Part II

Possibility Spaces

5 Invariance, Modality, and Modelling

Andreas Hüttemann

5.1 Introduction

Scientific modelling can have many purposes. Generating modal knowledge and justifying modal claims are two such purposes. Such claims may concern, for instance, what is physically, biologically, and so on, conceivable, or they may concern possible explanations. In this chapter, I will focus on the relation between scientific modelling and *objective* or *de re* modal features of systems. More precisely, I will discuss how scientific modelling gives us knowledge about *possible* states of systems and about the ways in which the behaviour of target systems is *constrained*. I will argue that the concept of invariance is particularly helpful for exploring this relationship. After some stage setting (section 5.1), I will discuss the concept of invariance (of laws) and will argue that such invariances are typically empirically accessible (section 5.2). In section 5.3, I will argue that (some) modal features of the behaviour of systems can be understood in terms of invariance relations. Section 5.4 then concludes by discussing the connections between empirically accessible modal features of the behaviour of systems and some aspects of our modelling practices such as abstractions and idealizations.

5.2 Stage Setting: Models and Laws[1]

5.2.1 Models

In order to situate the following discussion, it will be useful to introduce some of the distinctions that have been discussed in recent literature on scientific modelling. Michael Weisberg stresses the flexibility of modelling by pointing to the variety of possible targets. Modelling can be used to study specific systems, for example, a specific fishery in the Adriatic Sea, clusters of targets, a generalized target (predator-prey-populations), or hypothetical targets such as species with more than two sexes. There is also targetless modelling when the features of models are studied without considering whether the model in question represents a target system (Weisberg 2013, chapters 5 and 7). In what follows, I will focus on models of generalized targets such

DOI: 10.4324/9781003342816-8

as harmonic oscillators, free-falling stones, or economies. (This chapter will thus be complementary to Michaela Massimi's paper (2019) on "Two kinds of exploratory models," which focuses on hypothetical models and targetless fictional models and their relation to modal knowledge.)

Another useful distinction has been introduced in a paper by Reutlinger, Hangleiter, and Hartmann (2018). Models are either embedded in empirically well-confirmed theories or they stand on their own and are thus autonomous. Quantum Mechanics and Newtonian mechanics are examples of theories within which models are embedded. Paradigm cases of embedded models are the harmonic oscillator, models of the hydrogen atom, and so on—the kind of models that Cartwright studied in *How the Laws of Physics Lie* (1983). These models typically serve the function of subsuming phenomena under abstract dynamic laws such as the Schrödinger equation. Autonomous models, by contrast, stand on their own. Well-known examples are Schelling's model of segregation and the Lotka-Volterra model. The focus of this chapter will be on embedded models~~, that is, how are models embedded in theories or laws?~~

5.2.2 Law Statements

With respect to laws of nature, their *content* (i.e., what the law statements state) can be distinguished from their *nomic status*. The latter will be discussed in section 5.3. Here I will focus on the content of law statements. Law statements (or theories) are not simply mathematical equations. Even if we know that in the equation $s = 1/2gt^2$ *s* stands for a path and *t* for a time, the equation cannot be taken to be Galileo's law of free fall. For instance, nobody takes Galileo's law to be disconfirmed by spheres rolling on a plane. Galileo's law is not simply a mathematical equation but the claim that the behaviour of a certain class of systems can be represented by the above equation:

Free-falling bodies behave according to the equation $s = ½ gt^2$.

It is essential for Galileo's law that it refers to free-falling bodies. They are the generalized target systems of the equation. For the purposes of this paper, I take the general form of a law statement to be the following:

(*L*) All physical systems of a certain kind *K* behave according to Σ

Σ is what I call the "law-predicate," which includes the mathematical equations that are meant to characterize the behaviour of the systems. (*L*) is ultimately of the time-honoured form "All Fs are Gs," but the latter hides all the interesting complexity in Σ that will be relevant for discussing the modal aspects of laws, models, and systems.

How are models and law statements related? Even though models have a plurality of possible functions (see Gelfert 2016 for an overview) within (*L*) models serve the function of representing a (generalized) target system. An example is the statement:

(H) All hydrogen atoms behave according to the Schrödinger equation with the Coulomb potential.

The hydrogen atoms are the generalized target systems. The Schrödinger equation with the Coulomb potential is meant to represent the behaviour of the system in question. The hydrogen atoms are modelled by the Coulomb potential. Modelling assumptions, for instance, idealizations and abstractions, are typically introduced when it comes to specifying the Hamilton operator. For instance, picking the Coulomb potential implies that certain variables are relevant while others are left out as irrelevant.

5.2.3 Internal and External Generalizations

Given the above characterization of law statements, we can draw a distinction between different kinds of generalizations that will play a role in what follows.

Law statements typically involve at least two different kinds of generalizations (see Scheibe 1991). A *system-external generalization* quantifies over systems. System-external generalizations *explicitly* occur in (*L*): "all systems of a certain kind K." *System-internal generalizations*, by contrast, are not explicitly mentioned in (*L*), but they are assumed. They quantify over the values of the variables that occur in the law equations. To illustrate this distinction: in Galileo's law (*Free-falling bodies* behave according to the equation $s = ½ gt^2$), we can distinguish (1) a generalization that quantifies over systems—the equation is said to pertain to all systems of a certain kind *K*, namely to free-falling bodies. This is a system-external generalization. (2) generalizations that quantify over the values of the variables in the mathematical equation in Σ, in this case, *s* and *t*. The equation is meant to hold for all values of, say, the variable *t* (within a certain range). This is a system internal generalization.

It will turn out that many interesting aspects of the modality of laws are connected to internal generalizations.

Working with embedded models is an important aspect of scientific practice (at least in physics), and the question of how modelling practices in this context are related to modal features of the behaviour of the target systems is the issue I will address in the remainder of this chapter.

5.3 Invariance

5.3.1 Different Kinds of Invariance

In what follows, the notion of *invariance* will serve to link modelling assumptions such as idealizations and abstractions on the one hand and (objective) modal features of systems on the other. But what is an invariance? The basic

BK-TandF-KNUUTTILA_9781032379647-241020-Chp05.indd 105 12/11/24 3:47 PM

idea is that something remains the same while certain changes or transformations do or could take place. Invariance is a modal notion, so in arguing that modal features of systems are to be understood in terms of invariances I am not engaged in the project of explaining modal notions away.

In the philosophical literature, invariance has been linked to notions such as truth and objectivity. The fact that in the special theory of relativity, the space-time-interval (as opposed to the spatial interval) is invariant under Lorentz transformations has been taken to show that this is an objective non-relative feature of the world (see, e.g., Nozick 2001). These implications of (some kinds of) invariance will not be the focus of this chapter. I will instead distinguish various kinds of invariances and explore their role in scientific practice.

Wigner, in a famous paper on invariance, observed:

> The world is very complicated and it is clearly impossible for the human mind to understand it completely. Man has therefore devised an artifice which permits the complicated nature of the world to be blamed on something which is called accidental and thus permits him to abstract a domain in which simple laws can be found. The complications are called initial conditions; the domain of regularities, laws of nature. (Wigner 1949, 521)

So, according to Wigner, the very notion of a law presupposes a distinction of something that changes ~~(according to the law)~~ on the one hand and the law(-equation) itself, which remains invariant, on the other.

Let us illustrate this notion of invariance with Newton's second law:

(N2): All bodies behave according to the equations $F = ma$.

Despite changes in initial conditions, that is, in the values of the force *F*, (N2) and thus in particular the law equation remains the same for all values of these initial conditions. The law and thus the internal generalization (as well as the external generalization) remain invariant.

Another example is Galileo's law:

Free-falling bodies behave according to the equation $s = ½ gt^2$.

Despite changes in initial conditions (e.g., the values of *s*), Galileo's law and thus in particular the law equation remain the same. In this case, however, the law equation is invariant only for small *s* compared to the diameter of the earth. Invariance, in this case, is domain-restricted, that is, not universal as in the case of Newton's second law. Furthermore, the domain restriction may depend on pragmatic considerations, for example, on the question with which precision we need to know *s*. The more precise we want to be, the more the domain of invariance will be restricted.

This provides us with a first kind of invariance:

> Invariance *of the law equation* with respect to *the initial conditions*: The law equation holds independently of the values (within a certain domain) of the initial conditions.
>
> (see Woodward 2018, section 3 for discussion)

What is characteristic of the first kind of invariance of laws is that the quantities or variables that change are characterized in terms of variables which are explicitly mentioned in the law predicate. There are, however, also changes with respect to quantities or features of reality that are not explicitly mentioned in the law equation. This leads to a second kind of invariance:

> Invariance *of the law equation* with respect to *features of the target system that are not explicitly mentioned in the law equation*:

i same-level properties, for example, colour, shape, and mass in the case of free-falling bodies.
ii lower-level or constitutional properties, for example, the molecular structure of the gases in the case of the ideal gas law.

Closely related is a third kind of invariance:

> Invariance *of the law equation* with respect to *changes* (not of the target system but) *in the behaviour of other systems*.

Newton's laws of motion and his law of gravity hold for the solar system whether or not other systems in the universe undergo changes. Another example is the ideal gas law. Its equation holds whether or not I am on time when I go to the dentist or whether or not a particular tree loses its leaves in New Mexico. Of course, the initial or boundary conditions of the system under consideration will change in the case of the solar system due to changes of other systems in the universe, but the law equations remain the same.

To sum up, the various types of invariance we have discussed so far (the list I presented is not exhaustive) differ with respect to the kind of changes that are envisaged.

The law equation of the particular system under investigation may remain invariant under changes with respect to:

- the initial conditions (first kind of invariance),
- features of the target system that are not explicitly mentioned in the law equation (second kind of invariance),
- the behaviour of systems elsewhere in the universe (third kind of invariance).

Let me briefly flag that there is one kind of invariance that, despite its prominence in the philosophy of physics literature, will not play an important role in what follows (for reasons to be indicated later).

Given the first kind of invariance, the law equation remains the same under changes of initial conditions. In general, however, the *solutions* of these law equations will change if the initial conditions change. Given Newton's second law, if a different force is applied, the law equation will remain the same; the solution of the equation will, however, in general, be different.

Changes in initial conditions that not only leave the law equations unchanged but also the solution of these equations lead to what might be called "transformational invariance":

> Invariance not only of the law equation but also *of the solutions of the law equation* with respect to *certain changes of initial conditions* (which are called 'symmetry transformations' if they leave the solutions unchanged).

Simple examples are spatial transformations or velocity boosts in Newtonian mechanics. In this case, it is not only Newton's second law that remains the same but also the solutions of Newton's second law. (It does depend on the law equation(s) in question whether or not such symmetry transformations are allowed.) It is this kind of invariance that has been the focus of the philosophy of physics literature (Brading and Castellani 2003; Brading and Castellani 2007; Brading, Castellani, and Teh 2021; Wigner 1949).

5.3.2 Invariance as an Empirically Accessible Relation

Invariance relations are accessible by ordinary empirical methods. This will turn out to be relevant because it implies that other modal features of the behaviour of systems, which can be accounted for in terms of invariance relations, are empirically accessible too. Let me start with how we empirically investigate dependence claims. Suppose we are given claims like "The length of a metal rod depends on its temperature" or "The period of a simple pendulum depends on the length of the string L." Such dependence claims are (in principle) empirically accessible: we vary (in the first case) the temperature and figure out how the length of the rod changes as a result. Similarly, in the case of the simple pendulum's period and the length of its string.

Independence claims are empirically accessible in exactly the same way as dependence claims. Take, for example, the claim, "The period of a simple pendulum is independent of the mass of the pendulum." We now need to vary the mass of the pendulum and see how this affects the period. Provided other things have been kept equal, the period is independent of the mass if the period's values remain constant despite changes in the value of the mass. Of course, if we want to figure out dependence and independence claims for more complex systems in more complex settings, things will become more complicated. Background theories will play their part, and so on. But there is no reason to assume that independence claims are more difficult to ascertain than dependence claims or *vice versa*.

The essential point is that invariance claims are independence claims, and they are empirically accessible as other (in)dependence claims are. Whether or not Hooke's law or the Schrödinger equation continues to hold under changes of initial condition is something we can empirically check. In the case of Hooke's law (which states that the force F that pertains to a body attached to a spring extended by a distance x conforms to the equation $F = -kx$), it turns out that the law equation holds only for values of x that are small compared to the possible extension of the spring. Similarly, whether or not a certain law equation is invariant under changes of certain features of the target system is empirically accessible. In the case of Galilei's law, we can test whether or not colour, mass, and shape are irrelevant in free fall, and the same is true for changes in constitutive properties. Again, whether or not the behaviour of other systems leaves a law equation unchanged can be empirically accessed. The important point is that ascertaining whether or not law equations are invariant in any of the above-mentioned senses is not something that transcends our usual scientific methods.

5.3.3 Realism about Invariance

So far, we have talked about the invariance of laws or law equations. Within the frame of scientific realism (which I will assume here), we have good reasons to hold that the entities and properties quantified in well-corroborated laws or theories exist as mind-independent features of reality. As a consequence, invariances of laws—in general—translate into invariances of the behaviour of the target systems. Thus, for example, the fact that a law equation is invariant with respect to features of the target system that are not explicitly mentioned in the law equation can be translated into a claim about the target system: the behaviour of the target system (e.g., a falling body) is invariant with respect to features not explicitly mentioned in the relevant law equation (e.g., in colour of the body). This applies also to cases where the invariance relation pertains only to a restricted domain, as, for example, in the case of Hooke's law. Even if the boundaries of the domain are set by requirements on precision, which are ultimately due to pragmatic considerations, it is an objective and empirically determinable feature of the spring under consideration whether or not its behaviour (as described by Hooke's law) remains invariant within a certain domain for the elongation of the spring.

I hedged the claim that the translation from the invariance of laws to the invariance of the behaviour of systems holds by the "in general"-clause because there are specific reasons to be more cautious in the case of transformational invariance (the *solutions* of law equations are invariant with respect to certain kinds of symmetry transformations). There is an extended debate over whether or not this type of invariance is indeed indicative of invariances of the target system or whether it is due to mathematical surplus structures in the relevant theories. The question is whether, for instance, in the case of Newtonian mechanics the invariance of the solutions with respect to spatial transformation shows something about the behaviour of systems or whether it indicates that the mathematical structure of Newtonian mechanics allows one to distinguish situations that are in fact identical (see the essays in Brading and Castellani 2003 or Dewar 2019). So, according to this latter interpretation, symmetries like translational or rotational symmetries of space indicate that the theory has some mathematical surplus structure, which should not be interpreted realistically. In the case of Newton's Mechanics, absolute velocities or positions would have no physical meaning. I will not delve deeper into this debate. I have bracketed transformational invariance precisely because it is controversial as to whether it allows for a realist reading.

5.4 Nomological Modalities

Thus far, I have discussed the fact that laws and *a fortiori* systems that are characterized in terms of these laws may be invariant with respect to some changes. I have also argued that such invariance or independence relations are accessible by ordinary scientific methods. In this section, I argue that nomological necessity and nomological possibility—two modal features that are often associated with laws as well as the systems that are characterized in terms of these laws—can be understood in terms of invariance relations. This idea is not new; it has been put forward, among others, by Mitchell (2003, 140), Lange (2009) and Woodward (1992, 2018). The following section can be read as fleshing out Woodward's claim that "invariance-based accounts [of nomological necessity] provide a naturalistic, scientifically respectable, and non-mysterious treatment of what non-violability and physical necessity amount to" (Woodward 2018, 160).

To start, what qualifies as a modal feature of a system? A modal aspect of the behaviour of systems is an aspect that concerns not (only) the actual behaviour of systems but (also) possible behaviour or behaviour that takes place by some sort of necessity. To illustrate, laws tell us how systems might or would behave—provided certain conditions were to be met; that is, they tell us that this behaviour is possible. Laws furthermore tell us how the temporal evolution of systems is constrained, that is, that a certain necessary evolution is bound to happen. Nomologically modal features of systems are those modal features that are obtained simply by virtue of the fact that the behaviour of systems can be characterized in terms of statements of the form (*L*). Thus, laws attribute to the target systems, by virtue of the domain that the quantifier of the internal generalization is restricted to, a space of nomologically possible states. The law equation constrains, for example, how these states develop over time. Let us have a closer look at these modal claims.

5.4.1 A Space of Possibilities

By virtue of internal generalizations, laws attribute a space of possible states to systems. In the case of dynamical laws, it is assumed that the systems have a set of possible initial states. With respect to these states, we can distinguish two cases. Either the domain of quantification comprises all possible states (e.g., in the case of Newton's second law or the Schrödinger equation) or, as is the case of more specific laws, the domain of quantification comprises only a restricted range of states. Hooke's law, as we have seen above, holds only for a limited range of extensions of the spring.

The point to emphasize is that we are dealing with a modal claim because it is not only actual states or actual behaviour that the internal generalizations quantify. In fact, the internal generalization on its own does not even tell us which state of the system is the actual state.

The internal generalization's concern is possible behaviour only (whether actual or non-actual). Thus, the fact that internal generalizations come with a domain of values for variables requires the assumption that law statements attribute a space of possible (and mutually exclusive) states to systems. The fact that laws of nature attribute a space of possibilities to systems is essential for scientific practice (at least in physics) and the application of laws. It allows us to know what would happen if certain circumstances were to obtain (which we then might choose to bring about or to prevent to occur).

5.4.2 Constraints

Let me now turn to a second aspect of internal generalizations that is relevant for the examination of modal structure—the law equation. Law statements do not simply register the past, present, and future behaviour of systems; they describe how this behaviour is constrained. Internal generalizations put restrictions on the space of possible behaviour of systems by establishing relations between variables (i.e., law equations). These restrictions can either concern the synchronic co-possibility of values of variables that characterize the state of a system, as in the case of the ideal gas law, or the temporal evolution of the states of a system as in the case of the Schrödinger equation.

In the case of a synchronic law (law of co-existence), for example, the ideal gas law, the set of possible values for the variables p, V, and T is restricted to those that satisfy the equation $pV = \nu$ RT. Thus, the possible states of the gas are constrained to a two-dimensional hypersurface of the three-dimensional space that is generated by the variables p, V, and T. The internal generalization does not only provide information about how the actual state of a system (if known) is constrained. In addition, it tells us how all possible states of the system are constrained, whether or not they are actual. That the systems are constrained means that those states not on the hypersurface are not accessible for the system. They are classified as states the system cannot possibly occupy, given the law equation, that is, as nomologically impossible states.

The fact that the gas satisfies the equation of the gas law allows a scientist or an engineer who can manipulate pressure and volume to *ensure* that the gas will have a certain temperature. Similarly, the engineer might want to prevent certain situations, such as preventing a gas from having a certain temperature. In such cases, she will rely on the fact that the law tells us that certain combinations of pressure, volume, and temperature will not occur; by setting pressure and volume appropriately, we can make sure that a certain temperature value will not be obtained.

The same holds for internal generalizations that describe the temporal evolution of a state of a system. Provided we prepare the system under consideration in a certain state and provided the equation in question is deterministic, we can ensure that at a later time, the system is in a certain state, and we can also prevent the system from being in certain other states. In the case of prevention, it is not only that given certain combinations of, say, pressure and volume, certain values for T simply do not occur. These values *cannot* occur. The use scientists and engineers make of internal generalizations in scientific practice is best understood by assuming that internal generalizations represent modal, that is, nomologically necessary relations.

5.4.3 Nomological Modality and Invariance

In this section, I will argue that modal notions such as nomological necessity or nomological possibility can be understood in terms of invariance relations. The point is not that modal notions can be understood in terms of a non-modal notion. Invariance, as introduced in section 5.2, characterizes something as remaining the same while certain changes or transformations do or could take place. It is defined with respect not only to actual but also to counterfactual changes. Invariance is thus clearly a modal notion. The project is not an attempt to eliminate or reduce modal notions but rather that of showing that nomological necessity and nomological possibility can be understood in terms of something that is empirically accessible.

How is nomological necessity related to invariance? Let us approach this issue by contrasting nomologically necessary laws with accidental generalizations. Take, for instance, Reichenbach's famous example of an accidental generalization: "All gold cubes are smaller than one cubic mile" and contrast it with Newton's second law. The former is a paradigm case of an accidental truth; that is, it is an accidental matter that it has not been rendered false. There is enough gold in the universe to build large gold cubes, it would have been costly but otherwise easy to do this. It simply never happens, neither in the past nor in the future, but it could have happened, for example, by actors intervening appropriately. By contrast, there are no interventions or other natural changes (more about natural changes in a minute) that could render Newton's second law false. It will hold, come what may. In other words, the difference between these examples for an accidental generalization and for a law is a difference that can be spelt out in terms of invariance: while Newton's law is invariant with respect to natural changes, an accidental generalization, such as Reichenbach's, is not.

Understanding nomological necessity in terms of invariance furthermore explains why we can rely on laws (e.g., when it comes to their application in technological contexts) while we cannot rely on accidental generalizations. We can rely on laws because they will not be rendered false, given a large range of circumstances. When we rely on Newtonian mechanics in building a bridge, we assume that the equations will continue to hold under a wide range of actual and possible conditions.

If we, as suggested, spell out the nomological necessity of laws (and *a fortiori* of the behaviour of the target systems) in terms of invariance, the fact that invariance relations may be domain-restricted translates to a domain restriction of nomological necessity.

Newton's second law or Schrödinger's equation may be invariant under any natural changes. Hooke's law, as we have seen, is not. Even Reichenbach's accidental generalization is invariant with respect to some changes. But that should not be considered to be a bug of the account. The traditional categories—accidental generalization on the one hand and law (nomologically necessary generalization) on the other—are the endpoints of a spectrum. This spectrum needs to be explored empirically. It may very well turn out that certain invariances (e.g., with respect to initial conditions) turn out to be more relevant for law status than others. But that is to some extent uncharted territory.

Let me briefly address a worry that has been raised (Psillos 2002, 185). I argued that Newton's law is invariant under *natural* changes. But it seems that we need to spell out natural changes as changes that are in accordance with laws of nature, that is, that are nomologically possible. The account thus appears to be circular. There are two things to be said about this. The account would indeed be circular if the project were one of reducing modal to non-modal facts. But that is not the project envisaged here. The point is to draw a difference between the gold-cube generalization being accidental and Newton's second law being nomologically necessary by relying on a class of natural changes that have been identified antecedently. (The same strategy is used in the causal interventionist literature *vis-à-vis* the objection that definitions of causes in terms of interventions are circular (see, e.g., Woodward 2003, 20–22.) Furthermore, it may very well be possible to characterize the class of what I have called here "natural changes" in terms that do not rely on the notion of law or nomological necessity (see Lange 2009, chapter 1, for such an attempt). However, for the purposes of this chapter, I do not need to commit myself to this position.

As we discussed above, laws ascribe a space of nomologically possible states to target systems by specifying a domain to which the quantifier of internal generalizations is restricted. In other words, nomologically possible states are those states of the target systems that are compatible with the law equation. Changing the system's state, if restricted to this domain of states, leaves the law equation invariant. Nomological possibility, when understood in terms of a space of possible states for a system, can thus be understood in terms of an invariance relation. The same holds if we use the term "nomological possibility" in a broader sense as referring to scenarios that are compatible with a whole set of laws. Again, compatibility with laws can here be understood as follows: switching from the actual world to such a scenario leaves the laws invariant. Thus, nomological possibility, both in a narrower as well as in a broader sense, can be accounted for in terms of invariance relations.

To sum up, what I have argued for is that our notions of nomological necessity and nomological possibility be explicated in terms of invariance or independence relations. While both the former as well as the latter are modal relations, the important point is that the latter are empirically accessible relations.

5.4.4 Humeanism and Non-Humeanism

Can we say something more about the nature of the invariance relations? One may, for instance, ask "What underpins these invariances?" (Bird 2007, 5). As is well-known, there are different accounts of nomological modalities in the literature on laws of nature. On the one hand, there are non-Humean accounts of laws which postulate via an inference to the best explanation that a *necessitation relation* obtains, which accounts for nomological modalities (e.g., Armstrong 1983). The idea is that this relation explains why a certain system is necessitated to behave as it actually does. Pieces of copper, for instance, cannot help but be excellent electric conductors because a second-order necessitation relation obtains between the property of being copper and that of being a good electric conductor. Others (dispositional essentialists, e.g., Bird 2005) postulate that it is of the essence of copper to be a good electrical conductor. Thus, again, a piece of copper in virtue of its essence cannot avoid being a good electrical conductor. Humeans, by contrast, typically don't believe in essences or necessitation relations. Ultimately, they think there is only the actual behaviour of systems and that there are the theories (maybe one *ideal* theory) about the actual behaviour that might serve as a basis for various counterfactual claims, which in turn underpin our modal talk (see, e.g., Lewis 1973).

The non-Humean is motivated by the idea that there needs to be something in nature, that is, *de re* necessities, to account for scientific practice. The Humean is motivated by the idea that we should refrain from postulating what is empirically inaccessible and thus rejects accounts such as Armstrong's or Bird's (see Earman and Roberts 2005). Both of these motivations seem plausible, and the invariance account allows you to take into account both: it argues that there are *de re* necessities which are this-worldly, immanent, and empirically accessible. That seems to be all we need.[2]

5.5 Modelling the Modal

We are now in the position to explain how (successful) scientific modelling leads to modal knowledge, where I take modal knowledge to be knowledge of (objective) modal features of the behaviour of target systems.

The construction of models has many purposes or functions (see, e.g., Gelfert 2019; Grüne-Yanoff 2013; Knuuttila 2021; Knuuttila and Loettgers 2017). Representing the behaviour of systems is (only) one of these—the one I have focused on here. When models are used for this purpose, the use of idealizations and abstractions may seem puzzling. I take idealizations and abstractions to be intentional misrepresentations of the properties or the behaviour of a target system, for example, by omission of properties of the systems or factors that contribute to its behaviour (abstraction) or by distorting the characterization of the behaviour or the properties of the system (idealization). The terms "idealization" and "abstraction" may refer both to the process or activity as well as to the product or result of a misrepresentation. The fact that idealizations are *intentional* misrepresentations is important in order to distinguish idealizations from hypotheses that turn out to be false (Hüttemann 1997). An idealization has to be distinguished from the *hypothesis* that a certain body is indeed a point particle. Because idealizations are intentional misrepresentations (in contrast to the case of hypotheses that turn out to be false), the question for the rationale for idealizations arises.

Possible candidates for such a rationale can best be discussed in the context of an (apparent) paradox associated with idealizations. The paradox arises provided certain assumptions about the aim of science or at least the function of law statements such as (*L*) are made: if, for example, the aim of science is representation of the phenomena, the question arises of why scientists idealize. Similarly, if it is assumed that science aims at explanation and explanations invoke laws that need to be true~~. Again,~~ if understanding is what is sought, and understanding presupposes veridicality (see discussion in Gelfert (2019) and Reutlinger et al. (2018)). If the aim of science presupposes that the descriptions of the behaviour of systems which are invoked to realize this aim are truthful, the question arises of why scientists idealize. What is the purpose or rationale of idealizations and abstractions?

Various approaches to deal with this paradox can be distinguished. First, there is the pragmatic approach. It sticks with the assumption that truth or representation is a necessary component of what science aims at and accounts for the use of idealizations as a compromise (see Strevens (2008, 298) for this characterization). We distort the characterization of target systems or leave out certain features in order to make the characterization mathematically tractable or tractable by simple mathematics. Furthermore, it is assumed that we can always de-idealize and thus make our characterization of the target systems more truthful (see Knuuttila and Morgan (2019) and McMullin (1985) for a discussion of the prospects of de-idealization).

According to a second approach, truth is not a necessary condition for what science aims at (or for explanation in particular). One prominent example is Cartwright's simulacrum account of explanation, developed in her paper, "The truth does not explain much" (Cartwright 1983). Cartwright argues that explanatory power comes with unifying power rather than with truth only. As a consequence, laws (including the modelling assumptions that go into Σ) do not explain *in spite of* idealization but rather *in virtue of* idealizations. The tension is resolved by revising assumptions about what science aims at. A third approach distinguishes between the occurrent behaviour of systems on the one hand and an underlying more fundamental structure that gives rise to the occurrent behaviour on the other. While idealizations may misrepresent the occurrent behaviour of systems, they may nevertheless be instrumental in correctly identifying the underlying structure, for example, the dispositions that underly the overt behaviour of the systems. This approach has been discussed by Cartwright as well (Cartwright 1989; see also Hüttemann 1998, 2014). The tension between idealization and truth is resolved by revising what kind of truths science aims at (revising what laws/models are supposed to represent).

There is no reason to assume that the various kinds of idealizations and abstractions that have been distinguished and discussed in the literature (see, e.g., Hüttemann 1997, chapter 2; Weisberg 2007) can all be accounted for by the same approach. My aim in the remainder of this section is to argue for the claim that the use of some modelling practices indicates that scientists assume invariance claims about the behaviour of systems to hold. We employ idealizations and abstractions when we have reason to assume that certain invariance relations hold. This suggestion can be seen as falling under the third approach. Science is not (only) interested in the actual or occurrent behaviour of the target systems but rather in their modal profile: how the system might behave (provided the right conditions hold) and how its behaviour is constrained.

Let me illustrate this claim by discussing a simple example. Suppose the target of our modelling is the behaviour of a real physical pendulum. Let us furthermore suppose that we model the motion of the pendulum as a classical harmonic oscillator (Figure 5.1).

With F_{return} the return force, L the length of the pendulum, m its mass, T its period, and θ the angular displacement from the vertical equilibrium position

$$F_{\text{return}} = \text{mg} \sin\theta(t)$$

$$\text{ma}_{\text{return}} = \text{mL}\, \text{d}^2\theta(t)/\text{dt}^2$$

For small amplitudes $\sin\theta(t)$ can be replaced by θ, which yields

$$\text{d}^2\theta(t)/\text{dt}^2 + g/L\ \theta(t) = 0$$

and finally

$$T = 2\pi\sqrt{L/g}$$

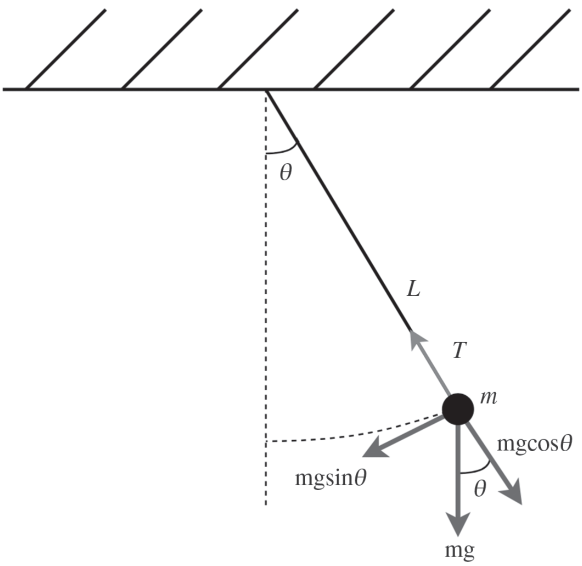

Figure 5.1 The harmonic oscillator.

If this model is empirically adequate (for small $\theta(t)$), it allows us to read off modal features of reality.

Why? First, the model is *abstract*. It leaves out features of the real pendulum and its environment. It does not mention the colour of the pendulum, the material it is made of, the exact form of the bob, the time of the day, the position of Jupiter, and so on. The fact that these things are not mentioned implies (for an empirically successful model) that the model is invariant with respect to these features. The equation with the force function provides an adequate account of the system, *whatever* the material, the colour, the time of the day, or the position of Jupiter. Abstractions in empirically successful models allow us to infer modal features of the target system.

Second, the model is idealized. It is assumed that the bob can effectively be treated as a point particle, even though it does have a finite extension. The fact that we can ascribe this feature to the bob, even though it does not have it, indicates that the extension of the bob is irrelevant to the behaviour of the pendulum (see Strevens 2008, Chapter 8). The law equation with the force function is—again—invariant with respect to this feature. Idealizations in empirically successful models allow us to infer modal features of the target system.

The following objection may be raised: models are empirically successful depending on pragmatic considerations, so that many models are approximate and simplifying yet very useful. But in those situations (which are ubiquitous in science), inferring modal features from the model invariances seems unjustified. This objection can be answered by pointing to the fact that even if one's measures of precision depend on pragmatic considerations and other purposes, it is an objective modal feature that the behaviour of the target system under consideration is or fails to be invariant relative to changes in a certain variable and provided the behaviour is characterized in this or that approximate way. It is, for instance, true that Galileo's law for free-falling bodies (and thus the fall of free-falling bodies) is invariant relative to changes in the height from which the body falls, given a certain accuracy of measurement.

Modelling practices assume that invariance relations hold. Conversely, if there is independent evidence (empirical or theoretical or both) for the obtaining of such invariance relations, the modelling practices in question may receive a *post facto* justification. Hüttemann, Kühn, and Terzidis (2015) have argued that the renormalization group approach to phase transitions provides a justification for abstractions that leave out the details of the systems' constitution.

5.6 Conclusion

Modelling the modal needs to be spelt out differently depending on the function of the models. In this chapter, I assumed that models sometimes have the function to represent target systems. Given this function, the role of idealizations and abstractions in modelling may seem puzzling. I argued that this puzzle can be resolved if what models and laws are meant to represent is not (only) the actual or occurrent behaviour but rather their modal profile. Idealizations and abstractions in empirically successful models allow us to infer modal features of the target systems—that their behaviour is invariant under certain natural changes.

Notes

1 This chapter uses and develops material from the first chapter of Hüttemann (2021).
2 In Hüttemann (2021, 74–78), I argue that while dispositions are an important ontological category to understand certain features of scientific practice, the modal nature of dispositions is not primitive but should rather be spelt out in terms of invariance relations.

References

Armstrong, David. M. 1983. *What Is a Law of Nature?* Cambridge: Cambridge University Press.
Bird, Alexander. 2005. "The Dispositionalist Conception of Laws." *Foundations of Science* 10: 353–70.
Bird, Alexander. 2007. *Nature's Metaphysics: Laws and Properties*. Oxford: Oxford University Press.
Brading, Katherine, and Elena Castellani eds. 2003. *Symmetries in Physics: Philosophical Reflections*. Cambridge: Cambridge University Press.

Brading, Katherine, and Elena Castellani. 2007. "Symmetries and Invariances in Classical Physics." In *Handbook of the Philosophy of Science. Philosophy of Physics*, edited by J. Butterfield, and J. Earman, 1331–67. Amsterdam, North Holland: Elsevier.

Brading, Katherine, Elena Castellani, and Nicholas Teh. 2021. "Symmetry and Symmetry Breaking." In *The Stanford Encyclopedia of Philosophy* (Fall 2021 Edition), edited by Edward N. Zalta (ed.). Stanford University. https://plato.stanford.edu/archives/fall2021/entries/symmetry-breaking/.

Cartwright, Nancy. 1983. *How the Laws of Physics Lie*. Oxford: Oxford University Press.

Cartwright, Nancy. 1989. *Nature's Capacities and Their Measurement*. Cambridge: Cambridge University Press.

Dewar, Neil. 2019. "Sophistication about Symmetries." *The British Journal for the Philosophy of Science* 70: 485–521.

Earman, John. 2002. "Laws, Symmetry, and Symmetry Breaking; Invariance, Conservation Principles, and Objectivity." (PSA 2002 Presidential Address)." *Philosophy of Science* 71(2004): 1227–41.

Earman, John, and John Roberts. 2005. "Contact with the Nomic: A Challenge for Deniers of Humean - Supervenience about Laws of Nature. Part I: Humean Supervenience." *Philosophy and Phenomenological Research* LXXI(1): 1–22.

Galilei, Galileo. 1954. Dialogues Concerning Two New Sciences. Transl. H. Crew and A. de Salvio. New York: Dover Publications.

Gelfert, Axel. 2016. *How to Do Things with Models*. Cham: Springer.

Gelfert, Axel. 2019. "Probing Possibilities: Toy Models, Minimal Models, and Exploratory Models." In *Model-Based Reasoning in Science and Technology*, edited by Matthieu Fontaine, Cristina Barés-Gómez, Francisco Salguero-Lamillar, Lorenzo Magnani, and Ángel Nepomuceno-Fernández, 3–19. Dordrecht: Springer.

Grüne-Yanoff, Till. 2013. "Appraising Models Nonrepresentationally." *Philosophy of Science* 80(5): 850–61.

Hüttemann, Andreas. 1997. *Idealisierungen Und Das Ziel Der Physik*. Berlin: de Gruyter.

Hüttemann, Andreas. 1998. "Laws and Dispositions." *Philosophy of Science* 65: 121–35.

Hüttemann, Andreas. 2014. "Ceteris Paribus Laws in Physics." *Erkenntnis* 79: 1715–28.

Hüttemann, Andreas. 2021. *Minimal Metaphysics for Scientific Practice*. Cambridge: Cambridge University Press.

Hüttemann, Andreas, Reimer Kühn, and Orestis Terzidis. 2015. "Stability, Emergence and Part-Whole-Reduction." In *Why More Is Different. Philosophical Issues in Condensed Matter Physics and Complex Systems*, edited by Brigitte Falkenburg, and Margret Morrison, 169–200. Springer: Dordrecht.

Knuuttila, Tarja. 2021. "Models, Fictions and Artifacts." In *Language and Scientific Research*, edited by Wenceslao J. Gonzalez, 199–22. Cham: Palgrave Macmillan.

Knuuttila, Tarja, and Andrea Loettgers. 2017. "Modelling as Indirect Representation? The Lotka–Volterra Model Revisited." *British Journal for the Philosophy of Science* 68(4): 1007–36.

Knuuttila, Tarja, and Mary S Morgan. 2019. "Deidealization: No Easy Reversals." *Philosophy of Science* 86(4): 641–61.

Lange, Marc. 2009. *Laws and Lawmakers: Science, Metaphysics, and the Laws of Nature*. Oxford: Oxford University Press.

Lewis, David. 1973. *Counterfactuals*. Oxford: Blackwell.

Massimi, Michaela. 2019. "Two Kinds of Exploratory Models." *Philosophy of Science* 86: 869–81.

McMullin, Ernan. 1985. "Galilean Idealization." *Studies in History and Philosophy of Science A* 16(3): 247–73.
Mitchell, Sandra. 2003. *Biological Complexity and Integrative Pluralism*. Cambridge: Cambridge University Press.
Nozick, Robert. 2001. *Invariances*. Boston: Belknap.
Psillos, Stathis. 2002. *Causation and Explanation*. Montreal: McGill-Queen's University Press.
Reutlinger, Alexander, Dominik Hangleiter, and Stephan Hartmann. 2018. "Understanding with (Toy) Models." *The British Journal for the Philosophy of Science* 69: 1069–99.
Scheibe, Erhard. 1991. "Predication and Physical Law." *Topoi* 10: 3–12.
Strevens, Michael. 2008. *Depth – An Account of Scientific Explanation*. Cambridge, MA: Harvard University Press.
Wigner, Eugene. 1949. "Invariance in Physical Theory." *Proceedings of the American Philosophical Society* 93(7): 521–6.
Weisberg, Michael. 2007. "Three Kinds of Idealization." *The Journal of Philosophy* 104: 639–59.
Weisberg, Michael. 2013. *Simulation and Similarity*. Oxford: Oxford University Press.
Woodward, James. 1992. "Realism about Laws." *Erkenntnis* 36: 181–218.
Woodward, James. 2003. *Making Things Happen*. Oxford: Oxford University Press.
Woodward, James. 2018. "Laws: An Invariance-Based Account." In *Laws of Nature*, edited by Walter Ott, and Lydia Patton, 158–80. Oxford: Oxford University Press.