Introduction to Pavel Tichý and Transparent Intensional Logic.

Andrew Holster. Original 2004. Reposted 2023 unchanged.
(An updated version on some issues in intensional logic is required, but I hope this is still useful.)

Life.

Pavel Tichý (1936-94) was born on 18 February 1936 in Brno, Czechoslovakia. He studied philosophy and mathematics at Charles University, Prague, from 1954-60, where he subsequently taught in the Department of Logic from 1961-1968. In 1968-1970 he was a Research Fellow in the Department of Philosophy, University of Exeter, England. In absence from the Socialist Republic of Czechoslovakia, he was sentenced to 5 years hard labor for illegally emigrating. He moved to New Zealand in 1971, where he taught for the rest of his life in the Department of Philosophy at the University of Otago, Dunedin, becoming a full professor in 1981. He spent a year (1976-1977) as Andrew Mellon Postdoctoral Fellow, Department of Philosophy, University of Pittsburgh. In 1995 he was to return to the Czech Republic, to become Head of Department of Logic at Charles University, Prague, but died tragically on 26 October, 1994, in Dunedin, New Zealand. He was married to Jindra, and they had two children.

Introduction: Work and Reputation.

Tichý was a highly original philosopher, semanticist, and logician. Perhaps his most enduring claim to fame lies in his theory called *Transparent Intensional Logic*, the culmination of his extensive work on semantics and logic. This has become the basis of an important research program, based in the Czech Republic and Slovakia, and I will concentrate in this essay mainly on explaining the central concepts of this theory. But his work includes important contributions to a wide range of subjects in the philosophy of language, science, and metaphysics. For instance, he is well known for his 1974 proof of the failure of Popper's theory of verisimilitude (or 'likeness to truth'). Characteristically, he was never satisfied with giving purely negative criticism, and in this case, with his student and collaborator Graham Oddie, he went on to develop a new approach to redress the flaws in Popper's initial conception.

Tichý's work in semantics and philosophical logic is firmly in the tradition of 'objectual semantics', generally regarded as originating with Frege. The first part of the 20th Century saw the first waves of this approach in the work of Russell, Church, Godel, Carnap, Tarski, and then a growing body of work from about 1950-1970. The second major watershed in the subject is generally regarded as the work of Richard Montague, who in 1970 published the first formal systems of what is now called *Montague Grammar*. The more general name for systems of this kind is *intensional logic*, or *intensional semantics.* This led to a proliferation of subsequent work in modern semantics, encompassing a variety of different approaches, including programs in AI (Artificial Intelligence), computational logic, philosophical logic, and new approaches to conceptual analysis and linguistics.

Remarkably, Tichý discovered intensional logic independently of Montague, and published his first system (in English) almost simultaneously. Those who know this work often regard Tichý's system as more elegant and perspicuous than Montague's,

although the essential idea is the same. But unfortunately for Tichý, his theory was first published in 1971, shortly after Montague's papers of 1970, and he has received little credit for his originality.

Tichý's first paper on this, in English at least, is "*An Approach to Intensional Analysis"* (1971). (I cannot judge whether the idea is formulated in earlier papers in Czech, although obviously, the ideas were some time in gestation[1]). After an elegant presentation of an intensional system of logic, Tichý concludes by discussing Montague's paper of 1969, "The Nature of Some Philosophical Entities", and demonstrates a problem with it, which his own theory solves. To this critique of Montague, he subsequently adds a footnote (fn. 10; p.294), "*Added in proof: This particular objection loses its weight in the light of Montague's 'Pragmatics and Intensional Logic', an article which… was not available when I was writing the present paper. …"*, with a brief but formally precise observation of how Montague escapes the particular problem raised at this point. Although Tichý partially withdraws his criticism, the system he presents here is decisively better than Montague's earlier attempts. Montague only solved the problems satisfactorily in his two subsequent landmark papers of 1970, "English as a formal language" and "Universal grammar", and Tichý was unaware of these papers when he wrote his (1971). This is one of the first critical discussions of Montague, and it shows Tichý's critical acumen, as well as the independence of their conceptions.

But while Montague's two main papers on intensional logic established his reputation as a seminal figure in the subject, Tichý has gained almost no recognition. Montague was murdered in 1971, but he rapidly became one of the most famous modern semanticists, whereas Tichý has remained obscure, despite making many further advances in the subject. A good gauge of this lack of recognition is evident in the influential source, *Handbook of Logic and Language*, (van Benthem and ter Meulen (Ed.), 1997). This begins with a long article on Montague Grammar by Barbara Partee, which contains a couple of acknowledgements of Tichý, (particularly p.74); but he is only mentioned in passing; and the only reference is to his 1988 book, *The Foundations of Frege's Logic.* His contributions to the treatment of time in intensional logic, and the concept of compositionality, are completely overlooked in the articles on *Temporality* (Steedman), and *Compositionality* (Jansen) in the same volume, despite his original and challenging articles on these subjects.

This brings us to the difficult subject of Tichý's reputation. Tichý's work is still not widely known, except perhaps in the Czech Republic and Slovakia, and among a few of his colleagues and students from his time in New Zealand, and his reputation has yet to be settled. However, in the light of his original discoveries, and the deep interest and high regard shown by a number of important contemporary philosophers, it seems inevitable that, like Frege before him, his achievements will eventually be reassessed in a far more positive light by scholars in the future.

A number of reasons can be found for this lack of recognition, but the most obvious is a polemical feature of Tichý's work, which sets him at odds with many influential contemporary writers. He wrote many critical and challenging commentaries on the approaches of other writers. These serve to establish the context for proposing solutions of his own. His critical studies are very important, and explain much of the rationale for his creative ideas. He has unique insights into earlier writers, such as Frege,

[1] Pavel Materna has informed me that the origins of TIL are first evident in Tichý's (1968) "Smysl a procedura" ("Sense and procedure"), and in his (1969) "Intensions in Terms of Turing Machines", but I have not read these papers.

Russell, Church, Carnap, Tarski, and Popper. He criticizes modern interpreters of Frege and Russell, in particular, arguing that authorities such as Dummett and Quine have misunderstood their ideas, and imposed their own views instead. He also gives sharp and succinct arguments against approaches of many of his contemporaries. He was especially opposed to a certain dominant line of development in mid-to-late 20th century logic and semantics, which he calls the 'linguistic' tradition, typified by the technique of 'semantic ascent' to solve semantic paradoxes, or interpret belief statements (see below). Among those whose views he criticizes on one point or another are Quine, Montague, Kripke, Hintikka, Dummett, Kleene, Putnam, Goodman, Prior, Lewis, Stalnaker, Cresswell, Richards, Dowty, Partee, Tenant - and Tichý himself. (He criticizes some of his own earlier views in exactly the same tone that he criticizes others). Materna (1994, p.2) observes that these authors almost never responded.

Tichý's criticism of much contemporary work in semantics may appear negative, in the first instance. But his critiques are by no means negative: they are rather his way of introducing problems, before offering new solutions of his own. Even his most original theories are introduced by first offering critiques of other approaches and other scholars, before presenting his own original contributions. His book *The Foundations of Frege's Logic* is an example - it might have been better entitled *The Theory of Transparent Intensional Logic*. And many of his papers are titled or introduced as critiques of other writers, although their greatest interest often lies in the original alternative analyses he offered.

Tichý seemed to lack any instinct for self-promotion, often a vital ingredient for obtaining recognition of new ideas. The social dimensions of intellectual success are well recognized in modern sociology of science – popular success or failure is not determined by the quality of intellectual work alone, especially in fields that are young and ideologically fragmented, like modern semantics, and Tichý would no doubt make a good case-study of this.

But while few targets of his criticism have responded, the problems he raised have remained central issues. And perhaps a dozen or so writers who have considered his theories seriously have been deeply impressed – or indeed, judged that he has decisively solved some problems of fundamental importance! Collaborators in the Czech TIL program have made important developments of his theories, and provided useful critical appraisals and presentations. Writers such as Pavel Materna, Graham Oddie, Marie Duzi, and others, have emphasized their debt to Tichý, whose theories have provided the original starting point for their approaches to many problems.

I will concentrate in this article on explaining some of the central ideas of Transparent Intensional Logic, with the aim of introducing the reader to Tichý's general approach. A few detailed examples are given, but these are only illustrative: it will be clear that this article is far from a full or adequate summary of his work.

Objectual Semantics.

The idea of *objectual semantics* is credited to Frege, and is fundamental to Tichý's approach. The starting point is factual language, in which we compose an endless variety of complex expressions, called *sentences* or *statements,* using a limited set of basic *terms*, or *words*. We use language to communicate information, referred to as 'expressing facts', or 'stating propositions'. Expressions such as sentences and words are called *linguistic* or *syntactic items.* These are what we find written on a page. We perceive them as physical symbols of distinct types. We refer to the symbols themselves

by using quotation marks, e.g. the word ‘moon’ is a type of symbol, not to be confused with the moon itself, which is a physical object. Using these symbols to communicate requires us to understand the meanings of the symbols: we have to learn that ‘the moon’ refers to an object, the moon. But the connection between the symbols and the objects, which is connected by the *meanings of the symbols,* is very mysterious - it almost seems magical! How do words ‘mean things’? How do they make connections to external reality?

The idea of *objectual semantics* is not really to explain *how* words mean things, in the sense of a giving a causal explanation, but just to define *what* they mean. The aim is to specify the direct connections between the *expressions* and the *things,* or ‘objects of reference’. But we should emphasize something of crucial importance here from the beginning: the *meanings* of expressions cannot be just the *physical things* that they happen to refer to. E.g. the meaning of ‘the moon’ is not just the physical object (the physical moon); rather, it appears to be something ‘conceptual’, that comes between the expression and the physical object. Objectual semantics does not assume that meanings are identified simply with *actual references.* The term ‘object of reference’ as it was used above is to be taken in a wide sense, to include ‘abstract objects’ – for instance, ‘concepts’ are regarded by Frege as objects of meaning.

This approach is to be contrasted with non-objectual theories of meaning, which typically appeal to the causal processes that underpin language use - but processes that are not explicitly mentioned by the expressions themselves. For instance, some philosophers explicate meanings as *mental states* (e.g. Hume), or *social behaviors* (e.g. the later Wittgenstein). These kinds of theories interpret the meanings of expressions as the mental states they conjure up, or as behaviorist systems of rules for using expressions – even though the expressions in question do not explicitly (or even implicitly) refer to any such mental states or rules of language behavior. E.g. when we say ‘It is raining’, we only refer to a fact about the weather – not to our mental image of ‘rain’, nor to behaviorist rules about the appropriate use of the expression.

Thus non-objectual theories involve detours into realms of objects or processes which underlie the use of language, but which are not mentioned by the expressions being analysed. This kind of detour is rejected in objectual semantics, which seeks a system of direct associations between expressions and ‘objects of reference’, to represent ‘literal meanings’ of expressions. It is this focus on the ‘literal meaning’ that distinguishes objectual semantics as a study of *the logic of meaning,* rather than a study in anthropology or psychology.

Now of course, as human beings, our use of language actually involves mental states, and learning language actually involves social behavior. But this does not mean that mental states or behaviors are a part of the *meanings* of expressions. Consider the analogy with describing a certain bridge. An *objectual description* can be given by an engineering specification of the materials and dimensions of the bridge. The bridge might consist of a certain arrangement of steel girders, of certain sizes, welded together in certain ways, and so on. Naturally, we also *build* the bridge, we *perceive* the bridge, and we *use* the bridge. But we don’t have to describe how a bridge is built, or how it looks to the eye, or what we use it for, to define it as an object.

Similarly, objectual semantics attempts to define meanings directly, without specifying how we learn meanings, or use meanings, or perceive meanings. Our ability to learn and use meanings is a fundamental constraint on what they can be: we can’t have a good theory if it implies that meanings are impossible to learn, or impossible to use. So an objectual theory must be consistent with a reasonable *epistemology of*

meaning. But objectual semantics separates the two: the epistemology of meaning is not part of the definition of what meaning is.

The Objects of Reference.

The obvious difference between bridges and meanings, in the analogy above, is that the former are concrete physical objects, while the latter are very 'abstract'. We want to define meaning as a system of direct connections between *expressions* and *objects:* but what are the objects, and what are the connections?

The *connections* are easiest to deal with: we will simply define them by direct 'functional connections' between expressions and objects. That is, we specify mathematical functions, which map expressions to objects, without worrying (initially at least) about how these are implemented in practice. Our practical grasp of meaning clearly involves thinking, and a complex cognitive machinery of mental states, functional states of the brain, and so forth, comes into play when we actually use language. But this detailed machinery is precisely what objectual semantics ignores. This is a common device in scientific theories: e.g. the classical theory of gases specifies a relationship: $PV = nRT$, connecting pressure, volume and temperature, without initially explaining *how* this connection is generated. This is a common kind of 'mathematical idealization', found throughout science.

The nature of the *objects* is much more problematic, however, partly because their 'abstract' quality makes them difficult to define. The example of the term 'the moon' referring to the (physical) moon may suggest that we can use *physical objects* as referents, but this simple example is misleading, as Frege showed. (Plato recognized this too in his *Theatatus* and other dialogues.) Here are three reasons the simple idea of 'referential meaning' fails.

First, language contains abstract logical terms: for instance, what 'objects' do terms like 'and', 'or', 'not', 'the', 'all', and 'some' refer to? These have essential functions in factual language: but they surely do not refer to *physical* objects. If we want to associate them with 'objects', these need to be rather 'abstract' objects. (In fact, the dominant 'formalist' tradition of C20th logic baulks at this problem, and treats 'logical terms' as different in kind to 'empirical' terms, and merely defines 'syntactic rules' for their use, without assigning any 'objectual meaning'; but in objectual theories, they are interpreted as abstract functions of one kind or another.)

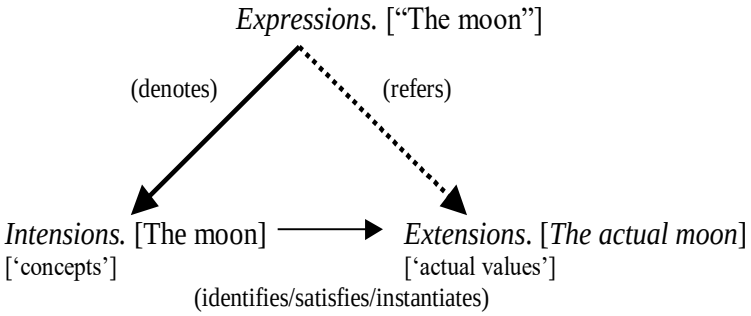
Second, even simple terms like 'the moon' are problematic, because although 'the moon' may refer to the physical moon, it connects to this object through what we colloquially call a 'concept'. This is more obvious with a term like 'unicorn': there are no physical unicorns, and 'unicorn' does not refer in fact to real, physical unicorns. But we clearly have a *concept* of unicorns, and it is most natural to take the term to refer in the first place to the concept. Similarly, we can have a concept of 'a moon of Jupiter', even if it has never been observed, or if it is unknown whether there is any such thing. We describe this concept in language, express a belief that there is a moon of Jupiter, and subsequently attempt to observe it. So even the simplest kinds of empirical terms seem to require a level of abstract objects – 'concepts' - to interpret. This is one of Frege's fundamental insights.

Third, we understand complex expressions, not just single words. *Statements* are the most important: we say that they *express propositions,* or that they *refer to facts.* But what are *propositions,* or *facts?* They are not just the statements or sentences – i.e. the linguistic items - they are what the statements or sentences *mean,* and in our objectual theory, they belong to the class of *objects* that we use to explicate the

meanings of the expressions. On the other hand, propositions can be made *about physical objects and events*: but they are not those physical objects or events themselves. Again, it is natural to say that propositions express *conceptions* – or *concepts of how the world might be*.

This suggests that there are *two levels of 'meaning':* expressions in a language first of all *denote* what we intuitively call 'concepts'; and these 'concepts' may or may not *refer to specific actual things,* or *actual values.* Frege proposed the first detailed theories of this kind, and his approach ultimately led to the development of *intensional logics*. In the simplest understanding of intensional logic, we can take the 'conceptual objects' to be represented by what are called *intensions,* and 'actual values' to be represented by *extensions.*

Two-level theory of meaning.



I should hasten to say that Tichý does *not* identify 'concepts' or 'meanings' as intensions. (In fact, he does not use 'concepts' as a technical term in his own theories, and it is only being used in an informal sense here). We will see shortly why he rejects this 'two-level' theory as too simplistic, but I will first explain the interpretation it initially suggests.

In this two-level theory, the solid line from *expressions* to *intensions* (or the *denotation)* indicates that this is taken as the primary relation of *meaning*; the relation to *extensions* or *actual values* (or the *reference)* is secondary. The actual reference is not itself part of the *conceptual meaning,* since the meaning of an expression is generally the same *whether or not it refers to an actual object*. E.g. we understand the meaning of terms like 'a moon of Jupiter' before we learn whether anything satisfies them. (The rejection of *references* as providing *meanings* is quite widely accepted – except perhaps for some special disputes about whether certain 'primitive meanings' might be based on a direct knowledge of the references).

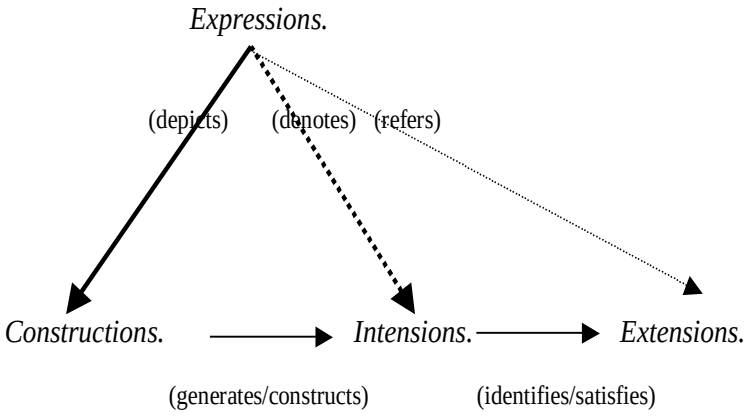
We need different names for the two kinds of relations: here I have used 'denotes' and 'refers', which is common, although there is still no widely agreed standard notation.

It should also be emphasized that *intensions* are not excluded from the class of *actual values* – for we can also refer to them *as things*. E.g. 'Tichý's favorite intension' denotes a higher-order concept, but *refers to* some 'actual intension' – perhaps that of 'political freedom', for instance. However, most kinds of *actual values* are not intensions (or concepts) – e.g. the actual moon, a physical object, is not a concept (although there is a concept of it).

In fact, most kinds of *actual values* are not simple concrete objects either: one example, to be examined shortly, are *truth values*, which are taken as the actual values of *propositions.*

This kind of two-level structure is common to many objectual theories, but Tichý rejected this scheme, and introduced a third level of structure, which looks something like this:

Tichy's three-level theory of meaning.



The main point here is that Tichý introduces a new level, which he calls *constructions.* This is his radical invention. In his view, complex expressions do not directly pick out *intensions*; instead, they represent ('depict') *ways of constructing or generating intensions – from simpler constructions and intensions*. For complex expressions are understood from their simpler parts; we *construct* the meaning of a complex term from the meanings of simple terms from it is composed; and it is this process of *construction* that Tichý focused on. It is a dimension of meaning that intensional logic alone does not describe.

The 'meaning' of an expression is now seen as the *construction*. Constructions are said to be *structured objects,* because rather than just grasping them as 'objects', they involve us in grasping structured 'procedures'. While this is still an objectual theory, meaning is no longer a simple denotation relation, as in the two-level theory.

Before going on to illustrate some details of Tichý's theory, I will briefly comment on some important background issues: what are concepts, or abstract objects? Is it plausible to have a theory that appeals to such things? And what is the aim of logic?

Abstract Objects and Logic.

We have appealed to a system of 'abstract objects' to provide objects of meaning – but what are they? Frege introduced a class of 'Concepts', and insisted they are neither physical nor mental - in particular, they are not 'thoughts', or mental images. The belief in the existence of abstract objects is called *Platonism.* And this brings us to a struggle philosophers have had over many centuries – ever since Plato – of understanding what 'abstract objects' could be. How do they exist? How do we perceive them? Are they necessary to our picture of the world – or are they merely figments of the philosophical imagination? This is one of the great historical divides in Western philosophy: philosophers who accept abstract objects in their scheme of things are called *Platonists,*

or sometimes *metaphysical realists*, while those who reject them are often called *nominalists*, or *anti-realists.*

Tichý, like Frege, was a Platonist. In fact, every serious attempt at giving an objectual theory of semantics seems to incorporate some form of Platonism – because the class of concrete existing things is not sufficient to represent the richness of the concepts that we use language to refer to and discuss. This remains a main ideological divide throughout 20th Century semantics. Those who reject objectual semantics – like the later Wittgenstein, Quine, and many others – do so primarily because they reject the idea of abstract objects, which they regard as viciously 'metaphysical'. Since objectual semantics requires abstract objects, they reject objectual semantics, and try to explain 'meanings' by appealing only to non-abstract things – typically, physical or mental objects and events, or actual human behavior, or linguistic use.

I will not go into the general dispute about metaphysics here. Instead, I suggest we should simply side-step it to begin with - just as we side-step it when we learn mathematics. For although mathematics is typically explained in a 'Platonist' manner (by appealing to the existence of the natural numbers for example), we rarely worry about metaphysics when we learn mathematics. Our first concern is just how it 'works', in a very practical sense.

We can regard an objectual theory of semantics in the first place as providing a *framework* for giving specific 'normative analyses' of interesting parts of real languages. The test is whether the framework allows an accurate analysis of our judgments about logical inferences, semantic relationships, computations of information, and so on. This is something we can check independently of whether we agree on metaphysics. We will soon see an argument, for instance, that the *intensional framework* alone is simply not rich enough to reflect the logic of propositional belief accurately – but this argument has nothing to do with abstract objects: it has to do with judgments about logical inferences, and the information we can sensibly extract from various kinds of propositions.

Now the general framework of any objectual theory inevitably seems to involve certain 'metaphysical' ideas, and we can hardly ignore this. But I think we can suspend judgment on the most *abstract* questions about 'metaphysics', which philosophers often like to begin with - at least until after we see whether an objectual theory 'works' or not as an effective tool for logical analysis. Does a given theory successfully capture the mechanics of meaning, evident in ordinary meaning computation, ordinary judgments about conceptual inferences and logical relationships, and so forth?

Tichý maintained the objective reality of certain kinds of abstract objects, and his arguments about this are very interesting, and worth studying in their own right. He also gives original analyses of various traditional metaphysical arguments. But I think his conclusions about metaphysical problems can be separated from the immediate aim of his semantic theories, which is to provide a framework for the study of meaning as we know it, rather than to advance a metaphysical theory for its own sake.

In this respect, Tichý's notion of what *logic* is about is also an important break from earlier traditions that dominated mid-20th Century logic and philosophy. He takes logic to be based on the study of *meaning.* While he introduces a number of formal systems to help perform logical analysis, he is not interested in the study of *formalism* for its own sake. Formal systems are only proposed to help with the analysis of meaning. He is particularly opposed to the 'formalist' program associated with Hilbert. This approach generally takes logic as a study of *formal systems,* or purely *symbolic systems*, or 'grammars', without reference to meaning as it is exemplified in real

language. This approach remains common in many modern approaches to logic as the study of 'abstract algebras'. These are often studied as purely formal structures, with the potential to represent information in one way or another, but divorced from the analysis of meaning in natural language. Tichý is opposed to this kind of approach, and insists that *logic is driven by meaning*, and should not be treated as an autonomous 'formal' discipline separated from the logical analysis of meaning.

This became very clear in the last major project he was engaged in before his death, which he called 'Meaning Driven Grammar' (MDG):

> "MDG is based on the hypothesis that form and meaning are inseparable and that an adequate grammar must generate not just well-formed sentences but sentence-meaning pairs. (Meanings are identified with logical constructions, a notion proposed and expounded in previous publications, especially *The Foundations of Frege's Logic.*)" [From: Abstract of Current Research Project, 1994].

This reflects Tichý's conviction that our *understanding of meanings* is a primary source of knowledge about the formal mechanics of language, and is essential to understanding the real processing of information in language. Formalist approaches reject this point of view, often because of philosophical objections to 'abstract objects'. Formalists often express the desire to 'clean up' logic by ejecting the 'ethereal' domain of meanings, or abstract objects, and sticking directly to the level of expressions and formal rules of syntax. They do want to study how formal systems can be used to represent *information*: but their concept of information is abstracted from the analysis of meaning in real languages.

Whether a formalist approach to information theory or to natural language processing can succeed without attention to a theory of meaning is not a question I will try to comment on. But I think the refusal to contemplate objectual semantics because of prior philosophical convictions against 'metaphysics' is premature, and adherents of the purely formal approach are missing out on some of the most exciting developments in 20th Century semantics and logic for the wrong reasons.

However, although we can usefully side-step general metaphysical controversies for a time, it should be emphasized that *conceptual epistemology* remains central in Tichý's theory. An account of meaning certainly needs support from a plausible account of conceptual knowledge. Many of Tichý's arguments appeal directly to principles or observations about conceptual knowledge and conceptual judgments. He also advances a more general theory of an 'epistemic framework', as a basis for his main theory of semantics. I will not try to give a general account of this part of his theory here. But one principle is of special importance to his approach, which I will describe next.

Compositionality.

After the Fregean notion of *objectual semantics*, perhaps the most central principle in Tichý's approach is called the *principle of compositionality.* Tichý also calls this the *Frege-Church principle,* and identifies it as originating with Frege, and being given precision by Alonzo Church (who after Russell was Frege's most important early interpreter and advocate).

This principle states that: *The meaning of a complex expression is a function of the meanings of its parts, and the way they are combined.*

This brings us back to a fundamental observation about language: we have only a limited number of basic terms – words – but we can combine them in an endless variety

of ways to produce meaningful complex expressions. And yet, we usually seem to understand the meanings of the complex expressions just from our knowledge of the meanings of component terms. Complex meanings are built up by combining meaningful parts. This is a kind of 'part-whole determinism': the combination of parts determines the whole, without adding anything extra or external.

Now this principle is not meant to rule out the fact that real language is often *ambiguous* – expressions do not always determine a *single* meaningful reading. Some expressions have multiple meanings – puns. But dealing with puns is considered a fairly trivial problem, and this hardly threatens the Principle of Compositionality. A more serious problem is with what are called *de dicto* and *de re* suppositions, where the same expression appears to take on alternative types of meaning in two different contexts. This was a stumbling point for Frege's theory – and it is a point where he appears to abandon the principle of 'compositionality'. This is a test-case for Tichý's theory, which solves the problem without abandoning compositionality, as we will see shortly.

We must recognize that natural language obviously has ambiguities and peculiarities, and we have to interpret our way through these. Natural language also has functions not connected with communicating information – but objectual semantics generally does not deal with these. It is intended to deal only with the primary communicative functions of language. The principle of Tichý's approach is that we should be able to provide *accurate and objective logical analyses of factual language.* To do this, we will often have to disambiguate natural language expressions, and explicate their intended content – which is why *analysis* is necessary in the first place.

To represent an explicit logical analysis, he introduces the 'idealized' logical language of TIL. This is a tool for explicitly representing the intended meanings of natural language expressions. Now the symbolic language of TIL itself has a strong property of 'compositionality' – i.e. it represents meanings in a way that adheres to the Principle of Compositionality. To apply it accurately to real language, we have to *analyse meanings accurately* – and we often have to disambiguate between possible readings of natural language expressions. This is by no means always trivial - as the treatment of simple 'puns' usually is – but this does not in itself represent a challenge to compositionality.

The serious challenge to 'compositionality' is found in the idea that meanings are only determined, in general, by aspects of the *context* in which statements are made, and that this context-dependence should itself be reflected in the 'logical language' used for analyzing meaning. In fact, this is the fundamental point on which Tichý criticizes both Frege's theory, and recent intensional semantics. As well as the *de dicto – de re* problem, we will also see a most important example of this in the treatment of statements about *propositional beliefs.* This problem is one reason Tichý found it necessary to introduce the concept of *constructions,* in addition to *intensions.*

This difficulty is now well-recognized in semantics – but Tichý was arguably the first to recognize the depth of the problem for intensional theories. And while some theorists have chosen to abandon compositionality, Tichý thinks this is a premature capitulation.

Focusing on compositionality also draws our attention to something that Tichý, again following Frege, took as of profound importance. The simple fact that simple terms are *combined with each other* to generate meaningful complex terms shows - in an objectual theory - that *the objects that provide the meanings of simple terms can be combined with each other, to generate other objects, which provide the meaning of the*

complex expression. For instance, 'The moon is yellow' combines the meanings of 'the moon' and 'is yellow', to make a complex proposition. How does this happen?

For it to happen at all, the objects that provide meanings of terms must be quite special: they must be able to *combine with other objects*. But what kinds of objects combine with each other?

The answer provided by Frege is that *functions* combine with *arguments* to produce *values,* or *results.* This is the essential nature of functions: they 'take' objects of one kind, and 'give' objects of another kind. We generally represent the combination of a function with some arguments by writing the terms for the functions and arguments next to each other: e.g. '1+2'. The syntactic juxtaposition of terms reflects something important: *functional application.*

Now the principle of Compositionality is routinely defined in terms like: "the meaning of a complex expression is a function of the meanings of the component terms and their *syntactic* combination." Their 'syntactic combination' means their order of juxtaposition in a sentence or phrase. But Tichý would not accept this: in his view, the *syntactic combination* also corresponds to something *objectual* – namely, functional application. His theory of constructions might very well be taken as an objectual theory of *the meanings* of syntactic combinations. Now this is something that is rarely explicated or even noticed – it is like an invisible framework of language – but a little reflection shows that placing two terms adjacent to each other in a sentence also has a meaning. E.g. placing 'Fred' adjacent to 'is asleep' *means that we combine their meanings in a definite way.* Or placing "+" in between "1" and "2" means we combine their meanings in a definite way. It is intuitively a functional application: '…+…" is a function; we can fill in the gaps with numbers, and calculate a result.

Tichý usually explained *constructions* by appealing to the idea that when understand complex expressions, we have 'calculate' what they mean somehow from their simple constituents; we go through some kind of 'procedure', where we put component meanings in one end, and get a newly constructed result out the other end. And he showed the value of this extension to ordinary intensional semantics by its success in solving certain stubborn semantic problems, for instance about individuating our beliefs about propositions. We will go on this shortly; but I think it is also useful to see this theory as a natural development in explicating the central notion of compositionality, which plays such an overriding role in his theory.

Seen in this way, Tichý's constructions can be taken an explicit theory of *the meanings of the kinds of combinations of meanings* required in a fully objectual theory. He proposed a formal theory, which identifies a few basic or primitive types of constructions, and iterative rules for combining them to give an open-ended hierarchy of constructions. The most intuitive examples of primitive constructions correspond to what we normally call *functional application* and *functional abstraction,* which he gives the technical names *composition* and *closure*. Other primitive constructions, which are not so intuitively obvious are called *trivialization, variables,* and *execution.* His precise formulation of this theory seems to me an indisputable stroke of genius. But first, let us start with a quick look at his theory of *intensions.*

Extensions and Intensions.

The notion of *intensionality,* or *intensional meaning,* is contrasted with *extensionality,* and was recognized by earlier writers. An important treatment is given in Carnap's (1947) system in *Meaning and Necessity,* and the concept was widely discussed in the 1960's. Tichý's first achievement, in creating an intensional logic, lay in reanalyzing

this notion, and formalizing it effectively. He proposes a system in which we use a small number of explicit categories of fundamental objects, essentially: *worlds, times, individuals,* and *truth values*, to construct *intensions.* The most revolutionary feature of this logic is that it introduces explicit quantification over *worlds*.

The idea is most obvious when we consider *propositions.* We regard factual propositions as being *true* or *false* (or sometimes as having no truth values, which we can ignore for the moment). For instance, the proposition that: *the author of Waverly is Scott* is *true*. But the truth values of typical propositions can *change with time,* and they are also generally only *contingent.* They are only the *values* that propositions take, at certain times, and in certain states of affairs. If the world were different, some propositions would have different values to those they actually have. Clearly, we understand the *meanings of propositions* in general before we know whether they are true or false. We have to investigate whether a given proposition is true or false by learning about the world.

Now if propositions take *true* and *false* as their values, *what are propositions themselves?* This is a characteristic kind of question that Tichý asks: he demands precise definitions of things like propositions, meanings, worlds, and so on. Tichý's first approach was to identify propositions as mappings or functions, which take us *from worlds* and *times* to *truth values.* To find the *value* of a proposition, we apply it first to a *world,* and then to a *time* in the world.

Worlds are conceived in the first instance as maximal classes of facts – including all historical past, present, and future facts about everything that ever happens. Of course we do not *know* what world we inhabit in this sense – we can never know all the facts about the world. But we can find out about some of the facts that hold in *our actual world.* This is generally what we must do to find out whether a given proposition is true or false. We can evaluate propositions at different times, because we can obtain knowledge about the actual world.

Thus, we come up with a kind of mathematical picture of *propositions.* If we let *P* be a proposition, then we identify it as a specific mapping, from worlds and times to truth values. Mathematicians indicate the structure of such mappings schematically, like:

$$P: (\omega,\tau)\rightarrow o$$

Or to be more precise, Tichý separates this mapping into two steps:

$$P: \omega \rightarrow(\tau\rightarrow o)$$

The symbols refer to basic categories of objects:

ω is the class of *possible worlds.*

τ is the class of *times.*

o is the class of *truth values,* i.e. {*True, False*}.

We evaluate an intensional proposition, *P,* in two steps: we first of all apply it to a *world, w,* (from the class ω) and this gives us *a mapping from times to truth-values*; we then apply this to a time, *t* (from the class τ) and this gives us *a truth-value.*

We can say that the actual truth-value of a proposition *P* at the present time is the *extension of P,* while *P* itself is the general mapping (an *intension*). This is called an *intensional theory of propositions,* and is the first major step in defining Tichý's system.

There is a similar duality between *intension* and *extension* for almost every semantic category. E.g. a *set* or *class* is defined by its specific members, and classes are

the *extensions* (or values) of *properties.* A class is defined by its members, whereas a property may pick out one set of things in one possible world, at one time, and another set in a different possible world, or at a different time. Thus a property *U* is an *intension* that looks like:

$$U\text{: } \omega \rightarrow (\tau \rightarrow \{\iota\})$$

Where we symbolize:

ι is the class of *individuals* (or objects)

Notice that the property maps to a *sub-class of individuals,* not just a single individual. This may be an empty class, as always the case for an impossible property, e.g. the property of *being round and square.*

Note that Tichý also treats *classes* themselves as mappings from *members* of the class to *truth-values*; hence the sub-class indicated above by: $\{\iota\}$ is defined as a mapping of the form: $(\iota \rightarrow o)$, where an individual *i* maps to *true* just in case *i* is in the sub-class.

A trickier case involves the distinction between *names,* like 'Scott', and *definite descriptions,* like 'The author of Waverly'. In the simplest case, we can take a *name* to refer directly to an *individual.* That is, 'Scott' is assumed to name the same individual in every possible world, and at every time (similar to what Kripke calls a *rigid designator*). A definite description, on the other hand, refers to different individuals or objects at different times or in different possible worlds. Tichý calls this kind of semantic category an *office* (from the notion of a 'political office', like 'the office of President').

The simplest kind of *office* is a mapping from worlds and times to individuals. It is different to a property, because the mapping is to a single individual, not to a class – i.e. the extension of an office is an individual. This mapping looks like:

$$\mathbf{A}\text{: } \omega \rightarrow (\tau \rightarrow \iota)$$

But there is also a question about whether proper names should be taken as 'hidden descriptions' – rather than being directly interpreted as particular world-independent individuals. If so, we may take them as offices as well (as with *the Moon* below). If we took *all* proper names as offices, we might be left only with variables for individuals: but we still need individuals in Tichý's system to provide references, and to interpret variables over individuals. But I will not go into this tricky question here.

A above is an example of an *individual office.* But we may also specify offices like "Scott's favorite proposition", for example, where the office is filled (if at all) by a proposition – which is already a complicated function. Thus, we can build up higher-order objects, with functions embedded in other functions.

Because of the need to do this, the basic theory of classes or functions that Tichý employs is a *hierarchical type theory,* like Russell and Whitehead developed in *Mathematica Principia.* The use of this theory, called the *ramified theory of types,* allows us to construct higher-order classes and functions in a consistent way, and to quantify over and refer to higher-order objects. This is the most unfamiliar part of the theory from the point of view of modern 'mathematical logic', which rarely goes beyond the study of first-order logic. First-order logic is preferred in mathematics, because it is deductively complete, while second and higher-order logics are not deductively complete. However, higher-order logics have far more expressive power

(for instance, Godel's famous theorem that there is no complete axiomatisation of arithmetic only applies to *first order axiomatisations:* in second order logic, we can indeed give a complete axiomatisation of arithmetic, but now the proof theory is incomplete).

This increased power is necessary for any theory like Tichý's. The deductive incompleteness may seem regrettable to mathematicians, who prefer to be able to prove all the logical consequences of their theories deductively (and consequently limit the range of theories they contemplate to ensure this); but it is logically unavoidable in the analysis of real languages, which allows reference to higher-order entities, such as properties-of-properties, properties-of-propositions, and so forth.

Another example involves the *propositional connectives,* such as *And, Or,* and *Not.* E.g. given two propositions, call them *P* and *Q,* we can form the new proposition: *P and Q,* meaning that *both P and Q are true.* An extensional treatment lets us define the operator *And* as a mapping from a pair of truth-values to a new truth-value. This has the functional form: *And: (o,o)→o.* It is world-independent, or analytic – it represents the same mapping in every world. In intensional logic, the propositions *P* and *Q* that we connect are not just truths values, but intensions, and the result is not just a truth-value, but another intension. However the term '*And'* is still analysed in Tichý's theory as just a simple truth-function, which operates on the truth-values, because we can define the *truth conditions* simply by: *P and Q* is true at a world *w* and time *t* just in case *P is true at w and t, and Q is true at w and t.* And similarly with other truth-connectives, like *not* and *or.*

The treatment of these 'logical terms' also illustrates the difference between 'formalist' conceptions of logic and objectual semantics. In the formalist view, which is very common in modern texts, logic is said to treat the formal or syntactic rules for a special category of 'logical symbols', such as 'and', 'or', 'not', 'all' and 'some'. These purely *syntactic* rules are often claimed to 'define the meanings' of these terms; and this is regarded as a completely different kind of 'meaning' to 'empirical meanings', of 'empirical terms'. But in objectual semantics, the meaning of logical terms is treated continuously with the meanings of empirical terms – they are all given objectual meanings. The difference is not that that there are 'two kinds of meanings', logical and empirical, but that the logical terms have *world-independent* meanings: they refer to the same functions in every possible situation, which makes it possible to define them exhaustively.

Tichý's Symbolism for Transparent Intensional Logic.

Tichý's term '*transparent* intensional logic' reflects the use of a symbolism to 'transparently' reflect the logical structures of expressions. Two main features are the definition of a system of *logical types,* and the use of *world-time indices*. For example, he uses the type symbolism (1988, p.202):

Office:	*Type:*	*Description:*
A	$\iota_{\tau\omega}$	the office of the author of *Waverly*
P	$o_{\tau\omega}$	the proposition that the author of *Waverly* is a poet.
U	$(o\iota)_{\tau\omega}$	the property of being a poet.

The type of *extension* of the object is indicated by the main symbols on the left; the subscripted world-time indices indicate the type of mapping. Note that a *class* in this system corresponds to a type of function: $(o\iota)$, which maps individuals from ι to truth-values from o - an individual is mapped to *True* if it belongs to the class.

More complex objects then have embedded types: e.g. the tricky relation of propositional belief: '... believes ...' is initially analysed (p. 202) with the type:

Office:	*Type:*	*Description:*
B	$(o\iota o_{\tau\omega})_{\tau\omega}$	the relation between individuals and propositions they believe.

$o_{\tau\omega}$ is the type of a proposition; $(o\iota o_{\tau\omega})$ represents a relation between individuals and propositions (i.e. the extension of 'beliefs' at a particular world-time); and $(o\iota o_{\tau\omega})_{\tau\omega}$ represents the *intension* of this relation.

This means, for instance, that the term '**A'** defined above names a function: it can take a *world, w,* and a *time, t,* as arguments, to give an individual, *i* – i.e. the author of Waverley in world *w* at time *t.* (This mapping has two distinct steps: first **A** is applied to the world *w,* to generate a mapping from times to individuals; then this is applied to *t* to generate an individual.)

Tichý writes functional applications with subscripted indices, like: $\mathbf{A}_{wt}$. Note that the Greek letters ω and τ are used for the *classes of worlds and times,* respectively; the Arabic *w* and *t* are variables over *particular worlds and times.*

In explicating the logical forms of sentences or phrases, Tichý then shows world-time dependences explicitly, by using abstractions on world-time indices to construct these functions, and give a vivid 'depiction' of the structures of the logical mappings involved. E.g. he explicates the sentence: "The author of *Waverley* is a poet" as:

$$\lambda w \lambda t.\mathbf{U}_{wt}\mathbf{A}_{wt}$$

The lambda is the abstraction operator: it is the inverse of functional application; so whereas: $\mathbf{A}_{wt}$ applies the world-time function **A** to the values *w* and *t* to generate a value (an individual in this case), $\lambda w \lambda t.\mathbf{A}_{wt}$ reverses the process, and returns us to the original function, **A**. We could just write: **UA**¸ but the expansion to: $\lambda w \lambda t.\mathbf{U}_{wt}\mathbf{A}_{wt}$ shows the functional construction, in a transparent and logical way, and this symbolic system is one of the keys to the success of *Transparent Intensional Logic.*

This symbolic system is quite different to Montague's system, which does not show world and time quantifications explicitly, and gives a much more 'opaque' symbolism.

The application of Tichý's theory is illustrated next with an important example.

The *De Dicto - De Re* Distinction.

An important application of TIL is to solve an old semantic problem that was first investigated by medieval philosophers, who defined a distinction between *de dicto* and *de re* occurrences of certain terms. For example, suppose that Edward has actually murdered John, and consider:

(1) The detective is arresting the murderer of John [*de re*]

(2) The detective is looking for the murderer of John [*de dicto*]

In the first sentence, it is natural to say that Edward (the person who actually murdered John) is being spoken about – even though his proper-name, 'Edward', has not been used. For given that the detective is arresting the murderer of John, and Edward is the murderer of John, it follows that *the detective is arresting Edward.* So (1) is *about Edward.* Note that this implies that we interpret the relation: *…is arresting…* as a relation between *individuals*. Here we say that the term 'the murderer of John' is used with a *de re supposition,* because it makes reference to the *individual who satisfies the office,* i.e. Edward.

But the second sentence is potentially ambiguous. There are two ways to understand it.

A. de dicto case. The detective *does not know who the murderer of John is,* and is trying to find out who the murderer is.

B. de re case. The detective knows who the murderer is, and is looking for that person – i.e. Edward.

In case (B), the situation is similar to (1): Edward is being spoken of. In this case, we clearly interpret the relation: *…is looking for…* as a relation between *individuals*.
But in case (A), Edward himself is not being spoken of at all. For instance, we may suppose the detective knows Edward, and knows where he lives, but does not know that Edward is the murderer. If he were looking for Edward, he would go to Edward's house; but he does not go to Edward's house, and we must conclude that *the detective is not looking for Edward* – even though the detective *is looking for the murderer of John*.

Thus, in case (B), it seems the term 'the murderer of John' is being used to refer to Edward, while in case (A), it is not used to refer to Edward. But this seems to contradict the Principle of Compositionality: the term 'the murderer of John' seems to be interpreted with two quite different meanings, and the ambiguity needs to be decided from the context, before the meaning can be decided.

But Tichý's theory allows a way of solving this kind of problem, without contradicting Compositionality, and without making the term "*the murderer of John"* ambiguous at all. In his view, the ambiguity is real, but it is found in the term *"is looking for"*. In case (B), we saw that: *…is looking for…* is a relation between two individuals (the detective and Edward); in case (A), *looking for* must be interpreted as a relation which holds between an individual (the detective) and an *office* (in this case, *the office of being the murderer of John*). It is a different relation altogether from that assumed in case (A).

It may first appear a little odd to say that a person *looks for an office,* rather than for a physical object. But it makes sense when we consider that 'looking for the murderer of John' in case (A) really involves something like *seeking an item of knowledge about the office,* rather than directly seeking to 'locate' the abstract object. It does not involve 'locating the office in space', which is impossible, because offices do not have locations in space. Rather, we would say that the detective has succeeded in his search for the murderer of John when he has correctly identified which individual satisfies the office. And this is a search to identify an 'abstract' object, not a physical object.

Let us symbolize the logical structures of these sentences, using **D** for *the detective* (which we will take to be an individual), **E** for Edward (an individual), **M** for *the murderer of John* (an office of individuals), **A** for *is arresting* (a relation between

individuals and individuals), and two different symbols for the two different meanings of: *is looking for*, $\mathbf{L}^{\mathbf{A}}$ for the first meaning (a relation between individuals and offices), $\mathbf{L}^{\mathbf{B}}$ for the first meaning (a relation between individuals and individuals). Then we analyse:

(1')	$\lambda w \lambda t.\mathbf{A}_{wt}\mathbf{DM}_{wt}$
(2'.A)	$\lambda w \lambda t.\mathbf{L}^{\mathbf{A}}{}_{wt}\mathbf{DM}$
(2'.B)	$\lambda w \lambda t.\mathbf{L}^{\mathbf{B}}{}_{wt}\mathbf{DM}_{wt}$

Tichy uses a Polish notation, without brackets. In a bracketed notation, we would write: $\lambda w \lambda t.\mathbf{A}_{wt}(\mathbf{D},\mathbf{M}_{wt})$, to indicate that **D** and $\mathbf{M}_{wt}$ are the arguments of the relation **A.** This can seem a little confusing if you are used to bracketed notation.

Now we have taken a single analysis of the term **M** – it is the office of the murderer of John. But there is a clear difference in the logical structures of (2'.A) and (2'.B).

Now neither proposition mentions Edward directly by name; but there is a clear sense in which (1') and (2'.B) are *about Edward (given that Edward is the murderer)*, whereas (2'.A) is *not about Edward at all (whether or not he is the murderer).* To see this, suppose that we *evaluate* these intensional propositions and objects at a specific world and time, *w, t*, and we get the *values:*

(1*)	The value of:	$\mathbf{A}_{wt}\mathbf{DM}_{wt}$	is *True*
(2*.A)	The value of:	$\mathbf{L}^{\mathbf{A}}{}_{wt}\mathbf{DM}$	is *True*
(2*.B)	The value of:	$\mathbf{L}^{\mathbf{B}}{}_{wt}\mathbf{DM}_{wt}$	is *True*

And we add that *Edward is the murderer of John* in world *w* at time *t*, so that:

(3*)	The value of:	$\mathbf{M}_{wt}$	is **E**

Now it follows from (1*) and (3*) that:

(1**)	The value of:	$\mathbf{A}_{wt}\mathbf{DE}$	is *True*

I.e. *the detective is arresting Edward* (in world *w* at time *t*). This process of evaluating the *references* of the terms is called *semantic descent:* we go from the *office*, **M,** to its *value* or *reference* in a world at a time, $\mathbf{M}_{wt}$.

In exactly the same way, it follows from (2*.B) and (3*) that:

(2.B**)	The value of:	$\mathbf{L}^{\mathbf{B}}{}_{wt}\mathbf{DE}$	is *True*

But it does *not* follow from (2*.A) and (3*) that:

(2.A**)	The value of:	$\mathbf{L}^{\mathbf{A}}{}_{wt}\mathbf{DE}$	is *True*

In fact, (2.A**) is badly formed (a null construction), because we have specified $\mathbf{L}^{\mathbf{A}}$ as a relation between individuals and *offices*, but **E** is an individual not an office. However, the meaningful statement, (2.B**), also does *not* follow from (2*.A) and (3*).

Thus Tichý's solution does not depend on any ambiguity in the meaning of the phrase 'the murderer of John'. It depends instead on an ambiguity in the phrase: "is looking for".

This ambiguity is real – it is an ambiguity of ordinary English. To analyse the statements accurately we have to disambiguate the intended meaning. In the accurate logical language of TIL, this ambiguity evaporates – TIL has perfect compositionality. Naturally, TIL is a more explicit logical language than ordinary English – since if English was perfectly explicit there would be no need for a separate logical language to explicate meanings.

The *failure* of compositionality occurs in other logical theories, like Frege's, for example, where the term 'the murderer of John' is taken to have two different meanings in two different contexts – acting as a name of an individual in case (B), and as the name of a Fregean 'sense' in case (A). Tichy's analysis seems to show that this is the wrong way to analyse the problem.

Inadequacy of Intensions to Serve as Objects of Propositional Belief.

Intensional logic is extremely useful and enlightening for semantic analysis: but Tichý seemed to recognize, even as he conceived it, that it has a serious limitation. The problem is that *intensions* alone are not fully capable of representing *propositional meaning,* seen through the logic of *propositional beliefs.* A simple argument shows this.

The *intensions* of any two true mathematical theorems are the same, because a true mathematical theorem is *true in every possible world, at every time.* Take for instance: *1+2=3,* and: *16x16 = 256*. These are both true in every world at every time, and hence their *intension* is simply the mapping from every world and time to the value *true.*

But many people believe that: *1+2=3* is a true proposition, without believing that: *16x16=256* is true. Hence, the *intensions* cannot completely represent the full 'propositional meaning', in the intuitive sense of that term, where we say that we 'believe one proposition but not the other'. There are far more mathematical theorems than available intensions.

The most popular response to this initially is called 'semantic ascent', advocated by Quine. This says that we individuate mathematical theorems in a more fine-grained way, on the basis of their expressions through mathematical sentences. Obviously it may be true that "Fred knows that *1+2=3",* while it is not true that "Fred knows that *16x16=256*." On the 'semantic ascent' view, we distinguish the two cases by observing that Fred knows that the *sentence*: "1+2=3" is true, but he does not know that the *sentence* "16x16=256" is true. On Quine's view, we are supposed to reduce all talk about our beliefs in propositions to talk about beliefs in the sentences or statements (linguistic items) that we use to express them.

But Tichý argues that this proposal fails for another reason. If Fred knows that *1+2=3,* then surely Fred knows that *one plus two equals three.* These are just two different ways of expressing the same item of mathematical knowledge, one in mathematical symbols, the other in English. Yet, suppose Fred is Czech, and does not know English. He does not recognize the meaning of the English sentence: "*one plus two equals three"*. So on the semantic ascent view, we would have to say that he does not know that *one plus two equals three,* while he does know that *1+2=3.* Yet these two statements seem to represent exactly the same item of mathematical knowledge.

The problem arises for empirical beliefs as well; the mathematical example is just a particularly simple one. The problem arises as long as we hold that *we can express the same item of propositional knowledge in different languages,* i.e. using different expressions. If propositional knowledge was differentiated not just by its 'objective content', but by the *expressions* used to state it, this would not be possible. Hence, Tichý concludes that:

> "Propositions (construed as [intensions]) are thus too coarse-grained, and sentences too fine-grained to serve as objects of mathematical beliefs. We obviously need a category of objects which falls between these two extremes. The category of constructions is an obvious candidate." (1988, p.222).

This mathematical example is used because it is very simple, but the same argument applies to empirical language generally, and Tichý's aim is to give a framework for semantics of factual natural language, not just mathematics. This point is a more general objection to what is known as the Tarskian view of meaning: that *the meaning of an expression is given by its truth-conditions.* This seems a sensible idea to begin with: to explain what an expression means, we need to explain what would have to be the case for it to be true. But in its simplest form, at least, it suffers the same problem as taking *intensions* to represent meanings: logical tautologies and mathematical theorems are *always true,* so their truth conditions are identical: *they are true.* But we do not explain what they *mean* just by saying that they are true. We have to explain what they are *about.*

Let us now consider Tichý's concept of constructions, which is used to solve this problem.

Constructions.

In intensional logic, a meaningful statement denotes an *intension* a mapping from worlds and times to truth values. But when we understand a complex expression, we do not just jump automatically to the correct intension: rather, we go through a procedure to 'calculate' it, from our knowledge of the meanings of the more basic parts of the expression. This is what the Principle of Compositionality tells us. Tichý identifies meaning with the procedures for 'calculating' meanings, which he calls *constructions*.

The *constructions* involve the objects mentioned in the complex expressions. So, for example, we have to calculate that: *16x16=256,* by considering all the objects involved: the numbers *16* and 256, and multiplication function, and equality. This is different from the calculation of *1+2=3.* On the other hand, the calculation of *1+2=3* is the same thing as the calculation of *one plus two equals three* – at least, it is the same as long as we identify the objects named by the terms '1', '2', '3', '+', and '=' as the same objects as 'one', 'two', 'three', 'plus' and 'equals' respectively.

Hence, if we take the 'calculations' we make into account, we may be able to satisfy both these requirements: (a) differentiate the meanings of true propositions that nonetheless involve different objects (*1+2=3* involves a different calculation to *16x16=256*), and also (b) identify the meanings of some propositions expressed in two different languages, or using two different expressions (*1+2=3* involves exactly the same calculation as *one plus two equals three*).

Tichý's theory of constructions is an explicit theory of what is involved in *the calculation of intensions*. We can easily see how we need to do this in specific cases. For instance, the intension of the statement: 'The moon is yellow' is arrived at by

starting with the basic objects: *the moon,* and *is yellow.* These must be taken as *intensions* to start with. We then construct a new intension by taking the combination of these two more basic intensional objects. This new intension is a proposition, i.e. a mapping from worlds and times to truth values. Let us call this '**P**'. The mapping for **P** is determined as follows:

For any world-time couple, (*w,t)*:

- **P** maps (*w,t*) to *true* just in case (i) the office *the moon* maps (*w,t*) to an individual object, *i,* and (ii) the office *yellow* maps (*w,t*) to a class of individuals, which contains *i* as a member.
- **P** maps (*w,t*) to *false* just in case (i) the office *the moon* maps (*w,t*) to an individual object, *i,* and (ii) the office *yellow* maps (*w,t*) to a class of individuals, which does not contain *i* as a member.
- **P** maps (*w,t*) to nothing (*null*) just in case (i) the office *the moon* fails to map (*w,t*) to any individual object, *i,* or (ii) the office *yellow* fails to map (*w,t*) to any class of individuals.

(The third case is important, and explains how some meaningful expressions may fail to have any value in certain circumstances. E.g. on this view, the proposition: *the King of France is bald* currently has no value (is *null*), rather than being false, because *the King of France* has no value. This differs from Russell's famous analysis of such statements, which would make this proposition *false* at the present time. It also differs from accounts that attempt to introduce more than two truth-values: *null* is not a third truth-value, it is the lack of any truth-value.)

Tichý would symbolize the logical structure in intensional logic as:

$$\lambda w \lambda t . \mathbf{Y}_{wt} \mathbf{M}_{wt}$$

with **Y** representing the property *is yellow* and **M** representing the office of *the moon.* And this constructs the same object as: **YM**, but with the construction displayed more explicitly.

But Tichý's theory of constructions goes a step further, and explicitly analyses the *constructions represented by complex symbols.* In the present case, the construction is called *application,* or *functional application:* the intensional object **Y** is applied to the intensional object **M**, as a function is applied to an argument, to generate the intensional object: **YM**. The *construction itself* is explicitly symbolized in Tichý's theory as: $\lambda w \lambda t . [{}^0\mathbf{Y}_{wt}\ {}^0\mathbf{M}_{wt}]$ (see below).

It seems plausible that we actually grasp such 'logical constructions', somehow, as abstract 'procedures', and this is central to our grasp of meanings. But how many different kinds of constructions are there? Can we define them all? Given that we seem to understand how to perform complex constructions by combining simpler constructions, Tichý proposes that constructions can be defined *recursively*, from a few simple or primitive types, which can be applied to each other to build more complex constructions. Tichý's main theory of constructions proposes six types of primitive constructions:

(i) *Variables* are primitive constructions, denoted by terms like: '*t', 'w', 'i', 'x', 'y', 'z',* etc.

(ii) *Trivialization* is the simplest construction: it takes an object, *X*, and generates *the same object*. This construction is written: '^0X'.

(iii) *Composition* corresponds intuitively to functional application: if *F* is a function and *x* is an argument, we often write: *Fx* to indicate the application of *F* to *x.* This construction is written: '[*F x*]'.

(iv) *Closure* corresponds intuitively to what we call *functional abstraction:* if: '*Fx'* expresses the application of *F* to *x,* then we can return to *F* itself by leaving a 'gap' for the argument *x.* This construction is written as: 'λx.*Fx'*.

(v) *Execution* corresponds to 'carrying out' or 'executing' a construction. The execution of *X* is written: '$^1X'$.

(vi) *Double Execution* is used if *X* constructs a construction; it corresponds to the execution of the latter. This is written: '$^2X'$.

(There is some dispute about whether the last two types of construction are ultimately necessary; Materna (1998) dispenses with them).

This small set of constructions can be applied recursively, to build complex constructions. To define this consistently, Tichý was forced to adopt a *typed hierarchy of constructions,* and the full theory is quite complicated. (In fact, his first formulation of the theory, in his (1986) "Constructions", does not use a typed hierarchy, but consequently suffers from an inconsistency, similar to that which Russell found in Frege's theory of logic. He corrected this flaw in subsequent versions. Materna (1998), Chapter 3, gives a good presentation of the essential theory).

I will not try to explain the technical details here. But the conception of *variables as constructions* deserves special comment, as a most remarkable and unusual feature of the system.

Variables as Constructions.

The standard treatment of *variables* in mathematics and logic just takes them as *letters: 'x', 'y', 't',* etc. We write formulas using these letters, and we quantify over them. We evaluate their meanings through the notion of Tarskian *valuations,* which involves assigning objects or individuals as values to variables. E.g. a universal quantification: (For all *x*)(*Fx*) is true just in case: *Fx* is true on *every valuation of x.* Tichý uses the same concept of *valuations,* but he has found a way to interpret variables themselves *objectually,* rather than taking them as letters, or syntactic items. That is, he has found a way to identify variables *as objects.* The objects in question, however, are not ordinary objects, or functions, or anything contemplated in purely intensional semantics: variables are a special kind of *primitive construction.* Variable letters (*'x', 'y',* etc) are therefore treated as names of special objects.

Variables are called *incomplete constructions,* because they only construct specific objects when they are combined with *valuations.* We still use Tarski-type valuations to bring variables into play. Superficially, the mechanics of the system is little different to the ordinary treatment of variables. But there is a deep impact: expressions for variables, like all other expressions, are given an objectual interpretation, and this allows a fully unified objectual semantics.

I will now sketch how this system is used to analyse the previous problem about propositional beliefs.

Propositional Attitudes.

The role of constructions comes to the fore in Tichý's analysis of propositional attitudes, such as *beliefs about propositions*. I will not try to explain the mechanics of this in detail, because the theory of constructions is too involved, but the general idea of the solution is fairly simple.

Tichý analyses the proposition that: *John believes that 1+2=3* as stating a relation between *John* (the individual) and the *construction* represented by '1+2=3' (*not just the intension*). Now the *construction* is a complex, structured entity, in which the primitive constructions of *1, 2, 3, +,* and = are parts. The construction (when *executed)* takes these parts, and gives a *truth-value* as a result.

Since beliefs are taken as being *about constructions*, a belief about the construction: *1+2=3* is distinct from a belief about the construction: *16x16=256.* The first belief, for example, is in part *a belief about the number 3;* the second is not a belief about the number 3 at all.

This theory also allows us to infer that if: *John believes that 1+2=3,* then: *John believes that one plus two equals three,* because we would analyse the sentence: '1+2=3' as representing exactly the same construction as: 'One plus two equals three'.

This is how Tichý proposes to solve the problem of individuating propositional beliefs, in a purely objectual way. As he says, constructions provide a category of objects which falls between *intensions* (which are too coarse-grained to individuate propositional beliefs), and *sentences* (which are too fine-grained).

The fact that constructions: (i) seem to provide objects which individuate beliefs with exactly the right degree of detail, and (ii) seem intuitively correct for their role of explaining how complex meanings are formed from simpler meanings, and (iii) allow us to satisfy the Principle of Compositionality in a precise way, gives a strong case for adopting them as the fundamental objects of *meaning* in an objectual theory of semantics.

Further Topics.

I have sketched some details of Tichý's conception of TIL; but this is only a brief sketch, and the full development of his system involves a lot of fascinating problems. Tichý has pursued deep questions about time, worlds, truth, individuals, identity, logical possibility, logical paradoxes, logical limitations on languages, counterfactuals, conceptual epistemology, logical analysis, and many other concepts that are intimately involved in our systems of understanding meaning, logic, metaphysics, and empirical knowledge. He has done a considerable amount of technical work on foundational theories of logic, and his fluent use theories such as the ramified theory of types, recursive function theory, abstraction operators, and Godelisation techniques can be quite demanding at times. However the primary focus in most of his work is on conceptual analysis and arguments, rarely on formal results for their own sake, and although his arguments are quite detailed and dense at times, much of his work is approachable without requiring much specialized training in mathematical logic.

General References.

Materna, Pavel. 1998. *Concepts and Objects*. Acta Philosophica Fenica, vol. 63.
The best book on Tichý's theory of Transparent Intensional Logic to date, Materna presents many of the ideas of TIL, and develops his own original treatment of many points. Quite technical.

Montague, Richard. 1974. *Formal Philosophy: Selected Papers of Richard Montague.* (Edited by Richmond Thomason.) Yale University Press.
The classic collection of papers by the famous inventor of intensional logic.

Oddie, Graham. 1986. *Likeness to Truth*. D. Reidel.
This presents the theory of 'verisimilitude', originated by Oddie and Tichý , to try to solve the problem of how the veracity of scientific theories can be judged. Inspired by Tichý 's (1974) paper on Popper's theory, this contains clear and simple explanations of the concept of the 'logical space' employed by Tichý.

TIL (Transparent Intensional Logic) Website:
http://www.phil.muni.cz/fil/logika/til/index.html
This website is a primary reference point for scholars interested in Tichý, and contains an extensive collection of papers, references, and biographical details. Two good papers for the specialist to begin with:

Duží, Marie and Pavel Materna. "Constructions"
http://www.phil.muni.cz/fil/logika/til/constructions_duzi_materna.pdf
Duží, Marie and Pavel Materna. "Parmenides' principle"
http://www.phil.muni.cz/fil/logika/til/materna_duzi_parmenides.pdf

Tichý , Pavel. 1988. *The Foundations of Frege's Logic,* de Gruyter, Berlin and New York.
Tichý 's most important single work, this book introduces many of the ideas of TIL, although it does not replace many of the detailed discussions found in his earlier papers.

van Benthem, Johan and Alice ter Meulen, 1997. *Handbook of Logic and Language.* M.I.T. Press.
A generally outstanding scholarly reference on modern logic and intensional semantics, but unfortunately with little acknowledgement of Tichý 's work.

Bibliography: Pavel Tichý.

The following is a fairly complete bibliography of Tichý's work.

Books.

1. *Logika pro PI (Logic for Pedagogical Institutes),* Prague 1963, 191 pp.
2. *Logicka stavba vedeckeho jazyka (The Logical Structure of the Language of Science),* Prague 1968, 272 pp.
3. *The Foundations of Frege's Logic,* de Gruyter, Berlin and New York 1988, 333 pp.

Collected Papers in Logic and Philosophy.

1. Eine Exposition des Gödelischen Unvollständigkeitsbeweises in der einfachen Typentheorie*, Acta Universitatis Carolinae, Philosophica et Historica* 5*,* 95-7, 1962.
2. Mají logicky pravdivé věty obsah?*, Filosofický Časopis* 13*, 1965, 82-86.*
3. K pojmu problému a řešitelnosti*, Kybernetika* 3*, 1966, 105-109.*
4. K explikaci pojmu obsah věty*, Filosofický Časopis* 14*, 1966, 364-372.*
5. Smysl a procedura*, Filosofický Časopis, 1968.*
6. Intentions in Terms of Turing Machines, *Studia Logica* 26, 1969, 7-25.
7. An Approach to Intensional Analysis, *Noûs* 5, 273-97, 1971.
8. On the Vicious Circle in Definitions, *Studia Logica* 28, 19-40, 1971.
9. Synthetic Components of Infinite Classes of Postulates, *Archiv f. Mathematische Logik und Grundlagenforschung* 17, 167-78, 1971.
10. Plantinga on Essence: A Few Questions, *The Philosophical Review* 81, 82-93, 1972.

11. On de dicto Modalities in Quantified S5, *Journal of Philosophical Logic* 2, 387-92, 1973.
12. On Popper's Definitions of Verisimilitude, *British Journal for the Philosophy of Science* 25, 155-88, 1974.
13. What Do We Talk About?, *Philosophy of Science* 42, 80-93, 1975.
14. A Counterexample to the Stalnaker-Lewis Analysis of Counterfactuals, *Philosophical Studies* 29, 271-3, 1976.
15. Verisimilitude Redefined, *British Journal for the Philosophy of Science* 27, 25-42, 1976.
16. Verisimilitude Revisited, *Synthèse* 38, 175-96, 1978.
17. Two Kinds of Intensional Logic, *Epistemologia* 1, 143-64, 1978.
18. A New Theory of Subjunctive Conditionals, *Synthèse* 37, 433-57, 1978.
19. Questions, Answers, and Logic, *American Philosophical Quarterly* 15, 275-84, 1978.
20. De dicto and de re, *Philosophia* 8, 1-16, 1978.
21. Existence and God, *The Journal of Philosophy* 76, 403-20, 1979.
22. Merrill on What a Sentence Says, *Philosophical Studies* 37, 197-200, 1980.
23. The Transiency of Truth, *Theoria* 46, 165-82, 1980.
24. The Logic of Temporal Discourse, *Linguistics and Philosophy* 3, 373-369, 1980.
25. The Semantics of Episodic Verbs, *Theoretical Linguistics* 7, 264-96, 1980.
26. Foundations of Partial Type Theory, *Reports on Mathematical Logic* 14, 52-72, 1982.
27. The Logic of Ability, Freedom and Responsibility, *Studia Logica* 41, 227-48, 1982, co-author Graham Oddie.
28. Kripke on Necessity a posteriori, *Philosophical Studies* 43, 225-41, 1983.
29. Ability and Freedom, *American Philosophical Quarterly* 20, 135-47, 1983, co-author Graham Oddie.

30. Subjunctive Conditionals: Two Parameters vs. Three, *Philosophical Studies* 45, 1984.
31. Do We Need Interval Semantics?, *Linguistics and Philosophy* 8, 263-82, 1985.
32. Indiscernibility of Identicals, *Studia Logica* 45, 257-73, 1986.
33. Putnam on Brains in a Vat, *Philosophia* 16, 137-46, 1986.
34. Frege and the Case of the Missing Sense, *Grazer Philosophische Studien* 27, 27-47, 1986.
35. Constructions, *Philosophy of Science* 53, 514-34, 1986.
36. Two Fallacies of Formal Semantics, *Essays in Honour of Bob Durrant* Martin Frické (ed.), 156-177, Otago University, 1986.
37. Einzeldinge als Amtsinhaber*, Zeitschrift f. Semiotik 9, 13-50, 1987.*
38. Resplicing Properties in the Supervenience Base*, Philosophical Studies* 58, 259-69, 1990, co-author Graham Oddie.
39. Sinn und Bedeutung Revisited, *From the Logical Point of View* 1, 1-10, 1992.
40. The Scandal of Linguistics, *From the Logical Point of View* 1, 70-80, 1992.
41. The *Tractatus* in the Light of Intensional Logic, *From the Logical Point of View*, Vol. 3, No.2, 32-41, 1994.
42. Cracking the Natural Language Code, *From the Logical Point of View*, Vol. 3, 6-19, 1994.
43. The Myth of Non-Rigid Designators, *From the Logical Point of View*, Vol. 3, No.2 20-30, 1994.
44. The Analysis of Natural Language, From *the Logical Point of View*, Vol. 3, No.2, 42-80, 1994.
45. Constructions as the Subject-Matter of Mathematics, *The Foundational Debate*, Vienna, 1995.
46. On Inference, *The Logica Yearbook 1998*, Timothy Childers (ed.), Filosofia, Prague, 73-85, 1999, co-author Jindra Tichý.

Acknowledgements.

Grateful acknowledgements go to Dr. Marie Duzi and Professor Pavel Materna for detailed comments on earlier drafts, and providing an up-to-date bibliography of Tichý's work. Any remaining errors are the responsibility of the author. Value judgments expressed in this article are entirely the opinion of the author.

Andrew Holster.