Abstract: Jaakko Hintikka proposed treating objectual perception sentences, such as ‘Alice sees Bob’, as *de re* propositional perception sentences. Esa Saarinen extended Hintikka’s idea to eventive perception sentences, such as ‘Alice sees Bob smile’. These approaches, elegant as they may be, are not philosophically neutral, for they presuppose, controversially, that the content of all perceptual experiences is propositional in nature. The aim of this paper is to propose a formal treatment of objectual and eventive perception sentences that builds on Hintikka’s modal approach to propositional attitude ascriptions while avoiding controversial assumptions on the nature of perceptual experiences. Despite being simple and theoretically frugal, our approach is powerful enough to express a variety of interesting philosophical views about propositional, objectual, and eventive perception sentences, thus enabling the study of their inferential relationships.

Keywords: Perception, Events, Hintikka, Possible world semantics.
1 Introduction

The following are examples of perception sentences of different syntactic types:

1. Alice sees Bob;
2. Alice sees Bob smile;
3. Alice sees that Bob is smiling.

Sentences of the first kind, which we call *objectual* perception sentences, usually involve a perceptual verb like ‘see’, ‘hear’, ‘smell’, ‘taste’, or the more general ‘perceive’, followed by a noun phrase. Sentences of the second kind resort to the so called naked infinitives, to attribute a perceptual experience involving events, like the event of Bob smiling. We will call these *eventive* perception sentences. Finally, sentences of the third kind involve a perception verb followed by a that-clause and are naturally understood as propositional attitude ascriptions. Let us call these *propositional* perception sentences.

Hintikka (1969) suggested dealing with propositional perception sentences in analogy with his (1962) now standard treatment of knowledge ascriptions. Intuitively, just like a sentence of the form \( \text{⌜α knows that } \varphi \text{⌝} \) is true if and only if \( \varphi \) is true in every possible world compatible with all that \( α \) knows, so a sentence of the form \( \text{⌜α sees that } \varphi \text{⌝} \) is true if and only if \( \varphi \) is true in every possible world compatible with all that \( α \) sees (Hintikka 1969: 155).

In contrast, Hintikka did not offer any specific *semantic* treatment for objectual perception sentences. Rather, he held that they can be analysed as *de re* propositional perception sentences (1969: 160). For instance, in his mind, (1) can be informally

1In ordinary language, there are propositional perception sentences, like ‘I see that Trump has been lying again’, where the content of the that-clause is not – strictly speaking – a *perceptual* content. For reasons of simplicity, here we shall leave this topic aside. Some readers might think that the content of every that-clause in the complement of perceptual verbs bears at most an indirect relation to the perception reported, or they might even think that the content of perception is always non-conceptual, and hence, it is never the sort of thing that is expressed in a that-clause at all. We think that these readers might find our proposal especially interesting, for our view is consistent with regarding objectual and eventive perception sentences as logically independent of propositional perception sentences.
rephrased as:

\[(1') \exists x \text{ Alice sees that } x = \text{Bob.}\]

Esa Saarinen (1983: 119) proposed extending Hintikka’s paraphrase strategy to eventive perception sentences. In Saarinen’s view, a sentence like \[(2)\] is to be paraphrased as:

\[(2') \exists e (e = [\text{Bob smiles}] \land \text{Alice sees that } e \text{ takes place}),\]

where the term ‘[Bob smiles]’ refers to the event of Bob smiling.

Hintikka’s and Saarinen’s paraphrase strategies, elegant as they may be, are not philosophically neutral, for they presuppose that ascriptions of perceptual experiences are all reducible to sentences that only involve *propositional* perception operators. In turn, this presupposition is acceptable only to those who regard the content of all perceptual experiences as reducible to some kind of propositional content. Philosophically speaking, however, the view that the content of perceptual experiences is ultimately propositional in nature is highly controversial, and it has been recently questioned, among others, by Tim Crane (2009), Tyler Burge (2010), Christopher Gauker (2012), and Bencey Nanay (2013). In Nanay’s words:

> We have no reason to believe that all mental representations are linguistically or propositionally structured… Some (but not all) mental states have content. Some of these (but not all of them) have propositional content. But perceptual states don’t. (2013: 37)

In our mind, lack of philosophical neutrality is a serious problem for Hintikka’s and Saarinen’s paraphrase strategies. The nature of perceptual content is an extremely delicate issue, with deep ramifications in epistemology and the philosophy of mind, and it is not up to formal semanticists to settle it. What is reasonable to expect from formal semanticists in this area is, rather, that they devise general linguistic and

\[\text{The extension of Hintikka’s logic of perception to sentences involving events was first proposed by Niiniluoto 1982 and then elaborated in detail by Saarinen 1983.}\]
inferential frameworks within which alternative conceptions of perceptual content can be expressed and compared and their consequences studied.

In this paper, we provide a framework of this type. More specifically, we propose a formal treatment of objectual and eventive perception sentences that builds on Hintikka’s modal approach to propositional attitude ascriptions while remaining neutral on the relationships between perceptual and propositional content. Crucially, in contrast to Hintikka’s and Saarinen’s paraphrase treatments, we take objectual and eventive perception statements at face value, making no assumption as to whether they can be analysed in terms of propositional perception sentences. On the other hand, in contrast to ‘naive’ approaches (Vlach 1983), which equate objectual and eventive perception with extensional relations between agents and objects/events (see, e.g., Higginbotham 1983), our proposal allows one to retain a considerable semantic uniformity in the analysis of propositional and eventive/objectual perception sentences, by letting the complement of perception verbs always correspond to a class of worlds suitably related to the world of evaluation. This uniformity makes it very easy to assess the semantic consequences of a variety of principles concerning the relation between propositional, objectual, and eventive perception sentences. As a result, and as we shall see in detail, our framework is especially well suited for thoroughly studying the inferential import of different philosophical views on perception.

We would like to stress, once again, that the focus of the paper is the exploration of a frugal, philosophically neutral logic of perception. We believe that a number of remarkable points on the topic can be brought to the attention of the reader by only resorting to minimal semantic resources. So before presenting the details of our

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3 This feature of our account might trigger an objection. By letting the complement of perception verbs always correspond to sets of worlds, so the objection goes, we entail that perception is always directed towards propositions (qua sets of worlds). Thus, our semantics is not neutral as to whether the content of perceptual states is propositional in nature, contrary to what we claim. We think that the objection is misguided. The fact that a semantic theory assigns a certain kind of value to a given expression need not (and generally is not taken to) imply that the mundane denotatum of the expression belongs to that kind. For instance, in Montague’s 1970 classic semantics, proper names like ‘Jack’ denote sets of properties, but very few philosophers would take this semantic view to entail that Jack is a set.
proposal, it is important to underline that our aim is not to offer a full-fledged formal semantics, comparable in complexity to the quite sophisticated approaches available in the logic and linguistic literature on naked infinitives, perception reports, and related constructions (see, e.g., Barwise 1981, Higginbotham 1983, 1999, Asher & Bonevac 1985, Neale 1988, Aloni 2001, Zucchi 2015, Abusch & Rooth 2017). Rather, our goal is to elaborate on Hintikka’s approach to knowledge and perception – a well-established landmark in philosophical logic – while keeping the resulting semantics as simple and philosophically neutral as possible. The reader may also think of our proposal as a formal toolkit for helping philosophers gain a better grasp on the inferential relations between different views on perception.

2 Hintikka on knowledge

As mentioned above, Hintikka (1969) suggested dealing with propositional perception sentences in analogy with his (1962) treatment of knowledge ascriptions. Before discussing perception, let us recall his approach to knowledge.

Let $L$ be a standard propositional language enriched with a knowledge operator $K$. (Throughout the current paper, for the sake of readability, we use formal expressions autonomously, viz., without resorting to quotation devices.) The formulae of $L$ are defined recursively as follows: propositional letters $Q, R, \ldots$ are formulae; if $\phi$ is a formula, then its negation $\neg \phi$ is a formula; if $\phi$ and $\psi$ are formulae, then their conjunction $\phi \land \psi$ is a formula; if $\phi$ is a formula, then $K \phi$ is a formula; nothing else is a formula. Other Boolean connectives are defined as usual. $K \phi$ is the formal counterpart of a knowledge ascription of the form $\langle \text{A certain agent } \alpha \text{ knows that } \phi \rangle$ (or $\langle \text{It is known that } \phi \rangle$ if the identity of $\alpha$ is deemed irrelevant).

Formulae are evaluated on Kripke models $M = (W, R, \sigma)$, where $W$ is a non-empty set of possible worlds, $R$ is a binary reflexive relation on $W$, and $\sigma$ is a function that maps each letter to a subset of $W$. If $w$ and $v$ are worlds in $W$, then $w R v$ entails, intuitively, that $v$ is compatible with what the agent $\alpha$ knows in $w$.

We let $[\phi]_M$ indicate the intension of a formula $\phi$ in model $M$. Intuitively, $[\phi]_M$
is the set of worlds where $\phi$ is true in $M$. The notion of truth of a formula $\phi$ in a world $w$ in a model $M$, in symbols $w \in [\phi]_M$, is defined as follows:

1. For propositional letters $Q$, $w \in [Q]_M$ if and only if $w \in \sigma(Q)$;
2. $w \in [\neg \phi]_M$ if and only if it is not the case that $w \in [\phi]_M$;
3. $w \in [\phi \land \psi]_M$ if and only if $w \in [\phi]_M$ and $w \in [\psi]_M$;
4. $w \in [K\phi]_M$ if and only if $v \in [\phi]_M$ for all $v \in W$ such that $w R v$.

Intuitively, clause (iv) says that $\alpha$ knows that $\phi$ in $w$ if and only if $\phi$ is true in all the worlds compatible with what $\alpha$ knows in $w$. It is usual to assume that only true propositions can be objects of knowledge, that is to say, that knowledge is factive. This is the reason why reflexivity is imposed on the accessibility relation $R$, thus ensuring that the world $w$ is always included within the worlds compatible with what $\alpha$ knows in $w$. As a result, the following principle, which expresses the factivity of knowledge, is valid:

$K\phi \rightarrow \phi$.

### 3 Hintikka on propositional perception

We have just seen that in Hintikka’s treatment of knowledge ascriptions, a cognitive agent $\alpha$ in $w$ is associated with a set of possible worlds representing all the different ways reality could be, compatibly with $\alpha$’s propositional knowledge in $w$. These worlds are called epistemic alternatives to $w$. Hintikka suggested dealing with propositional perception in a strictly analogous way by associating a perceiving agent $\alpha$ in $w$ with a set of possible worlds representing all the different ways reality could be, compatibly with $\alpha$’s propositional perceptions in $w$ (Hintikka 1969: 155). Let us call these worlds propositional perceptual alternatives to $w$.

The language $L'$ is identical with $L$, except that the place of $K$ is taken by a perception operator $P$. Intuitively, $P$ is the formal counterpart of ‘perceive’ (or, interchangeably, of more specific verbs like ‘see’, ‘hear’ . . . ; see Hintikka 1969: 156). Accordingly, $P\phi$ corresponds to sentences of form “$\alpha$ perceives (see, hear . . .) that $\phi$”.
or, if you prefer, "It is perceived (seen, heard . . . ) that \( \phi \)."

We let a Kripke model \( \mathfrak{M}' \) be a triple \((W, R_P, \sigma)\), where \( W \) and \( \sigma \) are as above, and \( R_P \) is a binary relation on \( W \) that, intuitively, connects each world with its propositional perceptual alternatives. Truth in a world in a model \( \mathfrak{M}' \) is defined as follows:

(i')--(iii') similar to clauses (i)--(iii) above;
(iv') \( w \in \llbracket \mathbf{P}\phi \rrbracket_{\mathfrak{M}'} \) if and only if \( v \in \llbracket \phi \rrbracket_{\mathfrak{M}'} \) for all \( v \) such that \( w R_P v \).

It is common to assume that propositional perception is factive (see, e.g., Williamson 2000: 34). In the remainder of the current paper, we shall stick to this assumption and require that \( R_P \) be reflexive, so ensuring that no one can perceive (see, hear . . . ) that \( \phi \) in \( w \) if it is not the case that \( \phi \) in \( w \). As a result, the following principle, which expresses the factivity of propositional perception, turns out to be valid:

(B) \( \mathbf{P}\phi \rightarrow \phi \).

4 Semantics for objectual and eventive perception sentences

As mentioned above, Hintikka proposed extending his treatment of propositional perception sentences to objectual perception sentences by paraphrasing the latter in terms of the former. We will go in a different direction. Like Hintikka, in evaluating all kinds of perception sentences, we will resort to a set of possible worlds representing the alternative ways the world could be, compatibly with what a given agent perceives. However, unlike Hintikka, we will not reduce all perception sentences to (constructions over) propositional perception sentences.

We have seen (in great detail) how Hintikka suggested dealing with propositional perception in analogy with his treatment of knowledge. In Hintikka’s mind, what changes when passing from one propositional attitude to the other is the relation of
compatibility at play. Sentences of form ” α knows that φ ” are dealt with in terms of a relation of compatibility between worlds and the things that α knows; sentences of form ” α perceives that φ ”, in terms of a relation of compatibility between worlds and the things that α propositionally perceives. We think that the analogy goes further than Hintikka envisaged. It is not just propositional perception that can be profitably dealt with by appealing to a suitable relation of compatibility. Also non-propositional perception can be. More specifically, objectual perception can be dealt with by appealing to a notion of compatibility between worlds and the things that the relevant agent objectually perceives. To make this idea work, we must distil a reasonable notion of compatibility between worlds and non-propositional objects of perception, which mimics Hintikka’s notion of compatibility between worlds and propositions.

Now, when propositional objects of perception are at stake, compatibility relative to a world w (in Hintikka’s sense) requires truth: w is compatible with the perception that φ if and only if, in w, it is true that φ. Our guiding thought is that when dealing with non-propositional targets, truth is to be replaced with actuality. More specifically, we take w to be compatible with a perceived object o if and only if, in w, o is actual. To say that an object o is actual in a world w, in our sense, is tantamount to saying that o is located in w or, if you prefer, that o exists in w.

Here, we are treating actuality as a perspectival, world-relative feature. However, we do not attach any special philosophical significance to this choice. If you think that there exists an absolute, non-relative notion of actuality (see, e.g., Bricker 2006), you can treat actuality, in the sense relevant here, as a different, possibly derivative notion. Alternatively, in what follows, you can replace ‘is actual’ with ‘is located’.

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4 See (Iaquinto & Spolaore 2019) for a semantic analysis of ascriptions of knowledge of acquaintance based on a similar strategy.

5 Please do not understand this casual talk of ‘propositional objects’ as entailing a full-fledged ontological commitment towards semantic entities.

6 Here we are presupposing that, in some sense, objects can be identified across worlds. However, we take no definite stance as to how identity across worlds is cashed out from a philosophical viewpoint, and we rest content with the assumption that at least some objects/events are actual in more than one world.
or with ‘exists’, or what have you. All that we require is that actuality (locatedness, existence . . . ) be regarded as a possibly contingent feature, that is, as a feature that some object possesses in some worlds but not in all of them. Let us stress that for our purposes, actuality need not be taken as a property that only ‘external’, public entities can possess. Accordingly, our proposal can be made consistent with the view that perception is (primarily) directed towards private, phenomenal entities.

We have said that the notion of compatibility at play in objectual perception involves actuality. The same goes with eventive perception. We surmise that a world \( w \) is compatible with a perceived event \( e \) if and only if, in \( w \), \( e \) is actual. To say that an event \( e \) is actual in \( w \) is to say that \( e \) is located in \( w \) or (in Saarinen’s terminology) that it takes place in \( w \). Again, we assume that actuality is a possibly contingent feature of events.

### 4.1 Basic semantics

It is straightforward to turn the ideas outlined thus far into a formal semantics for perception sentences. In what follows, to make the analogy between our approach and Hintikka’s treatment of propositional perception sentences clearer, we stick with a language \( \mathcal{L}_p \) that is as similar as possible to the above propositional language \( \mathcal{L}' \).

Language \( \mathcal{L}_p \) includes a countable set \( \Omega \) of objectual letters \( o, o' \ldots \) and a countable set \( \Delta \) of eventive letters \( e, f \ldots \). Intuitively, the letters in \( \Omega \) correspond to objects (like Bob) and the letters in \( \Delta \) to events (like Bob smiling). In our informal moments, we shall also use these letters to indicate the corresponding object or event. For each objectual letter \( o \), \( \mathcal{L}_p \) includes one or more object-involveventive letters \( e_o, e'_o \ldots \). Object-involveventive (eventive) letters are the formal counterparts of naked infinitive clauses like ‘Bob smile’ – intuitively, they correspond to events having the indicated object as participant.

We collectively indicate objectual and eventive letters of \( \mathcal{L}_p \).

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\(^{7}\)This is a common view among both sense data theorists (e.g., Robinson 1994, Jackson 1977, Casullo 1987, Garcia-Carpintero 2001, O’Shaughnessy 2003) and internalist representationalists (e.g., Rey 1993, 1997, 1998).

\(^{8}\)Nothing forbids further extending \( \mathcal{L}_p \) by adding object-involveventive eventive letters that correspond to relational naked infinitives such as ‘Alice kiss Bob’. However, they have limited interest for our purposes, and we shall ignore them. In the next section, we shall define an extended language in
as terms. If \( x \) is a term, \( \mathcal{L}_p \) includes propositional atoms of forms \( Ax \) and \( Px \). \( Ax \) is called an actuality sentence. Informally, \( Ao \) says that the object \( o \) is actual (exists), while \( Ae \) says that the event \( e \) is actual (takes place). Intuitively, an atom of form \( Px \) says that \( \alpha \) perceives \( x \).

The formulae of \( \mathcal{L}_p \) can be defined recursively as follows: if \( x \) is a term of \( \mathcal{L}_p \), then \( Px \) and \( Ax \) are atoms of \( \mathcal{L}_p \); atoms are formulae; if \( \phi \) and \( \psi \) are formulae of \( \mathcal{L}_p \), then \( \neg \phi, \phi \land \psi \) and \( P \phi \) are formulae of \( \mathcal{L}_p \). Nothing else is a formula.

Three remarks about \( \mathcal{L}_p \) are in order. First, objectual or eventive letters can never occur as formulae unless they are prefixed with the operators \( A \) or \( P \). Thus, expressions like \( o \) and \( e \) are not formulae of \( \mathcal{L}_p \). Second, \( \mathcal{L}_p \) does not enrich \( \mathcal{L}' \) with any new perception operator. In \( \mathcal{L}_p \), like in English, the distinction between propositional, objectual, and eventive perception sentences is encoded by a difference in form and not by a difference in the lexicon of perception. Thus, just as \( P \phi \) corresponds to propositional perception sentences, so \( Po \) and \( Pe \) correspond, respectively, to objectual and eventive perception sentences. Third, if we think of an eventive letter \( e \) as the formal counterparts of a naked infinitive clause, we can think of the actuality sentence \( Ae \) as the formal counterpart of the corresponding indicative sentence. Thus, for instance, if \( e_o \) reads ‘Bob smile’, then \( Ae_o \) reads ‘Bob smiles’. Atoms of form \( Ax \) are the closest analogue in \( \mathcal{L}_p \) of the propositional letters of \( \mathcal{L} \) and \( \mathcal{L}' \).

We let a Kripke model \( \mathfrak{M}_p \) be a tuple \((W,R_P,R_O,R_E,\varepsilon)\), where \( W \) and \( R_P \) are as above. Intuitively, binary relation \( R_O \) (\( R_E \)) holds between worlds \( w, v \) if and only if \( v \) is compatible with the objectual (eventive) perceptions of the relevant agent \( \alpha \) in \( w \). The interpretation function \( \varepsilon \) is a mapping from objectual and eventive letters to subsets of \( W \). Intuitively, if \( x \) is a term, then \( \varepsilon(x) \) is the set of the worlds where the object or the event denoted by \( x \) is actual. We let the intension of a term \( x \) relative to \( \mathfrak{M}_p \), in symbol \([x]_{\mathfrak{M}_p}\), be a set of worlds in \( W \) – intuitively, the worlds where the denotation of \( x \) is actual in \( \mathfrak{M}_p \). We impose that \( \varepsilon(x) = [x]_{\mathfrak{M}_p} \) for all terms \( x \) of \( \mathcal{L}_p \).

The notion of truth in a world in a model \( \mathfrak{M}_p \) is defined as follows:

\[
\begin{align*}
(i'') & \quad a. \ w \in [Ax]_{\mathfrak{M}_p} \text{ if and only if } w \in [x]_{\mathfrak{M}_p}; \\
& \quad b. \ w \in [Px]_{\mathfrak{M}_p}, \text{ with } x \text{ objectual term, if and only if } v \in [x]_{\mathfrak{M}_p} \text{ for all worlds}
\end{align*}
\]

which certain specific examples of relational naked infinitives are expressible.
\[ v \in W \text{ such that } w R_O v; \]

c. \( w \in [P x]_{\mathfrak{M}_p}, \) with \( x \) eventive term, \( v \in [x]_{\mathfrak{M}_p} \) for all worlds \( v \in W \) such that \( w R_E v; \)

(ii’’)-(iv’’’) similar to clauses (ii’) (iv’) above.

Logical properties are defined in the usual way.

We assume, plausibly enough, that all participants to actual events are themselves actual. So, for any object-involving letter \( e_o \), we require that \( \varepsilon(e_o) \subseteq \varepsilon(o) \). The following schema is therefore valid:

\[
(C) \quad Ae_o \rightarrow Ao \quad \text{(e.g., ‘If Bob smiles, then Bob is actual’.)}
\]

We also take for granted that all the things (objects or events) we perceive are actual, that is, that we do not perceive merely possible things (see also, e.g., Crane & French 2017: esp. § 1.1.2). Accordingly, we require that \( R_O \) and \( R_E \) be reflexive, thus ensuring that the following principles are valid:

\[
(D) \quad Po \rightarrow Ao, \\
(E) \quad Pe \rightarrow Ae.
\]

Principles \( (D) \) and \( (E) \) can be seen as the objectual and eventive analogues of the above factivity principle \( (B) \). By \( (D) \) and \( (E) \), perceiving an object \( o \) (or an event \( e \)) requires that \( o (e) \) be actual, just like, by \( (B) \), perceiving that \( \phi \) requires \( \phi \) to be true.

If we drop the reflexivity of the accessibility relations, we are in a position to account also for non-factive uses of perception sentences, some of which are well attested in natural languages. For instance, a sentence like ‘Bob sees the stick bend’ can be used to report Bob’s impression that the stick bends while being immersed in water, even if the stick is not actually bending.\(^9\) Moreover, as Hintikka (1975: 68) himself highlights, avoiding the assumption that the accessibility relation is reflexive allows to ‘discuss such epistemologically interesting problems as illusions, hallucinations, perceptual mistakes, impossible objects, etc.’. Dropping the factivity constrain

\(^9\)See Bourget 2017 for a recent defence of the view that non-factive uses of perception sentences are to be taken seriously from a semantic viewpoint.
on the objectual perception operator, for instance, allows to model what we could describe as its weak counterpart, to be employed in the formal treatment of seeing as statements (Howell 1972) and visual illusions and hallucinations (Niiniluoto 1982). Other interesting applications can be offered in dealing with objectual and eventive imagination sentences (Niiniluoto 1985) and with (non-veridical) perceptions in pictorial narratives (Abusch & Rooth 2017). Although we recognise the philosophical importance of such applications, in this paper we will focus only on factive uses of the perception operators, leaving the extension of our logic to other classes of sentences for future work.

4.2 A refinement: events of perception

It is reasonable to suppose that there are events of perception, such as the event of Alice seeing Bob. And it appears that events of perception can be perceived too. In order to account for this possibility, we need to (mildly) extend our language $L_p$.

Let $L^{+}_p$ be a language that is identical to $L_p$ except that it contains an additional, ‘naked infinitive’ functor $ni$. Intuitively, $ni$ turns certain sentences $\phi$ of $L^+_p$ into the corresponding naked infinitive phrases $ni\phi$. We take phrases of form $⌜ni\phi⌝$ to be (complex) terms for events. As such, they can be the object of eventive perception sentences of form $Pni\phi$.

Not all English sentences can be turned into naked infinitive phrases. For instance, conditionals (e.g., ‘If Alice smiles, Bob blinks’) have no naked infinitive counterpart. Moreover, whether and how negative naked infinitives (e.g., ‘Bob not smile’) can be made to correspond to (negative) events is a highly controversial issue, and one that is broadly independent of the logic and philosophy of perception (see, e.g., Casati & Varzi 2020 § 2.5, Bernard & Champollion 2018). Finally, it is unclear whether perception sentences involving conjunctive/disjunctive naked infinitives (e.g., ‘Alice sees Bob smile and/or Ed blink’) are anything but syntactical transformations of conjunctions/disjunctions of perception sentences (‘Alice sees Bob smile and/or she sees Ed blink’) (see, e.g., Vlach 1983: 137–138).

For these reasons, in what follows, we make the simplifying stipulation that the functor $ni$ does not apply to formulae whose main operator is a Boolean connective –
which means, \( \text{ni} \) only applies to formulae whose main operators are \( A \) or \( P \). Accordingly, the set of terms of \( \mathcal{L}_p^+ \) can be defined as follows: if \( x \) is an objectual or eventive letter of \( \mathcal{L}_p^+ \), then \( x \) is a term of \( \mathcal{L}_p^+ \); if \( P\phi \) is a (propositional, objectual, eventive) perception sentence of \( \mathcal{L}_p^+ \), then \( \text{ni}P\phi \) is a term of \( \mathcal{L}_p^+ \); if \( A\phi \) is an actuality sentence, then \( \text{ni}A\phi \) is a term of \( \mathcal{L}_p^+ \); nothing else is a term of \( \mathcal{L}_p^+ \).

In order to have a workable definition of truth in models \( \mathfrak{M}_p \) for sentences of \( \mathcal{L}_p^+ \), there is no need to introduce any modification in the above clauses \((i'')\)–\((iv'')\). All we need to do is to extend the notion of intension of terms to complex eventive terms of form \( \text{ni}\phi \). To this aim, we let the intension \( \llbracket \text{ni}\phi \rrbracket_{\mathfrak{M}_p} \) of a complex eventive term \( \text{ni}\phi \) coincide with the intension \( \llbracket \phi \rrbracket_{\mathfrak{M}_p} \) of the embedded sentence \( \phi \), that is, with the class of worlds where \( \phi \) is true. As a result, our semantics validates the biconditional \( Ani\phi \leftrightarrow \phi \). This is only natural: reasonably, the event of Alice perceiving Bob smile is actual (takes place) exactly in those worlds where Alice perceives Bob smile, and the event of Bob smiling is actual precisely in those worlds where Bob smiles.

Let us observe that the resulting semantics is philosophically neutral in that it does not validate any specific principle concerning (the perception of) events of perception, except for those principles that trivially follows from the factivity of perception, like:

\[(F) \quad P\text{ni}\phi \rightarrow \phi.\]

We introduced the operator \( \text{ni} \) to describe cases in which events of perception are themselves perceived. But our language \( \mathcal{L}_p^+ \) only allows us to speak about iterated perceptual events performed by a single agent (e.g., Alice perceiving Alice perceive something). Cases in which an agent perceives another agent perceiving something are not expressible in \( \mathcal{L}_p^+ \). It is possible to overcome this limitation by adopting a

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\(^{10}\) By the (idealising) assumption that propositional perception is closed under entailment, this principle implies the equivalence \( PA\text{ni}\phi \leftrightarrow P\phi \) (e.g., Alice perceives that the event of Bob smiling is actual if and only if she perceives that Bob smiles). This equivalence might sound problematic, for it seems to put strong constraints on the relation between propositional and eventive perception. Arguably, however, this impression is based on a confusion between \( PA\text{ni}\phi \leftrightarrow P\phi \) and the seemingly similar (but invalid) schema \( P\text{ni}\phi \leftrightarrow P\phi \) (e.g., Alice perceives Bob smile if and only if she perceives that Bob smile). The validity of \( P\text{ni}\phi \leftrightarrow P\phi \) would indeed lead to a collapse of propositional and eventive perception sentences. Nothing similar can be said about \( PA\text{ni}\phi \leftrightarrow P\phi \), which does not concern eventive perception and is not especially problematic in our idealised framework (see also the discussion about principle \((G)\) below, p.15).
more powerful language, which allows us to make implicit or explicit reference to a plurality of agents. However, the aim of dealing with perception in a multi-agent framework vastly exceeds the limited purposes of this paper. For this reason in what follows, we shall content ourselves with the limited expressive resources provided by our simple language $L_{p}^+$. 

5 Philosophical applications

As stressed above, we are not assuming that perception sentences can always be analysed in terms of propositional perception sentences. This does not prevent us, however, from exploring how propositional, objectual, and eventive perception sentences relate to one another. Indeed, our framework is especially well suited for formally studying these relations.

5.1 How propositional perception relates to objectual and eventive perception

Let us start by considering the relations between propositional and (factive) objectual perception sentences. To begin with, it is prima facie plausible to suppose that (factive) objectual perception has always some import on propositional perception. For instance, it is natural to think that Bob cannot see Alice without seeing that she is a physical thing, that she is close to him, or at least that she is actual. If this is correct, then it is natural to subscribe to the following principle:

(G) $P_o \rightarrow P\alpha o$ (viz., if an agent $\alpha$ perceives an object $o$, then $\alpha$ perceives that $o$ is actual).

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11 A natural choice would be a language $L_{p}^{++}$ including, in place of a unique perception operator $P$, a set of perception operators $\{P', P'' \ldots \}$, each corresponding (informally) to an agent. A semantics for $L_{p}^{++}$ can be defined by modifying the above semantics for $L_{p}^{+}$ along two lines. First, models for $L_{p}^{++}$ must include, in place of the three accessibility relations $R_P, R_O$ and $R_E$, three corresponding sets of agent-indexed accessibility relations $\{R'_{P}, R''_{P} \ldots \}, \{R'_{O}, R''_{O} \ldots \}, \{R'_{E}, R''_{E} \ldots \}$. Second, the above semantic clauses $[w''] [v'] \beta$ and $[v'] \beta$ must be modified by replacing any reference to the perceptual operator $P$ and to the accessibility relations $R_P, R_O$ and $R_E$ with a reference to their suitable agent-indexed counterparts.
Albeit natural, principle (G) is not philosophically uncontroversial. Consider, for instance, Dretske's (1969, 1981) distinction between epistemic and non-epistemic perception. According to Dretske, objectual perception can be non-epistemic in the sense that it need not come with any accompanying propositional attitude about the perceived object – someone, say, an infant, may perceive an object without perceiving that the object has any specific property. There is no general agreement as to whether objectual perception is non-epistemic in this sense (see Demircioglu 2017 for a recent discussion of the topic).

Our general framework is consistent both with the adoption of (G) and with its rejection, so offering a neutral tool with which one can contrast different views on perception. Let us note, moreover, that one may hold (with Dretske) that propositional perception and objectual perception are conceptually independent from one another, while insisting that (G) is a plausible principle when what is at stake is the actual behaviour of competent perceiving agents, who are very unlikely to perceive something without realising that the thing is actual. Alternatively, one can draw a distinction between explicit and implicit perception, in analogy with the distinction between implicit and explicit belief (and knowledge) discussed in the formal epistemology literature. For instance, by adapting to our system the core idea of awareness logic (Fagin & Halpern 1988), one can say that an agent α explicitly perceives that φ if and only if α perceives that φ while being aware of φ. The underlying thought is that, for example, Alice might perceive (implicitly) that Bob is actual without being aware of what she is perceiving and, so, without perceiving explicitly that Bob is actual. Explicit perception is thus stronger than implicit perception, and it need not be closed under entailment. If this distinction is adopted, then one can accept (G) as a valid principle when dealing with implicit perception, while rejecting it when it comes to explicit perception.

Within our framework, at any rate, it is easy to show that (G) corresponds to the following property (the proof is straightforward):

(G) For any worlds w, v, if w R_P v, then w R_O v (viz., R_P \subseteq R_O).

Intuitively, (G) says that all the propositional perceptual alternatives to a world are also (say) objectual perceptual alternatives to that world. If we think of the
magnitude of the information provided by a certain source as inversely proportional to the amount of worlds that are compatible with that information, we can conclude that objectual perception provides at most as much information as propositional perception. Let us observe that the equivalence between this view and principle (G) is far from obvious, and it is remarkable that the framework outlined thus far allows one to formally prove it.

It is well known that Hintikka’s framework allows one to model various principles of epistemic introspection. One can formalise, for instance, the idea that if \( \alpha \) knows that \( \phi \) then \( \alpha \) knows that \( \alpha \) knows that \( \phi \). Analogous principles, of course, can be introduced within Hintikka’s logic of perception attributions. Here, we are interested in an introspection principle connecting propositional perception with objectual perception. Consider the following schema:

(H) \( P_o \to PP_o \) (viz., if \( \alpha \) perceives \( o \), then \( \alpha \) perceives that \( \alpha \) perceives \( o \)).

Our framework allows us to highlight an interesting inferential relation between this introspection principle and the above principle (G) (viz., \( P_o \to PA_o \)): as far as objectual perception is taken to be factive, (H) is strictly stronger than (G). In other words, if \( R_O \) is reflexive, the introspection principle (H) entails (G) but not vice versa.\(^{12}\) Let us also stress that within our framework, no introspection principle involving only propositional perception (such as, for instance, \( P\phi \to PP\phi \)) immediately entails any introspection principle that, like (H), concerns non-propositional perception. This is as it should be, at least if one thinks that the issue whether propositional and non-propositional perceptions are independent of one another should not be adjudicated by logic or semantics alone (see also above, p. 3). Further principles of epistemic introspection that can be analysed within our framework include, among others, \( P\phi \to \)

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\(^{12}\)To prove this, it is sufficient to observe that, by standard correspondence theory, (H) corresponds to the following frame property:

(\[\text{(H')}\]\) If \( w R_P v \) and \( v R_O v' \), then \( w R_O v' \),

which, by the reflexivity of \( R_O \), clearly entails (G) (viz., if \( w R_P v \), then \( w R_O v \)). The converse entailment does not hold, however, for (G) does not exclude that, for some \( w, v, v' \), (H) is false — that is, that \( w R_P v, v R_O v' \), but \( w R_O v' \).
$PniP\phi$ (if $\alpha$ perceives that $\phi$, then $\alpha$ perceives $\alpha$ perceive that $\phi$), $Po \rightarrow PniPo$ (if $\alpha$ perceives $o$, then $\alpha$ perceives $\alpha$ perceive $o$), $Pe \rightarrow PniPe$ (if $\alpha$ perceives $e$, then $\alpha$ perceives $\alpha$ perceive $e$). For reasons of time, we leave the discussion of these further principles to another occasion.

As we underlined above, a crucial feature of our framework is that it allows us to express and compare different views on perception. It is thus interesting to note that to formalise Hintikka’s view that objectual perception sentences are ultimately reducible to propositional perception sentences, it suffices to take both $\square$ and its converse as axioms:

(I) $PA_o \rightarrow P_o$

(I) expresses the principle that no agent can perceive that an object $o$ is actual without perceiving $o$. Intuitively, taking the conjunction of $\square$ and (I) as valid leads to a collapse of the distinction between the alternatives associated with the agent in evaluating propositional perception sentences and those associated in evaluating objectual perception sentences. That is to say, the sets of alternatives against which we evaluate propositional perception sentences and objectual perception sentences are one and the same, just like in Hintikka’s proposal.

Virtually all that we have said so far on the relation between objectual and propositional perception also holds for the relation between eventive and propositional perception. Again, it is natural to maintain that if an agent $\alpha$ perceives an event $e$, then $\alpha$ also perceives that $e$ is actual – that is to say, that $e$ is happening. Thus, by parity of reasoning, we can conclude that the following eventive counterpart of (G) is plausible:

(J) $Pe \rightarrow PA_e$ (viz., if $\alpha$ perceives $e$, then $\alpha$ perceives that $e$ is actual).

The above remarks (p. 15) about principle (G) can be extended to (J). So, one might hold that (J), just like (G), is a reasonable principle when suitably restricted to certain classes of agents or to certain kinds of perception.

Within our framework, one can express the view that eventive perception sentences are reducible to propositional perception sentences by adding the converse of (J) as an axiom:
(K) \( PA_e \rightarrow Pe \)

As seen here in the case of \( G \) and \( U \) if one adopts both \( J \) and \( K \) then one is committed to evaluating eventive perception sentences relative to the same background of perceptual alternatives as propositional perception sentences.\(^{13}\)

5.2 Objectual and eventive perception

Consider the following inference:

(4) Alice sees Bob smile. Therefore, Alice sees Bob.

It is tempting to regard (4) as valid. For how is it possible to see a person smile without at the same time seeing that person? More generally, it appears that a sentence of form \( \langle \alpha \text{ sees } o \rangle \) take part in the event \( e \) cannot be true without the corresponding sentence of form \( \langle \alpha \text{ sees } o \rangle \) being true as well. (Among others things, this view is consistent with the thesis that events are somehow metaphysically dependent on objects; see, e.g., [Lombard 1986].) If that is correct, then the following principle is valid:

(L) \( Pe_o \rightarrow Po \)

For any object \( o \), in \( L_p^+ \) there is a co-intensional term \( niA_o \) that denotes the event (or the eventuality) of \( o \) existing – viz., an event that, by necessity, is actual if and only if \( o \) is. In our framework, it is natural to take \( niA_o \) to denote an event involving \( o \). Reasonably enough, then, principle (L) entails:

(M) \( PniA_o \rightarrow Po \)

Let us observe that principle (L) (along with (M)) is not generally valid within our framework (not even if we accept, in accordance with principle (C), that the

\(^{13}\)However, the converse conditional does not hold. For instance, although [Saarinen 1983] evaluates propositional and eventive perception sentences relative to the same background of alternatives, he would not accept the conjunction of \( J \) and \( K \) for he does not regard *de dicto* propositional perception sentences as equivalent to their *de re* counterparts. To adequately account for the relation between *de dicto* and *de re* perception sentences, a more expressive framework than the one introduced thus far would be needed (see also below, p. 20).
actuality of $e_o$ entails the actuality of $o$). Thus, if we want (L) to be valid, we have to impose some further constraint on our frames, namely, that they satisfy the following property:

\[ (L') \text{ For any worlds } w, v, \text{ if } w R_O v, \text{ then } w R_E v \text{ (viz., } R_O \subseteq R_E) \]

If, as above, we take the informativeness of a source as inversely proportional to the amount of worlds that are compatible with the information it provides, then (L) says that eventive perception provides, at most, as much information as objectual perception. This conclusion may sound surprising (if anything, it is eventive perception that looks informationally richer than objectual perception), and it is also surprising to note that it corresponds to a seemingly innocent principle like (L).

Also the converse of principle (M) may appear to be plausible, as it is tempting to suppose that, every time an agent $\alpha$ perceives an object $o$, $\alpha$ also perceives $\alpha$ exist:

\[ (N) \quad P o \rightarrow P niA o \]

Interestingly enough, principle (N) is consistent with the claim that objects metaphysically depend on the events in which they take part (see Parsons 1991).

Notoriously, there are philosophers who think that metaphysically speaking, there is no significant distinction between objects and events. They tend to conceive of ordinary objects (chairs, persons, cats, etc.) as temporally extended entities, as four-dimensional ‘worms’ that, in all respects, behave as events (see, among others, Quine 1950, Lewis 1986, Heller 1990, Sider 2001). Within such a framework, any object corresponds to an event (say, the object qua event), and it seems correct to say that an agent $\alpha$ perceives an object $o$ if and only if $\alpha$ perceives $o$ qua event. If we let $niA o$ represent such event, then the following principle turns out to be valid:

\[ (O) \quad P o \leftrightarrow P niA o \]

\[ ^{14} \text{To see this, it is enough to consider a sample counter-model } M_p = (W, R_P, R_O, R_E, \varepsilon) \text{ such that } W = \{w, w', w''\}, R_P \text{ is a suitable reflexive relation, } R_O \text{ is the reflexive closure of } \{(w, w'), (w, w'')\}, R_E \text{ is the reflexive closure of } \{(w, w')\}, \text{ and } [e_o]_{M_p} = [o]_{M_p} = \{w, w'\}. \]

\[ ^{15} \text{The proof is simple but not immediate, for the operators in the antecedent and in the consequent of } (L) \text{ apply to different letters. As for the } (L) \text{ to } (L') \text{ direction, suppose that the frame property } (L') \text{ does not hold. If so, we can use the counter-model just provided in note 14 to show that } (L) \text{ is not valid. The } (L') \text{ to } (L) \text{ direction is obvious, if one recalls that, by principle } (C) \quad [e_o]_{M_p} \subseteq [o]_{M_p}. \]
Principle \([O]\) follows from the conjunction of \([L]\) and \([N]\) \([O]\) is valid under the assumption that \(R_E = R_O\), that is to say, that the objectual and the eventive accessibility relations collapse into one another.

Other interesting principles can be analysed by extending our framework. For instance, by developing a first order version with identity, one can express principles like:

\[
(P) \quad P_o \rightarrow \exists x P x = o \\
(Q) \quad P e \rightarrow \exists x P x = e 
\]

Principle \([P]\) says that if \(\alpha\) perceives an object \(o\), then there is an object \(x\) such that \(\alpha\) perceives that \(x\) and \(o\) are the same object. Analogously, principle \([Q]\) says that if \(\alpha\) perceives an event \(e\), then there is an event \(x\) such that \(\alpha\) perceives that \(x\) and \(e\) are identical. A thorough examination of these and other related principles would be well beyond the scope of the present work.\(^{16}\) We leave it for another paper.

References


\(^{16}\)Arguably, due to the infamous problems raised by intensional de re constructions, to adequately deal with these principles one needs a modal semantics much more sophisticated than we one introduced here (see, e.g., Aloni, 2005).


