_x^2/c^2)}$$

This is closely related to the usual relativistic term γ, defined by:

$$\gamma = 1/\sqrt{(1-v_x^2/c_\infty^2)}$$ **Simple Gamma**

We explicitly write c_∞, because the speed of light at the field point, c, and in the direction, ϕ, is modified in our theory from c_∞ by the strain, W. We can write a more general gamma for a local point instead:

$$\gamma' = 1/\sqrt{(1-v_x^2/c^2)}$$ **Local (true) Gamma**

Then:

$$sin(\phi) = 1/\gamma'$$

$$cos(\phi) = \sqrt{(1-1/\gamma'^2)}$$

In weak gravitational fields, $W \approx W_\infty$, so: $c \approx c_\infty$, and:

$$sin(\phi) \approx 1/\gamma$$

As a first exercise, we will determine the centripetal acceleration of the circular path, stationary in *x*. This is a special case for comparison with the general solution.

Exercise: the centripetal acceleration to maintain stationary position in *x*.

Centripetal acceleration for a circular motion is: $a = V^2/R$. In this case, $V = c$, and the radius R is determined by: $\theta = W/R$. Hence:

$$a = c^2\theta/W$$

The angle θ is also given by the tangent, and since $dR/dx = 1$:

$$\theta = dW(x)/d/R = dW(x)\ dx$$

Hence in this special case we have:

$$a = (c^2/W)(dW/dx)$$

This is the 'external acceleration' required to maintain a point-particle at a constant position in *x* as it rotates in a perfect circle. A geodesic has no such external acceleration, and the geodesic with the same initial condition should have the opposite acceleration in *x*. We now work out a more general solution from first principles.

36.2 Acceleration of straight-line motion on the cone.

We can think of one rotation of *W* as producing an incremental change in *x*-velocity in an increment of time, $\Delta t_1 = \Delta s_1/c$. The *x-velocity* is initially $v_{x1} = c_1.\Delta x_1/\Delta s_1$. It increments in a period of time Δt_1 to $v_{x2} = c_2.\Delta x_2/\Delta s_2$. The exact *x-velocity* is given by:

$$v_x = dx/dt = c\cos(\phi) \qquad \textbf{\textit{x-velocity}}$$

Note this is along the *surface.* The surface itself is actually pointing slightly away from x in the Cartesian space. We note this point so we are aware of it, but we actually analyse the *surface velocity.*

Note on true *x-direction.*

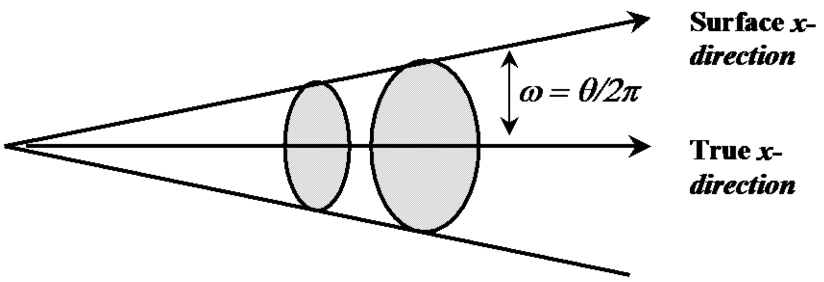


Figure 19. *True x distinguished from surface x.*

The *true x-component* is the central component of surface velocity. For an increment on the surface dx, the true increment is: $dx^* = dx\cos(\omega)$,where: ω is $\theta/2\pi$, or:

$$\omega = (dW/dx)/2\pi$$

The true *x-velocity* is:

$$v_x^* = dx/dt\cos(\omega) = c\cos(\phi)\cos(\omega)$$

However we work with the *surface velocity,* because this corresponds to the normal measurement variable that we can compare with Newtonian and GTR gravity. In any case the quantity: $dx^*/dx = \cos(\omega)$ is extremely close to 1 until we get very close to a source mass – i.e. to a gravitational hole – which we consider separately as a special region. We take the central distance as the true x however – if we follow x^* we never actually reach the central point (there is a singularity which we show how to remove later). In practice, x is measured in the true direction, e.g. by taking the *diameter* of a circle at r, and dividing by 2. We cannot physically follow the surface from the field point to the point mass.

The *x-acceleration* is therefore:

$$
\begin{aligned}
dv_x/dt &= d(c\cos(\phi))/dt \\
&= \cos(\phi)\,dc/dt + c\,d(\cos(\phi))/dt \\
&= \cos(\phi)\,dc/dt - c\sin(\phi)\,d\phi/dt
\end{aligned}
$$

We can replace the time differentials with space differentials by the chain rule:

$$dc/dt = \partial c/\partial x\;dx/dt + \partial c/\partial w\;dw/dt$$

By the model assumption: $\partial c/\partial w = 0$, so:

$$
\begin{aligned}
dc/dt &= \partial c/\partial x\;dx/dt \\
&= v_x\,\partial c/\partial x \\
&= c\cos(\phi)\,\partial c/\partial x
\end{aligned}
$$

`

And similarly:

$$d\phi/dt = \partial\phi/\partial x\;dx/dt + \partial\phi/\partial w\;dw/dt$$

By the model assumption: $\partial\phi/\partial x = 0$, so:

$$
\begin{aligned}
d\phi/dt &= \partial\phi/\partial w\;dw/dt \\
&= v_w\,\partial\phi/\partial w \\
&= c\sin(\phi)\,\partial\phi/\partial w
\end{aligned}
$$

So:

$$dv_x/dt = c\cos^2(\phi)\partial c/\partial x - c^2\sin^2(\phi)\partial\phi/\partial w$$

Or alternatively:

$$dv_x/dt = (v_x^2/c)\,\partial c/\partial x - v_w^2\,\partial\phi/\partial w$$

Note that in the geometric model the speed *c* varies with distance from the central mass, slowing as *W* increases.

36.3 Acceleration on two geodesics.

The *straight line* is not generally a *geodesic motion.* It would be if the *speed* did not vary. But as well known in classical physics (Snell's law of refraction), wave motion in a medium with a speed differential generates curved trajectories. This is independent of the spatial curvature. Thus the *straight line acceleration* stated above is not generally the *geodesic acceleration.* However it is the true geodesic acceleration in two special cases: the pure parallel and pure tangent motions.

The acceleration equation is simplified in these two cases, (A) where $\phi = \pi/2$, which represents a particle stationary in x, *i.e.* $v_x = 0$, and (B) where $\phi = 0$, which is a photon, i.e. $v_x = c$.

(A) $$dv_x/dt = -c^2 \partial\phi/\partial w$$

(B) $$dv_x/dt = c\, \partial c/\partial x$$

In case (A), acceleration is purely from the divergence of W, represented through changing angle ϕ. In case (B), acceleration is purely from the changing speed of light.

Note that $\partial\phi/\partial w$ is related to the angle θ:

$$\partial\phi/\partial w = -\theta/W$$

And θ is defined by:

$$\theta = dW/dx$$

So the general solution for the straight-line motion is:

(C) $$dv_x/dt = c \cos^2(\phi)\partial c/\partial x + c^2 \sin^2(\phi)(dW/dx)/W$$

Hence the solution (A) when the particle is stationary in x is:

(A) $$dv_x/dt = (c^2/W)\, dW/dx$$

36.4 The geodesic principle for the geometric model.

In simple Euclidean geometry, a straight line is a geodesic – the path of shortest *length* between two points. But in physics, the concept of a geodesic trajectory for a particle is different, because it also involves *time*, and the time taken to travel along a trajectory depends on the speed along the path. This is well known in classical physics, and we review the concept of the geodesic for the refraction of a light beam as it travels across the boundary of two materials with different light speed. The relationship is of course Snell's Law of Refraction.

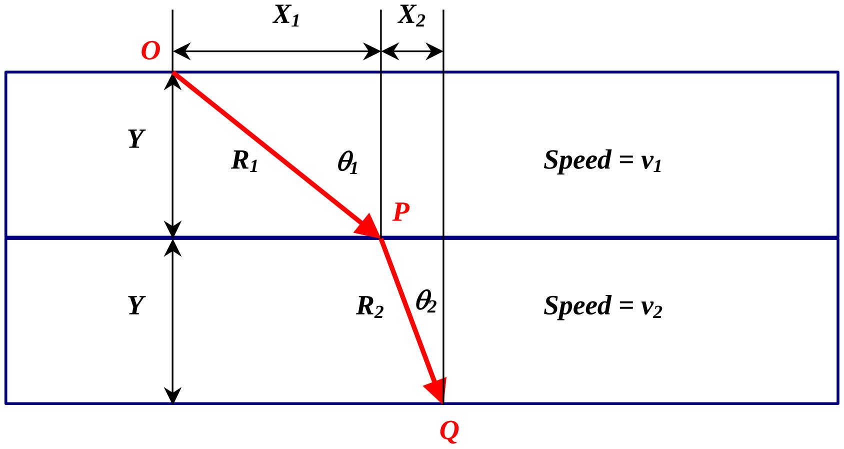


Figure 20. *A light beam crossing a speed boundary.*

This shows a light beam crossing from a material with light speed v_1 to a substance with light speed v_2, where $v_2 < v_1$. We take Y to be the same distance in both cases without loss of generality. Snell's Law tells us the relationship between the *angles* and the *speeds* in the two materials:

$$v_1/v_2 = \sin\theta_1/\sin\theta_2$$

Fermat showed that this is the *fastest path* light can take to travel between the two spatial points, O and Q. I.e. if we vary the intermediate point, P, to get different angles, the *time to travel from P to Q* will increase.

Exercise. Verify Fermat's Proof.

First define the geometric relationships:

$$R_1 = Y/\cos\theta_1 \qquad R_2 = Y/\cos\theta_2$$

$$X_1 = Y\tan\theta_1 \qquad X_2 = Y\tan\theta_2$$

Define the time to travel as sum of time to travel each leg:

$$\begin{aligned} T &= T_1 + T_2 \\ &= R_1/v_1 + R_2/v_2 \\ &= Y/v_1\cos\theta_1 + Y/v_2\cos\theta_2 \end{aligned}$$

The minimum *time T* is the stationary point where: $dT/d\theta_1 = 0$, so that:

$$\begin{aligned} dT/d\theta_1 &= (d/d\theta_1)(Y/v_1\cos\theta_1) + (d/d\theta_1)(Y/v_2\cos\theta_2) \\ &= -Y\sin\theta_1/v_1\cos^2\theta_1 - Y\sin\theta_2/v_2\cos^2\theta_2\,(d\theta_2/d\theta_1) \\ &= 0 \end{aligned}$$

Rearrange:

$$\sin\theta_1/v_1 = -(\sin\theta_2/v_2)(\cos^2\theta_1/\cos^2\theta_2)(d\theta_2/d\theta_1)$$

To determine $(d\theta_2/d\theta_1)$ define the geometric constraint:

$$dx_1 + dx_2 = dy\tan\theta_1 + dy\tan\theta_1 = constant$$

Differentiate both sides by θ_1:

$$(d/d\theta_1)\tan\theta_1 + (d/d\theta_1)\tan\theta_1 = 0$$

Differentiated:

$$1+\tan^2\theta_1 + 1+\tan^2\theta_2\,(d\theta_2/d\theta_1) = 0$$

Rearrange:

$$d\theta_2/d\theta_1 = -\cos^2\theta_2/\cos^2\theta_1$$

Replace in the time differential:

$$\sin\theta_1/v_1 = \sin\theta_2/v_2$$

This is Snell's Law of Refraction.

It is also important that this relationship holds *no matter how many layers of other material are inserted between the point of origin O with angle* θ_1 *and speed* v_1*, and the point of arrival Q with angle* θ_1 *and speed* v_2. This is easily seen because if there was a third layer of material between, then: $sin\theta_1/v_1 = sin\theta_3/v_3$ and $sin\theta_3/v_3 = sin\theta_2/v_2$, hence we still have: $sin\theta_1/v_1 = sin\theta_2/v_2$. The relationship holds if there is a continuous variation in speed. Thus for any two points, the final angle of the light beam is determined by the initial angle and the speed of light at each point:

$$sin\theta_2 = sin\theta_1\ (v_2/v_1)$$

Exactly the same principle applies in the geometric manifold: roughly stated, *geodesic paths represent paths of shortest real time, t.* More exactly, they represent local real time minima (since because of the cyclic nature of *W*, multiple paths can connect two given points – representing paths with different initial conditions, i.e. different directions). We need to use the calculus of variations, as in Riemannian geometry, to define the property of a geodesic more carefully as:

> **Geodesic Property.** If *r(t) is a geodesic path between two points on the geometric manifold, and the real time taken along the path is* Δt*, then an infinitesimal variation of the path:* $r(t) \rightarrow r(t) + \delta r(t)$ *will take a longer time,* $\Delta t + \delta t$.

We will take as a fundamental postulate of the geometric model that:

> **Postulate of physical geodesic paths.** *All possible physical paths are geodesic paths and all geodesic paths are possible physical paths.*

This can be justified in turn by the (Lagrangian) mechanics of the system, but we will take it here as fundamental, the connection being well established in classical physics.

This is the precise analogue of the geodesic postulate used in GTR, in the normal treatment of the *GTR space-time manifold* as a general Riemannian space. When we later specify the *metric equation* for a central mass in the geometric model, the solutions for physical paths are determined by essentially the same *geodesic principles* as used in Riemannian geometry, or GTR – but the metric itself has a slightly different form to the GTR model.

To put this in more formal terms, the *distance*, *dr* along a differential element of a path is related to the *time* by:

$$dr(t) \;=\; |\mathbf{r}(t+dt)-\mathbf{r}(t)| = c(\mathbf{r}(t))dt$$

where c is the *local speed on* **r** *at time t*. In general, this is determined by the position of $\mathbf{r}(t)$ and direction of the tangent vector at $\mathbf{r}(t)$. We examine this *speed function* in detail: it is the chief determinant of the geometric theory of gravity. Notice that paths are parametised by real time, t. They can also be parametised by a distance, s, along **r.** There are no physical singularities, as in GTR, to complicate this. The time along a segment of a path, in terms of a more general parameter s, is then the integral:

$$\Delta t = \int dt = \int 1/c(\mathbf{r}(s))\, dr(s)$$

The *geodesic principle* means that this is minimised on physical trajectories. Geodesics on our curved manifold are not generally straight lines, but *spatially curved paths*, because speed c varies with curvature.

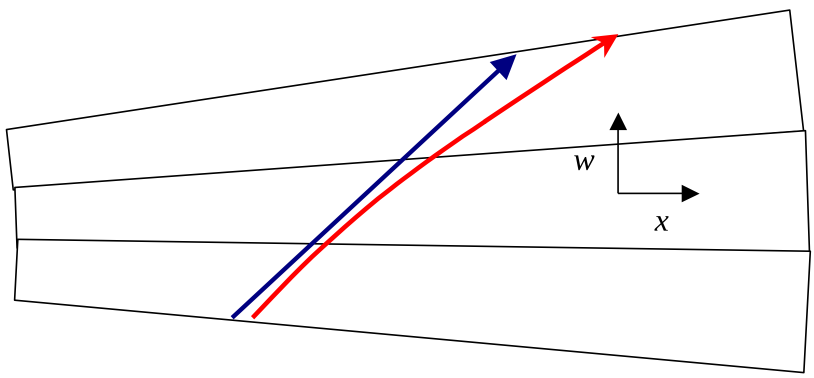


Figure 21. *A geodesic accelerates into the slower region of the manifold.*

Hence the acceleration in the x direction can be thought of as generally having two sources: the spatial curvature plus the speed differential.

Having clarified these general points, we now turn the geometric model itself.

36.5 The strain function for a central mass.

We now turn to the geometric manifold, and begin using the dashed (i.e. true) variables so our equations are valid over periods of cosmological expansion. This is done consistently throughout to avoid ambiguity. Thus we use: M'_∞,, G'_∞ , c'_∞ , h'_∞ which transform with cosmological time as the true constants, to avoid confusion with M_∞,, G_∞ , c_∞ , h_∞, which are the conventional constants. However we use: M_0, G_0 , c_0, h_0 for the *present values* of these, since the true and conventional constants coincide at the special moment selected as the *present* (which is the special moment when these values are set equal). We use *r'* instead of *x* as the radial distance, and we infer a solution for the *strain function, W'(r'),* to obtain Newtonian gravity as the limiting case. In Newtonian gravity, in the context of the geometric model variables, we have the inverse-square law:

$$dv'_r/dt' = M'_\infty G'_\infty/r'^2$$ **Newtonian acceleration law**

and $c' = c'_\infty$, and we can solve for: $-(c'^2_\infty/W')dW'/dr' = M'_\infty G'_\infty/r'^2$. I.e. solve the function *W'(r')* for the condition:

$$dW'/dr' = -W'M'_\infty G'_\infty/c'^2_\infty r^2$$

The simplest solution has the boundary condition that $W' \rightarrow W'\infty$ as $r \rightarrow \infty$. The solution for this is:

$$W'(r')^* = W'_\infty \exp(M'_\infty G'_\infty/c'^2_\infty r)$$ **Strain Function***

However the exact boundary condition requires that: $W' \rightarrow W'_\infty$ as $r \rightarrow \pi R'$. The point at $\pi R'$ is the point exactly on the opposite side of the hyper-sphere of the source mass. This is where the strain on *W'* induced by *M'* is a minimum. The solution is:

$$W'(r') = W'_\infty \exp((M'_\infty G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))$$

We condense this using the factor K: $K(r') = exp((M'_\infty G'_\infty/c'^2_\infty)(1/r' + 1/\pi R'))$, so that:

$$W'(r') = W'_\infty K$$

For approximate calculations we can just use: $K(r')^* = exp((M'_\infty G'_\infty/c'^2_\infty r)$, so:

$$W'(r')^* = W'_\infty K^*$$

Note that $W'(r')$ rearranges to:

$$\begin{aligned} W'(r') &= W'_\infty\, exp(-M'_\infty G'_\infty/c'^2_\infty \pi R')\, exp(M'_\infty G'_\infty/c'^2_\infty r) \\ &= W'_\infty{}^* exp(M'_\infty G'_\infty/c'^2_\infty r) \\ &= W'_\infty{}^* K^* \end{aligned}$$

where:

$$W'_\infty{}^* = W'_\infty\, exp(-M'_\infty G'_\infty/c'^2_\infty \pi R')$$

We can think of adding a new source mass M'_∞ to an otherwise perfectly smooth global manifold with minimum dimension W'_∞ as increasing the minimum to $W'_\infty{}^*$. For a sense of scale, adding one fundamental particle increases W'_∞ by about 1 part in 10^{80}:

$$\begin{aligned} W'_\infty{}^* &\approx W'_\infty\, exp(-M'_\infty G'^2_\infty m^3_\infty/c'^2_\infty h^2) \\ &\approx W'_\infty\, (1+(M'_\infty/m'_\infty)(1/D^2)) \\ &\approx W'_\infty\, (1+N/10^{80}) \end{aligned}$$

Exercise: demonstrate this solution works.

Inserting the b.c.: $r = \pi R$ gives:

$$W'(\pi R') = W'_\infty exp(0) = W'_\infty$$

Differentiating by r gives:

$$\begin{aligned} dW'/dr' &= d/dr'(W'_\infty\, exp((M'_\infty G'_\infty/c'^2_\infty)(1/r' - 1/\pi R')) \\ &= -W'_\infty M'_\infty G'_\infty/c'^2_\infty r^2\, exp((M'_\infty G'_\infty/c'^2_\infty)(1/r' - 1/\pi R')) \\ &= -W' M'_\infty G'_\infty/c'^2_\infty r^2 \end{aligned}$$

36.6 Solution for central acceleration on a straight line trajectory.

This verifies this solution matches Newtonian gravity in the limit – i.e. for weak gravitational fields, and particles with low radial velocity. We will take this as the essential form of the solution required for $W'(r')$. For stronger fields, or particles with radial velocity, the dynamics consequently departs from Newtonian gravity. The full solution for central acceleration is:

$$\text{(C)} \qquad dv'_x/dt' = c' \cos^2(\phi)\partial c'/\partial r' - (c'^2/c'^2_\infty) \sin^2(\phi)\, M'_\infty G'_\infty/r'^2$$

We can also write this with γ' replacing the *cos* and *sin* functions:

$$\text{(C)} \qquad dv'_x/dt' = c'/(1-\gamma'^2)\, \partial c'/\partial r' - (1/\gamma'^2)\, (c'^2/c'^2_\infty)\, M'_\infty G'_\infty/r'^2$$

The solution departs from Newtonian gravity for light, or particles with relativistic speeds, because: $\cos^2(\phi)\partial c'/\partial r'$ is not zero and $\sin^2(\phi) = 1/\gamma'^2$ is not 1. It departs for ordinary fields and non-relativistic speeds because (c'^2/c'^2_∞) is not exactly 1. We will see later that this is very similar to the GTR solution.

36.7 The speed of light function.

The solution for acceleration requires a solution for the speed of light, c', since this changes in the strained space. The cosmological model, which determines global dependence of fundamental constants, determines $\partial c'/\partial r'$ through the relationship of c' to W' in flat space:

$$c'(W') = c'_\perp(W') = c'_\infty W'_\infty/W'$$

This is the speed of light *in directions with no divergence in* W' – i.e. orthogonal to r. This is required because in the limit of large distance from a central mass, space is strained (W' is larger than W'_∞) and this relationship is needed for consistency with the cosmological solution.

But in the direction (*r'*) with divergence in *W',* the speed of light is not determined by this alone, as the *divergence of W'* is relevant. We develop this more generally later, but to jump ahead, the relation is:

$$c'_{=}(W') = c'_{\perp}(W')W'_{\infty}/W' = c'_{\infty}W'^{2}_{\infty}/W'^{2}$$

This is the speed of light parallel to *r'.* The primary relationship with the exponential solution for the strain function, above, gives:

$$c'(W') = c'_{\perp}(W') = c'_{\infty}W'_{\infty}/W' \;= c'_{\infty}\, exp(-(M'_{\infty}G'_{\infty}/c'^{2}_{\infty})(1/r' - 1/\pi R'))$$

This is the 'background speed of light' induced by straining *W'.* Differentiating:

$$\begin{aligned}\partial c'_{\perp}/\partial r' &= (M'_{\infty}G'_{\infty}/c'_{\infty}r'^{2})\, exp(-(M'_{\infty}G'_{\infty}/c'^{2}_{\infty})(1/r' - 1/\pi R')) \\ &= c'_{\perp}\, M'_{\infty}G'_{\infty}/c'^{2}_{\infty}r'^{2}\end{aligned}$$

Hence the acceleration for motion orthogonal to *r'*:

(C) $$dv'_{x}/dt' = c'^{2}_{\perp}\, cos^{2}(\phi)\, (M'_{\infty}G'_{\infty}/c'^{2}_{\infty}r'^{2}) - c'^{2}_{\perp}\, sin^{2}(\phi)\, M'_{\infty}G'_{\infty}/c'^{2}_{\infty}r'^{2})$$

For non-relativistic particle speeds this converges to Newtonian gravity.

For motion parallel to *r:*

$$\begin{aligned}\partial c_{=}(W')/\partial r' &= \partial(c'_{\infty}W'^{2}_{\infty}/W'^{2})/\partial r' \\ &= -2c'_{\infty}W'^{2}_{\infty}/W'^{3}\, \partial W'/\partial r' \\ &= 2c'_{\infty}W'^{2}_{\infty}/W'^{2}\, (M'_{\infty}G'_{\infty}/c'^{2}_{\infty}r'^{2}) \\ &= 2c'_{=}\, M'_{\infty}G'_{\infty}/c'^{2}_{\infty}r'^{2}\end{aligned}$$

Hence the acceleration for motion parallel to *r*:

(D) $$dv'_{x}/dt' = 2c'^{2}_{=}cos^{2}(\phi)(M'_{\infty}G'_{\infty}/c'^{2}_{\infty}r'^{2}) - c'^{2}_{=}sin^{2}(\phi)M'_{\infty}G'_{\infty}/c'^{2}_{\infty}r'^{2})$$

Note we justify this in more detail later – and represent it concisely as a line metric equation comparable to the G'TR central mass solution. But again, for non-relativistic particle speeds this converges to Newtonian gravity.

36.8 Invariance with respect to W'_∞.

A crucial feature is that the accelerations are invariant w.r.t. the dimension W'_∞. For this quantity does not appear in the acceleration solutions. This means is that gravity is precisely the same for electrons and protons – or any two particles hosted in two different-sized micro-dimensions. The reason is that the factor W' does not appear in the acceleration. This is critically important. It means that gravity will work the same for all particles, even if we adopt a different basic topology to the micro-torus topology – e.g. if we added yet more dimensions to host fundamental particles (although we do not do this).

36.9 Superposition of stationary masses.

So far this is for a single central mass. We must derive principles for the superposition of masses for a full theory. The simplest superposition is to combine two point masses in the same position, equivalent to taking a larger central point mass. This is equivalent to taking the strain from one mass, and then superposing the strain induced by the second mass on it. Thus:

$W'(r',M'_1) = W'_\infty \exp((M'_1G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))$ First mass strain by itself

$W'(r',M'_2) = W'_\infty \exp((M'_2G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))$ Second mass strain by itself

$$W'(r',(M'_1+M'_2)) = W'_\infty \exp(((M'_1+M'_2)G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))$$
$$= W'_\infty \exp((M'_1G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))\exp((M'_1G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))$$

This means that the *strain function* is generalised *linearly*, taking an additional mass M'_1 to induce strain at any field-point at distance r' on the existing strain function

$W'(r')$ simply as the product of $W'(r')$ with the function: $K(r,M'_1) = exp((M'_1G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))$.

$$W'(r',M'_1) = W'(r')\ exp((M'_1G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))$$

This applies to the addition of masses anywhere in space – not merely to a central mass point. For instance, if we take two masses, M'_1 and M'_2, at points specified by vectors: $\boldsymbol{r'_1}$ and $\boldsymbol{r'_2}$, with distances to a general field point $\boldsymbol{r'}$ defined by:

$$r'_{01} = |\boldsymbol{r'_1} - \boldsymbol{r'}| \text{ and } \qquad r'_{02} = |\boldsymbol{r'_2} - \boldsymbol{r'}|$$

then the strain function at $\boldsymbol{r'}$ is given by:

$$\begin{aligned} W'(r') &= W'_\infty exp((M'_1G'_\infty/c'^2_\infty)(1/r'_{01}-1/\pi R'))exp((M'_2G'_\infty/c'^2_\infty)(1/r'_{02} - 1/\pi R')) \\ &= W'_\infty exp(\ (G'_\infty/c'^2_\infty)\ (\ M'_1(1/r'_{01}\ -1/\pi R') + M'_2(1/r'_{02} - 1/\pi R')\ )\ ) \end{aligned}$$

The key feature is the effect of superposition of masses on the *accelerations* generated. In the Newtonian (and GTR) limit, adding two masses at a central point should *add the accelerations the masses generate separately*. This is immediately confirmed by differentiating the superposition:

$$\begin{aligned} (\partial/\partial r')W'(r',(M'_1+M'_2)) &= (\partial/\partial r')(W'_\infty exp(((M'_1+M'_2)G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))) \\ &= -W'_\infty(M'_1+M'_2)G'_\infty/(c'^2_\infty r'^2)\ exp(((M'_1+M'_2)G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))) \\ &= (M'_1+M'_2)G'_\infty/(c'^2_\infty r'^2)\ W'(r',(M'_1+M'_2)) \end{aligned}$$

This means the strain function, $W'(r')$, reproduces the *additive effect of mass* of Newtonian gravity in the limit, and the acceleration $a_{1+2} \rightarrow (M'_1+M'_2)G'_\infty/r'^2$.

36.10 The strain function for a moving mass.

To complete the strain function, we must allow for moving masses. What is the strain function for a moving mass? To work this out in a relativistic context we can imagine a *moving observer*, moving past a stationary mass. The theory tells us the strain function for the stationary mass. If we can derive what this looks like to the moving observer, we know the strain function for the moving mass. The pivotal point is to obtain the valid transformations for converting from a stationary frame of reference to a moving frame. In the relativistic context, these are the *Lorentz transformations*. But can we assume these here? In particular, doesn't the geometric theory *contradict* the relativistic principle that there is no preferred frame of reference, and thus undermine the reasoning that the laws of nature have the same form in all frames?

On a global scale, yes: there is a unique stationary frame for the universe as a whole. A moving observer will notice they are moving *if they travel right around the hyper-sphere* – or if they observe they are moving w.r.t. the local MWBR (microwave background radiation), because this is an isotropic global radiation field. But on a local scale, a principle of *relativistic covariance* holds for ordinary kinematics in the limit as in conventional relativity theory. *Some* kinematic laws have the same form in the moving frame as in the true stationary frame – as long as the frames are connected by valid symmetry transformations. In the geometric theory, these transformations are the Lorentz transformations, insofar as the local metric for flat space has the same form as the relativistic metric. There will be fine differences at a certain limit, but we start with the conventional Lorentz transformation.

The Lorentz transformation.

Here we use undashed coordinates: (x, y, z, t) for a stationary system, and daggered variables $(x^\dagger, y^\dagger, x^\dagger, t^\dagger)$ for a velocity-boosted system, transformed by the usual Lorentz transformation. We use daggers for the Lorentz transformed system, so as not to confuse the velocity boost with the dashed symbols for the general system.

The (ordinary) Lorentz transformation for a coordinate frame $(x^\dagger, y^\dagger, x^\dagger, t^\dagger)$ in which a particle that is stationary in the original frame (x, y, z, t) moves at a speed V in the $x^\dagger$-*coordinate* is:

$$\gamma = 1/(1-V^2/c'^2_\infty)$$

$$x^\dagger = \gamma(x + Vt) \qquad y^\dagger = y \qquad z^\dagger = z \qquad W^\dagger = W'$$

$t^{\dagger} = \gamma(t + Vx/c'_{\infty}{}^{2})$

The inverse transformation is:

$x = \gamma(x^{\dagger} - Vt^{\dagger})$ $\quad y = y^{\dagger}$ $\quad z = z^{\dagger}$ $\quad W' = W^{\dagger}$

$t = \gamma(t^{\dagger} - Vx^{\dagger}/c'_{\infty}{}^{2})$

Space-time points are *event points,* and this transformation means that *the event at point: (x, y, z, t)* in the stationary coordinate frame is the same event as at: $(x^{\dagger}, y^{\dagger}, x^{\dagger}, t^{\dagger})$ in the moving frame. The *strain, W',* is a part of this event, and the value of *W'* is independent of the coordinate system.

Note the differential transformations:

$dx^{\dagger} = \gamma dx + \gamma V dt$

$dt^{\dagger} = \gamma dt + (\gamma V/c'_{\infty}{}^{2})dx$

which are obtained from the partial differential chain law:

$dx^{\dagger}(x,t) = (\partial x^{\dagger}/\partial x)dx + (\partial x^{\dagger}/\partial t)dt$

$dt^{\dagger}(x,t) = (\partial t^{\dagger}/\partial x)dx + (\partial t^{\dagger}/\partial t)dt$

We need to write the strain function in a coordinate-function form: $W' = W'(x, y, x, t)$, giving the strain, *W',* at a space-time point: $\boldsymbol{u} = (x, y, z, t)$. In the solution so far, we have taken a stationary central mass, and written a function for: $W'(r')$, because it is spherically symmetric. We can write the stationary solution in Cartesian coordinates, using: $r' = (x^2+y^2+z^2)^{½}$. We have seen how time enters with the accelerations for the two special cases of a *moving test-charge,* and we now use the Lorentz transformation to derive the effect on these for a *moving source mass*.

We consider a special case of a moving central mass, by starting with the stationary solution, and then transforming to a moving frame, $(x^{\dagger}, y^{\dagger}, z^{\dagger}, t^{\dagger})$, defined by the Lorentz transformation above. We know that the *strain*, given by the coordinate function $W^\dagger$, at an *event point* in the moving frame is identical to the strain, W', at the same *event point* in the stationary frame – because W' is orthogonal to the motion, and invariant. Hence we will write an identity between *coordinate functions*:

$$W^\dagger(x^\dagger, y^\dagger, z^\dagger, t^\dagger) = W'(x, y, z, t)$$

This holds when $(x^\dagger, y^\dagger, z^\dagger, t^\dagger)$ is the same event as (x, y, z, t), which means that the coordinates are related by the Lorentz transformation above. Substituting with the inverse Lorentz transformation:

$$\begin{aligned} W^\dagger(x^\dagger, y^\dagger, z^\dagger, t^\dagger) &= W'(x, y, z, t) \\ &= W'_\infty \exp((M'_\infty G'_\infty/c'^2_\infty)(1/(\gamma^2(x^\dagger\text{-}Vt^\dagger)^2+y^{\dagger 2}+z^{\dagger 2})^{1/2} - 1/\pi R')) \end{aligned}$$

Covariance in our context requires that *the function $W^\dagger$ for a moving source mass has the same form as the function W' for the stationary source mass.* I.e. the law for the strain function does not change form in a moving frame. What is important for our theory of gravity is that *the acceleration generated by W' is the Lorentz transformation of the acceleration generated by W'.* We must check that our proposed function, W', is consistent with this principle. If so, it is appropriately covariant.

36.11 Strain orthogonal to motion of the source mass.

We take a stationary test particle in the field of a stationary gravitational mass M'_∞ at a field point in space-time: $\boldsymbol{u} = (0,y,0,0)$, i.e. at $x=0$, $y=y$, $z=0$, $t=0$, so: $r = y$. We know the acceleration produced - it is the special case of (C) for the stationary test particle:

$$dv_y/dt \;\; = -(c'^2/W')(dW'/dy)$$

This acceleration is in the direction of $-\boldsymbol{y}$. We now take the velocity boost V_x, to generate the transformed system. The test particle is now at: $\boldsymbol{u}^\dagger = (0,y,0,0)$, since $y^\dagger = y$. It moves with the source mass M' at speed: $v_x^\dagger = V_x$ in the *x-direction.* The acceleration is obtained simply from the Lorentz transformation. The velocity is:

$$v_y^{\dagger} = dy^{\dagger}/dt^{\dagger} = v_y/\gamma$$

since: $dy^{\dagger} = dy$ and: $dt^{\dagger} = \gamma dt + (\gamma V/c'_{\infty}{}^2)dx = \gamma dt$ since $dx = 0$. Differentiating the velocity gives the acceleration:

$$a_y^{\dagger} = dv_y^{\dagger}/dt^{\dagger} = a_y/\gamma^2$$

since: $dv_y^{\dagger} = dv_y/\gamma$ and: $dt^{\dagger} = \gamma dt$. This what the transformed acceleration will appear to be under the Lorentz transformation.

We then determine what acceleration is generated *by the strain function, W'.* There is no *a priori* reason the two should match. If the two accelerations do *not* match, the function *W'* is *not covariant.* This would mean that the law for gravitational strain is dependant on absolute motion, and lacks invariance w.r.t. inertial motion. In general:

$$W^{\dagger}(r) = W'(x,y,z,t)$$
$$= W_{\infty} \exp((M'_{\infty}G'_{\infty}/c'_{\infty}{}^2)(1/(x^2+y^2+z^2)^{½} - 1/\pi R))$$

Substituting with the (inverse) Lorentz transformation:

$$W^{\dagger}(x, y, z, t) = W_{\infty} \exp((M'_{\infty}G'_{\infty}/c'_{\infty}{}^2)(1/(\gamma^2(x\text{-}Vtv)^2+y^2+z^2)^{½} - 1/\pi R))$$

Note the transformation of the global variable: $R \rightarrow R^{\dagger}$ is a point where covariance *does* come into question, because in the stationary frame: $R = R_x = R_y = R_z$, the same in all directions, but in the moving frame: $R = R_y^{\dagger} = R_z,^{\dagger}$, but: $R_x^{\dagger} = R_x/\gamma$. We take R to be in the direction of the radial vector, in this case y, and $R = R_y^{\dagger}$. But the fact that R is a *globally determined variable* means that in general it represents a *non-covariant* feature – something entirely absent from the purely local equations of Newtonian gravity of GTR – and something intrinsic to the model. However this factor it is extremely tiny for local gravity. In any case, it does not affect this case.

To simplify we define:

$$r = (\gamma^2(x\text{-}Vt)^2+y^2+z^2)^{½}$$

and write:

$$W^{\dagger}(x, y, z, t) = W_{\infty}\, exp((M'_{\infty}G'_{\infty}/c'^{2}_{\infty})(1/r - 1/\pi R'))$$

In our example, the divergence at the field point *(0,y,0,0)* is now:

$$dW^{\dagger}/dr = dW^{\dagger}/dy^{\dagger} = dW^{\dagger}/dy = -WM'_{\infty}G'_{\infty}/c'^{2}_{\infty}y^{2}$$

This is the same at time $t' = t = 0$ as in the stationary system – as expected, since both y and W are invariant. We can now write this in our general solution for the central acceleration with non-radial test particle velocity:

$$\begin{aligned} dv_{y}^{\dagger}/dt' &= (c'^{2}/\gamma'^{2})\,(dW/dr')/W \\ &= -(c'^{2}/\gamma'^{2})(M'_{\infty}G'_{\infty}/c'^{2}_{\infty}y'^{2}) \\ &= -(c'^{2}/c'^{2}_{\infty})\, M'_{\infty}G'_{\infty}/y'^{2}\gamma'^{2} \end{aligned}$$

Hence:

$$a_{y}' = dv_{y}'/dt' = (1/\gamma'^{2})\, dv_{y}/dt = a_{y}/\gamma'^{2}$$

This is *almost* the same as the ordinary Lorentz transform of the acceleration seen above – except it has the dashed gamma-factor: $1/\gamma'^{2} = 1- V_{x}^{2}/c'^{2}$, instead of the usual simple Lorentz factor: $1/\gamma^{2} = 1-V_{x}^{2}/c'^{2}_{\infty}$. This reflects the fact that the geometric model *modifies the speed of light* in the context of gravity, or curved space. The ordinary Lorentz transformations for flat space do not apply exactly. But we can substitute the modified transformation: $\gamma'=1/(1-V^{2}/c'^{2})$ into the Lorentz transformation, giving:

Local Model Lorentz transformation.

$$\gamma' = 1/(1-V^{2}/c'^{2})$$

$$x^{\dagger} = \gamma'(x - Vt) \qquad y^{\dagger} = y \qquad z^{\dagger} = z \qquad W^{\dagger} = W$$

$$t^{\dagger} = \gamma'(t - Vx/c'^{2}) \qquad M^{\dagger}_{\infty} = \gamma' M'_{\infty}$$

Using this instead of the simple Lorentz transformation for flat space, the strain for the moving mass is exactly covariant.

36.12 Strain parallel to motion of the source mass.

We now take a second special example, for a field point at: $(x,0,0,0)$. As with $a_y^{\dagger}$, we can first calculate the relativistically transformed *x-acceleration*. The ordinary velocity of a particle ($\boldsymbol{dr}/dt$) transforms generally by:[15]

$$\boldsymbol{v}^{\dagger} = (\boldsymbol{v}_{=} + \boldsymbol{V} + \boldsymbol{v}_{\perp}/\gamma)/(1+ v_{=}V/c'^2)$$

where $\boldsymbol{v}_{=}$ is the velocity parallel to the motion, and $\boldsymbol{v}_{\perp}$ is orthogonal. Taking: $\boldsymbol{V} = \boldsymbol{V_x}$ means $\boldsymbol{v}_{=} = \boldsymbol{v_x}$ and: $\boldsymbol{v}_{\perp} = \boldsymbol{v_y} + \boldsymbol{v_z}$. For this example, $v_y = v_z = 0$, and:

$$\boldsymbol{v}^{\dagger} = \boldsymbol{v_x}^{\dagger} = (\boldsymbol{v_x} + \boldsymbol{V})/(1+ v_xV/c'^2)$$

Since we assume $v_x = 0$ at $t = 0$, $\boldsymbol{v}^{\dagger} = \boldsymbol{V}$. Differentiating and using $v_x = 0$:

$$\begin{aligned} dv_x^{\dagger}/dt^{\dagger} &= d/dt^{\dagger}((v_x + V)/(1+ v_xV/c'^2)) \\ &= dv_x/dt^{\dagger} - dv_x/dt^{\dagger}\,(V^2/c'^2) \\ &= (dv_x/dt^{\dagger})/\gamma'^2 \end{aligned}$$

Since: $dt^{\dagger} = \gamma' dt$, this means:

$$a_x^{\dagger} = dv_x^{\dagger}/dt^{\dagger} = (dv_x/dt^{\dagger})/\gamma'^3 = a_x/\gamma'^3$$

We then calculate the transformation of the strain function and resulting acceleration. The transformed radial distance $r^{\dagger}$ is now:

$$r^{\dagger} = \gamma'(x^{\dagger}+Vt^{\dagger})$$

and $W^{\dagger}$ is:

$$W^{\dagger}(x^{\dagger},0,0,t^{\dagger}) = W_{\infty}\, exp((M'_{\infty}G'_{\infty}/c'^{\ 2}_{\infty})(1/\gamma'(x^{\dagger}+Vt^{\dagger}) - 1/\gamma'\pi R^{\dagger}))$$

At time $t^{\dagger} = 0$:

$$W^{\dagger}(x^{\dagger},0,0,0) = W_{\infty}\, exp((M'_{\infty}G'_{\infty}/\gamma' c_{\infty}^{2})(1/x^{\dagger} - 1/\gamma'\pi R^{\dagger}))$$

[15] Lorrain and Corson (1988), p.259.

The divergence is correspondingly reduced in magnitude, to:

$$dW^{\dagger}/dr^{\dagger} = dW^{\dagger}/dx^{\dagger} = -W^{\dagger}M'_{\infty}G'_{\infty}/\gamma' c'^{2}_{\infty}x^{\dagger 2}$$

Writing this in our general solution for the central acceleration:

$$dv_x^{\dagger}/dt^{\dagger} = (c'^2/\gamma'^2)(dW^{\dagger}/dr^{\dagger})/W^{\dagger}$$
$$= -(c'^2/\gamma'^2)(M'_{\infty}G'_{\infty}/\gamma' c'^{2}_{\infty}x^{\dagger 2})$$
$$= -(c'^2/c'^{2}_{\infty})\, M'_{\infty}G'_{\infty}/x^{\dagger 2}\gamma'^3 \qquad \text{[Rearranging]}$$

Hence:

$$a_x^{\dagger} = a_x/\gamma'^3$$

Hence this matches the prediction from the transformation of the acceleration, and confirms covariance.

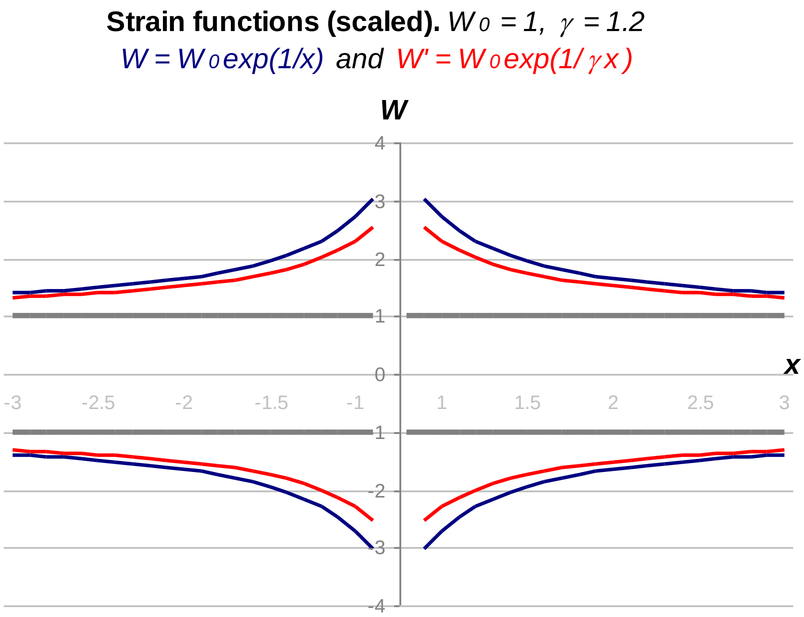


Figure 22. *Shape of the strain functions W and W'. The blue curve represents the strain function W(x) for the stationary mass. The red curve represents the strain function $W^{\dagger}(x)$ for the same mass moving with velocity V in x (as they overlap in space at a moment t=0). The velocity represents: γ = 1.2, or V = 0.553. The curves are scaled to W_{∞} =1, and $M'_{\infty}G'_{\infty}/c'^{2}_{\infty}$ = 1. Note the central part of the strain curve is left missing because it rapidly becomes very large for: x < $M'_{\infty}G'_{\infty}/c'^{2}_{\infty}$. There is a central singularity, but no event horizon singularity as with: $1/\sqrt{(1-2M'_{\infty}G'_{\infty}/c'^{2}_{\infty}r)}$.*

The strain for the mass moving *in the x-direction* has reduced by the (exponentiated) factor: $1/\gamma'$. For the stationary mass, $W(r)$ is spherically symmetric. For the moving mass it has effectively been (exponentially) compressed in the $x^{\dagger}$*-direction*. So it is no longer spherically symmetric. However it has *reflection symmetry:* $W^{\dagger}(r)= W^{\dagger}(-\mathbf{r})$.[16] For non-relativistic speeds $V<<c$, at distances $x>> M'_{\infty}G'_{\infty}/c'^{2}_{\infty}$, the difference between the curves becomes extremely small.

36.13 The speed metric and Schwarzschild solution analogue.

We now turn to a general solution, which gives us the equivalent of the Schwarzschild metric for GTR. We define the geometric model factor as K ('big K'):

$$K(r') = exp((M'_{\infty}G'_{\infty}/c'^{2}_{\infty})(1/r' + 1/\pi R'))$$

And the corresponding line metric in the geometric model is:

$$(K^2dr'^2 + dy'^2 + dz'^2 + dw'^2)/dt'^2 = c'^{2}_{\infty}/K^2$$ **Geometric Model Speed Metric**

This is the essential gravitational law for a central mass, to be compared with GTR. Note that the Schwarzschild solution is the covariant solution *when we write the mass-energy tensor for a point-mass* on one side of the GTR equation. It must be wondered how our solution can be covariant if it is different to this – even though it is only very slightly different. But our strain equation, W, no longer corresponds to a simple *point mass* in GTR. Rather, it corresponds to a 'smeared-out mass' – the energy is distributed through space. In fact it is consistent with treating the mass as a *quantum wave* – it has exactly the properties of a quantum wave. The geometric theory solution is slightly different to the Schwarzschild solution because *there is no point-mass in the geometric theory.* From our point of view, the Schwarzschild solution is not the correct physical representation of a central mass. We now turn to this.

---

[16] This also means it has a simple time-reversal symmetry. An accelerating source mass will not necessarily be symmetric under spatial reflection, and it will not be time-symmetric either, since like EM waves only the 'retarded solution' is physical. But we do not treat accelerated masses here.

We now compare this with GTR. The Schwarzschild metric for a central mass is equivalently written as:

$$(k^2dr'^2 + dy'^2 + dz'^2 + dw'^2)/dt'^2 = c'_\infty{}^2/k^2 \quad \textbf{GTR Speed Metric}$$

where *k* is the factor:

$$k = (1\text{-}2M'_\infty G'_\infty/c'_\infty{}^2r')^{-1/2}$$

Like *K*, $k > 1$, and $k \rightarrow 1$ as $r' \rightarrow infinity$. Thus for particles travelling in the *w*, *y* or *z* directions, i.e. orthogonal to *r'*, there is a common speed equal to:

$$c'_\infty/k = \sqrt{(dy'^2 + dz'^2 + dw'^2)}/dt'$$
$$= c'_w = c'_y = c'_z$$

For a particle travelling in *r*, the radial light trajectory, the speed is reduced by *k*:

$$c'_r = c'_\infty/k^2 = dr'/dt'$$

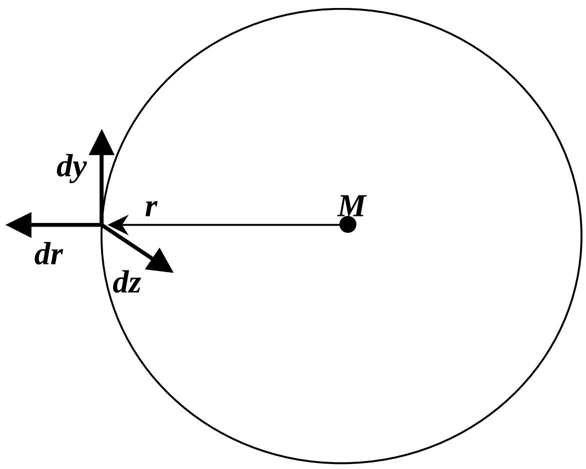


Figure 23. *Local Cartesian coordinates at r.*

Viewed in this way, GTR means that the *speed of light* changes depending on whether a photon is travelling radially (parallel with *r*), or orthogonally to *r*.

36.14 Comparison of K with k.

We compare: $1/k^2$ with $1/K^{*2}$.

$$1/k^2 = 1-2M'_\infty G'_\infty/c'^2_\infty r$$

Expanding $1/K^{*2}$ as a series:

$$1/K^{*2} = 1-2M'_\infty G'_\infty/c'^2_\infty r + (2M'_\infty G'_\infty/c'^2_\infty r)^2/2! - (2M'_\infty G'_\infty/c'^2_\infty r)^3/3! + \dots$$

Hence we see that $1/k^2$ is simply the series for $1/K^{*2}$ truncated after the first two terms:

$$1/K^{*2} = 1/k^2 + (2M'_\infty G'_\infty/c'^2_\infty r)^2/2! - (2M'_\infty G'_\infty/c'^2_\infty r)^3/3! + \dots$$

The first-order difference between k and K^* is approximately:

$$k/K^* \approx 1 + (M'_\infty G'_\infty/c'^2_\infty r)^2$$

In ordinary fields, $2M'_\infty G'_\infty/c'^2_\infty r << 1$ and: $r << R$, and this is the first-order difference between k and K.

$$k/K \approx 1 + (M'_\infty G'_\infty/c'^2_\infty r)^2$$

In any ordinary gravitational fields the difference is very small. Note the event horizon for a GTR black hole occurs at: $2M'_\infty G'_\infty/c'^2_\infty r = 1$, i.e.: $r = 2M'_\infty G'_\infty/c'^2_\infty$, and this is the region where the difference becomes large and the two theories diverge. E.g. in Earth gravity, the first-order term: $M'_\infty G'_\infty/c'^2_\infty r \approx 10^{-10}$, and the second-order term is about: $(M'_\infty G'_\infty/c'^2_\infty r)^2/2! \approx 10^{-20}$. When $r \rightarrow R$, $K \rightarrow 1$, and $k \approx K \approx 1$. See later sections comparison of gravitational holes and gravitational fields.

36.15 Divergence of k.

Using the term k instead of K in the strain function would be an alternative for the geometric model. I.e. can consider using: $W' = kW'_\infty$, instead of: $W' = KW'_\infty$. In this case the divergence is:

$$
\begin{aligned}
dk/dr' &= (d/dr')(1-2M'_\infty G'_\infty/c'^2_\infty r)^{-1/2} \\
&= -(M'_\infty G'_\infty/c'^2_\infty r^2)(1-2M'_\infty G'_\infty/c'^2_\infty r)^{-3/2} \\
&= -(M'_\infty G'_\infty/c'^2_\infty r^2)/k^3
\end{aligned}
$$

Setting: $W' = kW'_\infty$, we would have:

$$dW'/dr' = -W'M'_\infty G'_\infty/c'^2_\infty r'^2 k^2$$

Then using: $c' = c'_\infty W'/W'_\infty = c'_\infty k$, the solution (A) for a particle stationary in x' would be:

$$
\begin{aligned}
dv'_x/dt' &= (c'^2/W')\, dW'/dx' \\
&= -(c'^2/W')(M'_\infty G'_\infty/c'^2_\infty r^2)W'/k^2 \\
&= -(c'^2/c'^2_\infty k^2)(M'_\infty G'_\infty/r'^2) \\
&= -M'_\infty G'_\infty/r'^2
\end{aligned}
$$

This is also a plausible solution, since it matches Newtonian gravity in this non-relativistic limit, and it is very close numerically to the adopted solution. It corresponds to the GTR solution. However, it does not have the precise symmetry of the adopted solution, using K. We consider this shortly, but next we summarise the *speed function.*

36.16 The speed function.

The geometric model metric (just like the Schwarzschild metric) means that the *particle speed*, c, is not a simple scalar: instead it is a product of the *particle direction vector* with a *speed field,* and depends on the direction of the trajectory.

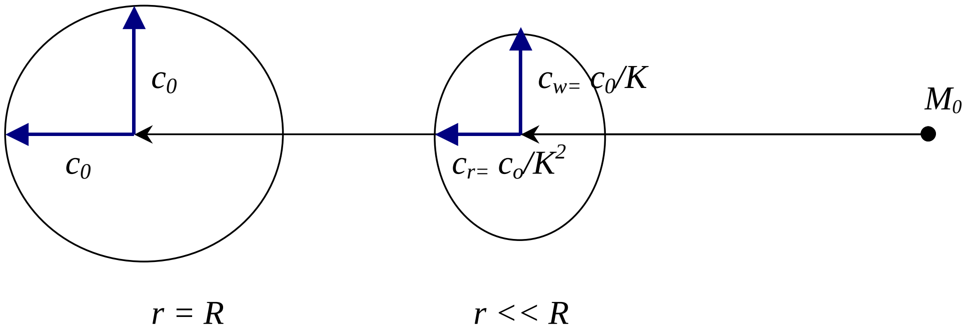


Figure 24. *The speed of light is different in directions parallel and orthogonal to r.*

As we approach: $r = R$, which is very distant from the mass M'_∞, the speed becomes c'_∞ in all directions. In the gravitational field, at $r << R$, the speed is attenuated to c'_∞/K for motion in the w, y and z directions, and to c'_∞/K^2 for motion in the r direction.

Another way to write this is to define a particle with a velocity vector: $\boldsymbol{c'} = c'\boldsymbol{s}$ in a specific direction with unit vector $\boldsymbol{s}$.

$$\boldsymbol{c'} = (\boldsymbol{v'}_r, \boldsymbol{v'}_y, \boldsymbol{v'}_z, \boldsymbol{v'}_w) = c'\boldsymbol{s}$$

where:

$$c'^2 = \boldsymbol{c'}.\boldsymbol{c'} = \boldsymbol{v'_r}.\boldsymbol{v'_r} + \boldsymbol{v'_y}.\boldsymbol{v'_y} + \boldsymbol{v'_z}.\boldsymbol{v'_z} + \boldsymbol{v'_w}.\boldsymbol{v'_w}$$
$$= v'^2_r + v'^2_y + v'^2_z + v'^2_w$$

Then:

$$\sqrt{((K^2v'_r)^2 + (Kv'_y)^2 + (Kv'_z)^2 + (Kv'_w)^2)}/dt' = c'_\infty$$

Or in parallel and perpendicular components:

$$C'^2 = \boldsymbol{c'}.\boldsymbol{c'}$$
$$= \boldsymbol{v'_=}.\boldsymbol{v'_=} + \boldsymbol{v'_\perp}\boldsymbol{v'_\perp}$$
$$= v'^2_= + v'^2_\perp$$

And:

$$\sqrt{((K^2v'_=)^2 + (Kv'_\perp)^2)}/dt' = c'_\infty$$

This is an elliptical function. This follows directly from the line metric.

Given the speed metric, the motion of a particle in a gravitational field is determined by almost exactly the same general principles as in GTR, as geodesic motions. We pass over this here since it is quite technical but well known.

36.17 Mass dilation in a gravitational field.

A test-particle with rest mass m'_∞ in the background space will have a lower rest-mass in the strained space generated by the gravitational field of a mass M'. This is determined by the more general transformations of quantities w.r.t. the spatial expansion. Or more directly, by the fact that fundamental particle rest-mass is determined by the relationship: $W' = h'/2m'c'$. In the flat space: $W'_\infty = h'_\infty/2m'_\infty c'_\infty$. Given that the dimension W' changes by: $W' = K(r')W'_\infty$, and the rest-speed changes inversely by: $c' = c'_\infty/K(r')$, we have: $W' = K(r')W'_\infty = h'/2m'c' = K(r')h'/2m'c'_\infty$, so that: $W'_\infty = h'/2m'c'_\infty$. Hence: $h'/m' = h'_\infty/m'_\infty$. The ratio of h' and m' is constant. For consistency with the general transformation, they must both change linearly with W': $m' = K(r')m'_\infty$, and: $h' = K(r')h'_\infty$.

$$W' = W'_\infty K(r')$$
$$c' = c'_\infty/K(r')$$
$$m' = m'_\infty K(r')$$
$$h' = h'_\infty K(r')$$

The ‘mass dilation’ reflects the analogous feature found in GTR. This is also why we are careful to specify the *fundamental constants* in our equations as m'_∞,, c'_∞, etc.

These relationships determine energy and angular momentum changes:

$$E' = m'c'^2 = m'_\infty c'^2_\infty/K(r') = E'_\infty/K(r')$$

Thus when a test-mass m'_∞ moves from a large distance away, to a radius r' close to M', it gives up a certain amount of its *rest-mass energy*, viz.

$$\Delta E' = m'_\infty c'^2_\infty - m'c'^2 = m'_\infty c'^2_\infty (K(r') - 1)/K(r')$$

To the first-order approximation, $(K(r') - 1)/K(r') \approx M'_\infty G'_\infty / c'^2_\infty r$, hence we have:

$$\Delta E' \approx m'_\infty M'_\infty G'_\infty / r'$$

This is the *classical potential gravitational energy*, representing the conformity with Newtonian gravity. If the particle undergoes free-fall, it loses this *potential gravitational energy*, and converts it to *kinetic energy*. This shows where the potential gravitational energy is *stored mechanically*: in the mass-energy. The classical theory of gravity has no analogue of this, for there is no mechanical model for the potential energy, and it is simply said to be stored in a 'field'. This classical 'gravitational field' is aethereal.

The angular momentum is determined by:

$$\begin{aligned} L' &= m'c'W'/2\pi \\ &= m_\infty' c_\infty' W_\infty'/2\pi \\ &= K(r') m_\infty' c_\infty' W_\infty'/2\pi \\ &= \hbar'/2 \\ &= \hbar'_\infty K(r')/2 \\ &= L'_\infty K(r') \end{aligned}$$

Thus *the intrinsic angular momentum (and h') increases like the mass in a gravitational field*. Where is this angular momentum exchanged from? I am not entirely sure. It is possible it exchanges with an orbital angular momentum, giving a tiny rotational force (like an analogue of the magnetic force). Another possibility is that it is stored locally in the stress tensor of the space itself, in which the mass-wave rotates. In practical terms, however, for a sizeable mass there are large numbers of particles involved, and their angular momenta are in random directions, and almost cancel (like electric charges) leaving gross matter as a whole almost neutral (as it is almost electrically neutral). The other possibility, for balancing momentum perfectly in general, is explored in the final section of the theory: the postulate of 'super-symmetry'.

We have now seen some of the main implications of the theory of gravity. We now go on to the important point of justifying this as the correct solution for the geometric model.

36.17 Justifying the solution from first principles of the model.

This theory of gravity represented by the line metric above matches GTR very closely, it matches empirical observation closely (see below), it is based on a nice function K, it is suitably covariant and coherent with the larger cosmological model. We can simply *postulate it* as the geometric theory of gravity, and go on to test it empirically, and that it is a valid approach pragmatically. But we need to justify it from principles of the geometric theory. It follows from three primary assumptions:

(i) **The strain function** has the form: $W'(r') = W'_\infty K$
(ii) **The speed function** for the pure tangential velocity is: $c'_w = c'_\infty W'_\infty / W' = c'_\infty K$
(iii) **The speed function** for the pure radial velocity is: $c'_r = c'_w K = c'_\infty K^2$

The second assumption follows from the first assumption combined with the general theory of variable transformations for the model, developed in detail in other sections. This needs no separate justification.

Assumptions (i) and (iii) are what need justification. In the next few sections we analyse (i). Following this, we see how the *speed function, (iii)* is justified.

In terms of (i), the main question remaining is why the model requires the strain to be based on K, rather than k, which is the only other highly plausible possibility at this point. The latter would give gravity corresponding almost exactly to GTR. The reason is that the K function reflects an appropriate *scale symmetry for the expanding manifold*. It is also reflects how *wave energy* is stored by the manifold. The k function is similar, but does not have the exact symmetries. The speed function, (iii), is related to covariance, and is shown by analysing the Lorentz transformation of the solution.

36.18 Scale symmetries of the strain function.

So far we have treated the strain for a system *at a single cosmological moment*, with a global radius *R'*. But we have written the equations generally so they are valid at any time. At the arbitrarily chosen *present moment,* conventional variables and constants used in physics match true variables and constants, except *R'* and *T'*. We now consider explicitly how the strain function behaves in the cosmological theory, with changing *R.*

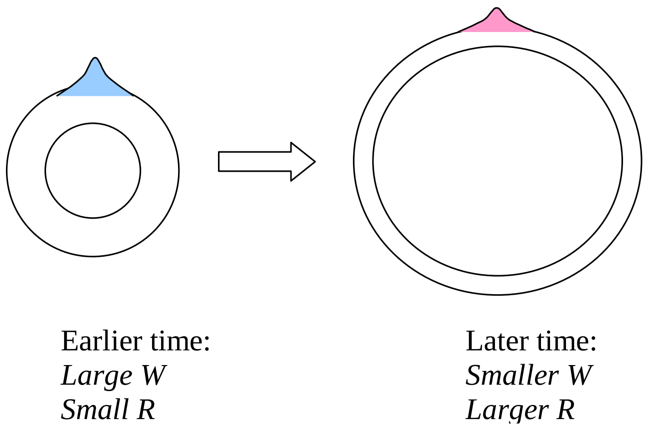
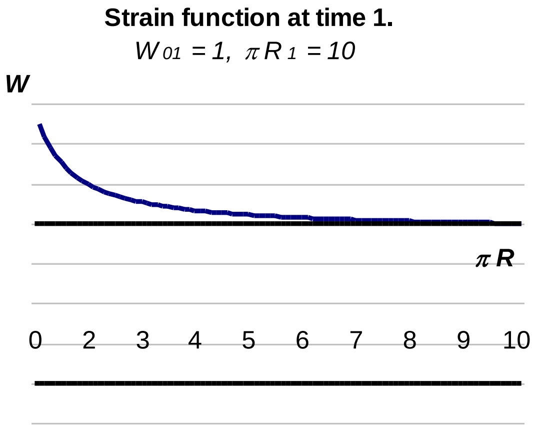


Figure 25. *The strain function thins out as the universe expands.*

As the universe expands, the strain function for a mass retains the same shape, *K*, but changes scale. We start by showing some properties of the *scale symmetry*. The strain function must be defined through *dimensionless variables,* because there is no 'absolute' metric scale for the background space in which the manifold is embedded. We show how the strain function is determined by a simple relationship between the *dimensionless divergence* and other dimensionless quantities. The surface tension of space increases, and the *energy* stored in the mass wave decreases as the manifold expands. We show how the *volume integral* behaves as the manifold stretches, and relate it to the energy.



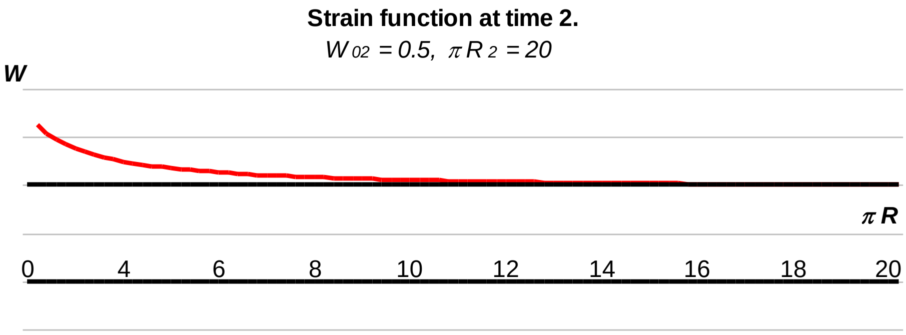


Figure 26. *A strain function as a universe doubles its radius.*

The key symmetry of the strain function relates to its change of shape in an expanding space, with changing R' and W'_∞. We start with the general equation:

$$W'(r') = W_\infty' \exp((M'_\infty G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))$$

This is the equation in the *true variable system*, where M'_∞, G'_∞, c_∞', R' are quantities for the universe at some time when we can assume the radius has changed (we can assume it has expanded) from an initial present radius R_0' to R', and the fundamental constants have likewise changed. We relate these to the *present variables* using [13.1] – [13.5], [14.1] – [14.5], [21.1]. The relevant relations are repeated here for convenience:

$\hat{R}' = R'/R_0'$	$\check{T}' = T'/T_0'$	
$\hat{R}'dx' = dx$	$\hat{R}'^2dt' = dt$	$\hat{R}'dm' = dm$
$c'_\infty = c_0\hat{R}'$	$h'_\infty = h_0/\hat{R}'$	$G'_\infty = G_0$
$m'_e = m_{e0}/\hat{R}'$	$m'_p = m_{p0}/\hat{R}'$	$M'_\infty = M_0/\hat{R}'$

$R' = R_0'\hat{R}'$ $\quad W'_\infty = W_0'/\hat{R}'$ $\quad R'W'_\infty = constant$

$D' = h'_\infty c'_\infty/m'^2_\infty G'_\infty$ $\quad D' = D_0\hat{R}'^2$ $\quad 2\pi R'/W' = D'$

The dashed variables refer to the *true variable system* for the *expanded space at a general time, T'*. Zero-subscripts refer to the *true variable system for at the original present time*, T_0'. Except for R_0' and T_0', the *true variables at the present time* match the conventional variables. For the universal constants, the *true present values and conventional present values* are identical, e.g.: $c_0 = c_0'$, so we can leave out the dashes on the present variables, except R_0' and T_0', which do not match R_0 and T_0.

We now observe some basic properties related to scale, i.e. relationships between W' and dW'/dr' at different r'.

The ratio of *W'(r')* at two different points.

The ratio $W'(r')$ at two different points, r_1' and r_2', at a moment of time is:

$$\frac{W'(r_1')}{W'(r_2')} = \frac{exp((M'_\infty G'_\infty/c'^2_\infty)(1/r_1' - 1/\pi R'))}{exp((M'_\infty G'_\infty/c'^2_\infty)(1/r_2' - 1/\pi R'))}$$

$$= exp((M'_0G'_0/\hat{R}'^3c'^2_0)(1/r_1' - 1/r_2'))$$

Hence comparing the ratio at a moment, T' with the ratio at the *conventional present moment*, T_0':

$$\frac{W_0'(r_1')/W_0'(r_2')}{W'(r_1')/W'(r_2')} = exp((1-1/\hat{R}'^3)(M'_0G'_0/c'^2_0)(1/r_1' - 1/r_2'))$$

The log of the ratio of *W'(r')* at two different points.

The log of the ratio of $W'(r')$ at two different points, r_1' and r_2' is:

$$ln(W'(r_1')/W'(r_2')) = (1/\hat{R}'^3)(M_0G_0/c_0^2)(1/r_1' - 1/r_2'))$$

This changes inversely with the radius scale of the universe, by $1/\hat{R}'^3$. Hence comparing the ratio at the *conventional present moment*, T_0' and at a moment, T':

$$\frac{\ln(W_0'(r_1')/W_0'(r_2'))}{\ln(W'(r_1')/W'(r_2'))} = \hat{R}'^3 = R'^3/R_0^3$$

This is a strong symmetry property, because *it is independent of* r_1' *and* r_2'. The symmetry is based on a log transformation of W'. This is the natural kind of symmetry for a system that has *no intrinsic scale*. We conceive the universe as a curved manifold set in an external 'empty' 6-D Euclidean space, but this external space has no scale properties, or metric. Distance is purely *relative to the scale of* W' *and* R', i.e. the dimensionless ratio D.

Note that the k function does not have any scale symmetry of this kind. Defining the alternative 'GTR strain function':

$$W_k'(r') = W_\infty' k = W_\infty'(1-2M'_\infty G'_\infty/c'^2_\infty r)^{-1/2}$$

We have:

$$\begin{aligned} \ln(W_k'(r_1')/W_k'(r_2')) &= \tfrac{1}{2}\ln((c'^2_\infty r_2 - 2M'_\infty G'_\infty)/c'^2_\infty r_2) \\ &\quad - \tfrac{1}{2}\ln((c'^2_\infty r_1 - 2M'_\infty G'_\infty/c'^2_\infty r_1) \\ &= \tfrac{1}{2}\ln((c'^2_0 r_2 - 2M'_0 G'_0/\hat{R}'^3) - \tfrac{1}{2}\ln(r_2) \\ &\quad - \tfrac{1}{2}\ln((c'^2_0 r_1 - 2M'_0 G'_0/\hat{R}'^3) + \tfrac{1}{2}\ln(r_1) \end{aligned}$$

and comparing this at at the *conventional present moment*, T_0' and at a moment, T':

$$\frac{\ln(W_{k0}'(r_1')/W_{k0}'(r_2'))}{\ln(W_k'(r_1')/W_k'(r_2'))}$$

$$= \frac{\tfrac{1}{2}\ln((c'^2_0 r_2 - 2M'_0 G'_0) - \tfrac{1}{2}\ln((c'^2_0 r_1 - 2M'_0 G'_0)}{\tfrac{1}{2}\ln((c'^2_0 r_2 - 2M'_0 G'_0/\hat{R}'^3) - \tfrac{1}{2}\ln((c'^2_0 r_1 - 2M'_0 G'_0/\hat{R}'^3)}$$

There is no simple property: it is a complex function of r_1' and r_2' and $\hat{R}$ - and the mass, M'_0. The failure of simple *logarithmic symmetries* of polynomial or power functions like k can be traced to the fact that if A and B are terms with the same physical dimension (e.g. length), the term: A/B is dimensionless, but: $\ln(A/B) = \ln(A) - \ln(B)$ is not dimensionless – it has the dimension of A and B. The log of a power function of a dimensionless term like: $2MG/c^2r$ is not dimensionless in turn, and a dependence on the transformation of dimensional quantities generally arises.

The log of an *exponential function of a dimensionless term* gives us back a dimensionless term, e.g. $ln(exp(MG/c^2r_2)) = MG/c^2r_2$ which is still dimensionless. This is really why a function like k is unsuited in the present theory, whereas the exponential function K has the right symmetry properties. We note a few more symmetry properties.

The ratio of the logarithms at two different points.

The ratio of the logarithms at two different points is:

$$\frac{ln(W'(r_1'))}{ln(W'(r_2'))} = \frac{(1/r_1' - 1/\pi R')}{(1/r_2' - 1/\pi R')}$$

$$= \frac{(\pi R'/r_1' - 1)}{(\pi R'/r_2' - 1)}$$

$$= \frac{r_2'\,(\pi R' - r_1)}{r_1'\,(\pi R' - r_2')}$$

The 'reflection' *of r' in πR'*.

If we define: $r'^* = (\pi R' - r')$, the 'reflection' *of* r' *in* $\pi R'$, then:

$$\frac{ln(W'(r'))}{ln(W'(r'^*))} = \frac{r'^{*2}}{r'^{\,2}}$$

This quantity is invariant in an expanding universe for a *commoving point*, i.e. a point defined at a certain fixed ratio of $\pi R'$, i.e. where: $r' = r_0'\hat{R}$. Again of course it is a symmetry of the log transformation of W'.

Transforming *r'* to an exponential scale.

Define a coordinate transformation that stretches r' exponentially:

$$q' = exp(1/r') \qquad Q' = exp(1/\pi R')$$

$$1/r' = ln(q') \qquad 1/\pi R' = ln(Q')$$

and define:

$$P_\infty' = M'_\infty G'_\infty/c'_\infty{}^2 \qquad\qquad P_0' = M'_0 G'_0/c'_0{}^2$$

Then:

$$\begin{aligned} W'(r') &= W_\infty' \exp((M'_\infty G'_\infty/c'_\infty{}^2)(1/r' - 1/\pi R')) \\ &= W_\infty' \exp(\ln(q') - \ln(\pi R'))^{P'} \\ &= W_\infty' q'^{P_\infty'}/Q'^{P_\infty'} \end{aligned}$$

So the ratio of W' at two different points: $1/r_1' = \ln(q_1')$ and: $1/r_2' = \ln(q_2')$ is:

$$W'(r_1')/\, W'(r_2') = (q_1'\,/\,q_2')^{P_\infty'}$$

Compression of *W* with expansion of *R'*.

$$\begin{aligned} W'(r') &= W'_\infty \exp((M'_\infty G'_\infty{}'/c'_\infty{}'^2)(1/r' - 1/\pi R')) \\ &= W_0'/\hat{R}' \exp(((M_0/\hat{R}')\, G_0\,/\,(c_0/\hat{R}')^2)(1/r' - 1/\pi R')) \\ &= W_0'/\hat{R}' \exp((M_0 G_0/c_0{}^2\hat{R}'))(1/r' - 1/\pi R')) \end{aligned}$$

Thus W' compared to the original W_0' is reduced by a *scale factor,* $1/\hat{R}'$, since the micro-dimension is reduced overall by this factor, and then compressed in the exponential factor by $1/\hat{R}'$.

Compression of divergence of *W'* with expansion of *R'*.

The *divergence* of W' at a point r' is:

$$dW'/dr' = -W' M_0 G_0/c_0{}^2\hat{R}' r'^2$$

Let us determine the radius, call it r_2', that gives the same value of dW'/dr' as at an original radius r_1'. This requires:

$$-W_1 M_0 G_0/c_0{}^2\hat{R}_1 r_1' = -W_2' M_0 G_0/c_0{}^2\hat{R}_2' r_2'^2$$

So:

$$W_2' R_1'\,/\,R_2' W_1' = r_2'^2/\,r_1'^2$$

But: $R'/W' = D'/2\pi$ so:

$$r_2'^2 / r_1'^2 = D_1/D_2'$$

Or equivalently, since: $D' = D_0\hat{R}'^2$:

$$r_2' / r_1' = R_1/R_2'$$

or more simply:

$$r_1' = r_0'/\hat{R}_1$$

where r_0' is for the original *present system.* This shows a scale symmetry property of the *divergence of W'*. It means that as we stretch the universe out in R', the divergence of W maps back onto its original values through the linear transformation: $r_0' \rightarrow r_0'/\hat{R}$.

This is the same as the transformation: $r_0' \rightarrow r_0'\hat{W}$. If we take W_0 as our fixed length scale, then *the divergence at a fixed point r does not change.* In fact this is our *conventional distance scale,* in which we have: $dW'(r)/dr = d/W_0'(r)/dr$, independent of R.

Comment.

We need a strain function that is concave, because the energy of the wave of the mass M' is physically stored by stretching the manifold outwards at a point. For any smooth concave function, the domain axis transformation mapping the divergence back to its original value must be compressive, i.e. compress the domain axis. This *linear compression* is about the simplest such function.

This is a type of *logically defined scale symmetry*. It is a *logically defined* symmetry in the sense that *there are no physical constants (or special points) in the divergence function.* The transformation is defined using only the *dimensionless ratio:* $\hat{R}$, which is a fundamental property of the manifold. It is independent of the *mass,* the values of the constants, or any other properties such as the speed of expansion of the universe. It is a purely logical spatial property. This gives us an initial visualisation of the symmetry. We now turn to a second way of viewing this symmetry that illustrates this as the result of a *dimensionless construction*.

36.19 Dimensionless construction of the strain function.

The fundamental feature of the geometric theory emphasised from the beginning is that *every property of the manifold is determined by a few dimensionless ratios, defined from the topology.* We saw this in detail in previous sections. Now introducing the gravitational strain gives us a new dimensionless property not previously considered: the *divergence, dW'/dr',* is dimensionless. If it is a new property then it should give rise to some new physical quantity not yet found in physics. If it is instead a logical feature of the manifold, then it should be logically determined by other dimensionless ratios. We now show this.

There are three other relevant dimensionless ratios we can construct that are related to *dW'/dr'*. First, *D*, the ratio of: $2\pi R'/W'_\infty$. Second, M'_∞/m'_∞, the *mass ratio* for the mass term in the *W'* function. And third, $r'^2/2\pi R'W'_\infty$. We know the *mass* must enter into the relationship since *W'* is the strain generated by a given mass. We know *r'* must enter because *dW'/dr'* is the strain *at radius r',* so *r'* must be relevant. We know *W'* must enter because the divergence in question is *at W'.* We know that *2πR'* must enter because *W'* is scaled depending on how much the universe is 'stretched'. We know that *D* must enter because the strain is scaled by this ratio, i.e. by: $2\pi R'/W'_\infty$. The relationship for the *gravitational strain W'* turns out to be simply this:

$$dW'/dr' = (2\pi R'W'/r'^2)(M'_\infty/m'_\infty)(1/D^2) \qquad \textbf{With } \boldsymbol{R, D}$$

Using: $D = 2\pi R'/W'_\infty$, this is equivalent to:

$$dW'/dr' = (W'_\infty W'/r'^2)\ (M'_\infty/m'_\infty)\ (1/D) \qquad \textbf{With } \boldsymbol{W_\infty, D}$$

and to:

$$dW'/dr' = (W'_\infty W'/r'^2)\ (M'_\infty/m'_\infty)\ (W'_\infty/2\pi R') \qquad \textbf{With } \boldsymbol{R, W_\infty}$$

Exercise: show this determines the strain function defined previously.

To show this determines our solution, just substitute fundamental constants (from the cosmological theory) for D, R and W_∞ and rearrange:

$$
\begin{aligned}
dW'/dr' &= (2\pi R'W'/r'^2)(M'_\infty/m'_\infty)(1/D^2) \\
&= (W'_\infty W'/r'^2)\,(M'_\infty/m'_\infty)\,(1/D) \\
&= (W'/r'^2)\,(h'_\infty/c'_\infty m'_\infty)\,(m'^2_\infty G'_\infty/h'_\infty c'_\infty)(M'_\infty/m'_\infty) \\
&= W'M'_\infty G'_\infty/c'^2_\infty r^2
\end{aligned}
$$

Hence this simple choice of a relationship based on *dimensionless ratios* determines the divergence of our strain function as: $dW'/dr' = W'M'_\infty G'_\infty/c'^2_\infty r^2$. We have seen that integrating this determines our exact solution for W', given the boundary condition that: $W'(\pi R') = W'_\infty$. Or we could just solve the equation in terms of fundamental ratios directly: $dW'/dr' = (W'_\infty W'/r'^2)(M'_\infty/m'_\infty)(1/D)$. We can derive the strain equation directly in various forms, using the *mass ratio:* $M'_\infty/m'_\infty = N$, the *radial variable,* r, and any two of $\{R', W'_\infty, D\}$.

$$
\begin{aligned}
W'(r') &= W'_\infty \exp((M'_\infty/m'_\infty)(W'_\infty/D)(1/r' - 1/\pi R')) && \textbf{With } \boldsymbol{R', W'_\infty, D} \\
&= W'_\infty \exp((NW'^2_\infty/2\pi R')\,(1/r' - 1/\pi R')) && \textbf{With } \boldsymbol{R', W'_\infty} \\
&= W'_\infty \exp((N/D^2)(W'_\infty D/r' - 2)) && \textbf{With } \boldsymbol{W'_\infty, D} \\
&= W'_\infty \exp((N/D^2)(2\pi R'/r' - 2)) && \textbf{With } \boldsymbol{R', D}
\end{aligned}
$$

But it is most convenient to convert these into terms of *present variables,* R_0', D_0, and the normalised radius, $\hat{R}'$:

$$
W'(r') = (W'_0/\hat{R}')\exp((N/D_0^2\hat{R}'^3)(2\pi R_0'/r' - 2/\hat{R})) \qquad \textbf{With } \boldsymbol{\hat{R}', R_0', D_0}
$$

This is generally the best way to write the equations, because the terms W'_0, D_0, R_0 are genuinely fixed constants, and all the dynamics is collected in the terms of $\hat{R}$.

This shows how tightly the *strain function* is bound up with the cosmological theory.

However it still only determines the solution for *W'* within alternative choices of simple combination of dimensionless ratios. We could multiply our original relationship by a factor of *D*, or a factor of α, the fine structure constant. In fact, these give us other possible forces. One is the form of the electromagnetic force, and two others should represent the strong and weak nuclear forces. These are dealt with elsewhere.

But why is the gravitational strain defined by the specific choice given here, and not these others? The reason for the gravitational strain function is ultimately bound up with the representation of *mass energy in the dimensional subspace.*

36.20 The volume integral of K.

The 6-D volume V_6 of the manifold is given by the integral of *dV* over the hyper-sphere, where *dV* is the 6-D differential volume element, and equal to the *spherical shell element times the micro-torus volume* at each point *r:*

$$dV'(r') = (4\pi R'^2 \sin^2(r/\pi R'))\ 2\pi^2(W'(r')/2\pi)^3\ dr'$$

[spherical shell] **[micro-torus]**

Note that *r* moves over the 3-D *hyper-surface,* not into the 6-D radius *R'.*
Rearranging, all the numerical constants cancel:

$$dV'(r') = R'^2 \sin^2(r/\pi R')\ W'(r')^3\ dr'$$

To cover the whole manifold this should be integrated over: $r = 0$ *to* $\pi R'$:

$$V' = \int_{r = 0 \text{ to } \pi R'} R'^2 \sin^2(r/\pi R') W'(r')^3\ dr'$$

In fact, for the mass function: $W'(r') = W'_0K$, there is a singularity at $r' = 0$, and the strain function actually physically ends at a small radius, $r+$, rather than at $r = 0$, so we limit the lower bound of the integral to $r+$. This is noted below. But when $W'(r')$

is uniformly flat, so that: $W'(r') = W'_\infty$ at all points, we integrate over the whole space, and have the volume:

$$
\begin{aligned}
V'_{Flat} &= R'^2 W_\infty{}^3 \int_{r=0 \text{ to } \pi R'} \sin^2(r/R')\, dr' \\
&= \tfrac{1}{2} R'^2 W'_\infty{}^3 \,[r - \pi R' \sin(r/\pi R')\cos(r/\pi R')]_{0 \text{ to } \pi R'} \\
&= (\pi/2) R'^3 W'_\infty{}^3 \\
&= (\pi/2) L_0{}^6
\end{aligned}
$$

where L_0 is defined as: $\sqrt{(R'W'_\infty)}$, the invariant quantity of *length*.

If we insert a mass $M'_\infty = Nm_\infty = N(m_e m_p{}^2)^{1/3}$, we get the strained W':

$$W' = W'_\infty \exp((NW'_\infty{}^2/2\pi R')\,(1/r' - 1/\pi R'))$$

Using: $N = M'_\infty/m'_\infty$ for the *mass ratio*, the volume is taken as the integral:

$$
\begin{aligned}
V'(N) &= \int_{r = r+ \text{ to } \pi R'} R'^2 \sin^2(r/\pi R')\, W'_\infty{}^3 \,(\exp((NW'_\infty{}^2/2\pi R')(1/r' - 1/\pi R')))^3 dr' \\
&= R'^2 W'_\infty{}^3 \int_{r = r+ \text{ to } \pi R'} \sin^2(r/\pi R') \exp((3NW'_\infty{}^2/2\pi R')(1/r' - 1/\pi R'))\, dr'
\end{aligned}
$$

We explain the lower bound: $r = r+$ before continuing.

36.21 The singularity at r = 0.

There is a singularity in W' at $r = 0$, where the strain function diverges to infinity. We examine this further when we look at gravitational holes. We take the meaningful volume integral from a lower bound close to 0, called r^+. For in the physical model, r never goes to 0, because when r becomes small enough so that: $W' \rightarrow R'$, it encounters the center of the hyper-sphere. It turns into a very thin ‘string’ connecting across the inside of the hyper-sphere. In the full theory, its continuation evolves into a reflection, connecting to an identical particle on the opposite side. This is the physical limit of the volume. This happens when:

$$W'_\infty \, exp((NW'^2_\infty/2\pi R')\,(1/r^+ - 1/\pi R')) = R'$$

So that:

$$1/r+ \;\; = ln(R'/W'_\infty)(2\pi R'/NW'^2_\infty) + 1/\pi R'$$
$$\approx ln(R'/W'_\infty)(2\pi R'/NW'^2_\infty)$$

The approximation is valid when R'/W'_∞ >> 1, which is true after the very early 'Big Bang'. So we take:

$$r+ \quad \approx \; NW'^2_\infty/(2\pi R') \,/\, ln(R'/W'_\infty)$$
$$= \; (NW'_\infty/D) \,/\; ln(D/2\pi)$$

In the universal constants, this is:

$$r+ \quad \approx \; (M'_\infty G'_\infty/c'^2_\infty) \,/\, ln(h_\infty c'_\infty/4\pi m^2_\infty G'_\infty)$$

At present, this is approximately 1/100th the normal black-hole event horizon for the mass M' (the Schwarzschild radius at: $r = 2MG/c^2r$).

36.22 Approximate solution for volume integrals.

I have solved volume integrals numerically, and confirmed a theoretically expected functional solution, without analytically proving this yet (or only approximately). For the fundamental mass: $N = 1$, we first find numerically, to a close approximation:

$$V'_W/V'_{Flat} = (1+\beta/D^2) = (1+\beta/D_0^2\hat{R}^4)$$

Or equivalently:

$$\Delta V'_W = V'_W - V'_{Flat} = \; V'_{Flat}\,\beta/D^2 = \; V'_{Flat}\,\beta/D_0^2\hat{R}^4$$

where β is a numerical constant of integration. β is very stable as R' changes, but reduces slightly with increasing R, and appears to converge to πe as D increases. At $D \approx 10^8$, $\beta \approx 1.003\pi e$. We can use πe as an approximation, and this reminds us that β is a number obtained logically as an integration constant, not an extra physical constant. It should emphasised that this is *for a single mass: $N = 1$.*

However this linear relationship is only an approximation to an exponential relationship:

$$V'_W/V'_{Flat} = exp(\beta/D^2) = exp(\beta/D_0^2\hat{R}^4)$$

Then for a multiple mass, *N*, the relationship is:

$$V'_W/V'_{Flat} = exp(N\beta/D^2) = exp(N\beta/D_0^2\hat{R}^4) = exp(\beta/D_0^2\hat{R}^4)^N$$

This is determined by the superposition principle, by recursively applying *N* masses to the flat space. The series expansion is:

$$exp(N\beta/D^2) = 1 + N\beta/D^2 + (N\beta/D^2)^2/2! + (N\beta/D^2)^3/3! \ldots$$

It is approximately linear in *N* when $N\beta/D^2 << 1$, and becomes non-linear as $N\beta/D^2 \rightarrow 1$. When $N \rightarrow D^2/\beta$, we have: $V'_W/V'_{Flat} = e$.

When $N\beta/D^2 << 1$, (remembering $D^2 \approx 10^{80}$ for the observable history of our universe), we have to a very close approximation:

$$V'_W = V'_{Flat}\, exp(N\beta/D_0^2\hat{R}^4) \approx V'_{Flat}\,(1+N\beta/D_0^2\hat{R}^4)$$

So the *volume difference* from adding *N* masses is, to a very close approximation:

$$\Delta V'_W = V'_W - V'_{Flat} \approx V'_{Flat}N\beta/D_0^2\hat{R}^4$$

In fact, if the masses were added *at different places,* widely separated in the manifold, this is more accurate. When the masses are added *close together,* the volume is slightly increased, by the extra terms in the exponential series. This slight increase should represent the negative potential energy of the gravitational field – transferred to kinetic energy when two masses fall towards each other.

Exercise: show the general relationship for multiple mass *N* assuming the simple linear relationship instead of the exponential relationship.

The general relationship for mass N consistent with the linear relationship noted initially is:

$$V'_W/V'_{Flat} = (1+\beta/D^2)^N$$

Or:

$$\Delta V'_W = V'_W - V'_{Flat} = V'_{Flat}\,((1+\beta/D^2)^N - 1)$$

This is obtained from the *superposition principle.* If we imagine adding one mass ($N = 1$) first, we get a space with volume: $V'_W = V'_{Flat}\,(1+\beta/D^2)$. We can *superpose* a second mass on this using the recursive superposition principle: $V''_W/ = V'_W\,(1+\beta/D^2) = V'_{Flat}\,(1+\beta/D^2)^2$. Adding N *masses* gives $V'_W/V'_{Flat} = (1+\beta/D^2)^N$. We can expand $(1+\beta/D^2)^N$ as the binomial series:

$$(1+\beta/D^2)^N = 1 + N\beta/D^2 + N(N-1)(\beta/D^2)^2/2! + N(N-1)(N-2)(\beta/D^2)^3/3!\ \ldots$$

For large D and small N this is dominated by the linear term: $(1+\beta/D^2)^N \approx 1 + N\beta/D^2$. Thus in the first order, with large D, adding extra masses increases the volume almost linearly. When $N \rightarrow D^2/\beta$, since: $ln(1+\beta/D^2) \approx \beta/D^2$, we have: $ln(V'_W/V'_{Flat}) \approx 1$, or: $V'_W/V'_{Flat} \approx e$, which departs from the linear approximation, of: $V'_W/V'_{Flat} \approx 2$.

This solution is numerically similar to the exponential solution for small masses, but it lacks the right symmetry properties to work in the broader theory.

36.23 The volume distribution across r’.

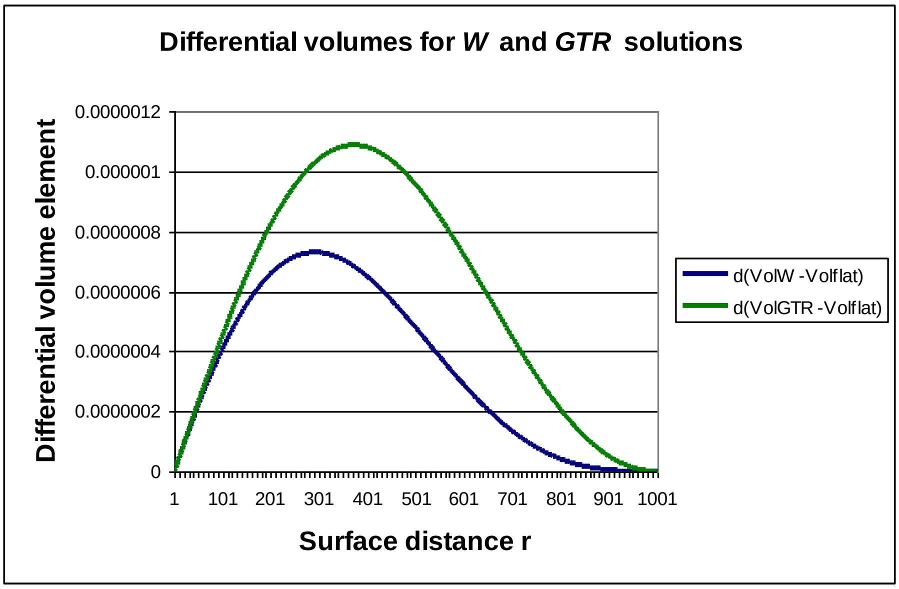


Figure 27. *Volume element density around $\pi R'$ for a mass, using the W function K (blue), and the GTR function k (green).*

This illustrates how the volume increase due to a single mass is distributed across r', for the $W' = KW'_\infty$ strain function, and the kW'_∞ (GTR) function. The scale of this model is: $R = 1{,}000$. The distribution shapes are essentially independent of scale as R' expands. The total volume increase using the GTR function k is about *1.7* times that from the exponential function K.

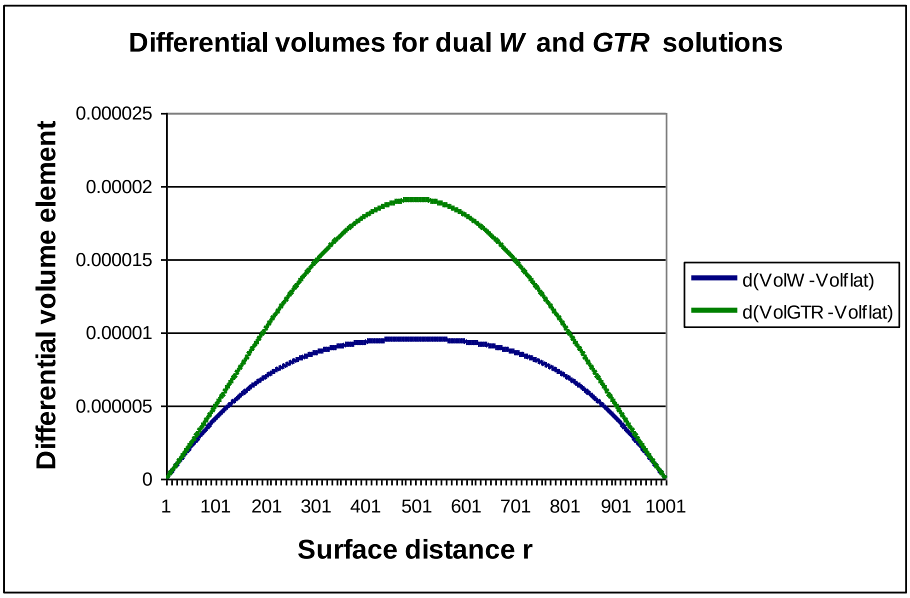


Figure 28. *Volume element density around $\pi R'$ for dual mass, using the W function K (blue), and the GTR function k (green).*

This illustrates how the volume increase due to a *dual mass system* is distributed across r', for the $W' = KW'_\infty$ strain function, and the $kW'\infty$ (GTR) function. The scale of this model is $R = 1{,}000$. The *dual mass system* has a mass at $r = 0$, and the same mass at $r = \pi R'$, on the opposite side of the hyper-sphere. Or more generally, a mass at a surface point, $\boldsymbol{R}$, and its reflection at $-\boldsymbol{R}$. This relates to the third section, where we impose a reflection symmetry on the universe. Hence, every particle has a reflected particle, and the total mass volume of both needs to be multiplied to get the real effect of inserting a particle. The function is symmetric w.r.t. reflection through the equator at $\pi R'/2$. The dual mass has the effect of making the hyper-sphere slightly ellipsoid. Note the *divergence* of the dual mass function close to either particle is extremely close to the divergence of the single particle system.

The distributions are essentially independent of scale as R' expands. The total volume increase using the GTR function k is about 1.7 times that from the exponential function K.

36.24 The rate of volume change.

The important thing is the functional dependence of V'_W on $1/D^2$. Given: $V'_W = V'_{Flat}exp(N\beta/D_0^2\hat{R}^4)$, the change of volume with spatial expansion is negative:

$$d(V'_W)/d\hat{R} = d(\Delta V'_W)/d\hat{R} = (-4N\beta/D_0^2\hat{R}^5)V'_W$$

To the previous very close approximation for $\Delta V'_W$ this is:

$$d(\Delta V'_W)/d\hat{R} \quad = -4\Delta V'_W/\hat{R}$$

In the physical picture, the manifold increases its surface tension with the expansion, and *compresses* the extra volume created by the strain of the mass. If we think of a volume of ordinary gas, increasing the pressure linearly decreases the volume: $PV = nRT$. But the manifold is *not* like this: the bulk volume of ‘flat space’ remains constant with expansion, although the surface tension increases. For an ordinary liquid, like water, placing it under pressure very slightly decreases the volume, and some energy is stored mechanically in the compressed water. The flat manifold is not like this either: it does not compress any further. It is like a substance that is already *completely compressed* – or rather, in its lowest possible energy state. But the extra volume representing a mass added to the flat manifold *does* compress further, as the surface tension increases. This volume decrease should correspond to the mass-energy increase as *R'* expands. Remembering that:

$$E' = M_0 c_0^2 \hat{R}'$$

This means that the *mass energy of the mass M increases with expanding R'*. (In the expansion solution for *R'*, this is precisely balanced by the deceleration of the expansion speed). In the gravitational picture, we can visualise the mass-energy of *M* is *physically stored* in the extra strain (distortion) of the manifold. Hence:

$$E'\, d(\Delta V'_W)/d\hat{R} = -4\Delta V'_W\, M_0' c_0'^2$$

Exercise: compare with the rate of volume increase on the linear approximation.
Given: $\Delta V'_W = V'_{Flat}\, \beta/D_0^2\hat{R}^4$, the rate of volume increase for a single mass is exactly:

$$\begin{aligned} d(\Delta V'_W)/d\hat{R} &= -4V'_{Flat}\, \beta/D_0^2\hat{R}^5 \\ &= -4\Delta V'_W/\hat{R} \end{aligned}$$

But for a multiple mass *N*, this is:

$$\begin{aligned} d(\Delta V'_W)/d\hat{R} &= (d/d\hat{R})(V'_{Flat}(1+\beta/D_0^2\hat{R}^4)^N - V'_{Flat}) \\ &= -(4N/\hat{R})\, V'_{Flat}(\beta/D_0^2\hat{R}^4)(1+\beta/D_0^2\hat{R}^4)^{N-1} \end{aligned}$$

This departs significantly from the relationship: $d(\Delta V'_W)/d\hat{R} = -4\Delta V'_W/\hat{R}$. The appearance of multiple powers in $\hat{R}$ reflect the lack of appropriate symmetry in this model.

Exercise. Confirm the relationship analytically.

Note that although we cannot integrate the volume analytically, we can differentiate the volume element dV directly by $d/d\hat{R}$ to confirm that the main dependency should give the relationship: $d(V'_W)/d\hat{R} \approx -4\Delta V'_W/\hat{R}$. This involves quite a bit of algebra: not added in this draft.

Note we can consider this as a time differential instead of a space differential. At a time T': $D = D\hat{R}'^2$, and the time differential is:

$$d(\Delta V'_W)/dt' = dD/dt'\ (-2V'_{Flat}\,\beta/D^3)$$
$$= -4\ d\hat{R}'/dt'\ (V'_{Flat}\,\beta/D^2)$$
$$= -4\Delta V'_W\ d\hat{R}'/dt'$$

This is opposite to the time evolution of the *mass energy:*

$$dE'/dt' = M_0'c_0'^2\ d\hat{R}'/dt' = E_0'd\hat{R}'/dt'$$

Thus we can equate the rate of *particle energy evolution* with the negative rate of *particle volume evolution:*

$$dE'/dt' = -(E'/4\Delta V'_W)\ d\Delta V'_W/dt'$$

Or:

$$d\Delta V'_W/dE' = -4\Delta V'_W/E'$$

36.25 Energy and pressure in the manifold.

Ordinary pressure is *force per unit of surface.* This is equivalent to *energy per volume.* The ideal gas relation: $PV = NRT$ has units of energy. In our manifold, the volume is six dimensional, and we have the analogue of *pressure* as:

$$P' = E'/V'$$

If we take the pressure for the flat manifold, we have:

$$P'_{flat} = E'_{flat}/V'_{flat}$$

We then add a mass *m'*, and get:

$$
\begin{aligned}
P'_W &= (E'_{flat} + m_0'c_0'^2)/V'_W \\
&= (E'_{flat} + m_0'c_0'^2)/(V'_{flat} + \Delta V') \\
&\approx E'_{flat}(V'_{flat} - \Delta V')/V'^2_{flat} + m_0'c_0'^2/V'_{flat} \\
&= E'_{flat}/V'_{flat} - E'_{flat}\Delta V'/V'^2_{flat} + m_0'c_0'^2/V'_{flat} \\
&= P'_{flat} + m_0'c_0'^2/V'_{flat} - E'_{flat}\Delta V'/V'^2_{flat}
\end{aligned}
$$

If we assume the *pressure* is invariant, so that: $P'_W = P'_{flat}$, then we have:

$$m_0'c_0'^2 = E'_{flat}\Delta V'/V'_{flat} = E'_{Flat}\,\beta/D^2$$

If we identify the initial energy as the mass energy: $E'_{Flat} = M'c_0'^2$ then we have:

$$M' \approx m_0'D^2/\beta \approx m_0'D^2/\pi e$$

Or equivalently, the number of fundamental mass particles is:

$$N' \approx M'/m_0' \approx D^2/\pi e$$

This is the simplest way to derive a relationship of the form: $N' \approx D^2$ for the particle number of the universe. Such a relationship is hard to avoid at least speculating, because it is on the right general scale for our universe, and Dirac thought there was a relationship of this kind. (It has sometimes been appealed by steady-state theorists to propose that matter is continually created.) However, I briefly explain the difficulty in this relationship.

For D_0 in our own present time, this would give: $N' \approx 2 \times 10^{82}/8 \approx 10^{81}$ fundamental particles. There are two issues with this. First, although the number is about the right scale, it is a bit large. The real number is estimated at around: $N' \approx 10^{78}$ *to* 10^{82}, but in the geometric model, the lower estimate should be more accurate (the universe being predicted to have about ¼ of the radius estimated on the conventional model, with about 1/64th the volume). Thus this relationship would predict N' around 1,000 - 10,000 times too large. Second, it implies that the number of fundamental particles increases with D^2. But this does not seem empirically plausible today, and it does not fit with the cosmological model we have adopted here, where the particle number is essentially stable.

In the geometric model, the natural assumption is instead that:

- The particle number N' was fixed, along with the mass ratio, after the Big Bang, when the main particle creation occurred.

This should be at the time when the radiation decoupled from matter. Before this time, matter was freely created from radiation energy: photon energies were on the scale of electron and proton particle energies. The universe at this time is thought to be have been about 1,000 times smaller than its present size, in the conventional model (the last scattering shell for the CMB). If we take the value 1/1,000 for $\hat{R}'$, then $D \approx 10^{35}$, so $N' \approx 10^{70}$ which is far too small. If we take the value 1/1,000 for $\hat{R}$, but use what should be the correct relation: $\hat{R}' = \sqrt{\hat{R}}$, then: $N' \approx 10^{77}$, which is close to what we expect.

But it is not clear what the correct assumptions about this are yet. The particle number may also have continued to multiple substantially after the last scattering, and may have increased by another factor of 10 in the early hot universe. The assumption that the *particle number is related to* D^2 *in the main phase of particle creation* is difficult to avoid, and it does indicate a plausible prediction for the total particle numbers, allowing the universe has a smaller conventional radius on the geometric model than in conventional Big Bang cosmology.

36.26 Total energy stored in the manifold.

To be completed.

36.27 Extra exercise: the strain function and the evolution of mass energy.

We may ask why we couldn't we adopt a different shape for W'? For instance, where A and B are positive:

- A *straight line*: $W = -Ar + B$ $\quad dW/dr = -A$
- A *reciprocal*: $W = A/r + B$ $\quad dW/dr = -A/r^2$
- A *reciprocal quadratic:* $W = A/r^2 + B$ $\quad dW/dr = -2A/r^3$
- A *reciprocal log*: $W = A/ln(r) + B$ $\quad dW/dr = - A/ln(r)^2 r$
- A *k function*: $W = W_\infty\ (1-A/r)^{-1/2}$ $\quad dW/dr = -WA/k^2r^2$
- A *k function*: $W = W_\infty\ (1-A/r +A/\pi R)^{-1/2}$ $\quad dW/dr = -WA/k^2r^2$

Note the divergence must be negative, and remain finite as r increases to the limit at R. Boundary conditions on the real solution include:

(i) $W'(\pi R') = W'_\infty$

(ii) $R'W'_\infty = R'W'_\infty = L_0{}'^2$ *(constant as R changes)*

(iii) $d/dt' \int_{space} dV_W' = -\beta dE'/dt'$ *(energy conservation)*

(iv) A and B contains only terms of R', W'_∞, D and M'_∞/m'_∞.

The first *k function* cannot satisfy the first BC, but the second *k function* is possible.

The second BC is a special case of a third BC: *the total volume of flat space remains constant as R changes*. The third condition means that *the rate of change of the mass volume is proportional to the negative rate of change of the mass energy*. This gives a critical extra condition on the function for W'. The fourth BC is the functional condition.

Exercise: solve an example for the linear function.

To illustrate, let us solve for a *straight line solution.* $W = -Ar + B$, so: $dW/dr = -A$, i.e. the divergence is constant. The first BC means: $W_\infty = -A\pi R' + B$, or: $B = W\infty + A\pi R'$. Hence:

$$W = -A(r - \pi R') + W_\infty$$

The second BC means that $W_\infty R'$ is constant. We just have to determine *A (or B).*

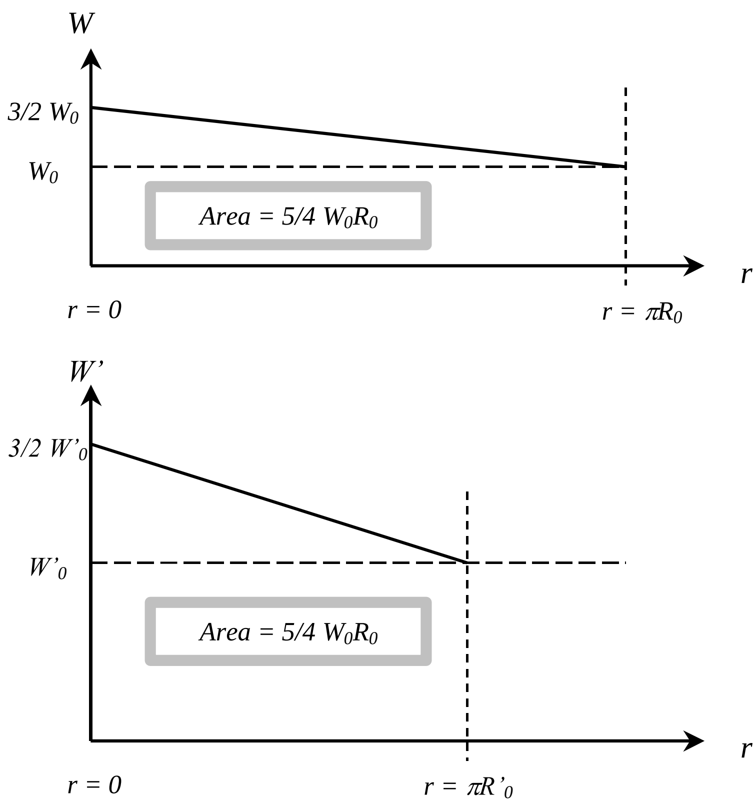


Figure 29. *A linear strain function, as a 2-D universe changes radius.*

Without the third BC, there are different possible choices for *A.* But once we have imposed a condition on the *volume integral*, choices are limited. To illustrate, let us choose a *volume conservation* principle equivalent to area conservation, so the total area remains constant. Thus:

$$dV = Wdr$$

Hence:

$$\begin{aligned} V &= \int_{(r=0 \text{ to } \pi R')} -A(r - \pi R') + W_\infty \, dr \\ &= [-Ar^2/2 + (A\pi R' + W_\infty)r\,]_0^{\pi R} \\ &= -A\pi^2 R'^2/2 + A\pi^2 R'^2 + W_\infty \pi R' \\ &= A\pi^2 R'^2/2 + W_\infty \pi R' \end{aligned}$$

Hence:

$$A\pi^2 R'^2/2 = Constant * W_\infty R'$$

So:

$$A = Constant * W_\infty R'/R'^2$$

$$= Constant * W_\infty/R'$$

The simplest choice for the *Constant* is $1/2\pi$ to give:

$$A = W_\infty/2\pi R' = 1/D$$

Giving:

$$W = -r/D + \pi R'/D + W_\infty$$

Thus we see that the *volume conservation*, or third BC, determines in this case that the dimensionless factor *A* must be a multiple of 1/*D*. This satisfies the BCs above, but it still needs a mass term, which we could add as:

$$W = (M/m)(-r/D) + (M/m)(\pi R'/D) + W_\infty$$

This of course does not generate a realistic *gravitational acceleration* – shown by the fact the acceleration function is not linear with mass, and tends to *1/r*.

$$(c^2/W)\, dW/dx = c^2(M/m/D)/[(M/m)(-r/D) + (M/m)(\pi R'/D) + W_\infty]$$
$$\approx c^2/r \qquad \textbf{At small } \boldsymbol{r}$$

This is wildly wrong as a model for gravity - but it satisfies the BCs we set for the *strain function.* However, this is only because we chose a simplistic volume conservation principle, i.e. conserving the *area integral:* $\int dV = \int W dr$, to make a simple example. This is *not* the volume integral condition for the *geometric manifold* that must be satisfied. The real volume integral element is six dimensional:

$$dV' = 4\pi(R'/\pi)^2 \sin^2(\pi r/R')\, 2\pi^2 (W'/2\pi)^3\, dr$$
$$= (R'^2/\pi^2)\, W'^3 \sin^2(\pi r/R')\, dr$$

It is impossible to make *this* volume invariant (w.r.t. changing *R')* using the linear function for *W'*. For in this example:

$$W^3 = (-Ar + B)^3 = -(Ar)^3 + 3(Ar)^2B - 3ArB^2 + B^3$$

It is seen to be impossible for the integral to be invariant (without having to do the full integration) because the definite integral must be a sum of multiple different powers in *R'* and *A*. *A* must be dimensionless, and thus a power of *D*. The only combinations that are *invariant* are powers of: $R'W_\infty$. This is equivalent to powers of: R'^2/D. Consider one term in the integral, say the: A^3r^4 term, then we need: $A = 1/D^{2/3}$. Consider another term, say the A^2r^3 term, then we need: $A = 1/D^{1/2}$. And so on.

This happens in the first three examples of functions above, because there are multiple terms with different powers in *r* in the integral, but only one degree of freedom in the function *W*, viz. the parameter *A*. The only kind of polynomial functions that can satisfy such a condition are infinite series of a single dimensionless term. The (Taylor expansion of) the *exponential function* is an infinite series, and so is the *k* function, so either of these might satisfy the conservation principle for the volume integral. We next consider the real volume integral for *W'*.

36.28 Deriving the speed function.

The second assumption to justify the gravitational model in 36.15 is (ii) that the pure radial velocity is: $c_r = c_w K = c_\infty K^2$. Why should travelling towards a diverging *W* have the effect of slowing the speed by an additional factor *K?* Without this assumption, we could obtain a simple metric: $c = c_\infty{}^2/K^2 = (dr^2 + dy^2 + dz^2 + dw^2)/dt^2$, but it would consequently depart from GTR for radial light beams - it would halve the gravitational red shift of light predicted by GTR. It is justified by an analysis of the Lorentz transformation of the manifold.

To be completed.

36.29 Gravitational holes.

The geometric model rejects GTR 'black holes' as the result of extrapolating a theoretical model beyond its limits. But it predicts gravitational holes in the generic sense of large-scale aggregations of matter that effectively trap particles and light –

and direct them into a direction orthogonal to ordinary space. This happens around the radius where: $dW'/dr' = -1$, which occurs when:

$$-dW'/dr' = (W'_\infty M'_\infty G'_\infty/c'^2_\infty r^2)\ exp((M'_\infty G'_\infty/c'^2_\infty)(1/r' - 1/\pi R')) = 1$$

Hence:

$$ln((W'_\infty M'_\infty G'_\infty/c'^2_\infty r^2) = -\ (M'_\infty G'_\infty/c'^2_\infty)(1/r' - 1/\pi R'))$$

Since: $1/\pi R' << 1/r'$, approximately:

$$M'_\infty G'_\infty/c'^2_\infty r' + ln((W'_\infty M'_\infty G'_\infty/c'^2_\infty) = 2\ ln(r)$$

Hence:

$$M'_\infty G'_\infty/c'^2_\infty = 2r'(\ ln(r) - ln(M'_\infty) + 94.224)$$

For a typical star, with mass around 10^{32} kg, this means:

$$M'_\infty G'_\infty/c'^2_\infty \approx 2r'(\ ln(r) + 20)$$

So: $r' < r_{GTR}/80$, and in this case, $r' \approx r_{GTR}/100$. The GTR event horizon is instead:

$$r_{GTR} = 2MG/c^2$$
$$= M\ 1.4851E\text{-}27$$

Given an average star has a mass of around 10^{32} kg, the GTR event horizon for this mass would be about 10 km. The geometric model boundary would instead be at around 100 meters. It is impossible for stars to compact to such small radii. However, in the geometric theory, every fundamental particle actually is a 'gravitational hole', and this is the string radius for a *fundamental particle with the mass of a sun.*

36.30 Empirical differences between Model Gravity and GTR Gravity.

E.g. in Earth gravity, M_0G_0/r^2 is about 10 m/s^2, r is about 10,000 km = 10^7 m, $c^2 \approx 10^{17}$, so for the first-order term: $M_0G_0/c^2r \approx 10^{-10}$. The second-order term is about:

$(M_0G_0/c^2r)^2/2! \approx 10^{-20}$, modifying the first order term by a factor of about 10^{-10}. Hence the second-order modification is very small. So far there are few ways to experimentally detect such a fine departure from GTR. Equally, there are no ways to study strong gravitational fields directly. Even for gravity at the surface of the sun, the strongest g-field in our solar system, the term: $2M_0G_0/c^2r$ is only about 10^{-6}, and we cannot observe free-fall experiments at such regions because the sun produces so much radiation and electromagnetic fields it disturbs any bodies in this region far more than gravity. Rather than study strong fields directly, we can only study weak fields directly *over a long period of time.*

Hence the Model factor K is generally a very small modification of the GTR factor k in almost all empirical situations we can observe. The resulting difference between Model Gravity and GTR Gravity is barely detectible. But a difference should show up in one very delicate long-time solar-system scale study, viz. the Pioneer spacecraft trajectories, which represent free-fall experiments over a period of decades. In this case, GTR makes the wrong prediction, to the consternation of NASA physicists. I have calculated the correction using the geometric theory, and it appears to account closely for the anomaly. However the analysis is a little complex, and it not been independently verified yet.

Other solar-system scale observations are just on the verge of precision to show empirical differences implied by the geometric theory. When we consider very strong fields (approaching gravitational holes), the differences between k and K become large of course. The gravitational hole predicted by the present theory is very different to the GTR 'black hole'. But no empirical observations of 'black holes' sufficient to confirm the GTR event horizon has ever been performed – claims GTR black holes exist are theoretical speculation. Hence currently the difference between using the factor k versus K in the GTR central mass solution is not clearly distinguishable except in one experiment, the Pioneer spacecraft trajectories, where GTR is wrong, and the evidence so far is that the geometric theory is correct.

39. A Unified Quantum-Gravity Wave Equation.

We have now seen two partial wave solutions, one for the complex quantum wave and one for the strain or displacement component of a mass-energy source, which produces gravity. For a fully unified *wave theory* we really want to combine the two in one wave-function. To do this however we have to interpret the *complex-valued wave function* in the physical model. The usual interpretation as carrying the *probability distribution* is discussed the third part of this paper. I just note here that if we assume real-valued displacement (strain) is the *amplitude, A, of the quantum wave* just obtained, we can write a full wave equation in a form like:

[39.1] *Prototype for the Unified Wave Equation.*

$$\Theta(x,w,t)=\frac{W_0}{2\pi}\exp\left(\frac{m_0 G}{c_0{}^2\sqrt{\left(\frac{W(x,t)}{2\pi}\right)^2+x'(t)^2}}\right)\exp\left(\frac{i}{\hbar}\left(p_x x+p_w w-Et\right)\right)$$

$$=\frac{W_0}{2\pi}\exp\left(\frac{i}{\hbar}\left(p_x x+p_w w-Et\right)+\frac{m_0 G}{c_0{}^2\sqrt{\left(\frac{W(x,t)}{2\pi}\right)^2+x'(t)^2}}\right)$$

[39.1] *Simple prototype unified wave equation.*

$$\Theta(x.w.t) = (W_0/2\pi)\ \exp(MG/c^2x + (i/\hbar)(p_x x + p_w w - Et))$$

This combines *G* with *h* in a single wave equation. The surface of *W* is ‘stretched’ by the mass-energy to the radius: $W = (W_0/2\pi)\exp(MG/c^2x)$. But what is the imaginary part of the wave? It is like a two dimensional amplitude distortion. The surface of W can be thought of as being ‘twisted’ at each point, by: $(p_x x + p_w w - Et)/\hbar$ radians.

Properties need further treatment.

SECTION 6. PREDICTION OF ELECTROMAGNETISM.

We now derive the electromagnetic force, interpret Maxwell's equations and the model for photons.

39. Electromagnetism

[To be added]

SECTION 7. POSTULATE OF PARTICLE-STRINGS.

The final major postulate is an extension of the model, dealing with the connectivity of particles across space, and the nature of quantum particles. This amounts to the proposal that each individual particle-wave on the surface has a 'worm hole' or 'string' that individuates it. The 'strings' are just the natural extension of the manifold on the very small scale, appearing at the event-horizon scale for a fundamental point particle. Each particle effectively has a 'wormhole' at its center, which emerges from the 3-D surface, extending into the center of the hyper-sphere as a very long thin 'string' of manifold, and connecting to the opposite side. We begin with conservation of momentum.

40. Conservation of Momentum.

The Cardiod solution was entailed by a simple local conservation of energy, which ensured that *total energy of the manifold is conserved* as well. Momentum also appears to be conserved locally if we work in the frame of the manifold. But momentum is not conserved locally in the expansion process when we take the motion in the extra expansion dimension into account. Take a free particle *m*, then its total speed is the vector addition of *c'* and *dR'/dt'*.

[40.1] $L' = m'c'$ [momentum of particle in local frame of manifold]

[40.2] $P' = m' \sqrt{(c'^2 + (dR'/dt')^2)}$ [momentum in the global frame]
$= m'V'$ [Definition of *V'*, 25.2]

These quantities change with the expansion as follows.

[40.3] $L' = m'c' = c_0\hat{R}'\, m_0/\hat{R}' = c_0 m_0$ [sub 14.1, 14.4 in 40.1]

Hence momentum appears constant in the local frame. However since the radial expansion is changing, the manifold surface itself is accelerating (radially), and the local momentum can only appear constant in an *accelerating frame*. It cannot be constant *in any non-accelerating frame.* Choosing the stationary center-of-the-hyper-sphere frame as the global frame, the Cardiod solution (or any similar concave part of a solution) means that d^2R'/dt'^2 is negative for all the main cycle. Using the definition: $V'^2 = c'^2 + (dR'/dt')^2$ and noting that:

$$[40.4]\quad dV'/dt' = (1/V')\,(c'dc'/dt + (dR'/dt)\,(d^2R'/dt'^2) \qquad \text{[differentiate } V'\text{]}$$

$$= (1/V')\,(c_0'^2\,\hat{R}'\,d\hat{R}'/dt + R_0'^2(d\hat{R}'/dt)(d^2\hat{R}'/dt'^2)$$

$$= (\hat{R}'_{,t}/V')\,(c_0'^2\,\hat{R}' + R_0'^2\,\hat{R}'_{,t,t})$$

Differentiating 40.2 gives:

$$[40.5]\quad dP'/dt' = d/dt'(m'\sqrt{(c'^2 + (dR'/dt')^2)})$$

$$= V'dm'/dt' + m'dV'/dt$$

$$= -V'm_0\,\hat{R}'_{,t}/\hat{R}'^2 + m_0\,(\hat{R}'_{,t}/\hat{R}'V')(c_0'^2\,\hat{R}' + R_0'^2\,\hat{R}'_{,t,t})$$

$$= m_0\,(\hat{R}'_{,t}/V')(c_0'^2) - m_0\,V'\hat{R}'_{,t}/\hat{R}'^2 + m_0\,(R_0'^2/V'\hat{R}')\hat{R}'_{,t}\,\hat{R}'_{,t,t}$$

$$= (m_0\,\hat{R}'_{,t}\,/V'\hat{R}'^2)(c'^2 - V'^2) + (m_0\,R_0'^2/V'\hat{R}')\hat{R}'_{,t}\,\hat{R}'_{,t,t}$$

$$= -m_0\,R_0'^2\hat{R}'^3_{,t}/V'\hat{R}'^2 + m_0\,(R_0'^2/V'\hat{R}')\hat{R}'_{,t}\,\hat{R}'_{,t,t}$$

$$= (m_0\,R_0'^2\hat{R}'_{,t}\,/V'\hat{R}'^2)\,(\hat{R}'\hat{R}'_{,t,t} - \hat{R}'^2_{,t})$$

The factor on the left: $m_0\,R_0'^2\hat{R}'_{,t}\,/V'\hat{R}'^2$ is non-zero unless $\hat{R}'_{,t}$ is zero. It has the same sign as $\hat{R}'_{,t}$. The factor on the right: $\hat{R}'\hat{R}'_{,t,t} - \hat{R}'^2_{,t}$ cannot generally be zero except when $\hat{R}'_{,t}$ is zero. As expected, local momentum is conserved in the global frame only in the stationary state.

This effect could be ignored, and we could abandon conservation of momentum except in the local frame. But a much stronger theory is determined if we can ensure conservation of momentum, and I propose the following symmetry to ensure this.

41. Strong Reflection Symmetry

The only obvious way to ensure global conservation of momentum appears to be to postulate an additional principle of *strong reflection symmetry*. This holds that each point on the manifold surface is perfectly reflected through the center, i.e. if: $\Psi(\boldsymbol{R'})$ =

ψ is the wave-function at a radial point ***R'***, then the point at ***-R'*** has the exact space-reflected state: $\Psi(\boldsymbol{-R'}) = \psi^R$, where ψ^R is the space-reversal of ψ.

[41.1] $\Psi(\boldsymbol{-R'}) = \Psi(\boldsymbol{R'})^R$ [Strong Reflection Symmetry]

.

Every particle is represented by a wave distortion of the manifold. Hence [41.1] means that every particle has an exact image of itself impinging exactly on the opposite side of the universe, with the same wave-function (spatially reflected).

This symmetry ensures global conservation of momentum because the momentum of each particle is always exactly equal and opposite that of its twin. This symmetry ensures global linear momentum equals zero in the central rest frame. Note that a finite global angular momentum is possible, since the universe might be spinning, but the present model does not try to include this.

[41.2] Global sum of linear momentum = 0 in the central frame of reference
Global sum of angular momentum is constant in the central frame

42. Postulate of Particle Strings

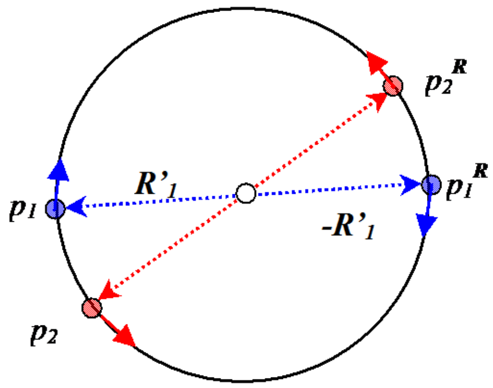


Figure 30. *Dual particles on opposite sides of the hypersphere.*

This illustrates the relationship 41.1, for two particles, P_1 and P_2, that have reflections on the other side of the universe. This means that the universe is coordinated as a whole. But do we understand this as a mathematical or a physical connection? These super-symmetric universes may be the only ones that are 'mathematically consistent' to form. But what keeps the two halves of the universe synchronised with each other? We still want to allow quantum probabilities and possible random events, so we should not rely on a Liebnitzian 'pre-arranged harmony' between the two reflected halves, since this is based on purely deterministic laws and an initial state that completely determines the future.

The particle strings provide a real causal connection across the universe, in the form of very thin *strings* of the manifold that connect pairs of particles on the outer surfaces. The strings themselves are formed as natural extensions of the curvature of the surface, at a tiny scale where the particle mass becomes a 'worm-hole'. They have the same (6D) volume and (5D) surface dimensionalities as the general manifold, but only extend significantly in one direction (***-R'***), with a tiny extension or circumference W_{string} in the remaining five dimensions.

On the micro-scale, each particle is now considered as a single *string*, which merges into its *quantum wave disturbance* on the opposite surfaces of our three-dimensional hyper-sphere.

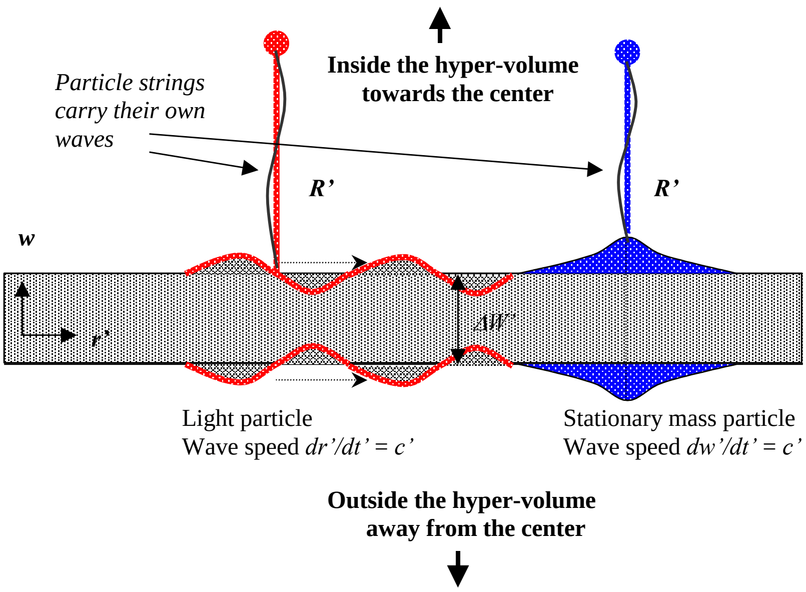


Figure 31. *Strings extending from particle-waves.*

The strings are very long and thin and carry waves at speeds, c'_{string}, of magnitude:

[42.1] $c'_{string} = c'R'/W' = c'D'/2\pi$ [Postulate of string wave speed]

Each particle string carries the pair of particle waves, which may superimpose two full waves across the length of $2R'$. The period of the simplest waves is:

[42.2] $\Delta T'_{string} = R'/(c'R'/W') = W'/c'$ [$n\Delta T$ = distance/speed]

This is basically the same as the period of the (rotating, quantum) wave in the surface. Hence the particle string waves and the particle surface waves can remain synchronised. The idea is that the surface waves drag the particle strings along with them, and the particle strings maintain the waves as single coherent particles so they do not disperse their energy throughout the manifold.

The circumference W_{string}' of the string is related to the wave speed by: $c'_{string}/c' = W'/W_{string}'$, or rearranged:

[42.3] $W_{string}' = W'c'/c'_{string}$

$$= W'^2/R'$$
$$= 2\pi W'/D' \approx 10^{-53}\ m$$

This is on the order of $10^{-53}\ m$. Substituting with fundamental constants:

[42.4] $$W_{string}' = 2\pi W'/D' = 2\pi(h'/m'c')\,(m'^2G'/h'c') = 2\pi m'G/c'^2$$

Or converting to the string radius: $R_{string}' = W_{string}'/2\pi$:

[42.5] $$R_{string}' = 2m'G/c'^2$$

This is the event horizon radius in GTR for a black hole of mass *m*. Hence single particles of mass *m* are like 'black holes', except the 'singularity' is an extended string into an extra dimension.

43. Particle Identity and Wave Function Coherence.

Quantum particles act like dispersive waves when we look at their interference properties and quantum wave description, but they act like single, quantised particles when we try to detect them or make them interact with other particles. We never find 'half a particle'. What keeps them coherent and causes them to act as integral particles? This is an abiding mystery in ordinary quantum mechanics, with wave-particle duality, wave function collapse and wave function coherence all subjects of fundamental dispute.

The present model provides an explicit mechanism. The *quantum particle* is identified with the *string* while the *quantum wave* is identified with the *surface perturbation.* The surface perturbation guides the string around with it in continuous evolution of the wave function. No matter how the wave disperses, there is always a string that keeps it 'whole'. The basic formalism for this is already specified by Bohm's deterministic theory of *pilot waves [1]*. In this theory, the wave function acts to guide a point-like particle along an apparently chaotic path, producing a trajectory that exactly matches the probability laws and produces interference effects from a deterministic mechanism. This possibility was a great surprise to physicists in the 1960's, being earlier thought to be ruled out by a mistaken proof by von Neumann which purports to show the impossibility of deterministic 'hidden variable' theories. Note also that the string itself (the point at which it touches the surface) may move at speeds greater than the speed of light, *c',* because it is only a 'node' of the wave.

The question of an underlying variable theory remains an area of lively interest among some researchers today, but Bohm's theory has not been extended beyond a simple particle model. The present model provides one possible means of extending the theory to a general treatment. The problem is not solved however unless a mechanism for *wave function collapse* is also provided.

44. Wave Function Collapse and Particle Interactions.

When two quantum particles interact they sometimes 'collapse' into another state, producing another set of particles, e.g. a photon is absorbed by an electron. There is no mechanism for such a collapse event in the conventional theory, which is only described probabilistically, and it there are problems with the apparent 'instantaneous' collapse of a spatially extended wave function into a more localised wave function. Any number of theories have been proposed to try to explain this, without general agreement being reached on any solution. All that is known with confidence is that collapse into new states is governed by the probabilistic laws of quantum mechanics.

On the present model, there are two distinct types of interaction possible. First is the usual continuous evolution of the wave function, with the surface waves deterministically attracting or repelling each other through purely mechanical wave propagation. E.g. gravity is like this, with no particle creation or destruction occurring, only the center of mass motion of the waves being accelerated. Second is the interaction between *strings* themselves. Particle creation on this model must result from *combinations of particle strings into new strings.* E.g. an electron and photon first have separate strings with separate wave functions. These may combine into a 'new' particle, an electron with the combined energy. In the model, for the photon to be 'absorbed' by the electron, the two strings must combine into one. We can initially count two distinct particle strings before the interaction, one for each particle, but only one after the interaction. (Of course particle interactions generally involve multiple particle products, not just two.) Sometimes lighter fundamental particles combine to create a heavy particle, requiring particle strings to merge, and conversely, sometimes heavy particles (e.g. unstable fundamental particles) may decay into multiple products.

The detailed mechanism for this in the model is not specified yet. But we know the essential probability laws governing the interactions from quantum field theory, and the model must conform to this. The *instantaneous wave function collapse* is also now more amenable, because the new particle string now forces a new coherent wave function to form around it. The 'action at a distance' problem changes because all model interactions or dynamics have 'localised' mechanisms mediated by the spatial manifold. Albeit space is now causally connected across large distances, but only by specific causal mechanisms, not by 'magic'.

45. Wave Function Entanglement.

A fundamental features of quantum mechanics is to allow multiple particles to be in 'entangled states'. Two electrons in a singlet state is the classic example. The electron spins remain coordinated no matter how far they are spatially separated, until one or other state is measured, when the state of the other is 'instantaneously collapsed' (to the opposite spin state).

The mechanism available in the model is for the *particle strings to remain separate at the surface (representing two electrons for instance), but merge together 'higher up' in the manifold,* creating a single multi-particle string, like a hair with split ends.

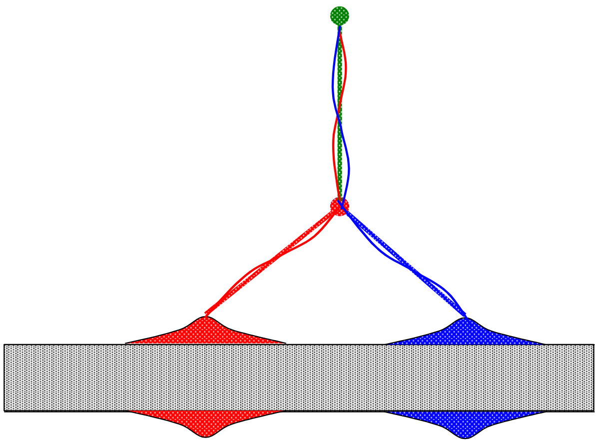


Figure 32. *Quantum entanglement occurs through string entanglement.*

Entangled particles have their *strings entangled* higher up in the hyper-sphere. This is what makes the two particles on the surface of space appear to have coordinated properties like spin. Note that the *information that represents the entanglement* is represented by the joining of the strings – it is not represented by the surface wave purturbations. More generally, there is information represented by physical structures of the strings *within the hyper-sphere* that is not represented by the surface wave alone. The quantum wave function for a *system of particles* does represent the entanglements among the particles, but only as a whole system – the individual particle wave functions do not encode the entanglement with other particles. The quantum wave functions represent the entanglements by coordinating complex-valued phases – evident in the representation using Hilbert spaces, which have more complex part-whole relationships than simple Cartesian spaces of (real-valued) properties.

What this means more generally is that there must be complex bundles of 'entangled strings', because all local particles that interact with each other are potentially entangled, but more and more 'distantly'.

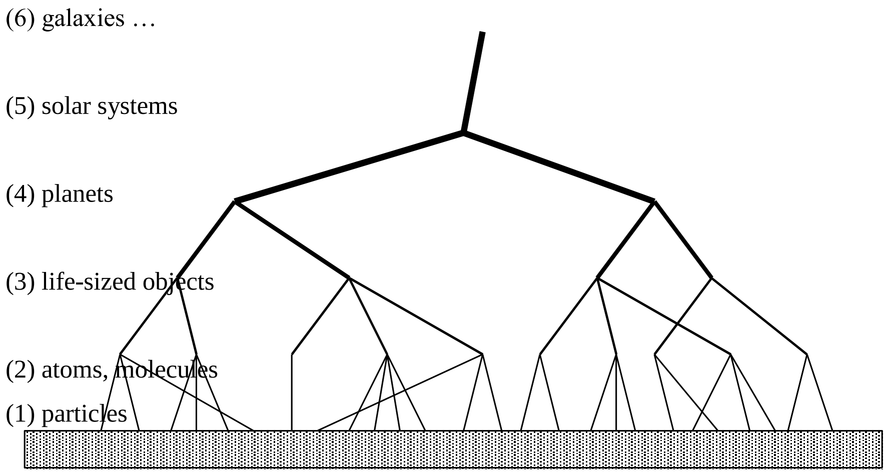


Figure 33. *Layers of string entanglement in the hyper-sphere reflect layers of holistically entangled systems on the surface.*

This illustrates the notion that the strings for (1) *fundamental individual particles* that impinge on the 3-D surface of space join into entangled states like (2) atoms or molecules, and on in increasing scale, to (3) life-sized objects (chairs, computers, brains …), then into (4) planet-sized objects (Moon, Earth), then into (5) solar-system sized objects, then (6) galaxies, and so on. Of course there may be far more layers. Separation in 3-D space is no obstacle to entanglement, although entanglements are thought to be created by local interactions.

As string are bundled together, their particle waves superimpose in the combined string. The 'entanglement' only leads to synchronised properties at lower levels (like spin of two electrons) if particle waves are synchronised – all the particles of the Earth and the Moon will be entangled at some level, for example, but there may be none or only very subtle effects at the particle level. But every *join* of two strings creates an 'interface' that contains 'extra-worldly' information that is *not reducible to the particle states taken separately.* This reflects the holistic nature of the quantum wave function. A quantum system with many particles is not reducible to its individual particle wave functions, in the way classical that materialist reductionism conceives physical objects to be reducible to their constituent atoms or 'smallest particles'. This reflects the common observation that the quantum system as a whole is not just the sum of it parts.

46. Entanglement of the whole Universe.

On a larger scale, the strings from larger-scale objects, such as galaxies, clusters of galaxies, super-clusters or 'walls' of galaxies, may be entangled, and ultimately form into a single 'string' or a new hyper-surface or a hyper-volume in the center.

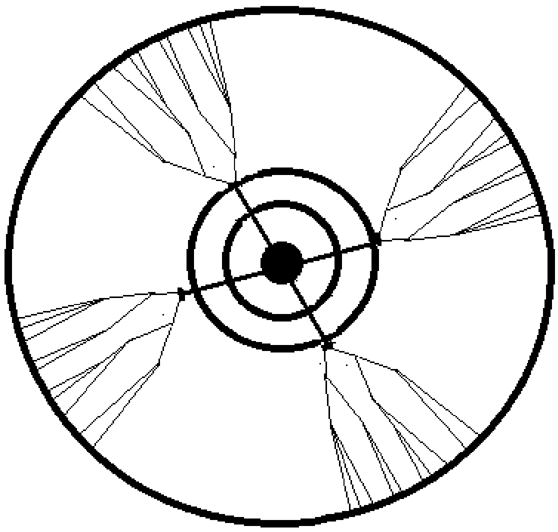


Figure 34. *The whole universe.*

We have learnt from quantum mechanics how local entanglements occur, but what about the large scale? What happens in the 'center of the universe' (hyper-sphere) where all the strings should meet and cross? Are there only thread-like strings connecting the surface particles, or are there more complex structures inside? Is information exchanged inside the hyper-sphere? Could we model another '3-D universe' as a hyper-surface nested inside the large hyper-sphere? These are open questions.

Given that the string structures inside the hyper-volume represent essential information about the surface particles and determine their behaviour and interactions, it is clear that the universe must contain more 'internal structure' than we suspected from local reductionist particle physics. The fact that information in the inner structures is ‘projected’ into the particle behaviour on the surface opens the door to any number of speculative possibilities about the source of causality in the world, the nature of information, and the existence of ‘non-physical’ entities.

47. What are We in the Model.

We can ask: what are *we* in the model? As conscious observers with physical bodies, able to do physics experiments, to begin with. We can think of our observable physical bodies represented on the particle surface and interacting causally with other physical bodies much as physicists think – but do we really *observe the particle surface*? When we ‘see the world’ do we really ‘look at’ the particle surface – or at larger inner ‘entangled objects’, that carry the *identity* of the life-sized things we see as *whole things*? In fact, we normally observe life-sized bodies, we cannot resolve our perception to fundamental particles or atoms or molecules. *Observation* reflects interactions among large holistic entities, which are larger bundles of strings in the model, joining higher up in the tree of strings in the hyper-sphere, and incorporating billions of billions of atoms at once in a single entanglement. Our ‘conscious brain state’ must have a holistic entity like this, in terms of the model, given that it acts like an ‘entangled whole’.

Then what correlates to consciousness? It seems most plausible that consciousness correlates to the *complex holistic information-carrying structures inside the hyper-sphere*, rather than to the ‘physical state’ of the particles impinging on the surface.

This view challenges materialist or reductionist theories of consciousness. Materialists might reply: “Its still all made out of matter, the matter is now just made out of other stuff we didn’t expect (the ‘manifold stuff’).” But that is just repeating the problem: *the stuff they didn’t expect* now has *a rich connectivity they didn’t expect*, and the materialists denied this possibility, as arcane or magical. The model does indeed make the world more genuinely ‘magical’, making it unclear where information originates, or what kind of effects might be projected onto the physical world from the internal structure.

The model leaves open the question of what consciousness is, what kinds of conscious interactions are possible, what kinds of conscious entities can exist. This remains mysterious, and it should be emphasised that it is not explained within the model by *another step of materialist reductionism*. The argument for reductionism in any case is only the claim of modern physicists that *there is nothing else in the universe except particle processes in space-time, and hence nothing else that consciousness could be.* But this claim is overthrown in the model, with strings and string structures, since there is now a whole lot more to the universe than just ‘particle physics in ordinary space-time’. Indeed there is a whole new *dimension of things,* and I think it would be surprising if this did not have a close connection to consciousness.

We must expect our view of own nature to be changed by the model if we take it seriously, for it means that we are multi-dimensional beings living in a multi-dimensional world, most of which is hidden from us.
The oddest thing perhaps is that we now all have reflections or twins on the other side of the universe! This is a classic philosophical thought experiment. Which twin are we? Which side of the universe is ‘my body’ on? Are we really *exactly identical* to our twin, or are there differences? Are there two conscious experiences, corresponding to two physical bodies, or just one? If just one, does consciousness ‘happen’ in the center, so it is symmetric with both bodies?

It should be remembered however that although the ‘two bodies’ are separated by a large distance in space, the speed of wave transmission on the strings connecting them makes them appear to us to be only a hydrogen-width (electron radius) away. So they appear like two coordinated shadows of each other, on two parallel surfaces separated by a thin gap, rather than two distantly-connected bodies. While it seems very odd to have a ‘twin’ there doesn’t seem to be anything wrong with it theoretically, and the image that they are ‘reflections’ of each other on two very thin surfaces rather than separated by a vast distance makes it easier to imagine.

48. Entanglement and Gravity.

A number of conventional features of gravity are modified by the model. For instance, it predicts that gravity should appear to be stronger in the past, and its effects differ very slightly from standard GTR to make it appear slightly 'stronger' in the solar-system. However these effects do not seem sufficient to explain the large galactic spin and formation anomalies that require dark matter and a cosmological constant and dark energy, with various anomalies remaining. Here I propose:

[48.1] Bundling of strings creates an inward pressure that counteracts gravity in large structures like galaxies.

[48.2] This pressure increases with radius *r'* from the center of mass, and with the number of strings.

In the model, the effect of the bundling of large strings to form galaxies is proposed as one mechanism for a large-scale quasi-gravitational effect, but no quantitative model is given. I have not modelled precisely what happens to space at the galactic boundary either, and in fact there are lots of effects to calculate. There are various possibilities for large-scale gravity effects in the model, and modelling this against the observed effects is a larger challenge.

49. Reversibility and Entropy.

The cyclic model has a time symmetric (or reversible) expansion and contraction trajectory. However the particle processes within the cycles are irreversible. The resulting laws of particle physics are time asymmetric, as evident in the time asymmetry of the quantum probability laws.

[49.1] $prob(\psi(t+dt)|\theta(t))$ <u>is not generally equal to</u> $prob(T\psi(t-dt)|T\theta(t))$
[criterion for probabilistic time asymmetry]

The irreversibility arises because the *cyclic behaviour* drives a cyclic change in the state-space, and entails a thermodynamic process as long as the relaxation time of the particle system into the changing state-space is slower than the expansion rate. All *structure* or *information* is assumed to be destroyed when the manifold is squashed into a 'blob' at the beginning. The *state space* in this configuration is very small. The expansion is faster than the thermodynamic relaxation rate, and it results in a massive expansion in the state-space. But the particles have started in a very special state, and only slowly evolve into higher-entropy states, creating information-rich structures in the process. These processes and structures persist through almost all the cycle, and result in much more highly ordered universe at the end of the cycle than at the beginning. This effectively lets us demonstrate (49.1) as a general result. See (12, 21, 33, 34) for background on probabilistic reversibility.

50. Time Flow.

The model assumes absolute time (and an absolute space manifold), with a universe-wide frame of simultaneity, and time flow through irreversible cycles of the universe that naturally generates irreversible probabilistic laws, and a thermodynamic time direction. This is contrary to the conventional philosophy of physics, which maintains the opposite, viz. that absolute time is impossible, that there is no unique frame of simultaneity, that time flow is meaningless, that physical laws are reversible. It is also widely assumed today that the universe is constantly expanding and not cyclic. The conventional metaphysics is a severe constraint on theory construction if it means the present type of model could not be considered. Yet none of the metaphysical assumptions about time flow made in the conventional philosophy has any bearing on the empirical viability or accuracy or coherence of the resulting model. The conventional 'relativistic metaphysics' based on relativistic invariance should be open to question, just as Galilean invariance became open to question in the late C19th, and no longer treated as a universally established principle of physics.

APPENDICES.

Appendix 1. Best-Fit Model Results. Simple Quantities.

Table 9. *Constants and simple calculations.*

Local variable	Value	Units	Dimensions	Name
c	2.9979E+08	m/s	X/T	speed of light
h	6.6261E-34	m^2kg/s	XXM/T	Planck's constant
G	6.6738E-11	m^3/s^2kg	XXX/TTM	Newton's constant
m_e	9.1094E-31	kg	M	mass of electron
m_p	1.6726E-27	kg	M	mass of proton
m_n	1.6749E-27	kg	M	mass of neutron
$m = (m_e m_p^2)^{1/3}$	1.3659E-28	kg	M	averaged mass PPE
ε_0	8.8542E-12	$s^2C^2m^{-3}kg^{-1}$	TTQQ/XXXM	electric force constant (permittivity)
q	1.6020E-19	C	Q	elementary electric charge
Local dimensionless constants	**Value**	**Units**	**Dimensions**	**Name**
$D = hc/Gm^2$	1.5953E+41	1	1	Dirac's constant
$D_e = hc/Gm_e^2$	3.5869E+45	1	1	Dirac's constant
$D_p = hc/Gm_p^2$	1.0639E+39	1	1	Dirac's constant
$D_{ep} = hc/Gm_e m_p$	1.9535E+42	1	1	Dirac's constant
		1	1	
$\alpha = q^2/2hc\varepsilon_0$	7.2957E-03	1	1	fine structure constant
$1/\alpha$	137.0665	1	1	inverse fsc
$\rho = m_p/m_e$	1836.1527	1	1	mass ratio
$\rho^{-1} = m_e/m_p$	5.4462E-04	1	1	mass ratio
$\gamma = \rho^{2/3}$	149.9475	1	1	normalised mass ratio
$\gamma = (m_e m_p^2)^{1/3}/m_e$	149.9475	1	1	mass ratio
$\rho^{1/3}$	12.2453	1	1	
Cosmological measurements	**Value**	**Units**	**Dimensions**	**Name**
T_1	1.3798E+10	yr	T	measured age since BB - min
T_2	1.3840E+10	yr	T	measured age since BB - max
$1/H$	1.3800E+10	yr	T	Hubble time
Fine Structure Dimensions - Model	**Value**	**Units**	**Dimension**	**Name**
W_e	1.2132E-12	m	X	electron circumference
W_p	6.6070E-16	m	X	proton circumference
W	8.0905E-15	m	X	normalised circumference PPE
Present Radius and Age - Model	**Value**	**Units**	**Dimension**	**Name**
WD	1.2907E+27	m	X	circumference - model
R_0'	2.0542E+26	m	X	radius of universe - model

R_0' in l.y.	2.1713E+10	l.y.	X	radius of universe - model
$R_0 = R_0'/2$ in l.y.	1.0856E+10	l.y.	X	radius of universe - model
$Z_0 = R_0'2/\pi$ in l.y.	1.3823E+10	l.y.		radius - conventional age units
Ratio: $R_0/2T_1$	1.0018			prediction is very accurate
Ratio: $R_0/2T_2$	0.9988			prediction is very accurate
Elementary electric charge	**Value**	**Units**	**Dimension**	***Name***
$(2hc\mu)^{1/2}\,(m_e/m_p)^{1/3}$	1.5316E-19	Coulombs	Q	elementary electric charge predicted
q measured	1.6020E-19	Coulombs	Q	elementary electric charge
Ratio	**0.9561**			prediction is quite accurate

Appendix 2. Best-Fit Model Results. Numeric Model.

Second is a numerical model, comparing with the solved equations. This sets parameters for a universe, including the present time, and calculates points in the history of a half-cycle, from big bang to full expansion, with 100 snap-shots at units of time: $T'/100$. In conventional time, these are about 138 million years per unit (although the T' scale is only approximately linear with the conventional scale T in our era). The numeric solutions are quite accurate to the equations. We set parameters to fix the model solution, through R_0' and R_{MAX}' or dR_0'/dT_0'. This determines the *present time*, which as a ratio of the half-cycle is predicted to be: $T_0'/T_{MAX}' = 0.81$ in the preferred model solution.

Table 10. *A best-fit model.*

Divide half-cycle into 100 parts	N	0	1	80	81	82	99	100
Time' - model	T' = N * time scale factor	0.0000	1.3650	109.2000	***110.5650***	111.9300	135.1350	136.5000
Radius' - Model	R' = R'max sin2(p t'/ 2 Tmax)	0.0000	0.0117	42.9973	***43.4266***	43.8366	47.5249	47.5367
Radius' - maximum	R'max = (V0'2/c0'2) R0'	47.5367	47.5367	47.5367	***47.5367***	47.5367	47.5367	47.5367
Time - Conventional	T = f(T')	0.0000	0.0000	30.6864	***32.0386***	33.4173	59.7267	61.3628
Radius - Conventional	R = R'^2/2Ro'	0.0000	0.0000	21.2909	***21.7182***	22.1302	26.0109	26.0237
measured age*c0	Z0 = 2R0/p	13.8200	13.8200	13.8200	***13.8200***	13.8200	13.8200	13.8200
time ratio	T/T'	0.0000	0.0000	0.2810	***0.2898***	0.2986	0.4420	0.4495
radius ratio R'-hat	R'/Ro'	0.0000	0.0003	0.9903	***1.0002***	1.0097	1.0946	1.0949
time differential ratio	dT/dT'	0.0000	0.0000	0.9807	***1.0003***	1.0193	1.1980	1.1986
time' - check	T' - 2R'o/c'0 arcsin(R'/R'0 co'/Vo')		1.3650	109.2000	***110.5650***	111.9300	135.1350	136.5000
speed total	V'=sqrt(c'^2+dR'/dt')^2)	0.0000	0.0172	1.0405	1.0457	1.0506	1.0939	1.0941
speed of light	C'	0.0000	0.0003	0.9896	***0.9995***	1.0089	1.0938	1.0941
speed	R'/T'	0.0000	0.0086	0.3937	***0.3928***	0.3916	0.3517	0.3483
speed	R/T	0.0000	79.5142	0.6938	***0.6779***	0.6622	0.4355	0.4241
speed	R/CoT	0.0000	79.5737	0.6943	***0.6784***	0.6627	0.4358	0.4244
time ratio	Slope of (dT/dT')	0.0000	0.0000	0.0064	***0.0064***	0.0064	0.0056	0.0055
factor	AT' = pi()/2 T'/Tmax	0.0000	0.0157	1.2566	***1.2723***	1.2881	1.5551	1.5708
expansion speed model - numeric	dR'/dT' Numeric	0.0000	0.0172	0.3215	***0.3074***	0.2931	0.0172	0.0000
expansion speed model - calculated	dR'/dT' = pi()/2 R'max/T'max sin(2t')	0.0000	0.0172	0.3215	***0.3075***	0.2931	0.0172	0.0000
expansion speed model - normalised	(dR'/dT')/R'	#DIV/0!	1.4648	0.0075	0.0071	0.0067	0.0004	0.0000
expansion speed - numeric	dR/dT	79.5142	39.7515	0.3246	***0.3073***	0.2902	0.0157	0.0000
expansion speed - calculated	dR/dT = Rmax'/R' co/2 sin(2AT')	#DIV/0!	63.6091	0.3247	***0.3074***	0.2903	0.0157	
expansion speed - normalised b.l.y.	(dR/dT)/R	#DIV/0!	2.14E+09	3.55E-04	3.26E-04	2.99E-04	1.27E-05	0.00E+00
expansion speed squared	dR'/dT'^2	6322.5128	1580.1825	0.1054	***0.0944***	0.0842	0.0002	0.0000
EM / Mass ratio sqrt	sqt(r2/3/a) = sqrt(150/137)	1.0464	1.0464	1.0464	***1.0464***	1.0464	1.0464	1.0464
EM / Mass ratio	r2/3/a = 150/137	1.0949	1.0949	1.0949	***1.0949***	1.0949	1.0949	1.0949
expansion ratio	R'max/Ro'	0.0000	4053.1807	1.1056	***1.0946***	1.0844	1.0002	1.0000
speed of light - invariant	Co = c	0.0000	0.9993	0.9993	***0.9993***	0.9993	0.9993	0.9993
hubble constant	H' = (dR'/dT') / R'	0.0000	1.4648	0.0075	***0.0071***	0.0067	0.0004	0.0000
hubble constant	H = (dR/dT) /R	0.0000	25094367.3920	0.0152	***0.0142***	0.0131	0.0006	0.0000
hubble constant	H'/H	0.0000	0.0000	0.4904	***0.5003***	0.5098	0.5991	#DIV/0!
hubble constant	R'/R	0.0000	7403.8101	2.0195	***1.9995***	1.9808	1.8271	1.8267
hubble constant	H'R'/HR	0.0000	0.0004	0.9904	***1.0003***	1.0098	1.0947	#DIV/0!
hubble constant	H'R'/Co'	**0.0000**	**0.0172**	**0.3217**	***0.3077***	0.2933	**0.0172**	**0.0000**
hubble constant	H'R'/C'	**0.0000**	**63.6463**	**0.3249**	***0.3076***	0.2905	**0.0157**	**0.0000**
hubble constant	HR/co	**0.0000**	**39.7813**	**0.3248**	***0.3076***	0.2904	**0.0157**	**0.0000**
hubble constant - true present value	T'H'	0.0000	1.9995	0.8165	***0.7827***	0.7483	0.0489	0.0000
hubble constant	T'H/2	0.0000		0.8324	***0.7823***	0.7340	0.0408	
hubble constant	TH	0.0000	0.4999	0.4678	***0.4534***	0.4383	0.0360	0.0000
hubble constant - 5% in the past	T'H/2-LAG 6			1.2785	***1.2065***	1.1382	0.2934	0.2521
hubble constant - 6% in the past	TH-LAG 6			**0.7185**	***0.6992***	**0.6796**	**0.2593**	**0.2267**
time	T'/Tmax'	0.0000	0.0100	0.8000	***0.8100***	0.8200	0.9900	1.0000
time	T'/T			3.5586	***3.4510***	3.3495	2.2626	2.2245
time	T/Tmax	0.0000	0.0000	0.5001	***0.5221***	0.5446	0.9733	1.0000
time	T'c'/R'		0.0314	2.5133	***2.5447***	2.5761	3.1102	3.1416
time	T'c0/R'		116.2988	2.5378	***2.5441***	2.5514	2.8413	2.8693
time	Tc0/R		0.0126	1.4402	***1.4741***	1.5089	2.2945	2.3562
gravity rate of change per b.y.	dG/dT = GoRo/R^2 dR/dT		2.325E+04	1.051E-12	**9.562E-13**	8.697E-13	3.404E-14	0.000E+00
gravity - normalised rate of change per year	(dG/dT)/G = Ro/R^2 *(dR/dT)		3.439E+05	1.554E-11	**1.414E-11**	1.286E-11	5.035E-13	0.000E+00
length - comoving	L' = 2R'arcsin(sqrt(R'/Rma')	0.0000	0.0004	108.0640	***110.5074***	112.9277	147.8109	149.3408
length - comoving	L'/R'	0.0000	0.0314	2.5133	***2.5447***	2.5761	3.1102	3.1416
length - comoving	L = L'/2	0.0000	0.0002	54.0320	***55.2537***	56.4638	73.9055	74.6704
length	DX' = coT''	0.0000	0.0001	45.7656	***47.1232***	48.4940	73.1771	74.6704
length	DX = coT	0.0000	0.0000	30.6634	***32.0147***	33.3923	59.6821	61.3169
length	DX/R	0.0000	0.0126	1.4402	***1.4741***	1.5089	2.2945	2.3562
length	dDX/dT	0.0000	0.9993	0.9993	***0.9993***	0.9993	0.9993	
co-moving acceleration	(dDX/dT)/X		50195812.1659	0.0326	***0.0312***	0.0299	0.0167	0.0000
length	(dDX/dT)/(dR/dT)	0.0000	0.0251	3.0785	***3.2514***	3.4429	63.6725	#DIV/0!
length	DX'/DX		6169.7547	1.4925	***1.4719***	1.4523	1.2261	1.2178

This best-fit model places our era at 81% through the expansion cycle.

Appendix 3. Estimating coincidence in the prediction of the age.

The model predicts a universe radius: $R' = h^2/4\pi m_e m_p^2 G = (\pi/2)13.823$ *billion l.y.* The two best measurements of the *current age of the universe* at time of writing (2013) are 13.798 and 13.84 billion years, determined experimentally by two different methods. The true 'measured age' is expected to fall between these two values. The model predicts *Z/c = 13.823 billion years,* which is within 0.999 to 1.002 of these two measured ages.

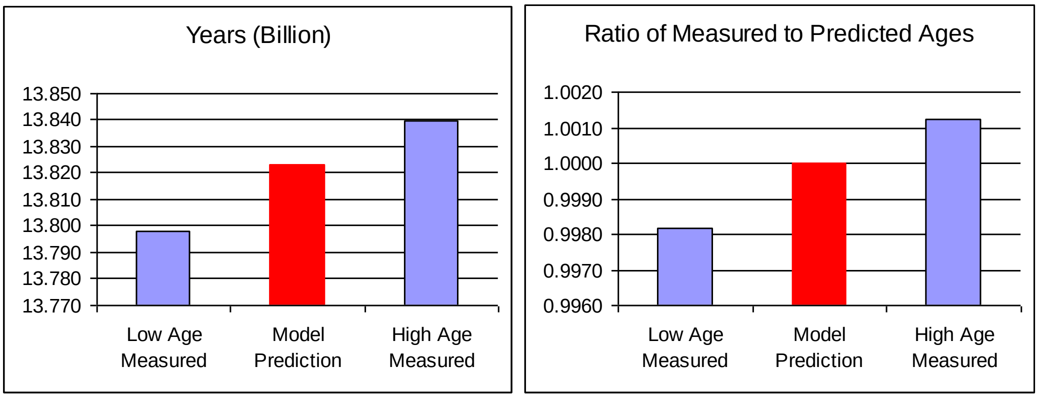


Figure 35. The model prediction exactly straddles the two best measurements of the age of the universe (as of April 2013). Graph spacings are 10 million years apart.

The empirical age estimates have altered substantially over the last 50 years, and the lower estimate was revised upwards by about 40 million years recently, whereas the model prediction has always been fixed by the constants. The constants on the RHS were known with enough precision for this prediction long before the age of the universe was determined accurately enough to test it. I note that some serious difficulties with cosmological age measurements have only been resolved in the last 10 years, since about 2003. When I first obtained this prediction it was probably inconsistent with some age estimates. The problem is that estimates of experimental accuracy of various methods may be accurate in their own terms, but are (unavoidably) made on the assumption that *there are no sources of unknown systematic error involved.* Age estimates are still changing, and the latest refinements from Planck now bring the estimate to 13.82 b.y., closer to the model prediction than a year ago when I wrote the first version of this paper, and indeed, converging on almost exactly the model prediction. I leave the original values from April 2013 in this version.

I briefly discuss the chance that this is merely a coincidence, and whether such a relation could be artificially constructed. Dirac and Eddington both recognised this and other 'large number coincidences' as profoundly interesting, even though they had only imprecise versions of the dimensionally correct relationship: $T \approx h^2/m^3Gc$ to go on, with T the age of the universe. We see next that freedom to take different combinations of the *electron or proton mass constants,* as well as small numeric constants (like 2π), produce enough variation to make coincidence more likely than may first appear. However the model prediction is accurate enough to be highly significant.

The relationship: $T \approx h^2/m^3Gc$ along with the other large number coincidences struck Dirac and Eddington as an unlikely coincidence, because the time T is calculated from unique combinations of constants with values (in normal units) on the scale of: $(10^{-34}*10^{-34})/(10^{-27}*10^{-27}*10^{-30}*10^{-11}*10^{8})$. What is the chance of a 'random combination' of such 'large numbers' falling within a factor of , say, a million ($10^{+/-6}$) of T by accident? We can estimate that fairly precisely. We can only construct *lengths* based on: (i) h/mc, and (ii) product of this with powers of the dimensionless constant: $D = hc/m^2G$. The latter is 1.6×10^{41}. The *scale of spacing* is represented by the natural logs of these ratios, and probabilities of a result being correct within a factor of: $Z+/-eZ$ where e is the 'error', is approximately: $\ln(e)/\ln(\sqrt{D})$ or: $2\ln(e)/\ln(D)$.

We can then construct a table showing the possible variations of the mass constants and small constants in D, and the corresponding probability that the result matches the empirical measurement by coincidence.

Table 11. *Possible variations of D.*

	Dimensionless constant *D* represents the spacing between possible values	Different Electron, Proton mass combinations:			MODEL PREDICTION = TRUE VALUE	
	D	EEE	EEP	EPP	EPP/2π^2	PPP
Predicted values *R*	1.60E+41	8.7030E+33	4.7398E+30	2.5814E+27	1.3077E+26	1.4059E+24
Correct value of *R*		**1.3094E+26**	**1.3094E+26**	**1.3094E+26**	**1.3094E+26**	**1.3094E+26**
Ratios	1.60E+41	6.65E+07	3.62E+04	**19.71470358**	**0.99875855**	**0.01073696**
Log base 10 (ratios)	**41.20**	**7.82**	**4.56**	**1.29**	**0.00**	**-1.97**
Ln(ratios)	**94.87**	**18.01**	**10.50**	**2.98**	**0.00**	**-4.53**
prob coincidence		**0.38**	**0.22**	**0.06**	**-0.00002619**	**-0.10**

Putting in different combinations of the electron and proton masses to make up the m^3 factor in h^2/m^3G, we see that the predicted values of $Z^* \approx h^2/m^3G$ might vary from the empirical value by about 66 million times (using EEE, i.e. *the electron mass cubed*) to about 20 times for EPP and 1/100th for PPP. Including the small numeric constant: $1/2\pi^2$ with the *EPP* combination gives the exact model prediction.

On this estimate, the *coincidence* represented by EEE, i.e. $m^3 = m_e^3$ has a 0.38 random chance. EPP and PPP produce the coincidence with a .06 to .10 chance. The exact model, with the factor $1/2\pi$, produces the coincidence with very low chance of 0.000026. Note that the combination: *EPP* is strongly determined by the model . Only the combination of small geometric constants, 2 and π, is potentially flexible, and there may be a model variation scaled by a factor of $\pi/2$ or similar. Note that removing this small constant results in changing the epistemological probability of coincidence from 0.000026 to 0.06. The general form of the relationship without the small constants gives a good indication that there is a stronger law-like relationship.

The spread of the *probability of coincidence* in Table 1 occurs because the combination of the ‘small numeric constants’, $2\pi^2 m_e m_p^2$, ranges over a scale of about 10^{11}, or: $20*1{,}836^3$, while the spacing of the values is about: $D = 1.6 * 10^{41}$. Because we compare the *natural logs* of the spacings, this kind of pattern has a reasonable chance of occurring randomly.

The relation known to Dirac and Eddington, that: $T^*_0 \approx h^2/m^3Gc$, taken by itself would represent only a ‘weak’ coincidence, with a chance on the scale of about 0.1, unless there is either: (i) an independent model prediction to fix the combination of constants and small constants and show their evolution with time, or (ii) a broader pattern of such coincidences, as Dirac believed. Indeed, given 3 such coincidences related to the same large number, *D*, if they have individual probability of 0.1 of being independently accidental, then their combination has a *prima facie* 0.001 chance of being coincidental, if they are independent – but less given that the common role of *D* points to a common cause and mechanism (like seeing multiple symptoms in a patient).

We see here why it is important to have: $Z = h^2/2\pi^2 m_e m_p^2 G$ in an exact form *determined by an underlying model.* The model determines the mass combination, and means we cannot concoct a result by manipulating the small constants. The accuracy of the prediction must give it a correspondingly low chance of being accidental.

Appendix 4. Note on Intrinsic Spin.

Intrinsic spin is different to simple orbital angular momentum, the difference being related to the dimensionality of the spin space. It requires more careful treatment and this is just a note that its quantisation requires further treatment. The spin-number N is quantised by ½, i.e. ½, 1, 1½ , 2, .. and L is quantised by units of momentum: $\hbar'/2$. The intrinsic angular momentum (or spin angular momentum) of the electron and proton and other particles are quantised by:

$$S = \hbar/2 \sqrt{n(n+2)}, \qquad n = 1, 2, 3, \ldots$$
$$n(n+2) = 3, 8, 15, 24, 35, 48, \ldots$$

The spin number is normally defined by: $s = n/2$, so $s = ½, 1, 1½, \ldots$ and: $S = \hbar/\sqrt{s(s+1)}$. This *small constant* modifies the basic unit of *orbital angular momentum*, $\hbar/2$. Note that the small constant rearranges to: $\sqrt{(n(n+2))} = \sqrt{(n^2 + 2n)} = \sqrt{((n+1)^2 - 1)}$. In our geometric model, we define:

$$n_i = d_i - 1$$

So the series has the form: $\sqrt{(d_i^2-1)}$. The model specifies d_i as *the dimensionality of the sub-space in which the lowest complex-spherically symmetric valued wave solution rotates*. To solve this analytically, we would need to find the series of lowest spherically symmetric solutions for the angular momentum in increasing dimensions. I just state the required result as a postulate here, after observing that:

- To match observation and quantum mechanics, the intrinsic spin solution must be quantised as the *square root of the square of the dimensionality minus 1:* d_i^2-1. This is the diagonal radius for a unit hyper-cube of d_i^2-1 dimensions: $r^2 = 1^2+1^2+1^2 \ldots 1^2 = (d_i^2-1)$

The intrinsic spin for particle type k is given by:

$$S = ½\, N_k\, (m_k' R_k' c')\, \sqrt{(d_i^2-1)} \qquad d_i = 1, 2, 3$$
$$= (\textit{wave-number}) \times (\textit{simple angular momentum}) \times (\textit{small geometric constant})$$

This is related to QM by interpreting the *spin number for a particle type* k *as:*

$s_k = (d_i\text{-}1)/2$

The model *electron* has intrinsic angular momentum in:

$d_p = 1$	*dimensions*
$n_p = 1$	*degrees of freedom*
$s_p = ½$	*spin number*
$N_p = 1$	*wave number*

References.

1. Bohm, David and Bub, J, 1966. "A Proposed Solution to the Measurement Problem in Quantum Mechanics by a Hidden Variable Theory." *Rev.Mod.Phys.* **38** (3) pp. 453-469.
2. Brown, James Ward and Churchill, Ruel V. 1993. *Fourier Series and Boundary Value Problems.* McGraw-Hill.
3. de Beauregard, Olivia Costa. 1980. "CPT Invariance and Interpretation of Quantum Mechanics". *Found.Phys.* **10** 7/8, pp. 513-531.

3.1 Corredoira, Martin Lopez. 2013. *The Twilight of the Scientific Age.* Brown Walker Press.

4. Dirac, P.A.M., 1958 [1930]. *The Principles of Quantum Mechanics.* Oxford.
5. Dirac, P.A.M., 1969. "Fundamental Constants and Their Development in Time". Symposium talk (*lost reference*).
6. Dyson, Freeman. 1977. “The fundamental constants and their time variation”, in Salam and Wigner, 1972.
7. Epstein, Lewis Carroll. 1983. *Relativity Visualised.* Insight Press, San Francisco.
8. Fine, Arthur. 1982. "Joint Distributions, Quantum Correlations, and Commuting Observables." *J.Math.Phy.* **23** pp. 1306-10.
9. Fine, Arthur. 1982. "Hidden Variables, Joint Probability, and the Bell Inequalities." *Phys.Rev.Let.* **48** pp. 291-5.
10. Fine, Arthur. 1986. *The Shaky Game: Einstein, Realism, and the Quantum Theory*. University of Chicago Press.
11. Gasiororowicz, S. 1966. *Elementary Particle Physics.* New York: John Wiley and Sons.
12. Holster, A.T. 2003. “The criterion for time symmetry of probabilistic theories and the reversibility of quantum mechanics", *New Journal of Physics,* (www.njp.org) http://stacks.iop.org/1367-2630/5/130. (Oct. 2003).
13. Holster, A.T. 2003. “The quantum mechanical time reversal operator.” PhilSci Archive pre-print. http://philsci-archive.pitt.edu/1449/
14. Jauch, J.M., and Rohrlich, F. 1955. *The Theory of Positrons and Electrons.* New York: Addison-Wesley. pp 88-96.

15. Kay, David C. 1976. *Theory and Problems of Tensor Calculus.* McGraw-Hill.

16. Kobayashi, S. and K. Nomizu. 1963. *Foundations of Differential Geometry*. Wiley and Sons.

17. Kuz'menko, V.A. 2007. “On the nature of entanglement”. arXiv preprint arXiv:0706.2488.

17. Kuz'menko, V.A. 2004. “Coherency is the ether of XXI century”. http://arxiv.org/abs/physics/0401051

18. Lord, Eric. 1976. *Tensors, Relativity and Cosmology*. Tata McGaw-Hill.

19. Lorrain, Paul, and Corson, Lorrain. 1988. *Electromagnetic Fields and Waves.* W.H. Freeman and Company.

20. Matthews, Philip S.C. 1986. *Quantum Chemistry of Atoms and Molecules.* Cambridge University Press.

21. McCall, Storrs. 1976. "Objective time flow". *Phil.Sci.* **43,** pp. 337-362.

22. Merzbacher, Eugene. 1970. *Quantum Mechnics.* John Wiley and Sons.

23. Messiah, A. *Quantum Mechanics.* 1966. John Wiley and Sons.

24. Misner, Thorne, Wheeler. 1973. *Gravitation.* W. H. Freeman.

25. Musser, George. 1998. “Pioneering Gas Leak?”, *Scientific American,* December 1998, pp.12-13.

26. NASA Website Pioneer homepage: http://spaceprojects.arc.nasa.gov/Space_projects/pioneer/Pnhome.html

27. Redhead, Michael. 1990. *Incompleteness, Nonlocality, and Realism.* Oxford.

28. Spiegel, Murray, R. 1974. *Theory and Problems of Complex Variable.* McGraw-Hill.

29. Spiegel, Murray, R. 1974. *Theory and Problems of Advanced Calculus.* McGraw-Hill.

30. Spivak, Michael. 1979. *A Comprehensible Introduction to Differential Geometry*. Publish or Perish.

31. Torretti, Roberto. 1983. *Relativity and Geometry*. Dover.

32. Wangsness, Roald K. 1979. *Electromagnetic Fields.* Wiley and Sons.

33. Watanabe, Satosi. 1955. "Symmetry of Physical Laws. Part 3. Prediction and Retrodiction." *Rev.Mod.Phys.* 27.

34. Watanabe, Satosi. 1965. "Conditional Probability in Physics". *Suppl.Prog.Theor.Phys. (Kyoto) Extra Number*, pp. 135-167.

35. Whitney, H. 1936. "The self-intersections of a smooth n-manifold in 2n space." *Ann.Math.* **45**, pp. 220-46.
36. Whitney, H. 1936. "Differentiable Manifolds". *Ann.Math*. **37**, pp. 645-80.
37. Kochen, S. and E. Specker. 1967. "The problem of hidden variables in quantum mechanics." *J.Math.Mech.* **17,** pp. 59-87.

Web References.

38. Wikipedia *Age Of Universe* http://en.wikipedia.org/wiki/Age_of_the_universe.
39. Wikipedia *Hubble Constant* http://en.wikipedia.org/wiki/Hubble_constant
40. Hazewinkel, Michiel, ed. (2001), "Klein-Gordon equation", *Encyclopedia of Mathematics*, Springer, ISBN 978-1-55608-010-4).
41. Holster, *The Time Flow Manifesto*. (2014).

    The Time Flow Manifesto Introduction. ***http://philpapers.org/rec/HOLTTF***

    The Time Flow Manifesto Chapter 1 Concepts of Time Direction. ***http://philpapers.org/rec/HOLTTF-5***

    The Time Flow Manifesto CHAPTER 2 TIME SYMMETRY IN PHYSICS. ***http://philpapers.org/rec/HOLTTF-6***

    The Time Flow Manifesto CHAPTER 3 REVERSIBILTY IN PHYSICS. ***http://philpapers.org/rec/HOLTTF-7***

    The Time Flow Manifesto Chapter 4 Metaphysical Time Flow. ***http://philpapers.org/rec/HOLTTF-2***

    The Time Flow Manifesto Chapter 5 Time Flow Physics. ***http://philpapers.org/rec/HOLTTF-3***

    The Time Flow Manifesto CHAPTER 6 PHILOSOPHICAL ISSUES. ***http://philpapers.org/rec/HOLTTF-4***

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