Philip Hugly and Charles Sayward

Null Sentences

1. Introduction

Consider the following classification of truth-valued sentences: First, *null* sentences: sentences which say nothing whatsoever about what is or is not the case—sentences which in no way assert anything about how things are. Second, *contentful* sentences: sentences which say something about what is or is not the case—sentences which assert something about how things are.

The notion of a null sentence has figured prominently in twentieth-century analytic philosophy. It plays a central role in logical positivism and in the writing of Wittgenstein. However, the notion is subject to a vexing difficulty, which can be put as follows: How can a sentence that says nothing be true? How can a sentence that says nothing be false? If it is true it must say something which is true; but if it says something which is true it says something. If it is false it must say something which is false; but if it says something which is false it says something. So how can there be any null sentences?

2. Null Sentences and Logical Positivism

It is natural to think of logical positivism as centrally including the following claim: if S is analytical, then if S is knowable, then S is knowable only a priori. This takes it that the notion of an analytic statement itself is central to positivism. In place of this we suggest that it is the notion of a null sentence which is central.

Some important theses of logical positivism were that mathematical sentences are null sentences, and that logically true sentences are null sentences, etc. Another thesis of logical positivism was that null sentences are analytic. By this two things were meant. First, that what made a sentence null was that it was analytic. Second, that what justified the assertion of a

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null sentence was an analysis of its terms or of the concepts they expressed.

We suggest that the notion of analyticity be viewed as one once put to the task either of explaining how it is that null sentences are null or of explaining one way we might come to be justified in asserting a null sentence. The notion of analyticity has both an epistemic and a purely logical aspect. Our own sense is that the notion of analyticity is a failed notion for the reason that it fails to provide an explanation either of what makes a null sentence null or of how we can come to be justified in asserting a null sentence.

Yet another thesis of logical positivism was this: That what justified the assertion of a null sentence was never due to experience. That is, it was held that experience is unable to justify any null sentence.

The thesis that experience is justificatorily irrelevant to null sentences seems to us unassailable. But it is not so obvious that there *are* such sentences or that, e.g., the sentences of arithmetic are among them. The loss of the notion of analyticity would in no way diminish the power of the claim that experience is justificatorily irrelevant to null sentences—or of the claim that some sentences are null.

3. Null Sentences and Wittgenstein

Wittgenstein held that there are null sentences—prominently including logical truths and the truths of arithmetic.

In the *Tractatus* (4.461) Wittgenstein says that tautologies and contradictions are without sense *(sinnlos),* while at 4.4611 he denies that they are nonsensical *(unsinnig).* This is also Wittgenstein's view of the theorems of mathematics in 6.2–6.22:

6.2 Mathematics is logical method.
The propositions of mathematics are equations, and therefore pseudo-propositions.

6.21 A proposition of mathematics does not express a thought.

6.211 In life it is never a mathematical proposition which we need, but we use mathematical propositions *only* in order to infer from propositions, which do not belong to mathematics, to others which equally do not belong to mathematics.
(In philosophy the question "Why do we really use that word, that proposition?" constantly leads to valuable results.)

6.22 The logic of the world, which the propositions of logic show in tautologies, mathematics shows in equations.

In his later lectures, as reported by G.E. Moore (1959, 266), he repeats the theme that the provable sentences of logic and mathematics say nothing.

Consider this passage from the *Tractatus* (4.461):

> The proposition shows what it says, the tautology and the contradiction that they say nothing.
> The tautology has no truth-conditions, for it is unconditionally true; and the contradiction is on no condition true.
> Tautology and contradiction are without sense.
> (Like the point from which two arrows go out in opposite directions.)
> (I know, e.g. nothing about the weather, when I know that it rains or does not rain.)

It is not obvious that truths of logic and mathematics cannot serve to inform. But our problem is not with this. Our problem is with Wittgenstein's claim that a tautology is true.

Wittgenstein says a tautology is unconditionally true. Well, whatever is unconditionally true is true. What is true or false? It is currently common to regard truth and falsity as traits of sentences relativized to particular languages. But surely it is what a sentence says which is true or false, and a sentence is true or false only insofar as what it says is true or false. So if a sentence says nothing, it is not true or false. Or so it seems.

We are struggling with these three thoughts Wittgenstein has in talking about logic and mathematics. He says of the theorems of logic and mathematics that they are true, that they are rules, that they say nothing. We are asking: If they say nothing, how can they be true? If they are rules, how can they be true? Do these thoughts cohere?

It might be thought that one could understand the truth of a theorem of logic or mathematics to consist in a certain rule of inference being valid. Wittgenstein suggests that we view the equations as rules of inference. For example, we can view the tautology

$$(p \,\&\, q) \supset p$$

as a rule of inference, namely:

$$\frac{p \,\&\, q}{p}$$

written out in the form of a sentence. To understand what it is for this sentence to be a truth of logic is to grasp that in virtue of which the inference is valid.

On Wittgenstein's view the equation

$$5+7=12$$

is a sentential formulation of this rule of inference:

$$\frac{\text{There are } 5+7 \text{ As}}{\text{There are } 12 \text{ As}}$$

To understand what it is for the equation to be a truth of arithmetic is to grasp that in virtue of which the inference is valid.

We have doubts about this explanation. Consider a truth table analysis of a tautology, e.g.,

Snow is white	If snow is white then snow is white
true	true
false	true

Presumably, 'true' means the same in all three occurrences. Well, then, 'If snow is white then snow is white' is true in the same sense that 'Snow is white' is true. Both sentences are true in virtue of the way the world is. What fact makes it the case if snow is white then snow is white? Why, of course, the fact that snow is white. That is clear from looking at the table.

This makes it clear that one cannot go from

That p makes it the case that S is true

to

S says that p

'If snow is white then snow is white', if it says anything at all, does not say that snow is white. But that snow is white is what makes it the case that 'If snow is white then snow is white' is true.

More generally, from the fact that a sentence is true in virtue of the way the world is, it does not follow that it says something about the world or that it says anything at all.

There is a problem with the idea that there are null sentences, and that tautologies are among them. Tautologies are paradigm examples of true sentences. But if a sentence is true it must say something which is true, in which case it must say something, in which case it is not null. Or so it seems.

4. Null Sentences and Disquotational Truth

The theme that the provable sentences of mathematics and logic say nothing was a theme of Wittgenstein early on. And it stayed with him throughout. But how important for him was it that they are true? What turns on their being true?

Something Wittgenstein says suggests that nothing turns on this. In *Remarks on the Foundations of Mathematics* (1956, 50e) he says: "For what does a proposition's *'being true'* mean? *'p' is true = p.* (That is the answer.)" Setting aside problems having to do with use and mention, this is the so-called disquotational theory of truth.

In *Word and Object* Quine (1960, 24) expresses this view as follows: "To say that the statement 'Brutus killed Caesar' is true... is in effect simply to say that Brutus killed Caesar." And in *Philosophy of Logic* he writes:

> The utility of the truth predicate is the cancellation of linguistic reference... this cancellatory force of the truth predicate is explicit in Tarski's paradigm
>
> 'Snow is white' is true iff snow is white
>
> Quotation makes all the difference between talking about words and talking about snow. By calling the sentence true we call snow white. The truth predicate is a device of disquotation. (Quine 1970, 12)

On this conception, attaching 'is true' to the quotation of a sentence has the same effect as would be obtained by simply erasing the quotation marks. This view is thus committed to the thesis that e.g., 'Snow is white' and "'Snow is white' is true" say the very same thing.

In Appendix I of *Remarks on the Foundations of Mathematics* (1956, 49e) Wittgenstein emphasizes that from the fact that a sentence is grammatically declarative it does not follow that it is assertoric, that is, that it says anything about how things are.

A simple argument shows this: Questions and commands are not assertoric. Questions and commands can take declarative form. Thus, declarative form does not entail being assertoric.

Now if 'is true' is disquotational then to say, for example,

'The King moves one space at a time' is true

is simply to say

The King moves one space at a time

So if the second sentence is a rule, as it appears to be, so is the first sentence. Similarly, to say

'Snow is white or it is not the case that snow is white' is true

is simply to say

Snow is white or it is not the case that snow is white

which leaves it open whether the last displayed sentence has the force of a rule or an assertion.

So, if the disquotational theory of truth is true, defenders of null sentences, such as Wittgenstein, have a way out of the difficulty posed by such questions as these: If the theorems of logic and mathematics say nothing, how can they be true? If they are rules, how can they be true? An answer gleaned from the disquotational theory of truth is that to say that S is true does not entail that S says something. To say S is true is simply to say what S says. If S says nothing then

S is true

also says nothing.

It is sometimes suggested that

Snow is white

and

'Snow is white' is true

do not say the same thing because they are about different things. The first is about snow, and the second is about the sentence 'Snow is white'. But there are ever so many pairs of sentences that say the same but are about different things. Do not

The number of whales which are not mammals = 0

and

All whales are mammals

fall into this category? Or take

Bill's pencil is sharp

and

Bill has exactly one pencil and it is sharp

These say the same thing; but the first is about Bill's pencil, and the second is about Bill. There are a lot of counterexamples to the principle that saying the same thing entails being about the same thing.

If it is the case that it is one and the same thing to say that snow is white and to say that 'Snow is white' is true, then 'Snow is white' and "'Snow is white' is true" must have the same truth-conditions in English. It is necessary that if sentences have the same truth-conditions in English that they form a biconditional which is necessarily true in English. For if a biconditional *can* fail to be true in English, then it is possible for the truth-conditions in English of just one of its sentences to be satisfied, in which case they are different truth-conditions.

Since snow is white, and coal is black, and people speak English, these things are compossible. Further, it is clearly possible for those conditions to continue to hold even if, as is also possible, English were to become the only language anyone speaks or understands and to become different from actual English just in having 'coal' denote snow and not coal and 'snow' denote coal and not snow.

Let w be a possible world representing these possibilities. Note first that the sentence

Snow is white

is true in English at w even though 'snow' denotes coal in w and coal is black in w. This is because snow is white in w, and what gets assessed *at* w is not the sentence 'Snow is white' as used *in* w, but rather that sentence as we English speakers actually use it.

But is the sentence

'Snow is white' is true

also true in English at w?

The truth-conditions which a sentence has in a world are just those which it has in the languages of that world of which it is a sentence. So, since English is the only language spoken in w, 'Snow is white' has in w only the truth-conditions it has in the English spoken in w. Thus, 'Snow is white' is true *in* (not *at*) w only if coal is white in w. Since coal is black and not white in w, the sentence 'Snow is white' is not true in w. Thus

'Snow is white' is true

is not true in English at w.

The conclusion is that the two sentences 'Snow is white' and "'Snow is white' is true" have different truth-conditions in English; hence, the disquotational theory of truth is false.[1]

5. *Objectivity and Truth*

Objectivity is an epistemic fact if there is agreement on what does and does not constitute justification for a certain range of propositions. In this epistemic sense, arithmetic, for example, is indisputably objective, since (on the whole) mathematicians agree on whether a certain set of inferences constitutes a proof of some arithmetical proposition.

But it is also possible to speak of the objectivity of a proposition in what appears to be a quite different sense. Here the idea is that a proposition is

[1] For more details see Hugly and Sayward (1993).

objective just in case it says that things are some way and owes its truth or falsity to how things actually are.

Some hold that there are truths in addition to *objective* truths. If this is so, then it is quite possible for a sentence to say nothing about how things are and to be true (or false). The idea now is that within the set of truth-valued statements there is a proper subset which consists of non-objective statements. Null sentences would be included in this subset. How plausible is this idea that there are non-objective truth-valued statements in addition to objective truth-valued statements? We shall consider two sources of this idea.

One Source. One source of this idea is what might be called the provability theory of mathematical truth.

Against the thought that there is non-objective truth is the thought that truth is the same sort of thing in each area in which we speak of truth (mathematics, logic, physics, politics, morality...), that knowledge is the same sort of thing in each area in which we speak of knowledge (mathematics, logic, physics, politics, morality...), and so forth. Certainly, many philosophers find it natural to think of the sentences of mathematics as truth-valued. Classically understood, for any definite (i.e., non-ambiguous, non-vague, etc.) sentence p and name *p* of p, the sentence

p is true iff p

itself is a truth. The sentences of mathematics are paradigms of definite sentences. Suppose then that some mathematical sentence p is neither true nor false. Then the above biconditional links sentences respectively false and not false. In that case, the biconditional is not true. So, if, for the domain of mathematical sentences, some are neither true nor false, mathematical sentences are not subject to the classical notion of truth (=objective truth). In this way application of the classical notion of truth to the sentences of mathematics presupposes that these sentences are one and all truth-valued.

If there are no such things as numbers, the theorems of mathematics are not true in virtue of how things are with numbers. So in what does their truth consist? The provability theory says that their truth consists in their provability. There is nothing else for their truth to consist in if they are not true in virtue of the way things are with numbers.

Outside of mathematics a proof establishes truth. But truth does not consist in proof. To prove that wild elephants still exist you have to search out one that the poachers or hunters or park managers have not yet slaughtered. That would establish the truth of the assertion. But the truth of the assertion does not consist in its having a proof; it is true in virtue of the way the world is with wild elephants.

Part of the content of the provability theory is that provability within mathematics is fundamentally different from provability outside mathematics. Outside of mathematics what establishes a sentence is not what makes it true. But within mathematics being true consists in having a proof. Outside mathematics proof establishes something beyond itself: truth. Within mathematics proof establishes nothing beyond itself.

Here is another way of putting the thought. A derivation from mathematical axioms cannot give an incorrect result if correctly carried out. If a verification method *can't* give an incorrect result if correctly carried out, then that result is not something the truth of which is due to the way things are. Instead, its truth *consists* in being the result of a correctly carried out verification procedure.

Here there is no "gap" between verification and truth. That is, for the sentences of pure mathematics truth *is* being the result of a correctly carried out proof. It thus becomes obvious that there is no need for a world of numbers to make for truth in mathematics. Against this it may be urged that the very possibility of defining truth for mathematical sentences requires a world of numbers.

There are two major objections to the provability theory.

(1) The first objection is that Gödel showed that mathematical truth cannot be identified with provability. For example, Richard Jeffrey writes: "Gödel's theorem dealt a deathblow to the theory which identified mathematical truth with provability" (Jeffrey 1967, 196). This theme is echoed in one logic text after another.[2]

The result of Gödel of which Jeffrey speaks is actually pretty simple to understand. The complexities lie on the side of the proof. Let us put that to the side and just think about *what* he proved.

It comes to this: That for any effective and consistent axiomatization of a theory including at least elementary arithmetic there are sentences in the language of the theory such that neither they nor their negations are derivable from the axioms.[3]

[2] See, for example: Stoll 1961, 167; Pollock 1969, 229; Massey 1970, 129; Mates 1972, 229.

[3] The Gödel result referred to is that if arithmetic is omega-consistent (if, that is, $\neg(\forall x)A(x)$ is unprovable if each $A(n)$ is provable) then it is incomplete (there are

This is a syntactical result. The notions of truth and falsity do not enter into it at all, either by way of the content of the theorem itself or by way of its proof. In particular, that truth and falsity in mathematics go beyond proof and disproof is no part of what Gödel proves.

So we do not think this objection poses a serious problem for the provability theory.

(2) The second objection goes thus: A proof in mathematics is a derivation from axioms. So, according to the provability theory, the truth of an axiom consists in its being derivable from itself. Is it not just obvious how implausible that is? Consider one of the Peano axioms:

$$\forall x\ (0 \neq sx)$$

How do we know that is true? The answer that it is derivable from itself is not likely to satisfy anyone. And it should not satisfy anyone since every statement is derivable from itself.

And why is not one consistent set of axioms as good as any other on the account offered by the provability theory? Suppose that instead of the Peano axioms we had as our only axiom for arithmetic

$$\forall x(x=0)$$

Relative to this axiom a wholly different set of sentences is true.

In *Remarks on the Foundations of Mathematics* Wittgenstein writes: "I should like to say mathematics is a *motley* of techniques of proof"(1956, 84; his emphasis). We are sure Wittgenstein would have denied that the motley of techniques of proof all reduce to derivations from axioms. But he gives no other account. Lacking such an account, it is impossible to say what the provability theory comes to. It is insufficiently clear to be accepted.

So we think this second objection does pose a serious obstacle to the provability theory.

A Second Source. A second source of the idea that there is non-objective truth and falsity is that fictional truth or falsity is non-objective truth or falsity.

Consider the sentence

Pegasus was a winged horse.

sentences such that neither they nor their negations are provable). Rosser extended this: if arithmetic is consistent (if, that is, not every sentence is provable) then it is incomplete.

Isn't *it* a true sentence?

There certainly are contexts in which we would regard a denial of that sentence as an error, and its affirmation as correct. And we pretty well know which these contexts are—the ones in which we are retelling or talking about the ancient Greek myths.

In these contexts the correctness of what is said is tied to how the myths go, and so the sentence 'Pegasus was a winged horse' makes for a correct utterance, for it gets the ancient *myths* right. But then what it possesses is only rightness relative to those myths—not truth. That rightness, not truth, is what the generalization 'There were once winged horses' inherits.

More generally, lots of sentences which we perfectly well recognize to fall short of truth yet are, as we sometimes put it, "true in fiction." The sentence 'Pegasus was a horse with wings' is such a sentence. Its truth in fiction requires not that there once were horses with wings but only that the ancient Greek Pegasus stories are ones that speak of horses with wings, and call one of them Pegasus.

To this it might be replied that what is true *in* fiction is a sentence as uttered in the course of story-telling, and that we do not require for the truth of 'Pegasus was a winged horse' that *this* sentence or any other sentence of which it is a translation actually occur in any telling of any Pegasus story. So *it,* since it is not a sentence used *in* fiction, cannot be *true* in fiction. Still, it is true. Further, it is not a sentence that says that such and such a sentence belongs to some story, for it is not a sentence which says anything *about* any stories. So it also is not true *of* fiction. Still, it is true. So surely the right thing to say of it is just that it is *true*!

But it is an error to suppose that a sentence is true in fiction only if it occurs in the telling of some fiction. A sentence might also be true in fiction if it were entailed by sentences actually so used. If in the telling of a Greek myth we use the sentence 'And then that wonderful horse Pegasus leaped from the hillside and winged his way to Ithaca', then the sentences 'Pegasus winged his way to Ithaca' and 'A horse winged its way to Ithaca' *also* are true in that fiction even if *they* are not used in the story telling. Or, to move to an example of a quite different type, consider a play in which the characters say and do certain things. That also is something in virtue of which a sentence can be true and thus be true in fiction. For example, it is true in *Hamlet* that Hamlet loved his mother. And this holds independently of whether or not 'Mom, I love you' or any other sentence to the same effect is a line in the mouth of Hamlet in the play.

The sentence 'Hamlet loved his mother' could be used to say something about some actual fellow named 'Hamlet'. So used, it might well be true. But in certain contexts—the ones in which the play figures as a subject of discourse—the above sentence stands as one fit for correct utterance independently of whether there ever was any such person as the Hamlet of that play, or of how things stand with anyone actually named 'Hamlet'. Here the play's the thing. Is the sentence then *about* the play? Well, in a way it isn't. For the sentence contains no term designating the play. But also, in a way it is, for we would say that someone who used that sentence in a context concerned with that play, had made an indirect reference to it. But however we decide about 'about', *this* is clear: in the sorts of cases here at issue it would be by inquiring into the *play* that we would determine whether or not the claim that Hamlet loved his mother was true. It is relative to the play that our sentence has such truth as it possesses in kinds of contexts with which we are here concerned.

Suppose that in a story the author writes

Jones hated someone

Suppose also nothing in the story entails any instance of this generalization, i.e., nothing in the story entails

Jones hated A

for any singular term A. How can 'Jones hated someone' be true under these imagined circumstances? It is true in the story because the author wrote it down as part of the story, but it is not true since nothing satisfies the schema 'Jones hated A'.

Despite these remarks, there may remain a sense that the cases at hand do in a way really involve *truth*. And there is something to that. Consider for a moment not the sentence 'Hamlet loved his mother' but 'In *Hamlet* Hamlet loves his mother'. This seems to be straightforwardly true. (If you don't find this interpretation of *Hamlet* plausible, switch examples, e.g., to 'In the ancient myths Pegasus was a winged horse'.) And so it is. The relevant semantic operation is that of applying a connective 'In *Hamlet*' to a sentence, thereby yielding another sentence which is true or not depending on how the play goes, independently of how the play is said to go in such stories as there may be in which it figures. By application of the connective we construct a sentence explicitly about the play.

We could take the occurrences of 'Hamlet loved his mother' in contexts concerning the play as elliptical for 'In *Hamlet* Hamlet loves his mother'.

Were we to do so, the generalization of 'Hamlet loved his mother', namely 'Someone loved his mother', would be elliptical (in context) for 'In *Hamlet* someone loved his mother'—another sentence explicitly about the play. And so it could be that the truth felt in the sentence 'Pegasus was a winged horse' (in context) is the truth of 'In the ancient myths Pegasus was a winged horse', and the truth felt in the generalization 'There once were winged horses' is the truth of 'In the ancient myths there were winged horses'.

Let us sum up our reply to the thought that truth in fiction is an example of non-objective truth. There are contexts in which we would regard the denial of 'Pegasus was a winged horse' as an error and its affirmation as correct. But this correctness is not truth. 'Pegasus was a winged horse' is not true in virtue of how the world is in regard to Pegasus. The world is not in any way with regard to Pegasus since 'Pegasus' refers to nothing in the world. The correctness of 'Pegasus was a winged horse' is tied to how certain myths go. It is true in fiction or true in myth. Truth in fiction is not truth. We need to distinguish 'In the ancient myths Pegasus was a winged horse' and 'Pegasus was a winged horse'. The former sentence is true, and the urge to call the latter sentence true might well arise from confusing the two sentences.

6. Final Remarks

Wittgenstein was very clear that tautologies and contradictions are not nonsense in the way that "Twas brillig an the slithy toves did gyre and gimble in the wabe" is nonsense. His thought was that they are without sense *(sinnlos);* he denies that they are nonsensical *(unsinnig).* What does it mean to say they are without sense? It means that they say nothing about how things are. But he also says they are true and false, respectively. The thoughts do not cohere. For to say a sentence is true is to say that it says things are a certain way and that things are that way; and to say a sentence is false is to say that it says things are a certain way and that things are not that way.

University of Nebraska —Lincoln