Logic: inductive

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Logic is the study of the quality of arguments. An argument consists of a set of premises P_1, …, P_n and a conclusion C. The quality of an argument depends on at least two factors: the truth of the premises, and the strength with which the premises confirm the conclusion. The truth of the premises is a contingent factor that depends on the state of the world. The strength with which the premises confirm the conclusion is supposed to be independent of the state of the world. Logic is only concerned with this second, logical factor of the quality of arguments.

Deductive logic classifies arguments into two kinds: those where the truth of the premises guarantees the truth of the conclusion, and those where they do not. The former are called deductively valid, and the premises are said to logically imply the conclusion. The latter are called deductively invalid. So the deductive-logical explication of the logical factor of the quality of an argument is the qualitative yes-or-no concept of deductive validity.

Inductive logic aims at a more lenient explication of the logical factor of the quality of an argument. It comprises deductive validity as a special case. The reason is that the conclusions we are normally interested in are too informative to be logically implied by premises we can know. For instance, no set of premises about the past and present logically implies a conclusion about the future. Inductive logic usually aims at a quantitative explication of the logical factor of the quality of an argument, viz. the degree to which the premises confirm the conclusion.

Hempel (1945) made one of the earliest attempts to develop a formal logic of qualitative confirmation. His goal of constructing a purely syntactical definition of confirmation is shared by Carnap (1962), who goes beyond Hempel by aiming at a quantitative concept of degree of confirmation. Carnap bases his inductive logic on the theory of probability (Kolmogorov 1956). Due to Goodman's (1983) "new riddle of induction" there is consensus nowadays that a purely syntactical definition of (degree of) confirmation cannot be adequate. However, the use of probability theory has been a central feature of inductive logic ever since.

Here is the definition. A function Pr from a field of propositions **A** over a set of possibilities W into the real numbers is a (finitely additive and unconditional) *probability measure on* **A** if and only if for all propositions in A, B in **A**:

(1) $\Pr(A) \geq 0$

(2) $\Pr(W) = 1$

(3) $\Pr(A \cup B) = \Pr(A) + \Pr(B)$ if $A \cap B = \varnothing$

The field of propositions **A** over the set of possibilities W is sometimes replaced by a language ***L***, where tautologies and contradictions play the role of W and $\varnothing$, respectively. The conditional probability measure $\Pr(\cdot|\cdot)$ (based on the unconditional probability measure Pr on **A**) is defined for all A, B in **A** where $\Pr(B) > 0$ as follows:

(4) $\Pr(A|B) = \Pr(A \cap B) \div \Pr(B)$

In inductive logic conditional probability is usually put to use in the following way (Carnap 1962, Hawthorne 2005, Skyrms 2000). The *degree of absolute confirmation of* a conclusion C *by* premises $P_1, \ldots, P_n$ *relative to* the probability measure Pr on the field **A** is defined as the conditional probability of C given the conjunction $P = P_1 \cap \ldots \cap P_n$, $\Pr(C|P)$. For more see Huber (2006).

It is important to note that this definition renders degree of confirmation relative to a probability measure on a language or field of propositions that include the premises and the conclusion. The difference between the Carnapian approach (Carnap 1962) and more modern approaches (Hawthorne 2005, Skyrms 2000) now can be put as follows. Carnap sought to come up with one single logical probability measure, whereas modern writers consider (almost) any probability measure as admissible from a purely logical point of view.

The notion of deductive validity is a three-place relation between a set of premises, a conclusion, and a language that includes the premises and the conclusion. By trying to define a unique logical probability measure for each language, Carnap in effect tried to define degree of confirmation in a similar fashion as a three-place relation between a set of premises, a conclusion, and a language. Modern theories of confirmation differ in this respect, because they construe confirmation as a four-place relation, thus making explicit the probability measure. Fitelson (2005) still considers this to be a logical relation.

Carnap (1962) also proposed a definition of qualitative confirmation, where the idea is that premises confirm a conclusion if the conjunction of the premises raises the probability of the conclusion. A conclusion C *is incrementally confirmed by* premises $P_1, \ldots, P_n$ *relative to* the probability measure Pr on the field **A** if and only if $\Pr(C|P) > \Pr(C)$.

As indicated by the qualifiers *absolute* and *incremental*, we have here two different concepts of confirmation. The quantitative concept of absolute confirmation is explicated by the conditional probability of the conclusion given the premises. Absolute confirmation thus consists in high conditional probability, and the qualitative concept of absolute confirmation is to be defined as follows. C is *absolutely confirmed* by $P_1, \ldots, P_n$ *relative to* Pr on **A** if and only if $\Pr(C|P) > r$, for some specified r in [1/2,1). Incremental confirmation, on the other hand, focuses on increase in probability. Therefore the quantitative concept of incremental confirmation is to be defined as the degree to which the premises increase the probability of the conclusion, i.e. the difference between $\Pr(C)$ and $\Pr(C|P)$.

As noted by Fitelson (1999), there are many non-equivalent ways to measure degree of incremental confirmation. Earman (1992) discusses the distance measure $d = \Pr(C|P) - \Pr(C)$, whereas Joyce (1999) and Christensen (1999) propose $s = \Pr(C|P) - \Pr(C|\text{not-}P)$. In a different context, Carnap & Bar-Hillel (1952) propose to measure the informativeness of the conclusion C by $\Pr(\text{not-}C)$, whereas Hempel & Oppenheim (1948) suggest measuring the extent to which C informs us about P by $\Pr(\text{not-}C|\text{not-}P)$. It turns out that the measures of incremental confirmation d and s are aggregates of the degree of absolute confirmation, $\Pr(C|P)$, and the informativeness in the sense of $\Pr(\text{not-}C)$ and $\Pr(\text{not-}C|\text{not-}P)$, respectively. More precisely,

$$d = \Pr(C|P) + \Pr(\text{not-}C) - 1 = \Pr(\text{not-}C)\Pr(C|P) + \Pr(C)\Pr(\text{not-}C|P)$$

$$s = \Pr(C|P) + \Pr(\text{not-}C|\text{not-}P) - 1 = \Pr(\text{not-}C|\text{not-}P)\Pr(C|P) + \Pr(C|\text{not-}P)\Pr(\text{not-}C|P)$$

In other words, incremental confirmation is proportional to expected informativeness. Different measures of incremental confirmation differ in the way they measure informativeness.

We have thus detected a third factor of the quality of an argument: the informativeness of the conclusion. This is not surprising. After all, the informativeness of the conclusion was the very reason why we were considering more lenient standards than deductive validity in the first place. Note also that the informativeness of the conclusion is as much a logical factor as is the degree to which the premises confirm the conclusion. For both factors are determined once the premises, the conlusion, and the probability measure on the field of propositions are specified. In fact, this opens the door to render all factors of the quality of an argument to be logical; for we can now also consider the probability that the premises are true.

So far we have been engaged in conceptual analysis, where we appeal to intuitions as the data against which to test various proposals for a definition of confirmation. The assumption is, of course, that the concept we are explicating is important. Surely it is a good thing for a hypothesis to be confirmed by the available data. Surely we should strive to list premises that confirm the conclusion we are arguing for. Inductive logic is important, because it is a normative theory. Yet conceptual analysis does not provide the resources to justify a normative theory. Appeals to intuitions do not show why we should prefer "well confirmed" hypotheses to other hypotheses, and why we should provide inductively strong rather than any other arguments.

The analogy to deductive logic again proves helpful. The rules of deductive logic are norms that tell us how we should argue deductively. As any other set of norms, it needs to be justified. Contrary to Goodman (1983), the rules of deductive logic are not justified, because they adequately describe our deductive practices. They do not. The rules of deductive logic are justified *relative to* the goal of arguing truth preservingly, i.e. in such a way that the truth of the premises guarantees the truth of the conclusion. The results that provide the justification are known as soundness and completeness. Soundness says that every argument we obtain from the rules of deductive logic is such that truth is preserved when we go from the premises to the conclusion. Completeness states the converse. Every argument that has this property of truth preservation can be obtained from the rules of deductive logic. So the rules of deductive logic are justified relative to the goal of truth preservation. The reason is that they further this goal insofar as all and only deductively valid arguments are truth preserving.

What is the goal inductive logic is supposed to further – relative to which it can be justified? Surely it includes truth. However, as Hume (1739) argues, it is impossible to justify induction relative to the goal of truth if this justification of induction means providing a deductively valid or an inductively strong argument with knowable premises for the conclusion that induction will always lead to true conclusions. However, as noted by Reichenbach (1938), there are deductively valid arguments for other conclusions that may show that induction furthers the goal of truth to the extent this is possible. Similar results obtain for absolute confirmation, where it can be shown that the conditional probability of a conclusion given the premises converges to its truth value when more and more premises are learned.

However, if obtaining true conclusions were the only goal induction is supposed to further, induction could be replaced by deduction. All that is logically implied by what we know is guaranteed to be true. We do not need to go beyond the premises to satisfy the goal of truth. The reason we nevertheless do go beyond what is logically implied by the premises is that we aim at more than mere truth: we aim at informative truth. It is this very feature that makes us strive for a more lenient explication of the logical factor of the quality of arguments in the first place; and without it Hume's problem of the justification of induction would not even get off the ground. Thus, the important question is whether and in which sense inductive logic can be justified relative to the goal of informative truth. One answer is given by Huber (2005). There it is shown that incremental confirmation in the sense of d or s converges to the most informative among all true conclusions when more and more premises are learned.

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