Scientific Th[illegible]ies as Intervening Representations[1]

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BIBLID [0495-4548 (2006) 21: 55; pp. 21-38]

ABSTRACT: In this paper some clas[illegible]sical representational ideas of Hertz and Duhem are used to show how the dichotomy between represe[illegible]ntation and intervention can be overcome. More precisely, scientific theories are reconstructed as complex[illegible] networks of intervening representations (or representational interventions). The formal apparatus develope[illegible] is applied to elucidate various theoretical and practical aspects of the in vivo/in vitro problem of biochem[illegible]istry. Moreover, adjoint situations (Galois connections) are used to explain the relation between empirical f[illegible]s and theoretical laws in a new way.

Key words: Representation, adjoint si[illegible] in vitro/in vivo problem, Hertz, Duhem.

1. Introduction

The concept of representat[illegible] has not yet been secured on the agenda of philosophy of science. Some philosoph[illegible] flatly deny that it could be of any use in epistemology or philosophy of science. I[illegible]nstead, they claim, the concept of representation leads us into a hopeless maze of ps[illegible]eudo-questions without answers. This is the case of Rorty and his antirepresentational[illegible] followers. According to them, epistemology based on the notions of negotiation [illegible] interpretation should replace epistemological accounts based on 'representation'. [illegible] this paper we will not address this kind of radical antirepresentationalism. But su[illegible]ffice to say, it is based on a rather primitive conception of representation identifying r[illegible]epresentation with some kind of copying or mirroring.

In this paper we want [illegible] elaborate some classical representational ideas of Hertz and Duhem in order to sho[illegible]w that a diagrammatical or combinatorial account of representations can be useful f[illegible] elucidating the role of representations in describing the practice of representational[illegible] reasoning in science.

The outline of this pap[illegible]er is as follows. In section 2, we outline some ideas of Hertz and Duhem concerning th[illegible]e structure of scientific reasoning that can be used to understand how representati[illegible]ons in science work. More precisely, following Hertz the idea of a commutative dia[illegible] of interconnected representations is introduced, and Duhem's account of empi[illegible] theories will lead us to the idea that the theoretical and the empirical are correlate[illegible] a so called adjoint situation. In section 3, the rudiments of a combinatorial theory c[illegible] representations are introduced, and are put to use in section 4 for the representati[illegible] of the in vitro/in vivo problem in biochemistry. In section 5, it is sh[illegible]own that Duhem's account of an empirical theory as a cor-

[1] We would like to thank two an[illegible]onymous referees for their detailed and penetrating criticisms that helped us to correct some major bl[illegible]unders in the first version of this paper. Further we'd like to express our sincere gratitude to the guest[illegible] editor José Antonio Díez, who pointed out some conceptual obscurities and infelicitous formulations[illegible] in the original text. Of course, we are responsible for all remaining errors.

relation of symbolical and empirical facts leads to the conception of an empirical theory as a Galois connection (or, more generally, an adjoint situation) in the sense of mathematical category theory. We close with some general remarks on the role of representational concepts in philosophy of science.

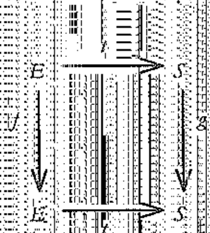
2. Classical Ideas of Representation

Let us start with some basic ideas on scientific representations put forward by the classical philosopher-scientists Hertz and Duhem. These ideas naturally lead towards an interesting conception of scientific theories as representations.

As our first classical intuition pump for the development of a comprehensive account of representation, we take Hertz's well known 'symbolical account' put forward in his *The Principles of Mechanics presented in a New Form* (Hertz 1894) where he described the general process of scientific representations as follows:

> We form for ourselves images or symbols of external objects; and the form which we give them is such that the necessary consequents of the images in thought are always the images of the necessary consequents in nature of the things pictured. In order that this requirement may be satisfied, there must be a certain conformity between nature and our thought. Experience teaches us that the requirement can be satisfied, and hence that such a conformity does in fact exist. […]
>
> The images which we may form of things are not determined without ambiguity by the requirement that the consequents of the images must be the images of the consequents. Various images of the same objects are possible, and the images may differ in various respects. […]
>
> Of two ima[illegible] equal distinctness the more appropriate is the one which contains, in addition to the essential characteristics, the smaller number of superfluous or empty relations, —the simpler of the two. Empty relations cannot be altogether avoided: they enter into the images because they are simply images … produced by our mind and necessarily affected by the characteristics of its mode of portrayal. (Hertz 1894, pp. 1f.)

We propose to restate Hertz's informal description of the representational activity of science in a diagrammatical language as follows. Let the set of "external objects" be denoted by E and denote the set of "images" by S. The following diagram may be used to capture the essential structure of Hertz's account:



The details are as follows: the horizontal arrow t corresponds to Hertz's formation of mental images. More precisely, if $e \in E$ is an external object, $t(e) \in S$ is the image corresponding to it. In other words, $t(e)$ may be considered as the theoretical counterpart of e. The left vertical arrow f in Hertz's diagram is to be conceived as a process or an experiment that 'necessarily' brings about the external fact that e is changed to another

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external fact $f(e) \in E$. In Hertz's terms, $f(e)$ is the 'necessary consequent' of e. Analogously, the vertical arrow g [illegible] right may be interpreted as a mathematical calculation or a logical argument [illegible] leads from a 'symbol' $s \in S$ to another symbol $g(s)$. It is to be interpreted as the [illegible] of the conclusion of the symbolical transaction g. In Hertz's terms, $g(s)$ is the [illegible] necessary consequent' of s. These ingredients of Hertz's diagram are of course not independent of each other; rather, as is informally stated in his *Principles*, they form a commutative diagram in 'that necessary consequents of the images in thought are always [illegible] images of the necessary consequents in nature', which in our diagrammatical language just amounts to the commutativity of the diagram:

(2.1) *Commutativity of Hertz's Diagram.* Assume t, f, and g as characterized above. They are assumed to satisfy the following concatenation law:

$$g \circ t = t \circ f$$

This equation is to be interpreted as follows: If we start with an empirical fact e in the left upper corner of the Hertz diagram, translate it to its theoretical counterpart $t(e)$, and use $t(e)$ as the input [illegible] calculation or a logical argument that leads to $g \circ t(e)$, then this outcome is the same as if we had submitted the empirical fact e to an experimental transformation [illegible] f arriving at $f(e)$, and translated this experimental fact $f(e)$ by t finally yielding $t \circ f(e) = g \circ t(e)$. In other words, the two paths in Hertz's diagram are strictly equivalent in [illegible] they may be considered as paths that lead to one and the same destination. As an [illegible] elementary example consider e to be some chemical substance that is submitted to a [illegible] chemical experiment f which, say, oxidizes e thereby yielding as outcome another [illegible] chemical substance $f(e)$. For this transaction a chemical theory has to provide a [illegible] formula $t(e)$ for e, and a theoretical transformation $g(t(e))$ of $t(e)$ such that $t(f(e)) = g(t(e))$. As is emphasized by Hertz, given E there may be different 'symbolic con[illegible]stellations' S, S'. The choice between them is a pragmatic matter of simplicity and [illegible] fitness. It may be that for different purposes different 'images' may be appropriate (cf. Hertz 1894, p. 3).

Second, let us come to Duhem's contribution to a modern representational account of scientific theories [illegible] which is found in his classic *The Aim and Structure of Physical Theory* (Duhem 1906). At various occasions in his *opus magnum* he asserts that scientific theories are to be conceived as representations. More precisely, he considers a physical theory 'as an e[illegible]conomical representation' that

> establishes an order a[illegible]nd a classification among [the experimental laws]. It brings some laws together, closely arrange[illegible] the same group; it separates some others by placing them into two groups very far apart. Theory gives, so to speak, the table of contents and the chapter headings under which the science [illegible] will be methodologically divided. (Duhem 1906, pp. 23f).

Later he goes on to explain this 'representation' as a correspondence between 'practical facts' and 'theoretical [illegible] or 'symbolical facts'. It is certainly not too far-fetched to consider Duhem's account as presented up to now as just another version of Hertz's structural approach. But there is one feature in Duhem's representational approach that is novel and not present in Hertz. In describing a physical theory as a correspondence between practical and symbolical facts he insists that

> a symbolic formula … can be translated into concrete facts in an infinity of different ways, *because all these disparate facts admit the same theoretical interpretation.* (*Ibid.*, p. 150)

And, in an analogous vein:

> The same practical fact may correspond to an infinity of logically incompatible theoretical facts; the same group of concrete facts may be made to correspond in general not with a single symbolic judgment but with an infinity of judgments different from one another and logically in contradiction with one another. (*Ibid.*, p. 152)

Duhem's account is rather informal, and he is not very clear about what is to be understood by 'theoretical fact'. In particular, one should not interpret him as conceiving a 'theoretical fact' as a fact 'belonging' to a specific theory. Rather, the most appropriate interpretation of Duhemian theoretical facts is to take a theoretical fact as one that asserts a physical state of affairs in precise mathematical terms, as is explained by Duhem. A typical example of a theoretical fact (or statement) is the following: 'An increased pressure of 100 atmospheres causes the electromotive force of a given gas battery to increase by 0.0844 volts.' (*Ibid.*, p. 152) Other 'logically incompatible' theoretical statements would be obtained by replacing '0.0844' by '0.0845' or '0.0846'. Hence, Duhem's account of an empirical theory can be formulated in relational terms as follows:

(2.2) *Duhem's Relational Account of Empirical Theories.* Denote the class of symbolic facts by S and the class of practical or empirical facts by E. Then a theory T is to be conceived as a relation

$$T \subseteq E \times S.$$

If $(e, s) \in T$ then this is to be interpreted as the empirical fact that e is related to s, or, to put it the other way round, that the symbolic fact s is related to the empirical fact e.

It is important to note that Duhem insisted that this relation is multi-valued: to a single e there may correspond many symbolic facts s, and, vice versa, to a single s, there may correspond many empirical facts e. This double ambiguity of the relation between empirical and symbolical facts is characteristic of Duhem's account and has no counterpart in Hertz's approach. As we shall show in the next section, this feature may be combined with the representational insights of Hertz to yield a complex representational account of empirical theories.

3. Representational Combinatorics

Following Hertz and Duhem in conceiving the practice of science as engaged in producing and manipulating representations of various kinds, the impression that comes to mind is that scientific representations do not live in isolation, rather they may be combined and concatenated in various ways (Ibarra, Mormann 2000). Hence, investigating these combinatorial aspects of representations is a central task of a general theory of representation (Ibarra, Mormann 1997 a, b).

Regardless of what kind of representations we consider, they are
with each other, rather, they form a representational network. One
tity A may be represented by several different entities B, C, D etc. s
representations $A \xrightarrow{r} B$, $A \xrightarrow{s} C$, $A \xrightarrow{t} D$, etc. On the oth
happen that one and the same entity E appears as the representative
ent entities A, B, C etc. That is to say we have representations $A \longrightarrow$
$C \longrightarrow E$. Furthermore, it can be the case that representations such
$B \xrightarrow{s} C$ are concatenated yielding an indirect or combined
$A \xrightarrow{s \circ r} C$.

As the result of these considerations, we can see that any theory
should comprise a combinatorial part, which describes the vario
combinations and iterations of representations. In the following we
this combination or concatenation of representations is associative, i.
f, g, and h, which 'match', satisfy the following law of associativity:

$$f \circ (g \circ h) = (f \circ g) \circ h. \tag{3.1}$$

The combination or iteration of representations is of utmost in
practice of science. For instance, in the standard representational th
ment the numerical measurement of an empirical domain D is co
representation r: $D \longrightarrow \mathfrak{R}$ of D into the real numbers $\mathfrak{R}$. This is
description. Actually, by a closer inspection the representation D —
regarded as a more or less extended chain of representations:

$$D \longrightarrow E \longrightarrow F \longrightarrow \ldots \longrightarrow \mathfrak{R} \tag{3.2}$$

In most cases, numerical or, more generally, mathematical repre
pirical data cannot be 'read off' directly; usually they have to consid
which have been built by a more or less complicated constructio
long way from data to theory shows that the standard dichotomic
idealized picture. Dealing with an example from general relativit
gives a detailed account of the 'long computational path from data t
mon 1982). Other examples of complex 'long distance' representat
in detail in Latour (1999): Latour tells us in detail the long story fr
theoretically digestible data in the case of 'botanical pedology' (*ibid*
withstanding important differences, all these accounts rely —implic
on what may be called a combinatorics of representations.

The combination of representations is not restricted, however,
tions. As will be shown by the *in vivo/in vitro* example of biochemist
combinatorial account of representations only comes to the fore if

our attention to linear chains of representations but, instead, also take into account non-linear net-like configurations of representations.

The importance of representational nets (diagrams) is evidenced by the fact that in the last forty years or so mathematics (and parts of other sciences as well) has been successfully reformulated in terms of representational networks. Here we refer, of course, to the mathematical theory of categories founded by Eilenberg and Mac Lane in the forties and presented for the general public in books such as Mac Lane's *Mathematics – Form and Function* (1986) or Lawvere and Schanuel's *Conceptual Mathematics – A First Introduction to Categories* (1996). In category theory, representations appear under the names morphisms, functors, and natural transformations. In the last decades it has been shown that not only the bulk of mathematics can be reconstructed in these terms, but also that this representational reconstruction has lead to new and fruitful lines of mathematical research. We take this fact, together with the representational ideas of Hertz and Duhem as evidence that combinations of various kinds of representations play an indispensable role for a representational theory of scientific knowledge. This claim is substantiated in the next section in which we propose to study in some detail various combinations of representations that arise from the so-called *in vitro/in vitro* problem in biochemistry.

4. *A Representational Account of the In Vivo/In Vitro Problem*

In this section we are going to apply the formal apparatus sketched so far to a specific problem of a scientific discipline that up to now has not received too much attention from philosophy of science, to wit, the so called '*in vitro/in vivo*' problem of biochemistry (cf. Strand, Fjelland, and Flatmark 1996 and Strand 1999). For the information on biochemical matters we heavily rely on these papers. Our purpose is to show that the rudiments of a theory of meaningful representations set out in the previous sections may be used to elucidate the problems of the representational practice biochemistry have to cope with. We chose the approach of Strand *et al.* as our starting point since it seemed to us particularly well suited for our purposes: on the one hand, it is sufficiently complex to require the employment of some non-trivial representational tools; on the other hand, it is conceptually not too complex as to be inaccessible for non-experts in biochemistry.

First, let us recall the basic ingredients of the *in vitro/in vivo* problem as it presents itself in biochemistry. The first point to note is that although biochemistry may be defined as 'the field of science concerned with the chemical substances and processes that occur in plants, animals, and microorganisms' it would be misleading to assume that 'biochemists study processes that occur in living organisms' (cf. Strand 1999, p. 273). The reason is that normally

> it is impossible to perform a chemical analysis of an intact organism. A biochemical analysis is typically preceded by an isolation procedure, in which the organism of interest is disrupted and a specific component of it is isolated. To put another way, almost all biochemical evidence is obtained *in vitro* under artificial experimental conditions. … [Nevertheless] Biochemists are concerned with the chemistry of the living organism *in vivo*. (Strand 1999, p. 273)

[redacted]

an *in vivo* system S and a corresponding *in vitro* system S^* as a representational relation $S \xrightarrow{d} S^*$ contending that the *in vivo* system S is represented by the *in vitro* system S^*. It should be noted that this representation is a material long distance representation par excellence. Usually the representing system S^* is obtained from S by a variety of massive, often destructive interventions of various kinds (cf. Strand *et al.* 1996, Strand 1999). The representing system S^* is far from being similar to S, and it is neither natural nor necessary to represent S by S^*. There may be many other ways of representing S by other $S^{*\prime}$, $S^{*\prime\prime}$, ..., depending on the representational interests and capacities of those who are engaged in the construction of these *intervening* representations. Thus, as the first outcome of considering the IVIV problem in biochemistry we contend that the dichotomy between representing and intervening put forward by some philosophers such as Hacking is pointless in the case of biochemistry, and, regarding biochemistry as a paradigmatic case for science in general, for other sciences as well (cf. Hacking 1983).

As has been fully explained by Strand *et al.*, there is much more in the IVIV problem than the statement that it gives rise to an intervening representation $S \longrightarrow S^*$. To deal with the more fine-grained aspects of the IVIV problem, let us introduce the following terminological conventions: properties, objects, relations, procedures etc. belonging to the realm of *in vivo* systems are denoted by $E, F, a, b, R, p, \ldots$, while the corresponding properties, objects, etc. belonging to the *in vitro* realm are denoted by $E^*, F^*, a^*, b^*, \ldots$. Our first purpose is to show that IVIV problems give rise in a natural way to a plurality of Hertz's diagrams. Given systems S and S^*, and important task of the biochemist's work is to study how these systems behave under certain perturbations p and p^*. Here, a perturbation p of S may be considered as a map: $S \xrightarrow{p} S$. More precisely, $p(s)$ is to be understood that for $s \in S$ the state $p(s) \in S$ is the state that resulted from s when submitted to the perturbation p. Analogously for *in vitro* states S^* and *in vitro* perturbations $p^*: S^* \longrightarrow S^*$. Then the systems and perturbations S, p, S^*, p^* may be said to be naturally correlated if the following Hertz diagram commutes:

$$\begin{array}{ccc} S & \xrightarrow{d} & S^* \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle p^*} \\ S & \xrightarrow{d} & S^* \end{array} \qquad p^* \circ d = d \circ p.$$

By definition, an artifact is an *in vitro* perturbation $d(s) \neq p^*(d(s))$ such that $s = p(s)$. If the Hertz diagram commutes, artifacts can be shown not to exist: Assume $d(s) \neq p^*(d(s))$ and $s = p(s)$. From Hertz we get $p^*(d(s)) = d(p(s))$. Hence we get the following proposition:

Proposition 1. If Hertz commutes, then there are no artifacts.

In a similar vein, one obtains that the non-existence of artifacts implies that the Hertz diagram commutes for states s that are invariant under the perturbation p, i.e., states for which $s = p(s)$:

Proposition 2. If s is invariant under p AND there are no artifacts, then HERTZ commutes for s.

Proof. Assume $s = p(s)$. Then $d(s) = d(p(s))$. Assume that HERTZ does not commute for s. That is to say $p^*(d(s)) \neq d(p(s))$. Then $p^*(d(s)) \neq d(s)$. Since there are no artifacts one infers $s \neq p(s)$. This is a contradiction. ■

In sum, the diagrammatically natural requirement that Hertz diagrams commute is a bit stronger than the claim that no artifacts exist. The existence of artifacts is, however, not the only problem that may arise when studying the relation between *in vivo* and *in vitro* systems. It may well happen that the combination of *in vitro* perturbation $p^*: S^* \longrightarrow S^*$ and the intervening representation $d: S \longrightarrow S^*$ are jointly too invasive and too coarse, such that a salient *in vivo* perturbation p fails to be detected by them. This is the case if it happens that $s \neq p(s)$ but $d(s) = p^*(d(s))$. This may be called an artificial null effect. Artificial null effects and the commuting of the Hertz diagram are related as follows:

Proposition 3. If the Hertz diagram commutes and the representation $d: S \longrightarrow S^*$ is mono, i.e., $d(a) = d(b)$ implies $a = b$, then no artificial null effects occur. ■

In this implication, the second clause of the antecedent is clearly necessary. This may be more conspicuously expressed by contraposition:

Proposition 4. If artificial null effects occur, then either the Hertz diagram does not commute or the IVIV representation $d: S \longrightarrow S^*$ is not mono. ■

One may ask whether the converse holds: If no artificial null effects occur, does the Hertz diagram commute and is d mono? As is easily checked by examples, this is not the case. In other words, the conjunctive assumption that the Hertz diagram is commutative and the IVIV representation d is mono is strictly stronger than the non-existence of artificial null effects.

As has been pointed by Strand *et al.,* the IVIV problem is not completely described by a Hertz diagram connecting an *in vivo* systems S and an *in vitro* systems S^*. Usually these systems are accompanied by what may be called their model systems M and M^* respectively. That is to say, for the *in vivo* system S there is a theoretical (or maybe sometimes a computer) model M, and for the *in vitro* system S^* there is a theoretical (computer model) model M^*. Then it is natural to assume that M is an appropriate representation of S, and M^* is an appropriate representation of S^*. These may be ex-

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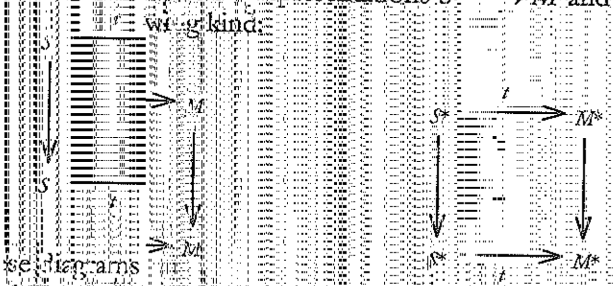

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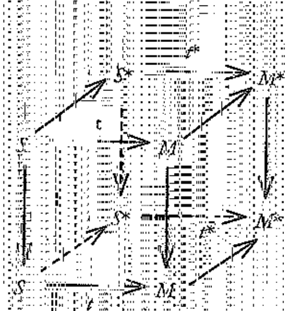

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[illegible] various kinds may [illegible] useful as an idealized model to spot where precisely
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[illegible] states s of S may have [illegible] simple theoretical model of (*in vivo* or *in vitro*) systems,
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plex net of representational and interve[illegible]ng links. The basic building block of this net, which intertwines theoretical represen[illegible] and practical interventions are various kinds of Hertz diagrams. Thus, taking [illegible] the IVIV problem in biochemistry as paradigmatical for empirical theories in genera[illegible] we contend that representations and interventions should be treated together, sinc[illegible] both may be characterized as moves in the complex network of an empirical theor[illegible]

The IVIV problem of biochemistry [illegible] is particularly interesting for a representational philosophy of science as it shows the [illegible] necessity of considering iterations and combinations of various kinds of interventio[illegible]s and representations. The language of representational diagrams is particularly apt [illegible] dealing with the various kinds of connections. We think that the opposition be[illegible]tween the representative and the performative perspective in philosophy of science is [illegible] an artefact of a misinformed philosophy of science. One does not have to choose bet[illegible]ween [illegible]. In some sense, every representation has an interventional aspect, [illegible] at least indirectly, and every intervention leads to a representation.

5. Adjoint Situations

In this section we are going to show th[illegible] Duhem's relational account of theories that conceives a theory T as a relation $T \subseteq S \times E$ between symbolic and empirical facts may be elucidated by using so called [illegible] connections or, more generally, adjoint situations in the sense of category theo[illegible]ry. This part of the paper is the most speculative one, and some readers may objec[illegible] a heavy formal apparatus without real justification. Thus the foll[illegible] preliminary remark may be in order: Our point is this: conceiving an empirical th[illegible] as a certain relation between empirical and theoretical facts seems to us quite a na[illegible] and intuitive approach. Otherwise Duhem, who certainly was not interested in fo[illegible], would not have endorsed it. Now, as soon as a theory is given as a [illegible] $T \subseteq S \times E$, the whole apparatus of Galois relations is available. One may eve[illegible] a Galois relation between PS and PE is nothing but a relation. Since Galois [illegible] connections have turned out to be a useful tool in the study of binary relations in m[illegible]athematics, computer science and elsewhere. Hence, one may suspect that they coul[illegible] some useful work in formal philosophy of science as well. This conjecture is fur[illegible] supported by the fact that Galois connections are just a very special case of ad[illegible]joint situations that may be characterized as *the* fundamental concept of category theo[illegible]ry. Hence, there is some hope that these conceptual tools have some applications in [illegible] philosophy of science as well.

By conceiving a theory as a relation $T \subseteq E \times S$ of empirical and symbolical facts in the sense of Duhem's *The Aim and Structure of Physical Theory*, it is not claimed, of course, that any relation $X \subseteq S \times E$ [illegible] as a genuine theory. There are countless relations between the two classes of f[illegible] that make no sense at all. Further restrictions will have to be imposed on T in [illegible] order that T can be acknowledged as a genuine theory. As will be shown later, for th[illegible] the representational ideas of Hertz turn out to be useful.

For the moment we only want to emphasize Du
any given empirical fact $f \in E$ there may be many sy
s are theoretically correlated, i.e., that $(f, s) \in T$, and
may be many empirical facts $f \in E$ such that (f, s)
Formally, this means that $T \subseteq E \times S$ is a relation and

This multivalued correlation between empirical
plausible that a single fact, be it symbolical or emp
That is to say, a single $s \in S$ or $f \in E$ is an object
Rather, what shows up in the practice of real scienc
cal and theoretical facts. Thus, we propose to co
$B \subseteq E$ as the real building blocks of scientific theor
symbolic facts $s \in S$ are auxiliary concepts introd
Replacing elements by subsets in this way is a natu
'elementary' facts of type s and f may be considered
and B by identifying s and f with their singletons
from elementary facts to subsets of elementary facts
Austrian colleague Ernst Mach proposed long time
task of science to describe the functional relations
ters of elements in the most economical way poss
elements to subsets facilitates to get started the form
in order to elucidate Duhem's relational account of
paratory remarks we are now ready to set up the for
cast Duhem's relational account of empirical theo
connections. First, let us deal with the necessary tech

Denote by PS and PE the power sets of S and E
us assume that PS and PE are endowed with their n
tures $(PS, \subseteq)$ and $(PE, \subseteq)$. A theory $T \subseteq E \times S$ gives
tween PS and PE by the following recipe:

(5.1) *Proposition.* Let $T \subseteq E \times S$ be a theory.
$PS \xrightarrow{e} PE$ by:

(a) For $Y \in PE$ define $e(Y)$ by $e(Y)$: = $\{s; \exists y\ (y \in Y$

(b) For $X \in PS$ define $t(x)$ by $t(X)$: = $\{y; (e(\{y\}) \subseteq$

Then the maps e and t are order preserving.

Proof. Check the definitions of e and t.

Obviously, e and t are not unrelated to each other
completely determined by e, and vice versa. Actuall
by the following proposition:

(5.2) *Proposition.* Let e and t be de
lowing holds:

In technical jargon, the orde
order structures PS and PE (cf.
(or right) adjoint, and e is called
lois connection (t, e) is *not* a sym
ally (e, t) fails to be a Galois cor
joint is reflected in the notationa
or 'upper' side of $\leq$, while e as th
tion $\leq$. This asymmetry is esser
between the domain of empirica

Proof (5.2). The proof naturally s
Then one has to show $z \in Y$. B
That is to say $z \in e(s)$. By presu
$z \in Y$; (ii) Assume $e(X) \subseteq Y$ and
and this just means $s \in t(Y)$.

(5.3) *Corollary.* The map PS —
$X \subseteq S$, and the map $PE \xrightarrow{t \circ e}$
$Y \subseteq E$.

After having presented these
start now with the task of el
amounts to an interpretation of
nection (t, e), and an explanation
of philosophy of science.

For this task it is expedient
sets $X \subseteq S$ and subsets $Y \subseteq E$
pirical facts. By definition $e(X)$ i
empirically correlated to at leas
preted as that the empirical fact
sense, i.e., it may be that the e
have theoretical correlates s tha
empirical facts of $e(X)$ in the ser

Analogously, the map t may
$Y \subseteq E$ into a related theoretic
$t(Y)$, i.e., $s \in t(Y)$ if and only

fined as above. Then for all $X \subseteq$
$X \subseteq t(Y)$ IFF $e(X) \subseteq Y$.
red pair (t, e) is called a *Galois*
(Herz *et al.* 2003). More precise
the lower (or left) adjoint. One s
metric notion, i.e., if (t, e) is a G
nection. The difference betwee
convention that t as the upper
e lower adjoint is on the "lower"
tial in the following to set up a
facts E and the domain of sym
splits into two parts: (i) assume z
by definition of $e(X)$ there is an
position $s \in t(Y)$. This means e
t $s \in X$. One has to show $s \in t(Y$
$\rightarrow PS$ is a kernel operator, i.e.
$\rightarrow PE$ is a closure operator, i.e.
fundaments of the theory of Gal
dicating the intuitive meaning
the components t and e, which
of their most important prope
to start with the map $e: PS$ —
are to be interpreted as symboli
the collection of all 'atomic' e
one 'atomic' symbolic fact $s \in$
$e(X)$ provides an empirical reali
empirical facts z realizing the sy
to do not belong to X but at lea
se that $t(e(X)) \supseteq X$.
be interpreted as a recipe to tra
fact $t(Y)$ such that each theor
ll empirical correlates z of s

$e \circ t(X) \subseteq X$, for all
$Y \subseteq t \circ e(Y)$ for all
this connection, let us
at this gadget. This
form the Galois con-
ties in informal terms
$\rightarrow PE$. Recall that sub-
(theoretical) and em-
empirical facts z that are
X. This may be inter-
zation of X in a broad
into facts on X may
s: X is covered by the
s are in empirical fact
tical fact s belongs to
belong to Y. In other
S and $Y \subseteq E$ the fol-
connection between the
y, t is called the upper
to note that a Ga-
is connection, its
upper and lower ad-
joint is on the right
side of the order rela-
symmetric relation
bolical facts S.
$X \subseteq t(Y)$ and $z \in e(X)$.
$\in X$ with $(s, z) \in T$
$(s) \subseteq Y$, and therefore
But $e(s) \subseteq e(X) \subseteq Y$,

words, $t(Y)$ is the most comprehensive theoretical fact for which Y provides a complete empirical realization.

We hasten to add that this relational account of empirical theories as a relation $T \subseteq E \times S$ is seriously incomplete. Its essential flaw is that it does not allow us to distinguish between approximately true theories and false theories, i.e., theories that are completely off the mark. If a theory T is just a relation $T \subseteq E \times S$ relating symbolic and empirical facts, there is no room for asking if T is (approximately) correct or not. This is clearly not sufficient to model the way of how theories relate theoretical facts to often recalcitrant empirical facts. To overcome this shortcoming, it is expedient to rely once more on the insights encapsulated in Hertz's diagram. In other words, we propose to combine the insights of Hertz and Duhem to obtain a better model of scientific theorizing that comprises the advantages of both the Hertzian and the Duhemian accounts.

This is done as follows: Let us start over again from the domains PS and PE of theoretical facts and symbolic facts, respectively, endowed with maps e: $PS \xrightarrow{e} PE$ and $PE \xrightarrow{t} PS$ as before. That is to say, e and t are to be interpreted as Duhemian maps correlating symbolic facts and empirical facts as explained above. The new ingredient we are going to introduce in order to distinguish between (approximately) true theories and those that are plainly false is provided by the replacement of the trivial set theoretical order relation $\subseteq_S$ on S and $\subseteq_E$ on E by appropriate non-trivial order relations $\leq_S$ and $\leq_E$ on PS and PE, respectively, which reflect some theoretical or empirical intervention and processes as explained in our discussion of the Hertz diagram in section 2. More precisely this is explained in the following definition:

(5.4) *Definition.* (a) Assume $Y, Y^* \in PE$. Assume that there is an empirical process P or intervention such that the empirical fact Y is the initial state $P(i)$ of P, and Y^* is the final state $P(f)$ of P. It is further assumed that processes or interventions P, P', P'' can be concatenated associatively. Define $Y \leq Y^*$:= there is a process P with initial state Y and final state Y^*.

(b) Assume $X, X^* \in PS$. Assume that there is a symbolic process P or intervention such that the symbolic fact X is the initial state $P(i)$ of P, and X^* is the final state $P(f)$ of P. It is further assumed that processes or interventions P, P', P'' can be concatenated associatively. Define $X \leq X^*$:= there is a process P with initial state X and final state X^*.

The class of processes or interventions defined for symbolic and empirical facts render PS and PE order structures, to be denoted by $(PS, \leq_S)$ and $(PE, \leq_E)$, respectively. From now on, PS and PE are assumed to be endowed with these interventional orders which differ from the set-theoretical orders $\subseteq_S$ and $\subseteq_E$. In Hertz's terms, then, $X \leq X'$ is to read as 'X' is a necessary consequent of X', and analogously $Y \leq Y'$ is to is read as 'Y' is a necessary consequent of Y, [illegible] the following Duhem-Hertz requirement makes sense:

(5.5) *Definition.* Let $T \subseteq E \times S$ be a relation of [illegible] and empirical facts. Assume PE and PS endowed with interventional orde[illegible] $\leq_S$ and $\leq_E$ respectively, $X \in PS$, $Y \in PE$. Let $PS \xrightarrow{e} PE$ and $PE \xrightarrow{t} PS$ define[illegible] by T. Then the theory T is said to satisfy the Duhem-Hertz condition iff for all $X \in PS$, $Y \in PE$ the following equivalence holds:

(5.6) $$e(X) \leq_E Y \text{ IFF } X \leq_S t(Y)$$

In other words, the pair (t, e) is a Galois connection between $(PS, \leq_S)$ and $(PE, \leq_E)$. More precisely, t is the upper (or right) adjoint [illegible] and e is the lower (or left) adjoint of this Galois connection.

Before we explain in some detail why theo[illegible]s satisfying (5.6) should be considered as (approximately) true let us note that instea[illegible] the set theoretical structures PS and PE it may be more expedient, more intuitive, a[illegible] less clumsy to replace PS and PE by ordered domains $(U, \leq_U)$ and $(V, \leq_V)$. [illegible] the Duhem-Hertz condition (5.6) simply requires that there are order-preservin[illegible] $U \xrightarrow{e} V$ and $V \xrightarrow{t} U$ such that (t, e) defines a Galois connection between U and V in the sense of (5.2). This may be even further generalized by the assumptio[illegible] that U and V are categories in an adjoint situation (cf. Goldblatt 1978). That is to [illegible] conceiving an empirical theory as an adjoint situation (F, G) between a category o[illegible] symbolic facts [illegible] and a category V of empirical facts combines in a neat and natur[illegible] way the classical insights of Hertz and Duhem.

As a summary of this section let us ref[illegible] once more the basic thesis in somewhat different terms, assuming that an [illegible] theory is given as a Galois connection (t, e) between an ordered domain $(U, \leq)$ of symbolic facts and an ordered domain $(V, \leq)$ of empirical facts, i.e. the map[illegible] $U \xrightarrow{e} V$ and $V \xrightarrow{t} U$ satisfy the Galois equivalence

(5.7) $$e(x) \leq a \quad \text{IFF} \quad x \leq t(a), \quad x \in U, a \in V.$$

Then we may conceive x as a *theoretical law* th[illegible] may be considered as the blueprint for the building of a nomological machine or e[illegible] apparatus $e(x)$ that produces the empirical fact a as its outcome. Then the [illegible] Galois connection states.

The nomological machine $e(x)$ brings about the empirical fact a IFF The theoretical law x implies an idealized version $t(a)$ of a.

This yields another interpretation of the formal apparatus of Galois connection that renders plausible the claim why theories which satisfy the Galois connection should be considered as (approximately) true theories: such theories are approximately true since they ensure a relation between the empirical and the theoretical that captures the idea that an approximately true theory should approximately correspond to the facts.

6. Concluding Remarks

The leitmotif of this paper was the thesis that scientific theories are to be considered as *representations*, and, more generally, that the practice of science may be conceptualized as a representational practice. This idea is not new, and many have put forward it in many different ways. Philosopher-scientists such as Hertz and Duhem provide distinguished examples. Tapping some of their essential insights we hope to have rendered plausible the following theses: (i) representation is a complex concept in need of a theory, (ii) representations do not live in isolation. Rather, they may be *iterated* and *combined* in various ways, and (iii) representations do not 'speak for themselves'. Rather, representations are in need of interpretation. A large part of scientific practice consists in interpreting and reinterpreting representations.

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