Book Review

Everything, More or Less: A Defence of Generality Relativism, by J. P. Studd. Oxford: Oxford University Press, 2019. Pp. xiv + 279.

The long-standing dispute between absolutists and relativists traditionally focuses on whether there are absolute truths, absolute epistemic norms, and absolute moral and aesthetic values. The last two decades have witnessed a revival of the dispute. One strand in this revival has been the attempt to recast the dispute as concerning whether a specific kind of context-dependence needs to be postulated in order to explain certain linguistic phenomena (MacFarlane 2014). Another strand has focused on a lesser-known area of application of the dispute, namely, whether it is possible to attain absolute *generality*—that is, whether it possible to quantify over an absolutely comprehensive domain.

This book belongs to the second strand. It is the most sustained articulation and defence of generality relativism to date. Its key novel claims are the following: generality relativism may be coherently formulated by means of schemata and modal operators; partly in virtue of this, standard objections to generality relativism may be satisfactorily addressed; using schemata and modal operators, we may also provide a compelling argument for generality relativism; and a revenge version of this argument for relativism undermines attempts to secure absolute generality by means of modal operators. Although the topic of generality relativism is a specialized one, the book also contains material that will be of interest beyond its more direct audience, for instance, on the use of modal operators in the philosophy of mathematics and the connections with issues in natural language semantics, in particular the theory of generalized quantifiers.

As an initial rough pass, generality absolutism is the thesis that quantifiers sometimes range over absolutely everything, and generality relativism is the denial of absolutism—quantifiers never range over absolutely everything. Relativists have offered several arguments against absolutism, and in chapter 1 Studd offers a brief survey, concluding that the only promising argument—and the one that is going to be his master argument—is the Dummettian argument from indefinite extensibility based on Russell-style reasoning. Suppose for *reductio* that the quantifiers do range over absolutely everything, and take $\mathcal{D}$ to be the domain of quantification. Using Russell-style reasoning,

Mind, Vol. 00 . 0 . 2021

the argument goes, we can diagonalize out of $\mathcal{D}$. Any purported attempt to quantify over absolutely everything is bound to fail. On the other hand, generality relativism—much like relativism about truth—is often rejected on the grounds of being mysterious (what prevents us from quantifying over absolutely everything?) and even ineffable (note the apparent use of absolutely general quantification in saying that the quantifiers never range over absolutely everything).

Among the arguments for and against absolutism, Studd's discussion of the argument from metaphysical realism deserves closer scrutiny. Studd understands the argument as showing that absolutism implies metaphysical realism in Hilary Putnam's (1981) sense. On the assumption that metaphysical realism is to be rejected, absolutism is to be rejected too. Studd points out that the implication from absolutism to metaphysical realism rests on the premiss that the absolutely general interpretation of the existential quantifier, if it exists, would be the privileged one. And, he argues, this premiss cannot be assumed without argument. However, there is a different way of understanding Putnam-style considerations in this context, namely, as challenging the *presupposition* behind the debate between absolutists and relativists that there is a God's eye point of view answer to the question of what the range of our quantifiers is. So understood, the argument appears to circumvent Studd's worries.

Having settled on the argument from indefinite extensibility as the most promising argument against absolutism, Studd proceeds in chapter 2 to take a first look at the argument and related Russell-inspired arguments for relativism. First we have Russell's claim that the phenomenon of indefinite extensibility 'seems to make the notion of a totality of all entities an impossible one' (p. 43). Since by 'totality' here Russell seems to mean a universal class, this is compatible with absolutism. In particular, standard set theory, ZFC, denies the existence of the universal set, but is interpreted by the absolutist as quantifying over absolutely all sets. Next we have Russell's argument for relativism based on his Vicious Circle Principle, but Studd argues that the principle is in fact compatible with absolutism. When we turn finally to the Dummettian argument from indefinite extensibility, the situation is more complex. One version of the argument appears to presuppose the claim that the domain is to be considered a single entity, a claim absolutists are likely to reject. If 'domain' is instead construed as a shorthand for plural talk, the argument appears to make use of the principle that every plurality forms a set, a principle inconsistent in standard plural logic. One of the main tasks of Studd's book, to be taken up in chapter 7, is to provide a consistent and compelling reconstruction of the Dummettian argument.

The dispute between absolutists and relativists concerns the possibility of quantifying over an absolutely comprehensive domain. What is it to quantify over a domain? This question is the topic of chapter 3. If we look at the standard model-theoretic semantics for the quantifiers, the absolutist faces a familiar difficulty. The semantics is cast in set theory, but according to standard set theory, there is no set of all sets. So when giving a semantics for the set-theoretic quantifiers, the absolutely comprehensive domain of quantification cannot feature among those countenanced by the semantics.

Work of Rayo and Uzquiano (1999) showed how to give a model-theoretic semantics for first-order set theory in a plural metalanguage. In this semantics, the domain is not a set, but a plurality. The advantage for the absolutist is clear: countenancing the absolutely comprehensive domain among those the semantics deals with no longer implies commitment to the existence of a set of all sets.

Studd points out that absolutists are not quite done yet. For besides providing a model-theoretic semantics for the language of first-order set theory, they should also be able to provide a semantics for the language of generalized quantifiers. The study of generalized quantifiers (Barwise and Cooper 1981) has been central to the development of natural language semantics over the last forty years, and Studd commendably brings the topic to bear on the debate on absolute generality. He observes that adopting a plural metalanguage will not suffice to provide a model-theoretic semantics for the language of generalized quantifiers. So called superplural resources are needed. But how to understand those?

The issue of whether English deploys superplural resources is a vexed one. Studd argues that the issue can be circumvented, since we can (following Williamson) take a different attitude towards the adoption of the expressively richer resources needed in the metalanguage. Rather than trying to validate them by appealing to their supposed natural language counterparts, we can acquiesce in the formalism. We can be *primitivists* about the logics belonging to the superplural hierarchy and let their intelligibility be vindicated by the possibility of using them 'in the right sort of way' (p. 79).

However, there is a character on the scene who is not yet out of the woods, the Quinean. The Quinean is unwilling to resort to resources going beyond those of first-order logic, let alone superplural resources. What can she do? As Studd observes, she may draw a distinction between semantics and model theory. For the semantics, the Quinean may resort to good old-fashioned Davidsonian semantics, which can be specified using homophonic clauses. Model theory can then be treated as a 'mere' branch of mathematical logic, which does not serve to provide the semantic values of natural language expressions. Given this job description, the intended model of set theory need not feature among those model theory is preoccupied with. However, Studd argues, the study of generalized quantifiers and of their features in terms of their model-theoretic renderings (such as monotonicity) cannot be so easily dismissed. To regard the study of generalized quantifiers as just a branch of mathematics does not do justice to its *raison d'être*.

Studd concludes that the Quinean ought either to accept superplural and perhaps further expressive resources in the end or to abandon absolutism.

However, he does not consider what is perhaps the most obvious strategy for the Quinean, namely, to reconstruct the theory of generalized quantifiers within a set theory with a universal set such as Quine's own system NF. NF has been profitably used to account for phenomena which seemingly involve large collections, most notably to give a foundation for category theory in set theory (Feferman 2013), so it is natural for the Quinean to look there. In private correspondence, Studd clarifies that he takes this option to be excluded by his naturalist assumption that 'semantic theories, cast in set theory' are 'broadly along the right track' (p. viii). The question then becomes whether a reconstruction of generalized quantifier theory in NF would be highly revisionary of standard semantic practice.

Having argued that absolutism faces substantial difficulties, Studd turns, in chapter 4, to generality relativism. Following Kit Fine (2006), he distinguishes between two varieties of generality relativism, *restrictionism* and *expansionism*, by appealing to another distinction, that between *domains* of discourse and *universes* of discourse. Domains are tied to quantifiers: a specific quantifier occurrence in a specific context has a specific domain. Universes are tied to languages: as well as the referents of singular terms and the members of the extensions of predicates, the 'universe of a language encompasses every object in the domain of any quantifier interpreted in the language, in any context' (p. 88). According to restrictionism, there can be languages with inexpandable universes, but domains are never universal because a language's quantifiers are always restricted. According to expansionism, there can be universal domains because quantifiers may be unrestricted, but universes can always expand.

It is natural for the restrictionist to take the restrictions that prevent domains from being universal to be supplied by context. Studd discusses three accounts of the mechanism responsible for these restrictions and argues that none of these accounts will serve the restrictionist's purposes. The difficulties of restrictionism set the scene for expansionism. Two versions of expansionism are considered. According to Fine's *postulationist* expansionism, universes expand in virtue of a change in the ontology under consideration, without new sets being created. Studd suggests that it is hard to make sense of Fine's proposal. According to Studd's *interpretationist* expansionism, universes expand in virtue of a change in the interpretation of the quantifiers. When we move from a given universe to a more encompassing one, we do not *create* a new ontology, but *recognize* it, by assigning more generous meanings to 'everything' and 'something'.

A standard objection to generality relativism is that it is ineffable. The generality relativist would seem to want to formulate her position via the following thesis:

(RT) No sentence quantifies over absolutely everything.

No matter what 'absolutely everything' in (RT) ranges over, the relativist thesis appears self-refuting: there is a sentence that quantifies over 'absolutely everything', namely (RT). Following Williamson (2003), Studd shows that several attempts to give a coherent formulation of generality relativism are either similarly self-refuting or compatible with absolutism.

One option (see Button 2010) is to understand relativism not as a thesis but as a form of *quietism*: any time she is presented with an attempt to quantify over absoutely everything, the relativist shows that it misfires. The absolutist is challenged with giving a non-trivial characterization of the domain of quantification, and the relativist's job is that of producing an object not in the domain (perhaps via a Russell-style *reductio*).

Studd does not consider the quietist option. Instead, he argues in chapter 5 that the relativist can do better by helping herself to *schemata*. Schemata play an essential role in the formulation of first-order theories of sets—for instance, ZFC cannot be finitely axiomatized. But, as Studd notes, Boolos claimed that our acceptance of ZFC is grounded in our acceptance of its second-order counterpart. Studd also mentions Kreisel (1967, p. 148), who famously wrote that 'A moment's reflection shows that the evidence of the first-order schema derives from the second-order [axiom]'. Studd replies that *everyone* must make sense of our capacity to be committed to schemata without being committed to the corresponding principle. For although second-order ZFC can be formulated using a finite number of non-logical axioms, this is because its underlying logic involves a schema with infinitely many instances, namely, Comprehension. Following Williamson (2006), Studd takes commitment to schemata to have a dispositional character: we are disposed to accept each instance of the schema. The point is well taken, although one might take Boolos's and Kreisel's concern to be of a different nature: even if we are disposed to accept each instance of a schema, what *justifies* this disposition?

Studd goes on to use schemata to offer a coherent formulation of relativism. The idea is that, given an interpretation of the current object language i (say, English$_i$), the relativist can express the limitations of her current domain by working in a metalanguage j, and state that no zero or more items$_i$ comprise every item$_j$. This latter sentence is understood as a schema: one takes i and j to be any language so long as j is obtained by a suitable relativist attempt to expand the universe of i. This is a coherent formulation of relativism. However, one should not overestimate what is being achieved. As Studd stresses, the relativist should be careful not to replace i and j with variables and bind them with universal quantifiers, lest the view relapse into incoherence. But then, it seems that the schema providing the coherent formulation of the relativist position is best understood as a template: given any purported universal interpretation of the domain, the relativist can ascend to a richer interpretation and show that the purported universal interpretation was not universal after all. This is, in effect, a formal restatement using schemata of the quietist strategy I outlined above: given any purported attempt to quantify over absolutely everything, the relativist can point out that there are more things in heaven and earth than are dreamt of in the absolutist's philosophy.

Studd sometimes appears to suggest that schemata allow the relativist to go further: 'Even if the quantifiers in each instance of a schema range over a limited domain, the relativist may take the collective effect of its instances to schematically generalize about absolutely every domain' (p. 142). But the crucial question here is whether the relativist can help herself to the notion of a 'collective effect' of the instances of a schema. As we saw, Studd had argued that there is a way of accepting a schema which is not parasitic on accepting the corresponding universally quantified principle, namely, being disposed to accept each instance of the schema. This is the sense in which relativists can and should accept schemata. But this appears to fall short of having a grasp of the collective effect of the instances of a schema: such a grasp seems to involve a grasp of the totality of the instances of the schema, which in effect suffices to ground the corresponding universally quantified principle. To be clear: it is compatible with the relativist view that schemata may serve as a good surrogate for absolute quantification; but it is far from clear that the relativist can see that they do. The fact that the relativist would find herself in this predicament is not entirely surprising: even if Russellian typical ambiguity is, as a matter of fact, a good surrogate for quantification over all types, it is not clear that the committed type-theorist can see that it is, since this would appear to require comparison with what is achieved by quantification over all types, which is precisely what is supposed to be impossible.

As Studd notes, schemata are in any case bound to fail as a surrogate for *all* cases of quantification: schemata cannot be negated—or if they can, their negation results in the negation of all of their instances. Consider, for instance, mereological nihilism, the thesis that everything is mereologically simple. By appealing to schemata, the relativist can express this thesis by indicating commitment to the schema 'Everything$_i$ is simple', where i is a schematic subscript for an interpretation of the language. (Note, however, that there is an issue here about how exactly commitment to the schema can be indicated without using metalinguistic quantification.) But how can the relativist express mereological anti-nihilism, the thesis that mereological nihilism is false? Simply negating the schema will not do: in saying that 'Not everything$_i$ is simple', the mereological anti-nihilist would be committed to the view that no matter how the language is interpreted, there are things that aren't simple. But the mereological anti-nihilist may simply believe this for *some* interpretation of the language.

The problem is that the generality relativist does not have a surrogate for existential quantification. In chapter 6 Studd argues that the relativist can obtain the required expressive power by enriching her language with modal operators. The mereological nihilist can indicate her commitment to the thesis that everything is simple by uttering 'Necessarily, everything is simple', where 'necessarily' is to be read as 'however the lexicon is admissibly interpreted'. And the mereological anti-nihilist can indicate her refusal to commit to the thesis that everything is simple by uttering 'Not necessarily, everything is simple'. More generally, modal operators, unlike schemata, can be embedded under, for example, negation and conditional antecedents.

One central question for the relativist becomes how to interpret the modality. We need to understand what it means to say that any totality of sets could be expanded with new sets. If it is not metaphysically possible for there to be more sets than there actually are, the modality cannot be circumstantial. Studd's answer is that the modality is *interpretational*: 'interpretational modal operators generalize about how the interpretation could admissibly be' (p. 147).

Formally, Studd orders interpretations as follows: an interpretation j succeeds an interpretation i if j is obtained by one or more relativist attempts to liberalize i. On this basis, he defines two modal operators: a *forward-looking* operator, glossed as 'however the lexicon is interpreted by succeeding interpretations', and a *backward-looking* operator, glossed as 'however the lexicon is interpreted by preceding interpretations'. The operators can be given a Kripke semantics taking interpretations to be points in a Kripke model. Given the constraints placed on the order between interpretations, the forward- and backward-looking operators are *S*4.3 modalities, and the defined operator $\Box$ reading 'however the lexicon is interpreted' is an *S*5-modality.

The modalities are therefore quite strong. In particular, *S*4.3 is the modality of linear *S*4 frames. But what reason is there for thinking that interpretations are linearly ordered? Under the ordering Studd gives, this is quite plausible: if an interpretation of the lexicon is always obtained by liberalizing the current, most encompassing interpretation, counterexamples to linearity are not going to be forthcoming. But this may not be a realistic model of how re-interpretation of the lexicon works. In particular, why can't we conceive of two communities of set theorists that start from the same interpretation of the lexicon and reinterpret it in different ways? This issue does not necessarily affect all interpretational accounts of the modalities. Øystein Linnebo's (2013) modalities sanction the weaker modal logic *S*4.2, the modal logic of directed *S*4 frames. This only implies that given any two interpretations, there is a third interpretation that subsumes them.

Studd goes on to consider several applications of his bimodal logic. One is that the relativist can now formulate relativism by means of a single thesis rather than a schema. Incidentally, this would provide a nice response to the worry I raised earlier in passing: although one can be committed to schemata without being committed to the corresponding universally quantified statement, it is less clear that one can indicate such commitment by means of schemata alone without some kind of metalinguistic quantification. Modal vocabulary addresses this problem.

Another important application of the bimodal logic is that the relativist can interpret theories with absolute generality by taking the universal quantifier to be prefixed by the defined modal operator $\Box$, and the existential quantifier (as well as atomic formulae) to be prefixed by its dual $\Diamond$. Thus when the set theorist says 'Every set has a power set', the relativist can take her to mean '$\Box\forall x\Diamond\exists x\mathcal{P}x$'. A metatheorem establishes that nothing is lost in doing so: φ is derivable from a set of premisses Γ in first-order logic just in case the modalization of φ is derivable from the modalization of Γ. This result is often taken to mean that taking formulae of set theory to have an implicit modal character does not interfere with mathematical practice. But the result cuts both ways: no genuinely new set-theoretic results can be obtained by working with modalized versions of the set-theoretic axioms.

Studd is, of course, explicit about the dialectical situation. His primary motivation for the use of modalities is provided by the supposed advantages of relativism over absolutism. Nonetheless, Studd argues that when we look at the motivation for the axioms of set theory, the use of modalities offers further benefits. He provides an elegant and simple bimodal axiomatization of the iterative conception, and shows that a formula is a theorem of ZFC just in case its modalized counterpart is a theorem of his modal set theory. Linnebo (2013) obtained a similar result on the basis of a plural unimodal axiomatization, and Studd makes a convincing case that the added expressive power of bimodality is beneficial, since it allows him to derive as theorems sentences that Linnebo needs to lay down as axioms, and without using plural resources. Studd concludes that the use of modal operators enables the relativist to provide an attractive motivation for standard set theory.

Having introduced schemata and modalities as relativist tools to attain greater expressibility, Studd shows in chapter 7 that the Russellian *reductio*-style argument from indefinite extensibility to relativism can now be formulated without relying on inconsistent premisses or on the domain being a single entity. Using schemata, the relativist can say that any plurality$_i$ forms a set$_j$. Using modalities, she can say that every plurality *can* form a set.

How can absolutists respond? Studd argues that denying that every plurality can form a set will saddle absolutists with unsatisfactory explanations (either the limitation of size explanation or the iterative one) of when a plurality forms a set. In particular, Studd claims that the absolutist can take the iterative explanation to be that some items are collectable just in case their rank is bounded by some ordinal. But, he continues, this is circular when we consider ordinals themselves. For instance, it means that the finite ordinals are collectable—for the absolutist: they are members of the ordinal ω—because they are bounded by ω, and that ordinals are not collectable—for the absolutist: they are not members of any ordinal—because they are not bounded by any ordinal.

This argument can be resisted. It is possible to formulate the iterative explanation without relying on an antecedently given progression of ordinals—either by taking the progression to be autonomous or by dispensing with ordinals altogether in the articulation of the iterative conception (see Incurvati 2020, §3.2). However, suppose one does take the iterative explanation to rely on an antecedently given progression of ordinals. Then the ordinals presupposed will be *Cantorian* ordinals, which may be *represented* as, but are different from, von Neumann ordinals. There is therefore no circularity involved in saying that the finite von Neumann ordinals are collectable because they are bounded by the Cantorian ordinal ω, or in saying that the von Neumann ordinals are not collectable because they are not bounded by any Cantorian ordinal. Of course, the iterative explanation would then need to be supplemented with principles generating the progression of ordinals. Basic principles about the structure of the ordinal progression will ensure that the von Neumann ordinals are not collectable, and further principles—such as a principle stating that the ordinal series is absolutely infinite—will ensure that the finite ordinals are collectable. But the iterative absolutist is not worse off from this point of view than Studd's relativist, who appeals to basic principles about the structure of interpretations and a Reflection Principle to derive the Axiom of Infinity.

The relativist has appealed to modality in her search for expressive power. Do the modalized quantifiers $\Box\forall$ and $\Diamond\exists$ attain absolute generality, even though their non-modalized counterparts do not? *Hybrid* relativists such as Linnebo answer this question positively. Studd recommends a negative answer: *thoroughgoing* relativism.

So far, hybrid relativists seem to do better. While the schematic version of the argument from indefinite extensibility can be used by thoroughgoing and hybrid relativists alike, only the latter are entitled to use the modal version. According to Studd, however, hybrid relativists have a revenge problem: a version of the Russellian *reductio*-style argument can be run against their view. The discussion is technical, but the idea is that we can use Russell-style reasoning to identify sets that lie outside every universe in the hybrid relativist's potential hierarchy. Hybrid relativists would baulk at this. But setting the issue aside, it now appears that modal vocabulary was not sufficient to capture the intended strength of the relativist thesis—at least of its thoroughgoing version. And if we are thoroughgoing about modalized quantifiers, we should have the same attitude towards the further modalities Studd introduces to articulate the differences between hybrid and thoroughgoing relativism. What the relativist seems to be doing is simply showing that any purported attempt to attain absolute generality, no matter the expressive resources involved, is bound to fail. The moral I am inclined to draw is that thoroughgoing relativism is, after all, best understood as a form of quietism.

I mentioned at the outset two objections to generality relativism: mysteriousness and ineffability. The latter Studd has tackled by discussing schemata and modalities. The former is addressed in the very interesting chapter 8 by explaining how universes expand. Studd asks how a quantifierless community may come to quantify, and then leverages the answer to show how a community quantifying over a certain universe may come to reinterpret the quantifiers so as to quantify over a wider universe. He shows that, assuming that the semantic values of the quantifiers are extensional and compositional, suitable open-ended inference rules for the quantifiers fix their intended interpretation. Here, it would have been interesting to situate the findings within the context of Carnap's (1943) categoricity problem and indicate how they relate to Bonnay and Westerståhl's (2016) result that the standard natural deduction rules for the quantifiers are not categorical and to their use of compositionality to secure categoricity.

As I hope my review will have conveyed, this is a book rich in both argument and detail. It will demand serious engagement by absolutists and relativists of all brands.*

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