Ockhamism and Quantified Modal Logic

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This paper outlines a formal account of tensed sentences that is consistent with Ockhamism, a view according to which future contingents are either true or false. The account outlined substantively differs from the attempts that have been made so far to provide a formal apparatus for such a view in terms of some expressly modified version of branching time semantics. The system on which it is based is the simplest quantified modal logic.

1 Preliminary clarifications

Let us start with some preliminary clarifications about the term 'Ockhamism'. This term will be used to designate a view according to which future contingents are either true or false. Consider the following sentence:

(1) It will snow

On this view, (1) is either true or false as uttered now. Whether it is true or false depends on whether it will actually snow or not. That is, (1) is true if it will actually snow, false otherwise; hence, it is either true or false because either it will actually snow or it will not. In this respect, (1) is exactly like the following sentences:

(2) It is snowing
(3) It snowed

(2) is either true or false as uttered now, because either it is actually snowing or it is not. Similarly, (3) is either true or false as uttered now, because either it did actually snow or it did not. The truth-value of (1), like the truth-value of (2) and (3), depends on what happens in one among the many possible courses of events, that is, the actual course of events.

More generally, the view is centered on the thought that a future contingent uttered at a time t has a truth-value which depends on what happens after t in the actual course of events. That is,

(O) A future contingent (uttered at t) is true if and only if it is true in the actual course of events.

The view is called 'Ockhamism' because this thought can be ascribed to William of Ockham. According to Ockham, the truth-value of every future contingent is known to God, in that it depends on what happens in the "true" future, which is one among the many possible futures. The "true" future is nothing but the actual future, namely, the future part of the actual course of events[1].

An important feature of Ockhamism so understood is that it leaves room for a distinction between *truth* and *settled truth*, if the latter is defined in the usual way as truth in all possible courses of events. (O) entails that a future contingent uttered at t is true if it is true in the actual course of events. But the actual course of events is simply one among the many courses of events that are possible at t, so the truth of the future contingent is consistent with its falsity in some of them. This means that truth and settled truth are not the same thing: settled truth entails truth, but not the other way round. For example, (1) as uttered now may be true without being settledly true, if it will snow in the actual course of events but not in some other possible course of events[2].

The thesis that future contingents are either true or false is controversial. According to an influential line of reasoning that goes back to Aristotle, this thesis must be rejected. For the supposition that future contingents are true or false leads to the unacceptable conclusion that the future is settled. That is, if (1) is true now then it is settled that it will snow, and if it is false now then it is settled that it will not snow. The distinction between truth and settled truth provides a coherent way to reject this line of reasoning. If truth does not amount to settled truth, there is no reason to assume that if (1) is true now then it is settled that it will snow, or that if it is false now then it is settled that it will not snow[3].

Certainly, this leaves open the question of whether Ockhamism is ten-

[1] Øhrstrøm and Hasle [?] outlines Ockham's doctrine and its historical context, pp. 6-10. Although the view considered is reminiscent of Ockham's doctrine of divine foreknowledge, this does not make it the only view that deserves to be called Ockhamism. The term 'Ockhamism' may be used in different ways, and this paper is not intended to question the legitimacy of any of them.

[2] The expression 'determinate truth' is often used as synonymous of 'settled truth', so the distinction might equally be phrased in terms of that expression.

[3] The origin of the line of reasoning considered is Aristotle's discussion of future contingents in *De interpretatione* 9. Iacona [?] provides a more thorough explanation of the divergence between Ockhamism and the Aristotelian tradition, pp. 31-37.

able from a metaphysical point of view. Some philosophers are inclined to think that the very notion of the actual future is at odds with the hypothesis that many futures are equally possible. But that question will not be addressed here. The scope of this paper is limited to the logical implications of Ockhamism, in that its main concern is the issue of how a suitable formal semantics can be defined if (O) is accepted. So the paper takes for granted that it is at least consistent to think that future contingents are true or false in virtue of what happens in the actual future[4].

2 Three assumptions

The formal account of tensed sentences that will be outlined rests on three main assumptions. The first concerns the formal representation of future contingency. Ockhamism takes for granted that there is a plurality of possible courses of events, and that the future contingency of sentences such as (1) amounts to their being true in some but not in all of them. In what follows it will be assumed that this plurality of possible courses of events can formally be represented as a set of worlds in accordance with the standard semantics of modal logic.

This assumption is not trivial. There is a widespread tendency to think that a suitable semantics for a genuinely indeterministic view of future contingents is provided by branching time models, that is, models based on a set of times and a non-linear order on the set. In such a model, possible courses of events are represented as "histories", that is, maximal linearly ordered sets of times. The underlying idea is that, according to indeterminism, possible courses of events are exactly like histories, in that they overlap up to a certain point, the present, and then branch towards the future. So if Ockhamism is a genuinely indeterministic view, one may think, then it should be accommodated in a formal semantics based on branching time models[5].

However, this tendency may be resisted. First of all, it is questionable that indeterminism entails branching. Certainly, indeterminism may be understood in many ways, and presumably on some of them branching holds by definition. But there is at least one plausible understanding of indeterminism on which it does not entail branching, namely, that according to which indeterminism is the negation of the claim that the state of the universe at any time is determined by its state at earlier times. Indeterminism so understood does not entail branching, as it simply requires that two possible courses of events are in the same state at t, just as at any earlier

[4] Iacona [?] dispels some misunderstandings that may lead to think that Ockhamism is untenable. Rosenkranz [?] considers different objections to Ockhamism and shows how can they be rejected.

[5] This line of thought emerges clearly in Belnap, Perloff and Xu [?], pp. 133-141, and in MacFarlane [?], pp. 323-324.

time, but in two different states at t', neither of which is determined by their state at t. In other words, indeterminism may equally be framed in terms of the conception that Lewis calls "divergence". On that conception, possible courses of events do not overlap, even though they can have qualitatively identical temporal parts. Possible futures may be conceived as parts of possible worlds that are wholly distinct, rather than branches that depart from a common trunk[6].

Secondly, and more importantly, the issue of whether possible courses of events overlap is a metaphysical issue, so its implications on the formal semantics are far from obvious. Even if it were granted that possible courses of events do overlap, it would still be an open question whether they must formally be represented *as* overlapping, that is, whether a model in which they are represented must somehow display that they have some temporal parts in common. The convention adopted here, that a plurality of possible courses of events is represented in a model simply as a set of worlds, implies a negative answer to this question. The formal semantics that will be presented is intended to be neutral with respect to any metaphysical issue concerning overlap.

The second assumption concerns the analysis of tensed sentences. In what follows it will be assumed that the logical form of tensed sentences involves quantification over times. For example, (1) is correctly paraphrased as 'For some t such that t is later than the present time, it snows at t'. Similarly, (2) is correctly paraphrased as 'For some t such that t is the present time, it snows at t', and (3) is correctly paraphrased as 'For some time t such that t is earlier than the present time, it snows at t'. According to this analysis, which has been called the *extensional* analysis, tensed sentences can be formalized by using the quantificational apparatus of first-order logic[7].

As it is well known, the extensional analysis is not universally accepted. The classical alternative to it is the analysis in terms of operators which underlies tense logic. On the latter analysis, if a formula α stands for (2), then (1) is to be represented as $F\alpha$, where F stands for 'It will be the case that', and (3) is to be represented as $P\alpha$, where P stands for 'It was the case that'. One major argument that has been given in support of this analysis is that the hypothesis that the logical form of (2) involves a quantification over times makes (2) structurally more complex than it appears. According to Blackburn and Recanati, the problem lies in the departure from the "internal" perspective on time, that is, the perspective we have as speakers

[6] Hoefer [?] presents a conception of determinism where the state of the universe at any time is determined by its state at earlier times. Lewis [?] spells out the difference between branching and divergence, pp. 206-209. Iacona [?] provides a more articulated discussion of the claim that indeterminism entails branching, pp. 39-44.

[7] The extensional analysis, which was originally suggested for philosophical reasons, has later been shown to comply with linguistic evidence. King [?] is one of the recent works in which this analysis is defended.

situated inside the temporal flow. Blackburn and Recanati argue that the present tense is not a tense like the past or the future: it is more primitive and, in a sense, temporally neutral[8].

However, much in this line of reasoning depends on what kind of thing logical form is expected to be. If logical form is understood as a property of sentences that is determined by their truth-conditions independently of their surface grammar, it is conceivable that (2) is structurally more complex than it appears. In that case, it may be contended that no consideration concerning the apparent structure of (2) or the cognitive aspects pertaining to its use is able to undermine the hypothesis that the logical form of (2) involves quantification over times. Truth-conditions may not be detectable from surface grammar, and certainly transcend the "internal" perspective on time. On the account that will be adopted, there is an important sense in which the logical form of (2) involves something "external", namely, that in which the truth-condition of (2) as uttered at t involves reference to t. More generally, our leading idea will be that logical form is a matter of truth-conditions, so that a formula can represent a tensed sentence as it is uttered at a given time insofar as it expresses the truth-condition that the sentence has at that time.

The third assumption concerns the relation between plain truth, understood as a property of utterances, and truth in a world, which is a property defined in rigorous way for a class of formulas of a formal language. Ockhamism provides an analysis of plain truth, that is, (O). According to (O), plain truth concerns one in particular among the many courses of events that are possible, the actual course of events. Therefore, what will be assumed is that a model based on a set of worlds offers a formal characterization of plain truth to the extent that one of the worlds in the set is taken to represent that course of events. To illustrate, suppose that (1) is uttered now. The following instance of (O) provides the truth-condition of the utterance:

(4) (1) as uttered now is true if and only if it is true in the actual course of events.

The right-hand side of (4) says that (i) there is a possible course of events c such that (1) as uttered now is true in c, and (ii) c is the actual course of events. A formal semantics offers a characterization of (i) to the extent that, for some model in which a given world represents c, a formula that expresses the truth condition of (1) as uttered now is true in that world. Given such characterization, (ii) may be understood as a hypothesis about the model. That is, the model gives us an account of the truth of (1) as uttered now on the hypothesis that the world in question is the actual course of events.

[8]The analysis of tenses in terms of operators goes back to Prior [**?**]. The argument mentioned appears in Blackburn [**?**], p. 83, and Recanati [**?**], p. 70.

This understanding of the relation between plain truth and truth in a world implies that a formal semantics consistent with Ockhamism does not require that the actual course of events is represented as a distinguished object in the model. That is, (ii) need not be expressed in the semantics, because it is a hypothesis about the semantics. So the account differs from the attempts that have been made so far to provide a formal semantics for Ockhamism. For those attempts rest on the hypothesis that actuality must be represented in the semantics, that is, something must be included in the model to make sense of (O). If branching time models are adopted, this means that the definition of truth must involve reference to a distinguished history as the actual history, the "Thin Red Line". Here, instead, the only formal notion of truth that will be adopted is the familiar notion of truth in a world. This notion can be adopted as part of the Ockhamist analysis of plain truth, assuming that the remaining part concerns a fact about natural language that need not be expressed in a formal semantics, namely, the fact that the utterance of a future contingent involves reference to the actual course of events[9].

The perspective on the role of formal semantics in an analysis of plain truth that emerges from the third assumption is quite standard, so it should cause no trouble. Consider the elementary case of propositional modal logic. Anyone agrees that the notion of truth at a world that features in the semantics of propositional modal logic does not provide an account of plain truth. Plain truth involves something more than what is represented in the semantics in terms of that notion, namely, actuality. Nonetheless, it is usually taken for granted that there is no need to represent the actual world in the semantics, as long as it is granted that truth in a world in a model offers a representation of plain truth on the hypothesis that the world in question is the actual world.

3 A First-Order Modal Language

This section shows how a formal account of tensed sentences may be framed in a first-order modal language. First, the language will be defined with the semantics that characterizes the simplest quantified modal logic, or SQLM. Then, it will be explained how a minimal restriction on that semantics makes the language suitable to formalize tensed sentences.

Let L be a language whose vocabulary includes the symbols $\sim, \supset, \forall, \Box$, a set of variables $x, y, z...$, a set of constants $a, b, c...$, a set of predicates $P, Q, R...$ and the two-place predicate $=$. Variables and constants are terms.

[9]The assumption that Ockhamism requires a formal definition of truth that involves reference to a Thin Red Line is adopted in Belnap and Green [?], pp. 379-381, in Belnap, Perloff and Xu [?], pp. 160-170, in Øhrstrøm [?], and in Malpass and Wawer [?]. Iacona [?] presents various kind of definitions based on that assumption and draws attentions to some problems that arise in connection with them.

The atomic formulas of L have the form $P\tau_1, ..., \tau_n$, where P is n-place and $\tau_1, ...\tau_n$ are terms. The other formulas are defined in the customary way: if α and β are formulas, then $\sim \alpha$, $\Box\alpha$ and $\alpha \supset \beta$ are formulas; if α is a formula and x is a variable, then $\forall x\alpha$ is a formula.

Let $\mathcal{M}$ be the set of the models for L defined as follows:

Definition 1 *A model is a triple $\langle W, D, I\rangle$, where W and D are sets, I is a function such that for each constant a, $I(a) \in D$, and for each n-place predicate P, $I(P)$ is a function that assigns to each member w of W a set $I(P)_w$ of n-tuples of members of D, with $I(=)_w$ being identity on D.*

W is a set of possible worlds, D is the domain of quantification, and I is the interpretation of the constants and predicates of L.

Given a model $\mathcal{A}$ in $\mathcal{M}$, an assignment υ is a function such that, for each variable x, $\upsilon(x) \in D$. The denotation of a term τ relative to υ in $\mathcal{A}$, indicated as $[\tau]_{\mathcal{A},\upsilon}$, is defined as follows: if τ is a variable, $[\tau]_{\mathcal{A},\upsilon} = \upsilon(\tau)$; if τ is a constant, $[\tau]_{\mathcal{A},\upsilon} = I(\tau)$.

Definition 2

1 *υ satisfies $P\tau_1, ..., \tau_n$ in w if and only if $\langle[\tau_1]_{\mathcal{A},\upsilon}, ..., [\tau_n]_{\mathcal{A},\upsilon}\rangle \in I(P)_w$;*

2 *υ satisfies $\sim \alpha$ in w if and only if it does not satisfy α in w;*

3 *υ satisfies $\alpha \supset \beta$ in w if and only if it does not satisfy α or it satisfies β in w;*

4 *υ satisfies $\forall x\alpha$ in w if and only if every assignment that differs from υ at most in the denotation of x satisfies α in w;*

5 *υ satisfies $\Box\alpha$ in w if and only if, for every w', υ satisfies α in w'.*

The satisfaction conditions for $\wedge, \vee, \exists$ and $\Diamond$ are specified in the usual way, in accordance with those of $\sim, \supset, \forall$ and $\Box$.

Truth is defined relative to a world in a model:

Definition 3 *α is true in w if and only if α is satisfied by all assignments in w.*

Finally, validity is defined as follows:

Definition 4 *α is valid if and only if α is satisfied by all assignments in all worlds in all models.*

This is the semantics of SQML. The hypothesis that will now be entertained is that, in order to handle tensed sentences, a special kind of model is to be considered:

Definition 5 *A T-model is a model $\langle W, D, I \rangle$ such that I assigns to some two-place predicate $<$ a constant function that maps members of W to a linear order R on a set T such that $T \subseteq D$.*

T is to be understood as a set of times. Since $T \subseteq D$, times are but objects among other objects. This means that the terms of L can denote both times and ordinary objects. What distinguishes times from ordinary objects is that they are ordered by R: for any $x \in D$, x is a time if and only if, for some y, $\langle x, y \rangle \in R$ or $\langle y, x \rangle \in R$. Accordingly, reference to times is indicated in L by the occurrence of the symbol $<$[10].

If definition 5 is adopted, the truth-conditions of (1)-(3) as uttered now can be expressed in L. If a is a name of the present time, and P is a one-place predicate that stands for 'It snows at', then (1)-(3) become

(5) $\exists x(a < x \wedge Px)$
(6) $\exists x(x = a \wedge Px)$
(7) $\exists x(x < a \wedge Px)$

Let t be the denotation of a. (5) is true in w if and only if Px is satisfied in w by some time later than t. (6) is true in w if and only if Px is satisfied in w by t. (7) is true in w if and only if Px is satisfied in w by some time earlier than t. More generally, any tensed sentence that can be formalized in L can be treated along these lines. As in the case of (1)-(3), the formalization rests on the hypothesis that the truth-condition of a tensed sentence as uttered at a given time is expressed by a formula that involves reference to that time.

One question that might be raised about T-models is whether it is right to assume that one and the same domain contains both times and ordinary objects. For it might be contended that this mixture of times and ordinary objects generates an undesirable referential promiscuity in L. Certainly, if the symbol $<$ occurs in a formula, it constrains the reference of the terms that flank it in the formula. For example, the occurrence of $<$ in the formula $a < x \wedge Px$ guarantees that a and x denote times in any assignment that satisfies the formula. But since $<$ does not occur in every formula, it is natural to wonder whether its absence can cause troubles. For example, in the formula $x = a \wedge Px$, no symbol guarantees that a refers to a time: the denotation of a could be Socrates. So, even granting that (5) is an adequate formalization of (1), it may be asked why should we regard (6) as an adequate formalization of (2), given that it represents equally well 'Socrates is a philosopher'.

This question may be addressed in at least two ways. In the first place, it may be suggested that the worry about referential promiscuity is easily

[10] The simplest case is that in which $T = D$. In this case, all the predicates of L may be understood as expressing properties of times, and we get something very close to the treatment of tensed sentences as predicates suggested in Prior [?].

dispelled. Let τ be a term that occurs in a formula α. Clearly, if $<$ does not occur in α, no constraint on the reference of τ is imposed by the syntactic properties of α itself. But this is not quite the same thing as to say that the reference of τ in α is unconstrained. For it may be constrained in virtue of the satisfaction conditions of *other* formulas in which $<$ does occur. If in a model $\mathcal{A}$ there is at least one assignment υ that satisfies some formula in which τ flanks $<$, this suffices for τ to denote a time in $\mathcal{A}$ relative to υ. More generally, the satisfaction conditions of the formulas containing $<$ entail that, for every model $\mathcal{A}$, there is a set of constants that specifically denote times in $\mathcal{A}$, and for every assignment υ in $\mathcal{A}$, there is a set of variables that specifically denote times in $\mathcal{A}$ relative to υ. Consider the constant a that occurs in the formula $x = a \wedge Px$. If at least one formula in which $<$ is flanked by a is satisfied by some assignment in a given model, say $a < x \wedge Px$, then a denotes a time in that model, hence the formula $a = x \wedge Px$ involves reference to times. The same goes for (6). Therefore, is seems correct to say that (6) is an adequate formalization of (2) on the assumption that a denotes a time. Of course, one can equally say that (6) is an adequate formalization of 'Socrates is a philosopher' on the assumption that a denotes Socrates. But there is nothing wrong with this, if the extensional analysis of tensed sentences is granted. According to that analysis, the logical form of tensed sentences does not substantially differ from the logical form of other sentences.

In the second place, it may be noted that, independently of what follows from the assumption that one and the same domain contains both times and ordinary objects, that assumption plays no essential role for the purposes at hand. An account of tensed sentences along the lines sketched in the previous sections could equally be framed in terms of a first-order modal language that includes two distinct kinds of terms, that is, terms for times and terms for ordinary objects. If such a language were adopted, models could be so defined as to include two separate domains, one for times and one for ordinary objects. Obviously, that would require more complexity both at the syntactic and at the semantic level. But at least *prima facie*, there seems to be no reason to doubt that a coherent formal apparatus could be provided. Therefore, even if one were not satisfied with the definition of T-models adopted here, one would still in a position to agree on the substance of the present proposal.

4 T-semantics and Ockhamism

Let $\mathcal{M}_T$ be the set of T-models. The semantics characterized by $\mathcal{M}_T$ may be called *T-semantics*. Since $\mathcal{M}_T \subset \mathcal{M}$, T-semantics is nothing but a restriction on the semantics of SQML. As it will be shown, T-semantics preserves some characteristic logical features of Ockhamism, and provides a notion of truth in a world that fits into the Ockhamist explanation of the truth-valuedness of future contingents.

One important logical feature of Ockhamism, which sets it apart from most views of future contingents, is that it complies with *bivalence*. According to Ockhamism, every future contingent is either true or false, because either the actual course of events makes it true or the actual course of events makes it false. It is easy to see that T-semantics preserves this feature. In every T-model, for every w and every α, either α is true in w or it is false in w. For example, in every T-model, for every w, either (5) is true or it is false in w.

A related feature, which Ockhamism shares with other views of future contingents, is that it entails the validity of *excluded middle*. According to the latter principle, the following sentence is true as uttered now:

(8) Either it will snow or it will not snow

Again, T-semantics preserves this feature. In every T-model, every formula of the form $\alpha \vee \sim \alpha$ is true in every w. So, for example, in every model, the following formula is true in every w:

(9) $\exists x(a < x \wedge Px) \vee \sim \exists x(a < x \wedge Px)$

Lastly, a logical feature that is commonly regarded as a distinctive trait of Ockhamism is the rejection of the principle called "necessity of the past". Usually, this principle is phrased in the language of tense logic as follows:

(NP) $P\alpha \supset \Box P\alpha$

According to Ockhamism, however, some truths about the past are not necessary because they involve reference to the future. For example, it may be true, yet not necessarily true, that, for some time in the past, there is a future time in which it snows. T-semantics preserves this feature. For some w, it may be the case that the antecedent of the following conditional is true in w but false in other worlds, so that its consequent is false in w:

(10) $\exists x(x < a \wedge \exists y(x < y \wedge Py)) \supset \Box\exists x(x < a \wedge \exists y(x < y \wedge Py))$

More generally, if α is a formula that contains a term τ, a counterpart of (NP) in L may be phrased as follows:

(NP′) $\exists x(x < \tau \wedge \alpha(x/\tau)) \supset \Box\exists x(x < \tau \wedge \alpha(x/\tau))$

T-semantics makes (NP′) invalid, in that there are T-models in which $\exists x(x <$

$\tau \wedge \alpha(x/\tau))$ is satisfied in some but not in all worlds[11].

It remains to be said how the notion of truth in a world adopted in T-semantics can fit into the Ockhamist explanation of the truth-valuedness of future contingents. We saw that the crucial distinction is between plain truth and truth in a world. Since plain truth concerns the actual course of events, T-semantics offers a formal characterization of plain truth to the extent that one of the worlds in a T-model is taken to represent that course of events. Consider again the right-hand side of (4), that is, (i) there is a possible course of events c such that (1) as uttered now is true in c, and (ii) c is the actual course of events. T-semantics offers a formal characterization of (i) to the extent that, in some T-model $\mathcal{A}$ in which a world w represents c, a formula that expresses the truth condition of (1) as uttered now is true in w. Given such characterization, (ii) may be understood as a hypothesis about $\mathcal{A}$. That is, $\mathcal{A}$ gives us an account of the truth of (1) as uttered now on the hypothesis that w is the actual course of events.

5 Necessity and Settledness

One last issue about T-semantics concerns the interpretation of the operator $\Box$. This operator expresses *necessity*, understood as truth in all worlds. For example, consider the following formulas:

(11) $\Box\exists x(a < x \wedge Px)$
(12) $\Box\exists x(x = a \wedge Px)$
(13) $\Box\exists x(x < a \wedge Px)$

These formulas are read as 'Necessarily, it will snow', 'Necessarily, it is snowing', and 'Necessarily, it snowed'. Necessity so understood is not the same thing as settledness: according to the usual distinction, the former involves unrestricted quantification over possible courses of events, while the latter involves quantification over possible courses of events that differ at most in what happens after a given time. T-semantics is insensitive to this distinction, for T-models may include worlds that differ at any time[12].

Is there a way to express settledness in T-semantics? Of course there

[11] As it is explained in Øhrstrøm and Hasle [**?**], the rejection of the necessity of the past characterizes the Ockhamist response to a classical argument for determinism that was widely debated in the Middle Ages.

[12] In this respect, T-semantics differs from the semantics that Prior calls 'Ockhamist', in Prior [**?**], pp. 126-127. To see the contrast, consider the principle of the necessity of the past. In Prior's Ockhamist semantics, (NP) is understood in terms of settledness rather in terms of necessity, so it is invalid due to counterexamples such as that expressed by (10), that is, to sentences about the past that involve reference to the future. The invalidity of (NP′) in T-semantics, instead, does not specifically depend on such counterexamples. So there is a sense in which (NP′) is not a proper counterpart of (NP).

is. T-models may be constrained in the following way. First of all, given a time t in a T-model $\mathcal{A}$, a kind of formula is characterized relative to t for an assignment υ:

Definition 6 *α is a t-formula on υ if and only if, for every term τ in α, if $[\tau]_{\mathcal{A},\upsilon} \in T$ then $[\tau]_{\mathcal{A},\upsilon} = t$.*

That is, a t-formula on υ is a formula in which no term denotes times other than t relative to υ. The next step is to define a relation between worlds:

Definition 7 *w and w' are t-equivalent if and only if for every υ and every t-formula α on υ, υ satisfies α in w if and only if υ satisfies α in w'.*

The relation of t-equivalence obtains between w and w' when w and w' have the same properties at t. In other words, w and w' are t-equivalent if they are in the same state at t. Finally, a constraint on T-models is specified as follows:

Definition 8 *A T-model is bound up to t if and only if for every w, w' and t' such that $t' = t$ or $\langle t', t\rangle \in R$, w and w' are t'-equivalent.*

Bound T-models warrant that $\Box$ expresses settledness. For example, in a T-model bound up to t such that $I(a) = t$, (11) may be read as 'It is settled that it will snow', (12) may be read as 'It is settled that it is snowing', and (13) may be read as 'It is settled that it snowed'. Thus, (11) is not true if (5) is true in some worlds but false in others, while (12) and (13) are either true in virtue of (6) and (7) being true in all worlds, or false in virtue of (6) and (7) being false in all worlds[13].

However, it must be clear that such a restriction on T-models is not essential in order to accommodate Ockhamism. As far as Ockhamism is concerned, the distinction between necessity and settledness plays no significant explanatory role. The central notion is the notion of truth in the actual course of events, which differs both from necessary truth and from settled truth. So the relevant contrast is between the former notion and any modal notion of truth, that is, any notion of truth defined in terms of quantification over possible courses of events. Consider future-tense sentences. Anyone agrees that there are clear examples of settled truth, such as (8). But these are also clear examples of necessary truth. So T-semantics accounts for them independently of whether $\Box$ expresses necessity or settledness. The following formula is true in every world in every T-model:

(14) $\Box(\exists x(a < x \wedge Px) \vee \sim \exists x(a < x \wedge Px))$

[13] Note that in this case the only kind of counterexample to (NP′) is provided by conditionals such as (10), in accordance with Prior's understanding of the Ockhamist rejection of (NP).

Similarly, anyone agrees that there are clear examples of lack of settled truth, such as (1). But these are also clear examples of lack of necessary truth. So, again, T-semantics accounts for them: (11) is not true in w if there is a w' such that (5) is false in w'. Now consider present-tense and past-tense sentences. It is plausible to think that there are settled truths that are not necessary truths. If it snows now, then (2) is true in all histories that differ at most in the future, even though it is not necessarily true. Similar considerations hold for (3). However, this is not a reason to think that $\Box$ must express settledness rather than necessity. Certainly, there are facts about the present or the past that are not necessary. But such facts can be explained without appealing to settledness. According to Ockhamism, saying that it is a fact that it snows now amounts to saying that it snows now in the actual course of events, and saying that it is a fact that it snowed in the past amounts to saying that it snowed in the past in the actual course of events. So the formulas that must occur in the explanation are (6) and (7), rather than (11) and (12).

Many of those who deal with the issue of future contingents are inclined to think that that settledness, rather than necessity, is the modal property to be represented in the formal semantics. Branching time models clearly focus on settledness. To ask what is true at a time relative to all the histories that include that time is to ask what is settled at that time. Presumably, if one assumes that truth and settled truth are the same thing, one will be apt to regard settled truth as a key property that must accurately be distinguished from necessary truth. However, according to Ockhamism truth and settled truth are *not* the same thing. Once the explanatory role of truth is recognized, no further reason seems to remain to appeal to settled truth as distinct from necessary truth.

6 Axiomatization

So far it has been suggested that, given T-semantics, the truth-conditions of tensed sentences can be expressed in L in accordance with Ockhamism. This last section shows how the favoured account looks from a proof-theoretic point of view. SQML is a system with the following axioms:

A1 $\alpha \supset (\beta \supset \alpha)$

A2 $(\alpha \supset (\beta \supset \gamma)) \supset ((\alpha \supset \beta) \supset (\alpha \supset \gamma))$

A3 $(\sim \alpha \supset \sim \beta) \supset (\beta \supset \alpha)$

A4 $\forall x\alpha \supset \alpha(\tau/x)$, if τ is substitutable for x in α.

A5 $\alpha \supset \forall x\alpha$, if x is not free in α.

A6 $\forall x(\alpha \supset \beta) \supset (\forall x\alpha \supset \forall x\beta)$

A7 $\forall x\alpha$, if α is an axiom.

A8 $x = x$

A9 $x = y \supset (\alpha \supset \beta)$, if β differs from α at most in that y replaces x in some free occurrence.

A10 $\Box(\alpha \supset \beta) \supset (\Box\alpha \supset \Box\beta)$

A11 $\Box\alpha \supset \alpha$

A12 $\Diamond\alpha \supset \Box\Diamond\alpha$

A1-A9 are standard axioms of first-order logic with identity. A10-A12 are characteristic axioms of S5. The rules of inference of SQML are Modus Ponens and Necessitation, with $\vdash$ expressing provability in SQML:

MP : if $\vdash \alpha$ and $\vdash \alpha \supset \beta$, then $\vdash \beta$.

N : if $\vdash \alpha$, then $\vdash \Box\alpha$.

SQML is sound and complete with respect to a semantics based on definition 1. Let $\models$ indicate validity on $\mathcal{M}$. Then $\vdash \alpha$ if and only if $\models \alpha$.

A straightforward extension of SQML is obtained by adding specific axioms for T-semantics. The following formulas express the conditions that characterize $\mathcal{M}_T$:

A13 $x < y \supset \sim y < x$

A14 $(x < y \wedge y < z) \supset x < z$

A15 $\exists z(z < x \wedge z < y) \supset (x \neq y \supset (x < y \vee y < x))$

A16 $x < y \supset \Box(x < y)$

A13-A15 define $<$ as a linear order. In A15, the antecedent is needed to characterize x and y as times. Without that, the axiom would turn every two distinct objects into times. A16 expresses the rigidity of $<$, namely, the fact that $<$ has the same extension in all worlds.

Let SQML$_T$ be the theory obtained by adding A13-A16 to SQML. On the assumption that SQML is sound and complete with respect to the semantics based on definition 1, it can be proved that SQML$_T$ is sound and complete with respect to T-semantics. A sketch of the proofs will now be provided, using the symbol $\vdash_T$ to indicate provability in SQML$_T$ and the symbol $\models_T$ to indicate validity on $\mathcal{M}_T$.

Theorem 1 *If $\vdash_T \alpha$, then $\models_T \alpha$.*

Proof. This result can be obtained by induction on the proof in $SQML_T$. To say that $\vdash_T \alpha$ is to say that in $SQML_T$ there is a proof $\beta_1, ...\beta_n$ of α, that is, a finite sequence of formulas such that $\beta_n = \alpha$ and for $1 \leq i \leq n-1$, each β_i is either an axiom or it is obtained from preceding formulas by MP or N. If β_i is an instance of A1-A12, then $\models \beta_i$, for the soundness of SQML. But since $\mathcal{M}_T \subset \mathcal{M}$, we get that $\models_T \beta_i$. If β_i is an instance of A13-A16, again $\models_T \beta_i$, as it is easy to verify. The same goes if β_i is obtained from preceding formulas by MP or N, for MP and N preserve validity on $\mathcal{M}_T$. So for $1 \leq i \leq n-1$, $\models_T \beta_i$. From this it can be concluded that the same holds for $i = n$, that is, $\models_T \alpha$.

Theorem 2 *If* $\models_T \alpha$*, then* $\vdash_T \alpha$.

Proof. This result can be obtained through the canonical model method. First, it is assumed that α is not a theorem of $SQML_T$, so that $\{\sim \alpha\}$ is a consistent set. Then a T-model $\mathcal{A}$ is constructed for $\{\sim \alpha\}$, in order to show that α is not satisfied by some assignment in some world in $\mathcal{A}$, hence that α is not valid on $\mathcal{M}_T$. It follows by contraposition that, if $\models_T \alpha$, then $\vdash_T \alpha$.

The construction of $\mathcal{A}$ involves three steps, because $\mathcal{A}$ is a triple $\langle W, D, I\rangle$. Let us start with the assumption that $\{\sim \alpha\}$ is consistent. Since every consistent set of formulas has a maximal consistent extension, there is a maximal consistent extension E of $\{\sim \alpha\}$. Moreover, if L$'$ is a language obtained from L by adding an infinite set of variables to its vocabulary, there is a consistent set E' of formulas of L$'$ such that $E \subseteq E'$ and E' has the $\forall$-property, that is, if $\alpha(\tau/x) \in E'$ for every τ, then $\forall x\alpha \in E'$. Therefore, there is a maximal consistent extension E'' of $\{\sim \alpha\}$ that has the $\forall$-property. E'' is a world, that is, a member of W. The other members of W are defined in terms of E'': every w in W is a maximal consistent set of formulas of L$'$ that has the $\forall$-property and is such that, if $\Box\alpha \in E''$, then $\alpha \in w$. Note that from this definition it turns out that, for every w in W, if $\Box\alpha \in w$, then, for all w' in W, $\alpha \in w'$. Suppose, for any w and w' in W, that α is not in w'. Then $\Box\alpha$ is not in E'', so $\sim \Box\alpha \in E''$. From A12 we get that $\Box \sim \Box\alpha \in E''$. This entails that $\sim \Box\alpha \in w$, so that $\Box\alpha$ is not in w[14].

The second step is the specification of D as a set of appropriately chosen terms of L$'$, where 'appropriately chosen' means that identity formulas are taken into account. First of all, note that $\vdash_T x = y \supset \Box(x = y)$, given that $\vdash x = y \supset \Box(x = y)$. This means that the worlds in W contain exactly the same identity formulas. Now consider an enumeration of the terms of L$'$. For any term τ, there is some τ' which is the earliest term in the enumeration such that $\tau = \tau' \in w$ for any w. Accordingly, D is defined as the set of all

[14] As usual, the $\forall$-property is understood as follows: if $\forall x\alpha$ is false in a given world, then there is some individual in that world which makes it false. For if a set has the $\forall x$-property and it does not include $\forall x\alpha$, then it must include $\alpha(\tau/x)$ for some τ.

terms τ such that τ is earlier than any other τ' such that $\tau = \tau' \in w$ for any w[15].

The third step is the specification of I as a function such that, for every constant a in L$'$, $I(a)$ is the earliest term τ in the enumeration such that $a = \tau \in w$ for any w, and for every n-place P and n-tuple $\tau_1, ...\tau_n$, $\langle \tau_1, ...\tau_n \rangle \in I(P)_w$ if and only if $P\tau_1, ..., \tau_n \in w$. Note that, given A13-A16, $<$ is interpreted as a constant function that maps members of W to a linear order on a set T such that $T \subseteq D$. For any two terms τ and τ' in D, τ precedes τ' in the order if and only if $\tau < \tau' \in w$ for any w. So we get that $\mathcal{A}$ is a T-model.

Let a canonical assignment υ in $\mathcal{A}$ be a function such that, for every variable x in L$'$, $\upsilon(x)$ is the earliest term τ in the enumeration such that $x = \tau \in w$ for any w. Given I, from this we get that, for every term τ in L$'$, $[\tau]_{\mathcal{A},\upsilon}$ is the earliest term τ' in the enumeration such that $\tau = \tau' \in w$ for any w. Now it can be proved that, for every α of L$'$ and every w in W, α is satisfied by υ in w if and only if $\alpha \in w$. The proof is by induction on the complexity of α.

1. $P\tau_1, ..., \tau_n$ is satisfied by υ in w if and only if $\langle [\tau_1]_{\mathcal{A},\upsilon}, ..., [\tau_n]_{\mathcal{A},\upsilon} \rangle \in I(P)_w$, that is, if and only if $\langle \tau_1', ..., \tau_n' \rangle \in I(P)_w$, where the formulas $\tau_1 = \tau_1', ..., \tau_n = \tau_n'$ are in w. But $\langle \tau_1', ..., \tau_n' \rangle \in I(P)_w$ if and only if $P\tau_1', ..., \tau_n' \in w$, so by A9 if and only if $P\tau_1, ..., \tau_n \in w$. Therefore, $P\tau_1, ..., \tau_n$ is satisfied by υ in w if and only if $P\tau_1, ..., \tau_n \in w$.

2. $\sim \alpha$ is satisfied by υ in w if and only if α is not satisfied by υ in w. By induction hypothesis, α is not satisfied by υ in w if and only if α is not in w, that is, if and only if $\sim \alpha \in w$.

3. $\alpha \supset \beta$ is satisfied by υ in w if and only if either α is not satisfied by υ in w or β is satisfied by υ in w. By induction hypothesis, this disjunction holds if and only if either α is not in w or $\beta \in w$, which means if and only if $\alpha \supset \beta \in w$.

4. Assume first that $\forall x\alpha \in w$. Let υ' be any assignment that differs from υ at most in the denotation of x. For any τ such that $[\tau]_{\mathcal{A},\upsilon} = [x]_{\mathcal{A},\upsilon'}$, from A4 we get that $\alpha(\tau/x) \in w$. By induction hypothesis, $\alpha(\tau/x)$ is satisfied by υ in w. It follows that α is satisfied by υ' in w. Therefore, $\forall x\alpha$ is satisfied by υ in w.

   Now assume that $\forall x\alpha$ is not in w. Since w has the $\forall$-property, there is some τ such that $\sim \alpha(\tau/x) \in w$. So there is some τ' in D such that $\tau = \tau' \in w$ and by A9 $\sim \alpha(\tau'/\tau) \in w$. It follows that $\alpha(\tau'/\tau)$ is not in w. By induction hypothesis, $\alpha(\tau'/\tau)$ is not satisfied by υ in w. Since A4 is valid, it follows that $\forall x\alpha$ is not satisfied by υ in w.

[15] This method is adopted in Hughes and Cresswell [**?**], pp. 316.

5. Assume first that $\Box\alpha \in w$. Then, for every w', $\alpha \in w'$. By induction hypothesis, α is satisfied by v in w'. Therefore, $\Box\alpha$ is satisfied by v in w.

   Now assume that $\Box\alpha$ is not in w. Then there is a consistent set A of formulas of L$'$ with the $\forall$-property such that $\sim \alpha \in A$ and, for every β such that $\Box\beta \in w$, $\beta \in A$. It follows that there is a w' - a maximal consistent extension of A with the $\forall$-property - such that $\sim \alpha \in w'$. So α is not in w'. By induction hypothesis, α is not satisfied by v in w. Therefore, $\Box\alpha$ is not satisfied by v in w[16].

Once it is proved that $\mathcal{A}$ is such that, for every α and every w in W, α is satisfied by v in w if and only if $\alpha \in w$, the desired result is obtained. That is, there is a w in W, namely E'', in which there is an assignment that does not satisfy α, namely, v. Therefore, on the assumption that α is not a theorem of SQML$_T$, we get that α is not valid on $\mathcal{M}_T$. This means that if $\models_T \alpha$ then $\vdash_T \alpha$[17].

[16] A proof by induction along these lines is provided in Hughes and Cresswell [**?**], pp. 261 and 317. A justification of the inference made in 5 from the assumption that $\Box\alpha$ is not in w to the existence of A is provided at pp. 259-251.

[17] Many thanks are due to Fabrice Correia for his very accurate and helpful comments on a previous version of this paper.