Vagueness, Truth and Varzi

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Is 'vague' vague? Is the meaning of 'true' vague? Is higher-order vagueness unavoidable? Is it possible to say precisely what it is to say something precisely? These questions, deeply interrelated and of fundamental importance to logic and semantics, have been addressed recently by Achille Varzi in articles focused on an ingenius attempt by Roy Sorensen ("An Argument for the Vagueness of 'Vague'") to demonstrate that 'vague' is vague.

I.

Defining 'n-small' to mean "small or less than n," Sorensen constructs the following sorites:

> (1) '1-small' is vague.
> If 'n-small' is vague, then 'n+1-small' is vague.
> Therefore, '10^{10}-small' is vague.

Since the plausibility of a sorites argument turns on the existence of borderline cases, he infers the vagueness of 'vague' from (1) on the basis of the following major premise

> ... predicates which possess borderline cases are vague predicates.

Sorensen's result has been criticized by Robert Deas ("Sorensen's Sorites"), who disputes that premise. He presents examples that suggest that

> (A) Borderline cases can be the result of vagueness in the subject term of a statement.

According to Deas, this is exactly what happens in Sorensen's sorites. In virtue of the truth of the biconditional

> (2) 'n-small' is vague iff n is small

(1) is equivalent to a standard sorites argument

> (3) 1 is small.
> If n is small, then n+1 is small.
> Therefore, 10^{10} is small.

Consequently, Deas concludes

> (B) In Sorensen's sorites the vagueness arises from the term 'small', implicit in 'n-small', and not from the predicate 'vague'.

In "Higher-Order Vagueness and the Vagueness of 'Vague',"Varzi criticizes this argument. The problem is with (2). Varzi points out that when ''n-small' is vague' is true, 'n is small' is indefinite; thus Deas' biconditional fails to be true (at least for any interpretation of biconditionality that plausibly accommodates indefiniteness). He concludes that, in the absence of support for (B), Sorensen's argument "is perfectly sound."

II.

My "Vagueness and 'Vague': A Reply to Varzi" was an attempt to resurrect Deas' position. Varzi had overlooked the importance of (A), so I adduced additional considerations in its defense. Obviously, if (A) is correct, then Sorensen's argument is not sound, whatever it is.

In place of (2), I gave an alternative justification for (B), based on a principle inferred from remarks of Kit Fine:

> The indefiniteness of a statement is due entirely to a particular term iff precisifying that term renders the statement definite.

But precisifying 'n-small' to mean "less than n or less than 100" is sufficient to render the statement

> (4) 'n-small' is vague

definitely false. Thus, I concluded, the plausibility of Sorensen's sorites is due, as Deas claimed, not to the vagueness of 'vague', but to the vagueness of 'n-small'.

In recent comments ("The Vagueness of 'Vague': Rejoinder to Hull"), Varzi concedes (A) – that vagueness in a statement can be due to its subject – though he has problems with some of the considerations adduced on its behalf (see below). But he rejects my defense of (B). He claims that, as with Deas, I have fallen victim to a use-mention confusion. In (4), the subject is not 'n-small' but ''n-small'', i.e. the name of 'n-small'; 'n-small' is not used in (4), but only mentioned. Hence, strictly speaking, the term 'n-small' does not occur in (4); consequently, Fine's principle is irrelevant.

Varzi argues, on the contrary, that it is "perfectly precise" what the subject of (4), viz. ''n-small'', refers to: it refers to the term 'n-small'. So any vagueness in (4) would have to be due to the predicate 'vague'. Varzi concludes that Sorensen's result is unimpeached: "At least so far as that argument is concerned, I say: 'vague' is vague."

III.

Varzi's criticism of the use of Fine's principle is well taken. Although the string 'n-small' does occur in (4), this is simply due to the convention of constructing the name of a term by putting the string associated with it inside quotes. But other ways of naming terms are possible, though less convenient: for example, we could simply list all terms in the English language alphabetically and name them 'term-1', 'term-2', etc. So the fact that the string 'n-small' occurs in (4) is accidental.

However, his claim that the name "n-small" is perfectly precise, because it is obvious what it refers to, is not as clear-cut as it may seem. Indeed, that name precisely identifies a particular string, viz. 'n-small'. But terms are more than just strings: they are strings associated with particular *meanings*. And whether "n-small" precisely identifies a particular meaning is not so obvious. On the one hand, one could argue that the string 'n-small' correlates with one and only one vague meaning, somewhat as a pointer can pick out a particular indeterminately-bounded-but-distinct blotch on the wall. On the other hand, one could argue that the string 'n-small' is associated with a whole swarm of different precise meanings, none of which it picks out in particular. In this latter scenario, one would have to say that "n-small" only *vaguely* identifies the term 'n-small' because it only vaguely identifies its meaning. Consequently, the purported precision of the subject of (4) would evaporate.

It is not inconceivable, therefore, that the truth status of a statement, which does not use but only mentions a term, may nonetheless be affected by that term's vagueness. Indeed, we would expect this when the statement is *about* its vagueness, as in (4). As both Sorensen and Varzi make clear [quotes?], it is in fact the higher-order vagueness of 'small' (via 'n-small') that results in the indefiniteness of (4). For those values of n for which 'small' is indefinite, it is clear that (4) is true. But for those values of n for which it is indefinite whether 'small' is indefinite – borderline borderline cases – the application of 'vague' to "n-small" is indefinite.

Hence, if we accept the existence of higher-order vagueness, it is the higher-order vagueness of 'small' that results in the plausibility of Sorensen's sorites. Consequently, there are no grounds for supposing that 'vague' contributes anything to the indefiniteness of (4). (Of course, if we reject higher-order vagueness, there will be a particular value of n at which 'n-small' ceases to be vague and begins to be false, hence the vagueness of 'n-small' would provide no assurance of the vagueness of 'n+1-small'.)

IV.

But Varzi has another argument. He claims that, just because the plausibility of Sorensen's sorites turns out to be parasitic on the sorites for 'small' – that is to say, on the vagueness of 'small' – it does not follow that 'vague' is not vague. In fact, he claims this shows that it is *because* 'small' is vague that 'vague' is vague. He introduces the following sorites for consideration:

(5) '1 is small' is true.
If 'n is small' is true, then 'n+1 is small' is true.
Therefore, '10^{10} is small' is true.

Given the Tarskian schema

'p' is true iff p

it appears that (5) is in some sense equivalent to (3), the sorites for 'small'. But just as clearly it is different: it is an argument about 'true', not about 'small'. On the assumption that (5) is no less plausible than (3), Varzi concludes that his sorites shows that 'true' is vague, and vague *as a consequence* of the vagueness of 'small'; and that the same may be assumed with respect to 'vague' and 'small' in Sorensen's sorites:

> it is (among other reasons) *because* 'small' is vague that 'true' is vague, and it is (among other reasons) *because* 'small' is vague – in fact higher-order vague – that 'vague' is vague. [Varzi's italics]

However, this simply begs the question with respect to (A), which Varzi elsewhere concedes: the fact that a term appears as the predicate in a plausible sorites does not entail that it is vague. Indeed, (5) is given by Deas as an example in which the inference that 'true' is vague is *prima facie* questionable.

And it *is* questionable. Note that the Tarskian schema falters as soon as indefiniteness is admitted as an alternative to truth and falsehood:

for when 'p' is indefinite, ''p' is true' will be false. So (5) is not really equivalent to (3): Varzi's biconditional fares no better than Deas'. In fact, as with Sorensen's sorites, it is the higher-order vagueness of 'small' that accounts for the plausibility of (5). Suppose that we knew the value of n at which 'n is small' becomes indefinite; then we would know when ''n is small' is true' becomes false. Consequently, (5) would cease to be soritical: there would be no indefiniteness forcing us to swallow the major premise. On the other hand, if 'small' is higher-order vague we will not able to say when 'n is small' ceases to be true: i.e., there will be borderline borderline cases of smallness.

So for both Sorensen's sorites and Varzi's sorites, it is the supposition of higher-order vagueness in 'small' that accounts for their plausibility. Hence, it is gratuitous to suppose that 'vague' and 'true' are also vague, even more so to suppose that they are vague *because* 'small' is vague. 'Small' has already accounted for the vagueness: there is no need to postulate additional vagueness in 'vague' or 'true'. The idea that those terms somehow must also be indefinite seems a reflexive holdover of the exploded notion that it is the predicate that accounts for the vagueness in statements.

V.

Indeed, though Varzi explicitly concedes (A), it appears as if he is not entirely reconciled with it. Whatever the reason, his criticisms of some of the proffered counterexamples to Sorensen's major premise are unconvincing.

In response to the argument that the statements

> 1000 is a large number.
> A large number is 1000.

are equally vague, regardless of whether 'large' shows up in the subject or the predicate, Varzi rejoins

> I do not think that 'A large number is 1000' has the logical form of a subject-predicate statement in which 'large' occurs in the subject.

But surely 'A large number is 1000' has the form of an *ordinary English* statement in which 'large' occurs in the subject. Ostensibly, then, Varzi is suggesting some kind of distinction between English grammar and "true" logical grammar. However, isn't the importance of vagueness based on the idea that logic should accommodate ordinary speech, not the other way round? For example, Hyde in "Sorites Paradox" [Stanford online article] claims: "If logic is to have teeth it must be applicable to natural language as it stands." If one is allowed to be dogmatic about logical form, why not simply be dogmatic about definiteness? Consequently, sans a certain suspect logical elitism, the above example shows that the term responsible for vagueness in a statement can occur in the subject as well as the predicate.

A second example in support of (A) is

(6) Approximately 0 is less than 1000.
If approximately n is less than 1000, then so is
approximately n+1.
Therefore, approximately 10000 is less than 1000.

Since the predicate in (6) is paradigmatically precise, it follows that any plausibility in this sorites arises from vagueness in the subject term.

Varzi's response:

> In a Sorensen-type sorites, for a given predicate term, the subject terms occurring in the premises and in the conclusion must be precise.... We have a sorites, but not a Sorensen-type sorites for 'less than 1000'.

But again the question has been begged: whether Sorensen's sorites has precise subject terms, (B), is exactly the issue in contention. The whole point of (6), of course, is that it is a sorites that *doesn't* have precise subject terms. The supposition that Sorensen's sorites differs from this in some kind of principled way is unwarrented; i.e., it is disputable (indeed, under dispute) whether Sorensen's sorites itself is a "Sorensen-type sorites." We have already discovered good reasons for believing that the subject terms in Sorensen's sorites and Varzi's sorites are not only not precise, but are responsible, given higher-order vagueness, for the indefiniteness that makes them plausible.

VI.

The good news to come out of all this is that the danger posed to logic and semantics by vagueness may have been overstated. Sorensen concludes:

> it appears that the vagueness of 'vague' ensures that the worrisome features of vague predicates will also be features of our attempts to describe those predicates. Thus dissatisfaction with vagueness seems to commit us to dissatisfaction with our attempts to express that dissatisfaction.

But we have found, on the contrary, that even with higher-order vagueness, Sorensen's sorites has given us no reason to believe that 'vague' is vague. Nor has Varzi's sorites provided any grounds for fearing that 'true' is vague. For the nonce, logic and semantics are safe from that quarter.