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MINIMIZING MAXIMIN

Perhaps the most controversial area of decision theory is that of decision under ignorance. Many and diverse strategies have been proposed for dealing with such choices. For decisions made in the special choice situation of the Rawlsian original position Rawls argues for a maximin strategy (sometimes referred to as a minimax strategy). The idea behind this strategy is that one ought to choose that act which maximizes his minimum gain (or, what comes to the same thing, minimizes his maximum loss). You are, according to maximin, to select that action the worst possible outcome of which is better than any alternative action's worst possible outcome. If there is a tie, the principle would be applied in the redundant lexical fashion that Rawls recommends for the difference principle. It should be clear that maximin is the most cautious strategy possible. That is, it seeks exclusively to minimize possible loss without regard for maximizing possible gain. How can such a strategy be justified?

Rawls points to three considerations in favor of the maximin strategy for the contractors. First, the agents do not know the probabilities required to pursue the strategy of maximizing expected utility. Secondly, each contractor has "a conception of the good such that he cares very little, if anything, for what he might gain above the minimum stipend that he can, in fact, be sure of by following the maximin rule". (Rawls, 1971, p. 154) Finally, Rawls says the maximin strategy leads to institutions that are acceptable in real life.

The first consideration is not so much an argument for maximin as an argument against attempting to maximize expected utility. Maximin is not the only strategy that can be adopted to deal with radical uncertainty. Rawls has not ruled out other, incompatible approaches; indeed, he only discusses one other approach — that is the fixing of probabilities by the principle of insufficient reason (better known as the indifference principle). According to this method, probabilities are assigned to the various states by assuming that all possible states are equiprobable. Thus, if there are n possible states,

the assumed probability of each is $1/n$. Rawls thinks that this approach is ruled out by the other two considerations. Interestingly, he fails to note the most conclusive reason for rejecting this approach. The principle of insufficient reason is known to lead to contradiction when applied is an unrestricted fashion. (See for example, von Mises, 1957, p. 77.) No one has yet proposed an acceptable restriction which avoids this consequence, known as the Bertrand Paradox.

Though there is a definitive reason for rejecting the principle of insufficient reason, this does not allow us to conclude that maximin is a reasonable strategy. There are, in the first place, other ways of choosing under ignorance. One could, for just one example, adopt a maximax strategy. That is, one could select that option that has the highest high payoff. (And there are, of course, infinitely many other strategies one could adopt.) Rawls must show why his extremely cautious strategy is more reasonable than the daring maximax principle. Maximin is extensionally equivalent to (i.e., selects the same actions as) choosing on the assumption that the state of nature will be the worst one possible given your choice. Maximax is equivalent to choosing on the assumption that the state of nature will be the best one possible given your choice. Rawls must show why, from the point of view of the original position, pessimism is preferable to optimism — and to every other attitude one could adopt.

The second and third considerations are intended to do this, though, as we shall see, they are inadequate for the task. The third consideration is, I take it, an appeal to the stability of institutions which satisfy the difference principle as well as possibly being an attempt to test the maximin strategy by the process of reflective equilibrium.

It is the second argument that will mainly concern us here. I interpret Rawls as making two claims here. First, he is asserting that the contractors are dealing with goods (primary goods) which have rapidly diminishing marginal utility beyond a required minimum. And secondly, he claims that this minimum is assured by the difference principle and the maximin strategy that leads to it. I shall examine these claims shortly.

It is important to note that Rawls is not arguing for a maximin strategy with regard to utilities (as defined by contemporary decision theorists). It would be a straightforward contradiction to argue that utilities are subject to decreasing marginal utility (unless, of course, there are different orders of

utilities − a possibility I shall ignore). Rawls is arguing for a maximin strategy with regard to payoff in terms of primary goods.

This little noted point goes a long way toward making maximin an acceptable strategy *if* Rawls can establish that primary goods are subject to the requisite kind of decreasing marginal utility beyond a minimum floor and that maximin *and only maximin* guarantees that floor. If Rawls can establish these points, he is free to appeal, as he does, to a 'quasi-dominance' argument to justify the maximin strategy.

To see how this argument works, let us look at a possible payoff matrix suggested by Rawls. (Rawls, 1971, p. 153)

	c_1	c_2	c_3
d_1	−7	8	12
d_2	−8	7	14
d_3	5	6	8

Maximin recommends d_3 on the grounds that it has the highest worst payoff. Rawls rationalizes this by use of the two claims mentioned above. (i) The minimum required payoff is less than or equal to 5. (ii) The contractors care little (or not at all) for any gain above this minimum. Though, as we shall see, Rawls needs a further assumption, and the above two need to be made more precise, he has, nonetheless, laid the groundwork for what I have called a 'quasi-dominance argument' for the maximin strategy.

Before discussing the quasi-dominance argument, let us look at its *standard* counterpart, the dominance argument. A decision, d_n, is (weakly) dominant with regard to its competitors relative to a payoff matrix *iff* for each state of the world, the payoff of d_n is equal to or greater than each of its competitor's payoff and for *some* state of the world, the payoff of d_n is greater than that of any of its competitors. The dominance principle states that if a dominant act is available, it should be selected. It takes but a glance to see that there is no decision which is dominant in the above matrix. In what sense, then, is Rawls giving a quasi-dominance argument?

I shall say that one decision, d_n, is quasi-dominant with regard to a given matrix *iff* for each state of the world, the payoff of d_n is roughly equal to or greater than that of any alternative decision and for some state of the world, the payoff of d_n is vastly better than that of its competitors. Obviously, quasi-dominance is nothing like a precise mathematical property. Nevertheless,

the maxim that one ought to choose the quasi-dominant action (if one exists) seems like a reasonable one, and one that we all accept. There is, however, an important restriction that must be put on the use of the quasi-dominance principle. That is, it is only reasonable to adopt this strategy when it is reasonable to believe that the sum of the probabilities of those states in which the quasi-dominant action actually dominates the alternatives is non-negligible. When it is not reasonable to assume this, the quasi-dominance principle is undermined.

The dominance argument proceeds on the intuition that when you are confronted with a choice and by choosing one act you have nothing to loose no matter what happens and may gain something relative to your other options, you should prefer that alternative. Quasi-dominance proceeds on the intuition that if by preferring one act you have little to loose in any case and possibly much to gain, that is the preferred act. Since you never lose anything (vis-à-vis your alternatives) by choosing a dominant action, no assumption about the relative probabilities of the states need be made. However, since you may loose something (even if only a little) by choosing a merely quasi-dominant act, an assumption must be made about the probability of those states in which you stand to gain by selecting that action.

Now it should be clear that Rawls is giving a quasi-dominance argument for the maximin strategy. While the *payoff* matrix in terms of primary goods may, let us suppose, look like the above matrix, Rawls is arguing that the *utility* matrix would look more like:

	c_1	c_2	c_3
d_1	−7	5.3	5.7
d_2	−8	5.2	5.9
d_3	5	5.1	5.3

We now see the structure of Rawls' argument for the maximin strategy and hence for the claim that the contractors would opt for the difference principle. Three assumptions (in addition to the assumption about the probabilities of the states which is always required by the quasi-dominance principle) are required for this argument. They are:

(i) Primary goods are subject to rapidly diminishing marginal utility above a minimum required amount;

(ii) The maximin strategy guarantees that the required minimum will be met or exceeded (in the choice situation confronting the contractors); and,

(iii) The maximin strategy is the only strategy that guarantees that the required minimum will be achieved or exceeded.

Let us examine these claims and their veracity in some detail.

The first assumption is made by Rawls without argument. This is odd, for the required assumption is *very* strong. Not only must primary goods be subject to greatly diminished marginal utility beyond the minimum floor Rawls envisions, they must be subject to what I will call 'geometrically diminishing marginal utility'. A commodity is subject to geometrically diminishing marginal utility *iff* for each increase, q_i, in the payoff of that commodity, the marginal utility of q_i is greater than or equal to the sum of the marginal utility of all succeeding increases (i.e., $mu(q_i) \geqslant \Sigma_{j=i+1}^{\infty} mu(q_j)$). Rawls must be claiming that this is so for primary goods above the required floor. That is, he is claiming that the marginal utility curve for primary goods looks like:

$p = 0$ m
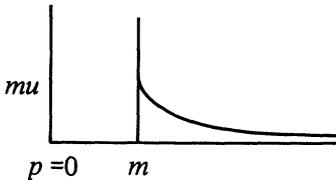
(where the horizontal axis represents the payoff in terms of primary goods, the vertical axis represents the marginal utility of increase of primary goods, and m is the minimum floor.) Rawls must be assuming this for it is only with this assumption that he can argue that the *total utility* of a quantity of primary goods greater than the minimum (no matter how much greater) is only insignificantly more than that of the minimum. (That is, Rawls must argue that $\lim_{p \to \infty} u(p) = n$; where, $n - m \leqslant \epsilon$.) If he is not assuming this, then he cannot argue that each contractor "cares very little, if anything, for what he might gain above the minimum stipend'. (Rawls, 1971, p. 154) This is absolutely required for the quasi-dominance argument Rawls gives to support maximin.

The assumption of geometrically diminishing marginal utility of primary goods (above the required minimum) is an implausible one not matter where the minimum is set. It is absolutely ludicrous if the minimum is set at any reasonable level. Imagine yourself with as much wealth or income as you choose. Does each dollar increase in your income increase your utility more than any number of dollars after that point? Of course not! The majority of

Americans have, no doubt, both income and wealth well above the required minimum (in any reasonable sense of that phrase). But we do not tend to consider a $5000 raise as insignificantly higher than a $500 raise.

If Rawls wants the minimum floor set at that point at which marginal utility diminishes geometrically, it will have to be high indeed – assuming that there is such a point. It seems that, if there is such a point, it will vary from one individual to the next depending on what life-plan each has. A required minimum of wealth and income for a person whose life-plan is to be a scholar may be quite different from that of someone whose life-plan includes winning the Americas Cup. If the contractors are of the conservative bent that Rawls supposes, they should assume that they will have the sort of life-plan that requires the *most* primary goods. This will result in the minimum being set at an impossibly high level.

But let us assume, *arguendo*, that Rawls can give an argument to establish the minimum at a reasonable level, and furthermore, contrary to what intuition and experience tell us, that above this minimum, primary goods are subject to geometrically diminishing marginal utility. Why does Rawls believe that adoption of the maximin strategy guarantees this minimum? Certainly, if there is a line of the payoff matrix the lowest value of which is above the required minimum, the maximin strategy will guarantee that this floor is attained. But the strategy one uses to select an action cannot affect the payoff that that action will have given various states of the world. It is not as if one can change the world by selecting a different decision principle for dealing with it. Rawls is, then, assuming that there must be at least one line of the payoff matrix every column of which has a value greater than or equal to the required minimum.

Rawls is nowhere clear about this, but this assumption appears to be grounded on the assumption that the circumstances of justice must obtain. Rawls appears to be assuming that since one of the conditions of justice is that the scarcity endured be moderate, rather than severe, there must be *some* principle of distribution which accords to each a minimally satisfactory share. This conclusion is immediate if Rawls understands by the moderate scarcity requirement what Hume did.

There is an obvious and apparently irreconcilable tension between the first two assumptions Rawls makes to justify the maximin strategy. If the minimum is set high enough to make the first assumption even plausible, then the circumstances of justice do not come close to ensuring that there will be

some distributive principle which will guarantee a satisfactory minimum for all. Hence, the second assumption is unwarranted. On the other hand, if the minimum is set low enough to justify the second assumption (given that the circumstances of justice obtain), the assumption of geometrically diminishing marginal utility is both absurd and utterly without justification.

To make matters worse, Rawls does not even take up the last assumption required to justify the maximin strategy. If what is really desired is to guarantee the required minimum, and the contractors care little or nothing for any gain beyond this, then there are infinitely many principles that will do this as well as maximin. Any strategy that is stated as follows will do this: "Choose that distributive principle *among those which guarantee the required minimum that*..." Fill in the blank as you will and the resulting strategy will guarantee the minimum if any strategy will. It is for this reason that R. M. Hare criticizes the difference principle. (Hare, July, 1973) He suggests a principle that, while guaranteeing the minimum, would simply maximize utility beyond that point. The reasoning that Rawls uses to justify the maximin strategy, cannot rule out the adopting of this sort of principle.

Of the three considerations Rawls gives in favor of maximin, the quasi-dominance argument is by far the most complex and intriguing. Nevertheless, there is good reason for thinking that Rawls does not take it to be essential to his justification of the maximin strategy. Rawls adopts what he calls the lexical difference principle. (Rawls, 1971, p. 83) The lexical difference principle requires that one select that system that leaves the worst-off better off than any alternative system. If there is a tie between several distinct patterns, then you look at the shares of the second worst-off and select the distribution that maximizes his share. This process is repeated until there is a single distributive schema selected or the population is exhausted (in which case the remaining schemas are extensionally equivalent).

The fact that Rawls adopts such a principle is telling. It means that he does not see the quasi-dominance argument as being crucial to the justification of maximin. To make this clear, consider the following primary good payoff matrix:

	c_1	c_2	c_3
d_3	5	6	8
d_4	4	22	36
d_5	5	5	36

The fact that Rawls adopts the lexical difference principle indicates that he accepts a generalized form of the maximin strategy. This generalized form can be stated as follows:

> Given a linear ordering of the payoffs of each act (Pa_i) from the lowest, $Pa_i 1$, to the highest, $Pa_i n$, and some action a_q and every alternative action, a_j $(j \neq q)$, if (i) for some k, $Pa_q k > Pa_j k$ and (ii) for all $m(m < K) Pa_q m = Pa_j m$, then select a_q.

It is obvious that maximin is just a special case of this strategy — that case in which (ii) is vacuously true.

Now the generalized maximin strategy leads to the selection of d_3 in the above matrix. This is, of course, in accordance with the lexical difference principle. Hare is asking why it is not more reasonable to select d_4 if we know that 4 is greater than or equal to the required minimum. It might be thought that Rawls could avoid this criticism by denying the contractors knowledge of quantity of goods that constitutes the minimum floor. All they would be allowed to know is that maximin guarantees this minimum. Though this extension of the veil of ignorance appears *ad hoc*, Rawls may yet employ it. He is, after all, quite candid about the fact that some of the assumptions made regarding the original position are justified primarily because they lead to the acceptance of intuititvely correct principles. Such a procedure is a simple application of Rawls' reflective equilibrium methodology.

Alas, for Rawls, this does not solve the problem. Hare is suggesting that the contractors agree to the principle that the minimum is to be met for every person, beyond this point, distribution is to be based on maximizing utility. To agree to such a principle, the contractors require no knowledge of the level at which the minimum is set. This would be decided at a later stage (presumably, the legislative stage). It appears that Rawls' argument about the ability of maximin to guarantee the required minimum does not give us a reason to prefer maximin to a principle such as the one Hare suggests.

When one notices that Rawls also prefers d_3 to d_5, though, it appears that his primary reason for accepting the maximin strategy is not based on this consideration at all. If the contractors know that the maximin strategy guarantees the required minimum, they surely know that d_5 does so as well (assuming that Rawls allows them the deductive consequences of their knowledge). But d_5 does not satisfy the generalized maximin strategy or the lexical difference principle. What argument can Rawls give against a strategy

of maximizing the worst payoff but deciding ties in some way other than that suggested by the lexical difference principle? What reason can Rawls give for accepting the generalized maximin strategy?

Rawls cannot give a quasi-dominance argument because, even assuming geometrically diminishing marginal utility, d_3 will not be quasi-dominant with respect to d_5 in the utility matrix. Assuming that 5 is the required minimum and that the utility of any payoff above that point is vastly smaller and geometrically diminishing, the utility matrix might look like this:

$$\begin{array}{cccc} & c_1 & c_2 & c_3 \\ d_3 & 5 & 5.5 & 5.875 \\ d_5 & 5 & 5 & \approx 6 \end{array}$$

Clearly, neither act is quasi-dominant. (Rawls cannot argue that the difference between 5.5 and 5 is significant to the contractors without undermining the assumption required to select d_3 over d_1 and d_2 is the earlier matrix.)

Rawls' acceptance of the lexical difference principle and his apparent acceptance of the generalized maximin strategy which leads to this principle suggest that Rawls' second argument for the maximin strategy is not crucial. Rawls intends the maximin strategy to prevail even when the contractors know that they are talking about payoffs which are all above the required minimum. The lexical difference principle shows this.

To this point, I have argued that Rawls can employ a quasi-dominance argument for the maximin strategy only if he can show that maximin, and only maximin, can guarantee a minimum of primary goods and that the contractors, while wanting very much to receive this amount, care little for any gain above it. I have shown that there are problems at every turn. These problems are, perhaps, not insurmountable, but they are formidable.

Let us now look at the last consideration Rawls gives in favor of the maximin strategy. He claims that maximin is justified because other strategies lead to institutions that are intolerable. I suggested earlier that Rawls is probably touching on two distinct considerations here. First, he is appealing to the stability of institutions satisfying the difference principle. Secondly, he is appealing to our considered moral judgments to give what I shall call a 'reflective equilibrium' argument for supposing that the contractors adopt a maximin strategy. When he presents the maximin strategy, he *claims* to be giving arguments for that strategy "from the standpoint of the original

position". (Rawls, 1971, p. 152) Clearly, the second argument is not such an argument. There is a way of interpreting the former so that it is. As we shall see, it is not successful.

One could interpret Rawls as claiming that since maximin leads to institutions that are stable, the contractors would have a reason to prefer it to its competitors. Now, even assuming what has not been shown – that maximin leads to the adoption of stable institutions – this argument will not do. If stability forms some part of the contractors' conception of the good – if they value it (perhaps because it leads to security) – then it must be a primary good. Rawls says that the contractors are motivated simply to attempt to "win for themselves the highest index of primary social goods". (Rawls, 1971, p. 155) Thus, if stability is desirable, this is already reflected in the primary goods payoff matrix. Choices that lead to instability have already been docked in setting up the matrix. In fairness, as well as for logical reasons, they ought not to be subject to "double jeopardy" for the single fault of instability. From the standpoint of the original position stability is a red herring in the defense of maximin.

This leaves us with only the reflective equilibrium argument. Though I find this argument no more successful than those already considered, I shall not undertake to criticize it here. Such arguments – involving as they do our entire moral theory and nexus of moral judgments – are not to be given or refuted quickly. But even if Rawls could make out a persuasive reflective equilibrium argument, he still would not have succeeded in giving an argument for maximin which is independent of our moral judgments. He would not yet have given us a tool to use against those with opposing moral convictions.*

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