The Paradoxical Theorem

Md. Shouvik Iqbal
shouvikiqbal@gmail.com

Abstract.

This paper presents the Paradoxical Theorem, which states that no imperfect being can determine the truth or falsity of any statement with absolute perfection. We then apply the theorem to itself for any imperfect being, which leads to a self-referential paradox.

1. Introduction

"Is an entirely imperfect being also imperfect in accepting their imperfections?"

The question above is difficult to answer. If an entirely imperfect being is imperfect in accepting their imperfections, then it implies a degree of perfection, creating a contradiction to their status of being an entirely imperfect being. On the other hand, if they are perfect in accepting their imperfections, then they cease to be entirely imperfect, creating another contradiction to their status of being an entirely imperfect being. And the loop continues.

Building upon this observation, we prove a theorem stating that any imperfect being cannot determine the truth or falsity of any statement with absolute perfection. We then demonstrate how the theorem applies to itself, creating a self-referential paradox—hence the name, The Paradoxical Theorem.

Definition 1 (proposition, P). A proposition is a declarative expression that is either definitely true or definitely false. Let "**P**" represents any proposition whatsoever.

Definition 2 ($\varepsilon_{\text{perf}}$). Let $\varepsilon_{\text{perf}}$ represent perfection of the highest order.

Definition 3 (set $\mathbb{B}$). Let $\mathbb{B} = \{\text{the set of all beings}\}$

Definition 4 (set $\mathbb{P}$).
Let $\mathbb{P} = \{b \in \mathbb{B} \mid b \text{ can determine whether } \mathbf{P} \text{ is true or false with } \varepsilon_{\text{perf}}\}$

Definition 5 (set $\mathbb{B} \setminus \mathbb{P}$). Let $\mathbb{B} \setminus \mathbb{P} = \{b \in \mathbb{B} \mid b \notin \mathbb{P}\}$

Axiom 1 (self-evident). $\mathbb{P} \subset \mathbb{B}$ and $\mathbb{B} \setminus \mathbb{P} \subset \mathbb{B}$

Theorem 1. $\forall b \in \mathbb{B} \setminus \mathbb{P}$, b cannot determine whether $\mathbf{P}$ is true or false with $\varepsilon_{\text{perf}}$.

Proof. We shall prove this by contradiction. Therefore, assume—for the sake of contradiction—that $\exists b \in \mathbb{B} \setminus \mathbb{P}$ s.t. b can determine whether $\mathbf{P}$ is true or false with $\varepsilon_{\text{perf}}$.

1. If $b \in \mathbb{B} \setminus \mathbb{P}$ can determine whether $\mathbf{P}$ is true or false with $\varepsilon_{\text{perf}}$, then b—by definition—must be an element of $\mathbb{P}$.

2. However, this creates a direct contradiction as b is already an element of $\mathbb{B} \setminus \mathbb{P}$ and $\mathbb{B} \setminus \mathbb{P} = \{b \in \mathbb{B} \mid b \notin \mathbb{P}\}$.

Therefore, the assumption that $\exists b \in \mathbb{B} \setminus \mathbb{P}$ s.t. b can determine whether $\mathbf{P}$ is true or false with $\varepsilon_{\text{perf}}$ must inherently be false and the proof is complete. ∎

2. The Paradox

Theorem 2. $\forall b \in \mathbb{B} \setminus \mathbb{P}$, b cannot determine whether **Theorem 1** is true or false with $\varepsilon_{\text{perf}}$.

Proof. We shall prove this directly as follows.

1. **Theorem 1** states that $\forall b \in \mathbb{B} \setminus \mathbb{P}$, b cannot determine whether $\mathbf{P}$ is true or false with $\varepsilon_{\text{perf}}$.

2. If so, then **Theorem 1** is also a $\mathbf{P}$.

3. Therefore, b also cannot determine whether **Theorem 1** is true or false with $\varepsilon_{\text{perf}}$.

This completes the proof. ∎

3. Conclusion

Any imperfect being cannot determine the truth or falsity of any proposition with absolute perfection, including the proposition that "any imperfect being cannot determine the truth or falsity of any proposition with absolute perfection."