Update Rules and Semantic Universals*

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Abstract

We discuss a well-known puzzle about the lexicalization of logical operators in natural language, in particular connectives and quantifiers. Of the many logically possible functions of the relevant type, only few appear in the lexicon of natural languages: the connectives in English, for example, are only *and*, *or*, and perhaps *nor* (expressing negated disjunction). The logically possible nand (negated conjunction) is not expressed by a lexical entry of English, or of any natural language.

The explanation we propose is based on the "dynamic" behaviour of connectives and quantifiers: we define *update potentials* for logical operator, under the assumption that the logical structure of a sentence p defines what type of update p contributes to context, together with the speech act performed (assertion or denial). We conjecture that the adequacy of update potentials determines the limits of lexicalizability for logical operators in natural language.

1 Background and Motivation

The present work contributes to the search of semantic universals in the functional domain (von Fintel and Matthewson, 2008): the area of the lexicon which includes expressions whose meanings are connectives and quantifiers. A fruitful line of research focused on a characterization of the set of possible quantifiers in natural language (Barwise and Cooper, 1981; van Benthem, 1984; Higginbotham and May, 1981; Keenan and Stavi, 1986; Keenan and Faltz, 1986), while similar questions about the connectives were initially pursued by Gazdar and Pullum (1976) and Gazdar (1979).

Quantifiers are relations between sets of individuals. A well-known generalization about *determiners*, i.e., natural language expressions whose meanings are quantifiers, concerns *monotonicity*. Let R, S, S' be sets of individuals (type et), with $S \subseteq S'$.

A quantifier Q is *upward monotone* iff $Q_R S \vDash Q_R S'$
downward monotone iff $Q_R S' \vDash Q_R S$
monotone iff it is either downward or upward monotone

If *some* expresses the upward monotone quantifier $\exists$, then (1a) should entail (1b) (since $[\![$ *run fast* $]\!] \subseteq [\![$ *run* $]\!]$), as is indeed the case. Similarly, if *no* expresses the downward monotone quantifier no ($= \neg\exists$), we should expect (2b) to entail (2a), as again is the case.

1. (a) Some students run fast

* **Acknowledgments**:

(b) Some students run

2. (a) No students run fast

   (b) No students run

Barwise and Cooper's (1981) Monotonicity Universal is the hypothesis, so far unfalsified, that natural language determiners express monotone quantifiers. The Monotonicity Universal is empirically significant. It entails, for example, that quantifiers which are typically expressed (compositionally) as *exactly five* in (3) cannot be expressed by a single lexical entry, since (3a) does not entail and is not entailed by (3b).

3. (a) Exactly five students run fast

   (b) Exactly five students run

Numerous logically possible quantifiers are ruled out by the Monotonicity Universal. Consider, however, a well-known version of the question of lexicalization due to Horn (1972). The functions $\forall, \exists$, no and nall ($= \neg\forall$) form an Aristotelian Square of Oppositions. Horn observed that, across languages, only three of the four corners are ever occupied. For example, the English lexical determiners are so distributed:

	Assertion	Denial
Universal	$\forall$ *all*	no *no*
Particular	$\exists$ *some*	nall ?

The corners are traditionally labelled **A** (universal assertion), **E** (universal denial), **I** (particular assertion), and **O** (particular denial). Consider the quantifier nall in the **O** corner. Since $\mathsf{nall}_R S$ is the contradictory of $\forall_R S$, and the latter is true iff $R \subseteq S$, it follows that $\mathsf{nall}_R S$ is true iff $R \not\subseteq S$. If so, then if $S \subseteq S'$, then $\mathsf{nall}_R S' \vDash \mathsf{nall}_R S$. Thus nall is (downward) monotone. However, it is not the denotation of any lexical entry in natural language: it can, of course, be expressed compositionally, e.g., in English by *not all*. The Monotonicity Universal does not rule out a hypothetical (but unattested) **nall*.

Other possible universals have been proposed in this tradition.

A quantifier Q is *conservative* iff $Q_R S \equiv Q_R(R \cap S)$

The Conservativity Universal is the hypothesis that all natural language quantifiers are conservative (Barwise and Cooper, 1981); this would be an empirically significant generalization (von Fintel and Keenan, 2019), although see Dekker (2015). In any case, nall is conservative, hence conservativity wouldn't explain the missing **O** operator. For if $R \not\subseteq S$ then $R \not\subseteq (R \cap S)$, and vice-versa.

Interestingly, a similar pattern applies to "coordinated quantifiers", connectives, and temporal adverbs (Horn, 1972). English includes {*both, either, neither*}, {*and, or, nor*}, and {*always, sometimes, never*}, expressing all but the **O** corner in their respective table.

	Assertion	Denial
Universal	$\wedge$, both, $\forall_t$	nor, $\mathsf{neither}$, no_t
Particular	$\vee$, either, $\exists_t$	nand, nboth, nall_t

There are different formulations of Horn's (1972; 1989) well-known account of the **O** gap; see Katzir and Singh (2013) for a recent discussion. Horn's account relies, first, on filtering out all logical operators that don't stand in the relevant Square of functions of their type—these are presumably excluded by some combination of B&C-style universals. Of the remaining 4 operators, Horn's account predicts that, in principle, all can be lexicalized. However, there are pragmatic reasons why this doesn't happen, summarized by (i)/(iv).

i. *Scalarity*: Sentences whose main operator is **O** are scalar implicatures of sentences whose main operator is **I**;

ii. *Markedness*: a preference for {**A,I**} operators;

iii. *Speaker's Economy*: a preference for fewer lexical elements;

iv. *Hearer's Economy*: a preference for no semantic gaps.

First, observe that there is a scalar implicature from **I** to **O** (i):

4. (a) Some girls swim $\rightsquigarrow$ Not all girls swim

   (b) John is either at school or at work $\rightsquigarrow$ John is not both at school and at work

Presumably, there is a scalar implicature in the opposite direction as well, from **O** to **I** (Katzir and Singh, 2013). It appears to follow that languages don't need all four corners occupied to satisfy Hearer's Economy (iv), since the semantic space can be covered by pragmatic reasoning. A principle of Speaker's Economy (iii) puts pressure for dispensing with the **I** or the **O** operator. If we finally assume a Markedness condition against **E** and **O** (ii), it is the {**A,I,E**} pattern that is predicted to obtain.

We have some concern with Horn's implicature-based account, which we will discuss in the last section of the paper. Most of this paper is dedicated to an alternative account of the lexicalization of logical operators. We think that considerations of explanatory strength and generality weigh in favour of our proposal.

In conclusion of this introductory section, we note that what counts as a lexical unit is a somewhat loose question, even understanding lexical simplicity as e.g., monomorphemicity (Keenan and Stavi, 1986). In particular, the status of **E** expressions like *nor, no, neither, never*, was and remains controversial (Jespersen, 1917, 108). In many languages, and for many lexical categories, **E** operators follow the fate of **O** operators, and are not lexicalized at all. Moreover, split scope readings in Germanic suggest that **E** expressions might not be syntactic units (Sauerland, 2000; Zeijlstra, 2011; Landman, 2004)—tough see Geurts (1996) and De Swart (2000) for a different take. Following Keenan and Stavi (1986); Horn (1989); Katzir and Singh (2013), we proceed under the reasonable assumption that there is a level of description of the linguistic phenomena at which a lexical distinction can be drawn between, on the one hand, {*and, or, nor*}, {*all, some, no*}, {*both, either, neither*}, {*always, sometimes, never*}, and on the other hand *not and*, *not all*, *not both*, *not always*.[1]

[1] In some languages, such as Maricopa (Gil, 1991), and ASL (Davidson, 2013), not all of the operators discussed here are simultaneously realized. Our analysis is about lexicaliz*ability*. Horn-style pressures to lexicalize less than what's logically possible are well-motivated, and may induce languages to develop specific lexicalization strategies that require further hypotheses.

2 Update Potential and Force

From a "static" perspective, declarative sentences carry information: they (purport to) *represent* the world as being a certain way. An utterance of a declarative sentence takes place within a *context*: a body of information which represents the presuppositions shared by participants in a conversation (Stalnaker, 1978, 1999, 2002). From a "dynamic" perspective, declarative sentences are typically associated with the speech act of *assertion*, whose function is to *update* the context by adding the information conveyed by the asserted sentence. The account of lexicalizability we offer builds on this familiar background.

We model a context $c \subseteq W$ as a (non-empty) set of possible worlds. For simplicity, we work in a formal language L which includes a set of atomic sentences $s_1, \ldots, s_n$, and is closed under the signature

$$p := \ s \mid \neg p \mid p \wedge p \mid p \vee p \mid p \ \mathsf{nor} \ p \mid p \ \mathsf{nand} \ p$$

L does not contain context-sensitive expressions, nor presupposition triggers, to keep the semantics simple. More complexity could be added without affecting the main point. For background, we take a standard intensional semantics, in which the interpretation of a sentence p is a set of worlds $[\![p]\!] \subseteq W$. Interpretation is compositionally extended to all sentences of L in the familiar way:

Definition 1. Static Interpretation

$$\begin{aligned} [\![\neg p]\!] &= W \backslash [\![p]\!] \\ [\![p \wedge q]\!] &= [\![p]\!] \cap [\![q]\!] \\ [\![p \vee q]\!] &= [\![p]\!] \cup [\![q]\!] \\ [\![p \ \mathsf{nor} \ q]\!] &= (W \backslash [\![p]\!]) \cap (W \backslash [\![q]\!]) \\ [\![p \ \mathsf{nand} \ q]\!] &= (W \backslash [\![p]\!]) \cup (W \backslash [\![q]\!]) \end{aligned}$$

We define a *formula* as a sentence $p \in L$ prefixed by a force marker: either $+$ or $-$. Force markers are non-embeddable speech act operators, indicating assertion ($+$) or denial ($-$). We do not consider, here, other speech acts. For each formula $\pm p$, we define an *update potential* $[\pm p]$: a function from context to context. The statement

$$c[\pm p] = c'$$

says that $[\pm p]$, applied to an initial context c, yields the updated context c'. The update function $[\cdot]$ is not to be confused with the interpretation function $[\![\cdot]\!]$: the latter maps a sentence p to a set of worlds $[\![p]\!]$; the former maps a formula $\pm p$ to a function $[\pm p]$ from context to context.

Update potentials can be understood as *context change potentials*, in the tradition of Stalnaker (1978) and Heim (1983). Both UPs and CCPs are functions from context to context associated with a sentence and the speech act its utterance performs. We depart from the Stalnaker/Heim framework in some important respects: (1) we add a speech act of *denial* as a basic operation on context, distinct from assertion (Price, 1990; Smiley, 1996; Incurvati and Schlöder, 2017; Willer, 2019); (2) we do not focus on anaphora and presupposition—though, of course, the present work could be extended with the reader's preferred account of such phenomena; (3) UPs, unlike Heim's CCPs, are not 'fully compositional', in a sense explained below.

The main takeaway of the present paper is this: logical expressions denote operators on semantic content, but also on *force*. There are two kinds of force (assertion/denial), and two kinds of updates (sequential/joint), generating a 2x2 grid. We use this grid to interpret the Aristotelian Square, and show that the update potentials of denied conjunction (corresponding to nand) and denied universal (nall) are not adequate with respect to the static semantics of these operators. Our hypothesis is that this mismatch explains the failure of **O** operators to occur in the lexicon of natural languages.

2.1 Assertion and Denial

An assertion of p is 'a proposal to change the context by adding the content [of p] to the information presupposed' (Stalnaker, 1999, 10). As a type of update, Stalnaker's (1978) assertion can be seen as a way to map context c to context $c' = c \cap [\![p]\!]$. Denial is naturally understood along similar lines, as subtraction.

Assertion and Denial

$$c[\pm p] = \begin{cases} c \cap [\![p]\!] & \text{if } \pm = + \\ c \backslash [\![p]\!] & \text{if } \pm = - \end{cases}$$

Since $[\![\cdot]\!]$ is defined for p of arbitrary complexity, assertion and denial modify the context for all sentences of L in predictable ways. For example, given Def. 1, an assertion of p nand q results in,

$$c[+p \ \mathsf{nand}\ q] = c \cap [\![p \ \mathsf{nand}\ q]\!] = (c\backslash[\![p]\!]) \cup (c\backslash[\![q]\!])$$

which, from a static semantic perspective, is correct. However, this is because taking Assertion and Denial above as definitions of update potentials would give us a *non-compositional* account of update: a combination of the static interpretation, with a single positive or negative update for all sentences (Rothschild and Yalcin, 2016). Interpretation is itself fully compositional, and so such account extends to all formulas. More interestingly, however, we can show how to *derive* the Assertion and Denial equivalences from a *compositional* account of $[\pm p]$, given the logical structure of p and the force of the speech act. For this, we begin with a bilateral analysis of assertion and denial.

2.2 Acceptance and Rejection

Intuitively, a denial of p, paraphrasing Stalnaker (1978, 1999) on assertion, is *a proposal to change the context by rejecting the content of* p. There is nothing in the Assertion and Denial equivalences of the previous section that formalizes the notion of *rejection*. The notion of *acceptance* typically goes unanalysed as well, when assertion is technically treated as intersection.

However, both acceptance and rejection are empirically real stages in the conversation. Acceptance can be explicitly signalled in speech by the utterance of a positive particle, e.g., *Yes!*, or by a nod of assent, although it is by default unmarked, as tacit approval. Interlocutors who don't make a fuss about what's been said are taken to be willing to accept the move and continue on with the conversation.

Likewise, rejection is explicitly signalled by the utterance of a negative particle, e.g., *No!*, headshakes, or some other manifestation of dissent. Some languages have dedicated expressions for sentential rejection (Frana and Rawlins, 2019). In the Stalnakerian dynamics of conversational update, rejection is the operation of *update-blocking* (Stalnaker, 1978; Incurvati and Schlöder, 2017; Incurvati and Sbardolini, ms).[2]

[2]There are further considerations against conflating assertion and denial. These considerations may be of a linguistic (Horn, 1989; De Swart, 2010), logical (Smiley, 1996; Berto and Restall, 2019), philosophical (Price, 1990), and cognitive nature (Dimroth, 2010). We do not review the immense literature on this topic.

We propose to analyse update potentials as combinations of two operations: first, there is a *proposal* to *accept* or *reject* some possibilities, and second, possibilities are *filtered out* of the input context accordingly. This gives us a function from an initial context c to the updated context uc.

Definition 2. Update Potential

$$c[\pm p] = \begin{cases} 1.\ \text{Proposal about } p \text{ in } c \\ 2.\ \text{Filtering} \end{cases}$$

A proposal is the assignment of a numerical value (+1 or −1) to a given possibility in context. We assume that worlds are either *accepted* (+1), *rejected* (−1), or *in the background* (0), and that, initially, they're all in the background. We also assume that, after the new context is defined, all worlds are zeroed—so that the context is ready for a new conversational move.

> **Stipulation**. Worlds in the input context (prior to Proposal), and in the updated context (after Filtering), have value 0.

Intuitively, when p is asserted (denied) the p-worlds are accepted (rejected): they are assigned +1 (−1). For this, we introduce a *highlight* function $h : W \to \pm 1$ from worlds w to numerical values. Acceptance and rejection are then proposals to update the context.

Definition 3. Proposal about p in c

$$h(w) = \pm 1 \text{ if } w \in c \cap [\![p]\!]$$

Acceptance of p in c is given by +; rejection of p in c by −. By the previous stipulation, after an acceptance of p, $w \in c \cap [\![p]\!]$ iff $h(w) = +1$, and after a rejection of p, $w \in c \cap [\![p]\!]$ iff $h(w) = -1$. Next, we define the updated context uc as a *filtering* of c: the subset of worlds with highest h-value.[3]

Definition 4. Filter

$$uc := \{w \in c : \forall w' \in c, h(w) \geq h(w')\}$$

Henceforth, we abbreviate the *definiens* in Def. 4 as f^c. For an atomic sentence s, an update potential $[\pm s]$ is simply a (positive or negative) proposal concerning $[\![s]\!]$ followed by a filtering of the context. The updates thus found for atomic sentences are intuitively correct, since they match the notions of Assertion and Denial from §2.1.

Adequacy

$$c[+s] = \begin{cases} 1.\ h(w) = +1 \text{ if } w \in c \cap [\![s]\!] \\ 2.\ uc := f^c \end{cases} = c \cap [\![s]\!]$$

$$c[-s] = \begin{cases} 1.\ h(w) = -1 \text{ if } w \in c \cap [\![s]\!] \\ 2.\ uc := f^c \end{cases} = c \backslash [\![s]\!]$$

[3] The highlight function h on a set S defines a poset $(S, \leq)$ by ordering the worlds in S by their h-value. A filter is a (i) non-empty, (ii) downward directed, and (iii) downward closed subset of a poset. Since $S \neq \varnothing$, $f^S \neq \varnothing$ (i). For every $w, v \in f^S$, there is a world $u \in f^S$ such that $u \leq w$ and $u \leq v$ (ii). Finally, for every $w \in f^S$ and $v \in S$, if $w \leq v$ then $v \in f^S$ (iii). So f^S is a filter on S. Moreover, a filter is *proper* if it is a proper subset. It is obvious that f^S is a proper filter, and the only proper filter, on S, so long as $[\![p]\!] \neq S$, and $[\![p]\!] \neq \varnothing$, i.e., aside for contradictions and tautologies.

A consequence of Def.s 3 and 4 is that positive and negative update potentials of atomic sentences commute with respect to complementation: a property we call Inversion.

Inversion (Force)

$$c[-s] = c \backslash c[+s]$$

We proceed to extend this framework to complex formulas.

2.3 Negation

Intuitively, $[+\neg p]$ must be different from $[+p]$. From a static perspective, negation is a truth-value switcher: it takes a true (false) sentence and forms a false (true) sentence. From a dynamic perspective, negation is intuitively a *force* switcher: an assertion (denial) of $\neg p$ is a proposal to reject (accept) p, followed by a filtering of the context. For force sign $\pm$, we indicate by $\mp$ the opposite sign.

Definition 5. Negation

$$c[\pm\neg p] = \begin{cases} 1.\ h(w) = \mp 1 \text{ if } w \in c \cap [\![p]\!] \\ 2.\ uc := f^c \end{cases}$$

Def. 5 is intuitively adequate, since the updates it defines are equivalent to those expected from §2.1.

Adequacy

$$c[+\neg p] = \begin{cases} 1.\ h(w) = -1 \text{ if } w \in c \cap [\![p]\!] \\ 2.\ uc := f^c \end{cases} \qquad = c \backslash [\![p]\!] = c \cap [\![\neg p]\!]$$

$$c[-\neg p] = \begin{cases} 1.\ h(w) = +1 \text{ if } w \in c \cap [\![p]\!] \\ 2.\ uc := f^c \end{cases} \qquad = c \cap [\![p]\!] = c \backslash [\![\neg p]\!]$$

Moreover, denial and negation cancel each other out.

Equivalence

$$c[+\neg p] = c[-p] \qquad\qquad c[-\neg p] = c[+p]$$

These equivalences are sometimes taken to provide the means for analysing denial away—a tendency that goes back to Frege (1919) and Geach (1965), and earlier still (see Horn, 1989, for references). By contrast, we don't treat denial as the assertion of negation: we derive denial (as well as assertion) for all sentences from more basic primitives. This delivers a more natural account of complex updates.

Finally, negation preserves inversion:

Inversion (Negation)

$$c[-\neg p] = c \backslash c[+\neg p]$$

In the next paragraph, we generalize the notion of adequacy and set the agenda for the rest of the paper.

2.4 Compositionality, Adequacy, Admissibility

Let k be the main operator in a complex sentence $k(\ldots, \psi_i, \ldots)$, e.g., a connective or a quantifier, with $1 \leq i \leq n$. As we mentioned, the interpretation function is *fully* compositional, since for a complex sentence $k(\ldots, \psi_i, \ldots)$,

Full compositionality of static interpretation

$$[\![k(\ldots, \psi_i, \ldots)]\!] = [\![k]\!](..., [\![\psi_i]\!], \ldots)$$

We define update potentials *compositionally*, but not in the sense that the update potential of a complex sentence is a function of the update potentials of its parts. This hardly makes sense in our framework, since update potentials are tied to the *force* of a speech act (assertion/denial), and the subsentential components of an utterance perform no speech act.[4] There is a weaker sense of compositionality.

Weak compositionality of update potential

$$[\pm k(\ldots, \psi_i, \ldots)] = z(..., [\![\psi_i]\!], \ldots)$$

where z depends on k and $\pm$. Def. 5, for example, defines weakly compositional updates for $\pm\neg p$. (For brevity, we will call them *compositional*, rather than weakly compositional.) We have shown above that the updates from Def. 5 are adequate in the sense that they deliver the same restriction of the input context as Assertion and Denial from §2.1. As we define (weakly) compositional updates for all sentences of L, we use this notion of adequacy as a benchmark. Finally, we say that if $[+p]$ is adequate, k is admissible.

(*Def.*) $[+p]$ is *adequate* iff $c[+p] = c \cap [\![p]\!]$

$[-p]$ is *adequate* iff $c[-p] = c \backslash [\![p]\!]$

For $p = k(\ldots, \psi_i, \ldots)$, $[+p]$ is adequate iff k is *admissible*.

Admissibility of an operator is a matter of adequacy of its positive update potential. In this sense, assertion does retain some kind of notional priority over denial—even in a bilateral framework.

3 Conjunction and Disjunction

With (weakly) compositional update potentials, conjunction and disjunction differ not only at the static semantic level, but also in the type of update they generate. Update by conjunction is a *sequential* update of the conjuncts (Heim, 1983); update by disjunction is a *joint* update of the disjuncts. For generality, we introduce functions $[\cdot]^s$ and $[\cdot]^j$ from a set $F = \{\pm p_1, \ldots, \pm p_n\}$ of formulas (with the same force), to update potentials. These will be useful below, in discussing the quantifiers. For simplicity, the following definitions state the case for $F = \{\pm p, \pm q\}$, but the n-ary case is obvious.

[4] In contrast, Heim's (1983) CCPs are fully compositional. This is because CCPs are sentential meanings, albeit dynamic, whereas update potentials are operations on context obtained by combining propositional content and force, according to the logic of the operator. For a detailed discussion of compositionality, see Pagin and Westerståhl (2010a,b).

Definition 6. Sequential update

$$c[\pm p \pm q]^s = \begin{cases} 1.\ h(w) = \pm 1 \text{ if } w \in c \cap [\![p]\!] \\ 2.\ c' := f^c \\ 3.\ h(w) = \pm 1 \text{ if } w \in c' \cap [\![q]\!] \\ 4.\ uc := f^{c'} \end{cases}$$

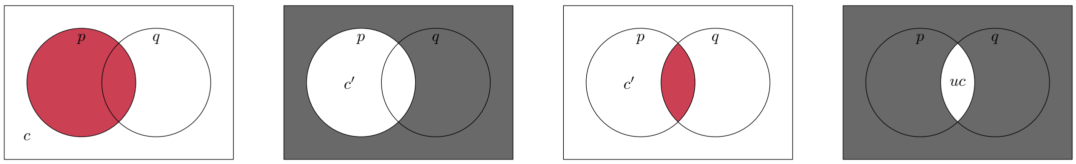

Figure 1: Update by $c[+p+q]^s$. (1) The worlds in p of c are highlighted for acceptance (red); (2) Context $c' = c[+p]$ is defined (white); (3) The worlds in q of c' are highlighted for acceptance; (4) Context uc is defined.

Sequential update of assertions, $c[+p+q]^s$, is illustrated in Fig. 1. It is equivalent to Heim's definition of update by conjunction: first an update of c by $[+p]$, then of $c' = c[+p]$ by $[+q]$. (Given initial stipulations (§2.2), h-values are set to 0 after step 2.) Joint update is defined next, and shown, for assertion, in Fig. 2.

Definition 7. Joint update

$$c[\pm p \pm q]^j = \begin{cases} 1.\ h(w) = \pm 1 \text{ if } w \in c \cap [\![p]\!] \\ 2.\ h(w) = \pm 1 \text{ if } w \in c \cap [\![q]\!] \\ 3.\ uc := f^c \end{cases}$$

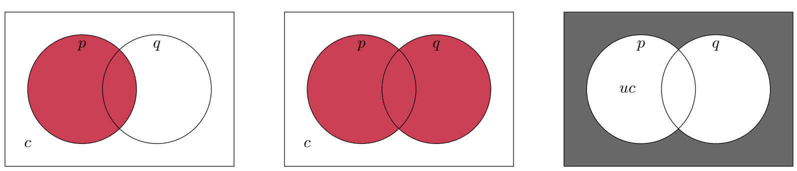

Figure 2: Update by $c[+p+q]^j$. (1) The worlds in p of c are highlighted for acceptance; (2) The worlds in q of c are highlighted for acceptance; (3) Context uc is defined.

Sequential and joint update are essentially the same, but for a missing step. With the former, the process is: (1) acceptance of p; (2) filter; (3) acceptance of q; (4) filter. With the latter, the process is: (1) acceptance of p; (2) acceptance of q; (3) filter. We now refer to sequential and joint updates for an analysis of the update potential of conjunction and disjunction, respectively.

Definition 8. Conjunction and Disjunction (assertion)

$$c[+p \wedge q] = c[+p+q]^s \qquad\qquad c[+p \vee q] = c[+p+q]^j$$

These definitions are adequate. Assertions of $p \wedge q$ and $p \vee q$ are expected to be, respectively, $c \cap ([\![p]\!] \cap [\![q]\!])$, and $c \cap ([\![p]\!] \cup [\![q]\!])$. We define $[+p \wedge q]$ as $c[+p+q]^s$. This function delivers first a local context $c' = c[+p] = c \cap [\![p]\!]$, and then $uc = c' \cap [\![q]\!]$. Therefore, $c[+p+q]^s = (c \cap [\![p]\!]) \cap [\![q]\!] = c \cap ([\![p]\!] \cap [\![q]\!])$. Hence, $[+p \wedge q]$ is adequate, and so conjunction is admissible. We define $[+p \vee q]$ as $c[+p+q]^j$. By this operation, two proposals are made with reference to the same context: accordingly, $f^c = \{w \in c \mid w \in [\![p]\!]\} \cup \{w \in c \mid w \in [\![q]\!]\}$. Therefore $c[+p+q]^j = c \cap ([\![p]\!] \cup [\![q]\!])$. Hence, $[+p \vee q]$ is adequate, and so disjunction is admissible.[5]

3.1 Denials of Conjunction and Disjunction

For negative updates, we simply replace rejection for acceptance in joint and sequential updates.

Definition 9. Conjunction and Disjunction (denial)

$$c[-p \wedge q] = c[-p-q]^s \qquad c[-p \vee q] = c[-p-q]^j$$

Consider $c[-p-q]^j$ first: update by denied disjunction is a joint denial of the disjuncts. That is, (1) rejection of the first disjunct, (2) rejection of the second, (3) filter. This update is adequate, for by §2.1 the denial of disjunction is expected to update to the complement, with respect of the context, of the interpretation of disjunction: $c[-p \vee q] = c \backslash ([\![p]\!] \cup [\![q]\!])$. After the second rejection proposal, for all $w \in c$, $h(w) = -1$ iff $w \in [\![p]\!]$ or $w \in [\![q]\!]$. Since the remaining worlds have h-value 0, $f^c = \{w \in c : w \notin [\![p]\!]\} \cap \{w \in c : w \notin [\![q]\!]\}$. Hence $c[-p-q]^j = c \backslash ([\![p]\!] \cup [\![q]\!])$, and so $[-p \vee q]$ is adequate. It follows that the update potentials of disjunction invert.

Inversion (Disjunction)

$$c[-p \vee q] = c \backslash c[+p \vee q]$$

Intuitively, nor stands to denied disjunction as $\neg$ stands to denial: $+\neg p$ is equivalent to $-p$, and we may take $+p$ nor q to be equivalent to $-p \vee q$.

Definition 10. Nor

$$c[\pm p \text{ nor } q] = c[\mp p \mp q]^j$$

That is, $c[+p \text{ nor } q] = c[-p-q]^j$, and $c[-p \text{ nor } q] = c[+p+q]^j = c[+p \vee q]$. It is straightforward to show that nor is adequate. From Def. 1 and §2.1, $c[+p \text{ nor } q] = c \cap [\![p \text{ nor } q]\!] = (c \backslash [\![p]\!]) \cap (c \backslash [\![q]\!])$. By Def. 10, $c[+p \text{ nor } q] = c \backslash ([\![p]\!] \cup [\![q]\!])$. Hence, nor is admissible. The update is illustrated in Fig. 3.

We conclude the discussion of denied disjunction by observing that, unlike *and* and *or*, the use of *nor* requires that its first argument be marked as "negative": unlike (5a), (5b) is ungrammatical if *not* is omitted. Note that *nor* can follow other "negative" operators besides negation, e.g. *never* in (5c).

5. (a) Alex is rich, and/or Susan is famous.
   (b) Alex is #(not) rich, nor is Susan famous.

[5] The CCP of disjunction is a controversial topic in the literature on presupposition projection. Many candidate CCPs deliver the right truth-conditions, but make different predictions with respect to anaphora resolution. See Heim (1983); Soames (1989); Beaver (2001); Geurts (1999). For recent strategies on this problem, Schlenker (2009); Rothschild (2011). We leave for the future the job of integrating the present work with a satisfactory theory of presupposition projection.

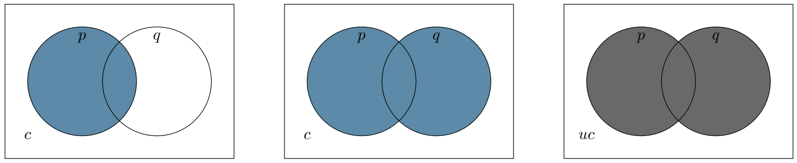

Figure 3: Update with nor (denied disjunction). (1) The worlds in p of c are highlighted for rejection (blue); (2) The worlds in q of c are highlighted for rejection; (3) Context uc is defined (white).

(c) Pat #(never) visited Bristol, nor did Sam live in Durham.

We take the ungrammatical versions of (5b) and (5c) to indicate that *nor* obligatorily coordinates denials, not assertions. That is, the arguments of (asserted) *p nor q* are rejected, not accepted. Since, plausibly, an utterance is by default understood as an assertion, the obligatory presence of a negative operator *not, never* in the first argument of *nor* is there to signal rejection.[6]

Finally, consider the denial of conjunction: a sequential update by denied conjuncts.

$$c[-p \wedge q] = c[-p - q]^s$$

In more details,

$$c[-p \wedge q] = \begin{cases} 1.\ h(w) = -1 \text{ if } w \in c \cap [\![p]\!] \\ 2.\ c' := f^c \\ 3.\ h(w) = -1 \text{ if } w \in c' \cap [\![q]\!] \\ 4.\ uc := f^{c'} \end{cases}$$

That is, (1) rejection of the first argument; (2) filter to a local context $c' = c \backslash [\![p]\!]$; (3) rejection of the second argument in c'; (4) filter. Consequently, $c[-p - q]^s = (c \backslash [\![p]\!]) \cap (c \backslash [\![q]\!])$. This result is not adequate, since we expect denied conjunction to deliver $(c \backslash [\![p]\!]) \cup (c \backslash [\![q]\!])$. Instead we find

$$c[-p - q]^s = c[-p - q]^j$$

There are two ways to define update potentials. The first is to take §2.1 as a definition, in which assertion and denial apply to the static interpretation of the whole sentence. The second is to consider (weakly) compositional updates, whose inputs, for a complex sentence, are its parts. For negation and disjunction, for both assertion and denial, we have shown equivalence, and concluded that the update potentials so defined are *adequate*. For conjunction, we have taken our model to be Heim's (1983) influential account of asserted conjunction as sequential update, which we have shown to be adequate, and finally applied it to its denied counterpart. The result is not adequate. Consequently, conjunction does not invert:

$$c[-p \wedge q] \neq c \backslash c[+p \wedge q]$$

Let's consider an operator, nand, that stands to denied conjunction as ¬ stands to denial and nor stands to denied disjunction.

$$c[+p \text{ nand } q] = c[-p - q]^s$$

[6] The observation that the arguments of *nor* are rejected, not accepted, is sometimes put in terms of the notions of *negative polarity* (Baker, 1970; Szabolcsi, 2004; Barker, 2018), or *non-veridicality* (Giannakidou and Mari, 2020). We keep the rigorously defined notion of rejection from §2.2, and not broach the vast topic of polarity/veridicality. We'll leave the connection between rejection and negative polarity further research.

It is straightforward to verify that $[+p \text{ nand } q]$ isn't adequate. See Fig. 4. We conclude that nand is not admissible. We conjecture that the failure of adequacy just observed is the reason why no natural language connective can express nand.

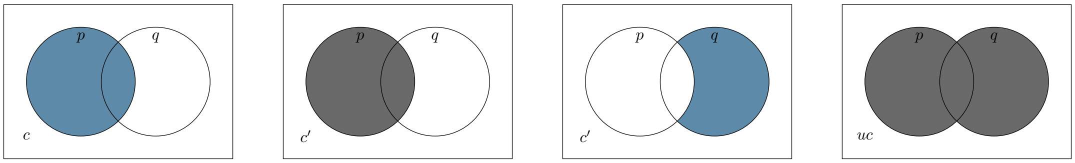

Figure 4: Update with nand. (1) The worlds in p of c are highlighted for rejection (blue); (2) Context c' is defined (white); (3) The worlds in q of c' are highlighted for rejection; (4) Context uc is defined.

We have shown that the update potential of denied conjunction is not definable compositionally in the present framework. This result depends on assumptions we have made about assertion, denial, and the assertoric updates of conjunction and disjunction. Of course, other systems could be designed: for example, by positing new primitives or making different assumptions about how updates work. However, the assumptions we have made are well-motivated, and independently supported. We take our result as evidence that in order to further understand semantic universals concerning logical operators, one ought to (a) separate assertion from denial, and (b) better understand the operators' update potentials.

3.2 Other Connectives

A somewhat curious prediction of our system of update potentials is that the trivial operator $\top$ is admissible. Semantically, let $[\![\top p]\!] = [\![p]\!]$. A compositional definition of $c[+\top p]$ might just be

$$c[+\top p] = c[+p]$$

Obviously, this is adequate, and so $\top$ is admissible. We find this result harmless. Perhaps, $\top$ is just a semantically null expression, such as uses of *really* in English. Otherwise, it would also be plausible to require that no semantically null operator be lexicalized, by independently motivated reasons of cognitive economy. The present framework can and should be embedded within larger assumptions about historical/evolutionary regularities deriving from functional pressure on grammars.

Of the remaining truth-functional connectives, none are admissible on the (quite modest) set of primitives we have assumed so far. Of course, all remaining boolean combinations of $\wedge, \vee, \neg$, nor, are definable, following e.g., Keenan and Stavi (1986), who conjecture that Boolean definability sets the limit of *expressibility* in natural language writ large (i.e., lexically or compositionally).

4 The Quantifiers

Determiners, e.g. *all, some, no* in English, are semantically analysed as quantifiers, namely relations between sets: a restriction R and a scope S. As we remarked in §1, the lexicalization patterns of determiners and coordinating expressions like *and, or, nor*, are parallel (Horn, 1989). The Aristotelian Square of Oppositions for quantifiers leaves the southeast corner empty, which would otherwise be occupied by a missing lexical item expressing $\text{nall} = \neg\forall$ (Horn, 1972).

	Assertion		Denial	
Universal	$\forall$	*all*	no	*no*
Particular	$\exists$	*some*	nall	?

Similar remarks hold for *both, either, neither*, and for temporal quantifiers *always, sometimes, never*. Expressions in the former group are often used in combination with the connectives *and, or, nor*, but may occur as determiners (see Hendriks (2004) for a detailed discussion):

6. (a) Both hands are needed to clap.
   (b) You may kick the ball with either foot.
   (c) I am deaf in neither ear.

We regard *both, either, neither*, when used as determiners, as $\forall, \exists, \mathsf{no}$, with a presupposition that their NP restrictor has cardinality 2 (e.g., as hands, feet, ears). We do not give a separate analysis for temporal and presuppositional determiners.

For the analysis of quantifiers, we extend L to include a set of m atomic n-place predicates $P_1, \ldots, P_m$, closed under the (type-shifted versions of the) connectives shown to be admissible in the previous section. We then add four inductive clauses to the definition of sentencehood: for R, S predicates,

$$p := \ \exists_R S \mid \forall_R S \mid \mathsf{no}_R S \mid \mathsf{nall}_R S$$

The notion of a formula is defined as above. We add a non-empty set d as *domain* of individuals, and assume, for simplicity, that predicate extensions (which are subsets of d) are defined at all worlds. For ease of notation, we indicate the extension of F at a world w, officially $[\![F]\!](w)$, by F^w, and its anti-extension, officially $d \backslash [\![F]\!](w)$, by $\overline{F^w}$. We assume uncontroversial truth-conditions for the quantifiers.

Definition 11. Static interpretation of quantifiers

$$\begin{aligned} [\![\forall_R S]\!] &= \{w \in W : R^w \cap \overline{S^w} = \varnothing\} \\ [\![\exists_R S]\!] &= \{w \in W : R^w \cap S^w \neq \varnothing\} \\ [\![\mathsf{no}_R S]\!] &= \{w \in W : R^w \cap S^w = \varnothing\} \\ [\![\mathsf{nall}_R S]\!] &= \{w \in W : R^w \cap \overline{S^w} \neq \varnothing\} \end{aligned}$$

As above, Assertion and Denial from §2.1 give us conditions of adequacy for the definitions of update potentials. In particular, for example, results such as the following are expected to follow:

$$c[+\exists_R S] = c \cap [\![\exists_R S]\!] = \{w \in c : R^w \cap S^w \neq \varnothing\}$$

We proceed to define compositional update potentials for the quantifiers, as functions of R^w and S^w. There are two formal aspects of the meanings of quantifiers that we should account for: (1) the quantificational "strength", i.e., what medieval logicians distinguished into Universal and Particular, and (2) the relation between the predicate arguments of the quantifier. For the former, we generalize the notions of *joint* and *sequential* update, which we used to explain the Universal/Particular distinction at the level of sentential connectives (that is, the distinction between $\wedge$ and $\vee$). For the latter, we introduce an additional layer of structure on updates, partly motivated by consideration of *conditionals*.

4.1 Generalized conjunction and disjunction

We interpret the two classical quantifiers as "generalized" conjunction and disjunction. The notions of *sequential* and *joint* update from §3 generalize to an arbitrary number of arguments, delivering predictions for the update potentials of conjunctions and disjunctions of length n:

$$c[+p_1 \wedge \ldots \wedge p_n] = c[+p_1 \ldots + p_n]^s$$

$$c[+p_1 \wedge \ldots \wedge p_n] = c[+p_1 \ldots + p_n]^j$$

which we abbreviate as $c[+p_i]^s$ and $c[+p_i]^j$, respectively, for $p_i \in \{p_1, \ldots, p_n\}$. Unpacking the definitions, adequacy still holds.

> **Adequacy** of sequential and joint updates
>
> For $p_i \in \{p_1, \ldots, p_n\}$,
>
> $$c[+p_i]^s = \bigcap_i \{c[+p_1], \ldots, c[+p_n]\}$$
>
> $$c[+p_i]^j = \bigcup_i \{c[+p_1], \ldots, c[+p_n]\}$$

This gives us a promising basis from which to define update potentials for operators that are expected to deliver $\forall$ and $\exists$.

4.2 Existential Quantification

Intuitively, if one asserts *Some R are S*, one thereby accepts that some x is R-and-S, i.e., x belongs to the intersection of predicates R and S. We treat an update by means of the existential quantifier as a joint update of its instances, one instance for each $x \in d$. We need to define what counts as an instance of a quantified sentence. For this, *intersection* must be introduced as a new primitive, to capture the familiar idea that an existential quantifier makes a claim "about" the intersection of its restriction and scope. We indicate the intersection of R and S by $R \sqcap S$. (It is important that $\sqcap$ not be confused with conjunction. Conjunction is sequential update, which *has the effect* of updating with the intersection of its arguments.) Consider the following definitions.

Definition 12. Existential quantification

$$c[\pm\exists_R S] = c[\pm R \sqcap S]^j_{x_i \in d}$$

Update by the existential quantifier consists in joint acceptance (rejection) of worlds in $R^w x_i \cap S^w x_i$ for each $x_i \in d$. That is, for $1 \leq i \leq n$,

$$c[\pm R \sqcap S]^j_{x_i \in d} = \begin{cases} \text{1. } h(w) = \pm 1 \text{ if } w \in c \cap (R^w x_1 \cap S^w x_1) \\ \ldots \\ \text{n. } h(w) = \pm 1 \text{ if } w \in c \cap (R^w x_n \cap S^w x_n) \\ \text{n+1. } uc := f^c \end{cases}$$

The proposal for $[+R\sqcap S]^j_{x_i\in d}$ is to assign h-value $+1$ to a world $w \in c$ if w is such that $x_i \in R^w \cap S^w$, for $x_i \in d$. The proposal for $[-R\sqcap S]^j_{x_i\in d}$ is to assign -1 to a world that satisfies to the same condition.

Consider a simple model in which $d = \{a, b\}$, as e.g., Fig. 5. The context c may be a set of 16 worlds, each represented by a numbered dot, so that all possible combinations of extensions of R and S are represented, and worlds are easily referred to. For example, under the leftmost column, all worlds 11 to 14 are such that $R^w = \varnothing$.

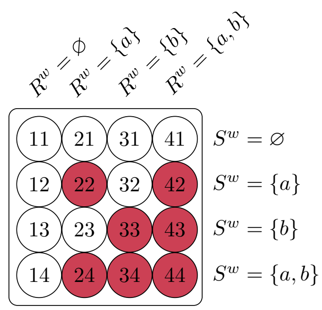

Figure 5: Update by Existential quantification (assertion)

The update for $[+R\sqcap S]^j_{x_i\in d}$ consists in (1) accepting worlds such that a belongs to $R^w \cap S^w$, (2) accepting worlds such that b belongs to $R^w \cap S^w$, (3) restrict by filtering. With respect to Fig. 5, the worlds in (1) are $\{22, 24, 42, 44\}$, and the worlds in (2) are $\{33, 34, 43, 44\}$. In Fig. 5, these are highlighted in red. The usual filtering of the initial context keeps only the red circles in the updated context. It follows that a world is accepted and kept in uc by $c[+R\sqcap S]^j_{x_i\in d}$ provided some x (not necessarily the same for all worlds) is in the intersection of R^w and S^w. Therefore, $c[+R\sqcap S]^j_{x_i\in d} = \{w \in c : R^w \cap S^w \neq \varnothing\}$. Therefore, $[+R\sqcap S]^j_{x_i\in d}$ is adequate, and $\exists$ is admissible.

In $c[-\exists_R S] = c[-R\sqcap S]^j_{x_i\in d}$, a world $w \in c$ is assigned -1 by a joint update of its instances. That is, the worlds coloured in red in Fig. 5 are assigned -1. By filtering, these worlds are discarded, and the final context will consist of the remaining white circles. Hence $[-R\sqcap S]^j_{x_i\in d}$ is adequate. It follows that the update potentials of the existential quantifier invert.

Inversion (Existential quantifier)

$$c[-\exists_R S] = c\backslash c[+\exists_R S]$$

Let no be a function that stands to denied existential as $\neg$ stands to denial and nor to denied disjunction.

$$c[\pm \mathsf{no}_R S] = c[\mp R\sqcap S]^j_{x_i\in d}$$

The adequacy of $[+\mathsf{no}_R S]$ with respect to the truth-conditions given in Def. 11 follows right away from the adequacy of denied existential. We conclude that the admissibility of $\exists$ and no in the present framework predicts that lexical items expressing these functions could be realized in natural language. This is, of course, as expected.[7]

[7]Even though the definitions are truth-conditionally adequate, it's admittedly a stretch of the imagination to think that Def. 12 represents a cognitive process in the speaker's mind (and similarly for the update by universal

4.3 The Conditional

Def. 12 states that $\pm\exists_R S$ generates an update of the context that can be analyzed as an iterated update by $\pm R \sqcap S$. The iteration is a joint update of $R \sqcap S$ for each individual in the domain. Therefore, update by $\pm\exists_R S$ can be seen as a generalization of

$$c[\pm p \sqcap q] = \begin{cases} 1.\ h(w) = \pm 1 \text{ if } w \in c \cap ([\![p]\!] \cap [\![q]\!]) \\ 2.\ uc := f^c \end{cases}$$

Indeed, this is Def. 12 for the case in which the domain d is a singleton. It is immediate to see that the pair of updates $[\pm p \sqcap q]$ on a singleton domain are adequate with respect to the semantics of both $\wedge$ and nand, for $+$ and $-$ respectively, and so if the quantificational structure of intersection $\sqcap$ was possible at the level of sentential connectives (that is, independently of the size of the domain), this would frustrate our account of the missing nand. We claim that the operations on updates that we introduced for the connectives are available for the quantifiers (e.g., joint and sequential update), but not *vice versa*.

Nevertheless, it is possible to define an operation on sentences that generates an update defined, in part, by the intersection of its arguments. The material conditional has the following truth-conditions:

$$[\![p \supset q]\!] = (W \backslash [\![p]\!]) \cup [\![q]\!]$$

We follow Heim (1983) and include an analysis of the material conditional in our account of update potentials. One way to understand the assertion of a conditional is as something like a "test": a conditional filters out worlds in the initial context so that the remaining worlds would satisfy q if p were asserted (Ramsey, 1926).[8] Roughly along these lines, we may take a conditional assertion to be a way to accept p-and-q, conditional on p. More precisely, $[+p \supset q]$ is a proposal to assign h-value $+1$ to a world $w \in [\![p]\!] \cap [\![q]\!]$, provided that the update keeps in uc the worlds in $c \backslash [\![p]\!]$, without accepting them. Similarly, with rejection replacing acceptance, for a denied conditional. That is, we can understand a conditional update as an update by the intersection of its arguments, such that filtering does not exclude possibilities in which the antecedent fails.

Definition 13. Conditional Update

$$c[\pm p \supset q] = \frac{c[\pm p \sqcap q]}{c \backslash [\![p]\!]}$$

In more details,

$$c[\pm p \supset q] = \begin{cases} 1.\ h(w) = \pm 1 \text{ if } w \in c \cap ([\![p]\!] \cap [\![q]\!]) \\ 2.\ uc := f^c \cup c \backslash [\![p]\!] \end{cases}$$

quantification, see below). It's implausible to think that speakers mentally go through all the elements in a domain to process an update, and so Def. 12 is not how things really work.

But what we have described is a *model* of the effects on the conversational context of asserting an existential quantifier or denying it. Admittedly, these effects need to be calculable (for things to work as predicted), but this doesn't mean that speakers need to be able to go through all elements of the domain in the model, in the same way in which speakers don't need to mentally go through all possible worlds in the context for an ordinary update.

[8]There are many different ways to understand the Ramsey test, and the literature on conditionals is vast, but with little consensus (Lewis, 1972; Kratzer, 2012; Willer, 2019; Goldstein, 2019). An influential option is to understand the conditional as a dynamic test, which either returns the initial context, or the empty context (Veltman, 1987; Gillies, 2004). Since one of the main motivations for our analysis is to use the structure of a conditional update for the definition of the universal quantifier (see §4), we set aside the dynamic test view of conditionals.

So the notion of intersection can appear at the level of sentential connectives, when matched with a different way of filtering the context (one that retains the worlds in which the antecedent is false), *if* one is willing to accept this analysis of the conditional. We note that it is not uncontroversial, but it's at least initially plausible.

Given the static interpretation of $p \supset q$, an assertion of the conditional should result in $(\llbracket p \rrbracket \cap \llbracket q \rrbracket) \cup (c \backslash \llbracket p \rrbracket)$. It is easy to verify that $[+p \supset q]$ is adequate, and so $\supset$ is admissible. But of course, the material conditional is not the meaning of any natural language expression! Certainly, $\supset$ is not the meaning of *if* (Kratzer, 1986, 2012). We do not dispute this claim. Indeed, in our view, the conditional is not an operator of the same kind as conjunction and disjunction: it requires additional structure on the update (as stated by Def. 13), and in this respect it is similar to the quuantifiers, which also require a specification of the structural relation between intersection and scope.

There may well be aspects of the meaning of *if* that should be captured by a more specific semantics, or by adding presuppositions (our account of update potential does not include a treatment of any presuppositions that, plausibly, may be attached to *if*). We remain neutral on the difficult question whether some of these aspects should be incorporated in the conditional's update potential, as opposed to its static semantics or its presuppositional meaning.

Is it possible to define additional logical operators by combining the notion of conditional update with rejection? As a matter of fact, none that are admissible. For $c \backslash \llbracket p \supset q \rrbracket = c \cap (\llbracket p \rrbracket \backslash \llbracket q \rrbracket)$. But by Def. 13, with $\pm = -$, $[-p \supset q]$ is not adequate. For $w \in c[-p \supset q]$ iff $w \in c \backslash \llbracket p \rrbracket \cap \llbracket q \rrbracket$, which is not the expected result. Other definitions are possible, of course, but none with the current list of primitives. Moreover, the definitions we have will prove useful for our analysis of the quantifiers. We conclude that there is no admissible operator the assertion of which has the effect of denying a conditional. Thus we expect that there should be no such expression in the lexicon of natural languages. This explanation is of the same kind as the one we have given for the missing nand. Even if $\supset$ only at best approximates the meaning of *if*, no natural language expression (under the same idealizations) can mean the contradictory of $\supset$.

4.4 Universal Quantification

If one asserts *All R are S*, intuitively, one accepts (i) that some x is R-and-S, (ii) *conditionally* upon x being R. For an update by universal quantification, we take $\forall$ to be a generalized conjunction, hence to consist in a *sequential* update of its instances, and the instances of a universal claim to be conditionals, as defined in §4.3.

Definition 14. Universal quantification

$$c[\pm \forall_R S] = c[\pm R \supset S]^s_{x_i \in d}$$

Update by the universal quantifier consists in sequential acceptance (rejection) of each of $R \supset S$ for $x_i \in d$. That is, for $1 \leq i \leq n$,

$$c[\pm R \supset S]^s_{x_i \in d} = \begin{cases} 1.\ h(w) = \pm 1 \text{ if } w \in c \cap (R^w x_1 \cap S^w x_1) \\ 2.\ c^1 := f^c \cup c \backslash R^w x_1 \\ \ldots \\ 2\text{n}.\ h(w) = \pm 1 \text{ if } w \in c^n \cap (R^w x_1 \cap S^w x_n) \\ 2\text{n+1}.\ uc := f^{c^n} \cup c^n \backslash R^w x_n \end{cases}$$

Consider Fig. 6, which represents a simple model consisting of 16 worlds, with domain $d = \{a, b\}$. In an update by $[+R \supset S]^s_{x_i \in d}$, first, a world in c is assigned h-value $+1$ if it belongs to $R^w a \cap S^w a$. These are $\{22, 42, 24, 44\}$. The conditional update keeps these in c' plus those in the complement of $R^w a$, namely the first and third column of Fig. 6. In c', thus defined, a world is assigned h-value $+1$ if it belongs to $R^w b \cap S^w b$. These are $\{33, 34, 44\}$. The conditional update keeps these in uc plus those in $c' \backslash R^w b$, which are the worlds in the first and second column that belong to c'. The result are the circles highlighted in red in Fig. 6. Since $c \cap [\![\forall_R S]\!] = \{w \in c : R^w \cap \overline{S^w} = \varnothing\}$, inspection of Fig. 6 shows that $[+R \supset S]^s_{x_i \in d}$ is adequate, and so that $\forall$ is admissible.

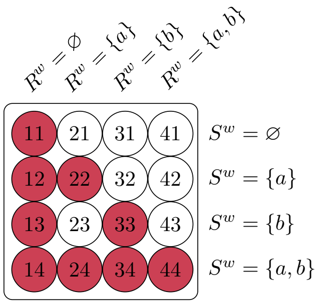

Figure 6: Update by Universal quantification (assertion)

Finally, consider the denial of universal quantification, in Fig. 7. First, a world w in c is assigned h-value -1 if it belongs to $R^w a \cap S^w a$, i.e., if $w \in \{22, 42, 24, 44\}$. The conditional update discards these worlds, keeping $\{21, 23, 41, 43\}$ which have h-value 0, alongside with the complement of $R^w a$. In c', thus defined, a world is assigned -1 if it belongs to $R^w b \cap S^w b = \{33, 43, 34\}$, and the updated context includes $c' \backslash R^w b$ as well as $c' \backslash (R^w b \cap S^w b)$. Consequently, the updated context consists of the white circles in Fig. 7, the blue ones having been rejected. The resulting update by denied universal is not adequate.

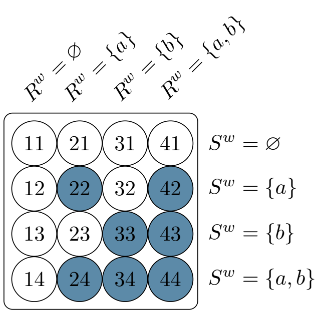

Figure 7: Update by Universal quantification (denial)

We observed above that, given a natural definition of conjunction as sequential update (Heim, 1983), $c[-p \wedge q] = c \backslash [\![p \vee q]\!]$. We concluded that $c[-p \wedge q]$ is not adequate. The analogous result obtains for the universal quantifier.

$$c[-\forall_R S] = c \backslash [\![\exists_R S]\!]$$

Hence, the update potentials of universal quantification do not invert.

$$c[-\forall_R S] \neq c \backslash c[+\forall_R S]$$

Consider an operator nall such that

$$c[+\mathsf{nall}_R S] = c[-R \supset S]^s_{x_i \in d}$$

Such operator would not be adequate. For by §2.1 we expect $c[+\mathsf{nall}_R S] = \{w \in c : R^w \cap \overline{S^w} \neq \varnothing\}$. But as we have just seen, $c[-R \supset S]^s_{x_i \in d} = \{w \in c : R^w \cap S^w \neq \varnothing\}$. It follows that nall is not admissible.

This completes our account of the update potentials of the familiar logical operators. All remaining Boolean combinations of $\forall, \exists$, and no, can be defined on the basis of the admissible quantifiers and connectives, following, e.g., Keenan and Stavi (1986). We have given compositional update potentials for assertion and denial, on the basis of two independently motivated notions of joint and sequential update. We have shown that, in this framework, denied conjunction, denied conditional, and denied universal, do not define adequate update potentials. This is a unified and general explanation for observed facts about lexicalization. We conclude the paper with a brief discussion of monotonicity, and of Horn's (1972; 1989) alternative explanation of the same evidence.

5 Monotonicity

As we mentioned in §1, Barwise and Cooper's (1981) Monotonicity Universal states that all natural language quantifiers are monotone.

Let R, S, S' be sets of individuals (type et), with $S \subseteq S'$.

A quantifier Q is *upward monotone* iff $Q_R S \vDash Q_R S'$
downward monotone iff $Q_R S' \vDash Q_R S$
monotone iff it is either downward or upward monotone

The Monotonicity Universal has stood the proverbial test of time. This generalization has recently been linked to *connectedness*, a possible universal in the predicate domain (Chemla et al., 2019). Moreover, Steinert-Threlkeld and Szymanik (2019) have argued that monotone quantifiers are easier to learn—a consideration that might explain the preference of natural languages for functions with this property.

Here we do not broach the vast topic of the cognitive significance of monotonicity, and of what might underlie the Monotonicity Universal at the level of linguistic processing. We note, however, that the system of update potentials presented here defines only monotone operators, for the good reason that *update itself*, by acceptance or rejection, is monotone.

We generalize the notion of monotonicity, following Peters and Westerståhl (2006). As usually defined, validity $\vDash$ is the subset relation in the semantics, thus the metalinguistic statement $\alpha \vDash \beta$ is understood as $[\![\alpha]\!] \subseteq [\![\beta]\!]$. For readability, we omit the interpretation function from the following. Let $u \in \{e, s\}$, with e for the type of individuals and s for the intensional type.

Definition 15. Monotonicity for arbitrary sets
Let A, A' be sets of type ut and $A \subseteq A'$. A one-place operator k is:
UMON iff $kA \subseteq kA'$
DMON iff $kA' \subseteq kA$

Thus if $k = Q_R$, Def. 15 is the standard one: $\exists_R$ and $\forall_R$ are UMON while no_R is DMON. Moreover, negation is DMON, since for $p \subseteq p'$, $\neg p' \subseteq \neg p$. Next, we generalize to n-ary operators.

Definition 16. General Monotonicity
Let $A_1, \ldots, A_n$ be sets of type ut and $A_i \subseteq A'_i$ for $1 \leq i \leq n$. An n-place operator k is:
UMON in its i-th argument iff $k(A_1, \ldots, A_i, \ldots, A_n) \subseteq k(A_1, \ldots, A'_i, \ldots, A_n)$
DMON in its i-th argument iff $k(A_1, \ldots, A'_i, \ldots, A_n) \subseteq k(A_1, \ldots, A_i, \ldots, A_n)$

We call an operator UMON (DMON) *simpliciter* just in case it is UMON (DMON) in all its arguments. It is easy to verify that $\wedge$ and $\vee$ are UMON, while nor is DMON, since the following statements hold (see also (Humberstone, 2011, 490)):

$$p \wedge p' \subseteq (p \vee q) \wedge p'$$
$$p \wedge p' \subseteq p \wedge (p' \vee q)$$
$$p \vee p' \subseteq (p \vee q) \vee p'$$
$$p \vee p' \subseteq p \vee (p' \vee q)$$
$$(p \vee q) \text{ nor } p' \subseteq p \text{ nor } p'$$
$$p \text{ nor } (p' \vee q) \subseteq p \text{ nor } p'$$

These logical connections are observable in natural language. For instance, (7a) entails (7b) while $[\![$ *lifted a finger* $]\!] \subseteq [\![$ *moved* $]\!]$, and not vice-versa.

7. (a) Julia hasn't moved nor spoken since yesterday.
   (b) Julia hasn't lifted a finger nor spoken since yesterday.

The material conditional $\supset$ is DMON in its 1st argument, and UMON in its 2nd argument.

$$(p \vee q) \supset p' \subseteq p \supset p'$$
$$p \supset p' \subseteq p \supset (p' \vee q)$$

In this paper, we have shown a few logical operators to be admissible: to have a truth-conditionally adequate positive update potential. Setting aside the conditional, we divide the admissible operators in two groups: those whose proposal is an acceptance, and those whose proposal is a rejection. In the former group, we have $\wedge, \vee, \exists, \forall, \top$; in the latter, $\neg$, nor, no. If **A** is a variable for operators in the first group, and **R** for operators in the second group, the following facts obtain.

Theorem 1. Monotonicity of Update
Let $A_1, \ldots, A_n$ be sets of type ut and $A_i \subseteq A'_i$ for $1 \leq i \leq n$.

$$c[+\boldsymbol{A}(A_1, \ldots, A_i, \ldots, A_n)] \subseteq c[+\boldsymbol{A}(A_1, \ldots, A'_i, \ldots, A_n)]$$

$$c[+\boldsymbol{R}(A_1, \ldots, A'_i, \ldots, A_n)] \subseteq c[+\boldsymbol{R}(A_1, \ldots, A_i, \ldots, A_n)]$$

Proof. The monotonicity properties of positive update potentials follow from the observation that positive update potentials for admissible operators are all adequate. We consider the **A** operators first. We write conjunctions and disjunctions with prefix notation. For $p_i \in \{p_1, \ldots, p_n\}$, $[+ \wedge$

$(p_i)]_{i\geq 2}, [+\vee(p_i)]_{i\geq 2}, [+\exists_R S], [+\forall_R S], [+\top p_i]$ are adequate. Since $\wedge, \vee, \exists, \forall$, and $\top$, are UMON, it follows that e.g., if $p_i \subseteq p'_i$, $c[+\wedge(p_i)] \subseteq c[+\wedge(p'_i)]$. Likewise for the remaining operators in this group. Therefore, **A** operators are UMON.

Consider the **R** operators. For $p_i \in \{p_1, \ldots, p_n\}$, update potentials $[+\neg p_i], [+\mathsf{nor}(p_i)]_{i\geq 2}$ and $[+\mathsf{no}_R S]$ are adequate. Since $\neg$, nor, and no are DMON, it follows that e.g., if $S \subseteq S'$, $c[+\mathsf{no}_R S'] \subseteq c[+\mathsf{no}_R S]$. Likewise for the remaining operators in this group. Therefore, **R** operators are DMON. □

We conclude with a generalization.

Theorem 2. Monotonicity of Admissible Logical Operators
Let $A_1, \ldots, A_n$ be sets of type ut and $A_i \subseteq A'_i$ for $1 \leq i \leq n$. If an operator k is admissible, $[+k(A_1, \ldots, A_n)]$ is monotone.

Proof. The result follows from Theorem 1, the adequacy of $[+p_1 \supset p_2]$, and two additional observations: (1) if $p_1 \subseteq p'_1$, $c[+p'_1 \supset p_2] \subseteq c[+p_1 \supset p_2]$, and (2) if $p_2 \subseteq p'_2$, $c[+p_1 \supset p_2] \subseteq c[+p_1 \supset p'_2]$. □

The converse, however, does not hold: nand, nall are monotone, but not admissible.

As we remarked, it is known since Barwise and Cooper (1981) that all natural language logical operators are monotone. It is also known that there are monotone operators that aren't realized in any natural language. Theorems 1 and 2 suggest a possible explanation for Barwise and Cooper's Monotonicity Universal: monotonicity is a by-product of the dynamics of conversation, given a basic distinction between acceptance and rejection. The explanation is direct, and based on the logic of natural language expressions. Of course, this may not be the place where explanations end—an investigation into our cognitive capacities might well be necessary to explain why conversational updates have the basic properties we have assumed they have. We leave this topic for further research.

6 Horn's account

By way of conclusion, we critically revisit Horn's (1972; 1989) account of lexicalization (see also Katzir and Singh (2013)). According to the account, languages may avoid lexicalizing **O**-corner operators because they can "fill the semantic gap" by scalar implicature from **I**-corner operators.

It is true that *Some A are B* typically implicates that *Not all A are B*, but this observation falls far short of providing a systematic mean to express nall by means of $\exists$. Suppose you are drawing 10 cards at random from a full 52 cards deck. After turning the first few cards, all Red, you can't utter (8a) to express (8b).

8. (a) Some cards are black.
   (b) Not all cards are black.

This is for the simple reason that $\mathsf{nall}_R S$ is true if $\mathsf{no}_R S$ is true, while of course $\exists_R S$ is not true if $\mathsf{no}_R S$ is true.

After you drew all the 10 cards, suppose you find out that they are all Red. (There is some probability that that happens.) Then you may continue (8b) with (9), but may not continue (8a) with (9) without contradiction:

9. In fact, no cards are black.

There is a general difficulty with the claim that a function such as nall can be expressed systematically by scalar implicature. It is consistent to strengthen $\exists$ to $\forall$ by cancelling the implicature, as in (10), and it is consistent to strengthen nall to no, since (8b) can be continued with (9). However, it is not consistent to strengthen $\exists$ (8a) to no (9).

10. Some students are asleep. In fact, all students are asleep.

So, while in some sense (8a) expresses (8b), relative to a context, it is not the case that **I**-corner operators can express **O**-corner meanings systematically.

We do not deny that functional pressure from pragmatic regularities, of the kind pointed out by Horn, may lead to stable grammatical generalizations. However, in light of the fact that (8b) is not exactly equivalent to the implicature of (8a), as shown by (9), we are unsure how to assess Horn's proposal. The context in which (8a) fails to be equivalent to (8b) is not particularly unusual. One might nonetheless insist that *some* does mean *not all* in the majority of cases, over the evolutionary history of these expressions, but this claim is difficult to evaluate. The account of lexicalization we have proposed avoids the need for such speculations.

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