Chapter 4

Motion in Leibniz's Physics and Metaphysics

4.1 Introduction

Leibniz's mechanics was, as we shall see, a theory of elastic collisions, not formulated like Huygens' in terms of rules explicitly covering every possible combination of relative masses and velocities, but in terms of three conservation principles, including (effectively) the conservation of momentum and kinetic energy. That is, he proposed what we now call (ironically enough) 'Newtonian' (or 'classical') elastic collision theory. While such a theory is, for instance, vital to the foundations of the kinetic theory of gases, it is not applicable to systems – like gravitational systems – in which fields of force are present. Thus, Leibniz's mechanical principles never led to developments of the order of Newton's in the *Principia* (additionally, he hamstrung their application by embedding them in a baroque philosophical system). All the same, I wish to demonstrate, against the tendency of many modern readers, that Leibniz's responses to the Newtonians must be understood in the context of his theory of motion, not in terms of Newtonian mechanics. As we shall see, his problems lie primarily in his own physics, *not* in misunderstanding Newton's.

Our ultimate goals are to understand Leibniz's views on relativity (kinematic and dynamical) and 'absolute' quantities of motion (i.e., those that cannot be understood in terms of the relative motions of bodies alone). Of course, such an analysis will require that we also study his metaphysics, concerning matter and space especially, and his mechanics; these will be the topics of the next four sections. The classic source of Leibniz's views on motion (and space) is a famous correspondence with the Newtonian, Samuel Clarke, in 1715-6. I want to show that to properly understand Leibniz's views there, one has to first study his writings on mechanics as it was developed, mainly 1689-95 – at least twenty years previously.[1]

[1] I want to acknowledge the strong influence of Garber (1995) on this chapter, the first successful attempt of which I am aware to explore Leibniz's physics sympathetically within the context of his metaphysics; the essay is compulsory reading for anyone with a serious interest in Leibniz's views on space and motion. However, I will give a considerably more detailed treatment of relativity, especially from the point of view of mechanics, and explain how Leibniz's account of motion can help make sense of several puzzling remarks that he makes in the *Leibniz-Clarke Correspondence* regarding Newton's views on absolute space.

4.2 Metaphysics

Leibniz's physical ideas cannot be fully understood independently of his larger, intricate philosophy. However, all that we can attempt here is a sketch of Leibnizian metaphysics, and (in the following sections) a more detailed discussion of those of Leibniz's ideas about mechanics that are most directly relevant to Newton. Despite the obstacles he is well worth discussing in detail because he was an influential and public critic of Newton – indeed, (with Mach) the paradigmatic (if misunderstood) opponent of absolute space.

Leibniz's views changed considerably during his career (see the essays in Jolley, 1995) but from around the mid-1680's he began developing the system of philosophy (including natural philosophy) that remained in place – with refinements – until the end of his life, and which forms the context of his response on Newton's absolute space. Very briefly, Leibniz believed that the world is not as it seems at all, but in 'reality' is composed, at the fundamental level, of non-spatial (i.e., not in space, or even spatially related) causally isolated (so unable to affect each other, only themselves), mental units, or 'monads'. Although monads are thus profoundly disconnected from one another, since they are mental they have states that represent changing 'external' states of affairs. In conscious monads, such as the souls of rational beings, these mental representations are what we take to be experiences of the world. Monadic representations are correlated in a 'pre-established harmony' so that each monad represents a world of material objects in motion in space, from the standpoint of a particular object moving in space. Conscious monads thus experience the world as we do, even though the experiences are produced by entirely internal causes, not by interactions with other things.

Thus, imagine, for instance, a (normal) world in which matter is distributed as a gas of particles bouncing off one another. And imagine an observer on each particle, viewing collisions and motions from different perspectives; naturally (ignoring the finite speed of light) they will see any collisions at the same time, and in the same order, but from different points of view. In a monadic world there are at base no real physical particles, but each monad (if conscious) in a collection could have exactly the same experiences as one of these particle observers, so that the collective appearances in either case were the same.

Within the scope of the present work it is impossible to investigate in detail how Leibniz arrived at such a view, so let us just acknowledge that his system was as well justified as one could imagine it to be, under the circumstances.[2]

In fact, Leibniz arguably developed his mechanics before his views on monads reached the final form sketched above, but it seems more than reasonable to assume that for our purposes it was essentially unchanged by his later work, and that Leibniz considered it integrated with the world of monads.[3] In such a metaphysics, the point of a science of mechanics is, of course, to give an

[2]For presentations of Leibniz's relevant views over time see *The Discourse on Metaphysics* (35-68 of Ariew and Garber, 1989 – henceforth 'AG'), *A New System of Nature* (AG 138-45) and *The Monadology* (AG 213-25), from 1686, 1695, and 1714 respectively (the term 'monad' only occurs in the last, and dates from the 1698 *On Nature Itself* [AG 155-167]). For a detailed discussion of Leibniz's metaphysics of this period, and further references, see Rutherford (1995); note in particular that I have glossed over difficult and controversial issues concerning how Leibniz thought extended bodies arose from monads.

[3]That is not to say that they are *successfully* integrated. Indeed, if one approaches the issues of the present chapter primarily from 'below', concentrating on Leibniz's metaphysics, rather than primarily from 'above', concentrating on his physics, one can reach different conclusions concerning Leibniz's views. Lodge (2003) is an example of the former approach: his differing conclusions about Leibniz's views on motion arise from Lodge's project to understand how motion could arise from the theory of monads in full detail. That said, *(i)* I do not yet despair that my account of Leibniz's motion, motivated by his mechanics, can be reconciled with his monadology, and *(ii)* the texts with which

instrumental account of the phenomenological world of appearances: the laws of mechanics describe an ideal material world, the one which it appears to monads that they inhabit. That said, Leibniz's work makes clear that he, quite reasonably, did not take all issues to be settled at the fundamental level of reality, and held that there could be meaningful investigations of the philosophical principles of the phenomenal level, though of course connected to fundamental metaphysics. In particular, when Leibniz engages with those, such as Descartes, Huygens and (through Clarke) Newton, who are primarily concerned with the phenomenal level, he does not have to abandon his metaphysics and adopt phenomenal principles that he thinks false, but merely bracket his metaphysics, and work from principles that he thinks hold in the limited but important phenomenal domain. It is with Leibniz's views of the phenomenal realm that we will primarily be concerned; this chapter should make clear that he had substantive views on the topic.

4.3 The Unreality of Space

To understand Leibniz's mechanics, we will follow much the same trajectory taken by Descartes and Newton: first (in this section) we consider the nature of space, next (in the following section) the nature of motion, and then (§4.5) we shall discuss the laws of motion. (And in following sections the question of relativity and absolute quantities of motion will be investigated.) Since space somehow arises from the non-spatial monads rather than an entity in its own right, it should not be a surprise that Leibniz is hostile to the reality of space. In fact, he views it as resulting from the relations between material bodies (themselves of course constructions from monads).[4]

In the *Leibniz-Clarke Correspondence* (1956, LIII.4 – the notation means Leibniz's Third Letter, section 4), he says (in the second of the two most quoted passages in the literature) that: 'I hold space to be something merely relative .... For space denotes, in terms of possibility, an order of things which exist at the same time, considered as existing together, without entering into their particular manners of existing.' Leibniz means that space is in some sense nothing but the relations between bodies, not something distinct, and he wants to distinguish his conception from Newton's absolute space (as he understands it). Just what this idea amounts to though is not clear here, and later in the *Correspondence* he explained his view more fully. So pleased is he with his explanation that he gives it three times in immediate succession, in more-or-less the same terms, though we shall quote just the first:

> And though many, or even all the co-existent things, should [move] according to certain known rules of direction and swiftness; yet one may always determine the relation of situation, which every co-existent acquires with respect to every other co-existent; and even that relation which any other co-existent would have to ... any other, if it had not changed, or if it had changed any otherwise. And supposing or feigning, that among

we are concerned are clearly primarily discussions of his physics, not monads, and so my interpretational stance seems completely justified here. (I am grateful to Lodge for a long and enjoyable discussion of our different approaches, and for a number of other useful comments.)

[4]The view of space described in this section comes from 1715-16, while following sections discuss Leibniz's views on motion, mechanics and relativity from the period 1689-95. I have permitted this anachronism to facilitate a logical development towards Leibniz's later writings – understanding his letters to Clarke is a main goal of this chapter. Despite the evolution of Leibniz's views concerning space (see, Cover and Hartz, 1988), jumping ahead in this way will not cause us problems because *(i)* even if the details change, Leibniz's general attitude of scepticism regarding space is consistent, and *(ii)* we shall take care to avoid importing any anachronistic ideas into our discussion of the earlier arguments in the following three sections.

> those co-existents, there is a sufficient number of them, which have undergone no change; then we may say, that those which have such a relation to fixed existents, as the others had to them before, have now the *same place* which those others had. And that which comprehends all those places, is called *space*. Which shows, that in order to have an idea of place, and consequently of space, it is sufficient to consider these relations, and the rules of their changes, without needing to fancy any absolute reality out of the things whose situation we consider. (LV.47)

We start by taking some bodies as our references – the 'fixed existents' that define a frame – and take them to be at rest (it follows of course that they are at mutual rest). Presumably the kind of thing Leibniz has in mind is the Earth if we are interested in terrestrial mechanics (or the motions of the fixed stars), or the Sun or fixed stars if we are interested in planetary mechanics. Then as time passes the particular body occupying a given set of relations to the reference body can change, but we say that the new body is in the same place as the old. One might infer that a place *is* a particular set of relations to reference bodies, but as we'll mention presently, things are not quite so simple.[5] Space then is just the collection of all places.

There is something puzzling about this account, for we have just seen that Leibniz denied the reality of any spatial relations; the monads are not spatially related, and everything is composed of monads. But there's really no contradiction, it's just that Leibniz is here talking in terms of the phenomenological world of appearances, a world in which apparent bodies stand in apparent spatial relations to one another. Mechanics is a system of laws for the phenomena, and the system of Euclidean spatial relations is a fundamental structure of the phenomena.[6]

Crucial for understanding Leibniz's views on space and motion is that he holds it to follow from his account of place that places (and hence space) are *ideal* – and not just in the sense that the world of appearances is nothing but the ideas of monads. This consequence is not obvious because one is tempted to see Leibniz as identifying places with sets of relations to reference bodies, like Descartes' positions and Newton's relative places, which were really possessed by bodies according to them. But the same account is not possible in Leibniz's logic because 'two different subjects . . . cannot have precisely the same individual affection; it being impossible, that the same individual accident should be in two subjects, or pass from one subject to another.' (LV.47) According to this doctrine, properties are 'particular' in the sense that two things cannot literally possess the same property; two things of exactly the same hue of redness do not literally share the same property but each

[5]In addition to the complication discussed below, Arthur (1994, §4-5) points out that according to Leibniz every part of matter is divided into parts in relative motion, so that there are no 'fixed existents' – hence the need to 'feign' a set of reference bodies. The non-existence of reference bodies is, according to Arthur, another reason that Leibniz thinks of even relative space as less than fully real.

[6]The passage quoted is also interesting because it contains the suggestion that 'rules of direction and swiftness' enable us to know how bodies would have been related if they had moved differently, and so (reading into the text) what bodies would then have had the same place as some body actually did given the actual motions. It's not very clear what 'rules' he has in mind: perhaps the laws of collision, perhaps merely a specification of velocities over time in some relative frame. However, the suggestion seems to be that we infer what relations are possible from the rules of how bodies move over times – from the *regularities* in the relations that are instantiated over time. (Similarly, we know what would have happened if the rock were dropped because we know what happens whenever rocks are dropped; and we know what would have happened to a body if had kept on moving as it was rather being struck, because we know what happens whenever bodies keep moving unimpeded.) It's impossible to say from this passage how Leibniz saw this idea working out, but it is worth pointing out because in later chapters I will advocate a (somewhat) similar approach to understanding Newton's laws of motion and the geometry of space and spacetime. (Note that for whatever reason, this idea that 'rules of direction and swiftness' are required for our idea of space does not appear in the two other formulations that Leibniz gives: indeed, this absence is the main difference them.)

possess its own redness, even though the two rednesses are identical. Similarly, even if two bodies are at numerically identical distances (and angles) to some reference bodies (at different times) they do not stand in literally the same relations, but in their own particular relations, which are 'in agreement'. Thus if a place were just a set of relations to a reference, two bodies in numerically equal distance (and angle) relations to reference bodies would still be at different (particular) places. Therefore, Leibniz concludes, one should take a place to be the set of 'particular places': the *set of* sets of particular relations *that are in agreement*. Leibniz claims that this set of sets is a mental construction, and hence an idea – that places are 'merely ideal'. Finally, since (in modern terms) space is the manifold of such sets (of sets), a manifold with Euclidean geometry: space 'can only be an ideal thing; containing a certain order, wherein the mind conceives the application of relations.' (LV.47)

Clearly the argument here is not anything to do with monads. Even in the world of phenomena – a world in the minds of monads – Leibniz, like most idealists, thinks that a distinction can be drawn between the real and the ideal. In that world, bodies and their particular relations are real, but places and space are not. (For further arguments to this conclusion, see Winterbourne, 1982, and Hartz and Cover, 1998.)[7]

Because Descartes and Leibniz seem to reduce space to bodies and their relations (and because both are on one end or the other of polemics concerning absolute space) they are typically placed together against Newton. Our discussion has shown however that in an important respect Descartes and Newton are far closer together than Leibniz concerning the metaphysics of space. That is, they agree against Leibniz that space is real; for Descartes space is space=matter (extension) while for Newton it is a pseudo-substance arising from God's existence – only Leibniz denies that space exists (other than ideally) at all.

There is one final aspect of Leibniz's account that needs to be stressed. When he says that space is 'something merely relative ... an order of things' one is tempted to take him to mean that space is nothing but (or supervenient on) the *actual* relations between bodies (perhaps, in the light of the previous discussion, at different times). However, in response to Clarke (CIII.16) Leibniz later clarifies: '[Space] does not depend on such or such a situation of bodies; but is that order, which renders bodies capable of being situated, and by which they have a situation among themselves when they exist together .... But if there were no [created things], space and time would be only in the ideas of God.' (LIV.41) The context indicates that Leibniz uses the term 'situation' to refer to a system of relations between bodies and so the order in question is some kind of 'meta-organizing scheme' which underwrites the possibility of spatial organisation. As such, space – that order – is not dependent on specific configurations: space is the same whatever the situation of bodies. Further, it does not even depend on there being any bodies at all; the possibility of situations would yet remain (imagine God thinking 'hmmm, I could arrange bodies in these ways, couldn't I'.)

In his fifth and final letter, Leibniz repeats this idea, and gives a more concrete story about how

[7] Leibniz backs up his argument with two examples, one analogising family members in an imagined genealogical tree to bodies in space, and one concerning ratios. It is important to see that they are only intended to explain the partly mental construction of places, not to show anything beyond this point. First, the argument for the ideality of place, and hence these two examples that illustrate it, *assume* that space is relative, and so Leibniz does not intend them to be further arguments against absolute space. Second, Nerlich (1976, 5-9; 1994, 14-8) is wrong to claim that Leibniz intends the family tree to show that all spatial properties should be reduced to non-spatial ones. Leibniz of course does fundamentally think that, but the passage here makes quite clear that his point to Clarke is that the ideality of space arises from the particularity of relations, not the metaphysics of monads – and given the phenomenal level of the debate, it would be quite inappropriate of him to bring in such considerations.

to think about this mysterious order. '[I say that space is an order] according to which situations are disposed; and that abstract space is that order of situations, when they are conceived as being possible. Space is therefore something ideal.' (LV.104) That is, space, the ideal manifold of places, represents the collection of *possible* situations. More specifically, since Leibniz takes space to be Euclidean, space is our idea (and perhaps God's) of Euclidean space, understood as determining the possible configurations of bodies – namely those that are embeddable in it.[8] Thus Leibniz's relational space does not supervene on the actual relations at a time, but on all possible relations (and, as his final sentence emphasises again, it is not a real thing that supervenes on the relations, but merely an idea).

Before we move on, a caution. Leibniz's views on space, motion and mechanics do not have in any obvious way a linear logical trajectory from some basic principle. Although the presentation here started with the relational nature of space and its consequent ideality, and now moves to the nature of motion and later to the laws and relativity, it would be a mistake to think that we are following a logical development in Leibniz's thought. Instead the ideas are better thought of as a collection of mutually consistent, mutually supporting and mutually sympathetic principles, none of which can claim priority. However, I am writing a book, not (like Leibniz) a series of inter-related works composed over a 30 year period, and so we have to start somewhere – and close the circle later.

4.4 Motion and Force

The fundamental problem for Leibniz in constructing a theory of mechanics is that he takes the phenomenological world of appearances to be the world described by Descartes, in which the only (fundamental) properties of matter are 'geometrical': shape, size and relative positions and motions. Early in his career, Leibniz developed a mechanics based on such a description of bodies (the 1671 *Hypothesis Physica Nova* – see AG 123, footnote 170) but his attempt failed when he convinced himself that the assumption that bodies only had geometrical properties would entail unacceptable mechanical laws. Consider, for instance, the *Specimen of Dynamics* of 1695 – this work is central to the understanding of force developed here. In it Leibniz claimed that the geometric properties of matter alone do not involve any resistance to motion, and so '. . . the largest body at rest would be carried away by the smallest body colliding with it . . . ' (Woolhouse and Francks, 1998, 161 – henceforth 'WF').

Thus Leibniz rejected the Cartesian analysis of matter as pure extension. Instead, he took the 'inmost nature of bodies' (WF 154) to be 'effort-exerting and counter-straining (that is, resisting)', which he terms '*forces*'. Leibniz offers a detailed metaphysical analysis of this innermost nature, but in outline he equates 'effort-exerting' with an 'active force' – the 'power' to move and move other things – and 'counter-striving' with a 'passive force' – impenetrability and a resistance to change in motion. Each of these forces he divides further into 'primitive' and 'derivative' kinds; the former is not quantifiable (possession of primitive active and passive forces is simply what it is to be a body). However, because bodies collide, they are not able to realise fully the primitive forces – to act and resist completely – but only partially – as derivative forces, which are quantifiable.

[8] To be specific, the condition of embeddability in Euclidean space will mean such things as that the distances and angles between any three bodies satisfy the generalised Pythagorean theorem: if the distances are a, b and c and the angle between the sides of length b and c is θ, then $a^2 = b^2 + c^2 - 2bc\cos\theta$.

(Perhaps the distinction that Leibniz is aiming at is close to that between determinable – such as colour – and value – such as bluey-green.)

The point of this detour through some of Leibniz's system is to be able to make sense of his claim that he was marrying the scholastic and mechanical philosophies, that each only had part of the story. First, Leibniz explicitly identified primitive passive force with primary matter, primitive active force with substantial form, and derivative passive force (the differing resistance to change in different kinds of material substance) with 'secondary matter'. Thus when he claims that the innermost nature of a body is the possession of primitive active and passive forces, he takes himself to be assenting to the Aristotelian analysis of material substance as the combination of matter and form.

Second, extension, the foundation of the mechanical philosophy, far from being the essence of matter, is nothing but 'the diffusion' of ontologically prior forces (WF 155).[9] That is, for Leibniz, the theory of forces expresses the truth of Aristotelian metaphysics, while the forces themselves provide the metaphysical foundation for the space and matter of the mechanical philosophy. Appreciating these two aspects of his philosophy will help us understand a number of positions that Leibniz takes.

Leibniz is now, however, apparently open to the pointed criticisms of the mechanists against Aristotelianism regarding the vacuity of the doctrine of substantial forms. As Leibniz himself says: the Scholastics failed by '... believing that they could account for the properties of bodies by talking about forms and qualities without taking the trouble to examine their manner of operation. It is as if we were content to say that a clock has a quality of [clockiness] derived from its form without considering in what all this exists ....' (AG 42) While it is only *primitive* active force that is identified with substantial forms, we see here one reason that Leibniz held that active force in general '... serves no purpose in the details of physics and must not be used to explain particular phenomena ...' (AG 42). However, neither is it possible, as we saw, to construct a theory of mechanics without reference to forces; the laws must reflect that matter is more than extension. So Leibniz's solution is that although the mechanical laws are 'derived' from forces, the particular forces involved should not be invoked as the causes of events. This claim seems to be strongly supported by the view, explained in following sections, that, according to Leibniz, forces cannot be determined from the phenomena.

So finally, what we observe in this discussion is that Leibniz's analysis of the world is naturally broken into various domains. First there is the fundamental metaphysical realm of the monads, and then there is the less fundamental dynamical domain of forces, primitive and derivative, and finally, grounded in them, the phenomenological world of appearances, especially of spatial phenomena. (See Rutherford 1995 for a more detailed analysis of the stratification of Leibniz's thought.) This picture is useful to bear in mind because, as we shall see, principles or arguments that Leibniz takes to hold in one domain, do not necessarily hold in another; and, for instance, he will take Newton's account of AM to be a (false) theory purely concerning the phenomenal world, while his theory of true motion concerns the dynamical too.

For now we turn to the question of how Leibniz quantified derivative active force – he abbreviates

[9] Derivative forces presuppose collisions, which in turn presuppose extension – if there is no space then nothing can collide, while extension, we have just seen, presupposes diffused forces. Thus Leibniz's account would be circular if the forces in question were derivative. Presumably then, extension is the diffusion of primitive forces.

See Garber and Rauzy for a more complete discussion of Leibniz's conception of matter during the period of the *Specimen of Dynamics*. See Rutherford 1995 for a discussion of how the metaphysics of force integrates with the metaphysics of monads; essentially, force is the *apparent* causal power of a monad.

it as 'force', and we will follow him for the remainder of this chapter.[10] One obvious candidate, in historical context, is of course Descartes' quantity of motion, *size* × *speed*, but using a very neat *reductio* (especially *Discourse on Metaphysics* §17, AG 49-51), which I've slightly modified here, he showed this could not be the case. The crucial premise is Galileo's result (immediately following Theorem II, Proposition II of Day Three of *Two New Sciences*, 1954, 175) that the speed of a falling body dropped from rest is proportional to the square root of the vertical distance fallen, and independent of the mass (the school-book formula is $v^2 = 2as$ where v is the final velocity, a the acceleration and s the distance). So if A is dropped from 16 times as high as B, whatever their masses, at the end of their falls, A will be moving four times as fast as B.

So suppose, Leibniz suggests, (a) that A is elevated to a height of four units, while B, which is four times heavier than A, is on the ground. The force required to produce this situation, Leibniz says, is the same as would be required to (b) raise just B to a height of 1 unit, while leaving A on the ground (since 'gravitational force' is simply height times mass); so we can substitute (b) for (a). (c) If B were now dropped to the ground, by Galileo's result it will have a speed of 1 unit, so the Cartesian quantity of motion is the same as if (d) B were at rest and A (which has 1/4 of B's mass) were moving with speed of 4 units, so we substitute (d) for (c). But inverting Galileo's relationship, a body with four times the speed will rise to 16 times the height, so A's Cartesian force is now sufficient (e) to raise it 16 units, four times its original height (a). Thus, if force is measured by Descartes' quantity of motion, by the equality of forces in each of the substitutions in this sequence, (a) and (e) maintain the same force, which is absurd (indeed, contrary to the principles of Cartesian physics); hence force is not measured by *size* × *speed*.

Worse still for the Cartesians, according to Leibniz (AG 110) the substitutions could actually be carried out mechanically, so if the Cartesian quantity of motion were conserved, a perpetual motion machine could be constructed, since in (e) A could be dropped back to its original position, work extracted in the processes, and the sequence repeated. Showing that the substitutions could actually be carried out was important for the argument, for it blocks the one Cartesian response; that quantity of motion and gravitational force are not fungible. (Indeed, Descartes explicitly denied such fungibility, as his defenders pointed out – see Costabel, 1973, 41-8.)

On the other hand, Leibniz showed that there is a quantity which can be consistently taken as a measure of force for a moving body in this thought experiment, namely $mass \times speed^2$. The essential step is that following (c); if force is proportional to $mass \times speed^2$ then we now have the same amount of force if (d′) B is at rest and A moves with speed *2 units*, and so we should in fact substitute (d′) not (d) for (c); Galileo's result tells us that a body dropped from 4 units will have speed of 2 units, which inverted means that, in (d′) A has the force to climb 4 units, back to the initial state (a). In modern terms, Leibniz discovered kinetic energy ($\frac{1}{2}$mass × $speed^2$) and used its fungibility with gravitational potential energy: that different kinds of energy could be interchanged, as long as the total was conserved. This argument was one of Leibniz's most important contributions to mechanics, and is indeed found in text-books to this day (Feynman 1963, §4.3, for instance, argues along these lines).

Leibniz engaged in a heated debate with the Cartesians over the correct measure of 'living force' or *vis viva.* While the case against Descartes' measure is fairly unavoidable, there remained

[10]In the *Specimen of Dynamics*, Leibniz is not explicit that he means derivative active force by 'force' in his derivation of the measure, rather than some other quantity, though it is clear that is his intention (it's especially clear that he intends neither primitive forces – since only derivative forces are quantifiable – nor passive forces – since 'force' measures motion not resistance). However, he is more explicit elsewhere: e.g., in a 1704 letter to de Volder he refers to his demonstration of the 'true way of measuring (derivative) forces' (AG180). My discussion thus takes him to mean 'derivative active force' by 'force'.

the question of whether mass$\times$*speed*^{2} or mass$\times\vec{v}$, whether $\vec{v}$ is not the speed of a body but its velocity (its directed speed), is the correct measure of force. In modern terms, is momentum (which is immune to Leibniz's argument) or kinetic energy more fundamental? As you might suspect, eventually (at least by 1743, and D'Alembert's *Traite de Dynamique*) it was realised that the question was largely futile, and that both were important quantities.[11] Indeed Leibniz, while arguing for the priority of his measure of force, also appreciated the importance of momentum, which played a role in his laws of mechanics. (In the *Essay on Dynamics on the Laws of Motion*, dated 1691, he denied that it was a suitably 'absolute' measure of motion on the grounds that a system of bodies in motion could have zero momentum: 1949, 658.)

Even though speed enters into the measure of force, Leibniz maintained a sharp distinction between motion and force: while force is real, mere change of relative position without force is unreal. (Of course this view is of a piece with his idealism about place, and his view that extension requires force.) Understanding this point is essential to our discussion because, as we shall see, it explains how Leibniz could simultaneously accept absolute quantities of motion and yet deny that they featured in mechanics. It is exactly this point that many readers of his *Correspondence* with Clarke have failed to appreciate.[12]

According to Leibniz in the *Specimen of Dynamics*, 'force is something fully real, even in created substances, whereas space, time and motion have something of the nature of beings of reason: they are not true or real in themselves, but only in so far as they involve the divine attributes ... or the force of created substances.' (WF 168). Leibniz gives several reasons for holding this view. For instance, 'motion, like time, does not really exist: for a whole never exists if it does not have coexistent parts.' (WF 155) That is, since motion involves being at different places *at different times* the different points on a trajectory do not exist simultaneously, and so, on a certain understanding of existence, never exist. Or, (in another essay) 'if we set forces aside, then nothing real remains in motion itself, since from change of place alone one cannot determine where the true motion ... really is.' (AG 256) The idea here seems to be that kinematic relativity means motion does not have a well-defined subject and hence is unreal (perhaps motion is a 'being of reason' insofar as reason freely attributes it to this or that body). The list could go on, but these points give a good flavour of Leibniz's view on the subject. For a comprehensive treatment the reader is referred to Cover and Hartz (1988). (Note that the unreality of merely relative motion does not seem to follow from the ideality of space: a body can really change its particular relations to a reference body.)

Given this sharp distinction, it should be clear that force is a frame-*in*dependent quantity according to Leibniz, otherwise one could not conceive of relative motion without force. Indeed, both the arguments given against the reality of motion would be arguments against understanding force – 'in reality' – as mass $\times$*speed*^{2}, with *speed* understood as a frame-dependent quantity. Moreover, it is incompatible with Leibniz's rejection of Descartes' metaphysics to take force to be merely phenomenological, as nothing more than a geometric property, like relative motion. For

[11] That is not to say that in historical context the debate was trivial, as commentators often suggest. As Papineau (1977) explains, what is at stake is whether the action on a body is proportional to distance – like impressed kinetic energy – or to time – like impressed momentum. This was a question the participants hoped to answer by understanding in detail the mechanisms by which bodies were accelerated: the *vis viva* controversy ultimately concerned the concrete issue of how bodies act on one another. What had occurred by 1743 was the realisation that such a detailed understanding of mechanisms was either not to be had or was irrelevant to the advance of mechanics.

[12] Recent exceptions include Lodge (2003) and Roberts (2003), both of which – like the present section – build on the discussion in Garber (1995), especially §4.2. Also important in revealing the importance of the concept of force for Leibniz are Gale (1973 and 1988). As I mentioned in footnote 3, Lodge discusses in detail (see especially §2) the place and status of force within Leibniz's metaphysics, and in particular its relationship to the monads that ultimately ground reality.

Leibniz argues, as we saw, that the laws of nature cannot be interpreted purely in geometrical and phenomenological terms, but require something else: specifically force.

Of course one can define a phenomenological, 'relative force' as mass $\times speed^2$ in this or that frame and, we shall see, such relative quantities play a role in Leibniz's mechanics. However, such a quantity should not be confused with the 'true', absolute measure of force; indeed, we shall see that that it is impossible to determine this quantity, although it is clearly possible to determine the phenomenal measure.[13] (In general, when I refer to 'force' I intend the true, not phenomenological sense, and I will only use appropriate modifiers to make the sense clear when there is any possibility of confusion.)

There is a question, however, of what Leibniz understands the 'degrees' of force – understood as a non-relative, 'absolute' quantity – to be. Even if the phenomenological measure is the real-valued mass $\times speed^2$, the true degrees could conceivably be just 'none' or 'some', rather than some number; in the other relevant passages Leibniz tends to talk not about degrees, but whether or not a body possesses force. Further, the idea that zero is just another possible value of a quantity, rather than indicating a qualitatively different state was not properly appreciated at the time: certainly Descartes treats motion and its absence as distinct states of affairs.

The question is not insignificant: if the true degree of force is a real number, then it has an almost irresistible interpretation as a particular value of mass $\times speed^2$, commiting Leibniz to the attribution of a true speed to every body. An alternative is that true force is a binary quantity, in which case a distribution of forces will only determine the speeds of bodies if one of them has zero force and hence is at rest. However, Leibniz repeatedly insists that '*there is never any true rest in bodies*' (WF 174) – such a state would imply the absence of force, and thus, since forces are the ground of body, the absence of body. So if true force is a binary quantity then it is just a metaphysical notion, not an unknown but physical measure of true motion. It seems to me that in the absence of some clear statement that the true degrees are binary, the natural interpretation is that they are numerical – indeed, that the phenomenological measure is the true measure. In that case, Leibniz does attributes a determinate force *and hence speed* to each body. Let us call this doctrine the 'uniqueness of force'.

If force is unique in this way – and assuming that the forces and relative motions of bodies are assigned in a compatible manner – then there is a unique frame (up to Euclidean transformations) in which bodies have their forces equal to mass $\times speed^2$. This preferred frame of course grounds a unique notion of TM in just the same way that the frame of absolute space did for Newton, although the metaphysical accounts of the two frames is completely different. Indeed, in the *Correspondence* with Clarke, Leibniz expresses such an idea (LV.47). The 'fixed existents' relative to which places are defined are those 'in which there has been no cause of change of the order of their co-existence with others; or (which is the same thing) in which there has been no motion.' Leibniz clearly means, in metaphysical terms, those bodies in which there are no forces; thus the space which he constructs is, assuming some bodies at rest, the very privileged frame in question. (There are no such bodies, and instead of modern talk of finding a frame in which forces and speeds agree, Leibniz employs the device of 'feigning' the fixed existents). In this sense, Leibniz's 'merely relative' space is not relational – it involves appeal to force as well as the relations between bodies.[14]

[13] This point is discussed at length, from different considerations, by Roberts (2003, §4).

[14] Several commentators have accepted the implicit existence of a preferred frame in Leibniz's system, and hence the uniqueness of force: for instance Garber (1988, 291-2) and (1995, 308) and Roberts (2003, §4). Note that frames do not supervene on the speeds and relative motions of bodies in those frames. For instance, two bodies at relative rest and with speeds u in a frame, could be moving in a straight line in the frame, or rotating about their centre of mass, and so on. Therefore, an assignment of speeds consistent with some frame is not always sufficient to pick out

One objection to this line of thought is based on Leibniz's claim no body is ever at rest. Because the universe contains an infinity of bodies in random collisions, it would seem unlikely that in any given frame no body would ever be at rest, even instantaneously – surely some bodies change direction? But according to our reading of Leibniz, if a body were ever at rest in the preferred frame, then it would have no force. To the extent that such a situation occurred to Leibniz, our reading is undermined.

Note in the first place that if collisions occur instantaneously, then a body's trajectory is not differentiable at the moment of impact, and hence there is literally no moment at which a body reversing its direction has zero velocity. Now, it is grossly anachronistic to attribute a modern understanding of analysis to Leibniz, but the point shows that making the objection stick would require showing that according to his understanding of collisions and the continuum – themselves vexed issues – colliding bodies could possess zero speed at any time. In the absence of a substantial objection, and given the evidence on the other side, I will table this question, and assume that our reading is correct – that the existence of a preferred frame is compatible with the perpetual motion of bodies.

To be clear about the results of this discussion we should distinguish two senses of motion in Leibniz's thought. First there is motion relative to arbitrary reference bodies, which we have called RM. Without force RM is not real, so we will understand 'Leibnizian motion' ('LM') to be the possession of non-zero force, and more specifically the particular speed implied by the value of the force. Clearly LM is the 'true' sense of motion for Leibniz, since motion without it is unreal – we will address the question of its role in mechanics later. Thus Leibniz not only held there to be a true sense of motion, like Descartes and Newton, he held it to be uniquely attributable to any body.

There are of course several surprising and seemingly paradoxical aspects to this view. First, it is natural to suppose that Leibniz took the speeds (and hence forces) of bodies to be highly correlated; in particular any system of bodies and speeds should be mathematically embeddable in Newton's absolute space and time such that the relations are all preserved and such that the speeds of bodies equal the moduli of the corresponding (fictive) absolute velocities. Then the speeds are correlated in all the ways that the moduli of absolute velocities are.

Second, since Leibniz accepts a privileged sense of motion and rest, like Descartes, like Huygens and like Newton, he accepts TM. We will see later that Clarke completely misunderstands this aspect of Leibniz's thought, as has almost every recent commentator on Leibniz (particularly in the philosophy of physics literature). Given Leibniz's account of space as a construction in terms of relations, it is very natural to expect that his account of motion is also purely relative; that to move is to be understood in terms of the relative motions of bodies. But Leibniz's LM simply cannot be understood in this way, while its prominent place in his writings on dynamics shows that it is to be taken very seriously in his system. So it is quite misleading to treat Leibniz as a relationist about

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that frame uniquely. Still, one imagines that with sufficiently many bodies and relative motions, and excluding cases of extreme symmetry, the frame determined is unique; every body with an independent speed places an additional constraint on the frames. Still, I do not know the general answer to the question of how many bodies are required to settle the matter – but certainly far fewer than composed Leibniz's plenum universe.

It follows that we have here good reasons not to foist any spacetime geometry on Leibniz, as contemporary philosophers of physics are fond of doing. For there is not one spacetime that will do justice to the points just made: in special cases (as Roberts, 2003, 559, proposes) we would need something stronger than Earman's (1989, 71-3) 'Leibnizian spacetime' in order to define true speeds (but not velocities or accelerations); while in general, the appropriate space is full Newtonian, in order to make sense of a rest frame. Not only is the appropriate spacetime ambiguous (and not because of any ambiguity in Leibniz) but it seems to do violence to Leibniz's views to capture contingent facts about matter in terms of spacetime geometry.

motion – as opposed to space – even if RM is a key concept for him.

Before we turn to the theory of mechanics then, let us review the basic outline of Leibniz's view. On the one hand space is purely relational (concerning possible arrangements) and ideal. Motion to the extent that it is purely relational – i.e., RM – is also unreal. However, contrary to Descartes, such a 'purely geometric' conception is too thin to ground a theory of mechanics, and so Leibniz introduces force as the missing metaphysical foundation. Then, motion, insofar as it corresponds to force – i.e., LM – is real. So Leibniz is a relationist regarding space but not, in its fundamental sense, about motion, and an idealist regarding space but not motion (again, in the fundamental sense). However, force, being form, while necessary for the laws, cannot play a role in particular mechanical explanations. Moreover, since force belongs to the dynamical realm, which ontologically precedes the phenomenological world to which spatial concepts belong, force *and hence motion in the sense of LM* are at root non-spatial. To repeat a quotation already given: 'force is something fully real, even in created substances, whereas space, time and motion have something of the nature of beings of reason: they are not true or real in themselves, but only in so far as they involve the divine attributes . . . or the force of created substances.' (WF 168) It is not surprising that the Newtonians had such a hard time getting to grips with such a convoluted view.

4.5 Mechanics

According to Leibniz, the only kind of interaction was collision (as we shall discuss, he held this view even more strictly than Descartes, denying rigidity to any body however small), so his laws treat freely moving bodies and impact.[15] The law for a freely moving body is 'to tend in a straight, tangent line' (AG177) – that is, assuming no change in speed, the Cartesian law of inertia.[16]

His account of collision, presented in the 1691 *Essay on Dynamics on the Laws of Motion* (1949, 657-70) consists of three conservation principles. Let m_i $(i = 1, 2)$ be the 'masses' of two bodies in collision and u_i and v_i be their initial and final velocities, respectively. Then:

(i) Conservation of Respective Speed: $u_1 - u_2 = v_2 - v_1$ along the line of collision.

(ii) Conservation of Common Progress: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$.

(iii) Conservation of Vis Viva: $m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$.

(ii) and (iii) we recognise as the conservation of momentum and kinetic energy respectively, and indeed they form the basis of classical collision theory one studies in school today – that is, ironically, 'Newtonian' collision theory! (i) Is the statement that the relative speed of two bodies remains the same after they collide, simply reversing direction. Now in the collision of two bodies (iii) may not hold as formulated; if the parts of one of the bodies are agitated, the total amount of kinetic energy after collision will also involve a term summing over the kinetic energies of the parts, and this quantity will be conserved, not that in (iii). Leibniz realised this possibility, and

[15]See Westfall (1971, Chapter 6) for a broader survey of Leibniz's dynamics.

[16]Note that in practice such a situation is impossible according to Leibniz, for the world is a plenum in which every particle is constantly in collision. The law is however not redundant for Leibniz treats the actual motion as the resultant of inertial motion plus whatever motion is imparted by collision. See WF 173-9.

so, as we do, restricted in laws to 'elastic collisions' in which the parts of the bodies are not set in independent motion.[17]

Leibniz also claimed that any pair of the laws entails the third, though there is a small (but possibly important for interpreting Leibniz) lacuna in his proof that (i) and (iii) entail (ii). He rearranges (i) and (iii) and divides the respective LHSs and RHSs to yield:

$$\frac{m_1(u_1^2 - v_1^2)}{u_1 + v_1} = \frac{m_2(u_2^2 - v_2^2)}{u_2 + v_2} \tag{4.1}$$

or, since $x^2 - y^2 = (x + y)(x - y)$, (ii). But of course the calculation is only valid as long as $u_i + v_i \neq 0$, to avoid division by zero. If this condition is not satisfied, then (ii) does not follow (put another way, in that case (iii) is not logically independent from (i)), a point that Leibniz does not acknowledge, likely because it seemed too trivial to be worth mentioning.

The condition fails in an inertial frame (i.e. one in which all three laws hold) if the bodies have equal masses ($m_1 = m_2$) and equal and opposite velocities ($u_1 = -u_2$), or more generally if $u_2 = -(m_1/m_2)u_1$. But it can also fail in non-inertial frames, in which case (i) and (iii) may hold while (ii) does not. Consider for instance a collision between identical bodies (this assumption simplifies the problem, but is irrelevant to the example) A and B described in a frame in which B is at rest throughout. In the inertial frame in which B is initially at rest and A moves with velocity V, the solution to (i)-(iii) is that afterwards A is at rest and B moves with velocity V. Thus in B's frame, A comes in with velocity V then bounces off with velocity $-V$: $u_1 + v_1 = V + (-V) = 0$. But $V - 0 = 0 - (-V)$ so (i) holds in B's frame, and $V^2 + 0^2 = (-V)^2 + 0^2$ so (iii) holds too; but (ii) obviously does not. We will see the possible significance of this point later.

We have then enough of a sketch of Leibniz's mechanical and metaphysical views to address the crucial issue for us, concerning the status of 'absolute' quantities of motion in Leibniz's thought. What we shall find is that there is no plausible reading of Leibniz's thought that completely saves him from incoherence, confusion over simple mathematics, dissembling, obscurity or some combination thereof. Given Leibniz's undoubted genius, this situation is troubling, but it seems that the best we can do is to identify the most likely loci of error. What is so hard to balance are his pronouncements and arguments concerning relativity, the nature of his mechanics (especially the laws of collision and his views on solidity), and his views on force.

4.6 Relativity

The question of kinematic relativity in Leibniz's system is now easily dealt with. RM is relative to what entirely arbitrary frame is chosen, but LM is unique, in the sense that (*modulo* footnote 14) every body has a unique velocity determined by the distribution of forces (i.e., kinetic energies/speeds). The question of dynamical relativity is far harder to determine. Behind his views, is the tension between his metaphysical account of motion, which appeals to forces, and his view that such forces are Aristotelian forms and so inadmissible in mechanical explanation. Leibniz deals with this tension with a relativity principle: the laws may be formulated in terms of the forces of bodies, but they must not depend on what the specific forces possessed by bodies are – the laws must be

[17]It is somewhat paradoxical that Leibniz proposed a theory of elastic collisions, since they seem impossible for him. On the one hand, Leibniz insightfully realised that inelastic collisions occur when force (i.e., kinetic energy) is lost to the parts of a body – as thermal energy we would say. On the other, he held that every body, however small, has smaller parts; it follows that every body is liable to lose force to its parts in any collision.

formulated so that they predict the same outcomes for systems that differ only in their distribution of forces. In this way, forces/forms can provide a foundation for mechanics without directly entering into them; the laws must be formulated in geometric, relational, terms. This outline of the view is clear enough, but exactly how Leibniz intended these ideas is, as we shall now discover, far from clear.

Leibniz repeatedly asserts a dynamical relativity principle for his mechanics, which he calls the 'equivalence of hypotheses' (or EH). (The name has its origin in the question of which hypothesis about the planets' motion, Ptolemy's, Copernicus' or Brahe's, is correct, but its use here refers to reference frames in a more general sense.) For instance:

> ... *motion [in the phenomenal sense] is nothing but a relationship* ... What we must say, therefore, is that given a number of bodies in motion, there is no way of determining from the phenomena which ones are in absolute determinate motion or rest. Any one of them you choose may be taken as being at rest, and yet the phenomena will be the same. It follows ... that *the equivalence of hypotheses still holds when there are collision between bodies*; consequently, we must work out the laws of motion [such that] there will be no way of determining from the phenomena after a collision which bodies before it had been at rest and which had been in absolute determinate motion. (*Specimen of Dynamics*, WF 168)[18]

Note that Leibniz does not use 'absolute motion' in Newton's sense here, but means some privileged, non-arbitrary, sense, which we have called in general 'TM'. For Leibniz, TM is LM, and so certainly his principle covers that notion. However, Leibniz also addresses Newton in his discussion of relativity, and so we can understand him to refer to TM generally, and hence to both LM and AM. It is worth emphasising that this proof is completely different from Newton's proof of the comparable Corollary V to the axioms; it proceeds, not from the laws of motion, but from metaphysical considerations. Indeed, here EH is not taken as a consequence of empirical laws at all, but as a constraint on such laws.[19]

The problem with EH is that it is very hard to determine its scope.[20] On the one hand the enunciation says that 'one can attribute rest to any [body]' and the system will evolve in the same way, suggesting that mechanical experiments cannot distinguish rest from *any* other state of motion.

[18]An issue to address immediately is Leibniz's claim in the *Discourse on Metaphysics* that on the basis of force we can 'know' which body is in motion (AG 51), which seems to be in tension with EH. However, this claim does not contradict EH: force, we have observed, is not part of the phenomena, so if we knew the distribution of forces without recourse to the phenomena, then we would have a *non-phenomenological* way to learn the TMs of bodies. But Leibniz gives little clue about how such forces should be determined, from the phenomena or otherwise (see, however, footnote 21.). (Immediately prior to the quotation, he measures force by conversion to elevation. But this method relies on changes in relative position, namely the motion of a body away from the Earth, and the EH implies that one cannot determine the LM and hence force of a body by its relative motions.) Perhaps Leibniz wished to leave open the possibility that forces and LMs could be known somehow, even if they could not be determined on the basis of mechanics and the phenomena. Our discussion concerns dynamical relativity in the context of mechanics and hence phenomena, so we shall bracket the issue of whether forces are knowable by other means.

[19]Leibniz does also offer a mechanical 'proof': since any motion is composed of linear motions, and since EH is satisfied for linear motions, by composition EH is satisfied for any motion. We will discuss this approach later, after we have had a chance to understand what the claim about the composition of motions might mean.

[20]My discussion of EH owes a considerable amount to Stein 1977, especially footnote 5. His work is by far the most sophisticated treatment of which I am aware (though see also Bernstein, 1984, §ii). Even careful readers of Leibniz's conception of force (e.g., Garber 1988, 290, Meli 1988, 26) often seem to take EH rather unquestioningly as a principle of general dynamical relativity. However, the interpretation that I ultimately propose differs from Stein's; mine seems to me to do a better job of fitting Leibniz's pronouncements.

Of course such a 'general' principle is far broader than either of Newton's corollaries on relativity, which are restricted to the unobservability of common velocities or accelerations, not, for instance, rotations. On the other hand, such a principle is easily seen to be incompatible with Leibniz's own mechanics, which satisfies the weaker statement given by Leibniz: that the *initial* velocity of a body be unobservable. As we shall see, Leibniz's pronouncements seem to swing between these interpretations of EH, and it is hard – though, I suggest, not impossible – to find a reading that simultaneously makes Leibniz consistent and not hopelessly confused.[21]

4.6.1 General Relativity?

One might hope that the 'proof' offered would help settle the issue. Leibniz argues first that motion, restricted to phenomena, and so independent of force, is RM. Why would that show that TM cannot be inferred from the phenomena? One possibility (which I will reject below) is that Leibniz implicitly appeals to the kinematic relativity of RM to infer dynamical relativity. If there is no 'absolute determinate motion', then obviously it cannot be observed; then since RM is a generally relativistic kinematic concept, this line of thought implies general dynamical relativity, as the enunciation suggests. Of course, this understanding fits extremely well with discussions of Leibniz that ignore LM, for then RM is the only kind of motion on the table, and EH seems necessarily to be general.

Tempting though this reading of the argument is, there are powerful reasons to resist it. First there is the amplification that Leibniz gives in the passage itself; he does not say that the laws should be fixed so that one cannot infer anything 'absolute' about the motion of the bodies, but only 'where there had been rest or determinate motion before the collision'. Admittedly the argument in this passage is directed against Descartes' laws of collision, which we saw in Chapter One to imply an observable standard of rest; indeed, Leibniz is keen to emphasise EH as a crucial difference between his work and Descartes'. So Leibniz might be drawing a restricted consequence of general EH sufficient to refute Descartes, but the fact that his own laws manifestly violate the general (but not restricted) principle argues that he is instead giving a precise statement of EH.

Given the importance that Leibniz attaches to EH – an explicit constraint on mechanical laws – it is simply impossible that he knew his laws to violate it; so either he did not intend EH to be a principle of general relativity, or he was seriously confused about some basic properties of his laws. I really can't believe that a mathematical genius of Leibniz's abilities could really have been so trivially confused, but let us consider one way that he might (just) have been. The discussion of this possibility will probably seem unnecessarily drawn out, beating up on a view long after it has ceased to have any plausibility; however, the interpretation of EH as general relativity has had such a hold on the literature that it is worth being clear just how confused and careless Leibniz would have to be to hold it.

[21] Lodge (2003, §2) discusses further important complications. First, he documents that at an early stage of his development of mechanics, in work dated 1678-81, Leibniz held that motions could be determined by mechanical experiments, potentially allowing true velocities to be measured. (Lodge correctly points out that it is hard to understand why Leibniz thought he was justified in so thinking.) Presumably by the time that EH made its appearance in Leibniz's system – 1689 seems to be the first occurrence – he had changed his mind. Second, Lodge suggests that even after 1689, EH notwithstanding, Leibniz held that it was possible to learn something about the TMs of bodies, particularly one's own body. Force can be equated with the (apparent) activity of monads, and in particular of the monads that constitutes each of our souls; then our direct introspection of our souls offers a way of determining whether we are moving (ourselves) – perhaps even something about the degree of motion. Even if Lodge is correct in this line of thought, it is not in conflict with EH, since that denies that motion can be determined by mechanical experiments, while direct introspection is clearly a *non-mechanical* experiment.

Consider again the example following equation 4.1, in which an elastic collision between identical bodies A and B is described relative to one of the colliding bodies – i.e., in a particular *non*-inertial frame.[22] In such a frame, the relative speed is conserved, since Law (i) holds in all frames, so the moving ball simply reverses its velocity, and maintains its speed. Of course it follows that it preserves its force, mass $\times velocity^2$. Now, if that was as far as Leibniz thought through the example (and I know no text where he considers such an analysis), and if he believed, as he said, that any two of the laws entailed the third (i.e., if he was unaware of the simple lacuna that we discussed), then he might (just) have concluded that all of his laws held in the frame of one of the colliding bodies. That is, he might have seen that (i) and (iii) hold, and mistakenly inferred that (ii) did as well.

But it just does not seem plausible that Leibniz could have been led by this line of thought to conclude the general relativity of his laws. First, the law of inertia obviously only holds in a restricted class of frames related by constant linear motions. Second, a moment's thought about the example shows that mass $\times velocity$ is not conserved in the given frame, but reverses direction. Third, reasoning on the basis of the claim that (i) and (iii) entail (ii) would require Leibniz to overlook the trivial lacuna in his proof – more likely he simply thought it not worth mentioning. Fourth, it is only in the special kind of non-inertial frame selected – the rest frame of one of the bodies – that (iii) holds in the example, while finally, (iii) also fails to hold in the given frame if the interactions are any more complicated.[23] Thus any of the most trivial kind of further analysis of the problem would have shown immediately that the laws were not generally relativistic. It's just implausible that Leibniz was misled by this example.[24]

Since there are some frames in which the laws hold and some in which they do not, it follows immediately that the laws permit a privileged kind of motion to be determined from the phenomena – motion as measured in the former, inertial, frames. This kind of analysis applied to Descartes'

[22] Recall that the relevant class of frames is not that of arbitrary co-ordinatisations, but of Euclidean frames adapted to bodies and those obtained by arbitrary rigid transformations from them. In the discussion here we will, for simplicity, restrict attention to a sub-family of such frames related by rigid transformations: i.e., we will ignore changes of scale, since these make no difference to anything of substance.

[23] To see that (iii) fails to hold in non-inertial frames in more complex cases, consider another simple example. Imagine three identical collinear balls A B and C, the first two of which collide elastically. Suppose that relative to C the velocities of A and B before the collision are $+v$ and $-V$ respectively (v, V positive), while after the collision they are $-V$ and $+v$ respectively – that is, relative to C, A and B 'exchange' velocities. Then the laws of collision (and inertia) hold relative to C: for instance, Leibniz's force is $mv^2 + mV^2 + 0$ before and after (and momentum and relative speed of the colliding bodies are conserved). But relative to A the velocities of B and C before the collision are $-(v + V)$ and $-v$ respectively, while after they are $v + V$ and V respectively, so the total force relative to A is $0 + m(v+V)^2 + mv^2$ before and $0 + m(v+V)^2 + mV^2$ after, a difference of $m(v^2 - V^2)$ Thus force is not conserved relative to A – in the example, the laws do not hold relative to A.

[24] It's worth mentioning another important example which could (just) conceivably fool one into mistakenly inferring the general relativity of Leibniz's mechanics. Suppose, as Leibniz does, that the planets are swept around the Sun by an enormous vortex the size of the solar system, the angular speed of which decreases with the distance from the Sun (suppose too that the density of the vortex is constant, though a density that only depends on the distance from the Sun suffices for the following points). From the point of view of the Sun, the force is constant since at any time the field of velocities around the Sun is the same (similarly, the sum of linear momenta is always constant, and in fact zero, by the symmetry of the velocity distribution). But, perhaps unexpectedly, the same thing is true from the point of view of a frame located at a distance equal to the orbit of the Earth oriented towards the Sun: at any moment the distribution of velocities around the Earth is a constant and so force and momentum are conserved. So in this rather special but, as we shall see, important case, all three laws apparently hold in a non-inertial frame. One might, again very hastily, be tempted to infer from the fact that the laws hold according to both the Copernican and Tychonic hypotheses, they hold for any frames. However, this example doesn't explain away the errors already made, and moreover assumes circular orbits, while Leibniz knew the planets to move in ellipses, so is unlikely to have swayed him either.

laws led Leibniz to conclude that they violated EH. Immediately after the passage quoted (WF 168) Leibniz points to Descartes' rule that a stationary body cannot be moved by a smaller body (Rule 4), which in conjunction with the rules that allow a smaller body to change the motion of a non-stationary body (e.g., Rule 2) allows a preferred standard of rest to be observed: a body is at rest iff Rule 4 holds of it. Analogously, in Leibniz's mechanics a body has a constant speed (and velocity) iff it moves in a straight line at a constant speed in a frame in which all the laws hold. That is, almost the same line of thought that Leibniz uses to show that Descartes' laws allow a true speed to be determined by the phenomena shows that his own laws allow changes in true motion to be determined.

Thus it's almost inconceivable that Leibniz would have failed to notice that his laws violate his important EH principle if he had considered any examples at all – *if he intended EH as a principle of general relativity.* The laws obviously allow changes in true motion to be determined; and changes in force too according to the identification of TM with LM. And it's impossible that he knowingly adopted laws violating EH, so the only way to reconcile a general principle of EH with Leibniz's laws would be to say that he never bothered to consider any concrete applications. But to do so would be to attribute a truly shocking level of carelessness to such a profound thinker, particularly because he clearly did consider the violations of EH by Descartes' rules. In addition, he was aware of Huygens' laws of collisions, to which his owe much, and likely would have considered their relation to his laws, in the particular cases Huygens treats. Further, as we shall see later, he likely did consider such cases when he responded to Newton's arguments from causes.

Indeed, to think that general relativity holds is to think that the phenomena and laws give us no reason to think that A does not at remain rest – in some 'true' sense – whatever interactions it undergoes, and not just initially. But of course in the examples A is not at rest all through the experiment, for it is *struck* by B! Since collisions are part of the phenomenal realm, and hence observable, Leibniz could only think that EH was a principle of general relativity if he held that a collision with B might have no effect at all on A, and there is no reason to think that he would accept that. Indeed, there are at least two reasons in the *Specimen of Dynamics* for thinking that Leibniz denied such a doctrine. First, he is explicit that there is a difference between rectilinear and curvilinear motions, and that it is collisions which cause deviations from rectilinear motion (e.g., WF 173-4). Second, in a discussion of the role of elasticity and the action-reaction principle, Leibniz says that 'the action of a colliding body provides the occasion for [the change in the other]' (WF 172). The passage refers to the doctrine that strictly speaking bodies only act on themselves, but the point suffices: collisions lead to changes.

Since interpreting EH as a general principle of relativity is to attribute to Leibniz an extraordinary and basic confusion and a truly bizarre view about the effects of collisions, we would do well to consider other possibilities. The next most obvious suggestion is that EH is strictly equivalent to the amplification, 'one cannot tell, on the basis of the phenomena resulting from a collision, where there had been rest or determinate motion before the collision'. Assuming, in accordance with the law of inertia, that bodies before the collision are moving inertially, this statement is equivalent to Newton's Corollary V for elastic collisions, the principle of Galilean relativity: the mechanical undetectability of a common velocity. On this reading EH does permit the inference of true quantities of motion, not velocities or speeds, but accelerations.

Of course, EH could only be plausibly be taken to be Galilean relativity if there is a plausible reading of the proof given in the passage quoted that is consistent with such a principle. There is, and in fact it makes better sense in the context of Leibniz's general views than the one proposed earlier, and is more charitable. According to the original reading, Leibniz infers that TM – e.g., AM or LM – cannot be inferred from the phenomena *just because* phenomenal motion is RM, which is arbitrary; but this argument is a *non sequitur*. As long as there is a privileged sense of motion distinct from RM – as there is according to Leibniz – there is no logical reason why it should not be inferred from relative motions.

The way that Leibniz presents the argument elsewhere suggests another reading of the proof, which we have touched on in previous sections. Although forces – in his sense of mass$\times$*speed*^{2} – underwrite phenomenal, mechanical laws, such 'dynamical' – again in his sense – considerations cannot be invoked in mechanical explanations. '...once [mechanical laws] have been established, entelechies or souls have no place in discussions of the immediate and specific efficient causes of natural things, any more than do useless faculties and inexplicable sympathies.' (WF 163) (A little earlier Leibniz explicitly equates 'entelechies' with 'forces'). The problem then is that invoking real forces – rather than their frame dependent measures – would spoil the mechanical credentials of Leibniz's physics, by reintroducing 'occult' powers and the like. (Note however that Leibniz's Aristotelianism means that every mechanical phenomenon has a parallel explanation just in terms of forces, and that physicists should in fact be prepared to look for whichever is most convenient: WF 163-4.) Since the true force thus cannot play an explanatory role in mechanics, Leibniz infers that the true force, and hence true motion it picks out, cannot be determined from mechanical phenomena. That is, if the value of X makes no difference to the outcome of mechanical interactions, then the value of X cannot be inferred from mechanical interactions.

But if the grounds for the EH is found in this line of thought, then it is not so clear that *changes* in the force cannot be inferred from the phenomena. That is, even if one accepts that changes in speed because of collisions mean changes in the measure of force in a body, it is not clear that the mechanical philosophy would forbid such a role for 'entelechies'. So, especially if collisions must cause changes, as I argued above, it is possible that Leibniz only wished to makes 'forms' themselves – the actual degrees of force – unobservable, not their time derivatives. In that case, Galilean relativity becomes a more plausible reading – especially in comparison with general relativity.

4.6.2 Galilean Relativity?

However, there are also serious problems squaring this interpretation with everything that Leibniz says about EH, and so ultimately I will propose and defend a new, third reading – with a scope wider than Galilean relativity, but narrower than general relativity. The problems arise from Leibniz's views on rotation; first concerning the solidity of rotating bodies, and second concerning the equivalence of Ptolemaic and Copernican hypotheses. We'll discuss the former point in detail first, then turn to the latter later, after I have explained my reading of Leibniz, for it affects that too.

In his *Dynamics*, Leibniz makes a crucial comment on Newton's spheres argument from the *Scholium*. He says that 'a certain illustrious man' argued correctly that 'if there were anything in the nature of a cord or solidity, and therefore ... circular motion as it is commonly conceived' then the 'subject of motion [could] ... be discerned ... on the basis of curvilinear [motions]' (translated in Stein 1977, 42) – and that such a determination would be a violation of EH. Leibniz, holds however, that there are no solid bodies (literally speaking – see below) and so no actual violation.

On the interpretation of EH as Galilean relativity, Leibniz's claim about Newton's argument is that observations of a solid rotating system would permit the determination of true (absolute or Leibnizian) *velocity* (or at least speed). But as Stein points out, it follows immediately by

Corollaries V and VI of Newton's *Principia* that such a determination is not possible, for they state that systems which differ by a common velocity are indiscernible.[25] Since Leibniz had read the *Principia* (in 1689, during a trip to Rome, by his own account [AG 309], but earlier according to Meli, 1993, 7-10) before the *Dynamics* was completed (1690-91), such a reading implies an implausible misunderstanding of some basic ideas of the *Principia* by Leibniz. In addition, Leibniz's explication in the *Dynamics* (Stein 1977, 43) of how such a determination could be undertaken for a solid body manifestly does not allow a true velocity to be inferred, and so could not plausibly be taken to refute mere Galilean relativity. (The idea is that the end of a solid body should be released, and observations made of whether it flies off at a tangent – showing that the body was rotating – or remains in place – showing that the body was not rotating. Both of these observations are compatible with any true velocity whatsoever for the centre of mass of the system, and so does not violate Corollary V.)

It seems then that we cannot understand EH as either general or Galilean relativity, and hence we have to attribute to Leibniz some other view that cannot be neatly parsed in terms of modern relativity principles. To understand what he may actually have thought, we will have to investigate more carefully his analysis of motion.

As a first step, we should consider Leibniz's views on solidity more carefully – after all, despite his denial, there do appear to be rotating solid bodies! What he rejects, more precisely, are bodies whose solidity arises from the intrinsic 'cohesion' of their parts, or from 'restraining cords' (p.42), or 'hooks' and 'handles', or from 'ropes or fibrous webs or other tangled textures' (p.44): i.e., bodies that are held together by unanalysed attractions. His mechanical philosophy leads him to hold instead that only collisions can explain motions, and in particular the collective motions of the parts of a solid body. Thus all reference to cords and so on solves nothing, for their solidity must also be explained, via the laws of collision: 'there are in Nature no other cords than these laws of motion themselves' (p.42).

Leibniz thus accounted for the (apparent) solidity of bodies as follows:

> . . . if we consider something which we call solid rotating about its centre, its parts will . . . begin to [fly off along the tangent]. But as each one's moving away from the others interferes with the motion of the bodies around it, they are repelled and pushed back together again, as if there were a magnetic force at the centre which was attracting them . . . otherwise it could not be the case that all curvilinear motion is composed only from rectilinear motions.' (*Specimen of Dynamics* WF 173-4 – a similar discussion is found in the *Dynamics*, Stein 1977 43-5.)

It's not clear from this text whether the 'mutual separation' is finite or infinitesimal. Levey (2003), discussing an earlier work from 1676, argues that for Leibniz the smallest geometric parts of space and time are unextended, but they are not points of $\Re^n$ but only exist as the limits of finite extensions. Motion then can be understood as involving 'leaps' from one end of a finite interval to the next, motion which nonetheless is not discrete, because any such leap can be resolved into smaller leaps – though not into a mapping of instants to points, since instants and point have

[25]The proof of Corollary V, as we saw, explicitly concerns collisions, not rotating solid bodies. However, there is no such restriction in the enunciation. The proof of Corollary VI, of which V is a special case, is not restricted to collisions. I want to thank Stein for clarifying this point for me. However, I do want to note that there is nothing to suggest, as Stein does, that Leibniz intends strict *rigidity* in particular here. I pointed out in the previous chapter that, although Newton's spheres are often taken to be rigid, the text does not require this; I see no evidence here that Leibniz took Newton to intend rigidity.

no reality except as termini. Levey describes such a conception as 'fractal'. Clearly, if Leibniz has a picture like this in mind, his account of the structure of motion is neither that of modern analysis nor straight-forward. However, it is very interesting to note in the present context that an important part of this 'fractal' view is that no motion, for any period, is 'uniform', by which Leibniz means that during any finite interval there will be a discontinuous acceleration. (Again we see that Leibniz does not have the modern conception of a mapping $\Re \rightarrow \Re^n$, for so conceived the motion that Leibniz describes is nowhere differentiable.) If a fractal picture is intended in the account of solidity, then Leibniz holds that the interval which a body travels before colliding is neither zero nor a smallest finite amount, for during any finite period a collision will occur.[26]

We further find in the passage Leibniz's claim that all motions – linear or curvilinear, accelerated or unaccelerated – are composed of linear motions. What he has in mind is the kind of picture described: roughly speaking, the path is a polygon approximation of some kind to a curve, which is physically realised by a body moving inertially along the sides, with collisions occurring at the vertices.[27] One of Leibniz's arguments for EH is that because motions are so composed, and because linear motions satisfy EH, all motions do (WF 174-5, Stein 1977, 41). Those who interpret EH as a general principle of relativity, generally understand Leibniz to be reasoning that since no phenomena will distinguish one inertial motion from another, and since all motions are composed of inertial motions, the phenomena will not distinguish any motion, inertial *or accelerated* from any other. Since Leibniz's laws are manifestly not generally relativistic, it goes without saying that, this argument is manifestly fallacious – charity suggests that it is not Leibniz's.

4.6.3 The Scope of the 'Equivalence of Hypotheses'

But since we cannot take EH be Galilean relativity either, what does it amount to? I propose that the quotation from the *Dynamics* regarding Newton's *Scholium* argument holds the key. Recall: 'if there were anything in the nature of a cord or solidity [there would be] circular motion as it is commonly conceived' (and a violation of EH). Compare this statement with the claim in the *Specimen of Dynamics* that if solidity were not the result of collisions with surrounding bodies then 'it could not happen that all curvilinear motion is composed of pure rectilinear motions'. What we see is the idea that it is not the case, after all, that all conceivable motions can be decomposed into linear motions; decomposability is rather a property only of the actual motions of bodies, and would not be a property of bodies attached by true cords or cohesion. Leibniz does not argue that a circular motion 'as commonly conceived' can be *decomposed* into linear motions, but argues rather that bodies do not actually move as commonly conceived at all – they move in a series of linear motions *instead*.

That is, he apparently views the two kinds of motion – those that are decomposable and those

[26] I noted above the apparent oddity of Leibniz proposing laws of elastic collision, when he seems committed to the view that there are no elastic bodies. If the fractal reading is correct, it also follows that there are no intervals during which bodies move inertially.

[27] The passage thus again emphasises that Leibniz cannot have had general relativity in mind, for it presupposes the distinction between inertial and non-inertial motions in the following way. The distinction between rectilinear and curvilinear motion is not kinematically invariant, so that even a body which does not collide moves curvilinearly in some frame. But clearly such a 'curvilinear motion' could not be explained by the composition of rectilinear motions as Leibniz proposes, exactly because there is no collision. Thus the distinction between rectilinear and curvilinear to which Leibniz refers is dynamical – Leibniz admits a distinction between inertial and non-inertial motion, and supposes that such properties have subjects. In the context of a reading of Leibniz which acknowledges only RM, this passage becomes virtually unintelligible. But Leibniz is not, we have seen, grossly inconsistent, for there is every reason to suppose that he did not hold a general principle of relativity in either the kinematic or dynamical senses.

that are not – to be qualitatively distinct. In this case, a new understanding of EH arises, one which has the unique virtue of rendering consistent (nearly) everything Leibniz says about dynamical relativity: EH denies the possibility of determining the magnitude of each qualitatively distinct kind of motion separately. First the decomposable. Since the laws of collision are Galilean invariant, we cannot tell 'where there had been rest or determinate motion before the collision': whatever the outcome of an interaction, there is always an inertial frame in which any of the bodies involved is initially at rest. And even if a body is moving in an ambient medium and has a curvilinear motion because of a series of collisions in the manner that Leibniz suggests, since its motion is composed of a series of linear motions, EH as applied to it means the same thing: there is an inertial frame in which it is at rest before it collides, or perhaps in which it is at rest at any instant.[28] All Leibniz claims – correctly – for decomposable motions is Galilean relativity. He does not say that the ability to distinguish accelerating or curvilinear motions of this type from inertial motions, or from each other would violate EH. (In particular, in the 'proof' of EH for decomposable curvilinear motions in the dynamics, the claim does seem to be only that the initial velocity cannot be determined: see Stein 1977, 41-2.)

But there are also 'strictly' curvilinear motions, or curvilinear motions as 'commonly conceived' – the motions that intrinsically solid bodies could have. Leibniz says that they would also be distinguishable from inertial motions and from one another: for instance, one could determine whether a body was rotating. He further claims that if they existed, they would therefore violate EH. From a modern point of view, which draws no deep distinction between decomposable and non-decomposable motions, EH seems to be contradictory: it both does and does not rule out the observability of rotation (and curvilinear motion generally). But this apparent inconsistency can be resolved if we bear in mind that Leibniz took decomposable and strictly curvilinear motions to be qualitatively distinct – that he took strict rotation (rotation 'as commonly conceived') to be *sui generis*. I propose that if a body has a motion decomposable into linear motions, then EH means that it is impossible to determine its velocity, while if instead a body is in strict rotation, then EH means that it is impossible to determine its angular velocity – more generally, its acceleration.

On the basis of his division of motions into distinct kinds, and consequently of EH into separate principles, we can understand why Leibniz's response to the *Scholium* is not to argue that rotations are *undetectable* according to his mechanics – of course they aren't – but to *deny the existence* of strict rotation. If there were a body strictly rotating about its centre, then a point on its surface would be tracing out a pure circle, and contrary to EH it would be measurable. However, there are no such bodies to lead to violations of EH. Actual rotations – i.e., particles following a suitable series of linear motions, not strictly circular motions – are measurable, since they involve collisions, but that is not contrary to EH, which for linear motions only holds that instantaneous rest is indistinguishable from instantaneous motion.[29]

An analogy may help; in ruler and compass geometry, any circle can be approximated with

[28] Given Leibniz's plenum and the possible fractal nature of motion, it may be impossible to state this claim in terms of modern analysis – there may be neither any moment *before* collision, nor well-defined derivatives at any instant.

[29] Earman (1989, 71-2) seems to suggest something along these lines, though it is unclear to me what problem he thinks Leibniz would have with rigid motion. Stein (1977, 33) also makes use of Leibniz's denial of rigidity in his interpretation of EH. His proposal is that EH amounts to the claim that you can't distinguish distinct dynamical states of a system in two relationally identical configurations – because the lack of rigidity means that no such system can be in two relationally identical states when it is in two dynamically distinct states. '. . . the rotating earth has – necessarily – *a different [relational configuration]*, from that of a nonrotating earth.' I find it unlikely that Leibniz had such a view in mind, for it does not preclude the possibility of determining motions – the Earth rotates iff it is oblate – which is exactly what EH is supposed to rule out according to Stein's reading.

arbitrary accuracy by a series of straight lines, and indeed, in the sense that there is a straight line tangent at every point, a circle is nothing but an (uncountable) collection of straight-line 'parts'. But one might still say that a circle is distinct from any linear approximation, and indeed, if (like Leibniz) one did not understand infinitesimal differences in terms of limits, that an infinitesimal linear approximation was distinct. Then just as one might say that what can be constructed with a pair of compasses is qualitatively distinct from what can be constructed with a ruler, one could say that linear and circular motion are distinct species, to be treated separately under EH. While EH for the former is satisfied by the laws, the latter is unobservable because nothing is rigid – there are no true compasses, if you like. That is, it is the physical constitution of the world, not the form of the laws, that vindicates EH for curvilinear motion.[30]

We should ask whether we can find in Leibniz's views on geometrical curves and the mathematical representation of motion a distinction between 'pure' curvilinear motion and 'actual' curvilinear motion composed of linear motions. In the first place, I don't know of any text where Leibniz draws the distinction between the specific compass and ruler constructions that I used in my analogy (even in analytic rather than geometric terms). In the second place, his views on motion and the continuum are complex and intricate, and it would take us too far afield here for a detailed investigation. However, there are a couple of telling observations that we can make.

First, Leibniz's views cannot be straight-forwardly understood in terms of functions on $\Re^n$: for instance, I have already mentioned Levey's (2003) account of Leibniz's views on motion in 1676. Thus, one cannot, without anachronism, conceive the different kinds of motion as different mappings from a real time line to curves of Euclidean space, as one is inclined to do – we won't gain an understanding of Leibniz by asking 'which point set in $\Re^3$ is strictly curvilinear and which is composed of linear parts?'. Thus there is room for the kind of distinction suggested by my analogy; if one does not conceive the circle and its linear approximations as figures in Euclidean space, they need not, *even in the limit*, be identical.

Second, while I am not aware of any analysis given by Leibniz of what I am calling 'strictly' curvilinear motions, an important motivation for the account described by Levey is Leibniz's desire to avoid the problems and 'paradoxes' that a strictly continuous motion raises – the 'labyrinth of the continuum'. Conjecturing, reasonably, that motion as 'commonly conceived' would require negotiating the continuum, it is fair to say that a distinction between the two kinds of curvilinear motion is at very least in the spirit of Leibniz's views on the foundations of mathematics. And if there are two qualitatively different kinds of motion, it is not at all implausible to think that EH applies to them separately, in the way that I propose. (Further support for my proposal comes from the natural interpretation of 'non-uniformity' in the account described by Levey in terms of continual collisions; that suggests that it is not supposed to cover the case of the strict curvilinear motions of rigid bodies – that they indeed do require a separate understanding, and thus, arguably, a separate treatment under EH.)

Further evidence that Leibniz understands EH in the way that I propose comes from the fact

[30] Here we see another reason that it is not particularly illuminating of Leibniz's views to attribute some spacetime on him. Leibniz's laws lead to Galilean symmetries and spacetime, but as we noted in footnote 14, his pronouncements on motion do not unequivocally correspond to any spacetime – in ours the symmetries and spacetime are presumably Newtonian, but in some configurations of matter something weaker. EH, which concerns an indistinguishability transformation seems as if it should determine a spacetime with suitable automorphisms is also uncooperative: if there were rigid bodies then EH would amount to Galilean relativity, and point to Galilean spacetime. But there are not, and so rigid rotations in spacetime are also symmetries according to Leibniz, suggesting something like Earman's (1989, Chapter 2) Maxwellian spacetime. Once again, not only do we have an ambiguity, but it seems wrong to capture contingent facts about the distribution of matter in terms of a spacetime geometry. I suggest that philosophers recognise here the limitations of the spacetime approach.

that he principally deals with solid rotations in his response to the *Scholium*, while Newton of course discusses both solid and fluid rotations: the rotating spheres and the spinning bucket. It is only the former that seems to worry him at all, suggesting that the curved surface of the bucket is no worry for EH, which it would not be if he held only Galilean relativity for non-solid motions – Leibniz would accept that such inertial effects allow one to discern a non-inertial motion from the phenomena. (It is true that there is another way of reading Leibniz's lack of concern regarding the bucket which is more-or-less textually consistent with the interpretation of EH as general relativity. In the case of an actual, non-rigid, body, Leibniz says that if EH appears not to hold for a curvilinear motions it is 'on account of the imperceptibility of the ambient bodies' [Stein, 1977, 42] with which the body is interacting – with which its parts are colliding. If Leibniz mistakenly thought after all that collisions did not allow one to distinguish inertial from non-inertial motions, then he might have concluded that such interactions, even if observed would not allow one to conclude that the body was truly moving. Of course, such observations would allow one to draw a kinematic distinction between different states of rotation, and one wonders how Leibniz could have thought that such states were not dynamically distinguished. Of course on my proposed reading they are.)

The major challenge to this interpretation – and equally, as we mentioned earlier, to the interpretation of EH as Galilean relativity – are Leibniz's views concerning the Copernican and Tychonic models of the solar system. In 1689 Leibniz published his account of the motions of the planets, *Tentamen de Motuum Coelestium Causis* (translated in Meli 1993, Ch 6). He proposed a quantitative version of Cartesian vortecism in which the planets are driven by: (i) a vortex in which the rate of rotation is proportional to the orbital radius (specifically by a 'harmonic vortex' in which speed is inversely proportional to orbital radius), of course such motion is to be understood in terms of linear motions and collisions with ambient bodies; and (ii) a 'magnetic attraction' (also to be analysed in terms of collisions, though Leibniz offered a number of different such mechanisms in the months following publication). (See Meli for a detailed study of this work, especially for our purpose, §7.4.)

In an essay published in the same year (written significantly during a trip to Italy) Leibniz discusses the theological ramifications of his account (which he summarises), attempting to reconcile its Copernicanism with Church doctrine. (The essay is part of a concerted effort by Leibniz to have the censorship of Copernican views lifted [see Meli, 1988], itself part of his wider desire for a reconciliation between the churches; Leibniz himself was Lutheran.) He starts the essay with a familiar-looking statement of EH (for free and colliding bodies), then continues:

> ...it follows that not even an angel could determine with mathematical rigour which [body] is at rest, and which is the centre of motion[31] for the others. And ...it is a wondrous law of nature that no eye, wherever in matter it might be placed, has a sure criterion for telling from the phenomena where there is motion, how much motion there is, and of what sort it is, *or even whether God moves everything around it, or whether he moves that very eye itself.* (AG 91)

The problem for my reading is of course that Leibniz seems to say explicitly that no rotation of any kind can be determined by mechanical experiment: the centre of rotation, Sun or Earth, can't be determined by angels and it can't be determined by human observers on Earth either. But on my reading, since the motions of the planets and vortex are decomposable into linear motions, EH should only mean that a common velocity cannot be determined, while a common rotation can.

[31]Note that while Leibniz usually mean 'motion in general' by 'motus', it can also be translated as 'rotation".

After the passage quoted, Leibniz goes on to 'summarise' his view as being that motion *per se* (he says 'in mathematical rigour') is merely *arbitrary* relative motion, RM. However, since we do ordinarily attribute *particular* motions and rest to bodies, he thinks that there must be some rules according to which our ascriptions can said to be true or false. His assumption here seems to be rather similar to Descartes' in discussing of OM, although Leibniz's account turns out to be rather different from Descartes' definition in terms of 'force' (in addition, Leibniz's analysis is prescriptive where Descartes' is descriptive). What Leibniz says is that a hypothesis about which bodies are at rest is true when it is the most 'intelligible' in a given context. This view is interesting to me because the idea that which frame is preferred depends on which hypothesis is simplest (though not the context dependency of simplicity) is similar to the view that I will develop in Chapter Six.

Specifically, to describe the motions of the Sun and stars relative to the Earth – 'spherical astronomy' – it is simplest to adopt the Tychonic conception (as is still done in celestial navigation). But to actually understand how the planets move it is far simpler and more illuminating to adopt the Copernican hypothesis. Thus, according to Leibniz, each is true in the context in which it is the most intelligible: correct ordinary ascriptions of motion are pragmatic and context relative. Then, because the relevant biblical passages refer to the motion of the Sun relative to the Earth, the appropriate context for interpreting scripture is spherical astronomy and the most intelligible hypothesis in that context is Tycho's (or Ptolemy's, he says, AG 91): thus it is (pragmatically) true in the context of the bible that the Sun orbits the Earth. And to say that the Copernican hypothesis is true is to say no more than that that is the most intelligible hypothesis for understanding the planets. The pragmatic aspect means that both hypotheses are equally true, while context dependency means that they do not contradict each other, properly understood.[32]

We will leave aside the issue of whether the Inquisition would have been satisfied with the pragmatic truth of the Bible. Our question is what exactly Leibniz says here about EH in regard to rotation (and linear acceleration). Does he think that in *no* sense but the pragmatic can rotation ever be observed? To make matters worse for my reading, in addition to the passage quoted, he finishes by saying that the principles of his vortex theory are 'the simplest and clearest for the understanding, that is the best and in our sense truest hypothesis.' (AG 94) This seems to imply that whatever we observed about the planets and the matter of the vortex, and their relative motions and collisions, we could not infer whether the system rotated about the Earth or Sun.

If Leibniz meant by the EH that – as the passage quoted suggests – no motions were mechanically distinguished, then manifestly neither his mechanics nor vortex theory satisfies the principle. We saw that the way he understands rotation is in terms of bodies being deflected inwards towards their centre of rotation by collisions with surrounding bodies. So not only are collisions a criterion for accelerations as we discussed earlier, the centre of rotation can be determined, at least roughly, by seeing towards which point collisions are predominantly directed. So on the one hand we have the

[32]In our discussion we have distinguished the unknowable – because of EH – true motion (i.e., LM) from motion ascribed on the basis of intelligibility. However, in one place Leibniz blurs the distinction between the two. In a passage deleted from the manuscript of the *Specimen of Dynamics* before publication (it explicitly criticises Newton, which likely explains its deletion) Leibniz says 'we can, *with good reason*, attribute true motion to that subject, which would result in the simplest hypothesis . . . .' (AG 125, my emphasis). This passage seems to suggest that there is reason to believe that the simplest hypothesis is not just pragmatically true – which it is by definition – but a literally true ascription of LM. Some support for this view is that EH is usually qualified by Leibniz to concern 'mathematical' knowledge, perhaps intending to leave open the possibility that a lesser kind of knowledge – perhaps even moral certainty – might be possible. On the other hand, perhaps the qualification is just intended to distinguish mathematical truth from pragmatic truth, with no suggestion that the latter is indicative of the literal truth. The idea that simplicity might be any guide to literal truth is further undermined by the fact that simplicity is context dependent for Leibniz: simplicity in which context is supposed to be the guide to truth.

strict reading of his words here, and on the other the glaring, basic confusion that they represent on that reading (basically the same one we have discussed at length). And if it was an error, it was not one in which Leibniz seems to have persisted, for six years later, in the *Specimen of Dynamics* he is quite clear that the relativity principle proposed by Galileo is false, because inertia is linear not circular: we can distinguish rotational motion from rest because'. . . the circular motion of the earth or of the boat would not remain in common with the rectilinear motion given to [a body] by the rotation of the earth or ship.' (WF 175)[33]

Certainly we are justified in considering another possibility, namely that Leibniz is being disingenuous. At worst (morally speaking), he may simply be deliberately conflating a more restricted relativity principle with the claim that rotations are unobservable. But we need not go that far, for his words can be understood in a way consistent with his other beliefs. And that is to understand 'motion' in this essay to be merely RM – motion relative to arbitrary reference bodies – and not TM at all. That is to say, Leibniz is just making the point that RM is fully relativistic in the kinematic sense: since the reference bodies are assumed to be arbitrary, there is no fact of the matter about a correct hypothesis. EH, as we have understood it so far, on the contrary, concerns the observability of TM. Then we understand Leibniz to say merely that observations cannot distinguish Copernican and Tychonic hypotheses, in the sense that they correspond merely to different choices of reference body (Sun or Earth); he is not claiming that they are dynamically indistinguishable.

Of course, elsewhere EH specifically refers to dynamical relativity and so Leibniz is either here using a deviant sense, or we should read the passage (and the rest of the essay) not as amplifying EH but as stating the distinct principle of kinematic relativity. And indeed there are two places in the quotation where Leibniz seems to signal a shift from dynamical to kinematic relativity. First, when he says that angels cannot determine 'with mathematical rigour' which body is the centre of motion, for later he says that with mathematical rigour motion is 'nothing but change of [relative] position' (AG 91). The problem with placing the shift here is that the kinematical relativity of RM follows by definition, not by a dynamical principle of EH. The second place that the shift may occur is in the next sentence, in which Leibniz does not say that the 'wondrous law' follows from EH, but states it as a logically distinct proposition, suggesting that from this point on he has just kinematic relativity in mind.

This reading is (more-or-less) compatible with the text, but it is clearly disingenuous. In the first place, when Leibniz says that one can't infer from the phenomena which body is the centre of rotation, one naturally assumes that he means *on any basis*; but what he means is *ignoring the laws of mechanics*. Second, while on reflection it becomes natural to see the 'wondrous law' as logically distinct from EH, his presentation could lure the reader into thinking that it is a logical consequence of EH, which usually means some kind of dynamical relativity.

While a principle of charity should make us wary of attributing such disingenuousness, in the context of the *Tentamen* a little dissembling is in fact not surprising. As Meli (1993, 158-9) argues, during his time in Rome, Leibniz was likely trying to revise the Tentamen into a form acceptable to the Catholic Church. In the so-called *Zweite Bearbeitung*, the law of vortical circulation – i.e., mechanism (i) above – is downplayed. So the essay we have been discussing can be read as Leibniz's attempt to reconcile his physics with Church doctrine. Unlike Descartes, Leibniz does not try to

[33]To be true he also remarks that EH requires that we consider 'everything relevant to the phenomena'; perhaps, persisting in his error, he thought fuller analysis would show that there was no criterion at all. But he doesn't explain his remark in that way. Jauernig (2003, 262-9) gives a vigourous and cogent defence of this interpretation, which of necessity requires denying that EH places a constraint on the form of the laws of mechanics. Attractive though her position may be in other ways, I just don't see how such a view can plausibly be rendered consistent with the explicit statement we saw at the start of this section, that we must find laws of motion that satisfy EH (WF 168).

make the mechanical and biblical senses agree and so contort his physics, and unlike Newton, Leibniz doesn't simply take the biblical sense to be 'unphilosophical' vulgar use. So he has to make some compromise, and his involves shifting his sense of motion from the true (LM) sense to RM in a way that is not transparent to the reader, and arguably off the real point of the dispute. Still, he had to find some way out of the difficulty and a little dissembling was perhaps the best option available. Anyway, my reading is the only way I can see to render the essay consistent with Leibniz's other writings, as we have seen.[34]

To summarise the proposal of this section: because LM is defined in terms of force/form, which, like his contemporaries, Leibniz held to be inadmissible in mechanics, he demanded that LM play no direct role in mechanical explanations. This requirement was met by imposing a principle of relativity – the indistinguishability of rest from uniform linear motion *and* the indistinguishability of rest from curvilinear motion. The first part of the principle is satisfied because his laws are Galilean relativistic (in the sense as Newton's *Corollary V*); the second part is satisfied because there are no strictly rigid bodies and hence no curvilinear motions strictly speaking. The centrifugal effects exhibited by water in spinning buckets and rotating spheres do not violate the principle because they arise only because of the non-uniform linear motions of the parts of the bodies – that the laws of collision are Galilean relativistic, but not under rotations (or accelerations generally, *modulo* Newton's *Corollary VI.*)

This discussion reveals an important lacuna in Leibniz's account of motion. He never, as far as I am aware, clearly recognises the nature of motion as it is employed in his mechanics. On the one hand, he denies that it is LM, since that appeals to force/form – Galilean relativity is the device that removes LM from mechanics. But on the other hand, according to my reading he is aware (how could he fail to be?) that the laws are not relativistic in any wider sense, but in that case motion cannot be understood in mechanics as merely arbitrary relative motion – RM. Moreover, I have argued that Leibniz would have accepted that changes in the TM of a body could be inferred from its collision with another body, and this means that he accepted that the concept of motion as employed in mechanics is thicker than RM; whether a body changes its RM or not in a collision depends entirely on the arbitrary frame chosen. (That a collision has occurred is agreed on by all frames of course; if bodies change their relative motions in one frame, then they do in all.)

I know of no place where Leibniz articulates a third sense of motion, which is suitable for his theory of motion.[35] If he had, there are two tacks he might take. First, he could look to his metaphysics, and say that while the absolute value of LM plays no role in mechanics, the absolute value of changes in LM – the true acceleration – does. That is, motion is relative motion in any frame in which the changes in speed in all collisions are equal to the corresponding changes in LM.

[34] Meli, 1988, contains a detailed analysis of Leibniz's attempts to argue that Catholicism is compatible with Copernicanism (and even that censorship could be without admitting error in the condemnation of Galileo, by claiming that the action was temporary, pending further evidence). It's important to note that Leibniz's was not motivated (at least, not primarily motivated) by concerns about personal consequences – censorship, legal proceedings or ex-communication – since he was a Protestant, living and working substantially in Protestant countries. Instead, his writings (published and unpublished) on the matter consistently state that his concerns are on the one hand that the development of science will be stifled in certain countries (Italy and Spain in particular), and on the other that the Catholic Church will leave itself open to criticism that exaggerates the meaning of the censorship. That is, he does not use the issue to attack the Church but is concerned to defend it. (I should also note that Meli, p.26, does not understand Leibniz to be disingenuous, but takes him as meaning only a principle of kinematic relativity – for RM – here.)

[35] Roberts (2003) 'reconstructs' an account, but it seems to me to be a stretch to suggest that Leibniz intended to provide the account described, even if he might have been sympathetic (had he had appreciated its need).

The problem with this proposal is that it seems to involve the undesirable spread of force/form into mechanics. Second, he could use the law of inertia to specify the frames in which relative motions are properly taken – the collision laws hold in frames in which free bodies have constant linear motions. This proposal has the virtue of being compatible with Leibniz's assertion that mechanics should be understood only in terms of relations: it is after all a relational fact whether a body collides with any others. That it is possible reminds us again that RM is only one kind of relational motion.

However, Leibniz did not have a clear grasp of the problem, and, as we shall now see, when he was pressed on Newton's arguments concerning the role of motion in mechanics, he did not appreciate the full force of the objections.

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