What Is the Point of Confirmation?*

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Abstract

Philosophically, one of the most important questions in the enterprise termed confirmation theory is this: Why should one stick to well confirmed theories rather than to any other theories? This paper discusses the answers to this question one gets from absolute and incremental Bayesian confirmation theory. According to absolute confirmation, one should accept "absolutely well confirmed" theories, because absolute confirmation takes one to true theories. An examination of two popular measures of incremental confirmation suggests the view that one should stick to incrementally well confirmed theories, because incremental confirmation takes one to (the most) informative (among all) true theories. However, incremental confirmation does not further this goal in general. I close by presenting a necessary and sufficient condition for revealing the confirmational structure in almost every world when presented separating data.

1. Introduction

Philosophically, one of the most important questions in the enterprise traditionally termed confirmation theory is this: Why should one stick to well confirmed theories rather than to any other theories? In other and more mundane words: What is the point of confirmation? In what follows I will examine whether and how absolute and incremental Bayesian confirmation theory answer this question.

According to absolute Bayesian confirmation theory, an agent's *degree of absolute confirmation* of some hypothesis or theory H by a piece of evidence E relative to a body of background information B equals the probability of H given E and B, $\Pr(H \mid E \wedge B)$, where $\Pr : \mathcal{L} \to \Re$ is the agent's actual degree of belief function on some language $\mathcal{L}$ (see section 2). According to incremental Bayesian confirmation theory, an agent's *degree of incremental confirmation* of H by E relative to B is measured by a relevance measure $\mathrm{r}_{\Pr}$ based on the agent's actual degree of belief function $\Pr$; i.e. a possibly partial function $\mathrm{r}_{\Pr} : \mathcal{L} \times \mathcal{L} \times \mathcal{L} \to \Re$ such that for all $H, E, B \in \mathcal{L}$ with $\Pr(E \wedge B) > 0$:

$$\mathrm{r}_{\Pr}(H, E, B) \begin{array}{c} > \\ = \\ < \end{array} 0 \quad \Leftrightarrow \quad \Pr(H \mid E \wedge B) \begin{array}{c} > \\ = \\ < \end{array} \Pr(H \mid B)$$

2. The Point of Absolute Confirmation

The traditional answer to our question is something like this: Science aims at true theories, and one should accept well confirmed theories, because confirmation takes one to true theories. Indeed, if arriving at true theories is our (only) goal, then there is a point to absolute confirmation. In the long run, absolute confirmation almost surely takes one to true theories. This is the content of the following theorem (Gaifman and Snir 1982, 507):

Theorem 1 (Gaifman and Snir) *Let $S = \{A_i \in \mathcal{L} : i = 0, 1, \ldots\}$ separate Mod_L, let A_i^ω be A_i if $\omega \models A_i$ and $\neg A_i$ otherwise, and let $[B](\omega)$ be 1 if $\omega \models B$ and 0 otherwise. Then for every $B \in \mathcal{L}$,*

$$\Pr\left(B \mid \bigwedge_{0 \le i < n} A_i^\omega\right) \to [B](\omega) \text{ almost everywhere as } n \to \infty.$$

Here is the relevant technical background. L is obtained from a first-order language for arithmetic, L_0, by adding finitely many "empirical" predicates and function symbols (whose interpretation is not fixed). L_0 contains all numerals '1', ... as individual constants; countably many individual variables 'x_1', ... taking values in the set of natural numbers N; the common symbols '$+$', '$\cdot$', and '$=$' for addition, multiplication, and identity, respectively; and the standard quantifiers and connectives. In addition, there may be finitely many predicates and function symbols denoting certain fixed relations over N. The set of well formed formulas of L is denoted by '$\mathcal{L}$' and is also called a language.

A model for L consists of an interpretation φ of the empirical symbols which assigns every k-ary predicate 'P' a subset $\varphi\left(\text{'}P\text{'}\right) \subseteq N^k$, and every k-ary function symbol 'f' a function $\varphi\left(\text{'}f\text{'}\right)$ from N^k to N. The interpretation of the symbols in L_0 is the standard one and is kept the same in all models. Mod_L is the set of all models for L. '$\omega \models A$' says that formula A is true in model $\omega \in Mod_L$. $A\left[x_1, \ldots, x_k\right]$ is valid, $\models A\left[x_1, \ldots, x_k\right]$, iff $\omega \models A\left[n_1/x_1, \ldots, n_k/x_k\right]$ for all $\omega \in Mod_L$ and all $n_1, \ldots, n_k \in N$. Here, '$A\left[n_1/x_1, \ldots, n_k/x_k\right]$' results from '$A\left[x_1, \ldots, x_k\right]$' by uniformously substituting 'n_i' for 'x_i' in 'A', $1 \leq i \leq k$. '$A\left[x_1, \ldots, x_k\right]$' indicates that '$x_1$', ..., '$x_k$' are the only variables occurring free in 'A'.

A function $\Pr : \mathcal{L} \to \Re_{\geq 0}$ is a *probability on* $\mathcal{L}$ iff for all $A, B \in \mathcal{L}$:

1. $\models A \leftrightarrow B \quad \Rightarrow \quad \Pr(A) = \Pr(B)$
2. $\models A \quad \Rightarrow \quad \Pr(A) = 1$
3. $\models \neg(A \wedge B) \quad \Rightarrow \quad \Pr(A \vee B) = \Pr(A) + \Pr(B)$
4. $\Pr(\exists x A[x]) = \sup\left\{\Pr\left(A\left[n_1/x\right] \vee \ldots \vee A\left[n_k/x\right]\right) : n_1, \ldots, n_k, k \in N\right\}$

The conditional probability of A given B, $\Pr(A \mid B)$, is defined as

5. $\Pr(A \mid B) = \Pr(A \wedge B) / \Pr(B)$,

provided $\Pr(B) > 0$. Pr is *regular* iff the converse of 2. holds as well,

6. $\Pr(A) = 1 \quad \Rightarrow \quad \models A$.

A set of sentences $S \subseteq \mathcal{L}$ *separates* a set of models $X \subseteq Mod_L$ iff for any two distinct $\omega_1, \omega_2 \in X$ there exists $A \in S$ such that $\omega_1 \models A$ and $\omega_2 \not\models A$. The set of all atomic empirical sentences separates Mod_L (Gaifman and Snir 1982, 507).[1]

However, absolute confirmation has long been abandoned in favour of incremental confirmation. Is there another goal for incremental confirmation that is different from arriving at true theories? If so, what is this goal?

3. What Is the Point of Incremental Confirmation?

Two popular measures of incremental confirmation are the distance measure d (Earman 1992) and the Joyce-Christensen measure s (Joyce 1999, Christensen 1999):

$$
\begin{aligned}
d_{\Pr}(H, E, B) &= \Pr(H \mid E \wedge B) - \Pr(H \mid B) \\
s_{\Pr}(H, E, B) &= \Pr(H \mid E \wedge B) - \Pr(H \mid \neg E \wedge B)
\end{aligned}
$$

What do these measures measure? d increases with

- the *plausibility* of H given E and B, $p = \Pr(H \mid E \wedge B)$, and
- the evidence neglecting or data independent semantic *informativeness* of H relative to B, $i_0 = \Pr(\neg H \mid B)$.

Similarly, s increases with

- the plausibility of H given E and B, $p = \Pr(H \mid E \wedge B)$, and
- the evidence based or data dependent semantic informativeness of H relative to E and B, i.e. the amount to which H *informs about* E relative to B, $i_1 = \Pr(\neg H \mid \neg E \wedge B)$.

This is clearly seen by rewriting d and s as follows:

$$
\begin{aligned}
d_{\Pr}(H, E, B) &= \Pr(H \mid E \wedge B) + \Pr(\neg H \mid B) - 1 \\
s_{\Pr}(H, E, B) &= \Pr(H \mid E \wedge B) + \Pr(\neg H \mid \neg E \wedge B) - 1
\end{aligned}
$$

p and i_0 as well as p and i_1 are *conflicting* in the sense that p decreases, whereas i_0 and i_1 increase with the logical strength of the hypothesis to be assessed. So d and s weigh between two conflicting aspects, viz. the plausibility and the informativeness of the hypothesis to be assessed.

In section 4 I will argue in more detail that i_0 and i_1 measure two different, but equally sensible kinds of informativeness. Section 5 provides another argument for the thesis that (i) d and s do nothing but weigh between the two conflicting goals of plausibility and informativeness; (ii) that they are exactly alike in the way they weigh between these two aspects; and (iii) that they differ from each other just in the respect that d is based on data independent informativeness whereas s is based on informativeness about the data. All this suggests the following answer to our question: Science aims at informative true theories, and one should stick to incrementally well confirmed theories, because incremental confirmation takes one to (the most) informative (among all) true theories. However, as shown in section 6, incremental confirmation does not further this goal in general. I close by giving a necessary and sufficient condition for revealing the confirmational structure in almost every world when presented separating data.

4. Measuring Semantic Information

In a subjective Bayesian framework it is clear that $p = \Pr(H \mid E \wedge B)$ measures the *plausibility* of H in view of E and B. It is still rather obvious that $i_0 = \Pr(\neg H \mid B)$ measures the data independent informativeness of H relative to B. i_0 was already considered by Carnap and Bar-Hillel (1952), Bar-Hillel and Carnap (1953), Hempel (1960, 1962), and Hintikka and Pietarinen (1966) (for the notion of semantic information cf. Bar-Hillel 1952, 1955). The second measure that was discussed in this connection is

$$i_2 = -\log_2 \Pr(H \mid B) = \log_2 \frac{1}{\Pr(H \mid B)}.$$

i_2 is ordinally equivalent to i_0. For future reference it is convenient to define the analogous

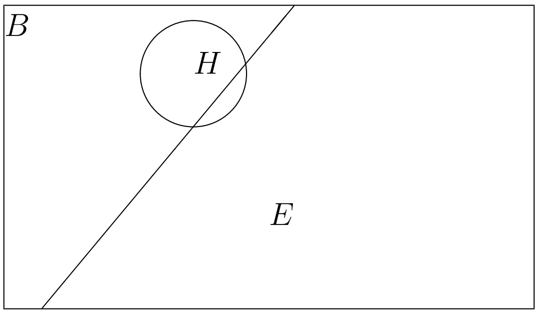
$$p_2 = \log_2 \Pr(H \mid E \wedge B),$$

which is ordinally equivalent to $p_0 = p$.

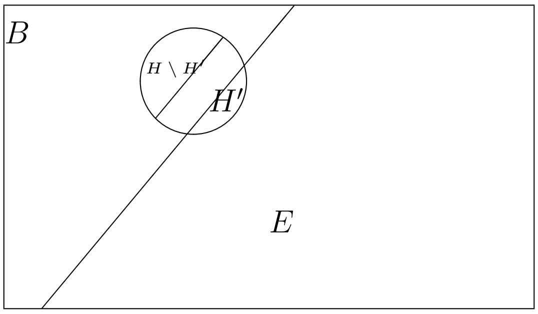
It is less obvious that $i_1 = \Pr(\neg H \mid \neg E \wedge B)$ measures how much H *informs about* the data E relative to background B (cf., however, Hilpinen 1970). Following the above mentioned literature, one would expect something like[2]:

$$\begin{aligned} i_3 &= \Pr(\neg H \mid E \wedge B) \\ i_4 &= \Pr(E) \cdot \Pr(\neg H \mid E \wedge B) \\ i_5 &= \log_2 \frac{1}{\Pr(H \mid E \wedge B)} = -\log_2 \Pr(H \mid E \wedge B) \end{aligned}$$

As is often the case, a picture is worth a 1000 words:



The background information B determines the set of possibilities and is nothing but a restriction on the set of possible worlds over which inquiry has to succeed (Hendricks 2005). H is the hypothesis whose informativeness about the data E is to be assessed (relative to B). Suppose you are asked to strengthen H by deleting possibilities verifying it, that is, by shrinking the area representing H. Would you not delete possibilities outside E? After all, given E, those are exactly the possibilities known not to be the actual one, whereas those possibilities inside E are still alive options. Indeed, i_1 increases when H shrinks to H' as depicted in the second figure, because it measures how much of $\neg E$ is occupied by $\neg H$.



As a consequence, the information H provides about E is maximal if H logically implies E (in this case H is completely within E, and so $\neg H$ covers all of $\neg E$). So according to i_1, two hypotheses both logically implying all of the data – say, a complete theory about the world, and a theory-like collection of the data – carry the same maximal amount of information about E. In a sense, this is odd, because one would like the complete theory to come out as more informative than the theory-like collection of the data. This is what i_0 yields. For i_0 it does not matter which possibilities one deletes in strengthening H (provided all possibilities have equal weight on the probability measure Pr). i_0 neglects whether they are inside or outside E. The other candidates for measuring semantic information do rather poorly on this count: they require the deletion of the possibilities inside E. (Another reason why i_3, i_4, and i_5 seem to be inappropriate in the present context is presented in the next section.)

The background information B plays a role different from that of the evidence E for i_0 and i_1, but not for i_3, i_4, or i_5. Clearly, there is a difference between data on the one hand and background assumptions on the other; and this difference should show up somewhere. Apart from the above mentioned point that B determines the set of possibilities over which inquiry has to succeed, whereas E is gathered in order to indicate which of these possibilities is the actual one, there is the following difference: Hypotheses are supposed to inform about the world, and hence about the data, but they are usually not supposed to inform about the background assumptions. (If one holds there should be no difference between E and B as far as measuring information is concerned, then one can nevertheless adopt the above measures by substituting $E' = E \wedge B$ and $B' = \top$ for E and B, respectively.)

In order to avoid that one has to take sides between i_0 and i_1 let us call a possibly partial function $i = f_{i_0,i_1} : \mathcal{L} \times \mathcal{L} \times \mathcal{L} \to [0,1]$ a *strength indicator* (based on i_0 and i_1) iff f is non-decreasing in both and increasing in at least one of its arguments i_0 and i_1, $f_{i_0,i_1} = 1$ for $i_0 = i_1 = 1$, and $f_{i_0,i_1} = 0$ for $i_0 = i_1 = 0$.

5. Expected Informativeness as One Way of Weighing

Having tried to make plausible that i_0 and i_1 measure informativeness *per se* and informativeness about the data, respectively, let us now turn back to the distance measure d and the Joyce-Christensen measure s. The two conflicting goals of informativeness and plausibility are equally important for d and s – and they are all what matters for them. Hence, other things being equal – these other things being the probabilities (plausibility values) of the hypotheses given the data E and the background information B – the overall d- or s-value of hypothesis H relative to E and B is the greater, the higher the informativeness of H (in the respective sense).

Clearly, if one knows the truth values of the theories one is assessing, then the plausibility of a theory's being true is of no interest anymore. In this case all what matters is how informative the theories are. Yet in general we do not know these truth values. Hence we consider how plausible it is that they are true in the world we are in, and how informative they are (about this world). Then we form their overall value by combining these two parameters in some suitable way. One such way immediately suggests itself: assign H as its overall value its *expected informativeness*:

$$\begin{aligned} E(i_0) &= \Pr(\neg H \mid B) \cdot \Pr(H \mid E \wedge B) - \Pr(\neg\neg H \mid B) \cdot \Pr(\neg H \mid E \wedge B) \\ E(i_1) &= \Pr(\neg H \mid \neg E \wedge B) \cdot \Pr(H \mid E \wedge B) - \\ &\quad - \Pr(\neg\neg H \mid \neg E \wedge B) \cdot \Pr(\neg H \mid E \wedge B) \end{aligned}$$

A little bit of reformulation shows that

$$E(i_0) = d_{\Pr}(H, E, B) \quad \text{and} \quad E(i_1) = s_{\Pr}(H, E, B).$$

So once again, d and s are exactly alike in the way they combine or weigh between informativeness and plausibility – which is to form the expected informativeness (cf. Hintikka and Pietarinen 1966 and Levi 1961, 1963, but also Hempel 1960). Their sole difference lies in the way they measure informativeness. In this sense, part of the discussion about the right measure of incremental confirmation is a discussion about the right measure of semantic information.

The measures i_3, i_4, and i_5 do again poorly:

$$E(i_3) = E(i_4) = 0$$

$$E(i_5) \begin{array}{c} > \\ = \\ < \end{array} 0 \quad \Leftrightarrow \quad \Pr(H \mid E \wedge B) \begin{array}{c} > \\ = \\ < \end{array} \Pr(\neg H \mid E \wedge B)$$

Hence only i_5 gives a non-trivial answer, viz. to maximize probability. But then we can simply stick to probabilities and need not employ i_5.

6. Revealing the Confirmational Structure

The preceding suggests the following answer to the question what goal incremental confirmation is supposed to further: Science aims at informative truth, and one should stick to incrementally well confirmed theories, because incremental confirmation takes one to (the most) informative (among all) true theories. The question is, of course, whether and in what sense this holds true.

When is one theory at least as informative as another? Well, if the first theory logically implies the second one, then the first theory is at least as informative as the second one. When else? In general, there is no further condition that applies equally to all probability measures $\Pr$. Just as the only $\Pr$-independent condition for H_1 to be at least as probable as H_2 is that H_2 logically implies H_1, so the above is the only $\Pr$-independent condition for H_1 to be at least as informative as H_2.

Hence, given a possible world $\omega \in Mod(B)$, H_1 is to be preferred over H_2 in ω if H_1 is true in ω, but H_2 is false in ω; or if H_1 and H_2 have the same truth value in ω, and H_1 logically implies H_2 but H_2 does not logically imply H_1. If H is logically true, then H is preferred in ω over any H_2 which is false in ω. On the other hand, any contingent H_1 that is true in ω is preferred over H, because these H_1s are not only true in ω; they are also more informative than H. Similarly, if H is logically false, then H is worse in ω than any theory that is true in ω, but better than any theory that is false in ω (because they are all less informative than H).

In this way each ω induces a partial order among the set of all (equivalence classes of axiomatizations of) theories: On the positive side one has all theories that are contingently true in ω, and on the negative side there are all theories that are contingently false in ω. In between there are the logically determined theories. Among the true theories on the positive side, the most informative, i.e. the complete theory about ω, is on top, followed by all true hypotheses it logically implies, partially ordered according to the logical consequence relation. This order goes all the way down to the least informative among all true theories, the tautology, which is placed at the bottom of the positive side. On that same level is the most informative among all false theories, the contradiction, followed by all contingently false theories, again partially ordered according to the logical consequence relation. Let us call this partial order *the confirmational structure of* ω.

For a given ω, we would like a function f to *stabilize* to the correct answer in the sense that f gets the confirmational structure of ω right after finitely many steps (data sentences from ω), and continues to do so forever without necessarily halting (or giving any other sign that it has arrived at the true answer) – cf. Kelly (1996). In general, stabilisation to the correct answer is a stronger requirement than convergence to the correct answer. However, the Gaifman and Snir convergence theorem actually gives rise to a measure 1 stabilisation result (assign 1 to H if its probability exceeds .5, and 0 otherwise).

Let $e_0, \ldots, e_n, \ldots$ be a sequence of sentences all of which are true in $\omega \in Mod(B)$. A possibly partial function $f : \mathcal{L} \times \mathcal{L} \times \mathcal{L} \rightarrow \Re$ *reveals the confirmational structure of* ω *when presented* $(e_i)_{i \in N}$ iff for any contingent $H_1, H_2 \in \mathcal{L}$, and any $H \in \mathcal{L}$:

1. $\omega \models H_1, \omega \not\models H_2 \Rightarrow \exists n \forall m \geq n$: $f(H_1, E_m, B) > 0 > f(H_2, E_m, B)$

2. $\begin{array}{c}\omega \models H_1, \omega \models H_2 \\ H_1 \vdash H_2 \nvdash H_1\end{array} \Rightarrow \exists n \forall m \geq n$: $f(H_1, E_m, B) > f(H_2, E_m, B) > 0$

3. $\begin{array}{c}\omega \not\models H_1, \omega \not\models H_2 \\ H_1 \vdash H_2 \nvdash H_1\end{array} \Rightarrow \exists n \forall m \geq n$: $0 > f(H_1, E_m, B) > f(H_2, E_m, B)$

4. $\models H$ or $\models \neg H \Rightarrow \forall m \geq n : f(H, E_m, B) = 0$,

where $E_m = \bigwedge_{0 \leq i < m} e_i$. An immediate consequence of the Gaifman and Snir convergence theorem is

Observation 1 *For any regular* $\Pr$ *on* $\mathcal{L}$ *and any* $\{e_i \in \mathcal{L} : i \in N\}$ *separating* Mod_L *there is* $X \subseteq Mod_L$ *with* $\Pr^*(X) = 1$ *such that for all* $\omega \in X$ *(and hence for all* $\omega \in X \cap Mod(B)$*, for any* $B \in \mathcal{L}$*):* $d_{\Pr}$*,* $s_{\Pr}$*, and* $c_{\Pr}$ *reveal the confirmational structure of* ω *when presented* $(e_i^\omega)_{i \in N}$*.*

$\Pr^*$ is the unique probability measure on the smallest σ-field $\mathcal{A}$ containing the field $\{Mod(A) : A \in \mathcal{L}\}$ such that $\Pr(A) = \Pr^*(Mod(A))$ for all $A \in \mathcal{L}$. c is the Carnap measure (Carnap 1962),

$$\begin{aligned} c_{\Pr}(H, E, B) &= \Pr(H \wedge E \wedge B) \cdot \Pr(B) - \Pr(H \wedge B) \cdot \Pr(E \wedge B) \\ &= (p + i_0 - 1) \cdot \Pr(B) \cdot \Pr(E \wedge B). \end{aligned}$$

However, observation 1 does not extend to all relevance measures. The log-ratio measure r (Milne 1996) and the log-likelihood ratio measure l (Fitelson 1999, 2001a, 2001b) do not reveal the confirmational structure of almost every $\omega \in Mod_L$ when presented separating data.

$$\begin{aligned} r_{\Pr}(H, E, B) &= \log\left[\frac{\Pr(H \mid E \wedge B)}{\Pr(H \mid B)}\right] \\ l_{\Pr}(H, E, B) &= \log\left[\frac{\Pr(E \mid H \wedge B)}{\Pr(E \mid \neg H \wedge B)}\right] \\ &= \log\left[\frac{\Pr(H \mid E \wedge B) \cdot \Pr(\neg H \mid B)}{\Pr(\neg H \mid E \wedge B) \cdot \Pr(H \mid B)}\right] \end{aligned}$$

Like all relevance measures, r and l separate contingently true from contingently false theories. More precisely, for any regular $\Pr$ on $\mathcal{L}$, any $\{e_i \in \mathcal{L} : i \in N\}$ separating Mod_L, any $B \in \mathcal{L}$, any $\omega \in X \cap Mod(B)$ (for some $X \subseteq Mod_L$ with $\Pr^*(X) = 1$), and any two contingent $H_1, H_2 \in \mathcal{L}$ such that $\omega \models H_1$ and $\omega \not\models H_2$ there exists n such that for all $m \geq n$:

$$\mathrm{r}_{\Pr}(H_1, E_m^\omega, B) > 0 > \mathrm{r}_{\Pr}(H_2, E_m^\omega, B), \quad \mathrm{r} = r, l$$

Furthermore, r and l also weigh between plausibility and informativeness:

$$\begin{aligned}
r_{\Pr}(H,E,B) &= \log \Pr(H \mid E \wedge B) - \log \Pr(H \mid B) \\
&= p_2 + i_2 \\
l_{\Pr}(H,E,B) &= \log \Pr(H \mid E \wedge B) - \log \Pr(H \mid B) - \\
&\quad - (\log \Pr(\neg H \mid E \wedge B) - \log \Pr(\neg H \mid B)) \\
&= p_2(H) + i_2(H) - (p_2(\neg H) + i_2(\neg H))
\end{aligned}$$

However, although r does distinguish between informative and uninformative true theories (in the sense of revealing part 2 of the confirmational structure of almost every world), it does not distinguish between informative and uninformative false theories. l performs even worse on this count, because it neither distinguishes between informative and uninformative true theories nor between informative and uninformative false theories. The reason is fairly obvious: If $p = 0$, then $p_2 = \log p = -\infty$, whence $p_2 + i_2 = -\infty$ for any finite value of i_2. This means in particular that informativeness does not matter anymore once a theory is falsified by the data. Similarly in case of r.

Which conditions are sufficient for a function to reveal the confirmational structure of almost every world when presented separating data? Let $f = f(i,p)$ be a function of, among others, $p = \Pr(H \mid E \wedge B)$ and some strength indicator $i = f_{i_0,i_1}$ based on $i_0 = \Pr(\neg H \mid B)$ and $i_1 = \Pr(\neg H \mid \neg E \wedge B)$. It is clearly necessary that $f(1,0) = f(0,1) = 0$; for $p = 0$ and $i = 1$, if H is logically false; and $p = 1$ and $i = 0$ if H is logically true – and in these cases H must be sent to 0, independently of what the data are.

1. Demarcation: $f(1,0) = f(0,1) = 0$

In conjunction with Demarcation, which is violated by r and l[3], the following is sufficient:

4. Continuity: Any surplus in informativeness succeeds, if the difference in plausibility is small enough.

   $$\begin{aligned}
   &\forall \varepsilon > 0 \quad \exists \delta_\varepsilon > 0 \quad \forall s_1, s_2, t_1, t_2 \in [0,1]: \\
   &s_1 > s_2 + \varepsilon \quad \& \quad t_1 > t_2 - \delta_\varepsilon \quad \Rightarrow \quad f(s_1,t_1) > f(s_2,t_2)
   \end{aligned}$$

(The s_i are possible values of i, and the t_i are possible values of p.) Indeed, it suffices that Demarcation be conjoined with

3. Continuity in Certainty: Any surplus in informativeness succeeds, if plausibility becomes certainty.

$$\forall \varepsilon > 0 \quad \forall \left(t_i\right)_{i\in N}, \left(t'_i\right)_{i\in N} \left(t_i, t'_i \in [0,1]\right) : t_i, t'_i \to_i \begin{cases} 1 \\ 0 \end{cases}$$
$$\exists n \forall m \geq n \forall s_m, s'_m \in [0,1] : s_m > s'_m + \varepsilon \quad \Rightarrow \quad f\left(s_m, t_m\right) > f\left(s'_m, t'_m\right)$$

Theorem 2 *Let* $\Pr$ *be a regular probability on* $\mathcal{L}$*, let* $\{e_i : i \in N\} \subseteq \mathcal{L}$ *separate* Mod_L*, let* f *be a function of, among others,* i *and* p *satisfying Continuity in Certainty and Demarcation, and let* $\Pr^*$ *be the unique probability measure on the smallest* σ*-field* $\mathcal{A}$ *containing the field* $\{Mod\,(A) : A \in \mathcal{L}\}$ *such that for all* $A \in \mathcal{L}$*:* $\Pr(A) = \Pr^*(Mod\,(A))$*, where* $Mod\,(A) = \{\omega \in Mod_L : \omega \models A\}$*. Then there exists* $X \in \mathcal{A}$ *with* $\Pr^*(X) = 1$ *such that the following holds for every* $\omega \in X$*, any two contingent* $H_1, H_2 \in \mathcal{L}$*, and every* $H \in \mathcal{L}$*:*

1. $\omega \models H_1, \omega \not\models H_2 \quad \Rightarrow \quad \exists n \forall m \geq n : f\left(H_1, E^\omega_m\right) > 0 > f\left(H_2, E^\omega_m\right)$

2. $\omega \models H_1, H_1 \vdash H_2 \nvdash H_1 \Rightarrow \exists n \forall m \geq n : f\left(H_1, E^\omega_m\right) > f\left(H_2, E^\omega_m\right) > 0$

3. $\omega \not\models H_2, H_1 \vdash H_2 \nvdash H_1 \Rightarrow \exists n \forall m \geq n : 0 > f\left(H_1, E^\omega_m\right) > f\left(H_2, E^\omega_m\right)$

4. $\models H$ *or* $\models \neg H \quad \Rightarrow \quad \forall m : f\left(H, E^\omega_m\right) = 0.$

However, even Continuity in Certainty is not necessary. The necessary and sufficient condition for revealing the confirmational structure in almost every world when presented separating data is this:

Definition 1 *A possibly partial function* $f : \mathcal{L} \times \mathcal{L} \times \mathcal{L} \to \Re$ *is a* Gaifman and Snir assessment function *iff for every probability* $\Pr$ *on a Gaifman and Snir language* $\mathcal{L}$ *(as described in section 2) and every* $\{e_i : i \in N\} \subseteq \mathcal{L}$ *separating* Mod_L *there is* $X \in \mathcal{A}$ *with* $\Pr^*(X) = 1$ *such that for all* $\omega \in X$ *and all* $m \in N$*:*

I. $$\begin{array}{c} H_1 \models H_2 \not\models H_1 \\ \Pr\left(H_1 \mid E^\omega_m\right) \to_m \begin{cases} 1 \\ 0 \end{cases} \end{array} \quad \Rightarrow \quad \exists n \forall m \geq n : f\left(H_1, E^\omega_m\right) > f\left(H_2, E^\omega_m\right)$$

II. $\models H_1, \quad \models \neg H_2, \quad \Pr\left(E^\omega_m\right) > 0 \quad \Rightarrow \quad f\left(H_1, E^\omega_m\right) = f\left(H_2, E^\omega_m\right) = 0$

I., and hence Continuity in Certainty, is violated by r and l.[4]

Definition 2 *Let* $\Pr$ *be a probability on a Gaifman and Snir language* $\mathcal{L}$ *and let* $\{e_i : i \in N\} \subseteq \mathcal{L}$ *separate* Mod_L. *A possibly partial function* $f : \mathcal{L} \times \mathcal{L} \times \mathcal{L} \to \Re$ reveals the confirmational structure of $\Pr^*$-almost every world $\omega \in Mod_L$ when presented separating $(e_i)_{i \in N}$ *iff there is* $X \in \mathcal{A}$ *with* $\Pr^*(X) = 1$ *such that for all* $\omega \in X$, *all contingent* $H_1, H_2 \in \mathcal{L}$, *and all* $H \in \mathcal{L}$:

1. $\omega \models H_1, \omega \not\models H_2 \quad \Rightarrow \quad \exists n \forall m \geq n : f(H_1, E_m^\omega) > 0 > f(H_2, E_m^\omega)$.
2. $\omega \models H_1, H_1 \models H_2 \not\models H_1 \Rightarrow \exists n \forall m \geq n : f(H_1, E_m^\omega) > f(H_2, E_m^\omega) > 0.$
3. $\omega \not\models H_2, H_1 \models H_2 \not\models H_1 \Rightarrow \exists n \forall m \geq n : 0 > f(H_1, E_m^\omega) > f(H_2, E_m^\omega)$.
4. $\models H \quad or \quad \models \neg H \quad \Rightarrow \quad \forall m : f(H, E_m^\omega) = 0.$

f reveals the confirmational structure of almost every world when presented separating data *iff for any probability* $\Pr$ *on a Gaifman and Snir language* $\mathcal{L}$ *and any* $\{e_i : i \in N\} \subseteq \mathcal{L}$ *separating* Mod_L: f *reveals the true assessment structure of* $\Pr^*$-*almost every world* $\omega \in Mod_L$ *when presented separating* $(e_i)_{i \in N}$.

Theorem 3 *A possibly partial function* $f : \mathcal{L} \times \mathcal{L} \times \mathcal{L} \to \Re$ *reveals the confirmational structure of almost every world when presented separating data iff* f *is a Gaifman and Snir assessment function.*

One reason why I still opt for the general Continuity condition is that it depends on the underlying convergence theorem which conditions are necessary and sufficient for revealing the confirmational structure in so and so many worlds when presented such and such data. More importantly, in the context of theory assessment (Huber 2006) the idea behind the use of these limit considerations is that they provide a theoretical justification for adopting the proposed conditions in the here and now. When assessing theories we cannot wait until we have arrived at the point of stabilisation for these theories. In fact, in general we will not know when we have reached that point. We need to make our evaluations here and now, where the probabilities are somewhere in between their maximal and minimal values, and we have no idea in which direction they will eventually converge (if they do so at all). Hence a theory of theory assessment needs to answer the question what to do when facing such a situation. Continuity gives an answer, but Continuity in Certainty does not. However, we also need to justify this answer – and we do so by appealing to the fact that when we satisfy Continuity in the special case when the probabilities converge (i.e. Continuity in Certainty), we reveal the confirmational structure in almost every world. As we usually do not know whether our probabilities have started to converge, we should always be prepared for this to happen – i.e. satisfy Continuity.

7. Conclusion

I started from the question: Why should one stick to well confirmed theories rather than to any other theories? The answer we got from absolute Bayesian confirmation theory is that one should stick to absolutely well confirmed theories, because absolute confirmation almost surely takes one to true theories. I continued by looking for an answer from incremental Bayesian confirmation theory. This answer should be different from the previous one in order for incremental confirmation to improve on absolute confirmation.

It turned out that three popular measures of incremental confirmation, viz. the distance measure d, the Joyce-Christensen measure s, and the Carnap measure c, give an interesting answer: One should stick to incrementally well confirmed theories, because incremental confirmation almost surely takes one to (the most) informative (among all) true theories.

However, although all measures of incremental confirmation separate contingently true from contingently false theories, not all of them distinguish between informative and uninformative true and false theories. The log-ratio measure r does not distinguish between informative and uninformative false theories, and log-likelihood ratio measure l neither distinguishes between informative and uninformative true nor between informative and uninformative false theories. A sufficient condition for revealing the confirmational structure of almost every world when presented separating data is the conjunction of Continuity and Demarcation, the core principle of the plausibility-informativeness theory of theory assessment (Huber 2006).

Notes

[1] The Gaifman and Snir framework is not rich enough for proper theory assessment. The reason is that the "theories" whose truth values one converges to by conditioning on appropriate data sentences are formulated within the same "empirical" vocabulary as are the data sentences. So there is no room for theoretical terms in the sense that the probability of a theory whose formulation contains theoretical terms not occurring in any data sentence does not necessarily converge to its truth value when one keeps conditionalizing on these data sentences. As an aside, note that this problem disappears if the realist goal of truth is replaced by the empiricist goal of empirical adequacy.

[2] In Levi (1967), i_3 is proposed as, roughly, a measure for the relief from agnos-

ticism afforded by accepting H as strongest relative to total evidence $E \wedge B$. For i_4 and i_5 the reader is referred to Hintikka and Pietarinen (1966).

[3] This defect can be repaired by sticking to the ordinally equivalent r^* and l^*, respectively:

$$\begin{aligned} r^*(H, E, B) &= \lim_{n \to \infty} \log \left[\frac{\Pr(H \mid E \wedge B) + 1/n}{\Pr(H \mid B) + 1/n} \right] \\ l^*(H, E, B) &= \lim_{n \to \infty} \log \left[\frac{\Pr(H \mid E \wedge B) \cdot \Pr(\neg H \mid B) + 1/n}{\Pr(\neg H \mid E \wedge B) \cdot \Pr(H \mid B) + 1/n} \right] \end{aligned}$$

[4] This defect cannot be repaired by sticking to r^* or l^*.

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