Edmund Husserl's *Philosophy of Arithmetic* in Reviews

Carlo Ierna[1]
Utrecht University, Utrecht
carlo.ierna@phil.uu.nl

Abstract: This present collection of (translations of) reviews is intended to help obtain a more balanced picture of the reception and impact of Edmund Husserl's first book, the 1891 *Philosophy of Arithmetic*. One of the insights to be gained from this non-exhaustive collection of reviews is that the *Philosophy of Arithmetic* had a much more widespread reception than hitherto assumed: in the present collection alone there already are fourteen, all published between 1891 and 1895. Three of the reviews appeared in mathematical journals (*Jahrbuch über die Fortschritte der Mathematik*, *Zeitschrift für Mathematik und Physik*, and *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*), three were published in English journals (*The Philosophical Review*, *The Monist*, *Mind*), two were written by other members of the School of Brentano (Franz Hillebrand and Alois Höfler). Some of the reviews and notices appear to be very superficial, consisting merely of paraphrases (often without references) and lists of topics taken from the table of contents, presenting barely acceptable summaries. Others, among which Höfler might be the most significant, engage much more deeply with the topics and problems that Husserl addresses, analyzing his approach in the context of the mathematics of his time and the School of Brentano.

Keywords: Edmund Husserl, philosophy of arithmetic, philosophy of mathematics, history of philosophy, history of mathematics

1. Carlo Ierna is a postdoctoral researcher at Utrecht University (The Netherlands), working on the renewal of the ideal of "Philosophy as Science" as a central project in the School of Brentano. After working at the Husserl-Archives since 2004, in 2012 he became a recipient of one of the Dutch NWO Innovational Research Incentives Scheme VENI grants. Recent publications focused on the philosophy of mathematics in Husserl's early works and the School of Brentano and he is in the process of completing a book on *The Beginnings of Husserl's Philosophy*.

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1. Carl Theodor Michaëlis, in *Jahrbuch über die Fortschritte der Mathematik* XXIII/1 (1891), 58–9.
2. Anonymous, in *Literarisches Centralblatt für Deutschland* 8 (Feb. 1892), 238–9.
3. Frank Thilly, in *The Philosophical Review* 1(3) (May 1892), 327–30.
4. Paul Carus, in *The Monist* II (July 1892), 627–9.
5. Anonymous, in *Mind* 1(4) (October 1892), 565–6.
6. Ernest Lindenthal, in *Zeitschrift für das Realschulwesen* (1893), S. 104–7.
7. Heinrich Schotten, in *Zeitschrift für Mathematik und Physik (Historisch-literarische Abtheilung)* 38 (1893), 88–90.
8. Franz Hillebrand, in *Göttingische gelehrte Anzeigen* 4 (1893), 175–80.
9. Albino Nagy, in *Rivista Italiana di Filosofia* VIII/II (1893), 243–5.
10. Alois Höfler, in *Zeitschrift für Psychologie und Physiologie der Sinnesorgane* VI (1894), 49–56.
11. Adolf Elsas, in *Philosophische Monatshefte* 30 (1894), 437–40.
12. Michael Glossner, in *Jahrbuch für Philosophie und spekulative Theologie* (1894), 235–9.
13. Friedrich Pietzker, in *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht* XXVI (1895), 512–17.
14. Władysław Heinrich, in *Vierteljahrsschrift für wissenschaftliche Philosophie* (1895), 436–9.

Introduction

The present collection of (translations of) reviews is intended to aid the assessment of the reception of Edmund Husserl's first major philosophical publication, the *Philosophy of Arithmetic*.[2] Husserl himself collected offprints of several of the reviews in a convolute conserved among his manuscripts: the reviews in *The Monist* and in the *Literarisches Zentralblatt* (with some underlined passages), the ones by Pietzker (with some lines in the margin), Michaelis, Lindenthal (with some lines in the text and margin) and Hillebrand.

Perhaps the first and most obvious insight to be gained from this collection of reviews is that the *Philosophy of Arithmetic* had a much more widespread reception than hitherto assumed. Consider, for example, Vilkko who compares the reception of Frege's *Begriffsschrift* with that of Husserl's *Philosophy of Arithmetic*:

2. Edmund Husserl. *Philosophie der Arithmetik (Psychologische und Logische Untersuchungen)* (Halle-Saale: C.E.M. Pfeffer [Robert Stricker], 1891), henceforth cited as *PA*; critical edition in Edmund Husserl, *Philosophie der Arithmetik: Mit Ergänzenden Texten (1890–1901)*, ed. Lothar Eley, Husserliana XII (The Hague: Martinus Nijhoff, 1970), henceforth cited as *Hua* XII. English translation in Edmund Husserl, *Philosophy of Arithmetic*, trans. Dallas Willard, Edmund Husserl Collected Works X (Dordrecht: Kluwer, 2003), henceforth cited as *Hua* CW X.

> Comparatively, I would say, in the early 1890s Husserl was just as well known in academic circles as Frege was in the late 1870s. Frege's book, though, received more reviews than Husserl's: apparently the *Philosophie der Arithmetik* was reviewed no more than twice![3]

The reviews Vilkko takes into account are only those of Elsas and Frege. However, in the present collection alone there already are fourteen, and we know of at least two more.[4] In addition to these, there is of course the one by Frege, which we chose not to include here, as it is already all too well-known and readily accessible both in original as well as in translation.[5] For those keeping score, this brings the number of reviews of Husserl's *Philosophy of Arithmetic* up to more than double the reviews of Frege's *Begriffsschrift.*[6] However, the really important element is not the sheer amount of reviews, but their content and distribution.

Three of the reviews appeared in mathematical journals (*Jahrbuch über die Fortschritte der Mathematik, Zeitschrift für Mathematik und Physik* and *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht)*, three were published in English journals (*The Philosophical Review*, *The Monist*, *Mind*),[7] two were written by members of the School of Brentano (Hillebrand and Höfler).

Many of the reviewers remark on the fact that the work under consideration is only the first volume and preparatory part, presenting interesting groundwork, but merely paving the way for the much more important second volume, whose core topics would have been general arithmetic and the arithmetical algorithm.[8] Indeed, Husserl sets such expectations quite explicitly in his preface, going so far as to announce the second volume for the next year. Unfortunately, as we know, while Husserl kept working on the second volume up to at least 1894, it was never completed, though large collections of material pertaining to and intended for this work have been subsequently made available in the *Husserliana* editions.[9]

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3. Risto Vilkko, "The Reception of Frege's Begriffsschrift," *Historia Mathematica* 25 (1998), 412–22, here 414.
4. One by Jules Tannery ("J.T.") in the *Bulletin de Science Mathematiques* XVI (1892), 239–45 and one by his brother Paul Tannery in the *Revue Philosophique* XXXVIII (1894), 59–62. The latter is done in the context of a "general review" regarding the field of "the theory of mathematical knowledge," also including a discussion of works by Milhaud, Renouvier, Poincaré, and Couturat.
5. I addressed some of the problems surrounding the reception of Husserl's *Philosophy of Arithmetic* based on Frege's review in Carlo Ierna, "Husserl's Psychology of Arithmetic," *Bulletin d'analyse phénoménologique* VIII(1) (2012), 97–120.
6. There is an additional review of the *Begriffsschrift* (again by Carl Theodor Michaelis, in *Jahrbuch über die Fortschritte der Mathematik* 11 [1881], 48–9), bringing Frege's total to eight.
7. The English reviews might be of interest to those who struggle with the translation of Husserl's terminology; for example, Thilly uses "sum" as translation for *Anzahl* and "multitude" for *Menge.*
8. Cf. Edmund Husserl, *Studien zur Arithmetik und Geometrie*, ed. Ingeborg Strohmeyer, Husserliana XXI (The Hague: Martinus Nijhoff, 1983), henceforth cited as *Hua* XXI, here xxii.
9. *Hua* XII; *Hua* XXI and Edmund Husserl, *Early Writings in the Philosophy of Logic and Mathematics*, trans. Dallas Willard, Edmund Husserl Collected Works V (Dordrecht: Kluwer, 1994); *Hua* CW X.

The *Philosophy of Arithmetic* would originally have consisted of two volumes with respectively two and three parts plus a (lengthy) appendix on semiotics:

> I. The Proper Concepts of Multiplicity, Unity and Amount. II. The Symbolic Concepts of Amount and the Logical Sources of the Arithmetic of Amounts. III. The General Arithmetic of Amount. IV. The Arithmetical Algorithm in Other Domains. V. Concluding Remarks. Appendix: The Investigations into Semiotics.[10]

As the reviewer in *The Monist* puts it: "The work is thus obviously one that can be dealt with critically only when it is complete." While many reviewers look forward with great interest to the second volume, none try to draw any implications from the first with respect to these topics, though they are already substantially hinted at, particularly in the later chapters. Even those reviewers that consider the formality of mathematics, the role of the algorithm in calculus, and so on as the more important topics, mostly merely mention them in brief quotes and paraphrases, if at all, but without a critical discussion. A partial exception to this is Glossner, who at least quotes Husserl's views on the mechanical operations and the essence of calculus at some length.

In general, the reviewers try to establish Husserl's position with respect to the various authors that he criticizes, such as Lange, Baumann, Jevons, Sigwart, Schuppe, and so on, mostly discussed in the second chapter, and Frege, Riemann, and Helmholtz, discussed in chapter VII and the appendix to part one of the *Philosophy of Arithmetic*. On the other hand, regarding his agreement with other authors' theories, only Höfler and Nagy seem to be able to place Husserl in the context of the School of Brentano,[11] being the only ones to even mention Brentano and Meinong. In this respect, Höfler's review is surely among the more significant, as it discusses Husserl in combination with some of the other Brentanists who wrote about mathematics at the time, Kerry and Von Ehrenfels, and because it cross-references other reviews of the work, showing that Höfler certainly had done more research than just reading (part of) the book itself. By contrast some of the other reviews and notices appear very superficial, consisting merely of paraphrases (often without references) and lists of topics taken from the table of contents, such as the very brief notices in the *Literarisches Centralblatt* and *Mind*, and the reviews by Michaelis, Thilly and Nagy, who present acceptable summaries, but contain hardly any analysis. Schotten mostly just reprints the table of contents, though he is the only one to have noticed a connection to Bolzano's *Paradoxien.* Other authors, such

10. Msc. K VI 2/18, see Karl Schuhmann, *Husserl – Chronik (Denk- und Lebensweg Edmund Husserls)*, Husserliana Dokumente I (The Hague: Martinus Nijhoff, 1977), henceforth cited as HC, here 30, which mistakenly refers to K IV 2. Incidentally, this notebook, titled "Logic, in particular logical calculus" and "formal arithmetic", contains more interesting material pertaining to the *Philosophy of Arithmetic*.
11. I discuss the approach to the foundations of mathematics in the School of Brentano more in general in Carlo Ierna, "Brentano and Mathematics," *Revue Roumaine de Philosophie* 55 (2011), 149–67.

as Elsas and Hillebrand, seem to take into account merely the first part of the book. The latter focuses on Husserl's psychological account of collectiva and *Gestalt*-qualities, no doubt owing to his own background in the School of Brentano.

Unfortunately it was not always possible to discover the identity of the reviewers, often only indicated by initials, in which case I tried to make an educated guess.

Carl Theodor Michaelis (1852–1914), identified only as "Mi.," that is, "Dr. Michaelis in Berlin," was the author of two short treatises *Über Kant's Zahlbegriff* (1884) and *Stuart Mill's Zahlbegriff* (1888).

Frank Thilly (1865–1934) had studied in Germany around 1889–90, *inter alia* in Berlin under Friedrich Paulsen (who had also been a teacher of Husserl). Later on, as professor at Cornell and Princeton, he popularized Paulsen's works in the USA through translations and reviews.

Paul Carus (1852–1919), identified only as "κρς," had studied under Hermann Grassmann at the gymnasium in Stettin. After obtaining his Ph.D. in Tübingen in 1876 he emigrated from Germany in the 1880s to Britain and the US where he co-founded Open Court Press and became life-long editor of *The Monist*.[12]

Ernest Lindenthal (1853–1922) was a high-school mathematics professor and author of a mathematics textbook *Rechenlehre für die I. und II. Realschulclasse* (1896).

Heinrich Schotten (1856–1939) was director of the *Oberrealschule* in Halle and from 1901 became editor of the *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht.*

Franz Hillebrand (1863–1926) was a student of Brentano and Marty, and author of *Die neuen Theorien der kategorischen Schlüsse* (1891), based on Brentano's logic lectures.

Albino Nagy (1866–1900), studied mathematics and philosophy in Vienna from 1884 to 1888, obtaining his doctorate with a thesis on "The Application of Mathematics to Logic," and taught mathematical logic at the University of Rome from 1893 to 1896. Most of his publications in the 1890s concern mathematical and logical calculus and in 1891, like Husserl, he had reviewed Schröder's *Vorlesungen.*

Alois Höfler (1853–1928) taught mathematics, physics, and philosophy at the Theresianum in Vienna from 1876 to 1903, obtained his Ph.D. with Meinong in 1886 and co-authored a book on logic with him in 1890. From 1894 he also was *Privatdozent* at the University of Vienna.

Adolf Elsas (1855–1895) was professor of physics in Marburg and author of *Über die Psychophysik. Physikalische und erkenntnistheoretische Betrachtungen* (1886), in which he advocated a Kantian approach to mathematics and measurement, criticizing both Fechner's psychophysics as well as formalism. In 1881, as an assistant at the institute for mathematics and physics, he had won a prize for an essay on Kantian foundations of mathematics in connection with psychophysics, which was probably the basis for this work.[13]

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12. I would like to thank Marcus Brainard and Barry Smith for helping me identify the anonymous κρς as Paul Carus.
13. See Ulrich Sieg, *Aufstieg und Niedergang des Marburger Neukantianismus* (Würzburg: Königshausen & Neumann, 1994), 131. Husserl wrote an emphatic rebuttal to Elsas's review (Ms. K I 52), which was not published, probably owing to the death of Elsas in 1895.

Michael Glossner (1837–1909) was a leading neo-thomist theologian and philosopher, frequent contributor and (often quite polemical) reviewer for the *Jahrbuch für Philosophie und spekulative Theologie.*

Friedrich Pietzker (1844–1916) taught mathematics and physics at the Gymnasium of Nordhausen and is the author of *Die Gestaltung des Raumes: Kritische Untersuchungen über die Grundlagen der Geometrie* (1891), in which he discusses among other things non-euclidean geometry and the theories of Riemann and Helmholtz.

Władysław Heinrich (1869–1957) studied mathematics, psychology, and philosophy at the Universities of Munich and Zürich, obtaining his Ph.D. under Avenarius with a work titled *Die moderne physiologische Psychologie in Deutschland* in 1894, published the following year. He was later among the first experimental psychologists at the Jagiellonian University in Poland.

1. Carl Theodor Michaëlis, in *Jahrbuch über die Fortschritte der Mathematik* XXIII/1 (1891), 58–9

In the first volume of his *Philosophy of Arithmetic*, Mr. Husserl establishes the meaning of a number statements by way of an exhaustive analysis of the concept of number [*Anzahl*]. He points out how in most cases we cannot rely on direct number constructions, performed on the objects themselves, but depend on indirect, symbolic ones, based on signs, and how the need for the development of an arithmetical science with its various domains of operation [*Operationskreisen*] is a consequence of this fact. The analysis of the concept of number leads through the analysis of the concepts multiplicity, unity, equinumerosity [*Gleichviel*], more and less. The multiplicity is a whole whose parts are united by collective connection [*collective Verbindung*]. The collective connection is an autonomous relation that cannot be derived from consciousness in general[14] or from the form of time[15] or space[16] or from the empty form of difference.[17] It is a relation that is constituted

14. [Ed.] Husserl's term actually is *Gesammtbewusstsein,* intended to convey that number cannot be derived from the simultaneous, but unarticulated, non-thematic presence of various contents in our "comprehensive consciousness."
15. [Ed.] Either collective unity would be given by contents simply being present at the same time or, exactly the opposite, multiplicity would be nothing but succession. See *PA* Ch. 2, and Carlo Ierna, "The Beginnings of Husserl's Philosophy. Part 1: From *Über den Begriff der Zahl* to *Philosophie der Arithmetik,*" *New Yearbook for Phenomenology and Phenomenological Philosophy* V (2005), 1–56, here 10.
16. [Ed.] According to Lange and Baumann, space would be the origin of discrete multiplicities and numbers.
17. [Ed.] While also mentioning Sigwart and Schuppe, Husserl mainly discusses Jevons under this heading, quoting his definition of (abstract) number as "empty form of difference." *PA* 52, *Hua* CW X 52. Also see Carlo Ierna, "The Beginnings of Husserl's Philosophy. Part 2: Mathematical and Philosophical Background," *New Yearbook for Phenomenology and Phenomenological Philosophy* VI (2006), 33–81, here 75.

in certain psychical acts, which encompass and thereby unify the contents, and whose linguistic expression is the conjunction "and."

By reflection on the psychical act that establishes the unity of the contents connected into a concept,[18] we obtain the abstract presentation [*Vorstellung*] of the collective connection, and by way of this we construct the concept of multiplicity as that of a whole which connects its parts merely collectively. The concept of multiplicity, with and in the concept of collective connection, also contains the concept "something," which is obtained by an abstraction whose main focus is on the collective connection. The general concept of number [*Anzahl*] originates from the concept of multiplicity by the distinction of the abstract forms of multiplicity. It is the genus-concept [*Gattungsbegriff*] that originates from the comparison of the already distinct and definite [*bestimmten*] forms of multiplicity or numbers as species-concepts. The concept of equivalence or equinumerosity [*Gleichzahligkeit*] does not contribute to the analysis of the concept of number. The number statement does not refer to the concept of the counted objects but to their *Inbegriff.* The origination of the natural number series and the development of a numerical system is founded on the concept of symbolic presentations and operations.

All operating that reaches beyond the very first numbers, is just a symbolic operating with symbolic presentations. This fact forces the development of the domain of numbers in the form of a number system together with the picking out of a symbolic construction which is given the systematic role, and with the reduction of all other forms of numbers to these through arithmetical operations. Husserl's investigations indicate themselves humbly as a preparation for a systematical philosophy of arithmetic. However, the author proceeds in such a clear, understandable and patiently detailed research and careful critique of the given theories and opinions from one problem to the next, that in his investigations there may be by far the best that has been written on the foundations of arithmetic in a long time.

2. Anonymous, in *Literarisches Centralblatt für Deutschland* 8 (Feb. 1892), 238–9

Three fields are involved in the same proportion in the kind of investigations that the author conducts and still plans to conduct: mathematics, philosophy and pedagogy, in particular the method of elementary mathematical teaching, which would, so to say, remain suspended in mid-air without the foundational investigations into the concept of number and the operations. With the exception of pedagogy, which only in the last years has felt the need to make the solution of certain questions about the method of calculation dependent on philosophical

18. [Ed.] *Zum Begriff verbundenen,* which means "connected into a concept," does not represent Husserl's position correctly. Rather, Husserl said *zu einem Inbegriffe vereinigt,* that is, "united into a collection"; see *PA* 10, *Hua* XII 16.

investigations, each of the sciences mentioned has long since listed attempts, sometimes very significant ones, to deal with the foundations of arithmetic. However, progress was only possible since the insight was gained that we are dealing here first and foremost with a psychological problem, and it is to be expected that we will now come considerably closer to "the true philosophy of the calculus, that desideratum of centuries" [*PA* VIII; *Hua* CW X 7]. This expectation is justified by the work under consideration, the author of which can claim the merit of having not only cleared the way for future elaborations of the problem, but also of having significantly furthered its solution by conscientious and penetrating investigations. The author believes that the time has not yet come to set up a "thoroughgoing system" [*PA* V; *Hua* CW X 5] of the philosophy of arithmetic; at the present stage of the science, something more than a preparation for a future development of the system would not be possible. Accordingly, he sets himself the goal, "to seek reliable foundations through patiently focused research, to verify the noteworthy theories in careful critique, to separate the correct from the erroneous, in order to, thus informed, set in their place new and possibly firmer ones" [*PA* V; *Hua* CW X 5]. The present first volume, to which a second one will follow after a year, contains in its first part (pp. 1–198) [*Hua* CW X 9–187] the analysis of the concepts of multiplicity, unity and number [*Anzahl*], and specifically, restricted to the proper, non-symbolical presentations of number. The second part (pp. 201–323) [*Hua* CW X 189–299] deals with the improper or symbolic presentations of quantity [*Mengenvorstellungen*] and the symbolic presentations of number that are possible on this foundation and tries to demonstrate the logical origin of a general arithmetic in the sense of a "general theory of operations" [*PA* 323; *Hua* CW X 299]. Here, we cannot omit to report the results, in which the investigations of the author culminate, *verbatim*: "The fact that in the overwhelming majority of cases we are restricted to *symbolic number formations* forces us to a rule governed elaboration of the number domain in the form of a *number system* (whether that of the natural number sequence, or that of the system in the narrower sense of the word), which according to a fixed principle always selects one from among the totality of the symbolic formations corresponding to each actual number concept and equivalent to it, and simultaneously assigns that one symbolic formation a systematic position. For all other conceivable number forms there then arises the problem of evaluation, i.e. of classificatory reduction to the number of the system that is equivalent to it. But a survey of the conceivable forms of number formation taught us that the invention of appropriate methods of evaluation is dependent upon the elaboration of a *general arithmetic*, in the sense of a general theory of operations."[19] As goal of these investigations, the author envisions the demonstration, to be given in the second volume (which would also address the higher symbolic methods), that "identically the same algorithm, the same *arithmetica universalis*, governs a series of conceptual domains that have to be carefully distinguished, and that by

19. [Ed.] Italics spaced in the original edition of the *PA*, in the review neither italicized nor spaced.

no means does a *single* type of concept, whether that of the cardinal or the ordinal number, or any other, mediate the application of it *everywhere.*"[20]

3. Frank Thilly, in *The Philosophical Review* 1(3) (May 1892), 327–30

We have here the first volume of what promises to be a very thorough and detailed account of the philosophy of arithmetic. The present installment is a valuable contribution to the understanding of the fundamental concepts underlying the science of number. The undertaking is a significant one, if only in the sense that it marks a new departure in logic. Hitherto the modern tendency to specialization has not appeared in this department, works on logic contenting themselves with a meager account of the philosophy of the sciences. Dr. Husserl describes the field of his research as a circle within many circles, and directs his attention to the principles of a single science.

The positive portions of the book display sound analytic judgment, while the critical parts, besides being keen and indicative of the author's wide range of reading, carefully restrict themselves to the essential points of the theories attacked. His consideration of the arguments advanced by mathematicians must acquit him of the charge of "onesidedness" frequently urged against logicians who discuss the philosophy of mathematics. Everywhere Dr. Husserl is clear, in thought as well as in expression, a characteristic which, when we remember the abstruseness of the subject and the traditional bent of the German mind for involved sentences, should be doubly appreciated. His intentional disregard of a terminology, which often repels those not skilled in the craft, renders the pages accessible to mathematicians as well as to philosophers. The first part of the work deals with psychological questions connected with the concepts plurality [*Vielheit*], unity [*Einheit*], and sum [*Anzahl*], while the second treats of the symbolical ideas of plurality and sum, and shows how the fact that we are restricted to symbolical number-concepts in arithmetic, determines its character. The author rejects the logical method, which is so strongly advocated by many writers. For him number is the result of psychical processes (p. 130) [*Hua* CW X 124–5]. Notions like unity and plurality cannot be logically defined, but rest upon ultimate psychical data. In this sense they may be designated as form-concepts or categories (p. 91) [*Hua* CW X 89]. Dr. Husserl examines the concepts, plurality, unity, and sum, which latter forms the fundamental notion of number. After investigating the time-succession theory, Lange's thesis that the synthesis upon which number is founded is a synthesis of space-intuitions, and the views of Baumann, Sigwart, Jevons, and Schuppe [in Chapter II, *PA* 17–67; *Hua* CW X 23–65], he finds the origin of the concepts, plurality, and sum, to be due to the "collective combination" of the mind, which

20. [Ed.] *PA* VIII, *Hua* CW X 7. Italics in the original spaced, in the review neither italicized nor spaced, only "arithmetica universalis" written in Latin instead of Gothic characters.

cognizes every member of a sum by itself and in connection with all the rest. The concrete phenomena, which serve as the basis for this abstraction, may be either physical or psychical. This explanation seems to me to be far more satisfactory than the superficial reasoning of Mill, who, like Bain, advocates the theory of physical abstraction. Of course, no concept can be conceived without being based on a concrete intuition, but the special nature of the particular object is of no account whatsoever. The notion of plurality ultimately rests upon that of the somewhat [*Etwas*], a concept which cannot be further analyzed, nor even explained in the way in which Dr. Husserl explains the other concepts. It seems to be a category in the Kantian sense of "function" or "form" of the intellect, a fact which the author does not, in my opinion, sufficiently appreciate.

Part II proposes to explain, psychologically and logically, the art of reckoning based on the notions hitherto analyzed, and to investigate its relation to the science of arithmetic. If arithmetic operated with the *actual* ideas of number, we should have to regard addition and division as its fundamental operations. But this is not the case. Logicians have overlooked the fact that all ideas of number beyond the first few are *symbolical* [*PA* 211–12; *Hua* CW X 200–201]. If we could have *real* ideas of all numbers, arithmetic would be superfluous. Only an infinite understanding, however, could possess such powers of abstraction. Arithmetic is merely an artificial means of overcoming the imperfections of a finite intellect. The most we can do is to cognize concrete pluralities composed of twelve elements [*PA* 213–14; *Hua* CW X 201–2]. When we present to ourselves a real idea of plurality, every member of the group is conceived in connection with all the rest. If we were restricted to this act, no conception of a multitude [*Menge*] would be possible. A hasty glance at a crowd of persons at once gives us the idea that it is a multitude. This is due not to a "collective combination," but to sensible quasi-qualities of the multitude itself, viz. to figural elements (row, heap, group), to the sensible contrasts existing between the members themselves, or between them and their background, to movements, etc. (pp. 227–240) [*Hua* CW X 215–26]. The psychological process, occurring in the formation of such a symbolical idea of multitude, is partly like that in the actual formation: there is psychical activity as regards some of the elements, and this serves as a sign that the process may be continued. Now symbolical numbers rest on the symbolical notion of multitude. Symbolically we may, therefore, speak of numbers whose actual ideation transcends the limits of human powers [*PA* 252; *Hua* CW X 236]. Signs or names are employed to designate groups that can be collectively combined. The sign remains as the fixed framework of the group; by means of it the latter may be reconstructed in thought. But a systematic principle is required for the formation of symbolical number-forms. If the advance from given numbers to new numbers results from the application of a transparent, simple principle, this only need be remembered. If the designations are appropriate, the signs will indicate the whole process. The following scheme, in which x represents the ground-number, embodies the principle underlying the logical formation of number [*PA* 261; *Hua* CW X 245]:

$1 \times 2 \times 3 \quad \ldots \quad x - 1$
$1x \times 2x \times 3x \quad \ldots \quad (x - 1)x$
$1x^2 \times 2x^2 \times 3x^2 \quad \ldots \quad (x - 1)x^2$
$1x^3 \times 2x^3 \times 3x^3 \quad \ldots \quad (x - 1)x^3$, etc.

The same system is expressed in the formation of sensible signs. Concepts are the sources from which the rules of all arithmetical operations spring, but the sensible signs only are taken account of in practice. With a chapter on the logical sources of arithmetic Dr. Husserl ends his first volume. The method of sensible signs is the logical method of arithmetic.[21] In the solution of a problem, the thought from which we proceed is first translated into signs, we operate with these signs according to the laws governing the system, and then translate the resulting signs back again into ideas. Hence, the task of arithmetic is to find general rules for the reduction of different forms to certain normal forms. Arithmetical operations will then signify no more than the methods of performing this reduction. With an examination of the processes of addition, multiplication, subtraction, and division the volume closes.

4. Paul Carus (κρς), in *The Monist* II (July 1892), 627–9

The present volume does not pretend to be a complete system of the philosophy of arithmetic, but it attempts to prepare, in a series of psychological and logical investigations, the scientific foundation for a future construction of this discipline, which would be of equal value to the mathematician and philosopher. The first volume which is now before us analyses in its first part the ideas plurality, unity, and number, so far as they are directly given us and not in their indirect symbolization. The second part considers the symbolical representations of plurality and number, and the author attempts to show that the fact of our being almost throughout limited to symbolical ideas of number determines the meaning and the purpose of that view which the author calls "Anzahlenarithmetik."

The author criticizes several theories which in different ways explain the origin of plurality and unity. There is one theory which explains the origin of the unit from the unity of consciousness; there is another one which explains the origin of number from a succession in time. F. A. Lange bases his theory of number upon space-conception, and Bauman declares there is something mathematical in the external world which corresponds to the mathematical in us. The theory of difference held by Jevons, Schuppe, and Sigwart is declared to be superior to all others, but even that is rejected by the author. Jevons says, "Number is but another name for diversity. Exact identity is unity, and with difference rises plurality. … Abstract number then, is the empty form of difference" [*PA* 51, 52; *Hua* CW X 51, 52]. Dr. Husserl objects: if numbers are all empty forms of difference, what makes the

21. [Ed.] *PA* 292, *Hua* CW X 272. Husserl emphasizes that it is *the* logical method.

difference between two, three, four, etc.? The contents of these numbers are very different. The inability of defining this difference shows the imperfection of the theory of difference. Dr. Husserl proposes what he calls "collection" as a special method of combination by which unities are formed.

Although the book contains many valuable suggestions, it is very hard reading. The author's views are not at all clearly set forth. Neither is the table of contents so systematically arranged as to give us a clue to the plan of the book, nor is there any index that might give us assistance in finding out the most characteristic passages. The reader is supposed to read the book right through, in order to understand detached chapters or even sentences. And even then we are not sure whether or not we have understood the author's propositions the consistency of which is not as apparent as it might be expected. For, after having criticized so many attempts at explaining and analyzing the ideas, plurality, unity and number, and after having proposed definitions, explanations, and analyses of his own, we find on p. 130 [*Hua* CW X 125] a passage where these ideas are incidentally declared to be incapable of definition. Speaking of Frege's theory, Dr. Husserl says, "As soon as we come down to elementary concepts, all definition has an end. Such concepts as quality, intensity, place, time, etc., cannot be defined. The same is true of elementary relations, and of those concepts upon which they are founded. Equality, similarity, gradation, whole and part, plurality and unity, etc., are concepts which are utterly incapable of a formal-logical definition. All we can do in such cases is to produce the concrete phenomena from which they have been abstracted, and to explain the method of this process of abstraction. One can, where it is necessary, exactly fence in (*umgrenzen*) by diverse circumscriptions, the concepts in question, and thus prevent confusion with kindred concepts." We must confess that we do not understand the author's idea; what is an act of defining if not an "umgrenzen," a fencing in of the concept? The book contains many similar passages, which, it seems to us, are not properly thought out by the author. But the subject is a difficult one, and, as the author says in the preface, "A work of this kind should, with regard to the difficulties of the problem it treats, be judged with leniency" [*PA* IX, *Hua* CW X 8].

5. Anonymous, in *Mind* 1(4) (October 1892), 565–6

We have here the first volume of an important work likely to be of especial interest to those who are concerned with the Theory of Knowledge. We hope to furnish a more extended notice of it when the second volume, which the author promises shortly, comes to hand. The whole work is to consist of four parts (of these the present volume contains the first two): (1) in the main psychological, the analysis of the concepts of plurality, unity, and number apart from symbolic forms of representation; (2) an examination of these symbolic forms and of the effect that our dependence upon symbols has in shaping the problems and methods of numerical arithmetic; (3) the logical investigation of arithmetical algorithmic, and, in particular, of the results of inverse operations-negative, imaginary, fractional and irrational numbers; and (4) the nature and scope of universal arithmetic. In an appendix to this volume the author also hopes to fill up a gap in our existing logic by treating generally of the logic of symbolic methods or "Semiotic." The work is thus obviously one that can be dealt with critically only when it is complete.

6. Ernest Lindenthal, in *Zeitschrift für das Realschulwesen* (1893), S. 104–7

This work does not claim to be a straightforward system of the philosophy of arithmetic; rather, it would merely prepare the foundations of this science. In the first of its two parts (200 pp.) the author discusses the origin of the concept of multiplicity by way of that of collective connection; he then illuminates in detail and with success the various attempts to explain the essence of number; he dissects the concept of number [*Anzahl*], and to that adds remarks on the concepts more and less, on equinumerosity and its criteria and finishes with an in-depth investigation of the concepts of unity and multiplicity. The second part is then dedicated to the so-called symbolic concepts of number and the logical sources of the arithmetic of numbers. Here we find a treatment of the four species.[22]

Neither in German nor in any other language has a work of such magnitude as this appeared, with such a detailed and encompassing exposition regarding the foundations of arithmetic together with so many real results. However, there will still scarcely be any question of significance with respect to which there would be any bearable harmony among the researchers trying to answer it.

In the critical developments of the first part, all prominent views on the essence of number undergo sharp scrutiny. Not without success. It is refuted that the presentation of the *Inbegriff* would consist merely in belonging to its encompassing consciousness, that it would contain nothing more than simultaneous contents, that temporal succession and nothing else would characterize a multiplicity, that arithmetic would rest on spatial intuitions, etc. The author rejects with full justification the nebulous and dependent opinions of Wundt regarding relative, fractional, and irrational numbers (p. 95) [*Hua* CW X 92–3] and the nominalistic attempts at an explanation of Kronecker and Helmholtz (pp. 190ff.) [*Hua* CW X 179–87]; however, he overshoots the target when he explains Lange's statement, "We originally obtain the concept of number as the sensuously [*sinnlich*] determined picture [*Bild*] of a group of objects" [*PA* 33; *Hua* CW X 36], in such a way that Lange would thereby have explained the general concept of number as the sensuously determined picture of a group of spatial things [*Raumdingen*]. Moreover, he refutes the positions that the number predicate would pertain to the counted things and that the equality of the units [*Einheiten*] would merely be an approximation, but not have an absolute value. The important fact, that

22. [Ed.] I.e. the four basic operations: addition, multiplication, subtraction, and division.

we are capable of conceiving one and the same object at will as one or as many without thereby rendering all arithmetic impossible, rather than the contrary, that all arithmetic rests on this, is appropriately brought forward and the plurivocity of the word unity (pp. 169ff.) [*Hua* CW X 159ff.] is clarified. With respect to the fifth meaning of this word given on p. 171 [*Hua* CW X 160–61], we might also add that imaginary units are nothing but countable unities. The word horse is not plurivocal because there are also white horses, and likewise the word unity is not plurivocal simply because there are also imaginary units.

The error appears to be removed, only to flee into other corners and take hold there. On p. 3 [*Hua* CW X 11] it says, "The concept of number is a multifarious one [*ein Vielfacher*]." Perhaps the author wanted to say that the meaning of the word number is multifarious. How are the basic, ordinal, variative, multiplicative, and partitive numbers dealt with here (p. 3)? Does the variative number not count kinds and the partitive number fractional parts? Is the ordinal even a number, does it not refer to a single object? Is Charles XII a multiplicity? Unwittingly the philosopher has become a grammarian; but where grammar begins, philosophy ends. The second part of the sentence (p. 9) [*Hua* CW X 15–16] "Wherever we speak of a definite number, we can also speak of a multiplicity, when we speak of a multiplicity, then we can also always speak of a definite number" is not true. In the sentences "there are many views," "many roads lead to Rome," multiplicities appear, but no definite numbers. One could at most claim that multiplicity can only be understood as finite multiplicity. On p. 146 [*Hua* CW X 140] we find the old catchphrase: "extension of the concept of number." However, by prepending zero and one to the number series, we extend neither the series of numbers, nor the concept of number; instead now the word number serves as an indication of the three concepts: counted multiplicity, one, and zero. By thinking things in one act (p. 79) [*Hua* CW X 77], we do not yet think a plurality. My friend, his love of order and indefatigability does not awaken the concept three when I think them simultaneously. The concept plurality cannot be separated from his twin-concept unity. Only when I direct my attention to the contrast of one and many, the concept of plurality arises. On p. 107 [actually *PA* 108; *Hua* CW X 105] the author states: "what it means, that two relatively simple contents are equal to each other, is neither capable nor in need of being explained." This is correct; but to make a distinction here between simple and composite contents, is not. One should not be misled by the unfortunately quite widespread negligent expression. When we say "these objects are equal [*gleich*] with respect to their color," then we mean "the color of these objects is the same [*gleich*]." Not the objects, but only their colors are the same. The comparison of multiplicities with respect to their numbers (p. 112) [*Hua* CW X 108] is reminiscent of the comparison of straight lines with respect to their straightness. If we would tolerate such negligent forms [of expression], then we can also say: these two persons are the same with respect to the buttons of their coats. In the case of quantities (multiplicities) a real and true equality should be noticeable in more than one respect (p. 109) [*Hua* CW X 106]. Two multiplicities can only be equal (p. 110) [*Hua* CW X 106–7] when to each element of one quantity (multiplicity) corresponds an equal one in the other quantity (multiplicity). Multiplicity and quantity are not strictly distinguished here. To two equal quantities does not need to correspond the same multiplicity and vice versa to two equal multiplicities not the same quantity. According to the opinion of the author 10 English feet and 10 meters would not be the same multiplicity. We are in complete agreement when (p. 128) [*Hua* CW X 123] he criticizes Schröder for taking tally marks [*Strichmengen,* lit. "quantities of lines"] as the natural numbers. However, after the well-thought-out discussion regarding one and something (pp. 90, 91) [*Hua* CW X 88–9] it is surprising to hear that the mark | could only mean for each content that it is something. Number is as little something, something, something etc., as it is apple, apple, apple etc. It is only the form here, which encompasses the somethings, and nothing else. To the question "How many are Jove, a contradiction and an angel?" (p. 161) [*Hua* CW X 152] we do not immediately reply three, but only after we brought them under a common concept; here under the concept thing. We are convinced of the fact that we postulate a concept before counting, e.g. by the answer that everyone gives to the question: how many is [*wie viel ist*][23] 1 ducat, 1 guilder, and l kreuzer?[24] How many is 1 third, 1 fourth, and 1 fifth? [25]How many is 1 dozen and 1? The author manages to meet Kerry's relevant views in a very perspicacious manner (p. 184) [*Hua* CW X 173], but this changes nothing of our opinion. Kerry's expression was infelicitous; instead of saying: without a leading concept [*Leitbegriff*] we risk counting something that we should not have counted; he should have held that without a leading concept we cannot count at all. Also where the author turns against Herbart and Frege (p. 186) [*Hua* CW X 174], notwithstanding all his acumen he does not refute the truth that all counting rests on a concept. Nevertheless, we concur that definitions of the concept equality advanced by Leibniz and Grassmann turn the true state of affairs on its head. Just as we can deduct the effect from the cause without failure, but not the cause from the effect, so we can substitute one thing for another in judgments, when they are completely equal; but we cannot conclude that they are equal because they appear in two equal judgments. The usual definition of equality of two numbers, as we find it also in Stolz, "Two multiplicities are said to be equal to one another, if to each thing of the first one of the latter can always be made to correspond and none remain unconnected,"[26] is rightly presented as criterion for the equality of two

23. [Ed.] The argument hinges on the precise formulation of the question: with "how many are" one would be inclined to say "three" (three coins), with "how many is" or "how much is" one would be inclined to respond with the total monetary value of the coins.
24. [Ed.] In 1892 there had been a monetary reform, introducing the gold standard in Austria, along with the new denominations of *krone* and *heller.* Previously it would have been 100 *kreuzer* to the guilder, while a ducat, consisting almost entirely of gold, was not considered legal tender. In Mozart's times, we would have had 4.5 guilders to the ducat and 60 *kreuzer* to the guilder, hence 331 *kreuzer* in total.
25. [Ed.] As 1/3 has an infinite decimal expansion, there is no finite fraction that could answer the question, it would equal 0.78333…, or three-quarters and a thirtieth.
26. [Ed.] Otto Stolz, *Vorlesungen über allgemeine Arithmetik* I (Leipzig, 1885), 9. See *PA* 105, *Hua* CW X 103.

numbers. While the introduction to Stolz's "Lectures on General Arithmetic" verily is a collection of samples of propositions that provoke decisive objection, we have to take the definition of more and less of this author under our protection against Dr. Husserl's attacks. In the definition of equality the presentation of more and less is certainly not already included, but that of inequality is. Equality and inequality, just like more and less, are twin-concepts; one illuminates the other. Therefore, there is no circularity at all to be found in Stolz's definition of "being greater than."

Despite the many truths pronounced in it, we only took up the second part with some reserve, despite the brilliant treatment of the matter. The author there becomes embroiled in the symbolic concepts of number, where it would have sufficed to distinguish concept and respective sign. How much we are children of our time, though we lift ourselves freely and boldly above the mainstream opinions and status, is betrayed by the sentence (p. 206) [*Hua* CW X 195]: Just as numbers serve as general signs, shortcuts to lighten our thoughts and speech, so they also function as multiplicators. On p. 216 [*Hua* CW X 206] we are told about a distinction between abstract and general concepts. In our opinion, however, there are only abstract presentations, i.e. concepts. Abstract concepts must be related to spherical spheres [*kugelrunden Kugeln*]. Calculating would be nothing but an operating with signs (p. 271) [*Hua* CW X 254]. Without doubt, the strict parallelism between the system of number concepts and that of their signs is extraordinarily useful when calculating. However, this does not justify the explanation of calculation as a rule-based deduction of signs from signs (p. 293) [*Hua* CW X 273]. It is correct to say that 7 + 5 means the number that encompasses all the units of 7 and all those of 5 (p. 205) [*Hua* CW X 194], and that the concept of addition does not contain anything of a temporal order (p. 210) [*Hua* CW X 199]. Nevertheless, it does not follow from the concept of multiplicative connection that the products *ab* and *ba* are equal (305) [*Hua* CW X 284], because this must be proven first. Moreover, multiplication does not have a single inverse operation, but two: partition and measurement.[27] The author disregards the latter, not mentioning it at all. As basic operations that we can apply to all numbers, and through which alone we can build new numbers from given ones, only addition and partition (p. 302)[28] are mentioned. Would it not have been better to speak of additive connection and decomposition, since the word partition means the decomposition into equal parts?

There is no room here to enter in detail into the main thoughts of the second part. The book, taken as a whole, is an excellent achievement on the border area of philosophy and arithmetic; but it is also suitable to reduce the dominant exaggerated fancies of the educational value of pure mathematics to their correct dimension. We look forward with anticipation to the publication of the second volume. The printing was well heeded. Only few errors, and among these only a single

27. [Ed.] Partitive division and quotative or measurement division.
28. [Ed.] Cp. *Hua* CW X 281, but it is unclear exactly which passage is meant. On *PA* 299, *Hua* CW X 279, Husserl says: "The forms of operation which the concept of number permits are addition and partition." Partition here includes both subtraction and division.

disturbing one, are missed by the errata, p. 283, l. 5 from the bottom,[29] where the two occurrences of the words "and ten" should both be eliminated. The error indicated for p. 116 is not to be found there.[30]

7. Heinrich Schotten, in *Zeitschrift für Mathematik und Physik (Historisch-literarische Abtheilung)* 38 (1893), 88–90

The volume under consideration is articulated in two main parts, of which the first "the proper concepts of multiplicity, unity and number [*Anzahl*]" discusses "mainly psychological questions," while the second is titled "The symbolic concepts of number [*Anzahlbegriffe*] and the logical sources of number-arithmetic [*Anzahlen-Arithmetik*]," in which the author "attempts to show how the fact that we are almost totally limited to symbolic concepts of number, determines the sense and objective of number-arithmetic."

After the introduction, the first part contains the following chapters: (1) The origination of the concept of multiplicity through that of the collective connection; (2) Critical developments; (3) The psychological nature of the collective connection; (4) Analysis of the concept of number in terms of its origin and content; (5) The relations More and Less; (6) The definition of equinumerosity [*Gleichzahligkeit*] through the concept of reciprocal one-to-one correlation; (7) Definitions of number in terms of equivalence; (8) Discussions concerning unity and multiplicity; (9) The sense of the statement of number; Appendix: The nominalist attempts of Helmholtz and Kronecker.

W. Unverzagt[31] says (in *Der Winkel als Grundlage mathematischer Untersuchungen*; Wiesbaden 1878), "The concept of number, in all its mutations, is perhaps the most interesting—though certainly also one of the most difficult" and provides a short historical survey of these mutations. However, his exposition concerns first and foremost the mathematical developments that the concept of number underwent. M. Simon,[32] whose manual of arithmetic met with the approval of excellent mathematicians, tells us in his programme "*Zu den Grundlagen der nichteuklidischen Geometrie*" (Strassburg 1891), that in the last ten years we have come to a certain agreement in arithmetic: "We broke with the Kantian subordination of number under time. The number is placed under the purely logical concept of order [*Zuordnung*]." This view is not shared by the author of the work under review here; he is rather of the opinion that "the attempts that deal with the fundamental questions of the domain treated are innumerable," and that a final decision has yet to be made. Admittedly, of these innumerable attempts, he wants to

29. [Ed.] Actually line 28, that is, 6 from the bottom, corrected in *Hua* CW X 264, line 15.
30. [Ed.] Indeed, it is on 166, line 9, corrected in *Hua* CW X 156, lines 21–2.
31. [Ed.] Wilhelm Unverzagt (1830–1885), high school teacher in Wiesbaden, among the first to introduce Hamilton's Quaternions in Germany.
32. [Ed.] Max Simon (1844–1918), graduated in Berlin with Weierstrass and Kummer.

consider only the most important "to seek reliable foundations through patiently focused research, to verify the noteworthy theories in careful critique, to separate the correct from the erroneous, in order to, thus informed, set in their place new and possibly firmer ones" [*PA* V; *Hua* CW X 5]. With these words he characterizes the intention of his work.

It seems to us as if the author would have underestimated certain works that are considered important by others. Thus it would have been appropriate to address G. Cantor's works and the related ones of Kerry right at the beginning. Likewise, Bolzano would have deserved to be mentioned among those who formerly worked on these problems, especially since we owe him the "*Inbegriff*."[33] Regarding the exposition itself, the reviewer must confess that the whole first part gives the impression of being hesitant, not having yet matured into a clear understanding, so that the reader also does not attain a satisfactory result. This is not to deny that, in the course of the investigations, there are a number of perspicacious arguments and sharp conceptual definitions.

It is not very pleasant that the author continually condemns other views, among them those of respected authorities, while particularly bringing to the fore the complete evidence of his own investigations, e.g. when he says: "As for the rest, it results from our analyses with incontestable clarity …; that the goal that Frege sets for himself must therefore be termed chimerical. It is therefore also no wonder if his work, in spite of all ingenuity, gets lost in unfruitful hyper-subtleties and concludes without positive results" [*PA* 131; *Hua* CW X 125–6]. There may be those who doubt the incontestable clarity and readers who also see unfruitful hyper-subtleties in the investigations of the author. In particular, we consider the statements of the author in chapter eight eminently contestable.

The second part comprises the following chapters: (10) Operations on numbers and the proper number concepts; (11) Symbolic presentations of multiplicities; (12) The symbolic presentations of numbers; (13) The logical sources of arithmetic.

This part begins with the words: "After the discussion and solution of the subtle questions …"; in general it is clearer and appears to be of greater significance. Chapters 11 and 12 especially, in the latter particularly the development of the number system, turned out well.

As a curiosity, we must mention that whenever the author reports calculations of the simplest kind, they are wrong (p. 150,[34] p. 283,[35] p. 296[36]), without being

33. [Ed.] See Bernard Bolzano, *Paradoxien des Unendlichen* (Berlin: Mayer & Müller, 1889), 2, cf. Carlo Ierna, "Husserl and the Infinite," *Studia Phaenomenologica* III(1–2) (2003), 179–94 and Ierna, "Beginnings (Part 2)," 45 for its influence on Husserl.

34. [Ed.] "three units plus five units yield seven units", not corrected in *Hua* XII 136, but corrected in *Hua* CW X 143.

35. [Ed.] "hundred and ten … or eleven times ten and ten (eleventy and ten)", 11 times 10, plus 10 or "eleventy and ten" would be 120 instead of 110. This was corrected in *Hua* XII 249 and *Hua* CW X 264.

36. [Ed.] "How much is 18+48? We answer 67". Corrected in *Hua* XII 261 and *Hua* CW X 276.

set right in the corrigenda at the end of the book. Also the spelling of the author is peculiar.

8. Franz Hillebrand, in *Göttingische gelehrte Anzeigen* 4 (1893), 175–80

In the present first volume the author takes up the task of investigating the psychological questions related to the analysis of the concepts multiplicity, unity, and amount [*Anzahl*], in so far as these are given to us in the *proper* [*eigentlich*] and not just the symbolic sense, and furthermore to discuss also the *symbolic* presentations of multiplicity and amount and to determine their function in the arithmetic of amounts [*Anzahlenarithmetik*]. This is the reason for the division of the present volume in two parts.

The psychological origin of the concept of multiplicity is the problem to which the author turns first.

The concept of multiplicity is attained through reflection on the peculiar manner of unification of contents, as present in every concrete *Inbegriff*. The author calls this manner of unification the *collective connection* and tries to give a more precise characterization of it.

The collective connection can be characterized neither by the fact that the elements are given *simultaneously* in consciousness, nor that they enter in consciousness in temporal succession. Likewise, the essence of the collective connection does not rest in the synthesis of spatial intuitions (as Albert Lange thought). The author also does not agree with the view of those who think that multiplicity would be the empty form of difference ("3 colors" would be identical with "3 different colors"); it is only relevant to notice the different (collected) contents in themselves, not to notice them *as different ones*.

Let us now ask, what is the collective connection? Up to now, with the exception of negative determinations, we only understood that it would be essential to it that the single partial contents are to be noticed in themselves.

Our author answers that it is a class of relations that is distinct from all others (69) [*Hua* CW X 69]. Relations are subdivided by the author to such that have the character of primary contents, i.e.—if I understand correctly—relations, in which two or more contents enter without any *psychical act* (such as e.g. presenting) being involved in establishing the relation (primary relations p. 72–3) [*Hua* CW X 71–2]—and in relations that are established through a unitary psychical act directed at a plurality of contents (psychical relations p. 73) [*Hua* CW X 72]. This kind of relation is not given by the contents as such, and hence cannot be detected in the contents as such. From another point of view, then, the author separates relations in simple and composite, the latter of which are characterized by the fact that they consist again of simple relations (p. 76) [*Hua* CW X 74].

The collective connection is a *psychical relation*, in so far as the partial contents are held together by a *unitary psychical act* (interest, noticing). "An *Inbegriff*," our author says, "originates in that a unitary interest and simultaneously in and with it a unitary noticing distinctly picks out and encompasses different contents" (p. 79) [*Hua* CW X 77].

Multiple *Inbegriffe* can then be again held together by a unitary psychical act of second order and hence generate an *Inbegriff* of higher order.

If with an *Inbegriff* we abstract from everything except the moment of collective connection, we attain the general concept of multiplicity and of the "one" (or "any one" [*irgend Eins*]), two correlative concepts.[37] The concept of *multiplicity*, however, is distinguished from the concept of *number* [*Anzahl*] by the fact that the latter already presupposes a distinction of the abstract forms of multiplicity.

The concepts more and less are founded on psychical acts of higher order; because here we deal with the insight [*Erkenntniß*] that an *Inbegriff* is equal to a part of another *Inbegriff* (p. 101) [*Hua* CW X 99], where equality is *not* identical with one-to-one correlation [bijection] of the single members, even though this correlation is a criterion for the equality (p. 114) [*Hua* CW X 110].

It is in accordance with the provided definition of the number concept that the author considers zero and one, not properly, but only figuratively as numbers (pp. 142ff.) [*Hua* CW X 136ff.].

As the subjects of number statements, the author indicates the *Inbegriff* of the collected objects itself, not their concept (p. 185) [*Hua* CW X 174].

Moreover, from the proposed characterization of the *proper* number concepts follows that addition and partition are the basic operations that we can apply to numbers. Both concepts have their origin in the fact that the comprehensive [*zusammenfassenden*] psychical acts can be of different orders. If the particular connections, which are given by a psychical act of first order, are dissolved, in such a way that now only that act remains that previously was an act of second order, or—as we may succinctly say—of an act of second order becomes an act of first order, then we speak of addition; in the opposite case [we speak] of partition. (Of course, this partition is not identical with division; because the equality of the parts does not belong to its concept).

After having psychologically analyzed those concepts that related to the *proper* numbers, and having described the basic operations on these, the author (in the second part of the first volume) turns to the investigation of the *symbolic* number concepts. The concept of a symbolic presentation in general is determined in the following way: "A *symbolic* or improper presentation is, as the name already says, a presentation through signs. When a content is not given directly to us as what it is, but only indirectly *through signs, that characterize it unambiguously*, we have, instead of a proper presentation, a symbolic presentation of it" (p. 215) [*Hua* CW X 205].

In most cases the concrete presentation of a multiplicity is not a proper one, but one that is symbolic in the indicated sense, as we are only capable of noticing few members each in itself, as required for a proper presentation of multiplicity.

37. Not unimportant are also the author's findings concerning the *equivocal* use of the name unity. He find no less than *eight* different meanings (p. 169 ff.) [Ed.] *Hua* CW X 159ff.

The intuitive presentation of quantity cannot overcome these narrow limits; if more elements are present, then only a symbolic presentation of quantity can be attained.

How are we to understand the construction [*Bildung*] of such a symbolic presentation? To only comprehend together [*zusammenzufassen*] a small part of the elements in a proper presentation of quantity does not suffice. Because, as the author correctly observes, "how can the two to three first steps of the process serve as sign for the allegedly intended full process? Whence do we know that the process of individual apprehension [*Sonderauffassung*] can be continued even just by a single step?" (p. 224) [*Hua* CW X 212]. The origination of a symbolic concept of quantity can only be explained if there are "immediately graspable indications in the intuition of the sensuous quantity through which the characteristics of being a quantity [*Mengencharakter*] can be recognized" (p. 225) [*Hua* CW X 213]. The author finds such characteristics in the so-called "figural moments" or, as Von Ehrenfels called them, in the "*Gestalt* -qualities." I prefer to take a general definition of this concept from a treatise by the latter author. He says: "By *Gestalt -qualities* we understand such positive contents of presentations, which are bound to the presence of complexes of presentations in consciousness, which on their turn consist in separable (i.e. individually presentable) elements."[38] Examples of complexes of presentations, that carry certain figural moments or "*Gestalt*-qualities" with them are: an avenue of trees, a row of soldiers, a chain of partridges, a flight of ducks. These "quasi-qualitative" moments are apprehended as something simple, not as a collectivum, and indeed immediately, i.e. without needing reflection on their constituting relations; indeed, the author even describes them as analoga to sense qualities. Now such figural moments are given everywhere we encounter quantities that are too big to construct a proper and intuitable concept of quantity, but in which we can bring to the fore single groups in an intuitable way and hence can at least *successively* arrive at a series of proper presentations of quantities, which then as a whole are equivalent to the originally given quantity. In this way a stable association is made between these processes and the figural moments; the idea [*Vorstellung*] that it would be possible to successively traverse a quantity in the described manner is then directly associated with the respectively given sensuous configurations.

In this way our author explains the character as well as the origination of the symbolic presentation of quantity.

On the basis of this result, the author then develops clearly and extensively the symbolic presentations of number, then the unsystematic, and finally the systematic numbers, specifically in our decimal number system.

At the end the author discusses the logical sources of arithmetic, by which he means the science of the symbolic derivation of numbers from numbers on the base of rule-based operations with sensuous signs.

38. v. Ehrenfels, "Ueber 'Gestaltqualitäten'," in der Vierteljahrsschrift für wissensch, Philos. Bd. XIV. 3, p. 262.

Hereby I think I have clearly brought forward the most important problems that are the concern of our author, and I hope that the reader will already gain the conviction from this that in Husserl's book he is dealing with a careful and sophisticated investigation of a problem area that men like Riemann and Helmholtz have not considered unworthy of their liveliest interest.

Nevertheless, I cannot suppress a remark, though it is in the form of a wish, that certainly also other readers of this book share with me and to fulfill which the author may take the occasion in the second volume (even if only parenthetically).

What I am aiming at is—in short—a somewhat deeper psychological analysis of the concept of a collectivum.

The author tries to characterize it by saying that it directs a "unitary act" at a plurality of given contents that "holds them together" (e.g. p. 78, p. 79 and elsewhere) [*Hua* CW X 76–7]. If we ask what is to be understood by such an "act," we obtain the answer (p. 79) [*Hua* CW X 77] that it would be "a unitary interest and in and with it at the same time a unitary noticing." Now these are already *two* things. Can they be present separately? Could perhaps noticing already be enough? And if not, which one of them is primary? Further, what do we have to understand by "noticing"? Does it belong to the acts of presentation? If yes, what distinguishes it from the mere presenting, since not every presenting is a noticing? Are these differences in intensity? And furthermore, we heard that besides the act of noticing that is directed at the whole collective, also every partial content would have to be noticed in itself. Do we then have to assume that one and the same content can become the object of a *double* noticing? Indeed, not just of a double, but even of a three-, four-, …, *n*-fold noticing, as there should be unifying acts of *higher order*?

I realize that here it is easier to ask ten questions than to answer one; but the need for an answer is not therefore less urgent.

If such questions cannot be given a conclusive answer in the end, then it is certainly good not to pursue the analysis farther than what one is completely certain of, and the reserves that our author imposes on himself here will find the support of any careful psychologist. There is no critique here, but the desire that the author, if he manages to close in further on the psychological analysis of the concept of a collective and to propound his results with as much persuasion as the preceding ones, would not deprive the readers of the soon-to-be-expected second volume of them.

9. Albino Nagy, in *Rivista Italiana di Filosofia* VIII/II (1893), 243–5

Dr. E. G. Husserl, *privatdozent* at the university of Halle, publishes a series of investigations regarding the fundamental problems that a future "philosophy of arithmetic" should deal with. Hence, these are merely preparatory contributions towards this discipline, of which a part has already been made public in 1887 in his *Habilitationsschrift*: "On the Concept of Number."

The present one is the first volume of the work and it mainly discusses two topics: (1) The true concept of multiplicity (*Vielheit*), of unity and of number

(*Anzahl*), 9 chapters; (2) The symbolic concepts of number and the logical sources of numeric arithmetic, 4 chapters.

Here the word "number" is intended to convey the everyday meaning and hence is meant precisely as cardinal number (*Anzahlen oder Grundzahlen–numeralia cardinalia*; p. 3).[39] The other numbers, i.e. the distinctions of positive and negative, rational and irrational, real and imaginary, quaternions, etc., that belong to arithmetical science will be discussed in the second volume.

Now the concept of number is derived from that of multiplicity and this from that of plurality, of a set [*insieme*] (*Inbegriff*) of certain objects, whatever they may be. The "set of a plurality of objects" is given by that psychological association that is also called "collection" or "collective connection" (*collective Verbindung*) due to which the set of given objects appears like a unity in which the presentations of individual objects are contained as partial presentations (p. 15) [*Hua* CW X 21]. This is the result that forms the pivotal point of all the detailed and careful observations, the subtle discussions, that constitute the first part of this volume: that is, the critical developments regarding the relation between collection and simultaneity, succession; regarding the collective and the spatial synthesis; and finally regarding the "*colligere,*" the numbering, the distinguishing (Ch. II).

In Ch. III he examines the psychological nature of the collective association, which, according to him, would consist in the fact that a unitary (*einheitliches*) interest and simultaneously with and in it a unitary noticing (*Bemerken*), picks out and encompasses various contents (*Inhalte*) for themselves (p. 79) [*Hua* CW X 77].

Having thus examined in these three chapters the concept of multiplicity, in the fourth he begins the treatment of the derivative concept of number by analyzing its origin and content. "The concept of multiplicity derives from the intuition [*percezione*] (*Anschauung*) of a concrete multiplicity, by way of a process of abstraction" [*PA* 84; *Hua* CW X 83] which consists in the following: "Certain individual contents are given in collective association; we do not consider them as thus and so determined contents, the main interest is instead concentrated on their collective connection while each of them is considered merely as something [*irgend Etwas*], as any thing [*irgend Eines*]" (p. 85).[40] The attention is directed at the collective connection; it is grammatically expressed by the conjunction "and." Hence "multiplicity" in general is nothing but "something and something etc." or "a thing and a thing …" or, simply "one and one …." The fifth chapter examines the relations of more and less, the sixth the definition of equivalence through the concept of one-to-one correlation, the seventh the definitions of numbers by equivalence. The eighth chapter contains various discussions regarding plurality and unity, the ninth a careful scrutiny of the meaning of number judgments (*Zahlenaussage*) and a reminder, due to the definition of number that was given, that they do not refer to the numbered objects but to their collection. In this, in a certain sense, he agrees

39. [Ed.] See *Hua* CW X 11, which has a slightly misleading translation.
40. [Ed.] *Hua* CW X 83. For the difficulties involved in translating these German expressions, see Ierna, "Beginnings (Part 1)," 12 n. 42.

with Herbart, who stated that "numbers refer to concepts not to things," against Mill who maintained that "numbers are names of objects."[41]

The second part begins with the tenth chapter, titled: the numerical operations and the true concepts of number. It starts with the observation that the numbers of arithmetic are not to be considered as abstract concepts—I would prefer to say that they are not to be considered formally but according to their content—due to which 5 + 5 does not mean "the concept 5" plus "the concept 5," which, logically, like "gold" + "gold" yields "gold," would give as a result "the concept 5," but rather means: "a quantity named 5 and another quantity of the same name, together yield a quantity named 10" (p. 202).[42] In the following he maintains that arithmetic does not operate with the true concepts of number, but with symbolic concepts of the same (ch. XI). The symbolic presentations are a popular topic in modern German psychology. I bring to mind the lectures I heard in Vienna from Prof. Brentano and the works of Meinong (*Humestudien* 1882), also quoted by Husserl.[43] In every case in which a content is not given directly to us, as it is, but only indirectly through signs that characterize it univocally, we have a symbolic presentation of it (p. 215) [*Hua* CW X 205].

The symbolic number concepts are given by the figural moments, that is, the various configurations and dispositions of the individual parts within the encompassing whole which constitutes the numbered quantity (e.g. groups, sequences, etc.).

The decimal number system is also based on these figural moments, that is, on groupings of the units by tens and tens. (Ch. XII).

The last chapter (XIII) finally examines the logical sources of arithmetic.

With this brief survey it was not possible to give more than a vague idea of the nature of the book, which is really unique due to its patient analytical and critical work. Moreover, the subject matter is still so difficult and rough, due to its scarcely progressed state, that it is vain to expect to gain a clear and systematic understanding of it even after an attentive and repeated reading of the book. The lack of a systematic order is also felt in the analysis of the initial concepts, which, as we saw, are not tight and straight, but diffuse and contorted. Hence, for example, in the exposition of the concept of multiplicity one is already forced to also discuss unity, parts, etc. which are notions barely explained in the successive paragraphs.

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41. [Ed.] Johann Friedrich Herbart, *Psychologie als Wissenschaft: Neu gegründet auf Erfahrung, Metaphysik und Mathematik. Zweyter analytischer Theil* (Königsberg: Unzer, 1825), 161 and James Mill, *Analysis of the Phenomena of the Human Mind*, ed. John Stuart Mill, 2nd ed., vol. II (London: Longmans, Green, Reader, and Dyer, 1878), 92 n. 22. The note is by the editor, i.e. J. S. Mill. See Husserl's discussion of these authors in *PA* 179–80, *Hua* CW X 169.
42. [Ed.] *Hua* CW X 191–2. Willard translates "*Menge*" with "group"; see Carlo Ierna, "Review of Edmund Husserl *Philosophy of Arithmetic*, trans. Dallas Willard (Husserliana Collected Works X)," *Husserl Studies* 24(1) (Apr. 2008), 53–8, here 55. On his part, Nagy appears to translate both "*Menge*" and "*Vielheit*" with "*moltitudine*."
43. [Ed.] Alexius Meinong, "Hume Studien II. Zur Relationstheorie," *Sitzungsbereiche der phil.-hist. Classe der kais. Akademie der Wissenschaften* CI(II) (1882), 573–752. Husserl quotes 656–8.

We attend with sure interest the publication of the second volume, which among other things will contain the principles of semiotic, in which the logical calculus will be treated, and which will probably contribute significantly to the recent polemic about this subject, which the author sustained in the *Vierteljahrsschrift für wissenschaftliche Philosophie* against Dr. Voigt,[44] which I also discuss in an "essay concerning the task of logic," shortly forthcoming.[45]

10. Alois Höfler in *Zeitschrift für Psychologie und Physiologie der Sinnesorgane* VI (1894), 49–56

The second of the works under consideration here,[46] Husserl's *Philosophy of Arithmetic*, testifies to a manifold influence by Kerry's work, or rather groundwork. The preface (p. VII.) [*Hua* CW X 6] says: "The first volume here[47] in the first of its two parts mainly deals with the psychological questions involved in the analysis of the concepts of multiplicity, unity and amount [*Anzahl*], insofar as they are given to us in the proper sense and not through indirect symbolizations. The second part then considers the symbolic presentations of multiplicity and amount, and attempts to show how the fact that we are almost totally limited to symbolic number concepts determines the sense and purpose of the arithmetic of amounts."

The first chapter, "The origination of the concept of multiplicity through that of collective connection," shows that we cannot say that "Collections [*Inbegriffe*] consist merely of the particular contents [*Einzelinhalten*]. However easy it is to overlook it, there still is present in them something more than the particular contents, something that can be noticed and that is necessarily present in all cases where we speak of a collection or a multiplicity: the connection of the single elements to the whole" (p. 13) [*Hua* CW X 19]. Moreover, it is "not [our intent to give] a definition of the concept multiplicity, but rather a *psychological characterization* of the phenomena upon which the abstraction of this concept rests."

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44. [Ed.] In *Vierteljahrsschrift für wissenschafliche Philosophie* 17 (1893): Husserl, 111–20; Voigt, 504–7; and Husserl again, 508–11. See *Hua* XXII, 73–82; 83–6; 87–91 and *Hua* CW V, 121–30; 131–4; 135–8. Voigt then published an additional note in the next issue of the *Vierteljahrsschrift* 18 (1894), 135, drawing in the editor of the journal, Richard Avenarius, who intervened on 135–6.

45. [Ed.] Probably *Sulla definizione e il compito della logica* (Rome: Balbi, 1894).

46. [Ed.] Together with Husserl's *Philosophy of Arithmetic*, Höfler reviewed Benno Kerry's series of articles "*Über Anschauung und ihre psychische Verarbeitung*" that appeared in the *Vierteljahrsschrift für wissenschafliche Philosophie* from 1885 to 1891 and Von Ehrenfels' article "*Zur Philosophie der Mathematik*," in *Vierteljahrsschrift* 15 (1891), 285–347.

47. Concerning the second volume, whose publication was announced for 1892 in the preface, the author had the regard to advise me by letter not long ago, that the preparation would be delayed beyond 1893. This circumstance may also in part count as exculpation for the lateness of the present review. [Ed.] See the letter from Höfler to Husserl of 16 February 1893 in Edmund Husserl, *Briefwechsel*, ed. Karl Schuhmann and Elisabeth Schuhmann, Husserliana Dokumente III (Dordrecht: Kluwer, 1994), vol. I, 63–4.

The second chapter, "Critical developments," besides a series of other theories, refutes especially those of Lange and Baumann. The author's detailed rendition of Jevons's, Sigwart's and Schuppe's attempts to reduce the presentation of number purely to that of difference is very interesting. Jevons: "Number is but another name for diversity. Exact identity is unity, and with difference arises plurality … Plurality arises when and only when we detect difference" [*PA* 51; *Hua* CW X 51–2]. To obtain the "pure" presentations of the numbers 2, 3, 4 …, we would have to become aware *in abstracto* of the relations of difference of first-, second- … order symbolized by $\frown$ in the "forms" for 2: $\overset{\frown}{AB}$, for 3: $\overset{\frown\frown}{ABC}$, for 4: $\overset{\frown\frown\frown}{ABCD}$ etc. "So, for example, the extremely rapidly increasing complication of those forms would make it understandable why we can attain a proper presentation only of the smaller numbers, while we can think the larger ones only symbolically, so to say, by detours" (p. 55) [*Hua* CW X 55]. Rendered in this way the theory is consistent. Despite its consistency and its further merits, the thusly completed theory of the reduction of numbers to presentations of difference is not tenable; because: "It is important to keep distinct: noticing two different contents and: noticing two contents *as different* from each other. In the first case we have, presupposing the simultaneous unitary comprehension [*einheitlich zusammengefasst*] of the contents, a presentation of a collection, in the second a presentation of a difference. … Only this is correct: where a plurality of objects is perceived, we are always justified, on the basis of the individual contents, in making evident judgments, to the effect that every one of the contents is different from each other one; but it is not correct that we must make these judgements" [*PA* 56–7; *Hua* CW X 55–6]. After these rejections, the "psychological nature of the collective connection" (p. 79) [*Hua* CW X 77] is introduced in the third chapter: "A collection [*Inbegriff*] originates in that a unitary interest and simultaneously in and with it a unitary noticing[48] distinctly picks out and encompasses different contents. Hence, the collective connection also can only be grasped by reflection on the psychical act through which the collection comes about. Again, the fullest confirmation for our view is offered by inner experience. If we inquire what the connection would consist in when we think a plurality of such disparate things as e.g. redness, the moon and Napoleon, we obtain the answer that it consists merely[49] in thinking these contents together, thinking them in one act" (p. 79) [*Hua* CW X 77].

The fourth chapter finally determines the "Relationship between the concepts amount and multiplicity," as the less determined "concept of multiplicity immediately falls apart into a manifold of determinate concepts that are most sharply

48. Compare to this double definition (interest and noticing) the objections of Hillebrand (*Göttingische gelehrte Anzeigen* 1893, No. 4, p. 180) [here 219].

49. Against this "merely" Ernest Lindenthal observes, in a review of Husserl's book in the *Zeitschrift für das Realschulwesen* (Wien 1893, year XVIII, second volume, p. 105 [here 211]): "By thinking things in one act, we do not yet think a plurality. My friend, his love of order and indefatigability do not awaken the concept three when I think them simultaneously." By pointing out the perspicacious review that could easily be overlooked, I deem it unnecessary to repeat its further objections above, in so far as I consider them applicable.

delimited with respect to each other: the numbers. There arise such concepts as: one and one; one, one and one; one, one, one and one etc., which by virtue of their extremely primitive character and their practical importance, at least within a limited range—namely in so far as they can be easily distinguished—have already been formed on the lowest levels of human mental development, so that their names two, three, four, etc. belong to the earliest creations of all languages" (p. 87) [*Hua* CW X 85]. From here on to the end of the first part (p. 198) [*Hua* CW X 187], follow mostly critical discussions, the refutation of the "nominalist attempts of Helmholtz and Kronecker," as in the case of Kerry, as an "appendix."

In turn, the reviewer is in the pleasant condition to be able to endorse by far most of the investigations indicated until now with respect to the content as well as—and this seems even more important in a certain sense—the method. This is not due to a superficial impression of these often subtle issues merely prompted by the book, but because the reviewer himself for many years has had a preference for the problem area discussed here and had arrived at solutions that are generally close to those in the work under consideration. Nevertheless, the most essential differences will not remain without mention here: one concerns in general what the author delivers in the section "On the theory of relations" (p. 70) [*Hua* CW X 69]: "Since I am not in a position to rely upon a firmly established and generally acknowledged theory of relations, I deem it necessary to insert a few general observations concerning this very dark chapter of descriptive psychology here." When the author considers the answer, that J. St. Mill gives to the question: "what is relation?," "intelligible and essentially adequate," then I fear that also the author's contribution to the theory of relations is not "firmly established" and cannot hope to be "acknowledged." Indeed, the author himself immediately admits "that Mill himself vacillates in his terminology," and e.g. also the author, while following Mill, spoke of the "fundament" and "the foundation" in the singular, but soon, following his own feeling for language, feels compelled to speak of "the foundations" (p. 71) [*Hua* CW X 70]. Likewise, the author feels that "it is somewhat awkward to designate a similarity, gradation, and the like as *physical* phenomenon" (p. 74 [n. 1]) [*Hua* CW X 73 n. 7], but without therefore freeing himself from the position adequately described by this term. The understanding among the researchers involved in the "theory of relations" would have been more effectively promoted, if a connection had been established—even if only polemically—to the first and up to now only extensive publication "on the theory of relations," Meinong's *Humestudien* II,[50] a book in which, as far as I know, the expression "theory of relations" itself appears for the first time.

A second objection concerns the fundamentally different treatment that the author reserves for the two, as one would think, simply coordinated moments

50. That the author knows the book follows from a reference at another point, p. 216, note [*Hua* CW X 205 n. 1], whose quotation, however, does not concern the "theory of relations" in general, but the concept of "*indirect* presentations" (compare p. 55, note 1 [of Höfler's own review, here 228, n. 56]).

in the constitution of the presentation of plurality, in particular the presentation of number, the *analyzing* and the *collecting*. Of the latter it was emphatically underscored, as we saw, that it would be a "*psychical act*";[51] of the first—the analyzing and at the same time also the comparing—it is just as emphatically denied. Due to the principled importance of this negative thesis, we allow ourselves to dwell on it a bit longer. At first we read on p. 42 [*Hua* CW X 43–4]: "the entire underlying intuition for Lange as for Kant, according to which a relational content is the result of a relational act, is psychologically untenable. Inner experience—and it alone is decisive here—shows nothing of such creative processes. Our mental activity does not make the relations; they are simply there, and, given an appropriate direction of interest, they are just as noticeable as any other content" (the author here quotes Stumpf, *Tonpsychologie* I, p. 104ff.).[52] Creative acts, properly speaking, that as a result produce some new content that is different from them, are psychological monstrosities [*undinge*]. Certainly one distinguishes in complete generality the relating mental activity from the relation itself (the comparing from the respect of comparison [*das Vergleichen von der Gleichheit*] etc.). But where one speaks of such a type of relating activity, one thereby understands either the grasping [*Auffassen*] of the relational content or the encompassing interest that picks out the points of relation, which is the indispensable precondition for the relations combining those contents becoming noticeable. But whatever is the case, one will never be able to maintain that the respective act creatively generates its content." Next, on p. 66 [*Hua* CW X 64] it says: "*analyzing* is not at all a psychical activity, properly speaking, i.e. one which would fall within the domain of reflection. Let us distinguish between a psychical event and a psychical act. Psychical acts are presenting, assenting, denying, loving, hating, willing, and so on, which are disclosed by inner perception (Locke's *reflection*). It is completely different in the case of analyzing. No one can inwardly perceive an analyzing activity. We can

51. It would be nice to know, with respect to these "*acts of collecting*", to which psychical class they would belong. Compare the review by Hillebrand [here 216–19] mentioned above, p. 50, note 1 [here 223, n. 48]: As the author by all means bases himself on [*auf dem Boden steht*] Brentano's psychology, then the choice would be first and foremost only that between presentations and judgments. Or should we believe in some effect of the "phenomena of love and hate" on arithmetic? Or are there yet any other intellectual "activities" besides presenting and judging? To the best of my knowledge, Zindler ("*Beiträge zur Theorie der mathematischen Erkenntnis*", *Sitzungsberichte der kaiserlichen Akademie der Wissenschaften* Wien, 1889; cf. my notice regarding this work in the *Vierteljahrsschrift für wissenschaftliche Philosophie,* 1890, p. 502) first publicly pointed out "certain activities, which are no judgments, but are also more than mere complexes of presentations, e.g. the 'comprehensive regard [*Zusammenfassung*]'," "the elementary thought operation of the fusion [*Verschmelzen*] of the unities of two whole numbers to a single number," "the coordination of number- and space-constructions" etc. This aside, already Zindler (*ibid.* second ch., §10) points to "relations with *more than two* foundations," which the author mentions on p. 71.
52. [Ed.] The parenthetical is Höfler's, referring to a footnote by Husserl. In *Hua* CW X the reference was changed to "105 ff.", though the fundamental question is indeed posed on 104 by Stumpf; also see Höfler's elaboration below.

have an experience where an at first unanalyzed content then becomes an analyzed one, and where earlier there was one content, now a multiplicity is noticed. But nothing more than this *post hoc* can be inwardly verified. Of a psychical activity through which the unanalyzed unity becomes the multiplicity, inner perception shows nothing. But we become aware of the fact that analysis has taken place by comparing the presentation, retained in memory, of the unanalyzed whole with the current [fact] of the analyzed [whole]. Such acts of comparing and distinguishing do occur, which however presuppose the completed analysis."

The reviewer confesses that the claims outlined in the above statements (as they admittedly are no foundations) regarding the finding or not finding of psychical facts are by no means confirmed without further ado in his own inner experience. "*No one can inwardly perceive an analyzing activity*." But who would not have believed up to now that analyzing can be innerly perceived at least as well as presenting? "where earlier there was one content, now a multiplicity is noticed." "Was"—so after the noticing the one content is no more? And yet we shouldn't say "that the multiplicity is generated from the unanalyzed unity" ... In the passage from Stumpf's *Tonpsychologie* (Vol. I, p. 104ff.) that the author quotes as support for his negative thesis, it is said that analyzing and comparing have no more claim to be called *activities* (in contrast to "passive events in the soul"), such as sensing [*Empfinden*]. Also with respect to this the reviewer can for the moment only confess that he already preferred the here-opposed theory of Lotze of an *active* engagement with those intellectual accomplishments as such,[53] to Stumpf's attempts at restricting the concept "psychical activity" to the intervention of the willpower that merely prepares such accomplishment. Nevertheless, let it be willingly acknowledged that this chapter concerning psychical activity (despite Kerry's contributions to the development of the concept of "psychical work" mentioned above) for the time being has itself been "worked" too little to exclude here the danger of mere terminological conflicts everywhere. Precisely for this reason we allow ourselves to point out to the author, who is still "at work" on these issues and who is called upon before others by what he has already achieved to proved further clarification of such obscurities, a series of passages of the first volume under consideration, which at least convey the impression that occasionally he would himself consider the "analyzing" (and the "comparing") as an "activity." We will emphasize the terms that in particular have conveyed this impression to us by *italicizing* them:

p. 77 [*Hua* CW X 75]. "Let us first compare the collection [*Inbegriff*] with any arbitrary primary presentational whole. In order to observe the connecting relations in such a case, *analysis is necessary*. If, for example, we are dealing with the

53. According to a remark by Meinong, precisely Stumpf's strict proof (I. p. 33) of a "*judgment threshold*" that even at the highest levels of attention is still different from a "sensation threshold" (p. 34) does not speak for the assumption of special *judgment-dispositions* that would not be reducible to presentation- and will-dispositions. The activation of *such* disposition (using potential psychical energy for psychical work) then would have also a claim to the name of "mental activity."

presentational whole that we call a rose, then we arrive at its various parts successively *by analysis*: the leaves, then stem, etc. (the physical parts); then the color, its intensity, the scent, etc. (the properties). Each part is *picked out by a distinct noticing* and is held together with the already separated parts. As the next *result* of analysis, as we see, we have a collection, namely, the collection of the parts of the whole that were noticed in themselves."

p. 80 [*Hua* CW X 77]. "For the *apprehension of each one* of the colligated contents there is required a distinct *psychical act*; their comprehension [*Zusammenfassung*] then requires a new act, which manifestly contains those *articulating* [*gliedernden*] *acts* within itself, and thus forms a psychical act of second order."

p. 96 [*Hua* CW X 93]. "Only this is correct, that the originally unarticulated [*ungeschiedene*] unity of a composite phenomenon passes over in a plurality that requires a plurality of *acts of thought* to be *picked out* [*Heraushebung*]."

p. 99 [–100, *Hua* CW X 97]: "… that the single contents are *picked out by distinct acts* and only then are encompassed by a common act which unites them all."

p. 162 [*Hua* CW X 153]: *Comparing* and *distinguishing*, collecting (the unification of concrete contents into collections), as well as counting (the abstraction of the general forms of collection) are well-distinguished *mental activities* that must be *held apart from each other*.

p. 218 [*Hua* CW X 207–8]: "However, in the sensuous[54] quantity the parts are precisely not contained in the manner of properties, but rather in the manner of discrete partial intuitions [*für sich gesonderter Teilanschauungen*], and these are indeed of such a kind that under given circumstances draw a dominant and unitary interest to themselves. Precisely because of this our original intention is directed toward the formation of a presentation of a collection that *apprehends each of these partial intuitions for itself* and comprehends it together [*zusammenbegreift*] unitarily with the others. Our intention is directed at this, but we lack the corresponding mental *capacity* to fully attain it in the case of greater quantities. While the successive singular apprehension [*Einzelauffassung*] of the members of the quantity is still possible, their comprehensive collection is not [possible] anymore…"

p. 219 [*Hua* CW X 208]: "For an actual presentation of quantity, according to the foregoing analyses, we need a *psychical act* which *presents every single member of the quantity for itself* and together with all the others; thus *just as many psychical acts as there are contents*, unified by a psychical act of second order."

p. 221 [*Hua* CW X 210]: "… in which, rather, there is accomplished by the required proper *psychical operations*, what is indeed to be accomplished, namely the

54. [Ed.] Höfler erroneously has "*nämlichen*" here, instead of "*sinnlichen.*"

successive *apprehension* [*Auffassung*] (even though not the unitary comprehension [*Zusammenfassung*]) of all the members of the quantity. …"

p. 231 [*Hua* CW X 218–19]: "… a consequence of fusion [*Verschmelzung*] is that in its higher degrees the total impression, other circumstances being equal, approximates that of a truly simple quality and becomes increasingly *difficult to analyze*." Etc.

There are enough examples of passages that to a greater or lesser degree seem to contradict that on p. 66 [*Hua* CW X 64]; the last one appears to even allow for differences in magnitude [*Grössenunterschiede*] in the "psychical work" available for the analyzing. Without doubt, the author will successfully manage to remove the semblance of contradiction from some of the passages, perhaps through even sharper formulations; but hardly in all cases.

The second part, "The symbolic number concepts and the logical sources of the arithmetic of amounts [*Anzahlen-Arithmetik*]," mentions "basic operations, which we can apply to all numbers and by which alone can form new numbers from given numbers, addition and partition." (The latter expression is not meant in the sense of arithmetic, which distinguishes division as partition [*Teilung*] from division as measurement, but in the sense of disaggregation [*Zerteilung*], in such a way that the above terms come closer to the general concepts for which the terms thetic—better synthetic—and lytic operations would be more usual (compare above p. 47)).[55]

In the eleventh chapter, "the distinction, fundamental for all further discussions, between symbolic and proper presentations" is clarified. "A symbolic or improper[56] presentation is, as the name already says, a presentation through signs. If a content is not directly given as what it is, but only indirectly through signs that characterize it univocally, then we have, instead of a proper, a symbolic presentation of it" (p. 215) [*Hua* CW X 205]. After dealing with various "attempts at an explanation of instantaneous apprehensions of quantities" (p. 219–227) [*Hua* CW X 208–15] he discusses among other things the figural moments (p. 227) [*Hua* CW X 215] and in the twelfth chapter "the symbolic presentations of number" (p. 250–290) [*Hua* CW X 235–69]. The latter exposition could have been provided in a less broad form, if the author would have presupposed the Indian positional number system as already known—as it indeed is for everyone—instead of constructing it step by step on the base of new and increasingly subtle logical

55. [Ed.] The reference is to a part of Höfler's triple review concerning Kerry's third article, where he discusses Grassmann's introduction of generalized thetic and lytic operations.

56. As much as the author appeals to Brentano's lectures for this distinction, he deviates from him in both the definition—cf. p. 215, note [*Hua* CW X 205 n. 1]—as well as the terminology, in that he just does not speak of "improper," but of symbolic presentations; and these are what he defines in the above phrasing by way of the Meinongian term "*indirect*" (to which the quotation reported above p. 52, n. 1 [p. 224, n. 50] refers). I myself preferred the meinongian term "indirect presentation" in my logic, as Meinong's analyses seemed to me to be the decisive ones in these matters, in particular concerning the part played by the relations in the content of such presentations.

postulates, and would have shortly listed its logical advantages one after another. Perhaps it would be better to come back to the thirteenth (last) chapter, the logical sources of arithmetic, after the publication of the second volume and we may postpone our overall assessment of the grandly planned and throughout thorough work, which already today is clearly by far the most comprehensive there is in the philosophy of arithmetic.

11. Adolf Elsas, in *Philosophische Monatshefte* 30 (1894), 437–40

The psychological and logical discussions that E. G. Husserl has brought together as basic building blocks for a philosophy of arithmetic are intended to serve, according to his own modest remark, merely as a preparation and scientific foundation for a future development. "In the present state of the science, nothing more than such a preparation could be attempted. I would not know how to indicate even one question of consequence where the response could sustain a merely passable harmony among the investigators concerned; This is sufficient proof that in our domain we are as of yet unable to speak even of a merely schematic articulation of truths already secured for knowledge. The task before us here is, rather: to seek reliable foundations through patiently focused research, to verify the noteworthy theories in careful critique, to separate the correct from the erroneous, in order to, thus informed, set in their place new and possibly firmer ones" (Preface) [*PA* V; *Hua* CW X 5].

Indeed we find conscientious, thorough, and detailed research in the book, making it into a scholarly work. Perhaps, though, to find an audience among the mathematically learned it would have been better to allow less room for the individual critical investigations and instead trace the historical development of the concept of number in mathematics and to propound one's own view in sharp lines. When the author admits that "I distance myself by not a little from the currently prevalent views," [*PA* VIII; *Hua* CW X 7] then he must endeavor first and foremost to win the mathematicians for his views.

Right from the start, Husserl sees the concept of amount [*Anzahl*] as the true and proper fundamental concept of arithmetic, underscoring that he agrees in this position with mathematicians of the caliber of Dedekind and Weierstrass, while on the other hand no less than the likes of Helmholtz, Kronecker, and W. R. Hamilton consider the concept of ordinal numbers as necessary for the foundation of the concept of amount. Against this, one might question whether it is really adequate for the philosopher to place the starting points of Weierstrass and Kronecker in such strict contradiction, instead of considering balance and mediation. In the field of physics e.g. it is not thought to be incompatible to sometimes use movement and sometimes energy as fundamental concept. Why should the arithmeticians not have some free choice to use cardinal numbers [*Cardinalzahlen*] or ordinal numbers as starting points, or even to base themselves on geometrical presentations and to start with a discussions of extended magnitudes [*extensiven Grösse*]?

The first treatment of the concept of number prompts the author to a strong rejection of a statement by J. St. Mill, which in the end turns out to be not at all so “manifestly wrong” [*handgreiflich falsch*; *PA* 12; *Hua* CW X 18], if one manages to bring out its correct sense. Mill says: “Each of the numbers two, three, four, &c., denotes physical phenomena and connotes a physical property of those phenomena. Two, for instance, denotes all pairs of things and twelve all dozens of things, connoting what makes them pairs, or dozens; and that which makes them so is something physical; since it cannot be denied that two apples are physically distinguishable from three apples, two horses from one horse, and so forth: that they are a different visible and tangible phenomenon” (p. 12).[57] To the contrary, Husserl thinks that the countability of psychical acts or states already reveals this thought as inadmissible; rather Leibniz’s view would be correct, according to which the number is a *universalissimum*, “originating through the unification of any things (entium)[58] whatsoever, e.g. of God, of an angel, of a man, of motion, which together are four” (p. 11).[59] Locke too expresses himself in a similar way, and Husserl arrives at the main point of his explanation in the statement: “The nature of the particular contents therefore makes no difference at all” (p. 11) [*Hua* CW X 17]. However, if we peel away at Mill’s statements, revealing their correct core, then we see that these mean that the concept of amount at first is developed in counting sensuously distinct objects and that something of this development remains attached to them, when we in the end learn to count mere concepts and psychical acts and states and rise with Leibniz to the concept of a quaternity [*Vierzahl*], under which four unrelated things can be united.

As soon as we admit that it does not “in any way” depend on the nature of the things or contents brought together under a concept of amount, then the epistemological significance of the number can only be understood “through reflection on the psychical act through which the collection [*Inbegriff*] comes about” (p. 79) [*Hua* CW X 77]. “A careful examination of the phenomena teaches the following: A collection originates in that a unitary interest, and simultaneously in and with it, a unitary noticing, distinctly picks out and encompasses different contents.” “If we inquire what the connection consists in when we e.g. think a plurality of such disparate things as redness, the moon and Napoleon, we obtain the answer that it consists merely in the fact that we think these contents together, we think them in one act.”

For this kind of connections Husserl made up a new name; he calls them “collective connections.” “The linguistic formulation of the circumstance that given contents are collectively connected … is accomplished in our language in a perfectly adequate manner by the conjunction ‘and’” (p. 81) [*Hua* CW X 79]. “… multiplicity in general, as we now can express ourselves quite simply and without

57. [Ed.] Husserl quotes Gomperz’s translation. Here I include the original from *A System of Logic*, book III, ch. XXIV, § 5. Willard provides his own translation in *Hua* CW X 18.

58. [Ed.] In *Hua* CW X 17 erroneously “entia.”

59. [Ed.] Translation from *Hua* CW X 17–18, the original reads: “Est enim numerus quasi figura quaedam incorporea, orta ex unione entium quorumcumque, v.g. DEI, Angeli, Hominis, Motus, qui simul sunt quatuor.” (Leibniz, *De Arte Combinatoria*, ed. Erdmann, 8.)

any circumlocution, is nothing other than: something and something and something etc., or: a thing and a thing and a thing etc., or shorter: one and one and one etc." (p. 85).[60]

This "one and one and one" brings us to the *Nestelschwab* in the pretty fairy tale: when he hears the bell toll, he connects the distinct oneses "collectively," obtaining the result that the bell had been always striking "one." Then why can the harebrained wretch still not count to three?[61] According to Husserl the concepts of amount are derivations from concepts, whose clarity and simple mutual distinctness appears to be beyond doubt; "one and one is sharply distinguished from one, one and one, and this in turn from one, one, one and one, etc." (p. 96) [*Hua* CW X 95]. One could also think the *Schwabe* would have lacked the capacity to distinguish one collective multiplicity from another. I rather think that he would lack the ordinal numbers, and would have to learn that it strikes two, when it strikes one and again one, and that it strikes three, when after the second another third one follows. But Helmholtz and Kronecker find no mercy from Husserl.

"The source of the noteworthy misconceptions into which these two illustrious investigators have lapsed," according to him, lies "in the misinterpretation of the symbolic counting process, which we carry out as a blind routine. Therein we proceed in such a way as to mechanically correlate the names of the numbers with the members of the quantity to be counted, and then take the last name required as that of the number sought" (p. 197) [*Hua* CW X 186]. "Now these great mathematicians have confined themselves to the external and blind process, have misunderstood its symbolic function and thus have confused sign and thing."

The first and mostly critical part of the volume under consideration ends with the critique of Helmholtz and Kronecker. Husserl's basic insight, developed on the base of such critical discussions, whose characteristics I have tried to bring to the fore above, is applied in the second half of the volume to clarify the symbolic concepts of amount and to reveal the logical sources of the arithmetic of amounts. The polemicizing critique here passes in the background; even the psychological foundations are seldom mentioned, and the main emphasis rests on the logically incontrovertible progression from established points to new ones. Hence, the study of the book requires a thorough attention to detail. We wish the author that not only philosophers, but also numerous mathematicians will not be put off undertaking such study.

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60. [Ed.] *Hua* CW X 83–4. This passage is quite hard to translate properly. Husserl starts out with "*irgend Etwas,*" literally "some something" or "any something," which in English would sound redundant. The second step is "*irgend Eines,*" which literally would be "any one" or "some one." The expression "*irgend Eines*" should be understood as the answer to the question "which one?": "any one." The last step is the passage from "*eines*" to "*eins,*" from "a thing" (article) to "one thing" (numeral). Also see Ierna, "Beginnings (Part 1)", 12 n. 42.

61. [Ed.] The popular fairy tale he is referring to is part of the stories about the *Schildbürger*: "As they once passed through the village, it struck three. So he counted the strikes, saying: one, one, one. When the clock was done, they asked him: what was it [*Wieviel es sei*]? to which he answered that he wouldn't know, as he didn't count it together [*zusammengerechnet*]."

12. Michael Glossner, in *Jahrbuch für Philosophie und spekulative Theologie* (1894), 235–9

Husserl, without doubt heeding a mighty current of the time, looks for a *psychological* foundation of the basic concepts [*Grundbegriffe*] of arithmetic. In the first volume of a "*Philosophy of Arithmetic*" (p. 7) [actually vii of the Preface, not 7 of the Introduction] lying before us, the author does not want to provide a regular[62] system, but a series of psychological and logical investigations as a preparation for the scientific foundations for a construction of the same. The negative, critical discussions are to be limited to those attempts that stand out because they are widespread or intrinsically significant, but the positive elaborations will encompass the psychological, logical and metaphysical aspects of the matter. Through investigations regarding the symbolic method, the author thinks to fill an essential gap in current logic. The first, present volume, in two parts, discusses the issues connected to the analysis of the concepts multiplicity, unity and amount [*Anzahl*], both in so far as they are given to us *properly* [*eigentlich*] as well as in so far as [they are given to us] by symbolization.

The author takes the concept of amount as the only basic arithmetical concept. This would presuppose the concept of multiplicity, whose origination on its turn is to be explained psychologically by reflection on the collective connection. An attempt is made to support the conception [*Auffassung*] of number as collective unity by refutation of dissenting views, specifically the "spatial" and the "temporal." True scientific worth belongs only to the *difference-theory*.[63] "It set out directly from certain psychical acts, but they were acts of distinguishing, which a more deeply penetrating critique could not acknowledge as the synthetic acts that enter into effect [*in Wirksamkeit tretenden*] in the case of collectivity and amount" (p. 68) [*Hua* CW X 68].

It suffices to superficially sketch the remaining content of the first part; it contains investigations regarding the psychological nature of the collective connection, the origin and content of the concept of amount, the relations "more" and "less," the definition of equinumerosity [*Gleichzahligkeit*], the definitions of number in terms of equivalence, regarding unity and multiplicity, the sense of the statement of number: topics that are discussed with deeply penetrating acumen.

The investigations of the second part elicit still greater interest and, for our part, greater sympathy. They discuss the difference of the proper and symbolic operations on numbers. One would become embroiled in irresolvable difficulties, when considering the arithmetical operations as being proper ones and not heeding the difference between the symbolic and the proper numbers. The different "phrasings" [*Wendungen*] in the arithmetical operations would be nothing but phrasings and

62. [Ed.] Glossner says *regelmäßig,* which means "regular" or "following a rule," while Husserl (p. V) actually says *regelrecht,* which in this context rather means "full and entire" or, as Willard translates, "thoroughgoing."

63. [Ed.] Also see footnote 15 and following on p. 203 above.

forms in the symbolism grounded upon the fact that all operating which reaches beyond the very first numbers is only a symbolic operating with symbolic presentations. If we had proper presentations of all numbers as we do of the first ones in the number series, then there would be no arithmetic, it would be completely superfluous (p. 212f.) [*Hua* CW X 201]. In order to substantiate this distinction there follows an extensive investigation regarding symbolic presentations of multiplicities [*symbolische Vielheitsvorstellungen*], whose results are applied to the symbolic presentations of numbers. We will have to content ourselves with reporting the following statements [selected] from the interesting discussions of the author. "So the number-systematic arrived at (specifically, our ordinary decimal system) is not a mere method to provide signs for given concepts [*gegebene Begriffe zu signieren*], but rather to construct new concepts and to simultaneously designate them with the construction [*mit der Konstruktion zugleich zu bezeichnen*]." … "All logical technique is directed toward the overcoming of the original limit of our natural mental abilities through the careful selection, application, combination and persistent repetition of the activities that they allow, and that, considered in isolation, are capable of accomplishing only very little" (p. 264f.) [*Hua* CW X 247–8].

The final chapter on the logical sources of arithmetic allows a deep insight into the essence of the mechanical operations of calculation as well as into the significance of the number designation for the direction and development of these operations. The logical method of arithmetic is the method of sensuous signs [*sinnliche Zeichen*]. The concept of calculation comprises every symbolic derivation of numbers from numbers, which rests mainly on rule-based operations with sensuous signs. Every instance of problem-solving decomposes into a mechanical[64] and two conceptual parts: conversion of the initial thoughts into calculation,[65] calculation and conversion of the resulting signs into thoughts. The indirect formations of the number system are the symbolic surrogates for the numbers in themselves [*Zahlen an Sich*; see *PA* 295; *Hua* CW X 275]. The first basic task of arithmetic is to separate all conceivable symbolic modes of formation of numbers into their distinct types, and to discover for each type a reliable and possibly simple method for carrying out the reduction [*PA* 297; *Hua* CW X 277]. The methods to perform this reduction are the arithmetical operations. Since the proper concepts of number are not accessible to us, and we can not classify, add, and subtract them, in their stead we operate with sharply determined symbolic surrogate concepts. "Instead of operating conceptually we have mechanical calculation, the logical soundness of which is guaranteed by means of the rigorous parallelism between the system of numbers and relations of numbers on the one hand, and that of the number signs and relations of number signs on the other" (p. 309) [*Hua* CW X 287]. The four kinds of basic operations are indirect methods for the classificatory subsumption of symbolic number compositions under the proxy [*stellvertretenden*]

64. [Ed.] Husserl actually says "*rechnerisch,*" i.e. "calculational." (*PA* 293; *Hua* CW X 273)
65. [Ed.] Husserl here says "*Umsetzung in Zeichen,*" that is, "conversion into signs." (*PA* 293, *Hua* CW X 273) Unlike the previous difference, I suppose this is simply a mistake.

number concept to which it belongs. From this we gain the solution to the problems of understanding the operations of calculation as proper ones performed on the real [*wirklichen*] number concepts. With the modified sense which the operations acquire in the domain of symbolic number formations, it seems fully intelligible why scientifically elaborated methods for carrying out the operations are here required, which seemed pointless there [see *PA* 309; *Hua* CW X 287].

In our view, we obtain the significant result that the mechanism of calculus does not have a greater power than the train of thought that it symbolizes and represents [*vertritt*], i.e. the subsumption under a certain number concept in the natural or systematic number series, and hence is not suitable in any way to solve problems that lie beyond this domain. Regarding 0 and ∞, to which metaphysical speculations are attached by preference, the author occasionally expresses himself correctly (p. 147) [*Hua* CW X 140–41]. He considers the actual infinite number as something contradictory. Regarding the negative, imaginary, fractional, and irrational numbers we find the short remark that through them there occurs in our domain of amounts [*Anzahlengebiete*] a calculational/formal, but by no means conceptual reduction of the inverse number forms to the direct ones. (p. 321 n.) [*Hua* CW X 298 n.]

With respect to the determination in principle of the essence of number, we are partially in agreement with the author, and in part we believe we must dissent with him. We agree with his opposition to the *nominalistic* conception of number; on the other hand, we contest the justification of his critique of the Aristotelian concept of number as well as the author's own definition of number as a collective concept. Regarding the first point, the author justly approaches the positions of Helmholtz and Kronecker (preceded by Berkeley), according to which not the amount [*Anzahl*] (cardinal number), but the *ordinal number* (in a modified sense) forms the prerequisite of the whole of arithmetic and which denies the *natural* development of the number series, with firm and substantiated criticism. The ordering, Helmholtz affirms, is in fact a norm given by man, our ancestors, who developed the language. "I stress this distinction, because the presumed naturalness is bound up with a defective analysis of the concept of number" (Helmholtz's words, p. 191) [*Hua* CW X 180]. In this view, the author correctly points out that the *concept*, which mediates each use of the signs and constitutes the unity of their meaning, is missing (p. 193) [*Hua* CW X 182]. He raises the question: "Wherever we use the name five, it occurs in *the same sense*. In what is it therefore grounded that different kinds of contents of presentation are designated in the same sense by these signs?" (*ibid.*) The number does not obtain its value and its meaning from the position that it takes in a series, but it takes a determined position because it contains this specifically determined amount [*Anzahl*], because it is this determined collection [*Inbegriff*] of units. "If I say, the amount of these apples is four, I certainly do not then have in mind the circumstance that, given some ordering of the apples, the last element is the fourth, but rather precisely that one and one and one and one apple is present" (p. 196) [*Hua* CW X 184–5]. The source of these misconceptions lies in the misinterpretation of the symbolic process of counting, which we employ blindly by habit and whose symbolic function is not acknowledged, which leads to the confusion of sign and thing [*Sache*] (concept). In particular, Helmholtz was misled into reinterpreting the concept of number nominalistically by the greater problems of general arithmetic (p. 197f.) [*Hua* CW X 186–7]. Cf. Cantor, *Mitteilungen zur Lehre vom Transfiniten* in *Zeitschrift für Philosophie und philosophische Kritik* [91 (1887)], p. 7.

Regarding the polemic against Aristotle, at variance with the author we consider the Aristotelian conception of number as one of the secondary objects of sense-perception, if understood correctly, to be right. The argument, advanced by the author, that the number in the mathematical sense would be a collective concept, could not convince us. The mathematical unit [*Einheit*] is an element of *magnitude* and is not to be confused with the unit in ontological sense. Only the former can be posited plurally [*ist mehrfach setzbar*] and is therefore an element of the augmentable and diminishable [*Vermehrbaren und Verminderbaren*] and is constituted in such a way that it does not fuse together with other units or transform into a continuum when connected to a number (the *numerus numerans*, which is to be distinguished from the *numerus numeratus*, the really existing, discrete quantitative multiplicity outside the soul). Hence, the Scholastics justly distinguish between the quantitative and the ontological multiplicity, depending on whether the unity in the quantitative or the ontological sense, the latter considered as the undividedness of being [*Ungeteiltheit des Seins*],[66] is taken as basis. The author misjudges the peculiar nature of the mathematical and its difference from the purely conceptual, which already prompted Plato to place the mathematical as a third world next to that of the senses and that of the ideas: a view that Aristotle brought back to its correct measure by his well-known theory of the levels of abstraction. Only in the improper sense, by applying the number abstracted from material things, we can also count psychical things [*geistige Dinge*]; however, their countability diminishes progressively as they become farther removed from materiality. Hence, the highest, divine Being, if we want to express ourselves precisely, cannot be counted together with any "something" [*Etwas*] (which, according to the author, would be the sense of the numerical unit), as the divine nature completely excludes the possibility of being posited plurally, which instead characterizes the numerical unit (the element of the number magnitude [*Zahlgröße*]). (Cf. St. Thomas q. 30 art. 1 ad 4. art. 2 ad 5. art 3.[67]) If one would object that e.g. angels can be counted just as well as stones, we respond that this can happen only in the sense that we *present*[68] spiritual beings [*Geistwesen*] as spatially separated beings and as discrete units of magnitude, hence in a *metaphorical* sense [*im übertragenen Sinne*]; in other words, angels form an ontological, not a numerical multiplicity. The non-psychological, but material

66. [Ed.] Probably a reference to Aristotle *Metaphysics* IV, 2.
67. [Ed.] Thomas Aquinas, *Summa Theologica*, First Part, Question 30: "The plurality of persons in God," article 1: "Whether there are several persons in God?," reply 4; article 2: "Whether there are more than three persons in God?," reply 5; article 3: "Whether the numeral terms denote anything real in God?"
68. [Ed.] *Vorstellen,* in this case used as meaning "imagine," or "consider."

origin of the number (i.e. the discrete multiplicity as a kind of magnitude) is confirmed by language, in that it often expresses the material unit of what it counted, i.e. e.g. pieces, heads, etc.

Against the conception of numbers as *collective unity* an excellent Thomist, Goudin, correctly observes that to the collection pertains only an external, accidental unity, but to the number pertains an internal unity, since every number is constituted as a new species by the added unit (Logic. maj. de quant. art 4).[69] It is not correct what the author says on p. 85 [*Hua* CW X 83], that in the formation of number the interest is directed only on the connection [established among the elements] by the mind, while the peculiarities of the content are merely not especially noticed. For the number *in abstracto* it is indeed irrelevant whether stones or stars are counted, but at the same time it does have a determined content in the plurally posited quantitative unity or the unity that is establihed by partition–material separation. The quantity in the mathematical sense or the number can hence only be understood as magnitude. The concept of number becomes totally confused when the "something," which according to the author should form the proper sense of the numerical unit, is conceived as a *relative* determination (p. 86).[70]

These critical observations are in no way intended to diminish or retract our recognition of the merit that the author attained by his in-depth insights in the essence of calculus. We look expectantly forward to the prosecution of the work.

13. Friedrich Pietzker, in *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht* XXVI (1895), 512–17

The here present first volume, the second volume of which, announced in the preface, has not yet appeared, is split in two main parts. The first discusses "the proper concepts of multiplicity, unity and number [*Anzahl*]," while the topic of the second are "the symbolic concepts of number and the logical sources of cardinal arithmetic [*Anzahlen-arithmetik*]."

The first part tries to find a foundation for the concept of number in the concept of multiplicity as the "concept of collective connection." Before the positive exposition of his own views, the author proceeds with extensive critical considerations of the various attempts of other researchers to provide a philosophical foundation

69. [Ed.] Antonio Goudin, *Philosophia Thomistica*, Vol. I *Logica*, Ch. *Logica Major: Prima Pars Logicae Majoris*, Quaestio III: *De quantitate*, Articulus IV: "*Utrum Locus, Tempus, Motus, & oratio sint propriae species quantitatis.*" However, Goudin actually makes this point in Articulus V: "*An numerus sit Species quantitatis?*," wherein he also discusses the distinction between transcendental and quantitative unity and multiplicity, the difference between *numerus numerans* and *numerus numeratus*, and then goes on to prove that "number is the true and properly predicated species of the quantity."

70. [Ed.] *Hua* CW X 79. Husserl speaks of the "something" as a "relative attribute," probably following Meinong, see Carlo Ierna, "Relations in the Early Works of Meinong and Husserl," *Meinong Studies* III (2009), 7–36, here 25.

for the concept of number, in which, next to the completeness, also and especially the extraordinary objectivity in the rendition of the intuitions that the author opposes deserves high praise.

The critique that he applies to these attempts is in my opinion, for the most part, justified, though I believe that he gives a too narrow interpretation of some of the opposing claims that was not intended by their originators. On the whole I concur with him, especially the refutation of the equivalence theory and of the substantiation of equinumerosity by the concept of one-to-one correlation I find felicitous. The intimate connection of the nominalistic explanatory attempts by Helmholtz and Kronecker, that take the ordinal number as the proper basic concept, with the just mentioned position was not given prominence in the appendix dealing with these Helmholtz–Kroneckerian ideas.

Against the idea that numbers would be a statement about the things themselves, the author fittingly raises—in the very sophisticated third chapter of the first part, dealing with the different kinds of relations—the issue that numbers do not belong to the primary relations, existing between the bodies themselves, but to the psychical relations, which only the mind [*Geist*] brings into the things.

However, I cannot agree with him, when the author in the positive statement of his position now claims that the counted things did not already have the equality that is concretely assigned to them in the process of counting by subsumption under a general concept, but in a sense obtain it only by the act of counting.

The examples that he appeals to are very infelicitous.

When he says: "my soul and a triangle are two" [*PA* 158; *Hua* CW X 150], or "Jove, a contradiction and an angel are three" [*PA* 161; *Hua* CW X 152], then he will first meet everywhere with puzzlement about where he got the idea to comprehend together such disparate things in one concept. He will not be able to do away with such puzzlement as to find a common aspect in such different kinds of things, which could consist e.g. also in these are three objects of thought (that just happen to catch my interest in succession), or even these are three individual representatives [*Einzelrepräsentanten*] of different kinds of conceptual categories.

The argumentation of the author is even less understandable when considered together with certain other of his statements. In the context of his very justified polemic against the confusion of equality and identity committed by Frege in the discussion of the concept of number, he emphasizes that by equality is never meant full congruence [*Übereinstimmung*], which would not be compatible with the being different of the objects themselves, but only congruence in a certain respect that happens to be the focus of interest [see e.g. *PA* 104, 109; *Hua* CW X 102, 105–6]. Then it would be quite obvious to reply that indeed one also only counts right from the start what is congruent with what happens to be of interest to the counting subject in this respect.

And when the author highlights that "not 1 = 1 is 2, but 1 and 1 is 2" [*PA* 160; *Hua* CW X 151], then we can reply that nobody means this in this way, who holds that a certain equality of the countable objects [*Zählobjekte*] is a necessary condition for the act of counting, as little as he would take the statement of difference valid for the content of the number statement, though he considers the difference of the objects as a moment in the multiplicity, to which he leads back the concept of number.

One even gets the feeling that, when the author only wants to consider as common moment of the things to be counted that each falls under the concept of "something," in the end he does not mean anything else than the advocates of the just sketched and by him decisively opposed view. This leads me to the avowal of the impression that the first part of the present volume as a whole made on me. The critique that the author moves against the theories of the in-part extraordinarily pre-eminent researchers that he mentions (Leibniz, Herbart, Wundt, and many others), often turns out to be that they would have been misled by equivocations or that their deductions would be factually correct, but would in no way advance knowledge. In truth we have to regretfully observe that so much acumen has been spent without any positive results, and we cannot be very surprised that we have to add: in so far as the practical results of the position of the author in the first section of this volume are incontestable, he also does not say much more than what is correctly intuitively felt by common sense [*der natürliche Verstand*], without such circuitous investigations. In the parts where the author goes beyond this, his claims are contestable. In particular, I think of his polemic against the view, defended by Frege, that no definition of number is acceptable that does not also fit zero and one [see *PA* 142–8; *Hua* CW X 136–41]. This is the only point in which I have to agree with Frege against him. The author does not want to consider zero and one as proper numbers because they do not fit into his derivation of the concept of number from the concept of collective connection. In my opinion he articulates his own ideas too narrowly and in a certain sense too literally. I have to take Frege's statement as completely correct: the number answers the question "how many?," which can also be answered by zero or one. The distinction advanced by the author, correct as it may be, between the unity in the multiplicity and the unity as opposed to the multiplicity [*PA* 148; *Hua* CW X 141], is not a conclusive argument here.

And if the author would really be right with his exclusion of zero and one from the series of proper number concepts, then consequently he would have to also banish them from the science of numbers, from arithmetic. That he yet wants to concede them citizenship in it is an inconsistency that is not refuted by his argumentation. Here a notion is propounded that finds a particularly significant expression at the beginning of the second part.

There the author finds an ambiguity in the use of the plus sign, which is used sometimes in the additive, sometimes in the collective sense; following this he lays down a difference—still often to be urged as he says—between logical and mathematical generality [*PA* 204; *Hua* CW X 193].

I cannot concede this ambiguity, it does not lie in the matter, but only in the far too narrow interpretation of the concept of multiplicity; but the distinction between logical and mathematical generality I find just as unmotivated, and additionally also highly dangerous.

Apart from this, the second part, which I will deal with now, is the more significant part of the volume, in so far as it leads to some notable positive results.

The actual content, which is also made explicit by the title, is constituted by a philosophical foundation of calculatory technique [*Rechenkunst*] ("cardinal arithmetic" [*Anzahlen-arithmetik*]). In a very subtle and deep discussion, whose details cannot be remotely exhausted in the context of a review, the author works out the thought that we do not operate with the numbers themselves in calculating, but with symbolic presentations; all presentations beyond the simplest numbers are symbolic; "if we would have a proper presentation of all numbers, the whole arithmetic would be superfluous" [*PA* 213; *Hua* CW X 201].

The process of "instantaneous apprehension of quantities," which is supported by the "figural moments" as the author calls them [see *PA* 227–36; *Hua* CW X 215–22], is of essential importance here. We generally apprehend quantities in a form that, due to certain outward characteristics, displays a certain quasi-qualitative moment.

This argumentation is extremely subtle and ingenious, but on one point I would like to raise an objection against the exposition of the author: he argues against the idea that one could obtain the awareness of the equality of the elements in a quantity by lightning-fast unconscious comparisons; he deems such comparisons impossible. I do not find that he gives any other satisfactory explanation. The figural moment does not warrant the relevant quality of all the single elements of the quantity and the attention directed at each of these single elements (the "individual apprehension [*Einzelauffassung*] of any of the members of the quantity") grants it just as little. Maybe here we just have a difference in the expression, I think it is possible that the role that the author assigns the sensuous impression in establishing the equality of the members of the quantity is basically the same as what the proponents of the unconscious comparison mean. The figural moments are of the most essential importance for the execution of the process of comparisons, they provide its practically possible form, but they alone do not constitute its essence.

Nevertheless, figural moments are the most important tool for mastering quantities, they are the proper source of the constructions of number that by nature are symbolic, we operate with the number series rather than with the number concepts.

One has to read for oneself in the book how the author arrives at the construction of number systems in general and the decimal system in particular; these and the following discussions belong to the best that have been written in the field in which the author operates.

In particular, he provides a very nice and apt characterization of the tasks of calculation on the one hand and arithmetic on the other.[71]

Calculation has the task of reducing unsystematic constructions to their corresponding systematical forms that classify them, whose type is like that of the equation 18 + 49 = 67. As the first basic task for arithmetic arises that of distinguishing

71. [Ed.] See e.g. *PA* 290–94, *Hua* CW X 271–4; and *PA* 297, *Hua* CW X 277.

all possible symbolic ways of constructing numbers into their types and of finding for each of these a sure and possibly simple method of reduction.

This leads the author to a consideration of the arithmetical operations, at first of the four basic kinds, where we have to notice that the dubious use of the word operation in mathematics has already been remarked on earlier. The author would like to consider as operations in the proper sense only addition and partition [see e.g. *PA* 299; *Hua* CW X 279]; I would rather say: composition and partition, also I cannot consider the view of subtraction as partition to be correct.

These discussions are preparations for the *second* volume of the book, which will cover the properly conceived arithmetic, which following the final arguments of the first volume is to be characterized as a general theory of operations.

This is the more significant and difficult part of the task that the author took upon himself. Considering the excellent acumen, the extensive expertise, and the autonomous approach that have been revealed in the first volume, one can rightly have great expectations of what the second volume will bring. I, for one, am especially curious to learn how the author will pronounce himself with respect to the formalistic view of arithmetic that now is allegedly dominant in more specialist circles.

14. Władysław Heinrich, in *Vierteljahrsschrift für wissenschaftliche Philosophie* (1895), 436–9

Concepts can be investigated in two ways: by following them through all their stages of development and trying to arrive in this way at a clear insight regarding their value and extension; or by limiting oneself merely to their present stage and trying to determine all the facets of the relations of the level under scrutiny. In his investigation of arithmetical concepts Husserl used the second method. According to our view, only the first can bring the full truth to light. Hence, if the current level represents the height of the developmental stage regarding what we want to assume about arithmetic within certain limits, then it can indeed discover the What, but never disclose anything about the How. And exactly in the case of mathematical concepts the How is of the greatest significance. With the analysis of mathematical concepts the situation of the researcher is similar to that of the anatomist. The latter can be fully aware of certain results in anatomical respect, but only the comparative–anatomical point of view can inform him about the full implications of the results.

It is entirely the same with the investigation of mathematical concepts. These too have undergone a long series of changes that have to be taken into account. And most of all, this is the case with the concept of number. The author derives the concept of number from the more general concept of multiplicity. For the latter he wants to establish an immediate foundation in the being-together [*Beisammensein*] of the objects, of sensuous or non-sensuous kind. “Those connections which, always the same in kind, are present in all cases where we speak of multiplicities are then the foundations for the formation of the general concept of multiplicity”

[*PA* 14; *Hua* CW X 20]. This peculiar kind of connection, which would underpin the number, is indicated by the author with the name "collective connection."

Psychologically the origination of the *Inbegriff* of the collective connection is explained as follows: "An *Inbegriff* originates in that a unitary interest, and simultaneously in and with it, a unitary noticing, distinctly picks out and encompasses different contents. Hence, the collective connection also can only be grasped through reflection on the psychical act, through which the *Inbegriff* comes about" [*PA* 79; *Hua* CW X 77]. According to the author, the concept of number [*Anzahl*] is distinguished from the concept of multiplicity only by the fact "that the concept of number already presupposes a distinction of the more abstract forms of multiplicity from one another, but that of multiplicity does not. The former is to be taken as the genus-concept [*Gattungsbegriff*], which originates from the comparison of the already distinguished, determined forms of multiplicity or numbers, as species-concepts; the concept of multiplicity, by contrast, arises directly out of the comparison of concrete *Inbegriffe*" [*PA* 89; *Hua* CW X 87].

However, since the number one cannot be subordinated to the concept of collective connection, Husserl is forced to accept, "that the designation of zero and one as numbers represents a transference of this name to concepts of a different kind, even though they stand in close relationship with the proper numbers [*eigentlichen Anzahlen*]" [*PA* 144; *Hua* CW X 138].

The extent to which the results would be changed if we would set up the investigation genetically, can be shown with some facts. For this purpose we only need to turn to anthropology. Let us take e.g. the excellent observation by K. v. Seinen as support:[72] it tells us: 1. The Bakaïri do not know of general concepts; e.g. they do not have the concept forest, but only the designation of single trees. 2. They count to six. What lies beyond, they simply refer to as many. 3. They reach even the numbers that they do have at their disposal by an enumeration starting from one. These simple facts, that do not stand alone, show without further ado, that the analysis of the concepts of number, that the author provides, cannot inform us as to the origination of the concepts of number. But they do indeed show, to the contrary, that they cannot be generated from the concept of multiplicity; the concept of multiplicity is originally coequal to the single numbers.

Once we gain the concept of number and we symbolize it by a sensuous sign, then it can enter into calculations as represented [*vertreten*] by this sign. We consider Husserl's acknowledgment of the distinctions between number-concept and number-symbol and his rightful emphasis on the role of the number-symbols as sensuous objects of mathematics an advance of great consequence. The formulation of the concept of calculation as "any rule governed mode of derivation of signs from signs within any algorithmic sign-system according to the 'laws', or better: the conventions, for combination, separation, and transformation peculiar to that system" [*PA* 293; *Hua* CW X 273] harbors a slew of implications, which

72. Unter den Naturvölkern Central-Brasiliens. Berlin 1894.

can finally illuminate the theory of mathematical knowledge. The author reserved this investigation for the second volume.

We will allow ourselves one last concluding remark: the author designates the reduction of the different forms of number construction to certain normal forms as the task of calculation; in the arithmetical operations he merely sees the methods to carry out these reductions. This designation is correct when used with respect to lower arithmetic, but inappropriate when speaking of general arithmetic. The investigation of the functional relation and hence the whole of functional analysis belongs to the field of general arithmetic, but does not aim at the reduction to a number, but at finding the mutual dependencies.