Valid Arguments as True Conditionals

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This paper explores an idea of Stoic descent that is largely neglected nowadays, the idea that an argument is valid when the conditional formed by the conjunction of its premises as antecedent and its conclusion as consequent is true. As it will be argued, once some basic features of our naïve understanding of validity are properly spelled out, and a suitable account of conditionals is adopted, the equivalence between valid arguments and true conditionals makes perfect sense. The account of validity outlined here, which displays one coherent way to articulate the Stoic intuition, accords with standard formal treatments of deductive validity and encompasses an independently grounded characterization of inductive validity.

1 OVERVIEW

The thought that some relation of equivalence holds between valid arguments and true conditionals goes back to the Stoics. The following passage from Sextus Empiricus provides a clear statement of their view:

> The conclusive argument is sound, then, when after we conjoin the premises and create a conditional that begins with the conjunction of the premises and finishes with the conclusion, this conditional is itself found to be true.[1]

Many logicians in the past have made claims that go in the same direction. For example, the medieval discussions on consequence took for granted that valid arguments are inherently related to true conditionals, as it emerges from the fact that the very term 'consequence' was applied both to arguments and to conditionals. Yet the *Stoic Thesis*, as we will call it, does not enjoy wide popularity nowadays, and most contemporary logicians would simply deny that validity and truth are so related.

Historically, this lack of popularity derives at least in part from the way modern logic has developed after Frege. On the one hand, logicians have been mainly concerned with formal validity, which is clearly too strong as a condition for the left-hand side: it would be patently wrong to expect that an argument is *formally* valid when the corresponding conditional is true. The only conditionals that could match formally valid arguments are logical truths. On the other hand, logicians have largely privileged the material reading of conditionals, according to which a conditional is true when it is not the case that its antecedent is true and its consequent is false. This

1. Sextus Empiricus, *Against the Logicians*, II, 417, edited and translated by R. Bett in Sextus Empiricus 2005, p. 171.

reading, which is traditionally attributed to Philo, is clearly too weak to provide a viable right-hand side for the equivalence: it would be patently wrong to expect that an argument is valid when the corresponding *material* conditional is true.[2]

A further motivation that may be acknowledged stems from the study of context within the philosophy of language of the twentieth century. As a result of extensive discussions on the semantics of natural language, it has become standard to assume that, for any sentence s, there are at least two ways in which the context in which s is used can be relevant to the ascription of truth to s: on the one hand it contributes to the determination of the content of s, on the other it provides the circumstances in which s — once its content is fixed — is evaluated. So it is natural to expect that the right-hand side of the equivalence is appropriately indexed. But since no parallel assumption is made in connection with validity, which is generally treated as *in*sensitive to such parameters, no similar expectation holds for the left-hand side. Thus it might seem that there is something wrong in the very idea that validity, an absolute property of arguments, can be reduced to a relative property of sentences.

Although these historical considerations can explain to some extent the present marginality of the Stoic Thesis, they do not provide reasons against the Thesis itself. In the first place, it is not obvious that the Stoic Thesis is to be framed in terms of formal validity, given that validity does not reduce to formal validity. In fact, and independently of what the Stoics had in mind, it is not even obvious that the left-hand side of the equivalence must be restricted to deductive validity. There is a naïve understanding of validity — the informal or pre-formal notion of validity that is intuitively prior to any well-defined notion of logical consequence — according to which an argument is valid when the inference from its premises to its conclusion is justified, or equivalently, when its premises provide reasons for accepting its conclusion. Obviously, the terms 'justified' and 'reason' can be made precise in different ways, and a crucial step towards a proper clarification of their meaning is the distinction between deductive and inductive validity. Still, it is entirely plausible to talk about validity in this broad sense, which informs our everyday practice of assessing arguments. The Stoic Thesis might be construed as a claim about validity so understood.[3]

In the second place, the material reading of conditionals is far from being uncontentious. Most contemporary theorists of conditionals tend to agree that a stronger reading is needed in order to adequately explain our use of 'if', although their views differ in many respects. At least three main options are on the table. One is the strict reading, according to which a conditional is true when it is impossible that its antecedent is true and its consequent is false. This reading, which goes back to Diodorus, was revived by C. I. Lewis at the beginning of the twentieth century and played a key role in the development of modal logic.[4] A second option is the suppositional reading

2. Sextus Empiricus ascribes the material reading to Philo in *Outlines of Scepticism*, II, 110, see Sextus Empiricus 2000, pp. 95-96. Although Philo's definition was discussed among the Stoics, there is no evidence for thinking that he endorsed the view about validity described above.

3. As explained in Barnes, Bobzien, and Mignucci 2008, p. 123, the Stoic notion of validity was not restricted to formal validity, and might easily have included inferences that are now classified as inductive rather than deductive.

4. Sextus Empiricus, in *Outlines of Scepticism*, II, 110-11, ascribes the strict reading, or at least a temporal version of it, to Diodorus, see Sextus Empiricus 2000, p. 96, and Kneale and Kneale 1962, pp. 132-133. C. I. Lewis 1914 is a seminal paper on the topic. More recently, the strict reading has been developed in different ways in Lycan 2001, Gillies 2009, Kratzer 2012, among other works.

adopted by Adams, Stalnaker, and Lewis, according to which a conditional holds when its consequent is very likely, or cannot easily be false, on the supposition that its antecendent holds.[5] Finally, in the last few years some attempts have been made to provide an analysis of conditionals based on the hypothesis that a conditional holds when its antecedent supports its consequent. This is precisely the kind of reading that will be suggested here. An account of conditionals in term of support can suit the Stoic Thesis, or so it is reasonable to expect.[6]

Lastly, the presumption that validity is an absolute property of arguments has some plausibility only insofar as one restricts consideration to formal validity, while it is no longer tenable when one looks at validity broadly understood. As it will be argued — and this is a key point of the paper — validity is relative exactly in the same sense in which truth is relative, so there is nothing conceptually wrong in the Stoic Thesis. In the formulation of the Thesis that will be proposed, both sides of the equivalence are indexed. This is to say that an argument is valid relative to a certain set of parameters just in case the corresponding conditional is true relative to that set of parameters.

The structure of the paper is as follows. Section 2 suggests that our naïve understanding of validity allows for two distinct sorts of relativity: one concerns the interpretation of arguments, the other concerns the circumstances in which they are evaluated. Section 3 claims that, as far as these two sorts of relativity are concerned, validity is essentially similar to truth. Section 4 spells out the equivalence between valid arguments and true conditionals. Section 5 outlines an independently grounded account of conditionals that fits its right-hand side. Sections 6 and 7 articulate the distinction between deductive validity and inductive validity. Finally, section 8 adds some concluding remarks.

2 TWO SORTS OF RELATIVITY

Ordinary judgements about validity seem to involve two distinct dimensions of contextual variation, that is, there seem to be two distinct ways in which an argument can be assessed as valid in some contexts but as invalid in other contexts. The following examples may be used to illustrate the distinction:

A
(1) Kevin is going to the bank
―――――――――――――
(2) Kevin is going to a financial institution

B
(3) This glass contains water
―――――――――――――
(4) This glass contains H_2O

C
(5) Angela was raised Catholic
―――――――――――――
(6) She does not fast during Ramadan

A first observation about A-C is that the question whether they are valid depends on how their premises and conclusion are understood. Consider A. As long as 'bank' denotes an institution that receives deposits and makes loans, it seems correct to infer (2) from (1). But if 'bank' is read as 'edge of

5. Adams 1965 outlines a probabilistic version of the suppositional reading. Stalnaker 1991 phrases it in modal terms, and D. Lewis 1973 develops a similar account for counterfactuals.

6. Accounts of conditionals along these lines are developed in Rott 1986, Douven 2008, Spohn 2013, Krzyżanowska, Wenmackers, and Douven 2013, Rooij and Schulz 2019, Crupi and Iacona 2020. A recent work that deserves special mention is Douven, Elqayam, and Krzyzanowska 2021, where an empirical case is made for "inferentialism" understood as the view that the truth of a conditional requires the existence of a compelling argument from the conditional's antecedent to its consequent.

a river', the inference is not justified. In the case of B, insofar as the two occurrences of 'this' refer to the same glass, it seems correct to infer (4) from (3). But if 'this' is used twice to indicate different glasses, the inference is not justified. Similar consideration hold for C, where the inference seems correct only if the word 'she' in (6) refers to Angela.

A second observation about A-C is that the question whether they are valid depends on whether some relevant facts warrant the inference from their premises to their conclusion. Consider A, and let it be granted that 'bank' denotes an institution that receives deposits and makes loans. The inference from (1) to (2) is warranted by the fact that every such institution is a financial institution. If this fact did not obtain, that is, if it were not true that every such institution is a financial institution, the inference would not be justified. Consider B, and let it be granted that the two occurrences of 'this' refer to the same glass. The inference from (3) to (4) is warranted by the fact that the chemical composition of water is H_2O. If water had a different chemical composition, the inference would not be justified. Similar considerations hold for C. Assuming that 'she' refers to Angela, if it were not the case that people who are raised Catholic mostly do not observe Ramadan, the inference from (5) to (6) would not be justified.[7]

Obviously, in each of these three cases, the facts that warrant the inference from the premise to the conclusion could be expressed by means of additional premises. For example, the following argument is obtained from A by adding a conditional premise:

D
(7) If Kevin is going to the bank, he is going to a financial institution
(1) Kevin is going to the bank
---
(2) Kevin is going to a financial institution

But the point here is about A-C as they are, that is, *without* additional premises. A and D are distinct arguments, and even though (7) may be treated as an implicit premise in most contexts, this does not prevent A from being intuitively valid.

Note that, as far as the second observation is concerned, it is not essential that the imagined situation in which the relevant facts do not obtain corresponds to a real possibility from the metaphysical point of view. For example, in the case of B it might be claimed that being H_2O is part of the very nature of water, so it is not possible that water has a different chemical composition. But the distinction between metaphysical necessity and other kinds of necessity, such as nomological necessity, does not really matter here. The same point can be made by using an example of argument which relies on nomologically necessary facts, such as the fact that nothing moves faster than light.

Note also that the second observation is neutral as to the distinction between deductive validity and inductive validity. B can plausibly be described as deductively valid, given the fact that water is H_2O. This fact warrants the step from (1) to (2) by ruling out the possibility that (1) is true and (2) is false. Instead, C can plausibly be described as inductively valid, given certain statistical data about Catholics, for those data provide at most a defeasible warrant for the step from (5) to (6). The notion of warrant adopted here is intended to cover both deductive and inductive justification.[8]

7. The term 'warrant', which goes back to Toulmin 1958, is often used in the informal logic literature to indicate the assumptions which justify an inference. In Douven, Elqayam, and Krzyzanowska 2021, where conditionals are explicitly defined in terms of arguments, the cogency of arguments is assumed to be relative to sets of background premises.

8. Toulmin 1958 deliberately and systematically uses the term 'warrant' in this broad sense.

The two observations just made suggest that ordinary judgments about validity involve two sorts of relativity. On the one hand, there is a sense in which arguments are assessed as valid or invalid relative to *interpretations*, that is, ways of understanding their premises and conclusion. On the other hand, there is a sense in which arguments are assessed as valid or invalid relative to *circumstances of evaluation*, that is, sets of facts that are taken for granted as background assumptions. These two sorts of relativity are independent of each other. Keeping fixed the circumstances of evaluation, an argument can be valid relative to one interpretation but invalid relative to another. For example, given all the relevant facts about financial institutions, A can be valid or invalid depending on how 'bank' is understood. Inversely, keeping fixed the interpretation, an argument can be valid relative to some circumstances of evaluation but invalid relative to others. For example, given an interpretation that assigns the same glass to 'this', B can be valid or invalid depending on whether water is H_2O.

Two further points are to be taken into account. The first is that the two sorts of relativity considered can be recognized independently of any grasp of logical form. As a matter of fact none of the arguments A-C is formally valid, and this is not accidental. Consider for example D as opposed to A. Insofar as 'bank' is read univocally in (7) and (1), D is adequately formalized as an instance of *Modus Ponens*, so it is formally valid. But in that case a specific interpretation is fixed. Moreover, D so interpreted is valid in virtue of its form, independently of any truth concerning banks. This contrast between informally valid and formally valid arguments, which is one of the data that an account of validity should explain, shows why it makes sense to treat validity as an absolute property of arguments when one restricts consideration to formal validity.[9]

The second point is that neither of the two sorts of relativity considered implies that the concept of validity is itself relative. For example, although A can be valid or invalid depending on how 'bank' is understood, and B can be valid or invalid depending on whether water is H_2O, this does not imply that 'valid' can be interpreted in different ways. Of course, it might be claimed that a third sort of relativity — logical relativity — must be acknowledged, in that 'valid' admits different readings. According to Beall and Restall, for example, the pretheoretical notion of validity can be made precise in more than one way, and this provides a *prima facie* case for logical pluralism.[10] But whether validity is relative in this third sense is not a question that needs be settled here, so it can be left out of the picture.

3 VALIDITY AND TRUTH

The distinction sketched above between interpretations and circumstances of evaluation evokes another distinction that is usually drawn in connection with truth. As noted in section 1, there are essentially two ways in which the context in which a sentence s is used can be relevant to the ascription of truth to s: one concerns the determination of the content of s, the other concerns the circumstances in which s — once its content is fixed — is evaluated. This duality emerges clearly in the standard treatment of indexicality, which goes back to Kaplan. Consider the following sentence:

9. Although the informal logic literature includes plenty of works on enthymemes and the standardization of arguments, the contrast between informally valid and formally valid arguments is hardly phrased in terms of the two sorts of relativity considered here.
10. Beall and Restall 2006, p. 30.

(8) I like ice cream

If (8) is uttered by Kevin, it expresses the proposition that Kevin likes ice cream, while if it is uttered by Angela, it expresses the proposition that Angela likes ice cream. In both cases the context determines the proposition expressed by (8), in that it fixes the denotation of 'I'. But it also provides the circumstances in which that proposition is evaluated. Kaplan identifies circumstances of evaluation with pairs of times and worlds, and takes propositions to have truth values relative to such pairs. For example, it may happen that the proposition that Kevin likes ice cream is true at the time of the utterance in the world of the utterance, while it is false at the time of the utterance in a world where Kevin does not like ice cream. Thus, a proper analysis of the truth of (8) in a given context involves two distinct sets of parameters: one fixes the content of (8), the other determines a truth value for that content.[11]

This double relativization, often called *double-indexing*, proves very useful in the formal treatment of natural language. The theoretical importance of double-indexing, among other things, lies in the fact that the semantics of some complex constructions can be explained by assuming that one of the two sets of parameters varies independently of the other. Consider for example the following sentence as uttered by Kevin:

(9) It might be the case that I do not like ice cream

To say that (9) is true in the context considered above is to say that there is a world in which the proposition expressed by (8) is false, that is, a world in which Kevin does not like ice cream. So the contribution of 'it might be the case that' to the truth conditions of (9) can properly be described by postulating a world shift at the level of circumstances of evaluation.[12]

The suggestion that emerges from section 2 is that validity is essentially similar to truth as far as double-indexing is concerned, for the context in which an argument is used exhibits the same kind of duality: on the one hand it determines the content of the argument, on the other it provides the circumstances in which the argument — once its content is fixed — is evaluated. As in the case of truth, double-indexing can play some important explanatory role. For example, there is an interesting analogy between (9) and the following statement about B:

(10) It might be the case that the inference from (3) to (4) is not justified.

This statement is clearly intelligible: if water were not H_2O — independently of whether its not being H_2O is merely conceivable or really possible — it would not be correct to infer (4) from (3). So it is reasonable to expect that the truth conditions of (10) can be phrased in terms of existence of non-actual circumstances in which B is invalid.

Note that, as in the case of truth, the possible invalidity of an argument is consistent with its actual validity, as the following sentence shows:

(11) The inference from (3) to (4) is justified. But if water were not H_2O, it would not be justified.

More generally, validity seems to be contingent in the same sense in which truth is contingent: an argument can be valid in some circumstances but invalid in other circumstances.

11. This is the view developed in Kaplan 1989.
12. D. Lewis 1980 is a classical discussion of double-indexing.

This should not come as a surprise, as our common use of arguments seems to involve some awareness of contingency so understood. In a dispute between two persons X and Y, it may happen that X makes a certain inference, and that Y, who does not find that inference compelling, is nonetheless willing to grant that the very same inference *would* be compelling if certain conditions obtained. In such a case, Y's point is that X's argument is not actually valid, although it is valid relative to some circumstances that X erroneusly takes to be actual.

4 THE STOIC THESIS

In order to provide a formulation of the Stoic Thesis that takes into account what has been just said about validity, some further clarifications are needed about interpretations and circumstances of evaluation. Let us start with interpretations. It is widely recognized that one and the same sentence can be interpreted in different ways, due to ambiguity and context sensitivity. For example, an interpretation of (1) provides a disambiguation for 'bank', an interpretation of (3) provides a denotation for 'this', and so on. More generally, we can assume that an interpretation of a sentence s assigns definite extensions to the ambiguous or context-sensitive expressions that occur in s.

Of course, ambiguity and context sensitivity are significantly different phenomena. Disambiguation is usually regarded as a pre-semantic process. In standard formal treatments of natural language, the properties called semantic are assigned to nonambiguous lexical items at some level of syntactic representation deeper than surface structure. Context sensitivity, by contrast, is often described as pragmatic or post-semantic, assuming that at least some effects of context on truth conditions are not traceable to assignments of semantic values to elements of syntactic structure. But such differences may be left aside, because all that is needed here is that an interpretation of s determines definite truth conditions for s by resolving the ambiguity or context-sensitivity that may affect s.

Since arguments are constituted by sentences, in that they can be represented as ordered pairs formed by sets of premises and single conclusions, interpretations of arguments are definable in terms of interpretations of sentences. More precisely, we will restrict consideration to arguments with finite sets of premises, so as to rule out the possibility of conditionals with infinite conjunctions as antecedents. Let $s_1, \ldots, s_{n-1}; s_n$ be an argument formed by a set of premises $s_1, \ldots, s_{n-1}$ and a conclusion s_n. Then,

DEFINITION 1 An interpretation i of $s_1, \ldots, s_{n-1}; s_n$ is an assignment of interpretations to each of the sentences $s_1, \ldots, s_n$.

A crucial feature of interpretations so defined is that they are not required to be univocal: it can happen that the same expression occurs in two distinct sentences in $s_1, \ldots, s_{n-1}; s_n$, and that in i the interpretations of these sentences differ as to that expression. Thus, for example, there are interpretations of B in which the two occurrences of 'this' denotes the same glass and interpretations of B in which they denote different glasses.

On the assumption that validity is relative to interpretations, definition 1 accounts for the first observation made in section 1, namely, that the question whether A-C are valid depends on how their premises and conclusion are understood. Each of those arguments admits different interpretations in the sense defined, so it can be described as valid in one interpretation but invalid in another interpretation.

Now consider circumstances of evaluation. In this case what we want to model is a set of facts that can warrant the inference from a set of premises to a conclusion. As it emerges from the examples discussed, these facts include not only information about what is actually the case, but also information about what is necessarily the case or what is likely to be the case. For example, the inference in B requires that water is necessarily H_2O, while the inference in D requires that one with a Catholic upbringing is likely not to observe Ramadan.

This mixed package of information can be represented in different ways. A basic theoretical choice to be made is between possible worlds and probabilities. Here the first option will be privileged: circumstances of evaluation will be identified with ordered sets of worlds, where the position of each world in the order indicates its estimated degree of proximity to the actual world. From now on, the index c — which abbreviates 'circumstances' — will refer to an ordered set of worlds. It is arguable, though, that the main points made in what follows could equally be phrased by representing circumstances of evaluation in terms of probabilities.

Let $s_1, \ldots, s_{n-1} \Rightarrow_{i,c} s_n$ mean that the argument $s_1, \ldots, s_{n-1}; s_n$ is valid in the interpretation i and circumstances c. The Stoic thesis can then be phrased as follows:

DIST $s_1, \ldots, s_{n-1} \Rightarrow_{i,c} s_n$ iff a conditional with the conjunction of $s_1, \ldots, s_{n-1}$ as antecedent and s_n as consequent is true in i and c.

The label DIST stands for *Double-Indexed Stoic Thesis*. Here 'conditional' is to be read as 'indicative conditional'. For our purposes it would make little sense to include counterfactuals, given that the conjunction of the premises of an argument, typically, is not assumed to be false. The same goes for specific categories of conditionals, such as concessive conditionals, which clearly rule out a relation of support between antecedent and consequent.

Now consider the role of i in DIST. i determines definite truth conditions for $s_1, \ldots, s_n$, the sentences that constitute both the argument and the conditional. So, fixing the content of the former amounts to fixing the content of the latter. Once contents are fixed, thus obtaining an interpreted argument and an interpreted conditional, what we get is an equivalence relative to c between the validity of the former and the truth of the latter.

This equivalence, which is the kernel of DIST, can be phrased in a straightforward way by relying on the plausible assumption that an adequate formalization of a set of sentences is a representation of their truth conditions. Since truth conditions vary as a function of interpretations, to say that a set of formulas represents the truth conditions of a set of sentences is to say that each formula in the set is assigned to the corresponding sentence in virtue of some interpretation of the sentence itself. In other words, on the assumption considered, formulas represent interpreted sentences. So the equivalence above can be phrased in more formal terms without making reference to interpretations.[13]

For any interpretation i, let α be a conjunction of formulas that represent $s_1, \ldots, s_{n-1}$ as understood in i. Let β be a formula that represents s_n as understood in i. Let $\triangleright$ by the symbol for the conditional. Then, from DIST we get what follows, where $\Rightarrow_c$ indicates validity relative to c:

13. Iacona 2018 articulates and defends the idea that an adequate formalization of a set of sentences is a representation of their truth conditions.

SIST $\alpha \Rightarrow_c \beta$ iff $\alpha \rhd \beta$ is true in c.

The label SIST, which stands for *Single-Indexed Stoic Thesis*, indicates that this formulation is obtained from DIST by fixing i through a formalization of $s_1, \ldots, s_n$. Since $\alpha; \beta$ represents an interpreted argument, and $\alpha \rhd \beta$ represents an interpreted conditional, SIST states the equivalence relative to c between the validity of the former and the truth of the latter. So it remains to be explained what it is for $\alpha \rhd \beta$ to be true in c. The next section suggests that there is a coherent account of conditionals that can provide such explanation.

5 THE EVIDENTIAL ACCOUNT OF CONDITIONALS

The Stoic discussions on conditionals did not revolve solely around the two views offered by Philo and Diodorus. A third view, which can plausibly be ascribed to Chrysippus, deserves special attention for our purposes because it was probably held in conjunction with the Stoic Thesis. Sextus Empiricus describes this view as follows:

> Those who introduce connectedness say that a conditional is sound when the opposite of its consequent conflicts with its antecedent.[14]

According to Chrysippus, the truth of a conditional requires some sort of incompatibility between its antecedent and the negation of its consequent. It is hard to tell what exactly he had in mind, given that we know very little about him. But it is reasonable to conjecture that his criterion was not restricted to logical impossibility, as it included various forms of incompatibility that would now be classified as non-logical.[15]

Crupi and Iacona have recently outlined an account of conditionals — the *evidential account* — which bears a very close resemblance to Chrysippus's view, in that it employs the notion of incompatibility to spell out the idea that a conditional holds when its antecedent supports its consequent. According to this account, which will be adopted here, α supports β if and only if α and $\neg\beta$ are incompatible. To say that α supports β is to say that the inference from α to β is justified, or equivalently, that α provides a reason for accepting β.[16]

What does it mean that α and $\neg\beta$ are incompatible? As Crupi and Iacona have suggested, there are at least two distinct routes that may be taken in order to provide a precise definition of incompatibility: one is to adopt a modal semantics, the other is to adopt a probabilistic semantics. Here we will focus on the modal version of the evidential account, but it is worth noting that there is a coherent probabilistic counterpart of the semantics set

14. Sextus Empiricus, *Outlines of Scepticism*, II, 111, edited and translated by J. Annas and J. Barnes, in Sextus Empiricus 2000, p. 96. Sextus lists this view as third after Philo's view and Diodorus' view. A fourth view he mentions, which will not be discussed here, is that according to which a true conditional is one whose consequent is contained implicitly in its antecedent. The attribution of the third view to Chrysippus is based on further sources, such as Cicero *De Fato*, 12, and Diogenes Laertius, *Lives of Eminent Philosophers*, vii, 73. On the connection between this view and the Stoic Thesis, see Barnes, Bobzien, and Mignucci 2008, p. 123. Diogenes Laertius, in *Lives of Eminent Philosophers*, vii, 77, seems to imply that validity was defined in accordance with the Chrysippean criterion.

15. As observed in Barnes, Bobzien, and Mignucci 2008, pp. 107-108, some evidence suggests that Chrysippus' criterion included formal incompatibility, analytical incompatibility, and perhaps some sort of empirical incompatibility.

16. Crupi and Iacona 2020.

out in this section, which provides assertibility conditions for $\alpha \rhd \beta$ in terms of a probabilistic measure of support.[17]

According to the modal version of the evidential account, α and $\neg\beta$ are incompatible if and only if the following disjunction holds: either there are no worlds in which α and $\neg\beta$ are both true, or (a) it is not the case that α and $\neg\beta$ have the same truth value in the closest worlds and (b) for every world in which α and $\neg\beta$ are both true, some strictly closer world is such that α is true and $\neg\beta$ is false and some strictly closer world is such that α is false and $\neg\beta$ is true. This disjunction — which may be called *Chrysippus Test* — can be rephrased as follows, using combinations of the numbers 1 and 0 to label worlds on the basis of the truth values of α and β: either there are no 10-worlds, or (a) some of the closest worlds are 11-worlds or 00-worlds and (b) for every 10-world, some 11-worlds and 00-worlds are strictly closer.[18]

Consider for example the following sentences, which are conditional counterparts of the arguments A-C:

(12) If Kevin is going to the bank, he is going to a financial institution

(13) If this glass contains water, it contains H_2O

(14) If Angela was raised Catholic, she does not fast during Ramadan

In each of these three cases, it seems that the antecedent and the negation of the consequent are incompatible in the sense suggested, that is, one of the disjuncts of the Chrysippus Test holds. The fact that this condition is satisfied seems to accord with the intuition that the antecedent supports the consequent.

Now the evidential account will be phrased in a proper formal framework in order to show how it can substantiate SIST. Let L be a language whose alphabet is formed by a set of sentence letters $p, q, r, ...$, the connectives $\neg, \supset, \Box, \rhd$, and the brackets $($, $)$. The formulas of L are defined by induction in the usual way: the sentence letters are atomic formulas; if α is a formula, so are $\neg\alpha$ and $\Box\alpha$; if α and β are formulas, so are $\alpha \supset \beta$ and $\alpha \rhd \beta$. The connectives $\wedge, \vee, \Diamond$ can then be defined in the usual way in terms of $\neg, \supset, \Box$.

An interpretation of L will include an ordered set of worlds of the kind mentioned in section 4, that is, it will allow for comparative measures of distance between worlds. Let us start with the following definition, which holds for every non-empty set of worlds W:

DEFINITION 2 A *proximity ordering* O on W is an assignment to every $w \in W$ of a binary relation $\preceq_w$ that satisfies the following conditions:

(i) for every $w', w'', w'' \in W$, if $w' \preceq_w w''$ and $w'' \preceq_w w'''$, then $w' \preceq_w w'''$;

(ii) for every $w', w'' \in W$, either $w' \preceq_w w''$ or $w'' \preceq_w w'$;

(iii) for every $w' \in W$, $w \preceq_w w'$.

Informally speaking, $w' \preceq_w w''$ means that, from the point of view of w, w' is at least as close as w''. Accordingly, its negation, $w' \npreceq_w w''$ means that, from the point of view of w, w'' is strictly closer than w'. (i) says that $\preceq_w$ is transitive. (ii) says that $\preceq_w$ is strongly connected. (iii) says that $\preceq_w$ includes

17. The modal version is developed in Crupi and Iacona 2020, and in Raidl, Iacona, and Crupi 2021. The probabilistic version is developed in Crupi and Iacona 2022b, and in Crupi and Iacona 2021.

18. This formulation of the Chrysippus Test slightly differs from the one originally provided in Crupi and Iacona 2020. The modification is explained in Crupi and Iacona 2022a.

w at its minimum, although it may not be the only world in that position. We will call *w-minimal* any w' such that $w' \preceq_w w''$ for every w''.[19]

A model of L is defined as follows:

DEFINITION 3 A *model* of L is a triple $\langle W, O, V \rangle$, where W is a non-empty set, O is a proximity ordering on W, and V is a function such that, for each atomic formula α of L and each $w \in W$, $V(\alpha, w) \in \{1, 0\}$.

The truth of a formula α in a world w in a model M — that is, the truth of α relative to M, w — is defined as follows, assuming that $M = \langle W, O, V \rangle$ and that $w, w', w'' \ldots \in W$:

DEFINITION 4

1 $[\alpha]_{M,w} = 1$ iff $V(\alpha, w) = 1$ when α is atomic;

2 $[\neg\alpha]_{M,w} = 1$ iff $[\alpha]_{M,w} = 0$;

3 $[\alpha \supset \beta]_{M,w} = 1$ iff $[\alpha]_{M,w} = 0$ or $[\beta]_{M,w} = 1$;

4 $[\Box\alpha]_{M,w} = 1$ iff $[\alpha]_{M,w'} = 1$ for every w';

5 $[\alpha \triangleright \beta]_{M,w} = 1$ iff for every w', if $[\alpha]_{M,w'} = 1$ and $[\beta]_{M,w'} = 0$ in w', then

   (a) some w-minimal w'' is such that $[\alpha]_{M,w''} = [\beta]_{M,w''}$,

   (b) for every w'' such that $[\alpha]_{M,w''} = 1$ and $[\beta]_{M,w''} = 0$, there are w''' and w'''' such that $w'' \not\preceq_w w'''$, $w'' \not\preceq_w w''''$, $[\alpha]_{M,w'''} = [\beta]_{M,w'''} = 1$, and $[\alpha]_{M,w''''} = [\beta]_{M,w''''} = 0$.

Clause 1-4 are standard. Clause 5 phrases the Chrysippus Test as a universal sentence. When there are no 10-worlds, this sentence is vacuously true because the antecedent of the embedded conditional is not satisfied. When there are 10-worlds, instead, (a) requires that some of the closest worlds are 11-worlds or 00-worlds, and (b) requires that, for every 10-world, some 11-worlds and 00-worlds are strictly closer.

In definition 4, the truth conditions of a formula are specified relative to a world w in model M, that is, relative to a model-world pair M, w. This relativity amounts to relativity to an ordered set of worlds of the kind desired. Since the proximity ordering in M assigns a binary relation to w, namely, $\preceq_w$, to say that α is true in M, w is to say that α is true relative to $\preceq_w$, which in turn can be replaced by our index c. So, clause 5 of definition 4 tells us what it is for $\alpha \triangleright \beta$ to be true in c.

Logical consequence, indicated by the symbol $\models$, is defined in the usual way as preservation of truth in every world in every model:

DEFINITION 5 $\Gamma \models \alpha$ iff there is no M, w such that $[\beta]_{M,w} = 1$ for every $\beta \in \Gamma$ and $[\alpha]_{M,w} = 1$.

The semantics just outlined determines the logical properties of $\triangleright$. At least three of these properties deserve attention. First, $\triangleright$ obeys *Supraclassicality*:

FACT 1 If $\alpha \models \beta$, then $[\alpha \triangleright \beta]_{M,w} = 1$ for every M, w.

Proof. Assume that $\alpha \models \beta$. Then, there is no M, w such that $[\alpha]_{M,w} = 1$ but $[\beta]_{M,w} = 0$. This entails that, for every M, w, clause 5 of definition 4 is vacuously satisfied. Therefore, $[\alpha \triangleright \beta]_{M,w} = 1$ for every M, w. □

19. Crupi and Iacona 2020 employs centered systems of spheres, where centering is stronger than w-minimality as required by (iii). But (i)-(iii) will suffice for our purposes.

It is plausible to regard logical consequence as the strongest form of support that the antecedent of a conditional can provide for its consequent: α and $\neg\beta$ are evidently incompatible when their conjunction is logically impossible. Note that Supraclassicality entails *Reflexivity*, that is, we get that $[\alpha \rhd \alpha]_{M,w} = 1$ for every M, w, given that $\alpha \models \alpha$. This is in line with the view attributed to Chrysippus.[20]

Second, $\rhd$ violates *Monotonicity*:

FACT 2 $\alpha \rhd \gamma \not\models (\alpha \wedge \beta) \rhd \gamma$

Proof. Let $M = \langle W, O, V \rangle$, where $W = \{w, w', w''\}$ and O is such that $w' \not\preceq_w w$ and $w'' \not\preceq w'$. Let $[\alpha]_{M,w} = 1$, $[\beta]_{M,w} = 0$, $[\gamma]_{M,w} = 1$, $[\alpha]_{M,w'} = 0$, $[\beta]_{M,w'} = 0$, $[\gamma]_{M,w'} = 0$, $[\alpha]_{M,w''} = 1$, $[\beta]_{M,w''} = 1$, $[\gamma]_{M,w''} = 0$. Then $[\alpha \rhd \gamma]_{M,w} = 1$ because conditions (a) and (b) of clause 5 of definition 4 are satisfied. Instead, $[(\alpha \wedge \beta) \rhd \gamma]_{M,w} = 0$, because those conditions are not satisfied. □

Third, *Contraposition* holds for $\rhd$, that is,

FACT 3 $\alpha \rhd \beta \models \neg\beta \rhd \neg\alpha$

Proof. Assume that $[\alpha \rhd \beta]_{M,w} = 1$. Then either clause 5 of definition 4 is vacuously satisfied or it isn't. If it is, there is no w' such that $[\neg\beta]_{M,w'} = 1$ and $[\neg\alpha]_{M,w'} = 0$. If it isn't, then (a) some w-minimal w'' is such that $[\neg\beta]_{M,w''} = [\neg\alpha]_{M,w''}$, and (b) for every w'' such that $[\neg\beta]_{M,w''} = 1$ and $[\neg\alpha]_{M,w''} = 0$, there is a w''' such that $w'' \not\preceq_w w'''$ and $[\neg\beta]_{M,w'''} = [\neg\alpha]_{M,w'''} = 1$, and there is a w'''' such that $w'' \not\preceq_w w''''$ and $[\neg\beta]_{M,w''''} = [\neg\alpha]_{M,w''''} = 0$. In both cases, $[\neg\beta \rhd \neg\alpha]_{M,w} = 1$. □

According to the evidential account, Contraposition makes sense because incompatibility is a symmetric relation: α is incompatible with $\neg\beta$ if and only if $\neg\beta$ is incompatible with α. Since 'α supports β' is defined as 'α is incompatible with $\neg\beta$', and '$\neg\beta$ supports $\neg\alpha$' is defined as '$\neg\beta$ is incompatible with $\neg\neg\alpha$', which is equivalent to '$\neg\beta$ is incompatible with α', we get that α supports β if and only if $\neg\beta$ supports $\neg\alpha$.

6 DEDUCTIVE VALIDITY

The view that emerges from sections 4 and 5, which concerns validity broadly understood, may be summarized as follows. For any argument $s_1, \ldots, s_{n-1}; s_n$ and any interpretation i, if α, β represents $s_1, \ldots, s_n$ as understood in i, then $\alpha \Rightarrow_c \beta$ if and only if $\alpha \rhd \beta$ is true in c. Now it will be shown how a principled distinction can be drawn between deductive validity and inductive validity.

As we have seen, clause 5 of definition 4, which expresses the Chrysippus Test, amounts to a disjunction: either there are no worlds in which α is true and β is false, or conditions (a) and (b) are jointly satisfied. When the first disjunct holds, α and $\neg\beta$ are *absolutely* incompatible, as it were, hence we can say that α provides a conclusive reason for accepting β. When the second disjunct holds, α and $\neg\beta$ are *relatively* incompatible, as it were, hence we can say that α provides a defeasible reason for accepting β. Deductive validity and inductive validity are definable in terms of these two sorts of incompatibility.

20. In *Outlines of Scepticism*, II, 111, Sextus Empiricus says that 'If it is day, it is day' is true according to the third view in his list, see Sextus Empiricus 2000, p. 96.

The definition of deductive validity goes as follows: $\alpha; \beta$ is deductively valid in c if and only if in c it is impossible that α is true and β is false. That is, using the symbol $\Longrightarrow$ for deductive validity, we have that

DEFINITION 6 $\alpha \Longrightarrow_c \beta$ iff $\Box(\alpha \supset \beta)$ is true in c.

This equation between the deductive validity of an argument and the truth of the corresponding strict conditional is exactly what one should expect if one accepts the characterization of 'implication' adopted by C. I. Lewis in his analysis of conditionals: 'that relation which is present when we "validly" pass from one assertion, or set of assertions, to another assertion, without any reference to additional "evidence"'.[21] A true strict conditional may be regarded as a conditional in which the antecedent implies the consequent in this sense.

At least two important facts must be noted about definition 6. One is that deductive validity is monotonic:

FACT 4 For every c, if $\alpha \Longrightarrow_c \gamma$, then $\alpha \land \beta \Longrightarrow_c \gamma$.

Proof. Given definition 6, this means that $\Box(\alpha \supset \gamma) \models \Box((\alpha \land \beta) \supset \gamma)$. Assume that $[\Box(\alpha \supset \gamma)]_{M,w} = 1$. By clauses 3 and 4 of definition 4, there is no w' such that $[\alpha]_{M,w'} = 1$ and $[\gamma]_{M,w'} = 0$. It follows that there is no w' such that $[\alpha \land \beta]_{M,w'} = 1$ and $[\gamma]_{M,w'} = 0$. Therefore, $[\Box(\alpha \land \beta) \supset \gamma)]_{M,w} = 1$. $\Box$

The other is that deductive validity is contingent in the way illustrated in section 3:

FACT 5 For some c and c', it can be the case that $\alpha \Longrightarrow_c \beta$ but not $\alpha \Longrightarrow_{c'} \beta$.

Proof. It suffices to show that, for some M, w and M', w', it can be the case that $[\Box(\alpha \supset \beta)]_{M,w} = 1$ but $[\Box(\alpha \supset \beta)]_{M',w'} = 0$. Trivially, it can be the case that no w'' is such that $[\alpha]_{M,w''} = 1$ but $[\beta]_{M,w''} = 0$, whereas some w''' is such that $[\alpha]_{M',w'''} = 1$ but $[\beta]_{M',w'''} = 0$. $\Box$

Fact 5 provides the key to the formal treatment of the arguments A and B. Let A be formalized as $p; q$, where p stands for 'Kevin is going to the bank' (in the financial sense of 'bank') and q stands for 'Kevin is going to a financial institution'. The observation that A seems valid relative to a set of background assumptions according to which banks are necessarily financial institutions is rendered as follows: $p; q$ is deductively valid in some M, w where there is no w' such that $[p]_{M,w'} = 1$ and $[q]_{M,w'} = 0$, that is, $[\Box(p \supset q)]_{M,w} = 1$. By contrast, the observation that A seems invalid relative to a set of background assumptions according to which banks may not be financial institution is rendered by noting that $p; q$ is deductively invalid in some M, w where some w' is such that $[p]_{M,w'} = 1$ and $[q]_{M,w'} = 0$, that is, $[\Box(p \supset q)]_{M,w} = 0$. The case of B is similar, given that B is formalized in the same way.

Formal validity, as distinct from deductive validity, is definable as follows:

DEFINITION 7 $\alpha; \beta$ is formally valid iff $\alpha \models \beta$.

This definition, unlike definition 6, does not involve relativization to circumstances of evaluation. The obvious reason is that, if the condition stated in its right-hand side holds, then it trivially holds for every circumstance of evaluation. Note that the relation $\models$ mentioned here — logical consequence in L — extends logical consequence as it is defined in classical

21. C. I. Lewis 1918, p. 324.

propositional logic. So, every argument that is formally valid according to classical propositional logic will be formally valid in our sense.

From definition 7 we get the following equivalence:

FACT 6 $\alpha;\beta$ is formally valid just in case $\alpha \Longrightarrow_c \beta$ in every c.

Proof. Given definitions 6 and 7, what needs be proved is that $\alpha \models \beta$ iff $[\Box(\alpha \supset \beta)]_{M,w} = 1$ for every M, w. Assume that $\alpha \models \beta$. Then, there is no M, w such that $[\alpha]_{M,w} = 1$ and $[\beta]_{M,w} = 0$. It follows that $[\Box(\alpha \supset \beta)]_{M,w} = 1$ for every M, w. By reasoning in the opposite direction we get that if $[\Box(\alpha \supset \beta)]_{M,w} = 1$ for every M, w, then $\alpha \models \beta$. $\Box$

Fact 6 implies that formal validity is an absolute property of arguments definable in terms of the relative notion of deductive validity by quantifying over circumstances of evaluation. This is a nice result, which explains, among other things, the observation made in section 2 that formally valid arguments are insensitive to variations in the circumstances of evaluation. The argument D is valid no matter whether it is possible that (1) is true and (2) is false, because it is valid in all circumstances of evaluation.

The traditional distinction between material validity and formal validity, introduced by medieval logicians, can be handled in similar way. According to Peter Abelard, materially valid arguments are valid in virtue of their content, while formally valid arguments are valid in virtue of their form. Thus, the following argument is materially valid, because its conclusion follows from its premise in virtue of a necessary truth concerning humans:[22]

E
(15) Socrates is a human being
―――――――――――――――――
(16) Socrates is an animal

Taking into account definitions 6 and 7, material validity can be identified with deductive validity relative to some circumstances of evaluation. Thus, E is valid in virtue of a necessary truth about humans that we are actually willing to accept. More generally, to say that an argument is valid in virtue of its content is to say that the inference from its premises to its conclusion is warranted by some fact which may be taken for granted in the context in which it is used. Consequently, formal validity is definable as material validity in all circumstances of evaluation.[23]

7 INDUCTIVE VALIDITY

Inductive validity may simply be identified with plain validity, so its definition boils down to SIST. From this we get that deductive validity entails inductive validity: if α conclusively entails β, then α defeasibly entails β.

FACT 7 For every c, if $\alpha \Longrightarrow_c \beta$, then $\alpha \Rightarrow_c \beta$.

Proof. Given definition 6, this means that $\Box(\alpha \supset \beta) \models \alpha \rhd \beta$. Assume that $[\Box(\alpha \supset \beta)]_{M,w} = 1$. By clauses 3 and 4 of definition 4, there is no w' such that $[\alpha]_{M,w'} = 1$ and $[\beta]_{M,w'} = 0$. It follows by clause 5 of definition 4 that $[\alpha \rhd \beta]_{M,w} = 1$. $\Box$

22. Abelard argued that "incomplete entailments" such as E — just as the corresponding conditionals — can be explained by a theory of the topics (to be forms of so-called topical inference), see King and Arlig 2018. But what matters here is the more general idea that materially valid arguments hinge on non-formal truths.

23. Contemporary accounts of the distinction between material validity and formal validity, such as Read 1994, do not differ from Abelard's as far as this explanation if concerned.

Moreover, formal validity entails inductive validity for every c:

FACT 8 If $\alpha; \beta$ is formally valid, then $\alpha \Rightarrow_c \beta$ for every c.

Proof. Directly from definition 7 and fact 1. $\square$

Note that the converse also holds:

FACT 9 If $\alpha \Rightarrow_c \beta$ for every c, then $\alpha; \beta$ is formally valid.

Proof. Given definition 7, what needs be proved is that, if $[\alpha \triangleright \beta]_{M,w} = 1$ for every M, w, then $\alpha \models \beta$. Assume that $\alpha \not\models \beta$. Then, for some M, w, $[\alpha]_{M,w} = 1$ and $[\beta]_{M,w} = 0$. By clause 5 of definition 4, condition (b), it follows that $[\alpha \triangleright \beta]_{M,w} = 0$. $\square$

The conjunction of facts 8 and 9 shows that formal validity is related to inductive validity exactly in the same way in which it is related to deductive validity, that is, in both cases formal validity is obtained by quantifying over circumstances of evaluation.

Inductive validity, by contrast, crucially differs from deductive validity in that it is not monotonic:

FACT 10 It is not the case that, for every c, if $\alpha \Rightarrow_c \gamma$, then $(\alpha \wedge \beta) \Rightarrow_c \gamma$.

Proof. Directly from fact 2. $\square$

This shows that the analysis of inductive validity suggested here is at least minimally plausible, for it is widely acknowledged that an adequate formal theory of defeasible reasoning must not include Monotonicity. Of course, different readings of $\triangleright$ are equally compatible with this basic constraint, and nothing of what has been said so far rules out such readings. A proper defence of the semantics set out in section 5 would require a thorough discussion of its logical implications, so it would go far beyond the scope of this paper. However, it is worth pointing out one distinctive feature of the reading of $\triangleright$ adopted here, namely, that inductive validity turns out to be contrapositive:

FACT 11 For every c, if $\alpha \Rightarrow_c \beta$, then $\neg\beta \Rightarrow_c \neg\alpha$.

Proof. Directly from fact 3. $\square$

As explained in section 5, Contraposition holds in virtue of the symmetry of incompatibility. In the evidential account, this symmetry is assumed to be conceptually basic, and therefore independent of the distinction between conclusive and defeasible reasons. Consider again (12) and (14). In (12), the antecedent provides a conclusive reason for accepting the consequent: the antecedent and the negation of the consequent are absolutely incompatible. In (14), instead, the reason stated by the antecedent is defeasible, because it does not rule out the falsity of the consequent. But this difference in strength does not prevent the incompatibility between the antecedent of (14) and the negation of its consequent from being symmetric, or so it appears. In fact, Contraposition seems to holds in both cases. The conditionals below are just as compelling as (12) and (14) respectively:

(17) If Kevin is not going to a financial institution, he is not going to a bank

(18) If Angela fasts during Ramadan, she was not raised Catholic

In this respect the evidential account crucially differs from most extant theories of conditionals. In the literature on conditionals it is often taken for granted that Monotonicity and Contraposition go together, so that a non-monotonic conditional connective must also be non-contrapositive. In fact Contraposition does not hold in the most established systems of conditional logic, such as the system CK due to Chellas, the system B proposed by Burgess, or the systems V and VC considered by Lewis.[24] The evidential account, by contrast, treats Contraposition as a basic principle, assuming that non-monotonicity does not entail non-contrapositivity.

The same difference emerges when the analysis of inductive validity suggested here is compared with formal theories that aim at providing an adequate characterization of a non-monotonic consequence relation. In a seminal paper on the topic, Gabbay initially suggested a restricted set of fundamental properties for such a relation.[25] His proposal has then been elaborated and refined in different ways. Notably, Kraus, Lehmann, and Magidor identified a set of properties of non-monotonic systems — known as KLM logic — which included Gabbay's properties.[26] The current literature on non-monotonic logic contains a wide variety of formal theories that develop similar ideas. However, one point on which most of these theories tend to agree is that Contrapostion must not be included among the basic properties of a non-monotonic consequence relation.

Of course, Contraposition could be ruled out on the basis of purely formal considerations about its interaction with other principles. In particular, as Kraus, Lehmann, and Magidor pointed out, if one rejects Monotonicity, one cannot retain both Contraposition and *Right Weakening*, the principle according to which, if α defeasibly entails β, and $\beta \models \gamma$, then α defeasibly entails γ. The reason is that Contraposition and Right Weakening jointly entail Monotonicity.[27] However, considerations of this sort do not show that defeasible reasoning is non-contrapositive, unless some independent argument is provided in support of the principles that are proved inconsistent with Contraposition. In the case of Right Weakening, the question is why we should regard it as fundamental. The analysis of inductive validity outlined here suggests that a non-monotonic consequence relation can coherently be defined by assuming that Contraposition, rather than Right Weakening, is an essential feature of defeasible reasoning. The resulting theory would be considerably different from the main theories currently debated in the literature on non-monotonic logic, but perhaps no less interesting.

8 FINAL REMARKS

The upshot of this paper is that the Stoic Thesis, once adequately spelled out, is appreciably more credible than is usually believed. If one is willing to grant some fairly innocuous assumptions about validity, and is sympathetic to the idea that a conditional holds when its antecedent supports its consequent, one can coherently accept the equivalence between valid arguments and true conditionals. Of course, the formal framework outlined in the foregoing sections is not immune to criticism, and might be questioned in various ways. But this does not necessarily undermine the Stoic Thesis. Even if a different

24. Chellas 1975, Burgess 1981, D. Lewis 1973.
25. Gabbay 1985.
26. Kraus, Lehmann, and Magidor 1990.
27. Kraus, Lehmann, and Magidor 1990, pp. 180-181.

framework were adopted, the equivalence between valid arguments and true conditionals could still be maintained.

In section 6 it is assumed that deductive validity amounts to necessary truth preservation, and that formal validity is adequately expressed by a classical relation of logical consequence, the relation $\models$ defined for L. But nothing essential depends on these assumptions. Deductive validity might be defined in a different way. Similarly, some alternative characterization of formal validity might be provided. In any case, the difference would not necessarily affect the main points made above. In particular, it could still be maintained that formal validity is definable in terms of a relative notion of deductive validity by quantifying over circumstaces of evaluation.

Similar considerations hold for inductive validity. In section 7, inductive validity is defined in terms of the Chrysippus Test, in accordance with the reading of $\triangleright$ suggested. However, as noted above, this does not rule out that some other definition of inductive validity is equally admissible. If the meaning of $\triangleright$ were specified by a different semantics, we would get a different logic.[28] However, the most important facts about inductive validity, such as facts 7-10, could still be maintained.

These considerations suggest that the credibility of the Stoic Thesis does not entirely depend on the acceptance of the formal framework adopted here, so at least some non-trivial variations in the framework would still be admissible. In order to provide a convincing refutation of the Stoic Thesis, instead, it should be argued that our best theories of validity and conditionals make it untenable. But it is far from obvious that this can be done. A sheer appeal to the centrality of formal validity or the usefulness of the material reading of conditionals would definitely not suffice. Unless such refutation is provided, nothing prevents us from thinking that there is a plausible sense in which valid arguments amount to true conditionals.[29]

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28. For example, Rott's theory of "difference-making" conditionals is similar to the evidential account in several respects, including the failure of Right Weakening, although it does not validate Contraposition.

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