Three Unique Virtues of Approval Voting

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Abstract

Approval Voting offers advantages over other voting systems for single-winner elections. This manuscript analyzes three unique virtues of Approval Voting. First, the procedure does not violate the independence of irrelevant alternatives criterion for rational choice. Second, it prevents manipulation of outcomes through agenda setting. Third, it avoids intransitive majority preference cycles like Condorcet paradoxes and so escapes Arrow's Impossibility Theorem constraints. As a result of these virtues, which are generally not shared by its best known competitors, Approval voting emerges as a strong option for realizing majoritarian democratic will.

Keywords: Approval voting, Social choice theory, Voting systems, Condorcet paradox, Arrow's theorem.

Three Unique Virtues of Approval Voting [1]

Approval Voting ("AV") has taken its place of late as a contemporary competitor to (the somewhat more complicated scheme of) Ranked Choice Voting as a candidate to replace first-past-the-post systems in single winner elections. While I do not think that frequent use of a simple quasi-majoritarian mechanism like AV[2] can alone completely capture the general will of an electorate (largely because, in my view, there must also be provisions for robust proportional representation, recall, referendum, and reversal), I do think that AV could make an important constituent of an authentically democratic system.[3]

It is vital to recognize that when one suggests that any particular voting scheme makes a good aggregation methodology for "capturing the general will," there are various ways to understand the claim. It may be, for example, that some system would be absolutely peerless if voters would only follow its instructions, but we have good evidence that they will not. Perhaps such rule violations are strategic, or they may simply result from voter misunderstanding or laziness. Obviously, whether lapses of that kind may be eliminated is a matter for social psychology and experts in education. In this paper, I confine myself to an (arguably fantastic) "ideal world" in which voters *do follow the voting rules.* This simplifying assumption may be thought by those interested in election reform to be illicit. It is my belief, however, that in order to know what would be good voting rules in the real world, it is essential to understand what sort of aggregation principles would be capable of the accurate capturing of group preferences in worlds in which voting rules were strictly followed.

Without that, election law reformers will reasonably be accused of being interested more in outcomes they like than in accurately capturing group preferences. Acceptance of this assumption means, however, that even if AV could be demonstrated to have all the virtues that any rule might exemplify—or at least more of them than any other rule—it might also be burdened with so many serious vices that others lack, that it should not be considered for real world use. That, I believe is a matter for empirical study: not only those involving the analysis of election results, but also those regarding what an electorate can be taught about the relation of election rules and getting the public policies they want. I consider neither of those matters here. The paper is, thus, a propaedeutic for such studies.

Because they are well known, I also ignore here what I consider to be the most basic virtues of AV. I discuss the importance of cardinality, simplicity, and several other important AV characteristics elsewhere,[4] and will not repeat any of that here. But, in light of the recent burgeoning of interest in AV, I think it is necessary to respond to a couple of important criticisms that have been made of the scheme[5] that do not involve the claim that AV voters will not follow instructions and will, e.g., simply bullet their favorite candidates rather than indicate every candidate they could live with. In addition, I believe that several of AV's merits have not received sufficient notice in the literature. The purpose of this paper is to try to address both of these matters. I will argue first that AV cannot be convicted of irrationality due to any alleged failure to guarantee results that are independent of "irrelevant alternatives." Second, I will show that AV eliminates the possibility of using agenda-setting to produce results that are inconsistent with majoritarianism and pluritarianism. Finally, I will explain why AV is not subject to majority voting cycles. Again, however, I make no claim that AV does not have demerits that have also failed to receive sufficient attention. My discussions of the three claimed virtues is prefaced by a general account of AV, of voting rules in general, and of the appropriate transformation of preference profiles to approval profiles.

I. AV and Voting Rules

Voting rules instruct voters how particular electoral mechanisms are intended to work, and we may infer from them the role of voters in elections utilizing these mechanisms. That is, they indicate what voters are supposed to do, how their votes will be aggregated, and how electoral winners will be determined from the results of those aggregations. The expression of such rules may include more or less information, however. For example, instructions regarding how AV works (to select single winners) might be accurately put by either of these two different rules:

1. Vote for all and only those candidates you minimally approve of by making a mark next to the name of such candidates—and by no other means. These votes/marks will be summed. The candidate getting the most votes wins the seat.
2. Vote by making a mark next to as many candidate names as you like—and by no other means. These votes/marks will be summed. The candidate getting the most votes wins the seat.

While these are similar and each can be said to correctly indicate the nature of AV, Rule (1) contains an instruction regarding the *basis* on which voters should be marking their ballots. It says that the electorate should vote in a way that will coherently map to their attitudes of approval or disapproval toward each candidate. That rule will be violated whenever a voter either puts a mark by a candidate of whom they don't approve, or fails to put a mark by a candidate they like.

Since Rule (2), the more latitudinarian version, contains no such instruction, it can be violated by a voter only when he or she performs actions such as crossing out the name of a candidate or ripping a ballot in half.

Most voting schemes are based on the theory that public attitudes are best obtained by aggregating the individual "preference profiles" (or rank orderings) of the voters who are called upon to assess the relevant candidates. And it is common to test election results by considering to what extent they seem to sensibly reflect the individual preference profiles of the populace, writ large.

AV, however, is not based on preference profiles, but rather on "approval profiles." Let us hypothesize a mayoral election involving six candidates: Austin ("A"), Bodin ("B"), Calhoun ("C"), Downs ("D"), Engels ("E"), and Fourier ("F"). And let us suppose further that there are three unanimous blocs of exactly 10,000 voters each that will participate in this mayoral election: the X, Y, and Z parties. Now suppose that bloc X approves of candidates A, B, and C, while disapproving of candidates D, E, and F, and both remaining blocs Y and Z approve of B, D and E, while disapproving of A, C, and F. If every voter in every bloc faithfully adheres to AV Rule (1) above, Austin and Calhoun will each receive 10,000 votes, Downs and Engels will each receive 20,000 votes, and Bodin will receive 30,000 votes and so win the election.[6]

However, if voters follow the less restrictive rule (2), they can vote in ways that produce a winner other than B. Why might this happen? Most likely it will be because, when they vote, they are moved by something other than their approval profiles, namely, their preference profiles. For example, perhaps some of these voters have a particular favorite among the group of three whom they approve, and those voters also believe (quite sensibly) that if they bullet their favorite candidate rather than follow the Rule 1 imprecation to vote for *all* the candidates they approve of, it will improve the chance that their particular favorite will win. This is a form of "strategic voting" that, while inconsistent with (1), is not violative of (2). Another bulleting strategy that comports with (2) but not (1) involves "bandwagoning."[7] This might be done by those who like always to be on the winning side. Finally, bulleting could simply result from voters simply being confused: e.g., they might vote for only one candidate because that is what they are used to doing. As such actions are perfectly consistent with Rule (2), there's a clear sense in which these voters may be claimed to be meticulously following AV rules, even though all of them would certainly be violating the more restrictive Rule (1).

Whenever some sort of utility calculation predictably moves voters to violate Rule (1), aggregated results of what are ostensibly AV elections will not produce credible expressions of the overall approval-attitude of the populace. As indicated above, the likelihood of such occurrences with respect to elections that are supposed to reflect the use of one or another of these rules is an empirical matter best left to social psychologists.[8] I simply note that if the desire to get one's favorite elected or to vote only for winners must, because of irresistible psychological causes, always prevail over compliance with (1), then, as Arrow has shown, aggregations of votes can never rationally map to individual attitudes. Why? Because Gibbard, 1973 has shown that where there are favorites, strategic voting is possible in any multi-winner scheme. If voters absolutely will not follow Rule (1), we must simply conclude that there cannot be any such thing as a rational ascertaining of any group's "general will" in a multi-seat election.

II. AV and "The Independence of Irrelevant Alternatives"

In his Impossibility Theorem proof, one of the criteria that Kenneth Arrow required for any rational aggregation of preference profiles is what he called *the independence of irrelevant alternatives* ("IIA").[9] This criterion is satisfied by a voting mechanism M if and only if it is the case that in any M election involving candidates X and Y, whatever relationship R (better, worse, or indifferent) that X has to Y according to voters in election results, that relationship will not be altered by changing the order of the preferences of any or all voters with respect to the relationships of candidates *other than* X and Y. For example, if at t1, X is an election winner and Y is not, no changes in voter preferences regarding, say, W and Z at t1—even a move from all preferring W to all preferring Z—can have any effect on the relationship between X and Y. If not one voter changes their t1 preference ordering of X and Y specifically, the aggregated results cannot change the group preference relationship of X and Y. X may no longer be a winner, or perhaps both X and Y will both win, but it cannot happen that Y is now a winner and X is not. This is intuitive, and it has seemed reasonable to many commentators to require that any good voting rule must comply with it. To the extent that AV is construed to follow Rule (1) above, even though it is not a ranking system, it seems clear that it will not fall afoul of a reasonably constructed approval analogue of this version of IIA. For those following Rule (1), newly approving one candidate on a list will not involve a change in one's approval-disapproval relationship to any other candidate on that list.

However, in discussions of social choice and voting, "the independence of irrelevant alternatives" is often used to mean something other than Arrow's "Condition 3." One popular understanding is generally intended to rule out alteration of any ordering by an attitude toward something *not previously on the list*—an alternative that is irrelevant for that specific reason.[10] Other sorts of alterations because of novel "irrelevancies" may occur when the list of candidates is altered, not by the inclusion of one or more new alternatives, but by the elimination of previously available alternatives. As these sorts of attitudinal reversals can seem irrational, it is worth investigating whether this quite common (but not strictly Arrovian) understanding of IIA (I shall call it "IIA2") is violated by AV.

AV looks at each candidate separately rather than relying on ordinal rankings, but it nevertheless at least *seems* like it is subject to its own version of the kind of allegedly irrational shifts outlined above. Consider our Mayoral race among the six candidates, A, B, C, D, E, and F, of whom some voter X approves just the three A, B, and C. Now imagine the late addition of candidate Gutmann ("G") whom X likes much more than any of the others. Would we really just expect X to add G to the three candidates she approves of, and not shift her take on A, B, or C into non-approval? Or suppose that, A, B, and C all suddenly drop out? Will X be likely to continue to disapprove all three of the only remaining candidates, or would that seem like relinquishing all input in this election? Finally, suppose that it is all three of the candidates whom X *doesn't* like who quit the race. Wouldn't it seem like a completely wasted vote if X were to still go to the polls to approve all three of A, B, and C?[11] While these attitudinal changes all seem reasonable, they also seem to be violative of an approval analogue of IIA2, since it is *other* options that have somehow produced the attitudinal changes toward A, B, C, D, E, and F.

Since AV is not a ranking system, to properly address these questions we will need to define IIA2 in a manner that can appropriately be applied to approval schemes. I will do this by giving a more precise definition of our IIA2 and then

indicating the relations between preference systems and AV that allow for reasonable transformations. I will then proceed to the provision of a definition of a version of IIA2 that can be used to assess the rationality of AV.

A. IIA2 for Preferentist Schemes

IIA2 = df. All rational judges J, options O, availability sets A and durations D are such that if O_x and O_y are $\in$ A and J judges $O_x > O_y$ during D, there can be neither an $O_n \neq (O_x$ or $O_{y)}$ in A nor any $O_m \notin$ A such that the subtraction of O_n or addition of O_m during D would cause J to judge Y > X.[12]

B. Entailment Relations between Preferentism and AV

All rational judges J options X and Y and durations of time D are such that, if J approves X and does not approve Y at D, then J judges X > Y at D.

All rational judges J, options X and Y, and durations of time D are such that if J judges X > Y at D, then, at D, either J approves both X and Y, J approves neither X nor Y, or J approves X but does not approve Y.

C. IIA2 for Approval Voting

IIA2* = df. All rational judges J, options X and Y, and durations of time D are such that

If O_x and O_y are $\in$ A and J approves O_x but not O_y during D, there can be can be neither an $O_n \neq (O_x$ or $O_{y)}$ in A nor any $O_m \notin$ A such that either the subtraction of O_n or addition of O_m during D would cause J to: approve of O_y but not O_x, approve of both O_x and O_y, or approve of neither O_x nor O_y; and

If J approves of neither O_x nor O_y at D, then there can be can be neither an O_n nor an O_m such that either the subtraction of O_n or addition of O_m during D would cause J to: approve of O_x but not O_y, to approve of O_y but not O_x, or to approve of both O_x and O_y; and

If J approves of both O_x and O_y at D, then there can be can be neither an O_n nor an O_m such that either the subtraction of O_n or addition of O_m during D would cause J to: approve of O_x but not O_y, to approve of O_y but not O_x or to approve of neither O_x nor O_y.

Once such definitions and inference rules are settled upon, social scientists can record the changes in preferential rankings or approval statuses that seem to have resulted from addition of one or more new options or elimination of one or more pre-existing options. One might then claim that preference rankings or approval statuses are violative of IIA2* and are thus irrational.

It is noteworthy that, pursuant to the definitions above, while the addition or subtraction of option Z is required to occur during duration D, nothing requires that the “effect” of this insertion or deletion also occur during D. Surely, however, if such effects occur after D, there need be nothing untoward about changes in either ranking or approval status. Rationality

does not require that a person's ranking of X and Y can never change over time. The simplest way to control for changes not issuing specifically from the "irrelevant alternatives" is to require that, for a violation of IIA2* to occur, the effects caused by the new appearance or disappearance of Z as an option must also happen during D. In that case, however, the irrationality emanating from the reversal is explicitly a contradiction: we are instructed, e.g., to assume that, during D, J prefers X to Y, and also asked to stipulate that, because of the addition or deletion of some Z *within that same period*, J concurrently believes Y > X.

It is quite clear, though, that the proposed counterexamples to IIA2* are not intended to show that some person J *simultaneously* holds both that X > Y and that Y > X (or simultaneously both approves and does not approve X). Rather, they are intended to show that *changes* in available options can change judges' attitudes in apparently irrational ways. Thus, controlling for extraneous causes will have to be achieved in some other way than by insisting that the conflicting attitudes occur during the same time period. Any requirement that all the measured attitudes occur within D would say only that those voters with explicitly self-contradictory beliefs can be accused of irrationality. That is, such a requirement would make IIA2 and IIA2* too weak: only the self-contradictory would be deemed irrational.

But there is also a difficulty if we leave out the "within D" constraint. Voters must be taken as they are at any point in time, i.e., presented with all and only such alternatives as are actually available to them *then*. There is no doubt that attitudinal changes might result from the addition or deletion of options, just as they can occur without any such insertions or removals. We cannot deem *all* attitudinal changes to be irrational.

One might try to rehabilitate IIA2* by putting it conditionally after we remove the temporal provision. So, e.g., we might put it in the spirit of Sen's property α,[13] as follows:

IIA2† = df. All rational judges J, options X and Y, and sets *T* and *S* are such that:

If X is in set *T* and J approves/disapproves of X when considering the members of *T* to be all the available options, then if X is an element of either superset/subset *S* of *T* then J would approve/disapprove of X when considering the members of *S*.

I don't doubt that AV—like most other voting schemes— is violative of IIA2†, but it does not seem to me that irrationality ought to be inferred from violation of such a principle. Suppose that you and your two children are apprehended by a foreign power and made to face a harrowing choice, though one that is somewhat less horrifying than that faced by a certain famous Sophie:

A. You may have both children killed.
B. You may pick one child to be killed and the other will remain alive.
C. You may keep both children and go home, but you are told that your current near-indigent status will not be allowed to change: you will never have quite enough resources to keep either child well-fed and healthy.

You approve of (C) only and may even do so happily. You may feel, in fact, that that option is much more generous than you'd ever have expected from this source. But now suppose that, just prior to when you must make your choice, the

following additional alternative is added to the list of available options:

D. You may keep both children and go home, and you are promised that your family will be kept wealthy and comfortable the rest of your days.

Is it irrational if, given your same set of priors, when presented with this longer list you approve only of (D), a situation that is better than you have ever known? Surely, it might seem inappropriate to any parent to approve of an option in which his or her children might starve when another one is made available which eliminates that possibility. Similarly, we may ask whether, if you were to begin with the set of four options and had it in mind to approve of (D) only, but then had (D) taken away, we must insist that you are irrational if you then approve of (C)? After all, (C) was your *status quo ante*—and you may not have been terribly unhappy with your lot. I think both supporters and critics of AV, at least those who understand that voters must be taken in the option environments in which they are found, will agree that shifts of this kind ought not to be declared irrational.[14] It follows those interpretations of rationality that yield rules like IIA2† are simply too stringent to be useful. One must remember, however, that neither IIA2* nor IIA2† is the principle that concerned Arrow.

III. AV and Defeat of Democracy by Strategic Agenda-Setting

Probably the most famous example of illicit manipulation by agenda control (i.e., the order in which questions are offered for vote) is the case of “The Powell Amendment” to a Federal aid-to-schools bill in 1956. First put forth as an example of how a majority might be thwarted through agenda-setting in Blydenburgh 1971, the suspicious Congressional activities of the mid-50s were used by William Riker in several works to illustrate what he took to be the futility of democracy.[15]

In the original case, an appropriation bill was amended to provide a certain amount of school aid. A modification of the amended legislation was then offered by Adam Clayton Powell which was intended to ensure that only those schools that were in compliance with the non-discrimination provisions of the *Brown v. Board of Education* decision could receive aid. According to a traditional manipulation theory, while the original amendment (without the Powell addition) was supported by a majority of the House, the addition of any anti-discrimination language in the legislation would have resulted in loss of support from a large number of Southern Democrats who would otherwise have backed the Federal subsidies.[16]

As indicated above, this analysis has been subject to repeated rebuttal. Extensive evidence has been adduced, much of it based on subsequent votes on a similar proposal, that supports the contention that an aidless bill would have defeated one containing school subsidies with or without the Powell amendment.[17]

It is easy to see how controversies of this nature can arise. Both the empirical and the theoretical questions are extremely difficult to answer. As examples of the former, consider these: “Did Congressperson X really know how Congresspersons Y and Z were intending to vote?” “What was Powell actually up to?” “Would the NAACP have opposed an aid bill that did not include the Powell Amendment?” And the theoretical questions might include these: “Can a vote for P actually reflect a desire for not-P?” “Is a desire to appear a certain way to future voters sufficient to indicate sincerity?” “Must sincere votes reflect the representatives’ own preferences or their constituents’ views”?) No doubt of these are fascinating questions,

but, fortunately, my goal here can be achieved without answering any of them. I want to show only that in a case in which agenda control surely could result in the defeat of a sincerely held majority view given present voting methods, no such result could occur under Approval Voting. As will be seen, that is a relatively simple task.

Let x = the aid bill with Powell Amendment; let y = the unamended aid bill; and let z = the status quo (no aid). Under Riker's (controversial) breakdown,

- 132 (31%) members supported the aid and the Powell Amendment, i.e., had a preference ordering of xyz.
- 67 (16%) supported aid, but preferred staying out of racial matters. i.e., they were yxz.
- 130 (30%) wanted aid, but only if schools in their districts could continue to discriminate, i.e., were yzx.
- 49 (12%) frowned on Federal subsidies but not *Brown v. Board of Education*, i.e., were zxy.
- 48 (11%) disapproved of both Federal school aid and anti-segregation mandates, i.e., were zyx.

The historical plausibility of every one of these propositions has been disputed, and, as indicated, I take no position on any of them. Obviously, it is quite difficult to determine with confidence whether a particular historical vote reflected a sincere preference, and this would be so even if the necessary and sufficient conditions for an action being sincerely supported were themselves agreed to by all parties. If, however, we assign noncontroversial interpretations to the numbers and percentages above, we will find that manipulation by agenda-setting is at least *possible*—and that is all we need. Let us therefore stipulate to the following scenario using the same numerical breakdown, regardless of its historical accuracy:

Let x = bill y as amended to triple the rate of taxes required to be paid in every tax bracket; let y = a bill to raise taxes in every bracket by 15%; and let z be no change in tax rates. Now, again,

- 31% want the larger tax increases. (They are xyz or "Big taxers")
- 16% prefer the smaller increase, but would support the bigger hike if necessary. (They are yxz or "Pragmatists")
- 30% support the 15% increase, but would balk at anything higher. (They are yxz or "Small Taxers")
- 23% (12% + 11%) won't support any increase. (They are zyx or "Anti-taxers")

Summing indicates that a plurality of deciders prefers a tax increase to the status quo. Let us now imagine, however, that the anti-taxers have met with their constituencies, have explained that they can kill the impending tax bill by pretending to support the bigger hike, and have been given a hearty go-ahead. These representatives see that if they vote with the big taxers to amend the bill and no other group votes strategically, they will be able to ensure the death of the original bill by making the taxes too high for the small-taxers. They would seem to have a winning, anti-democratic strategy.[18]

This should be allowed because, to repeat, the point of this exercise is not to make any claim about the frequency or likelihood of manipulation via agenda-setting, but simply to show how the possibility of any such subversion may be eliminated by moving to (our rule-following) AV and allowing no more than a single vote on all alternatives at once. To show this, however, we must, as discussed above, first convert the voter preference profiles to approval profiles. Obviously, we will need to proceed by stipulating to some vote breakdown, since no particular mapping to approvals is strictly entailed by any preference profile. So, let us say that of the 132 big taxers, 99 approve of both x and y; and 66

approve of x and z (or of x only, which will here make no difference).[19] Let us further assume that the 67 pragmatists approve of x and y only, the 130 small taxers approve of both y and z (or y only) and the 97 anti-taxers approve only of z. Given these hypothetical approval profiles, the result of a single simultaneous approval vote would yield this result:

- x receives 232 votes
- y receives 296 votes
- z receives 293 votes

This is a very narrow victory for the small tax increase, and it is certainly the case that a very small change in the breakdown (say by increasing the number of big taxers who want no part of a small increase) would flip it, allowing for the status quo to again prevail. Nevertheless, we have here a plausible scenario in which strategic sequential voting produces a minority victor but where the same result could not be obtained if a single AV vote were substituted. And this is not a fluke, the result is generalizable:

In any instance in which there are three approval profiles x, y, and z such that y is a compromise position between a more extreme position x and the status quo z; Ax (the number of approvals of x) is greater than either Ay or Az; and (Ay + Az) is greater than Ax, then y will prevail in a rule compliant AV election.

AV is not amenable to any strategy in which, by either voting for an option or candidate of which one really disapproves, or failing to vote for an option of which one actually approves, one can manufacture a desired, anti-democratic result. Any such disguise of an authentic attitude can only produce outcomes that are perceived as worse by the strategists.

This inoculation against the anti-democratic use of agenda-setting should be welcome to friends of majoritarian democracy. How often such manipulations actually take place in legislative settings, or whether they are even feasible under currently common legislative rules is controversial, but whatever may be the fact of those matters, such distortions of democracy cannot occur under AV.

IV. AV and Majority Cycles

The frequency of the occurrence of voting cycles in the real world has also been the subject of contentious debate. But what I believe will be generally agreed upon is that Condorcet was correct when he noted that, where there are three voters, P, Q, and R with the preference profiles x>y>z, y>z>x, and z>x>y, respectively, there are majorities in favor of each of the three alternatives. This sort of cycle, as Arrow has shown, is unavoidable under every type of minimally democratic preferentist voting mechanism whenever there are more than two candidates. Thus, if more is needed than a majority pick between two candidates to assess a “general will” or “the position” of an electorate, no ranking scheme can provide it. That essential defect of most voting systems used around the world is overcome by AV. The above preference profiles are consistent with all of the following approval profiles (where “A” means “approves of” and the double dashes indicate that none of the alternatives is approved):

P	Ax, Ay, Az	Ax, Ay	Ax, Az	Ax	--
Q	Ax, Ay, Az	Ax, Ay	Ay, Az	Ay	--
R	Ax, Ay, Az	Ax, Az	Ay, Az	Az	--

In other words, whatever one's preference ordering, one may approve all of the three alternatives, one's favorite and one other only, just one's favorite, or none of the three. That being the case, use of AV in this situation is quite likely to produce a tie, but it can also produce a legitimate winner. In fact, the preference array for A, B, and C set forth above is consistent with many coherent, (non-strategic) AV elections that will produce exactly one winner. That is because, as the above table shows, there are numerous different ways in which each candidate can rationally be approved by all three, two, one, or none of the voters.[20]

V. Conclusion

Because of the unique features of Approval Voting, at least where there is compliance with Rule (1) set forth above, any election where AV is used to select one winner from among any two or more alternatives provides several advantages not found within other systems. First, no interpretation of the independence of irrelevant alternatives according to which adherence to the principle is essential to rationality is violated by the legitimate (i.e., Rule (1)-compliant) use of AV. Second, democracy-destroying agenda-setting games are made impossible. Finally, by eschewing preferential rankings, AV eliminates all danger of the sort of intransitivity displayed in Cordorcet cycles—either in individual or group aggregations. Thus, it is not subject to Arrow's Impossibility Theorem.

Endnotes

[1] I am grateful to Jack Nagel for a number of helpful comments.

[2] At least in its single winner version. And I think there are several better mechanisms for the election of multiple winners in a single district, which I take to be the best way to provide proportional representation. I will not discuss that matter here.

[3] For my general take on the necessary and sufficient conditions for authentic democracy and my discussion of a number of other virtues of Approval Voting, see Horn, 2020 and 2021.

[4] Horn, 2020, Chapters 7 and 8.

[5] See, e.g., Niemi, 1984, Ohtsubo and Wantanabe, 2003, and Nagel, 2007.

[6] Fourier seems to have no support at all.

[7] See Niemi and Bartels, 1984.

[8] It is not, at any rate, resolvable by tinkering with electoral rules, as Gibbard, 1973 has demonstrated.

[9] Arrow, 1951.

[10] Such a shift is exemplified in the famous story about Sidney Morgenbesser, in which he pretended to reverse his preference for apple pie over blueberry pie upon learning that cherry pie was also available. See Ohtsubo and Watanabe, 2003 for a discussion of AV with respect to this sort of change in circumstances.

[11] I am grateful to Greg Dennis for pressing these criticisms. See also Nagel, 2007.

[12] The symbol ">" should be read "to be better than".

[13] Sen, 1970.

[14] However, I do not want to be thought to be denying what Jack Nagel has suggested to me is a dangerous possibility in the real world: "In considering AV as a voting or social choice mechanism, failure to guarantee IIA2† in the collective choice is important because it sets up the possibility of manipulation through agenda control—i.e., by adding or subtracting alternatives."

[15] See, e.g. Denzau, Riker and Shepsle, 1985 and Riker, 1986. The Rikerian analysis has been thoroughly criticized, by, among others, Mackie, 2003 and Gilmour, 2001, with the last-named author even offering an obituary for the claim. As I have no desire either to disinter or reinter this matter here, I will use some of the (ostensible) data of the case only for the purpose of simplifying it by altering various elements.

[16] Blydenburgh and Riker, 1971 flesh out the mechanics of the manipulation by claiming that a number of subsidy-opposing Republicans were able to kill all proposed school aid by craftily supporting the Powell amendment and thus setting the stage for a final vote between the Powell-amended proposal and the status quo ante (no aid at all).

[17] Some critics go further. In fact, while Riker has suggested that democracy-defeating manipulation via agenda-setting and strategic voting are nearly omnipresent in legislatures and committees, Mackie, 2003 has taken a contrary position according to which such shenanigans are extremely infrequent (if not entirely non-existent!) and, claims at any rate, that no single example of such behavior has ever been conclusively shown to exist in the halls of the U.S. Capitol.

[18] "Yes," It might be objected to this scenario, that "it is no doubt true that 53% of the voters is a sufficient number to kill new taxes and will do so if the votes are ordered in a certain way. But why would the other members not be just as strategic and so be able to defeat that effort?" This complaint might be answered by the hypothesis that the pro-tax constituents are not as flexible as those of the anti-taxers, or that the pro-tax members fear that a strategic vote in this instance might produce powerful opposition in an upcoming election. Alternatively, one might suggest that the pro-tax members were, at least in this case, simply not as clever as the anti-taxers. As we are here claiming only that the scenario is possible, and urge nothing about its likelihood, we can get where we want to simply taking the result as a

hypothetical possibility.

[19] The big taxers might take this latter position because they think that any smaller increase than that which is provided by x will do almost nothing and that tax bills can only be passed very infrequently.

[20] And those who believe that it would be odd for AV voters to bother to approve all or no alternatives should note that even if we remove the second and sixth columns based on that objection, there will remain multiple ways for an individual alternative to win. Subsequent to such (arguably inappropriate) truncations, there would be, for example, 12 different ways in which x could get exactly two votes. In those 12 scenarios, x would win in two, tie in eight, and lose to each of the other candidates once.

References

- Arrow, Kenneth. 1951. *Social Choice and Individual Values*. New York: Wiley & Sons.
- Blydenburgh, John C. 1971, "The Closed Rule and the Paradox of Voting." In*The Journal of Politics*, Vol. 33, No. 1: 57-71.
- Denzau, Arthur, William Riker and Kenneth Shepsle. 1985. "Farquharson and Fenno: Sophisticated Voting and Home Style." In *American Political Science Review*, Vol. 79, No. 4: 1117-1134.
- Gibbard, Allan. 1973. "Manipulation of Voting Schemes: A General Result." In*Econometrica*, Vol. 41, No. 4: 587-601.
- Horn, Walter. 2020. *Democratic Theory Naturalized: The Foundations of Distilled Populism*. Lanham, MD: Lexington Books.
- Horn. Walter. 2021. "Why Radical Democracy is Inconsistent with 'Mob Rule.'"*The Romanian Journal of Society and Politics*, Vol. 15, No.1: 7-22.
- Mackie, Gerry. 2003. *Democracy Defended*. Cambridge: Cambridge UP.
- Nagel, Jack H. 2007. "The Burr Dilemma in Approval Voting." In*Journal of Politics* Vol. 69, No. 1: 43-58.
- Niemi, Richard G. 1984. "The Problem of Strategic Behavior Under Approval Voting." In*American Political Science Review*, Vol. 78, No. 4: 952-958.
- Niemi, Richard G. and Larry M. Bartels, 1984. "The Responsiveness of Approval Voting to Political Circumstances." In *PS*, Vol. 17, No. 3: 571-577.
- Ohtsubo, Yohsuke and Yoriko Watanabe. 2003. "Contrast Effects and Approval Voting: An Illustration of a Systematic Violation of the Independence of Irrelevant Alternatives Condition." In *Political Psychology*, Vol. 24, No. 3: 549-559.
- Riker, William. 1986. *The Art of Political Manipulation.* New Haven: Yale UP.
- Sen, Amartya. 1970. *Collective Choice and Social Welfare*. San Francisco: Holden Day.