Concepts of physical directionality of time. Part 2. The interpretation of the quantum mechanical time reversal operator.

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Introduction.

This is Part 2 of a four part paper, intended as an introduction to key concepts and issues of *time directionality* for physicists and philosophers. It redresses some fundamental confusions in the subject. These need to be corrected in introductory courses for physics and philosophy of physics students. Here we analyze the quantum mechanical time reversal operator and the reversal of the deterministic Schrodinger equation. Time reversal is *the fundamental transformation, T: t* → *-t.* Here it is argued that:

- Quantum mechanics (classical) is anti-symmetric w.r.t. time reversal in its deterministic laws.

This contradicts the orthodox analysis, found throughout the conventional literature on physical time, which claims that quantum mechanics is *time symmetric* (*reversible*), and that we must adopt the anti-unitary operator (*T*)* instead of the unitary time reversal operator (*T)* for time reversal in quantum mechanics. This is widely claimed as settled scientific fact, and large metaphysical conclusions about the symmetry of time are drawn from it. But it is an error.

I have analyzed this problem previously in [Holster, 2003], so I do not want to repeat the points made there. But here I reply in some more detail to the usual objection to the use of *T,* argued for in that paper. This is the main reason the paper was unable to be published. I give the analysis in Section 1, and illustrate common objections made in peer reviews in Section 2.

Despite being unpublishable in several philosophy of science journals, [Holster 2003] is shown as having over 4,700 internet downloads from the *philsci-archive* site, and clearly it is being used for teaching. If really suffers from an elementary misunderstanding of quantum mechanics, as we will see claimed by peer reviewers, perhaps it should be removed, as it must be causing confusion to thousands of students? Alternatively, when we examine the objections, we might conclude that they are in error, and the many texts and online encyclopedia articles that explain the subject are causing even more confusion to even more thousands of students.

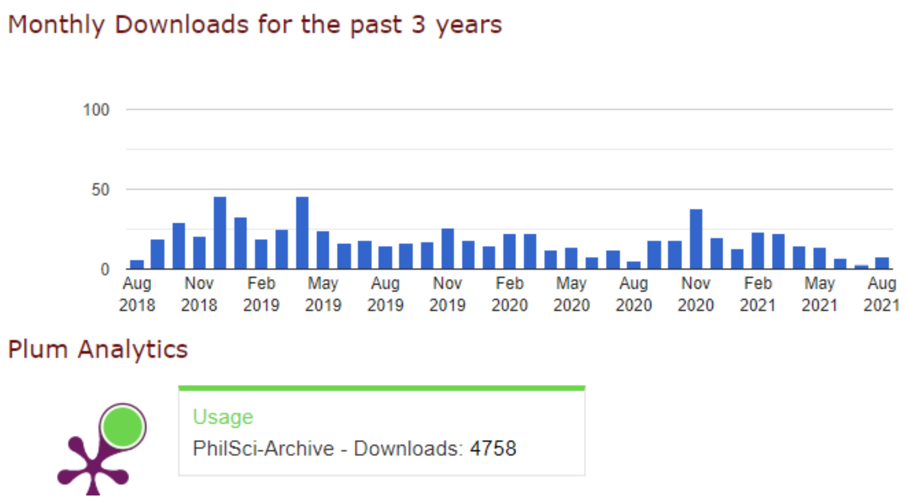

Figure 1. Monthly downloads of [Holster, 2003].

The analysis in that paper argues that the simple time reversal operator: *T*, which represents time reversal in every theory of physics except quantum mechanics, is also the correct choice for the time reversal operator in quantum mechanics. However this draws immediate objections from physicists, who believe that we are forced to use: *T**, the combination of the *time reversal and complex conjugation*, to be consistent with quantum mechanics. I will show here that this argument reflects a logical error.

Section 1.

It is readily seen that the time dependent Schrodinger equation is unchanged by the transformation *T**, but changed to an anti-symmetric form by *T* alone, and by * alone, by looking at the simple Schrodinger equation for a free particle, and its transformations:

	Theory	**Images of Schrodinger Equation**	**Simple Solutions**
(1)	*QM*	$\partial\Psi/\partial t = i\hbar/2m\ \partial^2\Psi/\partial x^2$	$A\ exp((i/\hbar)(px-p^2t/2m))$
(2)	*T(QM)*	$-\partial\Psi/\partial t = i\hbar/2m\ \partial^2\Psi/\partial x^2$	$A\ exp((i/\hbar)(px+p^2t/2m))$
(1)	*T*(QM)*	$-\partial\Psi/\partial t = -i\hbar/2m\ \partial^2\Psi/\partial x^2$	$A\ exp((i/\hbar)(-px-p^2t/2m))$
(2)	**(QM)*	$\partial\Psi/\partial t = -i\hbar/2m\ \partial^2\Psi/\partial x^2$	$A\ exp((i/\hbar)(-px+p^2t/2m))$

We see that the first and third equations or theories are identical, and anti-symmetric with the second and fourth equations.

The ‘simple solution’ here represents a particle with a precise momentum and kinetic energy, but with no position defined. More realistically, free particles are ‘wave packets’, represented by linear sums of simple solutions, with uncertainty in both momentum and position; but these have the same forms of transformation as illustrated by the simple solution, and the simple example suffices for the purposes of this paper. The class of these simple solutions for *T**(*QM*) is the same as for *QM* because *p* can be positive or negative. But the class of solutions for *(*QM*) (or equally *T*(*QM*)) is not the same as for *QM* because p^2 must be positive.

In the equations above, we see the use of the *time reversal operator on states: Ts*. In every other theory except quantum mechanics, this is simply taken as the *time reversal transformation: T: t* → *-t*, applied to states. However in quantum mechanics, the use of *T* is rejected, and a complex operator: *T** is adopted instead. This is called *Wigner time reversal*, symbolized here by: Θ = *T**, to avoid confusion with *T*. The operator: * is the complex conjugation transformation, mapping: **: a+bi* → *a-bi.*

But why should quantum mechanics, uniquely among all other physical theories, have to use this peculiar complex operator for *time reversal?* All quantum mechanics textbooks tell us that *T* must* be used, and give the reason in a complex mathematical argument (originally due to Wigner, 1932). They start by stating that any symmetry operator must be unitary or anti-unitary, and conclude that the time reversal operator must be the anti-unitary operator: $\Theta = T^*$, to preserve energies and reverse the sjgns of momenta under time reversal. But such arguments are conceptually opaque, and beneath the opaqueness lies a conceptual error.

I now set out the main form of the orthodox argument for the adoption of T^* as the *time reversal symmetry operator*. The first premise may be taken as the quantum mechanics law for the energy operator:[1]

(3) The *QM* Energy Operator:

> $\boldsymbol{H} = i\hbar\partial/\partial t$ is the energy operator in *QM*. This means that if the classically measured energy of a particle represented by a quantum wave function Ψ is E, *then:* $\boldsymbol{H}\Psi = E\Psi$

This is not disputed. The second premise is that:

(4) Classical energy is left unchanged by time reversal.

That is also not disputed. E.g. classical kinetic energy is ½ mv^2, and time reversal reverses the velocity, but it is squared to get the energy, so remains positive. (And equally with the relativistic generalization of energy).

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[1] I use *T* throughout as a generic *T*-transformation operator, which may be applied to all kinds of complex terms – e.g. terms for times, velocities, energies, momenta, states, wave functions, differential operators, propositions, laws and theories. I.e. the term: $T(z)$ constructs the image of z under the fundamental transformation: $T: t \rightarrow -t$, whatever z denotes. Thus we may write: $T(t) = -t$, $T(\boldsymbol{r}) = \boldsymbol{r}$, $T(d/dt) = -d/dt$, for the transformed images of these entities – times, position vectors, and the time differential operator. We may also write: $T(\boldsymbol{H})$ for the transformed image of a *QM* operator $\boldsymbol{H}$, *T(QM)* for the transformed image of the theory *QM*, *T(P)* for the transformed image of a proposition *P*. There is an important point here: there is only one concept of time reversal, which can be applied to many different kinds of entities – not, as commonly held, many different kinds of time reversal, for different kinds of entities or theories. Such a generic *T* operator can be constructed only because *T* is a *fundamental transformation*.

The third premise is:

(5*) The classical energy of a *time-reversed quantum state* must be the same as the classical energy of Ψ, so the time reversed quantum state also obeys (1).

This is the critical assumption of the argument: but it is false. What we might correctly say is that:

(5) *If QM is invariant under time reversal*, then the time reversal of a quantum state will have the same classical energy as the original state, and it will be given by (1). However*, if QM is not invariant under time reversal,* then the time reversal of a quantum state (i.e. wave function) will not necessarily obey the equations of *QM* at all! Indeed, it will not represent a *physically possible state* in *QM*! It will not obey (1).

It is this second possibility that the orthodox analysis overlooks – the very possibility that *quantum mechanics is not time reversible* – and this is the very possibility that we are trying to examine. What the orthodox analysis does is to *first assume that quantum mechanics is invariant under time reversal,* and then, because quantum mechanics is *not* invariant under the time reversal transformation: *T: t* → *-t,* it concludes that *T cannot be the time reversal transformation for quantum mechanics.* It goes for the next best thing – the *T** transformation, under which quantum mechanics is indeed invariant. With this circular logic, the analysis insists that *T** represents time reversal in quantum mechanics, and theory is reversible.

Instead, we may conclude that *T represents time reversal in quantum mechanics, and the theory is not reversible.*

What we can say is that:

(6) If Ψ is a wave function obeying *QM*, then its time reversal: *T*Ψ is wave function that must obey: *T*(*QM*), i.e. the time reversal of quantum mechanics.

This is essentially the *definition* of *T(QM)* (or any theory). I.e. solutions of the reversed theory are reversed solutions of the original theory. It does not follow of course that $T\Psi$ obeys *QM*. Rather:

(7) If *QM* is time reversible, then: *QM = T(QM).* If this is true, it follows that: *If Ψ obeys QM, its time reversal: $T\Psi$ also obeys QM.*

The hidden premise in the orthodox argument is precisely to *assume that the time reversal of Ψ must obey QM*. But to repeat: this true only if *QM* is indeed time reversible. If not, *$T\Psi$ will obey T(QM),* but it will not obey *QM*.

Now the energy operator in *T(QM)* must be the time reversal of the energy operator in *QM*, i.e: $T(\mathbf{H}) = T(i\hbar\partial/\partial t) = -i\hbar\partial/\partial t$. Thus we get:

(8) The *T(QM)* Energy Operator.

> $T(\mathbf{H}) = -i\hbar\partial/\partial t$ is the energy operator in *T(QM)*. This means that if the classically measured energy of a *T(QM)*-wave function $T\Psi$ is *E, then:*
> $T(\mathbf{H})T\Psi = ET\Psi$

This is true. $T\Psi$ is a *T(QM)*-wave-function just in case: Ψ is a *QM*-wave-function;

> which means that: $\mathbf{H}\Psi = E\Psi$;
>
> which means that: $\mathbf{H}T\Psi = -ET\Psi$;
>
> which means that: $T(\mathbf{H})T\Psi = ET\Psi$.

And this is the principle (8). It follows logically from (3). We see that there is no logical inconsistency in using *T* as the time reversal operator on *QM*: we can do this perfectly consistently – as long as we remember to use $T(\mathbf{H})$ and not $\mathbf{H}$ as the energy operator in *T(QM)*. (And similarly for other operators, like momentum, of course.)

In fact, this is fairly obvious, when you consider that *T(QM)* is perfectly isomorphic to *QM*, with exactly anti-symmetric solutions. Its solutions simply have the opposite complex phase rotation to *QM*, and these solutions *do not obey the QM equations*, they obey the *time reversal of those equations.* This is exactly what we expect from an *anti-symmetric irreversible theory*.

The Orthodox Argument Summarized.

I will repeat the orthodox argument again in the kind short form it normally takes. Suppose we take the time reversal of Ψ to be: $T\Psi$. Then applying the energy operator (3), we get: $\boldsymbol{H}(T\Psi) = i\hbar\partial/\partial t(T\Psi) = -E(T\Psi)$. But the energy has reversed – so $T\Psi$ has the wrong energy to be the time reversal of Ψ. To fix this, we note that: $\boldsymbol{H^*}(\Psi) = -i\hbar\partial/\partial t(\Psi) = -E(\Psi)$, i.e. the complex conjugate of $\boldsymbol{H}$ reverses the sign. Or equivalently: $\boldsymbol{H}(*\Psi) = i\hbar\partial/\partial t(*\Psi) = -E(*\Psi)$. Thus we realize that: $\boldsymbol{H}(T*\Psi) = i\hbar\partial/\partial t(T*\Psi) = E(T*\Psi)$. This is the correct energy law required for the reversed state. Hence we must take *T** as the *state reversal operator* in *QM*.

And to repeat the flaw in this argument: the fact that: $T\Psi$ has the wrong energy to be the time reversal of Ψ *in QM* does not show that *T* is not the time reversal symmetry operator: it shows that *QM is irreversible.* $T\Psi$'s are *not QM states.* If *QM* was invariant under time reversal, then $T\Psi$*'s* would be *QM* states, but they are not.

What does *T** represent in *QM*, if not *time reversal*? It represents the *spatial trajectory reversal operator*. E.g. if Ψ represents a particle with a certain spatial trajectory in *QM*, then $T*\Psi$ represents the particle with the reversed trajectory in *QM*. The fact that *QM* is symmetric under *T** represents the fact that there is a consistent logical representation in *QM* for reversed spatial trajectories. This is of course a very essential symmetry of the theory – without it, *QM* would be kinematically inadequate from the start. *But this does not show T* represents time reversal symmetry.*

To visualize this, *QM* particles are analogous to spinning tops. A top may follow a certain path, e.g. across a table. The trajectory reversal (analogous to *T**) will reverse its path – but not reverse its spin. The true time reversal (*T*) will fully reverse the sequence of states, including both trajectory and spin.

But what if a theory of tops states that *all tops can spin in only one direction?* Then the theory is not time reversible. It does not allow the real time reversal of tops, only the reversal of trajectories. This is exactly what *QM* is like.

It is should be emphasized that *T** symmetry of basic *QM* kinematics does not imply a reversible *dynamics* either. For that, specific dynamic laws (Hamiltonians) must be examined in detail. Just because we can represent a *kinematic reversal of trajectories* does not show that reversed trajectories will obey the dynamic laws of *QM* – i.e. the laws of forces. In fact the decay process of K-mesons has irreversible dynamics. The standard model of relativistic quantum field theory (the 'real quantum theory' in the present era) is *irreversible*, irrespective of the question of time reversal operator analyzed here. This is dismissed as merely a 'minor example of irreversibility' by orthodox writers in their eagerness to maintain claim (a). In fact, it is of profound importance.

I will emphasize a final point that should be of real interest to physicists here, that Costa de Beauregard (1980) was especially concerned with. This is that in *relativistic quantum field theory* (the real quantum theory), the *T* operator might be taken as the time reversal operator, *if time reversal can be taken to transform particles to anti-particles.* *T** reverses the trajectory of an electron, *and it remains an electron. T* reverses the trajectory and the complex phase rotation: might this be interpreted to turn the electron to turn into a positron – reversing the charge as well as the trajectory? This would correspond to Feynmann's 'zig-zag' theory that positrons are like electrons moving 'backwards in time'. On this view, *T** is not the time reversal transformation at all – and its use hides the fact that *real time reversal produces charge reversal*. Ordinary quantum mechanics does not have this interpretation – there were no positrons or anti-particles until relativistic quantum electrodynamics was introduced by Dirac.

This question revolves on whether *T* induces *charge reversal* in relativistic quantum field theory – or in a suitable interpretation of QED. Mei Xiaochun (2010) argues that

"The Current C, T Transformation Rules of Quantum Field Theory Must [be] Redefin[ed]", and argues that that time reversal should reverse the creation and annihilation operators in QED.

> "[A]ccording to the current *T* transformation of quantum field theory, creation operator of spinor particle is still creation operator and annihilation operator is also still annihilation operator … This result does not represent the real meaning of time reversal. In the interaction process, a particle's creation operator should become the annihilation operator and its annihilation operator should become the creation operator after time reversal." Mei Xiaochun (2010), p1.[2]

If de Beauregard is right, the irreversibility of simple (classical) quantum mechanics under *T* really points to the *incompleteness of this theory.* From this point of view, classical quantum mechanics should definitely be judged irreversible – its irreversibility reflects the fact that it lacks half the world of particles: the anti-particles that are the genuine time reversal of ordinary particles. The orthodox analysis falsely attempts to shoe-horn quantum mechanics into a 'reversible' theory by adopting *T** as time reversal. This is the real issue about *T** in quantum field theory.

Section 2.

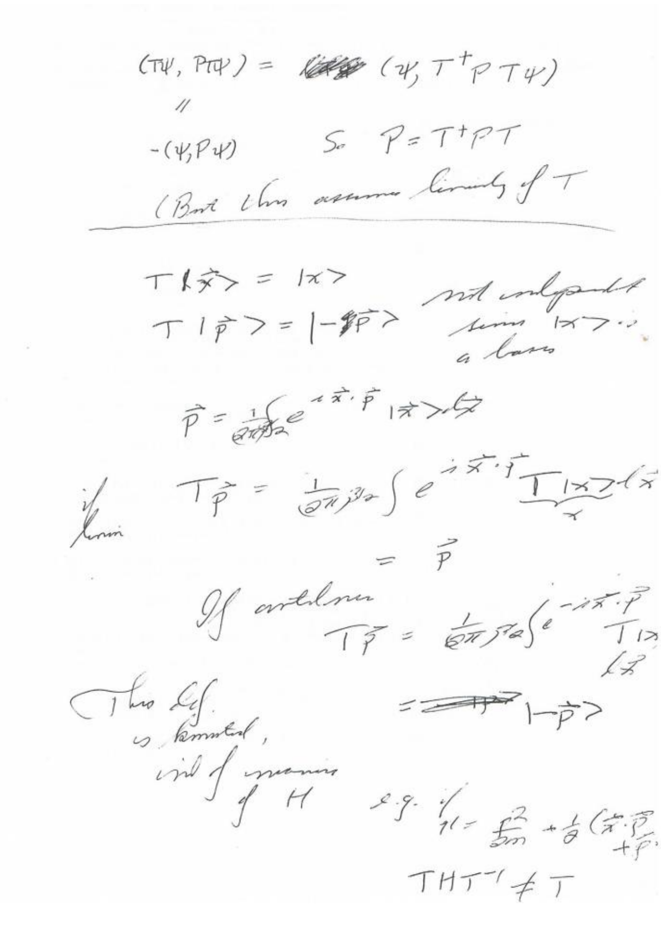
I hope this is a sufficient justification to at least take question raised in the paper seriously. Although this paper was rejected repeatedly by philosophy of science referees as not even a serious question, one referee did recommend publication, saying:

> "This is a very lucid paper, clarifying an important issue – or rather the point of departure of an investigation needing to be pursued. No additional explanation is recommended because it could only be too short or too long. But pursuit of the investigation on the following points is recommended. [Seven points listed]. The author's very <<down to earth >> method should

[2] By *T* he means the orthodox *T** transformation.

help clarifying deep issues in the philosophy of science.” (Personal Communication)

This was the late Olivia Costa de Beauregard, eminent French physicist-philosopher, who later wrote to me encouraging me to pursue the issue. But his positive review was overridden by a second referee, who summarized the paper as meeting the journal’s publication standards (original, well written, well referenced) but *“WRONG!”* and scrawled out the “right” answer, as reproduced below.


$(T\psi, PT\psi) = (\psi, T^{\dagger} P T\psi)$

$\parallel$

$-(\psi, P\psi)$ So $P = T^{\dagger} P T$

(But this assumes linearity of T

$T|\vec{x}\rangle = |x\rangle$

$T|\vec{p}\rangle = |-\vec{p}\rangle$ not only possible since $|x\rangle$ is a basis

$\vec{p} = \frac{1}{(2\pi)^{3/2}} \int e^{i\vec{x}\cdot\vec{p}} |\vec{x}\rangle d\vec{x}$

if linear $T\vec{p} = \frac{1}{(2\pi)^{3/2}} \int e^{i\vec{x}\cdot\vec{p}}\, \underbrace{T|x\rangle}_{x} d\vec{x}$

$= \vec{p}$

If antilinear $T\vec{p} = \frac{1}{(2\pi)^{3/2}} \int e^{-i\vec{x}\cdot\vec{p}}\, T|x\rangle d\vec{x}$

$= |-\vec{p}\rangle$

This def. is limited, ind. of meaning of H e.g. if $H = \frac{p^2}{2m} + \frac{1}{2}(\vec{x}\cdot\vec{p} + \vec{p}\cdot\vec{x})$

$THT^{-1} \neq T$

Figure 2. An argument by an anonymous referee (Synthese) given to dismiss the paper [Holster, 2003]. But the paper shows in detail why this kind of argument is unjustified. The arguments in the paper are never mentioned by the referee: only his own opinions.

The reader may wish to take this argument into account in case it really is the simple final solution to the whole question and no more needs to be said. But in fact it is incoherent. It concludes that: $\boldsymbol{T(H) \neq H}$¸ which I have emphasized. What is the objection to the argument that I actually give in the paper? This is never mentioned.

In any case, in the 18 months *Synthese* took to reply, Craig Callender (2000) managed to publish a paper raising the same issue. And in a parallel development, David Albert (2000) published a book, arguing that classical electromagnetic theory is irreversible. He pointed out *the lack of principled justification* for the choice of time reversal operation in EM theory. My arguments and conclusions are different to both: but these two writers made an important advance in legitimizing discussion of a serious issue, whatever the outcome of the debate may finally prove to be.

In 2002, I changed the introduction to my paper to reference Callender's paper as a published source for the debate, and sent it to some other journals. I will give just one typical response that any *realist* writing in this subject is likely to get. The referee for this prestigious British philosophy of science journal begins:

> "The premise is this: in implementing a desired transformation (in this case $t \rightarrow -t$) one is not at liberty to simultaneously transform other quantities (in this case $i \rightarrow -i$) appearing in the equations so as to obtain a symmetry. If this philosophy were correct, then neither are Galilean boosts symmetries of non-relativistic quantum mechanics, nor is time reversal a symmetry of Maxwell's equations (to give just two examples). The latter question was in fact recently raised by David Albert in his book "Time and Chance". David Malement gave a rebuttal of this claim at the recent Maryland conference, winning (I gather) wide agreement on the error of Albert's ways." (Reviewers Comments returned to author from BJPS).

He then insists I should pursue an "operationalist philosophy", beginning by first defining 'physically real' quantities as 'measurable quantities'. This will apparently "correct my mistakes". However he never comments on the arguments given in the paper, or says how his method leads to their corrections.

His initial reason for rejection is that my argument implies that "Galilean boosts [are not] symmetries of non-relativistic quantum mechanics". This is flippant nonsense, and he does not exert himself to explain it. His next reason is a report of gossip: *he has heard that a philosopher has won (in popular opinion?) an argument at a conference against another philosopher – but about a different argument and a different theory!*

These are absurd reasons to reject the paper. But his insistence that the analysis must pursue an "*operationalist philosophy*" is the fundamental point, common to peer reviewers from leading phil-sci journals. My paper specifically *criticizes* the typical operationalist or instrumentalist accounts for errors of analysis - yet he insists that I must start with definitions conforming to his own "operationalist philosophy". It is remarkable that this vague reference to a (discredited) positivistic theory of meaning can be taken as a conclusive argument to settle a complex issue about physics. Why can't the arguments given in the paper be taken on their own merits?

It should be emphasized that there is no need for a writer giving a realist analysis to justify their arguments other than by logical argument. Certainly a realist does not need to justify why they have not started with an 'operationalist' account. The simple fact is really that there is no common ground for realists to argue with the neo-positivist philosophers: we are engaged in different subjects. Realists are engaged in trying to find out the truth of the matter. Neo-positivists are engaged in an ideological agenda. The point of offering a realist analysis is *not* to convince neo-positivists.

The essential point is that *there is no real question about what time reversal symmetry means: it means symmetry under the transformation: T:* $t \rightarrow -t$*. The question is really about what quantum mechanics entails.* The orthodox position is that the definition of *time reversal symmetry* must be modified in quantum mechanics. But this is wrong: their only real argument is that, by taking a suitable ('operationalist') interpretation of quantum mechanics, we will see that *the physical content of quantum mechanics is unchanged under time reversal.* The onus then falls on them to demonstrate that they have such an interpretation of quantum mechanics, not that an 'alternative interpretation of time reversal' is required. But their claims to have such an interpretation are mere hand-waving: there is no such interpretation. What they claim to be an 'interpretation' has never been given any formal statement.

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Appendix. T-Symmetry Wikipedia.

Anti-unitary representation of time reversal [edit] **T-Symmetry WIKIPEDIA**

Eugene Wigner showed that a symmetry operation S of a Hamiltonian is represented, in quantum mechanics either by a unitary operator, $S = U$, or an antiunitary one, $S = UK$ where U is unitary, and K denotes complex conjugation. These are the only operations that act on Hilbert space so as to preserve the *length* of the projection of any one state-vector onto another state-vector.

Consider the parity operator. Acting on the position, it reverses the directions of space, so that $PxP^{-1} = -x$. Similarly, it reverses the direction of *momentum*, so that $PpP^{-1} = -p$, where x and p are the position and momentum operators. This preserves the canonical commutator $[x, p] = i\hbar$, where $\hbar$ is the reduced Planck constant, only if P is chosen to be unitary, $PiP^{-1} = i$.

On the other hand, the *time reversal* operator T, it does nothing to the x-operator, $TxT^{-1} = x$, but it reverses the direction of p, so that $TpT^{-1} = -p$. The canonical commutator is invariant only if T is chosen to be anti-unitary, i.e., $TiT^{-1} = -i$.

Another argument involves energy, the time-component of the four-momentum. If time reversal were implemented as a unitary operator, it would reverse the sign of the energy just as space-reversal reverses the sign of the momentum. This is not possible, because, unlike momentum, energy is always positive. Since energy in quantum mechanics is defined as the phase factor $\exp(-iEt)$ that one gets when one moves forward in time, the way to reverse time while preserving the sign of the energy is to also reverse the sense of "i", so that the sense of phases is reversed.

Similarly, any operation that reverses the sense of phase, which changes the sign of i, will turn positive energies into negative energies unless it also changes the direction of time. So every antiunitary symmetry in a theory with positive energy must reverse the direction of time. Every antiunitary operator can be written as the product of the time reversal operator and a unitary operator that does not reverse time.

For a particle with spin J, one can use the representation

$$T = e^{-i\pi J_y/\hbar} K,$$

where J_y is the y-component of the spin, and use of $TJT^{-1} = -J$ has been made.

The Wikipedia explanation of why T^* represents time reversal embodies the circular argument criticized in the text. It is not adopted because it is the transformation that represents time reversal in principle, but because it is a symmetry that satisfies quantum mechanics. T is the time reversal operator in principle, but it does not represent a symmetry of quantum mechanics. The orthodox analysis will not allow us to recognize that quantum mechanics fails to satisfy time symmetry.