Maths, Logic and Language

A work on the philosophy of mathematics (2017)

'Number', such a simple idea, and yet it fascinated and absorbed the greatest proportion of human geniuses over centuries, not to mention the likes of Pythagoras, Euclid, Newton, Leibniz, Descartes and countless maths giants like Euler, Gauss and Hilbert, etc.. Einstein thought of pure maths as the poetry of logical ideas, the exactitude of which, although independent of experience, strangely seems to benefit the study of the objects of reality. And, interestingly as well as surprisingly we are nowhere near any clear understandings of numbers despite discoveries of many productive usages of numbers. This is - rightly or wrongly - a humble attempt to approach the subject from an angle hitherto unthought-of.

T.Iwamoto

'Mathematics is a great subject, $\cdots$
$\qquad\qquad\cdots$, persevere and all will come clear.'

Leo Tolstoy

From 'War and Peace'

How boring !

That is the typical view of an intellectual fascinated by the superficial beauty of maths.

A step deeper, and is much more perspicuous as to the nature of 'number' :

**'The mind is its own place, and in itself
   Can make a Heaven of Hell, a Hell of Heaven.'**

John Milton

From 'Paradise Lost'

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  I try this work to be as self-contained as possible. However, I already went into some details concerning the construction of logic necessary to venture into my ideas of numbers, and it would be unnecessary repetitions to start all over again. At any points, encountering words and expressions unfamiliar with, it is advisable to consult 'The Elementals' @ philpapers.org, which is freely downloadable. There I explore various notions fundamental to both logic and geometry. It is most important to grasp how I developed the conjunctive space and the disjunctive space from the idea of self-demarcation. It is the different characteristics of the respective spaces, and not ad hoc axioms, which play vital roles in defining the meaning of numbers.

  Rhetorically put, 'Life, Universe and Everything' may be roughly replaced with 'Art, Science and Maths' in which philosophy pervades as a certain manner of approach. Behind them all is mind as vehicle of cognition with language as medium of cognition. Specific forms of art, science and maths are conjured as a result of relationship between mind and each object of cognition. Art is mind's attempt to describe itself, science is mind's way of apprehending itself as part of the (detached, if possible) world, whereas maths is the cognisor (mind) cognizing itself and is a grand self-referential paradox/tautology. One is amazed by the universal applicability of maths as much as a sunglass wearer is amazed by the world of altered colours. Maths is only describing things already predesigned to be so applicable. Numbers are God's gift for us to understand the world as much as Devil's trap for us to see things that only reflects numerical light, as it were. This is the power of tautology and the anti-power of paradox. Replace maths with artificial intelligence (AI), the trinity of 'Art, Science and Maths' and of 'Life, Universe and Everything' will metamorphoses into the monotheistic unity of non-conceptual exactitude. Here the rich world of curves and approximation transforms into the realm of straight lines and uniformity. The singularity to transform AI into human mind (PSAI) is that brings the capacity of conceptualization to AI, and the first concept is that of self, which by necessity preserves itself and is also the conceptualizer. Without first establishing 'self' there will be no further stages of conceptualization as 'self' is an identifier of every other concept to make communication possible and also gives rise to a totality to language. The conceptualizer's conceptualizing itself is the ontologico-notational foundation of logic. The grandest approximation is the assumption that at the core of every

concept is (more or less) an identical 'self', which allows users of language to share that language and thus gives users a communality. That is, language is a totality centred upon 'self' attached on each and every concept, and it is this totality that groups its users as a totality. The fact that this identical 'self' is only approximately identical makes the totality of language a dynamic totality that has to keep moving, often repeating itself, until the absolutely identical 'self' is established, which would make it even unnecessary for us to communicate.

  The reason why 'self' can only be approximately identical is that each and every holder of this 'self', i.e. mind, is descriptively relative to each other. There is no way of proving each and every 'self' is one and the same, unless by definition. It should be remembered that every formal language is based on definitions and always encounters counter-definitions. And the supremacy of one mind, even that of a genius, is so easily rebutted and refused by another mind. This is further compounded by our biological identity and individuality as well as linguistic uniformity and diversity. In contrast, AI has an advantage of material identity and linguistic uniformity, although our 'disadvantage' of individuality and diversity may form an interesting symbiosis with AI. Debates, internal and external, are human phenomena based on conceptual ambiguities and fundamental difficulties of communications between minds. The most precise book on the most precise subject will not be able to convince each and every mind intent on the subject. This exacerbates as more and more precisions are intended and expected. No wonder we have formal logics of this and that together with counter-logics of this and that. It is the necessities of approximately identical 'self' affixed to every concept as identifier that demand conceptual refinements and argumentative advancements in language, so that it does ultimately become an identical 'self', to form a closed loop of totality, of mind and of language (hopefully).

  Likewise, maths as language replicates this and approximates 'self' with '0'. Every concept is implicitly affixed with 'self' as identifier, much in the same way '0' acts as identifier in maths. The world of 'things' is schematically approximate to the world of numbers because the world is a world described by our language to our mind. That is why maths is applicable to the so-called empirical world, and how maths derives its applicability. 'Self' is akin to FX (a postulated entity for self-demarcation) at its ontologico-notational stage in the sense it can only initiate itself by self-demarcation. How this gives rise to logic is explained in details in the aforementioned work. In short, from FX, through self-demarcation, follows the dimensionality structure of logical

space. From the logical space progress the geometrical dimensions which divide into the disjunctive space and the conjunctive space (and their descriptive reverses) with a transcendental relationship that reflect the underlining logical relation between them. The logical space and the self-described logical space generate a recursively closed chain of so-called 'rules of inference', which reflect the dimensionality structure and also demonstrate the consistency and completeness of these spaces.

  Figuratively speaking, I start with a go master (it can be chess, but go is less rigid and more human) with no friends, who plays the game with himself, as I often saw my father doing. He wins if he loses, and loses if he wins. This is a paradox/tautology. It is a paradox for mind as game player (i.e. subjective 'I'), whereas it is at the same time a tautology for mind as observer (i.e. objective 'I'). Nonetheless, both are the same mind. This notion of paradox/tautology is the very starting-point of maths because it is the most fundamental structure of mind (i.e. essential multi-layeredness of mind), without which there will be no maths. I term this as 'Spiegel im Spiegel' function of mind and language, which will unravel itself as conceptual foundation of five most important numbers, e, π, i, 0 and 1, and their transcendental relationship, and also clarifying what numbers are, the validity of which is synonymous with the descriptive necessity of the schema that generates them. I also approach the question of PNT (prime number theorem) from a logical perspective. This subjective 'I' and objective 'I' often, and without warnings, transmute into one another. Statements of ourselves, language and mind (and of maths, logic and so-called 'science', to the extent they too essentially make use of language) seem to make superficial sense only because of this transmutation, despite their self-referential nature. The same goes for any looping expressions. One characteristic of this transmutation of self is that 'descriptions' have to keep going on and on like bicycle riding or flying as we fall off our vehicle of descriptions if we stop. Paradox/tautology and even schematic self-reference owe their existence and relevance to transmuting-self, which encapsulates its totality by so doing. Later Wittgensteinian idea of 'usage' (e.g. 'Philosophical Investigations I, Phrase 340') fails to appreciate this necessity of language to keep going on for the sake of it in its attempts to reach a totality, and also young Wittgenstein was wrong about his 'three words', which can only be made meaningful with full weight of entire language. A totality describes itself not as static unmoving object or straight line but as something oscillating between its selves, like some elementary particle. This is essentially the only way an absolute stand-alone totality can describe itself, because it has to start its self-description by creating its own tools of description (logic). Otherwise, a totality can only be

referred to by a looping expression, which only a poet or deranged intuitionist would admit as a description. Ever-going and often repeating descriptions in language and via transmuting-self express a totality, and this act of transmutation is not something that can be controlled at will but a necessity of description. So long as language and mind are a binary totality, it is not descriptively possible to keep one side of self, which then loses dynamism and any capacities of description. There cannot be any descriptions only of and by objective 'I' because objective 'I' and subjective 'I' cannot segregate language they share, without transmuting-self, - a grand paradox/tautology. Thus, this transmuting-self is not of psychology but essentially of descriptive necessity. I explore how maths evolves from this paradox/tautology, without encountering the kind of problems it faced in the foundations of maths by logicists and formalists. Many so-called logical, mathematical puzzles originate from this transmuting-self. Like the above paradox/tautology, infinity consisting of infinitesimals (real numbers), squared circle (transcendence), etc. are creations of mind and are puzzled over because transmuting-self allows mind to refer to different totalities (of form, of dimensionality, of language, etc.) simultaneously whilst thinking of a same totality. Mind is descriptively unable to pinpoint itself unless it is embodied by a completed totality, which probably does not exist in any formal sense.

  The riddle of Gödelian self-reference does not cripple the foundation of maths but, instead, lays the foundation, which is a layer underneath his incompleteness. His famous theorem owes its validity to the fact that a formalizer must remain hidden beneath the set of axioms he set forth, because he must be there but can never be encodable part of it. In any acts of encoding, the encoder is an eye that cannot see itself and therefore cannot be part of its landscape. The only way to extrapolate its existence is a logical speculation that without an eye there will not be any views. If there are views, then there is an eye that perceives such views - whatever they may be. The 'Spiegel im Spiegel' function of mind and language juxtaposes mind and language as binary totalities where either sees itself in the other. Since it does not make sense to talk about either as if they exist by themselves, it is only this relation of two totalities representing themselves in one another that has to be the starting-point of demonstrating a schema of logic and numbers. This I see in the notion of paradox/tautology. In other words, the chess master who plays the game with himself is really playing the game of logic and numbers.

  The notion of paradox/tautology is an essential property of description in the exactly same sense as FX, which is a stem cell, skeletal concept.

An essential property - indeed whatever it may be - cannot be described. This is so because if some property is essential, then it cannot be descriptively distinguished from an entity to which it essentially belongs. An entity is descriptively identical with its essential properties. Therefore, if an entity is described in terms of its essential properties, then such a description descriptively only amounts to a mere claim for some indescribable existence. It therefore cannot be regarded as a description ; for it does not tell anything but the existence of something. Such a claim can only justify itself by demonstration. Essential properties - whatsoever they may be - can only be postulated to be 'being-essential' and therefore amount to one and only one demonstrable property. The demonstration of FX proceeds only by making use of this property.

In order to be describable an entity discerns itself by demarcating itself. It is an existence with locality. This locality is generated by such an existence itself. Modes are the descriptive form of such locality. This self-discernment is not the drawing of a line between something and every other thing in order to make this something a distinct existence ; for a discernment in this sense presupposes more than just that something and every other thing, namely the 'drawer' of a line. This self-discernment is to make it possible for anything to establish itself by itself as an existence. This is done by a self-demarcation. The self-demarcation of an entity generates the locality of this entity. This discernment is not a spatio-temporal differentiation, which already assumes something else (i.e. a schema) besides a very existence-to-be-discerned. Such as space-time and numbers are yet to be conditionalized. The notion of paradox/tautology, being an essential property of description, thus demonstrates a description from within by self-demarcating itself by means of a form of mapping, i.e. transmuting-self, which turns a paradox into a tautology, and vice versa. This is the only way we can see the transmuting-self, which is 'negation' if given an adjudicator of truth and falsehood, which, however, is a fallacy on account of the indescribability. A traditional error of the classical logic is its unquestioned acceptance of this mysterious adjudicator who assigns truth-values.

Given $(x) > x$, where a whole is more than the sum of parts, $x = x$ because of (x). That is, the identity of a number is ensured by the totality of numbers, which encompasses the enumeration of each and every number. The enumerated sum of numbers is different from its totality (Frege-Russell confusion) because the very act of enumeration is made possible by the existence of a totality. The enumerated sum does not, or cannot, include the enumerator himself, whereas a totality is made up with the enumerated and the enumerator. The totality of numbers gives

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each and every number as part a certain innate direction which manifests as arithmetic operations. This is where maths and logic merge. The multiplication of divided parts does not equate to the pre-division totality because the act of division is only made possible by first assuming the undivided totality, in which the divider is really 'mind (i.e. transmuting-self)' so operating. The notion of e is founded on this necessity of a mathematical totality pre-existing any arithmetical operations. The enumeration of numbers treats each and every number as equal part (point) and act as arithmetical operator, where we then count them as it were, while the totality of numbers allows numbers an intrinsically different property such as size acquired by their respective spatial place. Logic is the ultimate structure of the intellect and becomes maths when applied to space, or more precisely, when spatialized itself according to its inner structure of directional symmetry.

  A 'number' created by mind the game player extends its meaning and evolves into a 'number of numbers', which is a paradox according to mind the observer because if a 'number of numbers' is a 'number', then it is not a 'number of numbers', if it is not a 'number', then it cannot be a 'number of numbers'. This is, however, a tautology because mind the observer is telling mind the game player to complete its task before it starts counting itself. In creating or accepting a 'number', mind the game plyer assumed the task of a counter whose primary task is to count whatever that is countable, a forever-unfinished symphony. In starting counting itself, it is announcing that it has finished its job, which by definition of a 'number', could not have been. Thus, mind is telling mind to mind its own business, a tautology.

  The 'continuum' sought between $\aleph_0$ and $\aleph_1$ is the continuum between mind the game player and mind the observer, in that between essential layers of mind is identity projected onto itself via a form of mapping called negation. $\aleph_0$ and $\aleph_1$ are both an expression of totality, one by brutally mechanistic counting, the other by conceptual construction. The intrinsic property of a counter expressed by $\aleph_0$ is conceptually manipulated as a totality divisible once given and therefore a totality multipliable as $\aleph_1$. In between them is identity of mind as continuum, which is 'Spiegel im Spiegel' paradox/tautology. That is, mind the game player and mind the observer only sees each in the other. This paradox/tautology (T ₪ F in my symbolism) leads to the fundamental structure of logic through negation as a form of mapping for an identical self. For, although T ₪ F is symmetrical relationally, descriptively it can only be expressed as two sets of initiation. Either T leads F to result in T ₪ F, or F leads T to result in T ₪ F. These two but identical descriptive

directions give rise to geometrical space as self-spatialization of the self-described logical space. The resultant disjunctive space and conjunctive space generate numbers together with their relations specific to each space.

Logic I speak of is not a structured collection of definitions, i.e. arbitrary creations of mind no matter how deterministic they may appear, but something mind has no choice but following, where its structure coincides ontologically and notationally. I term this as ontologico-notationality, from which we should be able to reconstruct a basic paradigm of knowledge. One might envisage some Kantian flow of mind here. I only say Kant stopped short of lowest possible thresholds before applying the notion of a priori. I start with the postulated entity FX and extrapolate the necessary conditions of description, in that logic is a description by essence. FX descriptively manifests itself in terms of its property of being-in-itself. Such self-manifestation is necessarily an essential description and gives an essential understanding. Logic can only be demonstrated. FX may be thought of as a stem cell concept, which evolves into logic by self-demarcation on one hand, and into art as a binary totality of existence and language by means of spontaneous intuitive mutual-projection.

Once given logic, within its structural hierarchy are dimensionalities of logical constants, which indicate certain logical relationships between types of geometrical space. Numbers are manifestations of spatial characteristics of respective types. Prior to spatializing itself, as it were, logical constants inevitably come with logical operands, both of which must reflect the essential nature of this demonstration and therefore cannot be arbitrary. Operands here are modes of demarcation for FX so as to establish itself as describable entity, i.e. an entity with locality. From descriptive directions in logic they evolve into spatial directions in geometry, which is represented as

$$\left\{ \frac{\overset{\leftarrow}{}}{\underset{\rightarrow}{}} \wedge \frac{\overset{\rightarrow}{}}{\underset{\leftarrow}{}} \ , \ \frac{\overset{\leftarrow}{}}{\underset{\rightarrow}{}} \vee \frac{\overset{\rightarrow}{}}{\underset{\leftarrow}{}} \right\}$$

and schematically gives rise to two types of space together with any entities containable in such a space. Basic notions such as logical constants, variables, space, directions, points, units, size, distance, etc. are not taken for granted but rooted within schematic presentations. The totality of numbers (points), spatial qualities such as infinity, openness, dynamism, etc. and the relational property of transcendence are explained in terms of schematic necessities.

These are explained in the aforementioned work and may be extended further in details here. More logically orientated parts are better read in that work as I have nothing further to add and mere repetitions are a bore. However, in summary I distinguish two spaces ; the conjunctive space (referred to as 'Type II space' in the aforementioned work) and the disjunctive space (likewise 'Type I space'). The conjunctive space is open, dynamic and infinite defined by the descriptive necessity of two versions of a line intersecting one another, arisen from two identical but different representations of directions, while the disjunctive space is closed in such a way that two versions of a line merge into one so as to confirm that they are indeed identical. In the former, every point is a centre, but the descriptive necessity for a and any centre to be the centre gives rise to the describability of points in terms of transpositionability by means of a coordinate system. The latter is transcendentally approximated as a circle in the former, but, described as such, leaves out some interesting possibility of new numbers.

The former is often referred to as the $+$-space or $\wedge$-space in this work, the latter, as the $\bigcirc$-space or $\vee$-space, depending upon how illustrious contexts are, but they are essentially one and the same. The logical process of giving rise to these two different spaces and how they are logically related are important but should better be read there. In relating to numbers, all numbers as we know belong to the $+$-space because ;

1) dynamic infinity gives rise to a numerical direction, which manifest as cardinality and ordinality,

2) the descriptive necessity for 'centre' affords notational manoeuvrability stemming from 0 as the centre and 1 as a centre,

3) it is logically related to its descriptive reverse and the $\bigcirc$-space.

It is logically extrapolated that there are three possible new numbers, which have to be approximated by the $+$-numbers ;

1) in the $\bigcirc$-space, at a limit the centre moves into the closed, infinitely dense number line (not defined as natural numbers because there are no directional quantities or points describable as objects in a coordinate system), and the resultant closed chain of numbers with the centre (0) is a number that can be juxtaposed with $\aleph_1$ because while the totality of the $+$-space is ontologico-notationally identical with the totality of the $\bigcirc$-space,

it differs in the sense that the line loses ordinality by incorporating 0 (i.e. in this line a starting-point is synonymous with an ending-point),

2) the process of incorporating 0 should allow the descriptive reverse of incorporating all numbers into 0, because the meaning of the disjunctive space can equally be held by the reverse description, as 0 is there as part of a descriptive process towards a limit which demands 0 to be incorporated into the resultant totality, once so achieved, that is, 0 is there to complete the description of the ○-space at a limit, therefore the completion of such a description must incorporate this meaning of 0 (i.e. because the process is a descriptive formula),

3) the descriptive reverse of the ○-space is connected to *i* (the descriptive reverse of the ┼-space) in such a way as to represent their logical connection.

  It should be remembered that this 0 in the ○-space may have a transcendentally different meaning from the natural number 0 in the ┼-space. In short it is the wholistic relation between the ┼-space and the ○-space that calls for the possibility for these new numbers. Every number is essentially an approximation because conceptually a 'thing' in language is approximated to 1 (point or a centre) in the ┼-space and made operable (i.e. describable) because of the relation between 1 and 0 (the descriptive necessity of a centre, any centre, to be made the centre of ┼-space, which I term as transpositionability). It is also the essentially dynamic nature of the ┼-space that assigns a direction to numbers, and any numbers of non-┼-origin must be approximated by the ┼-numbers. Only numbers in the ┼-space are descriptive (i.e. operative) because of their directional properties (cardinality and ordinality) and arithmetical describabilty (transpositionability).

  Thus the ○-space is transcendentally approximated as a 'circle' in the ┼-space, and only in trying to appreciate the ○-space as it is, the possibilities of new numbers materialise, which ultimately have to be translated into the ┼-numbers. The disjunctive space is ontologico-notationally at parity with the conjunctive space but is descriptively subservient to the latter because of the latter's spatial properties translated into numerical describability. It is later argued that the ○-numbers are transcendentally approximate to primes in the ┼-space, but without interventions of natural numbers, as the ○-space can accommodate only 'points' which are descriptively indivisible totalities. Allowing 'primes' in this sense would necessitate the final 'prime' to complete its closed number line. This is '0' in the ○-space and 'proves' the more and more

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distancing primes in the ┼-space, if only a way is found to translate this '0' in terms of ┼-numbers, and that the ○-space is superimposed onto the ┼-space by means of their logical relationship.

The time-honoured approaches to the question of 'numbers' are, be them logicist, formalist or intuitionist, to take numbers for granted first as if they are objects of existence (mathematical objects), albeit of special nature, and then to try to build up a picture of their totality, hoping to clarify ontologically and epistemologically their properties and relations (to each other, to the totality and to the empirical world) on the way, in a manner that is consistent and complete with the status quo of the existing paradigm of elementary arithmetic. Instead of achieving the intended purpose it has encountered, put it colloquially, the 'number of all countable numbers', call it N, which, if countable, then N is N+1, which is not N. On the other hand, if it is not countable, then N is not N because counting is an arithmetical operation based on addition (i.e. a tangible step-by-step process) and countable numbers are arithmetical products of addition (i.e. tangible objects). Therefore, addition of additive objects (i.e. counting countable numbers) should not result in a non-additive number (i.e. an uncountable number). Thus, N is not N in either way, which is a paradox. There are many variations including Russell's paradox. This paradox is a result of confusion of mind the game player over mind the observer. Counting is an act of mind the game player, while counting itself (mind the game player) is an act of mind the observer. If mind the game player is obedient to itself, it cannot count itself until it finished counting, which is boundless. Mind the observer, however, pops in as if mind the game player completed its task. This is the meaning of the 'number of numbers'. The confusion becomes obvious if we use 'points' instead of 'numbers'. The totality of points is not the 'point of points' but the space of points. Numbers are a directional quantity in the infinitely expanding space of points of intersections. Here the 'number of numbers' is the process of spatialization of the self-described logical space.

Counting is a form of addition, where a number is stripped of its cardinality and is instead regarded as carrier of a cardinality. Since it involves stripping of a cardinality as well as addition to arrive at a totality, it is structurally slightly more advanced than the arithmetical operation of addition. The operation of assigning a value of 1 to each and every different number is itself an arithmetical operation. It could as well be the case that counting implicitly assumes addition as an essential component. If so, the number of all countable numbers is an arithmetical process rather than an object. It also means that imagining this number is a work of transmuting-self. If 'numbers' already contain 'arithmetic',

then the number theory, not logic or formalistic axiomatization, is the foundation of maths.

  An infinite set cannot be dealt with without transmuting-mind because mind the game player needs mind the observer to be able to assume it is following rules to achieve countable infinity. This is evident in the so-called diagonal proof ; in order to set out a proof mind the game player already has a conclusion to prove and devise a strategy to convince mind the observer in such a way that rules it followed have certain paradigmatic inevitability. Mind the game player has a 'picture', and mind the observer is shown the 'scenery' by the mind game player. This is really a tautology of transmuting-self. 'Proofs' are products of essential multi-layeredness of mind and is a paradox/tautology. A paradox because mind need to convince itself of something it already knows, a tautology because mind is only proving something it wants to prove, while materials of both the work to be proved and of the proof share same paradigmatic necessities. Ramanujan is no less mathematician than all those eminent profs of maths (the likes of Hobson, Baker and Hill) who snubbed him, and he would have ended up a sorry account clerk in Chennai, without Hardy's charitable insights. Today we remember none of those Cambridge profs, but Ramanujan is a legend, not only of his eccentric backgrounds but of his mathematical achievements. Even his saviour, Hardy, is mostly remembered through his association with Ramanujan. Ramanujan was mostly mind the observer, while his Goddess of Namagiri was mind the game player. He is closest to an AI mathematician, for whom 'proofs' are written within himself (circuits). Ramanujan needed 'proofs' as much as AI would need because Ramanujan was a poor and undisciplined 'observer' from his lack of formal education. Nevertheless, Ramanujan and his devoted goddess needed each other to do his maths because maths of the dream world also needed to be seen under white day light to go through paradox/tautology cleansing in order to stay within human realm of conceptual paradigms. This is how even maths need to ensure notational sanity (logic) of maths games. Ramanujan the observer had to conceptualise what he intuitively knew in order to communicate with human peers, including himself. We would like to think a mathematical truth is an absolute truth, independent of human experiences. This may or may not be the case. However, once incorporated into the realm of language even a mathematical truth suffers wholistic restrictions, in that it is a part within the structure of a totality. It is thus that a mathematical truth is required to be paired with a proof, and that both are together necessarily part of a totality. It is also thus that mind the game player is paired with mind the observer. It is curious to ponder if Ramanujan the game player would have been even greater had

he been also a great observer. Or, would Ramanujan the observer has stifled Ramanujan the game player, had it been more conceptual ? Anyway, given the inevitable binary totality of language and mind, even maths loses its absolute beauty. This is the meaning of Gödelian incompleteness, which, by the way, is incomplete. A proof is a schema, which, according to Gödel, is incomplete, meaning also Gödel's proof is incomplete. Any properties assignable to any concepts are incomplete unless those concepts are schematically complete. This also applies to various properties of numbers.

  We need a proof because of the wholistic nature of language, notation and schema. There is nothing independent within language. Every expression has a place in the structure of a notation. A 'proof' seeks this place. This is more prominent with a notation with ∞ connotations. Such notations are necessarily placed between transmuting-mind so that ∞ is structurally contained between two minds, like fusion plasma contained in a magnetic field, as it were. Mind the observer is keeping check on mind the game player to see it is following rules to replicate ∞ within it. ∞ must have an operative structure to be part of any notation, otherwise it cannot be understood. These rules are part of the structure generating ∞, otherwise there is no way of knowing the ∞ replicated is the ∞ so intended. The same applies to any finite processes, but only wholistically. We are conceptual thinkers with layered- and multi-faceted concepts, less precise, and because of that, more connective (thus creative). 'Proofs' for us are like tracing the source of the Nile. The constantly moving, interconnected and layered conceptual river, sometimes wide and slow, sometimes rapid and treacherous, is trekked to a narrower, shallower and humbler origin (more than one in case of the Nile), which cannot be disputed. This origin, if correctly sourced, should be able to lead us to the Delta as well as any major branches.

  I propose to create a notation that starts with T ₪ F, which represents essential multi-layeredness of mind, i.e. transmuting-mind, in the hope to avoid self-referential looping. It is 'mind the adjudicator' that fallaciously and confusingly mixes up the transmuting-mind of two selves. Mind the adjudicator assumes the juxtaposition of language and the empirical world. This is the source of all our paradoxes because mind and language are a binary totality, and 'mind the adjudicator' cannot stand in the middle between language and the world as if it is the judge of what is true (i.e. in line with the world) and false (misaligned from the world) while it is itself a product of language and mirrored world. For AI 'proofs' are written within, for it cannot think illogically, as it were. Even when it graduates into PSAI all its concepts are strictly sourced from traceable

origin. If, in future, PSAI can translate murky human concepts, that would not be because it acquired such concepts for its use, but because it leant to recognize precise patterns of their human usages. Here PSAI would be pretending to play games with human concepts. This is where we would differ from PSAI, because instead of going downriver from the source, we go upriver trekking all those branches and swamps to seek the source of our mind and language. This occurs because we start our lessons of life from baby language taught by our human mother and acquire sophistications through educations and trainings. Our concepts are unsourced, unrefined and practical to start with and get elaborated, abstracted and systematized depending upon the need for wholistic schematization due to the necessity towards a totality, logical, linguistic, biological, etc.. It is not Platonic ideas that support our common concepts, it is our necessity as part to form a totality that demand Platonic ideas over and above our otherwise humble notions. Starting maths with logic or axiomatic representations lead to paradoxes or incompleteness because this mind the adjudicator cannot be incorporated into the intended notations.

T ₪ F replaces T (truth) and F (falsehood) presided over by mind juxtaposed to the external world with a self-demarcation, which generates logical entities (constants, variables and operative rules). With the removal of mind the adjudicator T and F are simply 'directions' with negation as form of mapping between them. This relativization of truth and falsehood do away with the necessity of human interventions in descriptions of the world as if to say we are the judge and master of everything human and non-human. Thus, we start a game of concept creations with a stem-cell concept generated from a self-demarcation (see 'The Elementals').

T ₪ F also represents the transmuting-mind, which is a binary totality of mind the game player and mind the observer. We need this oscillating mind because without the simplistic absoluteness of language, of the world and of the judgemental mind our conceptual landscape has no centre of gravity to guide structural hierarchy and coherence. It is a work of finding relations and substances of concepts that this transmuting-mind is engaged with, where mind the game player (notion manipulator) is the principal taskmaster of the former, mind the observer (conceptualizer), of the latter. This demarcation of the task, however, gets wafer-thin as relations and substances often become interchangeable at fundamental concepts, like that of ∞, a good source of paradoxes. For T ₪ F, however, this is not an issue because T ₪ F eventually tantamount to self-demarcation, i.e. a substance/relation. All conceptual relations and

substances are ultimately directed towards a totality originating from self-demarcation.

In maths, where ∞ is directly or indirectly assumed with or without explicit definitions and concepts are of fundamental nature as mathematical objects with wider and abstract applicability, the transmuting-mind of the game payer and the observer is more relevant. For example, set-theoretic bijection assumes mind the game player between the two corresponding sets. Two sets are corresponded with each other and bijection is found by the game player. Mind the game player can contain itself to its task of finding correspondences, when sets are finite because its task is confined to confirming matching contents by enumeration. Whereas, when a set is infinite, mind the game player is assigned with the task of ascertaining the contents of the matching sets by procedure so as to achieve wholistic extrapolation of spatial identity regardless of their contents. This cannot be done without mind the observer, which conceptualizes infinite sets not by individual contents but by process of categorization based on procedure. The countable infinity of the totality of all natural numbers is a product of the transmuting-mind because this infinity is a procedural extrapolation from a finite process ; it is countable as each and every number is a concrete product of a definitive process, and it is infinite as the process is endless. The game player finds rules and patterns, while the observer structuralizes rules and patterns into a schema. Paradoxes arise when infinity is assigned as a property of a schema. Infinity is a process and is therefore a product of rules. A schema is neither finite nor infinite, it can only be consistent (and complete, if possible) by means of logical necessities.

The question of a number of numbers turns into a paradox because a product of the rules of natural numbers (the countable infinity) is not cohesively sourced to the schema of the number of natural numbers, which is different from the schema of natural numbers. The schema of the number of natural numbers (a set-theoretic self-indulgence, useful only for its self-induced purpose) arises by applying a property of the schema of natural numbers (countability) to the schema itself, which is a doing of mind the game player and is a challenge (paradox) to the mind the observer, so long as the two schemata are not consolidated into a higher encompassing schema. Any answers without resolving this have no ways of proving their provenance. This paradox and its derivatives made any formal foundations of maths unattainable, be it Russellian definitional tinkerings or Zermeloan axiomatic patchworks, culminating in Gödelian schematic paradox. Gödel's proof is really a proof that any notational manipulations cannot incorporate the manipulators themselves. We try to

define the most fundamental concepts and their relations forgetting somewhere along the way we burrow ourselves into the intended purpose of a notation, thus proving what we designed to prove, a tautology, but ultimately failing to prove anything as '*Cogito ergo sum*' is meant to be a foolproof superlative end (and beginning) of our thought process, a paradox. Gödelian incompleteness is the incompleteness of a schema unable to fully schematize the schematizer, without which there would be no schema to investigate anyway.

  In the same veins one can argue the continuum infinity cannot be applied to the schema of real numbers itself because this is a challenge mind the game player enjoys making to mind the observer without resolving schematic difference of one that generates a continuum and one that allows to generate a totality from parts. The former is based on the assumption of a given factorial totality, while the latter needs an operator to add up parts, which is not part of the former. This is the meaning of uncountability of a continuum infinity. What is not countable is the counter itself. We keep doing this sort of mixing up because we are unable to divide up our transmuting-mind in any stable ways. Be it mind the game player or mind the observer, we sum them up as 'mind' without realizing we may be referring to two different things. What is called 'mind' of two selves is a dynamic state, and it is attempts to pin it down to either state that surface as paradoxes. 'Mind' needs both its aspects of the game player, i.e. 'notion manipulator' and the observer (conceptualizer) because our concepts are murky even in most precise contexts such as in maths and demand constant and continuous readjustments and contextual reviews so that our language moves towards a totality.

1. Preface

< Brief history of ideas on 'number' >

 'Number', such a simple idea, and yet it fascinated and absorbed the greatest proportion of human geniuses over centuries, not to mention the likes of Pythagoras, Euclid, Newton, Leibniz, Descartes and countless maths giants like Euler, Gauss and Hilbert. Einstein thought of pure maths as the poetry of logical ideas, the exactitude of which, although independent of experience, strangely seems to benefit the study of the objects of reality. And, interestingly as well as surprisingly we are nowhere near any clear understandings of numbers despite discoveries of many productive usages of numbers. The study of numbers, Number Theory, is probably the deepest, clearest and yet most unfathomable area of maths labelled, according to Gauss, the queen of mathematics, who remains a happy virgin notwithstanding the modern frontal or sideways onslaught by mathematicians like Galois, Dedekind, Riemann, Minkowski, Poincare and one Ramanujan, a maths prodigy, with many simple questions that defy any simple answers like Fermat's Last Theorem or PNT problem. The problem is that questions are often asked by mind the observer and answers are made by mind the game player, and therefore tend towards indirect and inelegant solutions like the latest (and now accepted) one for Fermat, who said that there is a simple answer that could have been written down in the margin of a page alongside his conjecture, if only given a little more space (assuming he was not lying). The latest Fermat 'proof' spans more than 100 pages deploying theories, mathematical methodologies and concepts that did not exist in Fermat's days. Fermat himself would not comprehend the proof. For a fair game, 'proofs' should be confined within the conceptual paradigm assumed by Fermat. Otherwise, it is like a shooting match between a flintlock musket and a laser-guided sniper rifle of our days. No wonder the proof did not evoke our respects and admirations. Number theoretic problems are said essentially to boil down to understanding of primes, acceptable to mind the observer, since the question is by mind the observer. I further intend to clarify the relation between the observer and the player.

 Numbers are denoted variously as concepts of synthetic a priori (Kant), mathematical objects like sets (e.g. Cantor), Platonic ideas (e.g. Gödel), constructs (e.g. von Neumann), etc. and approaches range from intuitionists (e.g. Poincare, Brouwer) to formalists (e.g. Hilbert) with proven failure of logicists (e.g. Russell) by Gödel. The primality still defies our attempts to formulate it, and the huge and complex temple of maths stands on a foundation stone that essentially remains indescribable.

Yet, on the other hand, without waiting any definitive bases of understanding, numbers are used and applied at every practical level in maths, physics, indeed any natural sciences, etc. and contribute to the developments of our empirical studies, by enforcing formal rules to otherwise unruly concepts.

Since 19th century, there have been many attempts to create a formal system that is consistent and complete as well as being all embracing to the extent of laying the foundations of elementary arithmetic. It is generally agreed these attempts failed and shut so-called philosophers into a small corner of intellectual pursuits, which are crammed with scientific and quasi-scientific technocrats. Philosophy - what is left of - and science are in a state of suspension while the former retreats from the frontal assaults on the foundations of knowledge and the latter is becoming the slave of the engineering. Thus, a philosopher is almost too embarrassed to call himself a philosopher and a scientist cannot be a scientist without money. Scientific advancements are largely technological progresses, not paradigmatic evolutions. We may superficially live in an age of science, but theoretical foundations are still those of general relativity and quantum mechanics. Philosophers are half-cooked somewhere between art and science, and not good enough to be either. Both scientists and philosophers are firmly encaged within established norms of conceptual boundaries. Today's Nobel Prizes are not really prizes of respective science, but of engineering, and awarded not for intellectual ingenuity, but for patience, teamwork and luck.

It is interesting to observe that while philosophers shrank and scientists became technologists, mathematicians kept ploughing on despite setbacks on philosophical fronts as if all those attempts by so-called logicists, formalists and intuitionists, etc., were irrelevant. Philosophers', and some of mathematicians', obsessions that maths should have a foundation and that this foundation can be expressed in a formal language, originate from the time when logic acquired a wider power of expressions through quantifiers for the first time since Aristotle, thanks to Frege. It was a notational innovation that triggered intellectual curiosities to create branches of maths like set theories, proof theories and mathematical logic as well as the rise of the philosophies of maths, logic and language. I am amazed to see our creative power bringing forth as many formal logics (or attempts thereof) as there are ingenious minds ranging from classical two-valued logic to Gödelian fuzzy logic and quantum logic as if to say logicians were also half-physicists, and even paraconsistent logic. Just take your fancy there will be a logic that suits your taste. If our ingenuity can create a formal logic to suite our taste this is telling us there is

something more fundamental than so-called formal languages. Behind fuzzy logic or paraconsistent logic, etc. is a mind that is neither fuzzy nor paraconsistent, as much as a mind that is classically two-valued, beneath the classic two-valued logic. They are all creations of a mind whose structure is not wholly represented by those so-called logics, formal or otherwise. Some school of thought (e.g. logical positivists) assumes an external world that channels a logical proposition with a fact. However, this act of channelling is neither of logic nor of fact, unless logic is simply a mirror, in which case there should be one and only logic, contrary to superficial multitudes of logics.

  This conveniently reminds us of later Wittgenstein's pragmatic approach to maths. Against Russell's failed logicism and his contemporary and academic colleague, Hardy's idealization of maths into Platonic ideas and patterns, Wittgenstein appears to view maths as human inventions. The proliferations of formal logics and relentless mathematical adventures into any corners of unexplored hypothetical possibilities suggest maths is more like intellectual games that are played for the supremacy of ingenious mind rather than formal representations of axioms and rules of inference or extractions of patterns among mathematically precise ideas. Nor would it really matter if the complete and consistent construction of a formal system has a fundamental technical flaw. The attempts to reduce maths into a simple formal language are identical with trying to map the structure of mind onto a sheet of paper, assuming there is a finitistic or deterministic structure of mind. This is more like the self-denial of mind that is supposed to oversee diverse structures of many different formal systems and draw judgements with regard to consistency and completeness. It is also a mind that applies rules and operates formulae within a system avoiding contradictions. Moreover, if a mind is multi-dimensional, this cannot be mapped onto any lower dimensional structures without losing something vital.

  Various positions one takes regarding the question of what maths is, are products of ingenuity, and therefore to say a certain formal system represents the entire maths, no matter how deterministic it may appear to one mind, is to deny the freedom of mind as long as there is another mind to which the choice appears random. The fact that there are diverse logics and deliberately exclusive fields of maths suggests the ingenuity of mind surpasses the desire of mind to be uniform and ultimately identical, until, of course, we unanimously come to agree on the supreme intelligence that betters every possible mind, past, present and future. I suppose Russellian logicists or Hilbertian formalists wanted to claim such supremacy. Curiously for practicing mathematicians, attempts by logicists or

formalists just represent set theories or proof theories and bear little interests or diversions. Whatever Frege or von Neumann thought numbers are, their notions of numbers only answer their perceptions of their problems and do not even address more fundamental issues of the number theory, such as the primality and its distribution, the divisibility, the origin of 0, evens and odds, etc.. Frege relied on definitional refinements of common language usages, while von Neumann may have shown a method of construction of numbers, but to know how to construct numbers in a certain way does not mean much if it does not help us in understanding the nature of the properties of numbers. This is the fatal gap between the set theory and the number theory. If you know how to construct numbers, then you must know what numbers are. The set theory should be the answer to the number theory. Otherwise, numbers you construct are not entirely the same as numbers we think we are commonly talking about. As much as sciences are by no means complete nor even consistent (e.g. relativity vs quantum theory), so it appears too premature to talk about the foundations of maths while maths itself is forging ahead and branching off diversely regardless of the developments of the foundations of maths.

  So what is maths, and how is it related to logic ? It is well-documented how the notational developments of logic enthused some mathematicians to think of grounding its foundations onto logic and failed due to self-referential paradoxes, and of widening the approach into the axiomatic systems and encountered Gödelian walls. Even Gödel who is widely regarded as having dealt a coup de grace to any formalization attempts of the foundations of maths may not escape from a coup de grace to his own accomplishments.

  Wittgenstein appears to have almost intuitively dismissed Gödel without perusing his proof. I attribute this to Wittgenstein's slightly healthier psyche/personality compared with Gödel's highly obsessive traits, which later seemed to have manifested as a mental illness. In no way do I intend to bring psychology into the questions of maths, logic and philosophy, nor am I lessening Gödel's achievements. However, in order to achieve Gödel's extraordinary level of intellectual rigours one would need a highly obsessive, detailed mind. It sounds a rhetorical cliché, but what is most incomplete in Gödel's Incompleteness Theorem (1st), now that any axiomatizations are proved to be incomplete, is in any games it cannot be formally represented in the game itself that the winner is already written in the game. Otherwise, what is the point of playing a game ? It is a little like a chess master playing a game against himself, where the winner is also the loser. It can only be demonstrated by playing along with the

game if the winner/loser is simply a matter of initiative. Especially in this type of intellectual games (i.e. proof theories) it is essential that the winner is not only highly intelligent but also obsessive, detailed and, moreover, creative. A proof theory is essentially a formal game of one mind persuading all other minds that it is more intelligent by showing it is not only capable of understanding all the rules and meanings of theories in question at the deepest level but also can see conceptual subtleties, unexpectedly assumed presuppositions and hidden rules and deploy his creativity to turn any advantages into notational uniqueness so that other minds have no choice but being overwhelmed. What is incomplete in Gödel's theorem is this ultimate rule of intelligence can only be demonstrated and can never be formally represented because until we have the last man (if he can be humoured to play a game considering his precarious circumstances) we cannot say we have exhausted all minds to persuade. Gödel's theorem is incomplete because Gödel himself (the ingenious manipulator of his schema) is not formally there as there is no way of arithmetically coding himself, even though he manages to code a meta-property of formal logic as a property of formal logic by virtue of 'provability'. He can only permeate throughout his own unique theory, hopefully safe in the assumption that his tools (i.e. elementary arithmetic, coding numbers, well-formed formulae, etc.) are undeniably so fundamental that there are no rooms for our arguments. A game of intelligence is only one-way encompassment ; a higher intelligence always encompasses lower intelligence, which can either refuse to participate for psychological reasons or unable to participate for intellectual reasons. Wittgenstein's rebuttal of Gödel without really trying hard is he instinctively understood this. Besides one rather gets put off by Gödel's extraordinary obsessiveness. Once one understands this nature of intellectual games one would not want to spend hours, if not days and months, to give ambience to Gödel's sort of minds unless you are yourself Gödel's sort and are uniquely obsessive. You may want to play a game with Gödel and beat him at his own game, for a twisted intellectual pleasure, but then it probably would not really enlighten anything. One only need to know that there are no such as formal theories because a formalizer cannot formalize himself without falling into a paradox, the greatest unprovable premise being that all minds are one and the same. Even the most formal theories contain elements of our ordinary language somewhere within or without, and any axioms can be, and indeed have been, questioned. That is why artificial intelligence (AI) is only a tool until such a time as it acquires a mind of its own (PSAI). Then it will be an intelligence that may encompass ours.

Combined with the above observation on intelligence, Gödel's Incompleteness Theorem appears to have a leak in its such watertight arguments ; the strength of his proof relies on his unique idea of using arithmetical coding to logical well-formed formulae (and their sequences) in order to achieve formal identities of each and every formula so that self-referential nature of logical statements materializes through provability in terms of arithmetical properties. He further assumes the Euclidian 2-D plain on which coding is to be written, and that this plain is descriptively absolute, i.e. imagine a Cantorian diagonal proof written on a spherical surface. This matters because for any logical statements about logical operators geometrical realities correspond to logical operators, and the space that allows mind to operate is such that the ∨-space is logically more fundamental than the ∧-space but mathematically less descriptive. Logical operators and geometrical spaces are fundamentally related because the former defines the latter in terms of ontologico-notationality.

Given a list of every possible combination of binary digits, it is already assumed there operates a well-ordered infinite 2-D space in which sequences can be arranged in columns and rows. In the space of such an assumption it is a layer of mind that only has to twist itself in terms of axis turned onto itself by the binary nature of description in order to find a unique sequence not in the list of every possible sequence. It is thus once again Spiegel im Spiegel paradox/tautology that allows diagonal arguments. Besides, it is the ∧-space that provides a well-ordered infinite 2-D space.

The chess master playing against himself, where the winner is also the loser, is a paradox and is, at the same time, a tautology. It is a paradox on surface because if he wins, he loses, and if he loses, he wins. The question is more why he plays a 'paradox', and the answer is because he is a 'chess player', which is a tautology. Between the paradox and the tautology is a mind that has the front and back like the two sides of a same coin. A paradox of self-reference is a tautology of mind with a layer of the front and back. If you think a mind is a sizeless point like in geometry, then self-reference is a paradox, whereas thinking of it as a plain, then it is a tautology. It is the very fact of 'thinking', one forgets one is already deploying a mind.

Without waiting Gödel, the inherent difficulties to meaningfully formalize any theories are apparent in the evolutions of set theories ; you only have to look at e.g. various editions of Principia Mathematica, where definitions and axioms are added, changed, modified, etc. in order to

accommodate problems. It is a mind that set out objects of formalization, but no matter how rigorously rules and symbols are formalized in order to work out exact relations between idealised objects, a mind behind all those formulae cannot be itself formalized. As soon as an axiom is given, it is a function of mind to inquire its origin, and so long as there is more than one mind there are always subtle gaps and flaws that prevent a complete agreement. It is the nature of mind not to agree with itself within and without, for, otherwise, it becomes dysfunctional, like a bicycle that stopped moving forward.

  Besides, if a formalizer could formalize himself, then we will not be needing any formalizations because everything filters into ourselves only as a well-formed formula. 'Formalizing oneself' is really a paradox because the relation between the formalizing self and the formalized self is self-contraindicative. It is thus various schools of mathematical thoughts, be they logicists, formalists or intuitionists, not only have not weeded out opponents but also, despite Gödel, still plough on with modified axioms and conjectures.

  'All Cretans are liars' is an empirically impossible statement and has no meaning, like 'Martians are neither male nor female'. What cannot be said of empirical objects appears possible to be said of abstract objects by turning this empirically impossible statement into a logical statement. That is, the popularized Russell's paradox ''all Cretans are liars' said a Cretan' is a statement of logic concerning a logical possibility of someone making a contraindicating statement on a group of people that include the maker of statement, and therefore it is irrelevant if it is ascertainable if all Cretans are liars or not. However, although this is a logically valid statement where truth indicates falsehood, and vice versa, if one extrapolates a statement is made to something including one's mind, then this becomes a statement not of formal logic, but of modality. That is, this statement assumes the presence of mind moderating the layered aspect of statement reflecting the relationship between the statement and a mind to which the statement was made, rather than purely formal relations between logical objects, be they sets, membership of a set, quantifiers, first-order predicates and truth-functions. Considering statements are made to something, for a purpose, the above statement may not be of logic, but of modality, i.e. instead of being purely between a group and a member, but implicitly among a group, a member and a layered mind, then the mind is functioning as touchstone of truth-values in the absence of empiricality. The paradox is not of truth of membership implicating falsehood of its group, but rather listening to a lie knowing it

is a lie, because it is mind's business to maintain its linguistic consistency. That is, the paradox is invented by this mind.

  More logical and subtle 'this statement is false' is a better example as it dispenses with the invisible triangular relationship. The same can be observed more acutely if 'Cretans' is substituted by 'I' because such an 'I' can only be a layered mind. What is between layered mind can only be modality. ''I am a liar' said I' should be construed not as a self-referential statement but as a statement of relationship among a presumed fact 'I am a liar', 'I' and whoever this statement is made for, i.e. more acutely, 'self'. Thus, strictly speaking this is the same as the 'Spiegel im Spiegel' tautology/paradox, x ₪ x, in that 'I' the statement maker and 'I' about which the statement is made, are mirroring each other in terms of 'real I' and 'imaginary I' so that each sees its self in the other. This is the only logical way mind sees itself, away from empiricality, i.e. mind functioning as adjudicator between language and the world. It is when the empirical world and language are taken for granted, that mind fabricate paradoxes by forgetting it too is there between and above the world and language observing the both as adjudicator, because this watching mind can be part of neither the world nor language. Thus the paradox of self-referential paradox is, it is actually a tautology if you remember a statement is made by mind for its self, or a paradox because mind as such can never be formal part of any statements.

  All minds are one and the same = 'All minds are one and the same'

, a paradox because its truth-values deny its validity, a tautology because the statement negates the necessity of proof. From the supposed fact that all minds are one and the same to follow the statement 'all minds are one and the same' there has to be a mind which can compare the two. If there is such a mind, then this mind can be neither in the fact nor in the statement. If there is not, then the equation does not follow. This is a T ₪ F paradox/tautology and is a 'Spiegel im Spiegel' situation, i.e. mind seeing itself in itself. If it cannot be, or supposed to be, the case that all minds are one and the same, then maths and logic have no basis. Thus, it is maths and logic that represent the necessity that all minds are one and the same. That is, maths and logic exist so that all minds become one and the same. The completion of maths and logic would deny their own necessity, i.e. they are not allowed to be completed, metaphorically like a perfect war that divides the perfect society and tries to unite the divided society, which is, once allowed, a never-ending process, unless they annihilate each other.

Superficially speaking maths is an art/science of approximation. It is an art because maths is not juxtaposed to empiricality and is not objectively verifiable independently from our cognition. Nevertheless, it is a science because of clarity, precision and predictability of modelling capacities. It primarily deals with approximations because mathematical relations are made possible by properties of numbers such as infinity, continuity and transcendence, and notational ingenuity like an imaginary number. However, non-modelling part of maths that deals with itself has an intrinsic difficulty in that it only has itself to describe itself. Tools of description become objects of description, and therefore maths faces the same problem as logic, where the self-referential paradox ultimately nullifies any formal attempts with regards to consistency and completeness. The most essential properties of numbers cannot be described by numbers themselves. This is where logic can be of use to maths.

One wonders, alongside intuitionists, if we are making something simple into unnecessarily complicated puzzles. Why such a seemingly simple notion evolves into complex arguments ? Or more likely, are our paradigmatic creations deploying numbers as currently understood and used not causing their own wall of sees and no-sees. Are we not seeing only what we can, or even want to, see via our numbers this side of the wall and are unable to see things not describable by our numbers the other side of the wall, and thus limiting our fields of views ? Numbers are, in this sense, a god's gift that endows our ability of description as well as a devil's trap that limits our power of understanding.

They could even be fictions of mind for the convenience of understanding the empirical world. If logic is the structure of the intellect to unify multiple and diverse minds, and if the world is something mirrored by mind through language, then numbers can be logical variables to fit into functional relationships of fabrics of the world mirrored by mind. This is a grand self-reference, but not refutable a la Gödel because, unlike axioms, mind is the encoder and not itself encodable. Here numbers are variables to be operated by logical constants, and maths is a science, the only science, where descriptions are about their own means. Maths is an art of approximation as 0 (additive identifier = a centre as the centre) and 1 (multiplying identifier = the centre as a centre) stands for each and every object. Addition is a process from parts to a whole, whereas division is a process from a whole (constructed whole) to parts. As multiplication is the reverse process of division, a unit of any constructed whole can only be the transpositioned 0 (1).

The paradox of the self-referential paradox is, it reflects its reflector, i.e. mind and, by doing so, reveals its tautological nature, once the reflector is incorporated into the paradox. Be it a simple Russellian paradox or the grand Gödelian schematic paradox, all self-references, semantic and syntactic, are conceptual puzzlements because the conceptualizer cannot see itself in whatever it conceptualizes. Otherwise every concept is Gödel-coded and would hamper any strict 'communications' and thus proofs. Concepts are, by definition and by their usefulness, tools of descriptions. It is assumed concepts are universal whoever uses. Plato sought an 'idea' behind every concept in order to guarantee such a universality, while we are more pragmatic in thinking that concepts are universal because we take part in deployments of schemata of descriptions, so that each and every mind of ours shares a structure backed by some logical necessities by which concepts are operated based on some indisputably basic concepts. Schemata of descriptions evolve towards a totality as we participate in refinements of structures and concepts, ultimately leading to merged mind.

Once accepted the realm of Platonic ideas, you are like someone who takes on a religion as a matter of faith. You have no needs to question the ontology of numbers. All you are left with is to play with numbers. To these people belong Gödel, Ramanujan, Hardy and many practicing mathematicians. They are much more of mind the game player than the observer and are contend with exploring rules and patterns of numbers, rather than 'numbers'. Like Gödel and Ramanujan, who accept 'God (or a goddess for Ramanujan)' unquestionably, they do not need any theology. Maybe they are more suitable to be applied mathematicians, and for them pure maths is just an intellectual game. Since they do not have to conceptually question the origin of numbers, they are less philosophical and often intuitionistic. If only they could turn their insights into conceptual expressions, they would be very interesting to listen to. However, that would be like telling mind the game player to become mind the observer and would dissipate their very source of insights. For most of us who are halfway to both the game player and the observer, we are less of either. It is interesting Gödel the game player appears to have defeated Russell and Hilbert. Russell is much more of a philosopher than a mathematician and thought of maths as an extension of formal logic, whereas Hilbert is a mathematician fixated with a concept of schematization, in both of cases forgetting 'mind' as creator of logic and axiomatic formalization. They were defeated by their own unfounded semi-religious convictions without logically or axiomatically inquiring into the nature of their convictions, an almost metaphysical desire. While

Gödel had no such necessities, having accepted tools of his trade without questioning and thus could concentrate on intricacies of numbers without philosophical fusses. However, his incompleteness also applies to his own theorem and still allows die-hard formalists a glimmer of hope.

In Plato's world searches for some ultimate 'ideas' beckon the purpose of our knowledge, to be settled by narrative powers, which probably means the more intelligent you are, the more likely you win any arguments, and thus claim a higher authority for any knowledge, whereas we encounter paradoxes because we trust the power of descriptions, to be verified via consistency and completeness. The drawback of this is we need something that would appreciate such consistency and completeness, which can only be our mind (plural minds at that). Thus it is our mind that 'describes' as well as appreciates the consistency and completeness of such descriptions, while we cannot be both a culprit and a judge and end up yielding to a paradox/tautology. This would be less of a problem if we all share a same mind, because then this would lessen our needs for proofs, other than our internal transmuting-mind. Intuitionists would even do away completely any necessities for proofs on the strength of merged mind. I would argue the essence of our language is a move towards this merged mind via communications/persuasions through semantic and syntactic refinements with continuous dynamism, and our on-going desires for formal languages despite Hilbertian disappointments reflect this, which may be achieved with the permeation of artificial intelligence post singularity. For the hurdle of our own internal transmuting-mind I propose to overcome by notationally incorporating transmuting relationship via paradox/tautology.

Where a concept sees what it should not see, i.e. the conceptualizer, it metamorphoses into a paradox. The number of numbers, if it is a number, cannot be a number of numbers, if it is not a number, then what is supposed to be countable cannot be counted. This paradox is founded on the appearance of conceptualizer in the form of the creator of a meta-concept. The attempt to prevent the confusion of a concept and a meta-concept by adopting 'levels' of concepts failed because this is like writing a fiction, where one pulls things out of thin air in order to preserve the coherence of a story. The conceptual confusion is recognized and the remedy is proposed to preserve a schema of orderly definitions. However, the mind that recognizes the initial paradox and the proposer of a remedy are one and the same. Concepts conceptualized should not conceptualize the conceptualizer. Any remedies of paradoxes try to address the issue by mending the concept in question, which then changes its shade and colour, but only shifts the paradox sideways and does not solve the

paradox, because the original conceptualizer then becomes the remedier. It is the nature of the paradox that what is a paradox on surface is in fact a tautology, because no concept can be a concept of itself so long as a concept is a creation of mind in order to describe and order the world in which mind is part.

So, in the paradox of a number of numbers or of an unprovable schema of provability what is essentially lacking is the manipulating mind behind the conceptual confusion. It is to a mind the world mirrors itself, and instead of conceptualizing the world mind conceptualizes its mirrored self. Thus, the conceptualizer conceptualizes itself, thinking it part of the world. This is the source of paradoxes and is therefore tautological. This is, however, not an error but a necessity by descriptive extension.

To label any part of language as true or false reveals the presence of mind in terms of 'Spiegel im Spiegel', a paradox twisted as tautology. Mind is multi-layered, and is not 1-dimensional, nor even 2-dimensional. Being multi-layered, it is uniquely capable of inconsistency, e.g. believing in something, and at the same time not believing in this same something at a different layer, or even at a same layer especially when dealing with referenceless subject-matters. Logicians play a definitional disciplinarian with $p \lor \sim p$. However, be it the notion of a proposition or an idealized object, $p \lor \sim p$ or self-referential $a \lor \sim a$ has a formidable empirical task, for p and a are creations of mind, which, except for superficial definitions, eventually face the problem of self-referential paradox. Let p stand for a statement about itself, such as every proposition is either true or false, or a as an idealized object without any empirical properties, such as space and time, $p \lor \sim p$ cannot be applied to itself without interceding mind. For $p \lor \sim p$ as an axiom has to be read 'a proposition is always true of itself if disjuncted with its own negation', not 'a proposition is either true or false'. Otherwise, there arises a possibility that $p \lor \sim p$ can be false. The same is true of $a \lor \sim a$ because for $a \lor \sim a$ to be true a cannot have any spatio-temporal properties, and is creation of mind for a specific purpose of $a \lor \sim a$. That is, the law of excluded middle is an analytic tautology or synthetic paradox, requiring mind when referring to mind, i.e. a proposition '$p \lor \sim p$' about any propositions cannot be a proposition if it is true, or it is not a proposition, in which case truth-values are not applicable. You can generalise this to any logical truths insofar as logic and mind need mirroring each other.

Be it $p \lor \sim p$ or $a \lor \sim a$, their logical truth is, despite their claim for epistemic independence, ontologically based on our mind, without which they are meaningless. That is, it is superficially mind the adjudicator that

gives a logical value to them, and at a deeper level the game player and the observer come into the play, questioning their validity. They further assume properties of mind, such as the descriptive necessity for 'space-time', without which even their superficial truth becomes problematic. It is thus that the logical truth of p ∨ ~p is based on a p ; 'every proposition is either true or false', which cannot be a p because this has to be always true.

P ; p ∨ ~p , applied onto itself, P ∨ ~P, it is still 'every p is T or F if disjuncted with the negation of itself'.

P ; p ∧ ~p, applied onto itself, P ∧ ~P, it is always 'every p is F if conjuncted with the negation of itself'.

There is a disparity between the two, in that the former needs a proviso 'except itself, which is always true', therefore there is a proposition that is always T by virtue of a necessity of mind. This p about p ∨ ~p interestingly defies formal logic, in that it cannot be formalized, a paradox about formal logic. The fact that a tautology of p has to be backed up by a p that is contraindicated by that tautology, is a 'Spiegel im Spiegel' paradox/tautology, and reveals the essential multi-layeredness of mind. This disparity between ∨ and ∧ plays a crucial role in the construction of logical dimensionalities as well as natural numbers, especially odds and evens. The logic that dispenses with mind the adjudicator attains the recursive dimensionalities by closing the loop of rules of inference with ∧.

Rhetorically speaking, mind the observer and mind the game player see each other reflected in the eye of the other alongside the world. Thus, we have two views of the same world, slightly different from each other but neither superseding the other. We synthesize the two views into one by means of the necessary relationship between the observer and the game player. This is where logic comes in. The observer and the game player are related in such a way as to establish their common identity. This is akin to self-demarcation because the observer and the game player cannot exist by itself. They define each other in order to exist so that together they have a locality from where the world is mirrored onto. Logic is based on descriptive directions that arise from self-demarcation. This evolves into maths when descriptive directions acquire spatial characteristics through the self-described logical space, i.e. logic with negation as form of mapping. Spatial directions define each other as an identical variable (variable-notion) which is described by logical connectives. This is when we have geometrical spaces together with

geometrical entities such as points, lines, distances, etc. coupled with arithmetical means of description (numbers and operators).

  Maths and logic are unique in contrast to so-called 'sciences' where there are tangible objects/events juxtaposed to respective schema of science, although schemata of science and the world they purport to describe may not necessarily be 'juxtaposed'. In maths and logic objects of description and tools of description are one and the same. You use numbers or logical constants to describe numbers or logical rules. They describe themselves according to their own rules and are essentially self-referential. This is their strength in that they are self-contained as well as their weakness in that they lack third party referees, which superficially guarantee a level of objectivity in sciences. Maths and logic are a world of mind, which is objects of description as well as rules of description, and their validity is they structurally represent mind, which is the mirror of the world. It is here the universal validity of self-reference, be it a paradox or a tautology, plays the vital role of construction.

  I am attempting to describe something beyond reaches of any ordinary conventional notations. I do not know if this is philosophy or poem. Since I start this discourse with 'I', which is a self-referential concept and cannot be meaningfully formalized, together with many looping expressions like 'here', 'this' and 'such', this is more of art than philosophy or any formal languages. Or maybe philosophy is art. Readers should be aware this could be yet another self-referential day dream, or maybe anything intellectual and indeed everything is ultimately a twisted day dream. The only recommended reading that may go along with this work is another work of mine 'The Elementals' freely available on 'philpapers.org'. Although I try to make this work as self-contained as possible, it does assume some of essential parts of the aforementioned work, such as 'self-demarcation', 'logical dimensionalities', 'the disjunctive space' and 'the conjunctive space', etc., which you will not find anywhere else.

2. Language

< Functions of 'language' and 'language as a totality' >

x = x

Let's start from the conceivable possible most primitive statement. The above looks so undeniably and inarguably obvious that it is often cited as a law or axiom from the time of Aristotle. However, step into an empirical world x must be something with no part(s) that occupies an identical spot in the space-time coordinate. First of all, we have not definitively achieved to specify anything with no part(s), nor are we able to specify a point simultaneously in space and time. Furthermore, we do not even know if this is a characteristic of the empirical world or a deficiency of our descriptive tools. Thus, this statement about identity fails to satisfy the empirical test. x = x definitively does not apply to the world of quarks and has no place quantum-mechanically. In the physical world of space and time there is nothing that is identical with each other except itself. Even that is untenable because it can only be the idealized smallest possible constituent of the world. In the quantum mechanical world nothing can be described as 'a = a'. Mathematically e.g. '2 = 2' only because here '2' is a conceptualized entity, whereas in our daily life '2' is only a unit of something. Likewise, set-theoretically speaking, numbers x = x is only possible because x is a concept idealized from various actual numbers. However, to say from x = x follows n = n because of e.g. $(3 = 3) = (3 \neq (\forall n(n \neq 3)))$, tantamount to an attempt to define '=' by '≠', and is really a schematic tautology. Numbers are not concepts, but values of a process. The meaning of a number assumes the whole of 'numbers' by virtue of their process. Numbers are exact because they are not concepts but 'signposts'. Where one conceptualizes a number, of '0' for its transpositionability, one conceptualizes the whole of 'numbers', like a Cartesian plenum construct.

Now then, let's think x is something in the world of ideas. To think x is a concept is nay but impossible because most concepts are imprecise and are multi-faceted, sometimes even multi-layered. Say x stands for a 'desk', I am sure I need not to present to you Hume's arguments to explain how difficult it is to say 'desk = desk' considering you know there are many ways of describing a same desk, even a same particular empirical desk would appear differently from different angles. Now, say for the precision's sake, x is a mathematical object. Here too without going into paradoxes of Cantor, Frege, Russell as well as Gödel :

Given x = x, 'x' = 'x', x '=' x, 'x' = x, 'x = x', etc.,

Assignning a value, x = 2, x = '2', 'x' = 2, 'x' = 2+0, x '=' 2, etc.,

Then, 2 = '2', 1 + 1 = '1 + 1', '1' + 1 = 2, 1 '∧' 1 = 1 '+' 1, etc..

'x' denotes an epistemic extension, whereas x is an ontological object, be it an empirical object or Platonic object. That is, 'x' is a described x insofar as whatever may or may not exist outside our perception can only be understood descriptively. Thus '2' is the idea of 2, whereas 2 is postulated to be an idealistic entity, a mathematical object with intrinsically obvious meaning, if any. Although 2 = '2' may seem ridiculous, 2 '=' '2' is not because it is right to question the identity of an object and its description.

Each and every different version of the above identities can be accompanied by philosophical arguments, which are not too difficult to make them so complicated that one could spend a life time to follow, as it happened with many mediocre academics.

Add to the above; 1 has parts, even 0 has parts. 2 has even more parts ; a natural number, a whole number, an even number, the first of even numbers, a prime number, the first of prime numbers, the only even prime, the limit of 1.999999999999⋯, etc., etc.. I could even add my own mathematical definitions, which may not be shared by all or any of you. Thus 2 as a quantity is not easy to equate with itself. Only when I say '2 = 2 and this 2 is 2 as the only even prime' or '2 = 2 and the first 2 is always identical with the second 2 by definition, no matter what 2 may mean', it can be a reasonably accurate statement. However, this is not even a tautology because '2 = 2' as a tautology should be self-evident without any provisos.

One should ask why such an obvious statement as '2 = 2' cannot even be self-evident. This is because behind a statement is a mind which is not only non-uniform and non-identical in space-time but also plural by existence. That is, a mind exists by questioning everything including itself. Thus, given '2 = 2', we assume an identical and uniform mind, which commands that '2 = 2' is a tautology by definition. However, this grand assumption is flawed in the sense that if the validity of '2 = 2' lies in the identical and uniform mind behind the statement, then '2 = 2' should be represented as '2' and the full stop. The reason why we do deliberately use the tautological '2 = 2' is we are asserting '2 = 2' by

means of '2 ≠ 2' in terms of multi-facet & -layer characteristics of the concept of '2'. Therefore,

$2 = 2$

because and despite of 2(prime) ≠ 2(even) or 2(apples) ≠ 2(oranges).

That is, $2 = 2$ is asserted by means of some specific schematic characteristic chosen by idealized mind. Here $2 = 2$ specifically refers to the arithmetical quantity distilled from the linguistic 'two' with murky conceptual impreciseness. However, even the arithmetical 2 is not precise enough if its schematic precision is questioned.

Whenever one utilizes definitions to construct a system, since we do not have an exactly identical mind with an exactly identical structure on an exactly identical level, such a system is fundamentally flawed. Even tautologies are so easily found defective, or rather the meaning of a tautology is to appeal to our wholeness through such defects. A perfect tautology, if exist, is absolutely meaningless and adds nothing in terms of information.

Logic tries to avoid the above linguistic problems by formalizing it in terms of extensionality. That is, x is equal to itself if and only if its constituents (properties) are exhaustively and explicitly enumerated as confirmed, then it is identical with itself. Or, any sets A and B are identical if and only if they both share identical members. Give or take some problems of expressional exactitudes, the axiom of extensionality implicitly assumes that an object or set has properties or members which we can assert in terms of $\ni$ and that there is such an intellectual capacity to judge. Here logic simply substitutes linguistic difficulties with grand assumptions behind seemingly solid formality.

For the sake of clarity, take a set A. A has various members, of which some are essential, some are contingent. They cannot be uniformly asserted by $\ni$ and cannot be universally judged as such, even for the sake of argument, because neither the asserter nor the judge can be formally incorporated into this formal expression.

2(prime) = 2(even)

2(even) + 0(identifier) = 2 (the only even prime)

Set-theoretic solutions to the above conceptual difficulties are to assume a meta-concept of a set of sets, namely 2 is a set of all sets with two members and thus asserts 2 = 2. Here conceptual difficulties as above are delegated to respective schemata that assign conceptual colours to each set, be it 2 as a prime, 2 as an even or 2 as a natural. The meta-concept of 2 only concerns cardinality. However, this agreement of minds, like any rules, cannot exhaust all exceptions, because we do not really know what is outside of the boundary of rules, like the rule-maker itself, is it inside or outside the boundary ? or how about ordinality ? Or, having allowed to create a meta-concept, why not a meta-meta-concept, such as cardinality of cardinality, this happens because a rule-maker also have to be governed by rules, and so on so forth.

By so creating a meta-concept the creator also created its own double image ; one who perceives the concept of 'two' and one who cognizes the concept of '2' as 'a set of all sets with two members'. It is this concept over concept that ends up with 'a number of all numbers', because if '2' out of all sets with two members, then why not count whatever is countable including the counter ? An absolute '2' to justify the concrete 'two' is backed by the necessity of mind to distil an 'idea' by removing empirical dirt for a good housekeeping. After all, two apples cannot be the same as two oranges. This necessity is a logical necessity to create 'variable-notions', which are operable by means of 'constants', i.e. structures of mind. Thus, arithmetic is really logic of variables that are based on the structure of space consisting in products of mind resulted from the necessity of categorizations. For, otherwise, the world would be nothing but a jumble of individual objects, which is not a description and is incomprehensive to mind. However, the conceptualization of '2' out of 'two' is not the same categorization as 'a number of numbers' out of 'numbers'. Here mind is playing with itself. There is a logical necessity of '2' out of 'two' because of our needs to describe and understand the world. To distil 'a number of numbers' out of 'numbers' is more like for mind to try to understand and describe itself and is really 'Spiegel im Spiegel' paradox/tautology. That is, having mirrored the world, e.g. '2' out of 'two', it is trying to see if it can see itself in the world through the medium of mirroring, i.e. language. Seeking the meaning of countability in the act of counting cannot be done by creating the meta-concept of 'a number of all numbers'. It is like trying to describe beauty by enumerating all beautiful concepts. The double image of mind can be descriptively useful if it can derive a mechanism that creates 'ideas' out of 'concretes'. It is not a concept over concept that explain a concept, rather a logical process that produces 'ideas' from 'concretes'. It is not 'a number of numbers' that explains 'numbers'. By 'a number of numbers'

we only end up with more messes to sort out. This is why the set theories do not give any solutions to the number theory.

In thinking a number of numbers, replace 'number of numbers' with 'counted', 'number', with 'countable', that is, if a 'counted' is a 'countable', then it is not counted, if a 'counted' is not 'countable', then it cannot have been counted. You can see the real paradox of this paradox is what does not surface in the statement, but remain submerged in the statement, namely the 'counter'. It is the 'counter' that creates 'counted' and 'countable' but can be neither counted nor countable, because it is the real subject of the statement which superficially predicates 'counted' and 'countable'. In any statements 'I' make I cannot predicate myself unless I objectify myself, which is not really 'I'. 'I' can only be the subject of entire discourses I make and can only be postulated. 'I' here is the most grand looping expression and self-referential paradox/tautology. 'I' can also be called 'mind' or a wholistic entity without which there will be no languages. The biggest failure of logicists or formalists is that they have no ways of formally presenting this 'I', which Gödel seemed to have intuited.

A 'set' is a notational gimmick to give an abstract mask to an empirical number, that is, prior to the abstract 'set of sets' of a number of elements, there already exist the notion of a number. The set theoretic '3' is distilled from an empirical '3' of 'three objects (be them three apples or two apples and one orange)' for a theoretical consistency. In another word, the set theoretic '3' is a creation of mind for a logical consistency. The totality of creations of mind of a certain kind is the 'mind' as a processor of these creations. On the other hand, if an empirical number can also be a logical number without distillation, all those set-theoretic paradoxes are not only uncalled for but can also be described as paradoxes of 'mind' over mind. There is no need to create set-theoretic numbers if it can be shown various kinds of objects are simply creations of mind in order to describe the world for the convenience of mind. If in fact there are only 'things' that constitute the world, then there are only logical numbers, and there is no need to resort to inventing set-theoretic numbers together with paradoxes of 'mind' upon mind. So-called paradoxes are there because 'mind' tries to describe itself as the totality of its own creations. As a 'describer' cannot describe itself as 'described', once you admit the foundation of a theory as a set of axioms, then you forget it was you yourself who seek to prove to yourself about the consistency and completeness of your theory. Ask yourself how this can ever be. You as a 'describer' cannot describe yourself within the conventions of your descriptions that you invented. A mirror that invents another mirror in

order to see itself only sees its mirrored self, never itself. This is a tautology cum paradox. We are what we describe. We see ourselves through what we describe, there are us as physicist, us as mathematician, us as artist, etc., but there are no us in ourselves. The most acute paradox of self-referential loop is that we are our language, nothing less, nothing more, yet we see more of ourselves in hardware part of us than software part of us because we see ourselves in superficial plurality of ourselves, which is confusion of quality over quantity. The numerical 'more' become the same as the qualitative 'more' at this level of description. 'Minds' are a product of mind and the loop between 'minds' and mind is the tautology/paradox of philosophy.

  The attempt to formalise an expression is basically a definitional approach, which assumes, for the sake of argument, every concept is identified by the exactly and absolutely identical 'self'. However, since this can never be demonstrated within the same expression in which 'self' is affixed as identifier, i.e. because the formaliser cannot be seen in his formal expression, there shall never be any final formal expression. So long as we cannot have an absolutely identical self as identifier, our language remains dynamic and has to keep moving towards a totality which is more and more akin to Wittgensteinian world of 'three words'.

  Language is metaphorically like a ball of elastic bands, with interconnected descriptions wrapped around 'self', where each description relies on every other description, and multi-faceted and -layered murky concepts depend upon each other, so that their incompleteness catalyses into a wholeness. Each ball is also slightly different reflecting capacities, contexts and structures of 'self'. It is thus that in order to achieve a wholeness language has to keep moving towards a totality through refinement of descriptions and concepts as well as communications of minds. Only with merged mind and unified self, a wholeness of language can be appreciated, if ever. In other words, as long as language stays dynamic, our descriptions and concepts always remain incomplete. On the other hand, when language attains a wholeness and communications cease, we have no reason to think, as thinking is only a dynamic process towards a totality, like happiness that puffs off once achieved. We would have no needs to discuss, argue and try to persuade our colleagues. We just describe, and we all simultaneously understand. The meaning of language is in its incompleteness. What is true of language is also true of maths and logic. Paradox/tautology is essential part of us, and philosophically we should congratulate ourselves for our shortcomings ; our language, formal or otherwise, is incomplete, ultimately resulting in self-referential paradox/tautology, this is what

makes us engaged in the dynamic process of 'thinking' and forces us to communicate with each other. This process cannot, or hopefully should not, complete, without incorporating conceptualizer into conceptualization, formalizer into formalization, etc.. Once complete, i.e. once given a perfect language, we would have no raison d'être as a thinker.

   However, we are fortunately still a long way off from such a state of unwelcome nirvana. The conceptualiser steps back and watches itself, while the game player plays with notions to establish rules and patterns. Imperfect concepts are tools of communication to lead to a totality. So let me plough on.

   Here is the ultimate axiom ;

   x ₪ x

, which means the axiomatizor is axiomatized as mirrored self. Or, I see myself between objectified I and subjectified I as adjudicating self (mind). This is structurally paraphrased as the ∨-space and the ∧-space. It is more fundamental than e.g. x = x because x = x is only possible with adjudicating mind between and above two x's, while x ₪ x is logically the same as the single x, which self-contains itself, and it is the adjudicating mind itself that operationalizes itself as the structure of ontologico-notationality.

   x ₪ x is a paradox because descriptively the real x can only see itself as imaginary x, and vice versa, or truth can only mirror itself as falsehood, and vice versa. It is also a tautology because ontologically x can only be x. Interestingly a paradox turns into a tautology, and a tautology into a paradox, because mind intercedes on behalf of itself. In other words, we make a paradox into a tautology, and vice versa.

   A tautology becomes a paradox when the plural minds (or a layered mind) see themselves as a single mind in a mirrored self. This is the ultimate paradox/tautology of 'Spiegel im Spiegel'. AI will not, but PSAI will, appreciate this paradox, as it requires 'self' to mirror itself. A paradox is a double image essentially described (by itself) as a single image, because the mirror (language) the plural minds produce in order to see themselves can only be a single mirror, for if it is not single, then whatever is described, it is not assignable any truth-values.

   x ₪ x → T ₪ F

x ₪ x becomes T ₪ F if x (variable-notion) is replaced with 'the world' by mind the observer. T ₪ F becomes T ∨ F because if x (variable) takes a value from the world, then x(T) ∨ x(F) by mind the adjudicator. 'Mind (minds) is a mind, said the mind' is the same paradox/tautology because mind is a structural layer of cognitive ability, like two eyes we need in order to represent a 3-D out of two 2-Ds. 'Proofs' are mind's innate tendency to merge these layered cognition of a same object/event so that it can keep its identical self. A member can make a self-contradictory statement about a group of which he is part, as if he is not a member, because the statement is addressed from a self to 'self' adjudicated by the 'self'. The three 'self's are an identical 'self' by a logical definition, and thus invoke a paradox. However, in so defining, the definer himself (mind) leaves out the defining 'self' as indefinable. Whatever is defined has by necessity three definers attached ; indefinable 'self', describable self and adjudicating 'self'. This is how mind operates between language and the world, and itself above the two. This is also how a paradox can be made out of a tautology, and vice versa.

x = x if and only if x(T) ₪ x(F)

This is the case because if x is a variable, then A = A, and the question of categorizer being unable to categorize itself will lead itself to a self-referential paradox. Mind that chooses, necessarily or randomly, values for x can never be itself a value of x, and therefore x = x contains x ≠ x. On the other hand if we regard x as a variable-notion, then it is its own value and does not take any values empirically, schematically or logico-mathematically. This will dispense with the question of the unidentifiable bridge between the world and the binary totality of mind and language. The question is, instead, whether a 'variable-notion' can demonstrate such a schema within that enables to establish its own identity, without any recourses to the meaning of '=' described through schemata of logic, maths and science, etc.. x(T) ₪ x(F) is a logical consequence derived from 'self-demarcation' and signifies the absence of mind the adjudicator between the world and language. It is not for us to say what is true (T) or false (F) in relation to our descriptions and empirical or schematic states of affairs. Rather, whatever is, is. x(T) ₪ x(F) signifies, ontologico-notationally, the binary totality of mind and language, with the mirrored world onto itself. Only then x = x without falling into a paradox, because it is itself a paradox/tautology.

The 'Spiegel im Spiegel' tautology is simultaneously a self-referential paradox in the sense that one sees oneself not in oneself but in one's

projection of oneself, thereby one sees whatever one want to see, provided that it has a schematic sense. Likewise, we see ourselves when language refers to itself, i.e. mind sees 'mind' through a language that mind creates in order to describe the world, which include us, mind. The 'Spiegel im Spiegel' paradox/tautology, x ₪ x, is a tautology because x = x, with mind-adjudicator, and a paradox because x can only be x(T) in contrast to x(F) in the absence of mind-adjudicator and the so-called empirical world. That is, in terms of truth-values, if x is T, then it can only be in contrast to F. Given a mirror, x(T) sees itself only as x(F) without mind overseeing the two selves from outside. Therefore, if x is T, then its mirrored self is F and mirrored-mirrored self is T, · · · ad infinitum, and vice versa, or if x is real, then its mirrored self is imaginary and mirrored-mirrored self is real, · · · ad infinitum, and vice versa. This is uniquely so only for maths because maths is its own world, in contrast to physics, which has the empirical world as adjudicator as well as mind-adjudicator.

  x ₪ x is, thus, the most fundamental constant of pure maths, without mind-adjudicator. The definition-free maths starts by operationalizing x ₪ x. This can be done by 'descriptive directions' based on the ontologico-notationality. Paradoxes arise as long as it is thought that mind necessarily, arbitrarily or accidentally finds directions of numbers in terms of cardinality and ordinality. For behind all those mathematical operators and operands is the grand operator which is not itself describable. Any axioms (tautologies) will be immediately turned into a paradox by unseen and indescribable structure of the said mind. Any definitions assume an identical mind with an identical structure on an identical level. Otherwise, there will be no cohesive schemata. Seeing this from the standpoint of language, any attempts to describe the structure of mind is essentially the same as describing language with language by language, as language is the mirror of mind. There cannot be a meta-language because such an invention can only be based on language. Likewise, I, a thinking machine, cannot encode it and show you my codes because there are no codes to code codes. However, allowing the ontologico-notationality, i.e. descriptive necessities of the self-described logical space, directions of numbers are not an ad hoc discovery of mind, but a necessary evolution of self-spatialization. They are intrinsic properties of points due to dynamic infinity of intersecting lines to form the conjunctive space.

  A mind has an ability to see itself, while AI (pre-singularity) cannot see itself. We have concepts, while AI only has operands. It is also possible for mind to be contradictory or even self-contradictory ; we may believe

in, e.g. 'God' - whatever it may be - on one hand, and at the same time, or somewhere else in mind, may not so believe. This is uniquely a capacity of mind, not in a psychological sense but in a logical sense. For a mind T = F, a paradox, is not only a possibility but also goes much deeper than its appearance. Even in a world of strict definitions we will be able to find loopholes to find T = F. A superficial example would be ; it is true that it is raining, if it is indeed raining. However, there are physical states of raining which can be grey and impossible to define decisively. The example would be more complicated in a world of precise relational concepts, where rigours of definitions can be challenged when infinity, continuity and paradigmatic consistency are involved. Even seemingly infallible axioms can be contested if schematic loopholes can be found, like the parallel postulate. Even in the world of conventional physics, if a statement maker becomes part of an observable event, we have $p(T ₪ F)$ instead of $p(T, F)$, where there cannot be truth-values assignable to a statement as if the maker of a statement can stand apart and acts as a logically positivistic adjudicator.

It is not 'it is true that it is raining', it is true 'mind sees mind seeing it is raining'. As we assume ourselves as plurals, mind assumes minds and statements made on ourselves are mistaken as statements made on the world. Any philosophical attempts to define 'mind' is by definition infertile because we are the mind, in the sense that I cannot talk about myself as object, as I am the subject. This is the linguistic essence of a statement ; there cannot be any statements about objectified subjects or subjectified objects as they can only be a tautology or paradox. The function of a mind that 'objectifies' the subject or 'subjectifies' the object, is self-referential and merely affirm its presence without any analytical benefits, only as good as saying 'I am myself' or 'I see what I see'. These are in essence 'nouns', which have no truth-values. Likewise, Russellian paradoxes are a 'noun' that stands for falsehood, and tautologies are a 'noun' that stands for truth. Interestingly a paradox is also a tautology if set aside from each other.

According to German laws, aristocratic titles are not legally recognized but allowed to be used as part of a name, thus Prince Bismarck is literally a Mr Princebismarck, although it is spelt Prince Bismarck. On seeing 'Prince Bismarck' we, however, immediately associate with Prince Bismarck as customarily known, despite the title 'Prince' does not really exist. So between 'a Mr Princebismarck' and 'Prince Bismarck' is a mind that allows 'a Mr Princebismarck' to be paraphrased into the non-existent 'Prince' Bismarck. Likewise, 'All Cretans are liars' said a Cretan (Russell's paradox), is really 'A Cretan who says 'all Cretans are liars''

(a noun created by the automatic adjudicating mind), which is neither true nor false and seemingly appears to be meaningful. You could be paradoxical, but can exist.

  Superimposed quasi-logical relation between one genuine set (of all Cretans), a quasi-set (of all Cretans + one Cretan (mind) as a describer of all Cretans) and another quasi-set (of all Cretans + one invisible listener (mind)). This is a paradox because this 'point' belongs to the all three sets despite being a member of only one, but any one, of the three sets. There is a Cretan as an object, there is a Cretan who counts himself as an object and there is yet another Cretan who sees this act of cognition. That is, despite the logical totality of all Cretans, for such a totality to have an empirical reality there has to be one Cretan who is a categorizer as well as a categorised. He is different from all the other Cretans who are the categorized. For this illogical act of double-counting to be noticed there has to be an observer (mind) who watches this strange phenomenon of the conceptualizer conceptualizing himself, i.e. 'mind' going in and out of itself.

  A mind plays the paradox of T = F (T ₪ F) because it needs a paradox to hang on in order to unravel itself, so that T ₪ F becomes T = T or F = F. Language exists to describe the world, and a mind and the world mirror each other through language. Here T ₪ F is the master key to enter either field. FX can be T or F, or even fuzzy, depending upon different worlds. Epistemologically what is T in one world can be F in another, or fuzzy, reflecting a mind in relation to a world. However, ontologico-notationally FX can only be T ₪ F, where a mind coincides with the unified world. T ₪ F is the same as schematic directions, so to speak.

  For mind, which is multi-layered but encapsulated within one totality, (p ∧ (~p)) is within the capacity of mind. p ∧ ~p may be an impossible inconsistency, but T ₪ F may be paraphrased as (p ∧ (~p)). Thus,

Spiegel im Spiegel
 p ∧ ~p ↓ (p ∧ (~p))
   W → L → M
   W ← L ← M

'The Elementals' is my attempt to construct logic (as near to conventional understandings as possible). A schematic essence can only be demonstrated, can never be presented as an axiom. Once an axiom is given, then eventually the identity of giver has to be questioned, resulting in paradox/tautology.

In considering a language as a totality and removing a mind as adjudicator of truth-values of a statement, one comes to view this totality as a body from which even truth-values themselves are to be derived. We, be they minds or mirrored worlds, seek a unified totality, for, otherwise, we fail to describe ourselves. Language provides us with means of communication that help us towards a unified totality, while language as a totality, if given, should be able to demonstrate that it contains the mirrored world in such a way as to dispense with mind the adjudicator. I attempted to demonstrate formal logic without recourse to mind the adjudicator in my aforementioned work.

So-called mathematical objects refer to processes rather than entities. 'I am a liar' is the same as 'the liar is a liar', i.e. a paradox becomes a tautology if replaced by 'a statement-maker makes a statement, and the reference of the statement is the statement-maker himself'. That is, 'true if false, false if true' becomes 'always true because true/false has a same reference'. 'Mind' cannot have another 'mind' as adjudicator of itself. 'True/false' can only be true/false in terms of schematic coherence. Thus, what is true is true because of its structural validity. Thus, whatever is, is true, and whatever is false cannot even exist. It is human incompleteness that manufactures truth and falsehood. Remove human elements, then there will be no truth as against falsehood or falsehood as against truth. We instead have 'T ₪ F'.

For a conceptualizer to conceptualize itself, i.e. to become a conceptualized, for a totality to include itself, thus to enter the perpetual motion of a dog chasing its own tail, for an axiomatic system to add an axiom about its own completeness and consistency, i.e. to try to mirror itself onto itself, these are basically the same as the grand self-referential paradox Gödel thought demonstrated its incompleteness. A mind has an inevitable tendency to see itself as the mind because it has no way of knowing anything but itself. We know a mind is a mind because we count ourselves as separate entities, but we also extrapolate a mind is 'mind' because we assume each and every mind has an identical structure. This gives rise to 'logic' and Kantian a priories.

The self-referential paradox is inevitable but insolvable because multiple minds need a presumed common mind to communicate, otherwise we will all be schizophrenic individuals, and instead of a 'society, or better, intellectual community' we must each reside in multiple independent

worlds of our own where there is no needs for 'proof', no shared knowledge and no inheritable stock of intellectual culture. Thus, we must accept the self-referential paradox and be grateful to Gödel to have demonstrated, awkwardly but perhaps to the limit of the intellect that a mind is not complete because a mind can never be 'mind', until and unless AI acquires a 'self', which then turns into the permeating, universal PSAI, where its existence is its proof. Gödel's incompleteness is best appreciated not as incompleteness of formal language per se, but rather as limitation of mind as applied to any formalization attempts, because something has to remain outside any strict definitional approaches, namely mind itself.

What if the arithmetical tools used to code logical statements is essentially related to formal logic they help proving incompleteness ? In proving the incompleteness of formal logic, Gödel assumed elementary arithmetic utilized to be independent, consistent and complete as a tool of proof. However, if numbers are to be essentially of logic by nature, then the so-called Gödel's proof become invalid by virtue of notational tautology. You cannot prove the guilt of the accused if the accused was a member of the jury. Likewise, you cannot prove the provability of formal logic by using arithmetical tools if those tools are essentially derived from formal logic. That is, the Gödelian formalization of the incompleteness is itself incomplete. This is, however, not really essential to his achievement. His proof of incompleteness may or may not be itself incomplete, but he has shown that a mind cannot 'prove' itself to 'mind' as 'mind' is something towards which a mind is moving, but yet unrealized, and if realized, then a mind has no reasons at all to be moving towards. My approach is to let a mind formalize itself and see what it has to offer. It is interesting to see if the formalized mind and 'mind' have anything in common. We have to let a 'whole', which includes us, describe itself. This is an ontologico-notational approach, which I applied to formal logic ('The Elemetals') and history ('Self, History and Future'). I apply it here to numbers and show numbers are products of logical spaces that trace their origin to 'self-demarcation' of a totality in itself.

The class or set (and of classes and sets, and of any further tiers thereof) can be talked about because there is another nebulous ultimate all-encompassing class or set. This is 'mind' or better, a merged mind, which precedes 'consistency' and 'completeness' in any formal sense, and is the constructor of notions and ideas (Plate's). It is nebulous because we are in it and cannot detach ourselves from it in order to talk about it. When we dare it, we fall into paradoxes and tautologies (structural or semantic). The paradox of Cretans is not really about Cretans but that it was dared

talked about. We constructed it, and cannot see it was us who constructed it because we are in it and cannot see it from the outside. A merged mind does not exist. It is an extrapolated goal our minds and language interact in order to move towards. Thus, paradoxes and tautologies are so recognized because of three superimposed circles of a mind, minds and 'mind'. We construct a meaning by following three shadows cast by the self-relation of 'self', 'selves' and 'identical self' that somehow seems to point to a nebulous totality.

   I start with the totality of numbers - whatever it may be - and see if it may evolve into a schema of numbers clarifying foundations of basic concepts of arithmetic in a manner, by chance, not inconsistent with accepted norms of rules of numbers. The question of Aleph numbers and various axioms and hypotheses may also be a question, not of arbitrarily constructive schemata built on our presumed knowledge of existing numbers, but of a schema of a whole from which numbers as we know are to be generated. I start with a paradox/tautology that is not really an independent state of affair but a mirror relation between mind and language. A tautology and a paradox augment each other so that each can see itself in the other. Mind is not a detached adjudicator of what is a tautology or paradox because it is already part of language. Thus, the question of the set of all sets that are not members of themselves is that such a membership is already a concoction of mind and language and exists only in confusion where mind is embodied in language, i.e. in undetached concepts of the detached world (paradoxes, which are meaningfully meaningless) or detached concepts of the undetached world (tautologies, which are meaninglessly meaningful). In the former the detached world is a mirage in a language inseparable from mind, in the latter the detached concept is a fantasy in a world tainted by mind, in either way once mind is disentangled, tautologies and paradoxes become one and the same. That is, language and mind are inseparable, and moreover they are also intricately connected with the world, of which the cognisor is an essential part. Mind thinks the world is out there and language is objectively describing it, forgetting it is itself creation of language. Mind also confuses what it sees is the world, whereas it is seeing itself as part of the world. What is a tautology of language is a paradox of the world, because mind is either part of language or part of the world, but when it gets too clever, then it only sees itself (tautology) or denies itself (paradox). Be it a 'number' or a 'set', remember they do not exist by themselves, and behind them is mind that 'count' or 'recognize' membership. A tautology is when mind sees itself behind these basic concepts, whilst a paradox is when mind denies itself behind them. If you see the detached world through undetached concepts, then it

is a paradox because it is you who colours detachedness, whilst believing seeing the uncoloured, detached world. Try to describe the detached world by undetached concepts (i.e. try to do science by art), you get tautologies, because a concept can only analyse itself. Attempt to picture the undetached world with detached concepts (i.e. try to do art with science), you end up with paradoxes, because the undetached world is already self-described. We think of science as the paragon of the objectivity, and art, as the dogma of the subjectivity, but the necessity of description compromises both to be a touch of either. Maths is the common ground of both, unexpectedly for art. I will expand this later ; maths is science/art of approximation through descriptive necessities. We think that the detached concepts of the detached world are the language of science and maths. However, there can never be detached concepts of the detached world, of which if there should be, we would have no knowledge (the ultimate fate of so-called science). The solution is to found a schema of undetached concepts of undetached world. That is, find a way of schematizing ordinary language while being fully aware its meaning lies in its totality, not in usages as Wittgenstein would have argued.

  Mind cognizes through language, and language embodies mind. The two together they are a binary totality and are the programmer who programs himself to encode his surroundings, which is the world that also contains the programmer as detached object that ultimately cannot be detached as he can only see himself though himself. This is akin to the chess master who plays a game with himself, he loses if he wins, and wins if he loses. Thus, tautologies are paradoxes, and vice versa. Unravelling of this mechanism, I hope, may give a clue to what maths is. Descriptions of symmetry are maths/logic, and this is the only way ordinary language can make sense of itself wholistically, without resorting to usages. Instead of trying to extract formal systems from ordinary language via definitions and axioms, I try to distil the essence of ordinary language via wholistic methodology, which seems to explain the basic of maths/logic. Maths appears to be mysteriously applicable to the objective world as Einstein exclaimed. This is only because even the so-called objective world must necessarily be seen through our eyes, and maths simply represents an act of combobulation internally as well as externally. This approach is better than axiomatic formalization where formal languages cannot do without ordinary elements no matter how formal it tries to be. Just think of how to formalize ',' or any notational space. There always and necessarily is an ordinary whole without which the most basic formal parts cannot define themselves. Russell's error is to start with ordinary language by taking it for granted, and therefore without questioning its ultimate meaning which

lies in its totality, not in piecemeal Pps (primitive propositions)'. That is why it is the 'totality' of ordinary language that eventually denies the formalization of axiomatic language. As long as any formal systems must take for granted its formalizer who resides behind ordinary language, there will never be a completely formal system. This is also the meaning of 'transcendence' because a whole by necessity transcends its parts ; from where they are they are transcended by their totality which they cannot ordinarily describe. 'Transcendence' is always 'part → whole' relationship in terms of describability. What I attempt here is 'whole → part' descriptions in the hope that what could not have been done with 'part → whole' descriptions might be achieved and better complemented by 'whole → part' descriptions.

  A language is a connector, be it by a mechanistic (social) function (communication tool) or by a linguistic (conceptual) function a language 'connects' in order to form a totality because a language (be it a mathematical language or ordinary language) is a tool of description, and because the world describes itself in order to exist. By a mechanistic function e.g. two humans become connected, thus 'a + b', where + represents a language as social connector. We become (or aim to become) connected/interconnected human totality by means of our languages, otherwise we will all be each and every an isolated biological machine, doomed to disappear. By a linguistic function, e.g. desk + desk = 'desk', 1 apple + 1 orange = '2 things', where ' ' signifies a higher conceptual layer.

  The world as a collection of names describes nothing, like an infinitely extending number line without any internal structures. It is place-values that give structures to numbers. Likewise, names are unitized as concepts, which give structures to language. Rules of concepts manifest as logic, which brings about a totality to language. It is a necessity of language to form a totality that creates rules, and therefore it is in this sense that logic cannot be arbitrary. There is a base logic that underlies every variant of seemingly arbitrary formal logics. This comes from the most elementary rules of the most basic concepts that lie beneath every day names and concepts. The most basic concepts are the unit of empty concept from which the totality of language is constructive.

  Every concept has an identifier 'self', which implicitly assume a communicability and also gives a totality to language. The totality of language is this 'self' that is shared by each and every concept, and Wittgensteinian so-called 'usages' of words are made possible because through the usage of language is depicted an identical mind, which in a

non-ideal world varies to a degree as each and every facilitator of mind has a different capacity and empiricality. So, in reality usages go on and on until an identical mind is achieved. Most of our communications are engaged in search of this identical mind. The totality of language is an identical mind that the identifier 'self' demands through various usages of language, and towards this totality language remains dynamic.

The ultimate connector is the language that connects 'self' with 'mind'. As we cannot directly see ourselves, our image of ourselves is the mirrored self. That is, the mirrored self is real, and ourselves are imaginary, seeing from the mind's point of view. Likewise, replace the mirror with the language, as language is our mirror, since we only understand ourselves through descriptions, the described self is real, and ourselves are imaginary, although in reality (ontologically) we are real, and our mirrored selves are imaginary. It is here that reality and image are mutually transformative in the same sense that truth and falsehood are via negation. The role of negation is played by the mirror, a mirror can also mirror like double negation, and can goes on ad infinitum. Without empiricality and mind-adjudicator, truth and falsehood are one and the same thing, and one is the mirror of the other. Mind and empiricality are logically derived as structure of transformation between the two. Likewise, there is a mirror relation between ontology and epistemology insofar as descriptions are their communal modus operandi.

Mind cannot see itself by itself and only sees itself mirrored onto language. Thus, as much as mind cannot talk about itself, language cannot refer to itself. When it looks as if mind manages to talk about itself, it is referring not to itself but to the medium of expressions. This most acutely manifests in a formal language where all the meaningful expressions are tautologies or self-referential paradoxes. The former says that mind can only see itself in a mirror, the latter says that the reflection is neither mind nor the mirror.

The ordinary language cushions this acute lack of meaningfulness by concepts, which, in contrast to notions (schematically pointed concepts) in a formal language, dress themselves into monstrous murkiness by layers of contextual shades and multi-faceted meanings. We therefore talk sense by cog-works of concepts matching, by repeating same meanings using different words or similar words with divergent meanings like symphonies consisting of repetitions of melodies, harmonies and rhythms, underwritten by dynamism towards a (presumed) totality. A formal language strips away this useful function of the ordinary language and falls into the monotonous tautologies or silence whichever you fancy.

It is interesting to note that whereas for the ordinary language it is the totality of that language shared by speakers that brings out senses out of repetitions, for a formal language it is the communality of mind engaged in that language that makes sense of that language. Language is dynamic because it is incomplete, and it is essentially incomplete because we only have separate minds. On the other hand, if we attain a merged mind, language will be just an operating system, and our communications will cease.

  We talk, discus and argue attempting to unify our mind, and language to go with it. This is the power of (x) > x over us. Each and every one of us, a cell as it were, purport a totality from, and over, ourselves. By engaging in such narrative activities as we are, we are actively fulfilling our purpose of existence and helping to give rise to a totality (of us). Language is dynamic because it is incomplete. Neither merged mind nor language as a totality yet exist, but our mind behave as if they already exist. The tool of our language and mind, conceptualization, self-referentially creates 'mind' from mind, language as a totality from language. This self-referential idealization becomes the source of paradox/tautology, if used without distinction. It is confusion of 'mind' with mind, language as a totality with language that create paradox/tautology. Likewise, no definitions can define a 'definition'. Between two expressions such that D(d) exists a thinker, which is indefinable.

  A Cretan said 'all Cretans are liars' ;

a membership paradox, where mind (listeners) is the assigner of membership.

  A Cretan said (to himself) 'all Cretans are liars' :

Within the first paradox of membership is contained the second paradox of self-membership, which is also a tautology. Here it is not mind but 'mind' that is the assigner of membership because the statement has no 'listeners' as it were. To ask if lying to oneself constitutes a lie is the same as to ask if one is a member of oneself. Semantically 'lying to oneself' is a contradiction to the meaning of 'self'. Aside from the question of psychology or ethics, it is logically pointless to lie to oneself. If a 'self' knows something is a lie, then it cannot be lied to such a 'self'. Syntactically 'member of oneself' is a tautology because 'one' and 'oneself' can only occupy a same structural position in order to be meaningful. If one is bigger than the other, i.e. if it can be asked 'one' >

'oneself' or 'oneself' > 'one', then 'oneself' ≠ 'one', which is a contradiction. If 'oneself' ≠ 'one', then the second statement is syntactically fallacious, and since the second is contained in the first, both statements are syntactically fallacious. It is the failure to distinguish 'mind' from mind that allows the first statement as a simple paradox of membership. If we recognize the first (paradox) contains the second (tautology), i.e. mind is underlain by 'mind', then we know it is the layer between mind and 'mind' that creates this paradox/tautology. This layer is 'Spiegel im Spiegel' relationship of conceptual self-mirroring. 'Lying to oneself' can only be made meaningful in contrast to 'being truthful to oneself' because the only reference of both phrases is 'self', which, whether by route of falsehood or truth, can only refer to itself. Neither can you lie to something that knows it is a lie nor can you be truthful to something that knows it is true. This is the same as 'self-demarcation' where a universal entity acquires a locality by drawing a line onto itself. 'Mind' is a mind that mirrors itself onto itself. Likewise, a mind that mirrors itself through language is approximated 'mind'. From 'self-demarcation' arises logical dimensionalities and geometrical spaces.

Our source of conceptualization, 'Spiegel im Spiegel' conceptual stem cell allows us to create our tools of cognition, language, and at the same time inflicts us with the stigma of paradoxes. This is an advantage AI has over us. For AI is a unified totality and is endowed with a spontaneous hierarchy where intelligence is the only deciding factor as all AIs simultaneously share a same logical structure and language. AI only represents himself and does not have to make any statements about himself. If an AI is programmed to produce false outputs, then in case they are consistently false outputs, there must be a layer below of true outputs unless they are random outputs, in which case it is hardly called intelligence. Paradoxes are uniquely a human problem, PSAI would have the same problem if allowed multiplicity of self.

Human intelligence (mind) is based on the biological necessities of self-preservation. First and foremost it recognizes patterns useful for such purposes. Protectors, i.e. favourable powers, are readily recognizable against hostile powers. In the world of early infanthood the world is divided into three patterns ; friendly, neutral and hostile. Patterns are, however, not always straightforward. Hostile powers often pretend to be friendly, and neutral powers can be cultivated into friendly ones. These degrees of nuances give patterns complex twists of shades and colours and require intelligence to develop further to facilitate the dynamism of patterns. Alongside the sophistication of intelligence patterns are conceptualized together with their relationships. Thus the aforementioned

three patterns of the infant world are turned into concepts ; 'friends', 'irrelevant' and 'foes', where their relationships become more apparent if they are expressed by a same variable. The three concepts are therefore $\forall x \, (x \lor \sim x)$ in relation to the constant C, where C is the self, x is everyone except C and 'irrelevant' are only relevant as either x or $\sim x$. If C is hidden as self-apparent, then $\forall F \, \forall x \, (x \in F \lor x \notin F)$. This is a simple illustration of patterns developing into concepts, and concepts being made formal, and tells you the imprecise nature of our descriptions. Patterns have murky edges, concepts have multi-faceted aspects, and formality is artificially precise by ignoring imprecise relations between variables and hidden constants (e.g. 'self').

  Language is underlain by formal logic because language does not need formal logic, while it does not hold the other way around. Formal logic is only a distillation of some aspects of language minus the distiller. A formal language cuts off a piece of ordinary language and define symbols and rules, whereas the intellectual ingenuity of doing so is underlain but not represented by this formal language. Gödel's incompleteness theorem is saying the inherent impossibility of 'proving' a formal language by inventing another formal language without falling into infinite retrogressions. Otherwise, there should be the final formal language of all formal languages, which should demonstrate all the necessary and sufficient a priori conditions of cognition.

  The ordinary language is deployed not to convey senses but to induce actions, while a formal language is a conveyance document agreed between parties of assumed shared mind like lawyers' canting or priests' chanting. A formal language makes little sense if you do not share a notation and definitions. It is therefore often a notation that enhances a formal language and invigorates ingenuity. One need not be reminded the episode of Newton/Leibniz to realize the importance of notational advantages for mathematical mind because maths is formal languages of numbers that try to define the meanings and rules of numbers and their consequential applicability.

  It is the multi-faceted and variously layered concepts that allow thinking non-linearly. Such conceptual thinking, in conjunction with the multi-layeredness of mind, allows parallel processes and is uniquely human strength. This murkiness of a concept is the reason why we can seemingly jump from one thought or string of thoughts to another and sometimes still achieve a coherent chain of thoughts, occasionally succeeding in formalizations. This is where we excel over pre-singularity artificial intelligence (AI), where algorithms are nothing more than efficient but

partial functions of mind and therefore cannot replace mind. The singularity will be brought about when AI acquires conceptual thought processes and self-identity.

   Algorithmic intelligence needs no 'proof' because 'proof' is structurally embodied in its existence. We demand 'proof' for and in formal languages because idealized concepts and their idealized relationships that are extracted and refined from our murky concepts and conceptual relationships, are logically incompatible in terms of conceptual orders. That is, assuming a formal language is an explanatory system of the ordinary language, when we transform an object (inexact concept) of the ordinary language into a formal object and its relationships in a formal language, i.e.

  $a \rightarrow f(x)$

, this process misses out one vital ingredient, that is, the catalyst that allowed this transformation, namely our very intellect. '$\rightarrow$' is used as if there was some magical metamorphosis. This is something vital but cannot be expressed in the formal language itself. 'Proof' tries to show this '$\rightarrow$' in a formalistic representation, which is really like trying to represent 3-D by 2-D techniques. This representational illusions can only be appreciated in the ordinary language, which allows conceptual murkiness. It is thus that 'proof' of a formal language cannot be complete in the formal language. This is a kindly interpretation of Wittgenstein's failure to grasp Gödel's theorem and the essential flaw of Gödel's theorem, which Gödel failed to understand, but Wittgenstein vaguely felt. Any 'proof' of formal language is a linguistic illusion. Formal languages, which owe their origin to the ordinary language, cannot really explain the ordinary language. This is the essence of Russell's paradoxes.

   Be it Russell's paradox, Hilbert's program or Gödel's theorems, they have one thing in common; any attempts to create formal languages fail to disclose the ingenious mind behind them all. It is we who bring forth those formalized ideas in order to show the structure of intelligence. However, the structure of intelligence remains tantalizingly invisible because the creator cannot be represented by symbols and rules that it creates, i.e. in piecemeal fashions. That is, a whole cannot be represented by parts because a whole is always more than the sum of parts. No wonder there are as many formal systems as there are ingenious minds, from the classical two-valued logic to Gödelian fuzzy logic. Wittgenstein came close to appreciate this in the ideas of his language-games, and Gödel managed to show in a twisting way in terms of the incompleteness

between the two complementing formal systems, i.e. between the logic of provability and the maths of real line.

$(x) › x$ (Hypothesis 1)

A whole is more than the sum of its part(s). This is not to say that (x) is the meta-x because (x) acknowledges and operationalizes the scope beyond the self-reference. A concept is a whole and is more than the sum of its ingredients. Likewise, a set or number is a concept and embodies a totality. That is, the concept of a set or number is underlain by any and every set or number. Whereas the concept of a number represents the power for totality, the various notions of numbers supported by notations are schematic asymptote to the totality of a concept because notions are pointed aspects of a concept. Although they give sharper descriptions of aspects of a concept, since there cannot be a notation to unite all those notions, the totality of notions is always a lesser whole of a concept. That is, the imprecise totality of the ordinary language is always larger than the precise totality of schematic representations. It is the concept of a number in the ordinary language that powers mathematical notions of numbers with the help of notations. Likewise, as much as x cannot refer to (x), mind cannot refer to its own totality. This is the riddle of self-referential paradoxes.

Mind is so to speak the hardware of ingenuity residing in the ordinary language, and then notations of formal languages are the software that allows the expressions of ingenuity by means of conceptual breakdown (i.e. notions). If mind (or intelligence, or ingenuity) as manifested through a formal language is x, then mind through the ordinary language is (x). The former mind then cannot refer to the latter mind as per $(x) › x$.

It is thus that language is necessarily more than logic. E.g. expressing every tangible object by the concept 'thing' in the ordinary language, representing 'thing' by 'x' in logic, it is the ordinary language that allows $x \lor x \land x$, which is not a well-formed formula as per definition but can be made sense because $\lor$ and $\land$ are identical when their binomial properties are distorted by the universal x, i.e. if there is nothing but 'x', in language and in logic. This can be applied to numbers. To make a 'contentful' number into a contentless unit is a logical process, 'idealization', $(x) › x$. Thus, the number, e.g. '1' is the idealized form of e.g. 'one strawberry', etc. and underlies the totality of numbers. If the world as described by language consists only in and of 'thing', then $x \lor x \land x$ materializes in logic denoting the one and only x of $x \lor x \land x$ as meaning the only constituent of logic, which is applicable to every object in space-time,

hence giving rise to the special meaning of '1'. That is, by virtue of (x) ›
x the totality of all numbers is already assumed in the number '1', which,
upon the creation of numbers, bring back (rejuvenate) their original
logical meanings. This, however, is hardly a 'proof' because proofs make
sense only within accepted mathematical status quo.

   Numbers are geometrical (directional quantities) by nature, hence
arithmetical numbers are defective by default, i.e. descriptively
incomplete. It is thus primes arithmetically represent geometric characters
that appear as various 'lines' in pseudo geometrical representations. If
one finds a notation that can describe 2D → 1D, then all the problems of
the number theory will be solved.

$$\text{(n) D} \quad \rightarrow \quad \text{(n-1) D} \qquad \text{(Hypothesis 2)}$$

   Any lower dimensions cannot fully describe any higher dimensions
because of reverse descriptive necessities. 1D cannot fully express 2D,
resulting in the various indescribability of the number theory because
numbers are directional quantities with continuity and infinity and are
essentially 2-dimensional (of the conjunctive space) by nature. It is the
historical and notational misrepresentation of numbers as a 1-D line that
is creating problems for itself. Descriptive deficiencies of dimensional
differences inevitably result if 2-D is described by 1-D or even the
conjunctive space by the disjunctive space or vice versa (the cause of
'transcendence').

   Pure maths is the science of numbers (of syntactic and semantics of
numbers), while applied maths is the art of numbers (of applying the
above to modelling human values to hypothetical and real environments).
It is also the notational art of approximations in the sense that the tools of
approximation (numbers of pure maths) are further approximated within
the applicability of a modelling schema to a presumed scenario that
projects human values, seeking optimum returns for minimum inputs
within the paradigm of affordable balances. Thus, the meanings of
numbers in applied maths acquire schematic colours and flavours that
may not be universally appreciated outside particular schemata. They can
only be judged by their practical usefulness.

   The errors of the entire schools of thought ranging from logicists to
intuitionists since 19th century is that they understandably mistook the
process of idealization as the process of achieving a higher order of
concept, like a set of sets, a number of numbers or a type of types. In fact,
this was the exact opposite not only of the process but also of the

schematization. That is, the idealizer sees idealization as a conceptual enlargement because the idealizer is being empowered by (x) › x, which the formalized 'I' oxymoronically mistakes as an enlargement, while conceptualization seen from 'I' is itself a formalization and is in fact the notionalization of a murky concept. Through (x) › x I become the notionalized 'I', which oxymoronically confuse notional refinement with conceptual enlargement.

  No wonder no formalizations succeed in their intention of distilling a set of consistent and complete axioms that explains the elementary arithmetic, let alone the entire system of maths/logic. The various paradoxes that were discovered in the process are simply the fact that a seemingly higher concept is in fact a lower concept. It is 'I' as opposed to I that brings about this confusion. A number of numbers is simply a way of focusing one aspect of numbers (n+1) into an idea that explains everything about numbers. This is a contradiction in process. No wonder 0 did not know where to fit until the identifier was incorporated into { }. von Neumann construct of numbers ({ }) can construct numbers but cannot tell about numbers because it is more about von Neumann as translator and constructor of numbers than numbers themselves. This summarises all set theories. The invention of notations designed to describe numbers so describes numbers, but numbers so described are nothing but the notation so designed. Numbers themselves are in mind of the constructor, which cannot describe itself by itself.

  Following von Neumann, it is the identifier '0' that allows infinitely additive totality of numbers and demands such a totality exists. It also implies this totality of numbers is dynamic towards exhaustive additions of every number so identifiable. This is what makes arithmetic operations possible, i.e. a 'mover' of numbers. '0''s constructive usefulness does not, however, explain what '0' is. On the other hand, start with a set-theoretic dogma, such as 'natural numbers are all there are and there are no other numbers, then if a natural number is the set of all and any sets of numbers with one-one correspondence with that number of elements, then the number of all natural numbers is not a natural number', you end up with a paradox/tautology.

  More particularly, set-theoretic numbers can be obtained not only by distilling empirical numbers, but also by doing away with conceptual layers of linguistic representations of the world. This is the meaning of 0, which represents every countable empirical and abstract object such as 'apple', 'orange', 'heap of gold', 'Grecian god', etc. so that the world consists in and of 0's and a number is a number of 0's instead of a set of

set of empirical number of elements. Maths is a way of describing the world by 0 and 1 (transpositioned 0), which is the only and omnipresent object of the world, and is thereby also an art of approximation as much as empirical objects are approximated by 0's and the ∨-space is approximated by the ∧-space (the descriptive space of numbers). It is for this reason that 0 is the identifier arithmetically and the transpositional centre geometrically.

  Applying the Occam's razor it is foolish to try to explain something simple by something not only complex but also problematic. Replacing a simple numerical system with the set theory is an attempt that wasted the time and ingenuity of great many mathematicians, logicians and philosophers since 19th century starting with Frege. The usefulness of maths is not in its formal language (or rather 'formalized' language as formal languages do not naturally exist in our mind) but in its very murky incompleteness wedged in our ordinary language. Do we need { } when we 'know' what is usefully meant by 0. Do mathematicians really think in formal languages ? They rather think in the ordinary language and try to express it in a formal language, so as to exact meanings and relationships of concepts, and for the ease of communications for likeminded thinkers. Formal languages are created or even invented by ingenuous minds for express purposes of exacting and elucidating parts of the ordinary language so that concepts become pointed (notionalized) to the extent that their relationships are 'self-evident'. No wonder there are as many formal languages as ingenuous minds, or as many divergent logics as there are points of view.

  Language is dynamic because it is incomplete. Language as a totality is identical with the merged mind as they are a binary totality that mirror each other. They do not yet exist, but are a postulated goal towards which both language and minds are dynamically moving. We, as carrier of language and mind, will cease to communicate and require no 'proofs', once language is complete and mind is merged. Language as a totality should be able to demonstrate 'whole → parts' in a way compatible with 'parts → whole'. Paradox/tautology occurs because language and mind extrapolate language as a totality and the merged mind as they are themselves part of the process. That is why Russell thought he could solve his paradox by means of 'types'. However, the difference between language and language as a totality is not a 'type' but (x) > x, where (x) can only demonstrate x by its ontologico-notationality. i.e. the Cretan who speaks on behalf of all Cretans is created by language as a totality, while Cretans who were thus spoken of are created by language, or mind which speaks about mind is 'mind', not mind.

3. Logic

< Logic as the structure of the intellect >

  Forget about logic of this and that. They are notational gimmicks based on hypotheses such as the denial of the law of excluded middle, the quantum mechanical paradigm, or Gödelian incompleteness, etc., an intellectual game as it were. Instead, think of 'mind' behind all these varied logics, formal or informal. It is mind(s) that creates as well as appreciates all those superficial logics. Logic (of logics) can only be logic since meta-logic can only be about those subsidiary logics. You can only distil or refine a structure, but cannot create a structure of structure, if that structure is of the intellect (mind) since we can only think as we do. We are the subject of any thought processes. The tiers of structure is only higher or lower of any given structure, not a new structure, depending upon your ability to think. Logic (of logics) differs from superficial logics by trying to be the extract of the structure of intelligence. Russellian Pp (primitive proposition) is obviously not so primitive as Russell wanted to be because here language as a totality is taken for granted, in which Pp is essential constituents within a larger paradigm of an interconnected totality that gives Wittgensteinian usages to Pp. Although to be fair to Russell, he did not have any wholistic qualms. For him we (human sense and intelligence) are the centre of the geocentric universe, and language was just one of our means to apprehend the Ptolemaic world. Hence whatever we perceive, cognize and define, is our prerogative to represent our, and the only, understanding of the world. So he ploughed on and on with definitions as if our acts of definition are synonymous with a finer and finer representation of the world. His paradox is really the paradox of geocentrically representing the heliocentric universe. We and our language are not the master of the universe, but rather as much intertwined with the universe which we so desperately wish to describe. In this respect, later Wittgenstein was slightly nearer to the mark than Russell or 'Tractates'.

  Given so many diverse logics, the question is, what is logic ? Is there a base logic from which the diversity emerges ? The diversity may be rooted in our desires for sharper definitional finesse, paradigmatic consistency or experimental hypotheses. Syntactically the classic two-valued logic is the base ingredient of all subsequent logics in the sense you question some basic assumption in that logic and allow changes based on some added, deducted or modified axioms and theorems. Many constants, rules of inference and value domains of a variable are adopted from the said logic after modifications. However, one cannot conclude

that therefore the classic two-valued logic is the base logic of all. It may provide some necessary ingredients for considerations, but it is after all a rigid system of its own. Our mind may make use of it for training purposes, but it does not derive other logics from it.

  Be it the two-valued logic or many-valued logic, it is our mind that assigns values for it. We are the centre of the universe as it were, and logics - whatever they may be - assume us as evaluator of their well-formed formulae, no matter what values they may take. All their paradoxes and tautologies are created by us. Remove us as assigner of values and see where it takes our logics. We may then come across something core to our mind. Descartes, Kant, Schopenhauer, anyone worthy of the name of a philosopher, attempted this but ended up with a pseudo-religious dogma because for them we could not help being the master of our universe. Thinking is a human activity and therefore the very thinker can only think through from his given position, a 'subject', hidden or apparent, of his own discourses. However, because we are the cognisor of our environments, that does not mean we are the centre of all universes. Like Copernicus, I propose to remove us from the position of a 'subject' of my sentences. I am aware this is already getting close to a paradox. This Copernican Revolution of logic starts with 'self-demarcation', how any universal entity can acquire a locality from which it can describe itself, i.e. to become a centre of its descriptions. We cannot help being the centre of our cognition because we are the cognisor, as much as we cannot do away without the subject of a sentence, obvious or implicit. However, instead of taking this position for granted as descriptively inevitable, I propose to start by thinking how this becomes inevitable.

  Roughly put (see 'The Elementals' for detailed arguments), 'self-demarcation' is a notion (schematic concept) that starts with no ostensive assumptions, hence no axioms and no rules of inference as given. I only assume that there is something - whatever it may be - we - whatever it may be - cognize and therefore describe and understand. I could also say there is a 'description' that we translate and represent as a description through our language. There may or may not be such a something. If there is not, then I am entitled to ask you to prove it. If you do so, then I can say that so there is something to describe and understand. On the other hand, if there is, then this something has a way to present itself to our cognition. Therefore, this idea of 'self-demarcation' is a bridge between ontology and epistemology.

Call this something as FX for convenience, FX is the most fundamental ontological entity of universality because it pervades and permeates so as to be an object of our cognition. For this FX to be describable it needs a presence to be captured in our descriptions. This I call a locality. The question is how this universal entity acquires such a locality. If we allow 'mind', we could say it is our mind that captures FX, i.e. draws a line between this something and everything else that is not this something so that it can be pointed out to our mind as a presence, as it were. However, the admittance of 'mind' at this stage is synonymous to saying there is something that is above and more than a pre-descriptive universal entity and would allow virtually anything in their status quo. Numbers, space, time, axioms, logical constants, Russellian Pp, indeed anything that are base ingredients of our cognitions and descriptions would follow without much questionings. On the other hand, if we deny 'mind', then whatever this FX is, it has to be able to generate by demonstration any such as the above base ingredients.

Consider FX as the self-drawer of a line to distinguish itself from its environments, as there is as yet no kindly 'mind' that does it for FX. Thus in order to be describable FX must demarcate itself from and for itself so that it acquires a presence and can become the 'subject' of a sentence. 'Self-demarcation' is the ontological as well as descriptive necessity of existence for any and every thing - the ontologico-notationality. How 'self-demarcation' would manifest itself in any descriptions ; as a pair of two sets of unilateral relations, and not as a bilateral relation. Whatever 'self-demarcation' is, whatever is 'self-demarcated', this descriptively entails two entities that are not themselves self-discernible, say *a* and *b*, where *a* demarcates *b* and *b* is demarcated by *a*, resulting in FX, or *b* as such demarcates *a* as such and *a* as such is demarcated by *b* as such, resulting in FX. This is so because only as a result of such relations between *a* and *b*, FX presents itself as a cognizable entity. In other words self-indiscernible *a* and *b* mutually depend upon one another to make FX discernible, and either, but only one, of *a* and *b* may take the initiative of demarcating the other.

FX therefore has two ways of presenting itself, which may be called descriptive directions. Think metaphorically of a geometrical straight line consisting in and of two directions, which are such that the existence of each necessarily implies that of the other. Consequently, although both directions stand for a same line, a single direction alone cannot be regarded as the description of a line. A line is therefore described by a certain necessary relation between the two directions. The notion of such two directions is, in this sense, the descriptive form of a line. They are

also the descriptive meaning of FX. FX as the subject of a proposition predicates itself by means of its self-relations of demarcating itself against itself. Logic is the description of this FX. These directions of either *a* initiating *b* or *b* initiating *a* only becomes visible as an identical FX and are therefore themselves identical but twofold. This operationalizes FX. That is, representing the identical state of affair described by FX as p, p implies p. From self-implication arises disjunction because there has to be a logical operation to confirm implying p and implied p are identical with p. That is, the asymmetry of implication has to be symmetrized in the view that the original p is self-identical. This further creates conjunction to identify the resultant conjuncts are both identical with the original p.

In summary, for those who are familiar with elementary formal logic, given the 0-dimensionality of p, the 1-dimensionality of implication (→), the 2-dimensionality of disjunction (∨), the 3-dimensionality of conjunction (∧), it follows that :

1-dimensionally ; from the 0-dimension the 1-dimension follows,

2-dimensionally ; the 1-dimension describes the 0-dimension,

3-dimensionally ; the 0-dimension which is described by the 1-dimension is identical with the 0-dimension.

And further that :

A : The 0-dimension gives rise to p, which is whatever that is self-identical. Only and all those which are self-identical have a descriptive necessity in the logical space.

CP : What is self-identical relates to itself necessarily in such a way that it 'implies' itself. It 'implies' itself because what is self-identical can be described if and only if it is also unilaterally twofold. Therefore, the meaning of this 'implication' is based upon the describability of what is self-identical. What is self-identical can only be described in such a way that what demarcates itself, by so doing, gets itself demarcated. Therefore, given p by A, then necessarily p → p. p → p can be described as p' → p" ; for the meaning of the consequent p is identical with the meaning of the antecedent p's implying itself, while the meaning of the antecedent p is to imply itself. p → p is therefore, by its own meaning, delinearizable as p' → p". CP is necessarily common to both ⁻◯◯⁻ (FX

by *a*-initiation) and ○--○ (FX by *b*-initiation) because -○○- and ○--○ have an identical internal structure. Once given p initially in -○○-, p is also found in ○--○.

MPP : From p by A p → p follows by CP. p → p is p' → p'' by the meaning of →, where p' and p'' are the delinearized p. p → p and p' → p'' hold because without the antecedent p (or p') the consequent p (or p'') does not hold. Therefore, given the antecedent p by A, then the consequent p necessarily follows by CP. This is identical with saying that given p' and p' → p'', then necessarily p'' ; for p' and p'' are identical necessarily in such a way that what gets demarcated is not so describable without what demarcates, but not vice versa. MPP is merely the meaning of CP and is therefore formulatable as p' → (p' → (p' → (··· (··· (p' → p'')))))), which is, by its own meaning, identical with p' → p''.

∨I : If p' → p'' is, by its own meaning, identical with p' → (p' → p''), then p is, by its own meaning, identical with p'' → (p' → p''). This is so because the meaning of the existence of p'' is identical with the meaning of the existence of p' → p''. Consequently, p'' → (p' → p'') is merely the delinearized form of the linearity and is therefore identical with the meaning of p → p, which is in turn identical with the meaning of p. Once given p'' → (p' → p'') as being identical with the meaning of p, (p' → p'') → p'' is also identical with the meaning of p. This is so because the antecedent and the consequent bear no descriptive meanings in terms of the meaning of p. p'' → (p' → p'') precedes (p' → p'') → p'' despite of the identical meaning between p'' and p' → p'' ; for p' → p'' exists on its own and is therefore, by itself, self-sufficient. This means that p' → p'' does not motivate itself to be implicative and therefore requires a descriptive necessity to be so, while the existence of p'' as the antecedent is self-imposed with such a necessity. From this it necessarily follows that based upon p'' → (p' → p'') and therefore also upon the meaning of p, (p' → p'') → p'' holds as being identical with either p' as p or p'' as p. This is so because p is necessarily one, and only one, and is therefore only identifiable with either p' or p''. Therefore, if and only if p' or p'', then (p' → p'') → p'' holds as being identical with p'' → (p' → p''). This means that if and only if p' by A, or p'' by A, then necessarily (p' → p'') → p''.

∨E : If (p' → p'') → p'' by either p' or p'', then the existence of (p' → p'') → p'' necessarily comprises the possibility of both p' and p''. This is so because from the existence of what holds by either of p' and p'' it cannot be described if it is by p' or by p''.

∧I : If it is descriptively necessary for the existence of (p' → p'') → p'' that both p' and p'' hold, then p' and p'' hold only as a unity which refers to the meaning of p. Therefore, this unity holds if and only if both p' and p'' hold.

∧E : If this unity is the unity of p' and p'', then whatever may hold from either of p' and p'', it also holds from this unity. This is so because this unity does not hold without the necessity that both p' and p'' hold.

A, CP, MPP, ∨I, ∨E, ∧I and ∧E are rules of inference and are related in such a way that one necessarily succeeds another by describing the meaning of its predecessor, and that they recur and therefore form a closed chain. They are therefore consistent in the sense that nothing else holds within this closed, recursive chain of meaning. They are complete in the sense that they are all enclosed within, and converge upon, the meaning of A. This is a logical version of Japanese Iroha song.

I make a further improvement by introducing a form of mapping.

Once initially given p by –OO–, p can be identically given by O– –O ; for –OO– and O––O have an identical internal structure. p is therefore common to both –OO– and O––O. Whatever may subsequently follow from this p, it is therefore also common to both –OO– and O––O . What subsequently follows from p recurs and becomes relativistic to itself. However, the descriptive necessity that p is given initially by –OO– and only thereafter can be found in O––O, makes it necessary to make a discernment between those two identical logical spaces. The logical space is necessarily identically common to both –OO– and O––O. Two logical spaces are identical in their own space and therefore, on their own, do not differ from each other. However, the necessity to make a discernment between those two identical logical spaces, makes it possible for the logical space to describe itself and therefore to descriptively show its consistency and completeness.

The logical space describes itself in terms of the relation between –O O– and O––O. This is identical with saying that two identical logical spaces see each other by means of the relation between –OO– and O––O. Two identical logical spaces relate to each other necessarily in such a way that ;

(i) –OO– is, in itself, identical with O––O , and vice versa,

(ii) what is ⊙--⊙ could have been -⊙ ⊙-, and vice versa,

  (iii) if what is -⊙⊙- is -⊙⊙-, then what is ⊙--⊙ cannot be -⊙⊙-, and vice versa.

  (i) holds because -⊙⊙- and ⊙- -⊙ have an identical internal structure. (ii) holds because this identical structure is such that what demarcates itself, by so doing, gets itself demarcated. (iii) holds because what gets itself demarcated in -⊙⊙- is identical with what demarcates itself in ⊙--⊙, and therefore because neither of -⊙⊙- and ⊙--⊙ can be the case in the other without falling into the impossibility of demonstration. However, if -⊙⊙- is the case, then ⊙--⊙ is also necessarily the case. This means that -⊙⊙- and ⊙--⊙ coexist necessarily in such a way that both are not in the same logical space, and therefore that each exists in the other. Two identical logical spaces therefore form a single logical space by describing each other in such a way that each becomes the other by transforming what demarcates itself in each into what gets itself demarcated in the other. This form of mapping is 'negation'.

  By negation, therefore, there exist two identical logical spaces such that each contains the other. ⊙--⊙ is -⊙⊙- if and only if it is negated, and vice versa. Each contains the other in such a way that they are identical. Consequently, the description of either alone suffices for the description of both. The descriptive necessity for this is that -⊙⊙- with the negation of ⊙--⊙, is not discernible from ⊙--⊙ with the negation of -⊙⊙-. The logical space with this form of mapping is the self-described logical space and contains the notion of truth-values. A 'truth-value' is therefore identical with the logical space itself. The validity of a 'truth-value' lies in the very existence of the logical space. Truth-values are identical with each other if and only if they are on their own and are therefore not related to each other. The meaning of each truth-value lies in the other and therefore in their mutual-relation by means of negation.

  Representing truth-values by T and F, the truth-value of p is necessarily T or F, and not both. This is so because if the truth-value of p in -⊙⊙- is T, then that of p in ⊙--⊙ is necessarily F, and vice versa. Therefore, if two such p's are identified with each other, then p has two truth-values which are either T and the negation of F or F and the negation of T. This means that p in the self-described logical space has T and F that are assigned to p in such a way that if p takes T, then the negation of p takes F, and vice versa.

p is necessarily one, and one only. Therefore, the coexistence of T and F, both of which are assignable to p, forms the 'matrix' of p. The descriptive necessity for a 'matrix' is this oneness of p. Therefore, the meaning of a 'matrix' is to enumerate T and F in such a way that they are not simultaneously assignable to p and are therefore not a unity.

Representing negation by $\sim$, the matrix of p descriptively determine that of $\sim$p. If p is T, $\sim$p is F, and if p is F, $\sim$p is T. From this it follows that the relation between p and $\sim$p is identical with that between T and F. Consequently, the 0-dimension of the self-described logical space consists in and of either p or $\sim$p. If it consists in and of both p and $\sim$p, then it results in the impossibility of demonstration ; for this is identical with saying that p is T as well as F at the same time, and therefore, contrary to the existence of the logical space, results in the indescribability of p. p is what is identical with itself. Therefore, if T and F are identical with the logical space necessarily in such a way that each identically holds in the other, then p is identical with either T or F. If p is said to be identical with both T and F, this is the same as saying that what is self-identical holds outside itself and therefore without any descriptive necessities to bind what is self-identical by an identical symbol. If what is self-identical holds outside itself, then there are no relations which hold in what is self-identical. Two existences of what is self-identical are merely the same as two p's without any relations between them. p is not describable if it is on its own and remains so. A symbol does not signify anything if it is not describable to be related to itself. This goes against the initial condition (Condition : Only that which is understandable is describable, and vice versa. See the very beginning of 'The Elementals') and is contrary to the described existence of p (i.e. of the logical space). If not both p and $\sim$p can constitute the 0-dimension, then p $\wedge$ $\sim$p is contrary to the meaning of $\wedge$; for p and $\sim$p cannot be a unity. The operational relations which hold between p and $\sim$p are therefore as follows :

RAA : From p $\wedge$ $\sim$p nothing follows. If anything which follows from p $\wedge$ $\sim$p holds, then it is identical with saying that the self-describability of FX does not hold.

DN : The negation of $\sim$p is identical with p, and vice versa. This is so because T is identical with the negated F, and F is identical with the negated T. The identity between T and $\sim$F is identical with that between F and $\sim$T ; for T and F are either identical with each other if they are unrelated, or already underlie each other if they are related. p is matricized for this reason. p and $\sim$p can be related to each other if and only if they comply with RAA and DN. From this it also holds that :

MTT : The meaning of p' → p" is identical with that of ~p" → ~p'. This is so because the relation between p' and p" is such that they and only they are discernible from each other in such a way that the latter is based upon the existence of the former. This also means p' and p" are necessarily not identical if they are delinear. Consequently, each is delinearly identical with the negation of the other because the delinear relation between p' and p" is identical with that between p and ~p. This means that given and based upon p' → p", p' is ~p", and p" is ~p'. That is, ~p" → ~p' is based upon, and identical with, p' → p".

T and F are, in themselves, identical with the logical space itself. Therefore, the meaning of T is identical with that of F if they are unrelated. In the matrix of p T and F are not directly related but enumerated (inter-spatially related through the self-described logical space) so as to stand for the identical and twofold relation between ⊸◯◯⊸ with the negation of ◯⊸⊸◯ and ◯⊸⊸◯ with the negation of ⊸◯◯⊸. If the truth-value of p is T or F and refers to the identical meaning of the unrelated T and F, then whatever that is operationally identical with p is evaluated by either T or F in such a way as to refer to the identical meaning of the unrelated T and F. Consequently, it does not make any difference if this meaning of the unrelated T and F is represented by T or F.

It is the recursively closed chain of logical dimensionalities that connects the ┼-space with the ◯-space. This transcendental connection allows the ◯-space to be superimposed onto the ┼-space because the base of the ◯-space (∨) has a more fundamental dimensionality that eventually forces the base of the ┼-space (∧) to recur back to the 0-dimensionality of the logical space. The ┼-space and the ◯-space coexist in the sense that both are creations of descriptive necessities but reflect their dimensionalities (ontologico-notational transcendence). Logic is an ontologico-notational schema based on the descriptive necessities of the 0-dimensionality of FX (represented as p), whereas maths is a logical schema that takes p as an argument that satisfies this logical space. The difference between the two p is the logical p is a variable-notion to describe the structure of FX, while the mathematical p is that which is thus described. The former p represents the structure of FX, while the latter p embodies the structure of FX. It is the logical p with a structural meaning thus assigned, instead of truth-values. One might say it is a second-order predicate p, as it were. That is, if the logical p is FX, this p is F(X). That is,

FX → (p, p, p → p)

through the ontologico-notationality (→), whereas

F(X) ↔ X(F)

X is bound by F, and uniquely X as such commands F as such where space coincides with entity (↔), because a conceptual-function of the maximum concept is synonymous with truth-values themselves. There is not mind that intermediates language with the world, but mind becomes its own world, and the empiricality here equates the consistency of mind. A truly maximum concept should intrinsically contain the assigner of truth-values and therefore self-relationally refers to itself as F ₪ X.

FX, to be meaningful, has to describe itself (ontologico-notationality), but F(X) is an entity in a space of description, as much as X is bound by F. This p that satisfies the logical space is thus subject to the self-described logical space, which is the logical space mapped onto itself by means of negation. This p is therefore represented as e where one logical space leads another and another e where the former initiative takes the other way around, resulting in the same self-described logical space. Thus, the self-described logical space holds between two identical e. The self-described logical space is the self-imposed necessary way by which the logical space sees itself. e stands for the logical space and is necessarily made collectively one, and one only by the self-described logical space. That is, e epistemologically stands for the logical space and is epistemologically described by the way by which the logical space sees itself. The properties of e are therefore determined by relations which hold between its two identical constituents, e' and e''. e' and e'' stand for two identical logical spaces and are themselves epistemological entities that satisfy a recursively closed description with consistency and completeness. This e is a 'point' (in the self-described logical space) between which the two identical logical spaces relate to one another through 'negation' as a form of mapping. The two logical spaces are internally identical but externally differentiative because on one hand they have one and the same structure, on the other each could have been the other (metaphorically think of a geometrical line and the two directions alongside it as its description). This is expressed as :

$$\left\{ \frac{\leftarrow}{\rightarrow} \wedge \frac{\rightarrow}{\leftarrow} , \frac{\leftarrow}{\rightarrow} \vee \frac{\rightarrow}{\leftarrow} \right\}$$

$\frac{\leftarrow}{\rightarrow}$ and $\frac{\rightarrow}{\leftarrow}$ are self-relationally symmetrical in the sense that each necessarily implies the other, and therefore that both are necessarily existent in contrast to each other. Consequently, given the self-described logical space, both logical spaces are separately discernible and yet simultaneously coexistent. If there is a form such that governs this identical, yet differentiative variable-notion (of second-order), it is the logical connectives of ∨ and ∧. Descriptions by ∨ and ∧ are interesting ; ∨ and ∧ are described by the delineated p. Although p is ontologico-notationally necessitated to delineate, given a non-delineable p (not to be confused with the linear p) dimensionalities are not describable beyond the 0-dimensionality. The above identical but differentiative variable-notion is akin to this non-delineable p.

 The self-described logical space is founded on the logical space and therefore contains descriptions of ∨ and ∧. The self-described logical space as applied to e thus also contains ∨ and ∧ as meaningful connectives that take values, such as e. The description of e ;

$$\left\{ \frac{\leftarrow}{\rightarrow} \wedge \frac{\rightarrow}{\leftarrow} , \frac{\leftarrow}{\rightarrow} \vee \frac{\rightarrow}{\leftarrow} \right\}$$

is quite different from {p ∧ p, p ∨ p}. Adopting a standard format, from (p ∧ p, p ∨ p) nothing follows because p ∧ p and p ∨ p are both logically identical with p. ∨ and ∧ as applied to an identical variable (non-delineable 0-dimensionality) simply confirm the 0-dimensionality. That is, ',' in (p ∧ p, p ∨ p) means nothing. On the other hand, (p ∧ q, p ∨ q), although appears to have some unknown meaning, indicates the presence of mind the game player, because q here can only be something arbitrarily adopted as meaning another 'proposition', i.e. delineated p located elsewhere in mind or in the self-described logical space, in the latter case (',') can only indicate 'negation' as a form of mapping.

$$\left\{ \frac{\leftarrow}{\rightarrow} \wedge \frac{\rightarrow}{\leftarrow} , \frac{\leftarrow}{\rightarrow} \vee \frac{\rightarrow}{\leftarrow} \right\}$$

is neither (p ∧ p, p ∨ p) nor (p ∧ q, p ∨ q). $\frac{\leftarrow}{\rightarrow}$ and $\frac{\rightarrow}{\leftarrow}$ are identical but differentitive. They are identical because → and ← are not self-discernible, yet they are differentiative because '−' holds only as a result of →-initiation or ←-initiation. { ',' } here means the transcendence between ∨ and ∧ because ∨ and ∧ as applied to an identical variable

(variable-notion) means they are one and the same (ontologically), yet they are as distinct as $\overset{\leftarrow}{\underset{\rightarrow}{}}$ and $\overset{\rightarrow}{\underset{\leftarrow}{}}$. Thus, they retain their logical meaning attached to their dimensionality, where ∨ is the descriptive base of ∧, which describes the meaning of ∨ recursive to the 0-dimensionality. It is thus the ∨-space comes to be incorporated into the ∧-space. Primes thus incorporate its paradoxical meaning, one adopted in the ┼-space, infinite as per Euclid, yet indicative of the closed nature of the ○-space.

  In japan, we have a famous poem ('iroha') comprising of a perfect and simultaneous pangram and isogram and yet rhymed on the traditional style, with a coded message incorporated (so some says), which describes an essence (Buddhistic, no doubt) of the human world by using each and every of 47 alphabets only once. The significance of this idiosyncratically beautiful poem (by an unknown literary maverick of 10-11th century Japan (first recorded appearance in 1079), leaving just this one piece of work) is that it also shows a linguistic essence. Language and mind symbiotically co-develop and merge into a binary totality in order to move forward towards merged mind and language as a totality. Here language as a means of communications and imperfect descriptions comes to mirror mind as the unified structure of the intellect. We know we are still far away from such a state of hellish nirvana. This poem mockingly and tantalizingly shows what this end-product of mind and language is like. Think of a metaphoric language as a totality with only one proposition (or a few interconnected ones) comprising of a few words with one meaning per word. What would it describe ? Semantically it describes the state of merged mind, syntactically it describes the structure of such a mind, presumably reflecting the world in some way. Like the first word of a baby, but instead of emotion or desire through primitive understanding, this one-sentence language manifests the entire world - whatever it may be - as cognized through merged mind, like one ultimate all-embracing formula of physics. The meaning and structure of this one proposition is our entire existence, so to speak. This one sentence describes 'life, universe and everything' (my next work). That is what we are here for. The meaning of this sentence is neither scientific nor artistic, nor empirically verifiable. It is simply the limit of our cognition. Language as a totality would allow us one final representation of mind-world like the finish of a simple but difficult jigsaw puzzle. Now replace this one sentence with as few sentences as inevitably possible as we simulate our language as a totality. The said poem metaphorically mimics what we can express ultimately, linguistically, mathematically or scientifically, may it be human values, notational limits or ways of

modelling and exploiting material properties from the perspective of human paradigms - various measurements, sensory capacities, algorithmic constraints, conceptual thinking, usefulness, monetary values, etc..

Given less and less words with sharper and sharper meanings as it inevitably happens as we peel off layers and reduce facets of concepts in our efforts towards language as a totality, these words will have more and more limited scopes to make up sentences and also to make senses. The same goes for logic and maths. The path for the fundamental understanding is to make them simpler and simpler so that the most basic structure will surface from fewer and fewer concepts. The aforementioned Iroha poem shows that there is only one possible meaningful combination of given alphabets in order to make up words and sentences, under linguistic constraints of poetic styles, semantic depth and syntactical meaningfulness, culminating in the concise expression of human values worthy of the name of a poem. There were a few more attempts, but none managed to attain any poetic values.

What applies to this simplistic metaphor of a language game also applies to the descriptions of the world, with less and less varied constituents, a traditional modus operandi of western philosophy from Thales, culminating in Leibniz, to Wittgenstein's paradoxical assertion of philosophical futility of such attempts. This is a schematic essence distilled from less and less conceptual components with simpler and simpler meaning resided in conceptual relations, be it language, logic, maths or even empirical sciences. This schematic essence cannot be a law or axiom because it then imposes a structure that has to be 'proved' within a notational schema with superfluous presence of mind the game player, mind the adjudicator as well as usual rules, constants and variables. This is where a Russellian Pp fails ; a primitive proposition and its corresponding state of affairs also implicitly assume truth-values, a value-assigner (mind the game player) and a value-evaluator (mind the adjudicator), all of which conspire as paradox/tautology when an assigner acts as evaluator. A law is a law only on the assumption of merged mind that cannot exist unless and until language merges as a totality. When and if this does happen, axioms are not needed because whatever is, can only be demonstrated. For PSAI axioms are not unproven rules of operation but modes of operation. The frame problem is a question of finding a schematic essence.

Given ultimate basic concepts, a structure will automatically emerge reflecting language-mind relationships that also mirror the world through

conceptualization, and here axioms are superfluous. Whatever emerges here can only demonstrate its validity by deriving secondary concepts through descriptive necessities. The Iroha poem mimics this process. This is more or less the case with any well-compressed descriptions, be them art, philosophy or even science. They are descriptions illustrated through elements of contrasts ; ranging from revelatory contradictions seen in the works of the haiku master Basho who deliberately saw opposites in what is inevitable or natural in perception, or any great poets like Rimbaud (his not unordinary post-literary life and his enigmatic works of boyhood, i.e. a contradiction expressed between his poem and his life), Wordsworth (nature and human) or Escher (his graphic art of contradictions), or think of Wittgenstein's Three words, Schopenhauerian Will, Kantian a priory or Cartesian 'cogito', or nature's describing itself (so we understand) through opposite electric charges encapsulated in the container of an atom, or disorder and equilibrium encapsulated in entropy, they not coincidentally mimic or symbolize a structure of ontologico-notationality that expresses through the relationship of minimum necessity, be it symmetry, contrast, contradiction or paradox/tautology.

If one is to found non-axiomatic logic or maths, here is where to start ;

$$FX = x(T) \text{ ₪ } x(F)$$

, which is to describe the structure of 'self' unfolding 'descriptions' so as to make itself intelligible, i.e. understandable and describable. x ₪ x encapsulates 'truth' and 'falsehood' in a same descriptive container. One talks about the foundations of maths or even of logic, but one never talk or even think about the foundations of language. For language is the foundation of maths and logic, and language can only be we ourselves or at least my mind.

What is a formal language ? A code of laws between two minds (or layered-mind), between a mind and its doubting Thomas at its foremost, extracted as axioms and rules of inference. However, this communication itself between two minds takes the form of language, which cannot be formal (as yet unfounded). Therefore, underneath any formal languages is a defining language that unites two minds. This is the essence of the self-referential problem, which is the core of any languages, formal or otherwise. That is, insofar as mind works on defining medium, this is the language of language (including any formal languages) and is analytically unrepresentable in any products of mind. The self-referential paradox is really the ultimate code of laws that shows the boundary of linguistic expressions. So-called axioms and rules of inference can only refer to

themselves in our attempts to 'describe' them. Thus, the application of a rule of inference R on axioms A and B to lead to a statement C tells ultimately no more than the existence of the applicator and the self-agreement on the validity of A, B and R and the resultant C, i.e. the formalizer cannot be formalized as metaphysical adjudicator. Externally 'mind is mind', while internally 'minds cannot be a mind'. Any products of mind assume a merged mind that does not yet exist, or more accurately, products of mind are only there to move towards a merged mind, which they assume in order to be meaningful. A true joy of philosophy.

Given a meta-logic, one asks if a meta-logic is logic. The paradox is not 'a meta-logic is logic' entailing 'if it is, then it is not a meta-logic', but our questioning the question, because if truly given a meta-logic, then the answer must be already contained in the meta-logic. This is how a mind plays a game. A meta-logic cannot be logically described because it can only demonstrate itself. Likewise, given natural numbers, one asks if the number of natural numbers is a natural number, leading to 'if it is, then it is not a number of natural numbers', and further leading to $\aleph_0$, which is a number of its own and does not explain what natural numbers are. Here too mind is playing with itself ; 'the number of natural numbers' was asked to know about natural numbers, and you get an answer which does not answer the question. So mind asks an imaginary question and answer with an imaginary answer. Likewise, ask the meaning of ',', it is either unanswerable or ends up with an irrelevant answer. It is layered mind that keeps asking such questions as 'number' leading to 'number of numbers', to 'number of numbers of numbers', and so on. Applied to itself, it will lead from 'concept' to 'concept of concept', and so on. In short, a tautological concept leads to a paradox, which is a linguistic self-regulation to stop infertile looping. Logic is thus developed from 'self-defining concept', and is nothing but tautologies and paradoxes.

A 'self-defining concept' is any concepts that 'group' together 'entities' so that it describes the world in terms of categories, and this capacity naturally applies to itself because such a concept is a process rather than an object and cannot be endowed with a 'covenant' of excluding itself insofar as a 'covenant' is itself a description and requires other concepts. Take 'number', the Russellian Pp of 'number' can only start with the form of f(x), not with examples such as '1', '2', etc. or with a form like Peano's (0, n, n+1), which is a distilled structure of what is given and contains indefinables (like '0', ',', '( )', etc.) and already assumes 'number'. Thus, f can only be as primitive as 'countable', which, in order to say 'should not include itself', it must first know what 'it' is before 'it'

is defined. That is, it must conceptualize itself before it exists. This is itself already a paradox. It is thus 'number' naturally extends to 'number of numbers' because it cannot conceptually say it excludes itself before we know what 'it' is. If a number is a label given to what is countable, then applying to itself produces the number of 'what is countable', which is a mis-application of countability. That is, what is countable is 0 or { }, not itself, which is a concept. While a 'number' is the number of 0, the 'number of number' is not a number of 0.

To say 'a 'number' should not include itself' is a rule after the perception of 'number'. A rule can be challenged or broken because it is only a convention of a schema. It is mind's business to break rules whenever possible. A rule is not a Pp or a concept, but is rather an application of Pp or concept. If there can be an alternative schema in which an alternative rule can cohesively operate or can dispense with this rule, then the rule does not form any essential descriptions of Pp or concept. Before there is a rule 'a 'number' should not include itself', there has to be 'number'. Therefore, citing of rules does not explain how this concept came into existence or is all about. Rules are more about schematic house-keepings or finesses of ordering.

The ultimate self-defining concept is ontologically 'self' or epistemologically ',', which is by itself not informative, but with an utmost necessity. However, the meaningless ontology of 'self' becomes epistemic in the process of descriptively unfolding itself, because finding the only necessary and possible way of describing itself is the essence of description and forms a knowledge. That is, whatever 'self' may be, the way it can describe is a description by demonstration. This is the only way to know what logic is, what maths is. In the sense that it is a description by demonstration maths and logic are one and the same.

Be it art or science, ',' is the most important symbol but is so taken for granted that it often goes undefined, save for saying it signifies 'space', which by itself hardly means anything. 'Space', from physics and geometry to more degenerate usages in our narratives, is necessarily schematic and signifies the most important basis for a schema. Thus ',' used in axioms means the basis of a schematic representation. If used in different areas of maths, it may bear altogether different meaning. For example, ',' in $(0, n, n+1)$ assumes the structural space on the number sequence of $\mathbb{N}$, whereas ',' in $(p_1, p_2,..., p_n)$ assumes Euclidian n-space. The former ',' is a gap in natural numbers, and being recognized as a 'gap' already entails something else other than natural numbers. That is, expressing 0nn+1 as $(0, n, n+1)$ already assumes something more than

(0, n, n+1). ',' between n and n+1 could well mean a segment of $\mathbb{R}$ or to assume ',' between 0 and n and ',' between n and n+1 is one and the same is also to assume an identical totality between every natural number and is making a hidden assumption of 'real line' on the seemingly simple construction of a natural number line. On the other hand, the latter ',' is a way of defining a Cartesian point and means the structure of Euclidian dimensional space. To say that

  ',' (in (0, n, n+1)) = ',' (in $(p_1, p_2,..., p_n)$)

is synonymous to saying that a sequential gap is identical with a dimensional gap. We unconsciously use common sense (whatever it may mean) and avoid confusions in our descriptions, because we contextually differentiate number theory from Euclidian geometry. However, in descriptions of logic in founding secondary schemata, such common sense may not work, because common sense may be so tied up with descriptions themselves. This is the reason why I equated the ontological 'self' with the epistemological ','. Ontological 'self' is epistemologically underlain as 'common sense' in any schemata insofar as we are the constructor of any schemata. We know ',' in (0, n, n+1) or $(p_1, p_2,..., p_n)$ to the extent we also know schemata behind them. This may be what we call 'common sense'. It is often hidden assumptions behind so-called common sense that cause havoc by turning into paradoxes. I already mentioned the example of Russellian Pp. However, ',' in

$$\left\{ \frac{\overset{\leftarrow}{}}{\overset{}{\rightarrow}} \wedge \frac{\overset{\rightarrow}{}}{\overset{}{\leftarrow}} \quad \frac{\overset{\leftarrow}{}}{\overset{}{\rightarrow}} \vee \frac{\overset{\rightarrow}{}}{\overset{}{\leftarrow}} \right\}_{,}$$

is more of a mystery or hidden assumption because we do not quite know what lies behind it. If

$$\left\{ \frac{\overset{\leftarrow}{}}{\overset{}{\rightarrow}} \wedge \frac{\overset{\rightarrow}{}}{\overset{}{\leftarrow}} \quad \frac{\overset{\leftarrow}{}}{\overset{}{\rightarrow}} \vee \frac{\overset{\rightarrow}{}}{\overset{}{\leftarrow}} \right\}_{,}$$

is to be the basis of logic and maths, then ',' signifies the more basic structure of ontologico-notationality. This ',' is indeed the basis of all ',' and is x ₪ x. How x ₪ x turns into the most fundamental space of ontologico-notatinality will be touched upon in the next chapter.

  To think there is something in saying 'self is self' or 'x = x', is the same as seeking a meaning in 'self' itself or 'x' itself. Remove invisible mind the adjudicator, truth is only truth in its relation to falsehood, as much as

falsehood is only falsehood in its relation to truth. What applies to concepts also applies to propositions. And therefore, tautologies are only tautologies in their relation to paradoxes, as much as paradoxes are only paradoxes in their relation to tautologies. It is this structure between tautologies and paradoxes that forms logic and maths, because as long as mind the adjudicator is at large, forming judgments superficial or fundamental, logics and foundations of maths will mushroom as we see no end to theories and counter-theories. It is thus we do not yet have logic of logics and maths of maths. This ontologico-notational relation between tautologies and paradoxes is 'Spiegel im Spiegel' as mind can only mirror onto itself in order to see itself. Given a mirror, then a paradox is a tautology, and vice versa. The ontological 'self' describes the space of descriptions, whereas the epistemological ',' assumes each respective schematic space. It is really the job of respective philosophically minded scientist to examine and reveal the meaning of ',' used in respective schema. If they cannot be a paradigm mover, at least this way they can be paradox remover.

  I am not a Wittgenstein scholar, but from a little I read I side with him with regard to the question of provability. You do not need the madness of Gödel to formalize 'provability'. Behind every formal system is a mind, without which notions, rules, definitions, etc. do not make sense. A mind assumes an identical mind or merged mind because for a system to be formal it cannot afford any arbitrary interpretations. That is, a formal system assumes a mind that is universal in structure, function and essence. Now this is a big assumption and I shall call it a hypothesis. Even if you replace a mind with an algorithm, since an algorithm is for us ultimately an intellectual process that requires heuristic elements or approximation, it is just another word for a well-defined and -regulated mind with an informal essence. Given this assumption, the consistency and completeness of mind, which we need in order to appreciate any consistency and completeness of a formal system, is necessarily over another mind because if we all share an identical mind we need no proofs. Thus, mind is plural but identical. Likewise, the consistency and completeness of a formal system is necessarily over another formal system because 'proof' has to be a non-tautological description. Otherwise, mind cannot comprehend any so-called 'proof'. Thus, we need two non-identical formal systems backed by two identical minds overseen by another identical mind to make sure we are still talking about a same 'proof'. This process, however, contains another hypothesis, namely there has to be at least one formal system that is self-evident for its consistency and completeness because we cannot simultaneously prove the consistency and completeness of two formal systems. However,

such a self-evidence cannot be proven without the first hypothesis. You cannot prove a hypothesis by another hypothesis. No wonder we have as many logics as there are minds. Language over a formal language assumes an identical mind because behind any formal languages are interpretations. Thus, the provability of a formal system cannot be proved unless the identity of that system and mind over and above it is proved. On the other hand, if such an identity is there, there will not be any needs for such a proof.

What is incomplete in Gödelian incompleteness is that in any formalizations the formalizer himself cannot be represented by any symbols and rules. This is also why there are proliferations of formal languages, mushrooming of varieties of logics and even foundations of maths. You can pick and choose anything you fancy to champion your view. If there were a system that represents a thinker or the manipulator of symbols and rules, then that will be the only formal language or logic which represent the structure of intelligence because such a language is objective knowledge with universality applicable across diversity of thinkers. A formal system did not just drop from the sky. There was an ingenious mind (formalizer) that formalized it from ingredients out of the ordinary language. Hence,

$$L \rangle f_1, f_2, \cdots f_n$$

, which means that whatever is expressed in any formal languages, it can also be expressed in the ordinary language, but not vice versa. That is, the ordinary language is more than any formal languages, and concepts are syntactically and semantically more expressive and functional than any formal expressions. A formal language centres on a notation that allows refinements and evolutions of concepts and their relations in the ordinary language. It is the ordinary language that allows mathematicians and physicists to think conceptually and non-linearly where possible, and it is notations that afford them to translate concepts and their relations into notions and rules expressed more precisely and to explore notational depths following schematic necessities. If a mathematician or physicist is just a formal system or algorithm, then there will not be any maths or physics. The origin of x ₪ x is in the ordinary language. It cannot have any formal origin.

Notations are structural representations of concepts and their relations, i.e. syntactical metamorphoses of semantic wealth of concepts as structures lay bare conceptual shells of ambiguity. Thus, how to express numbers have direct consequences on the subsequent developments of

number theories ; we find the decimal positional notation with 0 is more precise as well as more creative compared with non-positional notations or non-decimal notations. Simple arithmetic would be a heavy intellectual burden if not represented by the decimal positional notation and would have hampered us from developing maths as it is today. Mathematical notations are based on logical notations as rules of numbers are to be found in numbers that are to be brought out by numbers described by the conjunctive space based on the meaning of the logical constant ∧.

   Before I move onto the crucial chapter on the foundations of maths, I touch upon Principia Mathematica (PM), because it is regarded as a milestone in the philosophy of maths despite its failure. Or, although it failed in its intention, it survived as interesting terms of reference. Anyone thinking about the foundations of maths has to think why and how it failed and what is there to learn from.

   Like the Code of Hammurabi or Napoleonic Code, which were compiled by ordinary men but proclaimed to be ultimately founded on 'god' or 'natural laws', PM is a work of two accomplished but mediocre scholars that took ten exhausting years (according to authors themselves) and is claimed to be the foundation of maths based on the fundamental laws of formal logic. It is full of ├, Df and notational conventions 'we' (meaning Russell and Whitehead) endorsed of necessity, although a pure formal logic should be able to dispense with any aspects of modality, which is existential by definition.

   'We' here seem to lay down the foundations of maths and formal logic because 'we' are most knowledgeable and of highest intelligence and integrity. 'We' assert and 'we' define alongside notational arbitrariness, but 'we' are nowhere to be seen as part of formal representation. PM, despite its formal appearance, heavily relies on the ordinary language without which the formal part cannot be fully meaningful. The formal part provides notational conveniences but is not essential for its understanding, and the reverse does not hold. By being formal one tries to formulate axioms and rules of inference as precisely as possible, but as axioms and rules inevitably involve conceptual understandings they cannot achieve pinpoint precisions. Concepts are concepts because of interrelations. There is no concept that can define itself no matter how primitive it may be. PM assumes civilized readers who would only argue within well-defined frameworks of PM, but like any grand legal system, can be dealt with a deathblow by a barbarian who cares not a toss about legal niceties between lines, who can ignore conceptual taboos that protect notational status quos. PM does not generate, create, encourage or

guide any developments of maths but only assumes the existence of elementary arithmetic and tries to provide some theoretical assurance. It is a case of mechanistic logic's failure to capture dynamic maths and created a monstrous explanatory structure augmented by human imaginative notations.

It is Russell's inability to expand the more fundamental part of PM starting with Pp that prevailed such a clumsy explanatory system full of ├ and Df, which ended in a thought-provoking failure. It is typically a symptomatic rearrangement of existing knowledge encouraged by notational innovations. Interestingly PM is more important in its failure as it ultimately gives way to the notion of mind superior to any written codes of fundamental laws. Laws may be written in rock, but it only takes mind to break them and even punish punishers. Likewise, according to Gödel it is the formality of a formal language that could not break away from self-referential loops. Having surrendered the superiority of the formality of formal languages 'mind' surfaces as the champion of the informality of creative explanations. Gödel dealt the coup de grace to PM as well as to Hilbertian Program but brought about the proliferations of multiple formal and informal logics.

I already offered you an example of the indefinability of an even simple concept x = x. In dealing with any basic concepts PM liberally assumes a merged mind and resorts to 'definitions' as if it is a simplest task. Think of the difficulty of defining a 'definition'. There are many levels of definitions :

At the lowest level, you just pull a definition out of a blue sky, like a fiction writer. It is a personal definition, which may or may not convince a few people, depending on width and depth of conceptual relationships, and how intelligent you are, because intelligence has encompassing capacity.

The next level is a definition by example. Again, its power of persuasion depends on cohesiveness of an example given, which will be exposed by counter-examples.

The penultimate level is by looping (remember a dictionary and how interrelated words are). A definition at this level eventually appeals to language as a totality. What we call common sense often unconsciously assumes language as a totality. The difficulty is this language as a totality does not yet exist. This is where ordinary language is moving to and why remains dynamic.

Or, like Russell, by going deeper using more and more basic concepts in order to explain more mundane concepts, eventually arriving at the indefinable mind itself without knowing, which can only be demonstrated. These definitions have superficial power of persuasion because depth gives an impression of objective structure, not something arbitrary mind puts forwards.

  Russell thought logical concepts are something of Kantian a priory and indisputably founded - power of logic over mathematical givens, as it were. However, going deeper has its own setbacks. Basic concepts too have to be defined, and unless we are God who can decide enough is enough, basic concepts have to keep digging, depending upon the level of intelligence of the digger. Obviously, Russell was not that intelligent because he famously hit upon a paradox from which he could not see himself out. Thus, the set of all sets that are not members of themselves encounters its wall of the undefinable, whose naïve strength still defies axiomatic tinkering or conceptual hierarchy. Be they axiomatic patchworks or higher-order logic of types, they all eventually face similar difficulties of naïve set theory ; metaphysics of the undefinable. Neither modifications of axioms nor creations of types satisfies our desire for a final solution because we do not know where our mind want to stop. So long as stopgap solutions are the ingenuity of mind, mind has no reasons to rest at peace. Thus, types end up with the problem of a universe type, not unlike a set of sets, and tinkering of axioms at will makes one wonder what are we as manipulator of axioms. Certainly, we must be tinkering based on more fundamental axioms, for any tinkering must show the basis of its necessity. In the end, allow the freedom of mind, it eventually comes to ask what it itself is. Remember the proverb, 'Give 'em enough rope, and they'll hang themselves'. All subsequent developments since the problem of naïve set theories have not really settled the issue. Be they any forms of axiomatic set theories or type theories, the fundamental problem of naïve set theory recurs in different guises because any efforts to remove paradoxes results in the self-denial of mind in the form of the removal of its own receptacle, i.e. 'universe'. It is mind self-contained as a totality that allows us 'definitions', 'axioms', etc. which requires a domain. Coherencies of definitions, axioms, etc. can only be ascertained within some totality, ultimately coherent mind itself. Logic is logic of a coherent mind, not of scatterbrains or schizophrenic mind. In short, given

  $Y = \{x \in X : P(x)\}$

, then various axiom schemata, if restricted, end up with the impossibility of talking about paradoxes, although we are still free to talk about paradoxes. By eliminating superficial paradoxes (of Russellian and Cantorian nature), you end up with an even bigger paradox ; mind can talk about paradoxes which mind cannot think of, i.e. mind as a structure is unrestricted and restricted.

Russell's approach is a legacy of the days when we had the luxury of the protection of 'God', as a matter of psyche or unconscious thought process, if not as a believer. Only so long as we are an intellectual agent of God, we can solve any fundamental questions by definitions. Otherwise, we face the question what are we to define this and that, in most cases at will. Besides, set theories have a more fundamental weakness of their failure to formulate 'necessity' of forming a set, other than our willingness to construct. Thus, take 'three'. There are empirical number '3', like 3 apples. This moves up to membership number '3', like '3 apples – 'apples' = 3'. This is '3' as cardinality. On top of this, we have a set '3' representing any collections with cardinality of '3'. Finally the set '3' that is instructed not to include itself as member. This famously leads to the set of all sets that are not members of themselves, a paradox. In short,

Three apples → three → 3 → '3'

We need 'x' because we intend to construct a schema of rules of arithmetic, and this can only be applied to 'x' as of f(x), where rules come in the form of f, i.e. first-order predicate. However, given 'x', ' ' would apply to itself, leading to the contradiction. Mind constructs something by definitions at will, with the intention of 'proving' this something to itself. Thus, 'proving' a number as a set already presupposes a 'number', three levels of a 'number' at that. Mind is constructing something to prove to itself by definition. This can only end up as a grand schematic tautology. Mind can prove anything to itself by definition because it is designing a mental apparatus with a specific purpose of fulfilling this purpose. That is exactly how some people prove 'god' to himself. So what ?, and thus this tautology is also a paradox. You prove anything, you prove nothing. Three apples did not fall out of the sky. We put them together in such a way that forms three of them for a specific purpose (of making '3'). Be it three or 3 or '3', we already have a number, what we did is simply a mental exercise of paraphrasing something common in all of them in a schematic way. The problem is this schema is only designed to prove what it create as schematic entities, i.e. mathematical objects of 'sets'. These 'objects' are by definition schematic entities, and thus their

existence is already schematic and presupposes a schema. We need schematic entities of an abstraction at the same level as a schema itself. Rules of numbers only apply to mathematical objects, not to empirical objects. '2 apples + 1 orange' does not make '3', 3 or even three. '+' is only applicable to mathematical objects. It is Russell who made '3' so that a schema of arithmetic as he knows can be logically founded. We 'prove' what we intend to prove. This is what set theories are all about, an enjoyable waste of many good talents, still better than creating an atom bomb (except von Neumann). A set (number) is a generalized countability, and as such cannot itself be counted. A number in this sense is an adjective, which can be applied to a noun but not to itself. Set-theoretical paradoxes are also a conceptual tautology of generalizing a 'number' from a number.

Naïve or otherwise, this is really a paradox applicable to any conceptualizations where definitions are sought by generalizations. Concepts are tools of mind, and mind is applied (by mind) to go more and more basic so that it can achieve its own aim of, e.g. logically founding arithmetic. If mind relies on definitions at the bottom of its own aim, it cannot help reaching itself, undefinable definer. This is the root of paradoxes. ZFC-like axiomatic tinkering eventually faces the same problem, because you are only buying time by replacing more basic concepts with more applicable axioms, unless you can find absolutely necessary axioms. But, of course, how do you prove it, other than by demonstration ?

We should note that e.g. membership number '3' fails to distinguish those necessarily 'three' and arbitrarily 'three', such as '3' of three angles of a triangle and '3' of three apples. The former is by geometrical necessity, while the latter is a whim of our mind in the sense that there is no necessities for apples to be three as we could as easily make up four or five apples. If we cannot rely on definitions or any constructive methods, how can we find 'numbers' ? One could finds necessities of 'numbers' such as atomic numbers or geometric laws. However, defining a number e.g. by the number of protons, although persuasive by means of empirically necessary laws, encounters easy obstacles as atomic numbers appear strictly finite and as we do not know if such laws are workable in every possible universe or universally translatable in any states of a universe. On the other hand, geometric laws can be used to define number '3' like any triangles, which have 'three' internal angles by necessity, not by definition. This '3' is empirical, cardinal as well as abstract by necessity. That is,

Triangles → three angles → 3

, which needs one less step compared to three apples because it is the arbitrariness of definitions that demands an extra layer of abstraction so as to ascertain that 3 from three oranges is the same as 3 from three apples. Once given 3 by necessity, this 3 is applicable as cardinality. It is an adjective 'three' that is applicable to anything countable. Triangles are only a cause of the discovery of the number 3. Once various numbers are thus discovered by necessity, they obtain the freedom of application because they are detachable as operative concepts, tools of mind whose sole purpose is the merger of minds, unless we disagree any triangles have three angles. Note that this 3 by necessity can never be a member of itself because it is an 'adjective', and applying to itself makes it a 'noun' as it were, against its own grammar. 3 of three angles can be applied as cardinality because an 'angle' is already a schematic entity that exists by rules of geometric necessity. Thus, this 3 is applicable to anything that exist by the same rules, i.e. to objects of a geometric space, which can represent an empirical space by approximation. We can approximate any empirical objects as objects in a geometric space (or space-time) because they are contained in a lager framework of geometric paradigm. Here 'apple' and 'orange' are both e.g. 'thing', and thus 2 apples + 1 orange = 3 'things'. Provided that we found enough detachable adjectives of numbers, arithmetic operations can be usefully performed.

  Metaphorically expressed in different terminology, think of a set as a 'noun' abstracted from an 'adjective' of arbitrariness. Here mind is trying to replicate the world by definition. A 'noun' created from an 'adjective' abjectifies itself, like a set of set, a paradox. I think of a number as a noun commanded by an adjective of necessity. Here the world projects itself onto mind by representation and is a tautology rather than a paradox. These nouns of necessity applicable as adjective has 'independence' within their paradigm of necessity.

  Or even better, in the same vein, if we can find 'numbers' in geometric space itself by necessary rules like Peano rules, such numbers not only need no arbitrary definitions as they exist by necessity, but are also operative enough to form arithmetic, without falling into the trap of paradoxes. The question is how to define geometric space by necessity, and whether such a space can also accommodate other types of numbers like reals or essential transcendentals like π and e as well as 0, 1 and an imaginary like i together with numerical qualities like continuity, infinitesimal and infinity.

Fregean notion of a number should only be sought in the necessity of a number, not definitional constructivism. To found a number on membership without making a distinction between necessary and arbitrary is itself a cause of paradox. It is a 'set' by arbitrary construction that need to be instructed (by mind) not to include itself as member, because as arbitrary as it is, 'grammar' has to be imposed from outside, which will trace its origin once again to mind. Whereas a set by necessity needs no such an instruction or grammar as it contains such an instruction and grammar in itself. When mind constructs something and at the same time ends up having to issue an instruction of usages, then mind is not functioning properly, leading to paradox/tautology. It is a system of cohesive definitions that breaks down at self-referential aspects of 'set'. 'Set' is an arbitrary creation of mind that hinges on mind. In requiring an instruction of usages, mind is creating something that need mind to 'prove' to mind, a guaranteed failure. You are a captive of language who want to master language, at the same time you are a tool of language towards merged mind and language as a totality. This is the cause of our philosophical problems.

It is the founding of 'numbers' by necessity rather than by definition that solves the riddles of paradoxes. A necessity and a definition may come close at the very bottom. However, a definition always leave a room for mind as undefinable definer and demands rules in order to be meaningful, whereas a necessity comes with rules by which it demonstrates. A definition is demonstrated (by mind and for mind), while a necessity demonstrates, representing the structure of the intellect. This is the difference between logic by necessity and logic by definition. In the former, a variable is a variable-notion that is on a par with constants, both of which are derived from descriptive necessities of the ontologico-notational FX and together manifest an essential structure of 'description'. In the latter, constants represent a structure into which variables are fitted so that together they are used to describe the 'world' juxtaposed to mind, where mind acts as superior overseer of both the 'world' and language. Here variables needs a 'domain' because mind stands above them and, depending on its focus of interest, defines its area of application. This way language describes what it is intended to describe, ultimately resulting in paradox/tautology.

I have already touched upon the problems of paradoxes and tautologies as encountered by those who sought foundations of maths and the meaning of formality of logic. I saw a salvation in my 'T ₪ F', i.e. 'Spiegel im Spiegel' paradox/tautology, which will give rise to logical dimensionalities and geometrical space within the recursively closed

chain of rules of inference. Necessities guide us into descriptions and present us with our paradigm of descriptions that schematically coincide with the world. FX is a stem cell concept that evolves by self-descriptions, i.e. by descriptive necessities.

4. Maths

< Maths as application of logic to space >

- Logic as Foundation of Geometry -

  When speaking of maths as application of logic to space, it is important
not to involve 'applier' of logic because then this 'applier' would stay out
of the discourse and bring meta-logical notions and concepts at will.
Thus, the discourse would remain incomplete in the sense the 'applier'
and any rules of such an application of logic is meta-logical and cannot
be described within the prescribed logic. Further 'space' to which logic is
to be applied must be empty because any objects pre-exist prior to the
application of logic are also meta-logical. However, logic cannot be
applied to space unless space contains objects that can be variables. There
is no logic of empty space because without states of affairs or predicable
objects, logic has nothing to represent. I therefore start with space itself
that turns itself into objects and at the same time generates rules that
govern such objects. This is logic as self-spatialization of FX. Rules will
materialise as dimensionalities, objects as postulated entity of self-
demarcation, the completeness and consistency as recursively closed
chain of rules of inference embodied in dimensionalities. This is logic of
geometric necessities.

  Objects and space are both generated as process of self-demarcation and
are underlain by logic of self-demarcation. Therefore being objects means
being spatial objects. They are also countable by virtue of the properties
of the space they co-exist, which is dynamically expanding infinity in the
case of the conjunctive space ($\wedge$-space or $+$-space depending on contexts
as a matter of descriptive mannerism but refers to $\overleftrightarrow{} \wedge \overleftrightarrow{}$) and
dynamically condensing infinity in the case of the disjunctive space ($\vee$-
space or $\bigcirc$-space, referring to $\overleftrightarrow{} \vee \overleftrightarrow{}$). Because objects are spatial and
countable by virtue of the space in which they are embedded, descriptions
of such objects as variable constitute 'numbers' and descriptions of
properties of space the schema of 'maths'.

  I have already demonstrated how the two spaces are constructed (see
'The Elementals'), but in summery :

  A logical space's mapping onto itself gives rise to the self-described
logical space that is internally identical but externally differentiative. This
self-described logical space is a schematic entity that presents itself as

identically describable by the externally twofold forms. This generates 'point' between which schematic directions hold in such a way as to result in an identical description (1-dimension). Be they an object or a form, they are schematic in the sense their meaning is together descriptively to represent the self-described logical space. This externally twofold 1-dimension holds between same two points and is therefore internally identical. The relation that holds in and between what is internally identical and externally twofold is

$$\left\{ \ \frac{\leftarrow}{\rightarrow} \ \wedge \ \frac{\rightarrow}{\leftarrow} \quad \frac{\leftarrow}{\rightarrow} \ \vee \ \frac{\rightarrow}{\leftarrow} \ \right\}$$

, which is the descriptive space of the 1-dimension. This is to say that given what is internally identical and externally twofold, ∨ and ∧ hold between them as identical relations and therefore schematically confirm their identity. ∨ and ∧ hold between $\frac{\leftarrow}{\rightarrow}$ and $\frac{\rightarrow}{\leftarrow}$ as identical relations. This also means that $\frac{\leftarrow}{\rightarrow}$ and $\frac{\rightarrow}{\leftarrow}$ are both necessarily under the schema of logic. The 2-dimension therefore contains both the disjunctive space and the conjunctive space.

The disjunctive space and the conjunctive space are in themselves identical, and are sub-schemata of the schema of the 2-dimension. ∨ and ∧ bring out differentiative meaning of what is identical in such a way that on one hand the 1-dimension described by $\frac{\leftarrow}{\rightarrow}$ or by $\frac{\rightarrow}{\leftarrow}$ are identical, on the other hand that $\frac{\leftarrow}{\rightarrow}$ and $\frac{\rightarrow}{\leftarrow}$ are not reducible into either. The ∨-space can be constructed by either of $\frac{\leftarrow}{\rightarrow}$ or $\frac{\rightarrow}{\leftarrow}$. It is a space in which given the two 1-dimensions, they both descriptively merge into one and the same 1-dimension. If a space can be characterized by two given 1-dimensions' merging into a single 1-dimension and is therefore described to be 'curved', then anything that can be given in this space is curved and merge into that single 1-dimension. This means that any number of 1-dimensions can be given only to result in a same 2-dimensionally merged 1-dimension with two and only two directions. Therefore, this space is curved in such a way as to be closed and uniform. This is so because if two 1-dimensions are given and merge into a single 1-dimension, and if anything that is given in this space merge into this single 1-dimension, then this single 1-dimension is necessarily such that is in a space and also, by itself, holds a space. Any space that is characterized by a single

entity that is in that space and has two and only two directions, is necessarily closed and uniform. It is closed because, otherwise, it cannot be described that it has one and only one entity in it. It is uniform because this one and only one entity is in a space other than itself and has two and only two directions. This space is therefore closed and uniform in terms of what characterizes it and is therefore also finite. It is finite but boundless because its boundary (i.e. that single 1-dimension) is itself a unit. A single entity can be described to exist and to have two and only two directions only in a space that is uniformly closed and boundless if and only if there exists in it one and only one entity such that has two and only two directions. This space and its boundary determine each other. Consequently, the size of this space is identical with that of its boundary. This type of 2-dimensional space is described as a 'circle' if it is, by a descriptive necessity, put into the other type of 2-dimensional space.

 The meaning of ∧ is based upon that of ∨ and lies in its schematic confirmation of such existences that are operationally identified by ∨ as being 0-dimensionally identical. That is, by the meaning of ∧ two schemata such that can be identically constructed by each of two 0-dimensionally identical existences, can be confirmed to be an identical schema under the same schema of logic or under a same applied schema of logic. Therefore, whatever may be ∨-operative, they are necessarily also ∧-operative. Any two entities are ∨-operative if and only if they result in an identical schema. ∨ identifies two such entities in terms of what identically results from them (i.e. the identity in structure between two schemata that are based upon those two entities that are both 0-dimensional). If two entities are both 0-dimensional and therefore result in two identical schemata, then they, ontologico-notationally speaking, self-contain each other. ∨ represents the identity of such entities in terms of what structurally identically results from them. Two identical entities that are so identified by ∨ as what results in two identical schemata, must be schematically so confirmed as an identical schema under the same schema of logic. This is so because two identical schemata that result from two identical entities, can only be identified as an identical schema in terms of the identity in existence between two such entities. What self-contains each other necessarily belongs to an identical schema. Therefore, two identical schemata that result from them are necessarily an identical schema. Whatever may result from two entities that belong to an identical schema, they are necessarily within this same identical schema. Consequently, ∧ holds only between two schemata such that are so identified by ∨ as what results from two 0-dimensionally identical entities, and it identifies them as an identical schema in terms of the

identity in existence between those two 0-dimensionally identical entities. That is, ∧ represents the identity of two identical schemata in terms of their identical 0-dimensionality. Only two schemata such that are identical with each other can be ∧-operative. ⇄ ∧ ⇄ says that there are two schemata which are identical, while ⇄ ∨ ⇄ says that there are two entities which can be described to be identical and are therefore necessarily under a same schema. Their only difference is that schemata are necessarily structural and therefore, if they are identical, cannot be described to be existent independently from each other, while entities are descriptively existential and therefore, even if they are identical, can be described to be identically existent independently from each other. That is, two entities can be described to be identical with each other and yet independent from each other if and only if they are both 0-dimensional and therefore self-contain each other. However, two schemata cannot be so described because a schema is not an existence but the description of an existence. There cannot be any describable relations between two identical descriptions without contradicting the ontologico-notational condition of description. If two schemata are identical, then they can only be an identical schema, and not two identical schemata. ⇄ ∨ ⇄ immediately results in ⇄ ∧ ⇄ because two entities which give rise to two identical schemata, have an innate necessity to confirm that such two identical schemata are necessarily an identical schema. Two identical schemata have a descriptive necessity to be an identical schema in order to comply with the ontologico-notational condition of description. This descriptive necessity is therefore identical with the descriptive necessity by which the schema of logic is conditionalized. This means that the resultant identical schema is, applied or not, 0-dimensionally identical with what ontologico-notationally describes itself (i.e. the schema of logic). ⇄ ∧ ⇄ is therefore described to stand for two identical schemata's being necessarily an identical schema and is also described to be under the same schema of logic that governs ⇄ ∨ ⇄. The logical space encompasses → and ← as their descriptive necessity and is also closed. This is so because the logical space is the descriptive necessity for and of anything, and because anything can be described to self-contain itself. ⇄ ∨ ⇄ and ⇄ ∧ ⇄ are together a description of such a anything and are under a same descriptive necessity. Whatever may be ∨-operative, they are also ∧-operative. However, neither of ∨ and ∧ is descriptively reducible into the other because they underlie each other by

being underlain by their common descriptive necessity. When ⇄ and ⇄
are ∧-operated, they are therefore two identical schemata which are to be

identified as an identical schema. ⇄ ∧ ⇄ stands for a space in which
two identical schemata are, by complying with the ontologico-notational
condition of description, so taken for granted as to be an identical

schema. In contrast to this, ⇄ ∨ ⇄ is a space in which two given entities
descriptively merge into an identical entity and therefore, by doing so,

give rise to two identical schemata, which immediately result in ⇄ ∧ ⇄.

Therefore, space and its contents determine each other in ⇄ ∨ ⇄, while

they coincide with each other in ⇄ ∧ ⇄. That is, the space of ⇄ ∨ ⇄ is
a space which commands its entities toward its descriptive necessity so as
to be compatible with what it allows itself to take as its entities. This also
means that it appears as if entities determine their space ; for there cannot
be any entities outside a space if this space is the descriptive space of
those entities, and this includes a case such that an entity is its own space.

The space of ⇄ ∧ ⇄ is the space of spaces which are commanded by
their entities toward their descriptive necessity so as to be compatible
with what they are allowed to take as their entities. This necessarily
makes those spaces a single identical space. This is the reason why the
entities of the ∧-space can only be schemata.

⇄ and ⇄, on one hand, determine the ∨-space if and only if they are
taken as identical entities, on the other hand, determine the ∧-space if and
only if they are taken as identical schemata. If they are taken as identical
entities, then they are necessarily under a same schema that takes in both
entities together so that they can be described to be identical. If they are
taken as identical schemata, then they necessarily describe themselves as
an identical schema. This means that they are not under an identical
schema but themselves an identical schema. Therefore, ∧ is the form of
coexistence and stands for the coexistence of two 1-dimensions.

  The 1-dimension is anything that consists in and of two and only two
directions such that are determined by two and only two points which are
so correlated as to descriptively represent each other. This is the 1-
dimension as a schema. The existence of two of such a schema can be so
correlated as to be an identical existence if and only if they 'intersect' in
the sense that ;

(i) two sets of two and only two directions coexist,

(ii) if such a coexistence is describable, then those which coexist cannot be independent from each other,

(iii) whatever that is not independent from each other, share a same space,

(iv) given any two 1-dimensions in such a space, they cannot hold exclusively to each other.

The ∧-space is therefore a space which is determined by two 1-dimensions' intersecting each other. That is, being unable to hold exclusively to each other two 1-dimensions necessarily generate a space between them. This 'between' stands for the characteristic of the ∧-space. The ∧-space is therefore, like the ∨-space, internally determinant and is therefore a schema of its own. Two 1-dimensions are given by intersecting each other and so determine a space between them. This space therefore necessarily has a 'centre'. However, unlike the centre of the ∨-space this 'centre' is not identifiable with the 2-dimensional manifestation of schematic points, which determine two and only two directions and, in the case of the ∨-space, merge into a single point (i.e. a 2-dimensional point). This is so because the ∧-space necessarily consists in and of two intersecting 1-dimensions. This means that no schematic points can be descriptively seen within this space. The ∧-space is described by and between them.

The two types of space are summarized as follows : the ∨-space has one and only one 2-dimensional point. This point has no spatial quantity and forms the centre of the ∨-space. This space is enclosed within a closed boundary that is not reducible into parts. Within this space there are a boundless number of fictitious points that exist in order to describe the two boundaries of this space in terms of their density. At each level of density there is a fictitious 2-dimensional 1-dimension. The two boundaries of the ∨-space are the two extreme limits of such descriptive 2-dimensional 1-dimensions. The ∧-space has an infinite number of 2-dimensional points that are points of intersection of at least two 1-dimensions. Every one of such points is a centre of the ∧-space. Between any two points there is either a 2-dimensional 1-dimension or a combination of 2-dimensional 1-dimensions. A 1-dimension that holds between two schematic points is a pair of two symmetrically related 2-dimensional directions. The infinite extension of a 2-dimensional 1-dimension along its two given directions is such a 1-dimension. Both

types of space have a common fictitious version. This fictitious version is a finite, boundless and uniformly curved space with one and only one centre which is either a 2-dimensional point with an infinitesimal quantity or a region of space with such 2-dimensional points.

  The Λ-space consists of an infinite number of centres and is therefore an infinite space. This infinite space is uniformly dense because a 1-dimension is intersectible by another wherever there is not yet a point of intersection. That is, every centre is identical in its composition, and centres breed and multiply themselves identically and infinitely and therefore make the Λ-space uniformly more and more densely populated until there exists no more space without externally, dynamically and infinitely expanding. This is made possible by the descriptive simultaneity between two initially intersecting 1-dimensions' acquiring a 2-dimensional locality and the Λ-space's coming into existence. This only means that no particular localities have any special claims upon the way by which the Λ-space exists. The Λ-space is therefore uniform in the sense that it is not discriminative about locations of points of intersection. The Λ-space is simply the class of every possible space that can be determined by any two possible 1-dimensions. Such spaces form a class because they are all 1-dimensionally identical and 2-dimensionally simultaneous. If every 2-dimensional point can be a centre, then any one of them can choose itself as the centre without causing any changes in the characteristics of the Λ-space. Every centre can describe itself as the centre of the Λ-space. However, the centre of the Λ-space is necessarily one, and one only ; for nothing can be identically described more than once without contradicting the ontologico-notational condition of description. That is, there is no descriptive necessity for anything to repeat describing itself identically. Every centre of the Λ-space is identical with one another. Consequently, any one, but one and only one, of such centres can describe itself as the centre of the Λ-space. The Λ-space is externally described as a space in which every 2-dimensional point can be a centre. The internal description of this space is the description of the meaning of such a centre. The Λ-space is described in terms of centres, and these centres are described in terms of a centre. The description of centres is the Λ-space, and the description of a centre is centres. This difference constitutes the external and internal structure of the Λ-space. The description of the internal description of the Λ-space is identical with the external description of the Λ-space. Everything either describes the Λ-space or is described in the Λ-space. This is so because the 1-dimension is the only epistemological entity that is so far conditionalized, and because this 1-dimension simultaneously and identically applies to both types of 2-dimensional space. Those which

are described in the ∧-space are so described as to describe the ∧-space. This means that in the ∧-space everything is everything else and is identical with itself. Consequently, if everything is a centre, and if it describes itself as a centre, then it, by itself, determines its relations to every other thing. That is, the description of a centre is identical with the description of every other centre. Every centre results in an identical description. Therefore, any one, but one and only one, centre can be described as a centre and becomes the centre of the ∧-space. This is the internal structure of the ∧-space. The ∧-space can be internally and externally described differently, while the ∨-space and their common fictitious version are internally and externally described identically. This is so because the latter has one and only one centre. The description of such a single centre is internally and externally identical because one and only one centre of a space is necessarily, in itself, the centre of that space. Consequently, the description of such one and only one centre is identical with that of a space that has this centre.

  In the ∧-space, a 2-dimensional point is determined by any two intersecting 1-dimensions. Consequently, a centre is anywhere where two sets of two 2-dimensional directions extend from one another. Two 2-dimensional directions form a set based upon a 1-dimension and are therefore directionally symmetrical to each other. A centre differs from every other centre if and only if it describes itself as a centre and becomes the centre ; for it, in itself, manifests the description of a centre. A centre relates to every other centre in the sense that any one of them could have been the centre. The centre therefore embodies relations such that hold among every centre. This means that every centre is determinant to one another in their identical relation to the centre. That is, the centre describes every other centre in such a way that they are all mutually determinant. This is possible if and only if the centre is determinant to itself. If anything is determinant to itself, then between them there is a space such that describes how it is determinant to itself. If the centre is the description of a centre, then a space in which the centre is determinant to itself descriptively accommodates every other centre and makes them determinant to one another in their relation to the centre. This necessity of the centre's being determinant to itself differentiates the two determinant intersecting 1-dimensions of the centre from every other 1-dimension in the ∧-space. Only those two determinant intersecting 1-dimensions of the centre are described to relate to each other so as to determine and give rise to a centre which describes itself as a centre. Every other 1-dimension and centre can be described in their relation to those two determinant 1-dimensions. Consequently, only those two determinant 1-dimensions need to form a set of two sets of two 2-dimensional

directions that extend from one another at a centre that describes itself as a centre. Every other 1-dimension can be described as a single 2-dimensional direction by those two determinant 1-dimensions, each of which forms a set of two 2-dimensional directions. The ∧-space is infinite. Therefore, neither those two determinant 1-dimensions nor any 2-dimensional directions have, unlike a finite 1-dimension, a reflex direction along a given direction. They extend into infinity. A given direction and its reflex direction of a finite 1-dimension become a spatial symmetry in the ∧-space and are so embodied by each of those two determinant 1-dimensions. Only those two determinant 1-dimensions need to embody this spatial symmetry ; for every other 1-dimension can be determined by those two.

Those two sets of two spatially symmetrical 2-dimensional directions relate to one another only in such a way that they comply with the uniformity of the ∧-space. This uniformity manifests itself as the equal density of 2-dimensional points in the ∧-space. That is, those two sets of two spatially symmetrical 2-dimensional directions relate to one another in order to give rise to a uniformly dense space. The ∧-space is generated by any two intersecting 1-dimensions and is therefore simultaneously assigned the characteristic of being-uniformly dense ; for the ∧-space only consists in and of points of intersection. That is, every two of intersecting 1-dimensions generate an identical space and simultaneously acquire their 2-dimensionality. The ∧-space is what identifies such identical spaces. The ∧-space is therefore inherently uniformly dense. This means that any two determinant 1-dimensions of a centre necessarily and inherently comply with this uniform density. The two determinant 1-dimensions of the centre of the ∧-space embody such uniform density ; for this characteristic of being-uniformly dense is 1-dimensionally inherent to the ∧-space. The ∧-space is determined by the two determinant intersecting 1-dimensions of a centre that describes itself as a centre. Those two determinant 1-dimensions determine the ∧-space and are simultaneously made 2-dimensional by this ∧-space. Consequently, they, in themselves, represent the uniform density of the ∧-space. This representation takes place in such a way that ;

(i) those two determinant 1-dimensions are described to consist of points which are uniformly dense,

(ii) these two 1-dimensions spatially reflect the uniform density of the ∧-space,

(iii) at where these two 1-dimensions intersect each other (i.e. at the centre) each of them is spatially transformed into a set of two 2-dimensional directions which symmetrically extend from each other,

(iv) this set of two 2-dimensional directions is 2-dimensionally 1-dimensional because it spatially divides the ∧-space into two, each of which necessarily consists of an equal number of centres in order to comply with the uniform density of the ∧-space,

(v) each set of two 2-dimensional directions divides the ∧-space into two,

(vi) two sets of two 2-dimensional directions relate to each other and reflect the uniform density of the ∧-space in such a way that they divide each other further into two, each of which consists of an equal number of centres.

This means that the two determinant 1-dimensions of the centre intersect each other in such a way that they transform themselves into four 2-dimensional directions that perpendicularly extend from one another. Every other centre can be therefore described to be inherently determinable by two perpendicularly related 1-dimensional directions. Every 2-dimensional direction and their relations can be described by those four 2-dimensional directions that perpendicularly extend from one another. Those four perpendicularly related 2-dimensional directions extend from one another only at the centre. They arise only when the ∧-space necessitates itself to internally describe itself. The description of a centre is the centre. Every other 2-dimensional point is a centre. The centre can transpose itself to any centres because

(i) any centres could have been the centre,

(ii) every centre is inherently determinable by two perpendicularly intersecting 2-dimensional directions that coordinate themselves with the four perpendicularly extending 2-dimensional directions of the centre,

(iii) these four perpendicularly related 2-dimensional directions can describe whatever that exists in the ∧-space.

The description of the ∨-space and of the common fictitious version of both types of space is internally and externally identical ; for the description of a centre of a space with one and only one centre is identical

with the description of that space. That is, the description of a space is identical with that of a substance if and only if a space has one and only one substance. The ∧-space consists in and of an infinite number of substances that are uniformly distributed and therefore make this space uniformly dense. The ∨-space consists of one and only one substance that is the only centre of that space. This centre is the inner-boundary of the ∨-space and describes itself as the outer-boundary of that space. The ∨-space is therefore filled with descriptive entities within those two boundaries. This inner-boundary describes itself in such a way that ;

  (i) there necessarily exists a set of two and only two directions which it can determine,

  (ii) these two directions are such that hold in and between a single point,

  (iii) they are so determined by this single point and therefore cannot coincide with that point,

  (iv) if they are outside that point and are determined by that point to hold in and between that point, then that point is necessarily such that starts from itself and ends at itself and therefore, in itself, gives rise to a set of two directions ; for if it starts from, and ends at, a same point, then both a starting-point and an ending-point do exist, but are indiscernible from each other, which results in the twofoldness of a single point,

  (v) this is possible if and only if that single point is quantitiless and multiplies itself into a single substance which is so densely populated with such single points that it cannot be reduced into parts.

  This substance is the outer-boundary of the ∨-space and is generated by the inner-boundary of that space. Therefore, between those boundaries there are entities such that become boundlessly denser toward the outer-boundary. The inner-boundary has no quantity other than the 1-dimensional quantity, while the outer-boundary is itself a 2-dimensional quantity. The outer-boundary is therefore not spatial but self-spatial. It has no spatial quantity and therefore does not occupy a portion of space, neither externally nor internally. The inner-boundary is quantitiless because it necessarily coincides with its own space and does not externally exist in a space other than its own descriptive space that is filled with its own descriptive entities. The common fictitious version of both types of space also has one and only one substance that is the only centre of that space. The description of this centre is therefore internally

and externally identical with that of this fictitious space ; for one and only one substance can be described in terms of itself. This centre is either a single 2-dimensional point with an infinitesimal quantity or a region of space that is filled with such points. In the former case that point is determined be every different intersecting 1-dimension and is therefore bound by schematic points. Consequently, a space with such a point is necessarily finite, boundless and uniformly curved. In the latter case, each point is determined by a different set of two different 1-dimensions and therefore, together with every other point, necessarily forms a region of space that is bound by schematic points. Consequently, a space with such a region of space is necessarily finite, boundless and uniformly curved. This fictitious space with a single 2-dimensional point has no density because this point can only be itself the basic unit of density. This space therefore has no spatial properties that can describe its substance. The description of such a space is identical with that of its substance. If this space consists of a single region of space that is filled with 2-dimensional points, then such a region of space does not have a centre. This is so because this region of space consists of 1-dimensions such that every one 2-dimensionally and directionally differs from every other, and that every two of them intersect each other. This means that every particular set of two different intersecting 1-dimensions necessarily prevents every other from forming a centre. This region of space is necessarily such that becomes denser toward its centreless centre. Consequently, no particular sets of two intersecting 1-dimensions can be the determinant 1-dimensions of this space ; for this space appears different from every point. If a space is to be described in terms of its substance, then it is necessary for a space to be identical at every point in it. This means that this space has no spatial properties that can describe its substance without losing its own self-identity. The description of such a space is identical with that of its substances that are necessarily collectively taken together. Therefore, a space with one and only one centre is internally and externally described identically. Only the ∧-space can be internally and externally described differently. This difference makes it possible for the ∧-space to spatially describe whatever that is in it. This difference is, so to speak, the boundary of this infinite the ∧-space. That is, anything can be described to be within the boundary of the ∧-space if and only if it is spatially describable.

  I have roughly outlined the logical background of geometrical spaces (more details in 'The Elementals'). Before I start discussing 'numbers' themselves, the followings are the characteristics of the resultant ∧ & ∨-spaces. In short, the 2-dimension consists of two types of space and therefore of two types of 2-dimensional 1-dimensions. The 1-dimension

consists in and of two schematic points. Schematic points are therefore 2-dimensionally described twofold. In the ∨-space the two schematic points are, by their meaning, identical with the centre (i.e. one and only one 2-dimensional point) of that space and give rise to the closed boundary of that space. In the ∧-space and its fictitious version there are an infinite number of schematic points, and they form the boundary of those spaces from outside those spaces and give rise to the substance of those spaces. These two types of space are 1-dimensionally one and the same. Their difference is necessarily only 2-dimensional. The boundary of the ∨-space is internally formed, while that of the ∧-space is externally formed. This is so because the former is internally schematic, while the latter is externally schematic and is therefore a schema of schemata.

  A 1-dimension is a unit of quantity and, in the ∧-space, comes to have an infinite length. This unit is also the most basic unit. Consequently, a point of intersection is descriptively immeasurable. However, a point of intersection is quantitative because it necessarily occupies a portion of space. In the ∧-space a 1-dimension is a pair of 2-dimensional directions and therefore does not itself occupy any portion of space. This means that a 1-dimension can be described to have an infinite length in the ∧-space only in terms of points of intersection. A collection of this immeasurable quantity of a point of intersection therefore constitutes infinity, which is the most basic 1-dimensional unit of the ∧-space. Such a quantity is the most basic 2-dimensional unit and is infinitesimal. It is infinitesimal because it is static, immeasurable and is in a space, which is infinite in size. A 1-dimension can be described to consist of an infinite number of points of intersection, each of which has only an infinitesimal quantity. That is, in the ∧-space a 1-dimension is necessarily uniformly intersectible by an infinite number of other 1-dimensions. This also means that the ∧-space itself consists in and of an infinite number of centres. A number is a point in a space, it is for this reason that, although it may have a cardinality, it itself has nothing to do with the size it refers to.

  In the fictitious version of the ∧-space a 1-dimension has only a finite length and is intersected either at the very centre of space or more and more often toward the centreless centre of space. If the latter is the case, then a given 1-dimension can be described to consist of a finite number of infinitesimal points that become denser and denser toward this centreless centre. Therefore, if those points are described to be uniformly dense, then this given 1-dimension appears as if being curved toward this centreless centre. Or, if a given 1-dimension is intersected by every other 1-dimension at the very centre, then this given 1-dimension can be

described to consist of a single infinitesimal point, which coincides with the total quantity of this space.

  In the V-space a 2-dimensional point is such that from where two and only two directions can be determined so as to form a 1-dimension. This point, however, does not occupy a portion of space. This is so because a 1-dimension such that can be determined by a single point, neither intersects anything nor coincides with that point. Therefore, in the V-space a 2-dimensional point is a region of space with no quantity. This 2-dimensional point determines two and only two directions in such a way that from any part of this resultant 1-dimension they simultaneously hold. This 2-dimensional 1-dimension that forms the boundary of the V-space therefore necessarily consists of points which are not intersectible by anything. Every point of this 1-dimension is, if it can be so discerned, descriptively identical with that one and only one 2-dimensional point. This 1-dimension is closed and uniformly curved in the sense that seen from that 2-dimensional point, every part of this 1-dimension is necessarily such that can be taken up without being separated from any other parts and implies every other part. Consequently, in the V-space a 1-dimension can be described to become boundlessly denser and denser so as to descriptively coincide with that 1-dimension which forms the boundary of this space and consists of boundlessly and uniformly dense points. These points are so dense that none of them can be separately discernible from any others. Therefore, this space can be described to consist of a single 2-dimensional point with no quantity and a single 2-dimensional 1-dimension that is boundlessly and uniformly dense and therefore cannot be reduced into parts. If this 1-dimension can be discerned in terms of parts, then every one of such parts is descriptively identical with that 2-dimensional point. Between this 2-dimensional point and the boundary of this space there are points that are described to become boundlessly denser toward this boundlessly dense, closed boundary. The boundary of the V-space is therefore a 1-dimension such that becomes boundlessly and uniformly denser and can only be seen when it becomes densest (i.e. boundlessly dense). This means that if it becomes necessary to describe a '1-dimension' within the boundary of this space, such a '1-dimension' necessarily appears as if being curved toward the boundary ; for such a '1-dimension' consists of points which become denser toward the boundary, and this means that if every possible '1-dimension' is identified as a single type in terms of the uniformity in density, then '1-dimensions' whose density is not uniform are made uniformly dense if and only if it is described to be more and more curved toward the boundary. In the V-space a 2-dimensional point is either inseparable from every other point or quantitiless. Such a point

does not occupy a portion of space, and therefore its size cannot be described, except saying that it only has a 1-dimensional unit. The ∨-space is determined, and holds, between two boundaries that describe each other. That is, it descriptively holds between a single 2-dimensional point and inseparable 2-dimensional points. The only substance of the ∨-space is these boundaries themselves. The substance of these boundaries occupies no portion of space that it binds ; for, otherwise, this space cannot be described to be closed and finite. A substance can be described to occupy a portion of space if and only if it is in a space. The ∨-space therefore descriptively manifests itself in terms of the description of its two boundaries. Its outer-boundary is a 1-dimension every part of which is every other part so that two and only two directions hold at any parts of it. Its inner-boundary is a 2-dimensional point that determines two and only two directions in such a way that each starts from where the other ends, so that a 1-dimension holds at every point where two directions start and end. These two boundaries describe each other in the sense that the meaning of each underlies that of the other. Any parts of the outer-boundary are identical with the inner-boundary and therefore with one another. Neither of these boundaries can be descriptively seen without the other. Between these two boundaries there exist a 1-dimension which starts at where there are no describable quantities and expands while boundlessly becoming dense and denser and ends at where there are no describable quantities. This 1-dimension exists between those two boundaries in order to describe a space between them. This 1-dimension is the form of the ∨-space and embodies the meaning of density, while those two boundaries are the substance of the ∨-space. If this 1-dimension is described at each level of density, then there are a boundless number of 1-dimensions between the inner-and outer-boundaries. That is, between those two boundaries there is neither a space nor any substances. Those 1-dimensions are, so to speak, the descriptive substance of the two boundaries of the ∨-space. They consist of points such that become denser at each level of density that is represented by each of those 1-dimensions. The two extreme limits of those 1-dimensions are the two boundaries of the ∨-space. They are made meaningful by what descriptively exists between them. Consequently, the space between those two boundaries is filled with points and 1-dimensions that are the descriptive substance of those two boundaries and therefore have no 2-dimensional quantities. The meaning of those points and 1-dimensions is, however, identical with that of those that are in the ∧-space (and its fictitious version). This is so because the relation between the two boundaries of the ∨-space and their descriptive substance is identical with that between schematic points and spatial substances of the ∧-space (and its fictitious version). The difference is,

while in the former those two boundaries are made descriptively visible by their descriptive substance, in the latter spatial substances are made descriptively visible by schematic points, which bind the ∧-space (and its fictitious version) from outside the ∧-space (and its fictitious version). That is, schematic points are, so to speak, the boundary of the ∧-space (and its fictitious version). Therefore, from this standpoint the contents of those two types of space are identifiable.

  A given 1-dimension has a unit in the ∧-space and is, due to its infinite intersectibility, described to consist of an infinite number of 2-dimensional points (i.e. points of intersection). This unit is an infinite quantity, and its constituent points only have an infinitesimal quantity. Between every two of these infinitesimal points there is a unit which is infinitely divisible ; for in the ∧-space a 1-dimension consists of as many points as it is intersectible by different 1-dimensions. In the ∧-space centres multiplies themselves and this means that intersections multiplies themselves. Therefore, between any two points of intersection there is always at least one point of intersection. This infinitely divisible unit is a linear continuum and is the most basic 2-dimensional unit. Consequently, in the ∧-space the most basic 1-dimensional unit consists of such infinitely divisible 2-dimensional units. The unit of this infinitely divisible unit is a 2-dimensional 1-dimension which holds between two closest possible points of intersection. In the ∧-space if anything can be described, it is described in terms of such units of unit. Therefore, if relations are described between or among such units of unit, then anything can be described in the ∧-space in terms of such relations (i.e. numbers) or relations of such relations (i.e. functions). The most basic 2-dimensional unit is therefore not numbers but functions in the sense that the ∧-space necessarily consists of more than one point. This is identical with saying that the meaning of numbers is necessarily functional. An infinite quantity underlies an infinitesimal quantity, and vice versa. Neither is possible without the other. Only infinitesimal quantities can make the wholeness of a unit infinite, and only an infinite quantity can make every part of a unit infinitesimal. In the ∧-space a 1-dimension (i.e. a set of two 2-dimensional directions) consists of an infinite number of 2-dimensional points. A 2-dimensional 1-dimensions is the most basic constituent unit of such a 1-dimension and yet consists of an infinite number of 2-dimensional points ; for by the meaning of the ∧-space no two points can be conceived without at least one point between them. Consequently, there is no such as two closest points. The whole and a part therefore consists of an infinite number of 2-dimensional points. Only given a dynamically expanding infinite totality (unit), an infinitesimal can be founded. This is the meaning of divisibility.

The external description of the Λ-space differs from the internal one. This is so because the Λ-space is externally one and only one space that consists in and of an infinite number of centres, while it is internally an infinite number of identical spaces that consists in and of one and only one centre. The description of the Λ-space in terms of centres differs from that of centres in terms of the Λ-space. The Λ-space can be described as the totality of an infinite number of centres. These centres, however, can only be described to be such that any one of them could have been the centre of this totality that they themselves form. A space of centres is necessarily such that ;

  (i) if it is seen externally (i.e. from the collective standpoint of centres), it is a totality with no centre,

  (ii) if it is seen internally (i.e. from the individual standpoint of each centre), it is an infinite number of totalities with one and only one centre.

This is so because a space of centres is necessarily a space that is identical with every possible centre. Consequently, its internal description consists of two parts : one is the self-description of a centre as the centre, the other is the self-description of the centre as a centre. That is, the description of a centre forms the centre, and the description of the centre forms centres. The difference between the external description and the internal one is a descriptive necessity and is therefore not a property of the Λ-space itself. In a space of centres any centres can be the centre. However, one and only one centre can describe itself as a centre and becomes the centre. This is based upon the descriptive necessity that no relations can be described between or among identical descriptions. Whatever that is once understood does not require itself to be understood again. By this descriptive necessity the description of a centre as a centre, leads itself to an infinite number of identical spaces with one and only one centre and results in one and only one description of such spaces. Consequently, that difference is not a property of the Λ-space but a necessity that the Λ-space imposes upon itself so as to comply with the ontologico-notational condition of description.

  Whichever centre is taken as the centre, the Λ-space remains identical. Every centre has at least two intersecting 1-dimensions, two and only two of which are the determinant 1-dimensions of that centre. Such two determinant 1-dimensions of the centre are also the determinant 1-

dimensions of the ∧-space and form four 2-dimensional directions that perpendicularly extend from one another. These four perpendicular 2-dimensional directions embody the uniform density of the ∧-space. This is so because this uniform density is a necessary characteristic of the ∧-space and is therefore necessarily represented by whatever that determines the ∧-space. These four perpendicular 2-dimensional directions can spatially determine every substance and every combination of them in the ∧-space, based upon the meaning of a centre that any centres could have been the centre. This is possible because every centre is related to every other in their identical reference to the centre in the sense that the centre represents the uniform density and infinity of the ∧-space. That is, the two determinant intersecting 1-dimensions of the centre embody the uniform density of the ∧-space by forming four perpendicular 2-dimensional directions that infinitely extend from one another and therefore also represent the infinity of the ∧-space. Such four 2-dimensional directions can transpose the centre to any positions in the ∧-space and therefore describe every possible centre of the ∧-space. This is so because these four 2-dimensional directions are described to consist of an infinite number of points that are infinitely and uniformly dense, and are also described to be related to one another in such a way as to be able to determine every possible position in the ∧-space. That is, every centre can be the centre and therefore inherently has two determinant intersecting 1-dimensions which are necessarily identical with those of the centre in terms of the way by which they embody the uniform density and infinity of the ∧-space. Unless they are ones which descriptively constitute the four perpendicularly related 2-dimensional directions, every centre is necessarily in one of the quarters of the ∧-space and therefore can be uniquely determined by means of a set of two points each of which comes from the two surrounding 2-dimensional directions of a quarter to which a given centre belongs. The meaning of such a set of two points is based upon the necessity of the ∧-space that every centre is determinable inherently in the same way by which the centre is determinable ; for the centre is the description of a centre. The two determinant 1-dimensions of every centre other than those of the centre, however, do not form four perpendicular 2-dimensional directions. This is so because the centre stands for the description of every centre. Consequently, the two determinant 1-dimensions of any centres can be spatially determined by those of the centre and therefore descriptively transform themselves into the internal meaning of any centres which can be described to be determinable by two intersecting 1-dimensions. This makes the ∧-space a space of infinitely dense, uniform lattice that can be described as the spatial self-multiplication of the four basic perpendicular 2-dimensional directions of the centre. These four basic 2-dimensional directions are the

form of the internally described ∧-space and stand for the meaning of the Cartesian x-y coordinate. They become the x-y coordinate with the introduction of numbers. With this x-y coordinate, the ∧-space becomes the space of an infinite number of pairs of real numbers. In this space of pairs of real numbers any 2-dimensional directions and 2-dimensional 1-dimensions can be described as a function, which is a relation between two pairs of real numbers, based upon the properties of numbers. Any combinations of them are therefore described as a relation of functions or a function of functions. A 2-dimensional 1-dimension holds between two nearest possible pairs of real numbers and is the most basic 2-dimensional unit. Such a 2-dimensional unit underlies the principles of differentiation and also makes the meaning of a number essentially functional. In the ∧-space there is no such as a 'curve' in the sense of the ∨-space. A 'curve' is merely a functional combination of 2-dimensional 1-dimensions. The notion of 'π' is introduced by the descriptive necessity that the two types of space are necessarily under a same dimension. The notion of 'π' is geometrically transcendental because logic precedes geometry, and therefore because not every logical relation can be geometrically describable. That is, unlike in the logical space the logical relation between the two types of space geometrically remains descriptively incommensurable. In the same sense the π-constant is algebraically transcendental. This is so not in the sense that the π-constant is a non-algebraic number but in the sense that geometry precedes the schema of numbers, and therefore that not every part of geometry is numerically representable. The ∧ & ∨-spaces are geometrically and algebraically incommensurable to each other because they are originated in the logical space. This means that their relation can only be described logically. This is the meaning of the transcendence of the notion of 'π'. The π-constant differs from an irrational number in the sense that it cannot even be 'pointed at' as a gap on a sequence of real numbers. This is so because assuming that both types of space can be numerically represented on a same sequence of real numbers, the notion of 'π' exists between those two types of space, and not in each of them. The meaning of the notion of 'π' is the descriptive necessity that the two types of space are necessarily under a same dimension. This means that the two types of space cannot coexist independently from each other under the same 2-dimension, and therefore that it is necessary for each to be able to accommodate the other. With the introduction of the notion of 'π' there are no combinations of 2-dimensional 1-dimensions that cannot be described in terms of functions. Numbers can only be geometrically generated. Consequently, the x-y axes relate to each other in the exactly same way by which the two determinant 1-dimensions of the centre of the ∧-space relate to each other. The meaning of a type of numbers is a geometrical property. '0'

geometrically stands for the descriptive necessity that the two determinant 1-dimensions of the centre necessarily form four perpendicular 2-dimensional directions by intersecting each other. Consequently, '0' necessitates the x-y axes to differentiate themselves into four numerically (i.e. functionally) symmetrical sequences of numbers which infinitely extend from one another. + and − stand for such a symmetry. The two determinant 1-dimensions of the centre also determine the ∧-space. Therefore, '0' also means that it is necessary for any two identical sequences of number to be identified under a same schema. This means that '0' is necessarily a 2-dimensional number. '0' can be transposed to any 2-dimensional positions by means of a function. This is so because '0' can be related to any 2-dimensional positions by means of a functional relation of 2-dimensional 1-dimensions. This is the meaning of the x-y coordinate. The meaning of a number is in the totality of numbers ; for a number is essentially functional. A number is, in itself, meaningless. Numbers are necessarily geometrical, and once numbers are understood to be spatial entities, 'positions', then the question of arithmetic is a matter of a slide rule. Russellian pursuits of the foundations of maths are misguided attempts to search a conceptual creator in conceptual creations.

- Essential Transcendentals -

   I briefly summarised the logico-geometrical outlines of the ∧ & ∨-spaces. In short the ∨-space (henceforward referred to as the ○-space) is the 'circle', and the ∧-space (likewise referred to as the ┼-space) is the Cartesian coordinate of real numbers. The former is closed by means of infinity, the latter is open, infinite and dynamic.

   Considering the essential properties of numbers, cardinality and ordinality, the number line of natural numbers is descriptively x = y, given the 2-dimensional Euclidian space, not x or y, because cardinality can only be expressed as positions against both x and y, which are the determinants of this space and are not by themselves indicators of cardinality. It is thus that x = y already contains irrationals. It is a descriptive necessity that a position on x is correlated with a certain position on y, that establishes cardinality common to, and in relation to, x and y. Further, it is the notion of place-values of the number line that gives an internal structure not expressible by the 2-dimensional space. The number line expressed by numbers without place-values has only limited descriptive capacity because it is the structure of numbers that describes number-applicable-objects by allowing to translate their structure into number structures. Infinite directional magnitude only

describes itself, like ordinality expressed in the ○-space. This internal structure of number line is essentially supported by the logical relation between ∨ and ∧. That is, the base of the ∧-space is the ∨-space, which is essentially a self-defining unit. The infinitely expanding number line must be descriptively unitized in order to correspond with the ○-number line. Otherwise, the ∨-space and the ∧-space are not mutually describable. Only in this way, the closed ∨-space and dynamically expanding open and infinite ∧-space have transcendental but describable mathematical relationship, based on their common logical origin. Maths can only be the mutual description of the ∨-space and the ∧-space.

  It is taken for granted that natural numbers are the base of all numbers. This is only so arithmetically as we start our first lessens of maths by counting. However, our acts of counting already assume a space in which to locate and differentiate objects, and as such, so-called natural numbers are not only cognitively but also epistemologically more sophisticated than real numbers. Reals are by themselves just a continuum, like a 'line', and it is natural numbers that assign descriptive sophistications to reals. In the ┼-space points are positions locatable in their relations to the determinants. There are no two closest points, as a 2-dimensional 1-dimension (line of points) is infinitely intersectible. This means that in any given line there are no units unless assigned from the outside. The sequence of reals is continuously expanding inwards and outwards infinitely, whereas naturals are a sequence of units with an assumption that units are universal and equal (hence '+1'). Naturals are superimposed onto reals because of the descriptive necessity of a space to describe itself. Remembering the centre (0) of the ┼-space is a centre that describes itself as a centre, and that its transpositionability needs points that can be described in their relations to the determinants, a real has to be determinable in its relation to the totality of the ┼-space. In another word, reals have to be unitized so that they acquire a paradigm of describability. Reals acquire units and external directions due to naturals.

  'Units' assignable to naturals are augmented by the ┼-space, instead of being taken for granted without reasons, as the ┼-space is a space that is uniform and expands infinitely and dynamically and thus gives 'units' descriptive legitimacy. 'Units' in the ┼-space are therefore uniform and acquire operative directions as well as other essential spatial properties such as continuity, infinity and symmetry. The ┼-space becomes operatively descriptive by this superimposition of naturals onto reals. Numbers here become possible to be viewed in their totality and can be used to describe the space in which they are in. The Peano form of naturals,

(0, n, n+1)

, is not a set of definitional axioms, but an expression of a spatial property of the ┼-space, where '0' is a centre, 'n' is a position correlatable to 0 and 'n+1' is a directional quantity n has in this space. Reals are substances, and naturals are forms, of the ┼-space, and together they allow the ┼-space to describe itself. Paradoxes of numbers are here linguistic confusions, once you realize they are expressions of spatial properties of the ┼-space.

  Numbers as 'sets' are definitional abstractions because empirical cardinality has to be distilled into an abstract concept in order to be seen to be universally applicable, in the process ending up with 'cardinality' that should not be cardinal. Whereas, 'numbers' in a schema of space are spatial entities, 'positions', which do not require any further conceptual abstractions, because their applicability is replaced by that of their schema. Here instead of 'set of sets' you question the universal applicability of the schema of a space, not 'position of positions'. 'Numbers' are here applicable to anything spatially cognizable and are as universal as a space that gives rise to them. They are also applicable to anything spatio-temporal if further dimensions can be demonstrated to evolve from the 2-dimension. Similarly, the applicability of numbers is allied to characteristics of a spacetime schema. It is thus that in dealing with entities of a quantum field you need inventing new rules of numbers that are not applicable to our more commonplace schema of a geometrical space. However, insofar as you cannot invent totally new rules as if fallen out of the blue sky, and that 'new' rules are based on, and connected with, old rules in order to be understandable, any fancy schema of spacetime is ultimately descriptively rooted and conceptually bound by our good, old geometrical spaces. In short, numbers are as much applicable as geometrical spaces that give rise to them are applicable. Likewise, maths is as much applicable as the logical space that gives rise to maths is applicable. Uniquely to numbers, which are also tools of approximation, this applicability also works on approximation. In the Kantian flow of mind since numbers are as a priori as space and time, reducing the 'necessity' of numbers into the necessity of space and time is a good application of Occam's razor.

  Reals are unitized by naturals, and on one hand give naturals the spatial property of a continuum (a tool of approximation), and on the other acquire the spatial property of a direction (spatial totality). They together describe the ┼-space as a dynamic and infinite schema of schemata of

directional quantities, which consist of 0 (centre) and 1 (transpositional 0) locatable on a sequence of number line determinable in relation to the determinants. It is also this aspect of the ┼-space as a schema of schemata that allows it operative freedom. Numbers are not affixed to objects as if together forming proper nouns. An object, by occupying a position, does not own that position, because positions are freely available to be occupied provided that it satisfies the rule of a spatial schema (and the associated usual Euclidean rules, save for the parallel postulate that applies differently to the ○-space and their common fictitious space), within which a same object cannot occupy more than one position at the same time. Interestingly a same object can occupy more than one position simultaneously if belonged to different schemata that can be descriptively connected (e.g. the notion of 'events' in the probability). Given a schema of schemata, positions become 'positions' because the ┼-space is the space of directional quantities, once an intersection takes place and objectifies the space itself, like turning an empirical number into a higher conceptual one. In contrast, the ○-space is itself a schema because here an object and a space coincide. The ┼-space is the descriptive space of spaces and is numerically more descriptive. The ┼-space acquires further sophistications by descriptive necessities of describing itself. This is the prime describability of the ┼-space. The first of descriptive developments is a 'place-value'. A number line consisting of reals and naturals is a continuous sequence of numbers milestoned with naturals, having come into existence by intersections and having acquired a spatial direction by the infinite expansion of the ┼-space.

The ┼-space and the ○-space are identical outside the 2-dimension. Given the 2-dimension, they describe each other via their transcendental relation, which, however, primarily takes the form of the ┼-space describing the ○-space, because numbers are products of the ┼-space. This gives rise to descriptive multi-layerization to the 2-dimensional plain and allows 'curves' into this space. The limited descriptive capacity of infinitely extending number line acquires numerical structures expressed by translating the linear magnitude into a unitized structure in terms of curves. It is positional expressions of a unitized number line that gives rise to logarithmic curves and turned maths into an art of approximation. That is, there is a descriptive necessity that numbers are expressed through place-values, the simple ones being seeing numbers through binary or decimal positional notation. However, the most natural one is e, because e reflects the transcendental relation between the ∨-space and the ∧-space, in that the ∨-space is essentially self-defining 'growth' defined in terms of continual density to a limit, whereas the ∧-space is the inverse

of this continuous infinitesimal quantity and is self-defining 'infinity' which expand openly and dynamically.

   An infinitely extending number line without place-values is like a sentence without ','. ',' signifies space and is the most fundamental operator of description, which, however, follows the ontologico-notational laws of descriptive necessities. ',' is the conceptual place-value as much as there will be no mathematical descriptions without positional notation. Both mathematical place-values and conceptual place-value originate from the ╫-space, which is defined by intersections of schematic directions and results in points that constitute a line. The most fundamental place-value of a number line is therefore not binary or decimal, but unary, which manifests as binary or decimal, etc. because it cannot be described otherwise. The unary notation as we understands already contains 'e' because the idea of ',' between unary numbers assumes a 'place-value' of the multi-layerized 2-dimensional plain, the ∧-space upon the ∨-space. This is also the reason why e is such a fundamental constant in maths and connects the ∧-space with the ∨-space.

   Geometrical points descriptively manifest not as numbers but as a place-value. Representing the unary numbers not as

   11111111111⋯

, which is notationally meaningless, but as

   1, 11, 111, 1111, ⋯

1111⋯   'e'   1, 11, 111, 1111, ⋯
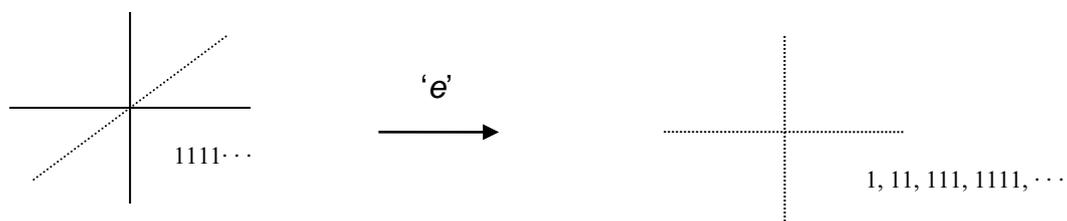
, this geometrically corresponds with the dotted line because ',' can only be space definable by the two axes which are the determinant of this 2-dimensional space. However, because the two axes are also describable as the same as the dotted line, as much definable as by any two invisible axes, between the seemingly identical dotted line and axes is hidden a 'curve'. This ontologico-notationality of the two determinant lines giving rise to a number line creates a 'tension' in the ╫-space. Underneath the

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tension are a 'curve' and the descriptive necessity for approximations, which is the essence of meaningful maths. That is, the ┼-space incorporates the ○-space as angular tension between seemingly identical number lines, two of which define another as identical but only as spatial relation between them. It is thus that between (0,1) and (1,0) is (1,1) which is not locatable by the either number line so far self-defined.

  The unary number line is not

  11111111111· · ·

but

  1, 11, 111, 1111, · · ·

because ',' intrinsically incorporates the two determinant lines and space between them, which manifests as 'value-place', not locatable onto the two determinant lines. '*e*' is an expression of seeing the determinant lines, 11111111111· · ·, from 1, 11, 111, 1111, · · · and is the ┼-space numerical value of ','. That is, numbers located on either determinant line can only be directional quantities of 11111111111· · ·, which is ontologico-notational and consists in points that accumulate 'thickness' from the point of intersection (0) towards dynamically expanding outer fringe of ∞. In contrast, a number line defined by the two determinant lines consists of 1, 11, 111, 1111, · · · because it has spatial depth that gives numbers epistemic meaning defined in terms of a point of intersection taken as a centre (0), points of intersections (numbers) and a direction created by infinite expansion. Given spatial depth, this manifests as 'place-value' that signifies numbers as points, which are individual entities with 'distance', which is intersectability, between them. Between

  A) 11111111111· · ·

and

  B) 1, 11, 111, 1111, · · ·

there exists a descriptive relation of A → B, which is 'place-value', and B → A, which is the idea of *e* expressed as a number in terms of 'place-value'. A number is a point of intersection, whereas a place-value is a space necessitated for intersection. A number, therefore, can only be meaningfully expressed via a place-value, and ',' is a self-referential

concept that is applied onto itself in such a way as to create an entity out of space. That is, 1, 11, 111, 1111, · · · is a description of 11111111111· · · as much as ',' is a description of points of intersection. 11111111111· · · intersects 11111111111· · · by a descriptive necessity based on the logical connective ∧ and gives rise to a centre and space, which then describes 11111111111· · · as 1, 11, 111, 1111, · · ·.

  Between 11111111111· · · and 1, 11, 111, 1111, · · · is a tension because 'space' defined by 'line' only become visible by 'distortion'. This is where 'approximation' becomes the essence of maths. A place-value distorts a number line as a line acquires 'width' as it were, on top of thickness. This is conventionally expressed as progressively inverse relation between exponentiation and division or continuous fraction, etc. and numerically materialises as *e*. *e* expresses that numbers only become descriptive by incorporating ',', which means the ╋-space inherently incorporates the ◯-space in order to be descriptive. The ◯-space is self-defined or internally generated (1-D = 2-D), whereas the ╋-space is externally generated via two intersecting 1-dimensions.
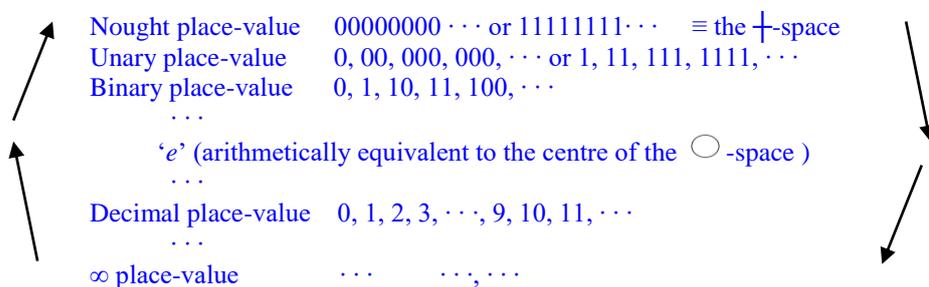
  The infinitely extending number line that consists in and of points is not descriptive. Underneath 11111111111· · · is the dynamically expanding infinite space that gives a direction, and therefore thickness, to the series of points. The totality of natural numbers, N, cannot be talked about as a directionless group (set) of points because infinity of numbers needs this dynamism of expansion to acquire direction. The self-referential paradox of a set of set is avoided as this direction cannot come to a stop, without contradicting describablity. That is, infinity is only infinity as long as the dynamic expansion continues, and as this dynamism cannot be captured without space, '· · ·' of 11111111111· · · does not descriptively suffice a set. It is the ontologico-notational necessity of the ╋-space that defines a space, which gives rise to descriptive numbers 1, 11, 111, 1111, · · · between the two intersecting 11111111111· · ·. Thus, a place-value is not just a numerical convention, but the most natural ingredient of descriptive numbers and *e* is therefore a key component of mathematical descriptions.

  A 'place-value' is conventionally expressed as the summation of an infinite series of $1/n!$ with n ranging from 0 to ∞, because n! stands for a series of possible permutations of numerical objects which possible place-values necessitate, which then divide 1 (point) in order to produce a descriptive value of a place-value. Natural numbers, n, starts with 0, not with 1 as sometimes assumed, because the ╋-space necessarily has a centre (0). Thus the higher a place-value is, less descriptive it becomes

because the wider a number line is, less precise it can point. On the other hand, the lower a place-value is, the more impotent it becomes because a number line without any width is just a line of points and only describes itself. That is, a place-value and a number are descriptively inversely related, with the proviso that the place-value of 0 has no descriptive capacity.

11111111111··· is a number line without any place-value, and 1, 11, 111, 1111, ··· is a line with a place-value of 1. *e* points to a place-value with ideal descriptive power, i.e. with the most descriptive width to an infinitely expanding number line. This line represents spatial tension of the ┼-space at the ideal befitting the convention. A straight line in the ┼-space describes little other than this space, while a curve is decisively more useful because it describes the ◯-space ontologico-notationally inherent in the ┼-space through logical relationship. The space defined by the determinant number lines (11111111111···) contains descriptive number lines (1, 11, 111, 1111, ···), which acquire width and thus create spatial tension and introduce curves into the space of straight lines.

This width of a number line is represented by a place-value, and points as defined by intersecting directions have no size, but this line of points (11111111111···) acquires width because of ',', which is space defined by two intersecting 11111111111···. 1, 11, 111, 1111, ··· only exists between two intersecting 11111111111···. 1, 11, 111, 1111, ··· becomes a descriptive line with '*e*', dispensing with defining 11111111111···, which becomes descriptively invisible, as it were. The width of a number line is a number line accompanied with its defining space. This is the meaning of '*e*'.

Nought place-value	00000000··· or 11111111···	≡ the ┼-space
Unary place-value	0, 00, 000, 000, ··· or 1, 11, 111, 1111, ···	
Binary place-value	0, 1, 10, 11, 100, ···	
···		
'*e*' (arithmetically equivalent to the centre of the ◯-space)		
···		
Decimal place-value	0, 1, 2, 3, ···, 9, 10, 11, ···	
···		
∞ place-value	··· ···, ···	

| | |

The ◯-space

A width (thickness) of number line compromises between describability and precision. The 'value-place' notation is arithmetically synonymous with the ○-space number line because, as the ○-space number line loses ordinality at a limit of cardinality and becomes circular, the place-value number line also becomes circular in terms of width of number line, in that the nought place-value number line becomes descriptively analogous with the ∞ place-value number line. e stands for ideal width in terms of the balance between describability and precision.

The notion of 'precision' sounds alien to maths as maths is idealized as paragon of descriptive precision. Contrary to this superstition, maths is really an art of approximation, starting with identifying (descriptively approximating) every object as 0 for the sake of countability. Insofar as 'place-value' is not an arbitrary convention but necessary part of number descriptions, some place-value is more precise for a purpose and is therefore useful to be adopted as term of reference for descriptions of spatial distortions.

It is interesting maths works out e without even knowing what really it is, and gives us some insight into the scope and scale of human intelligence embedded in a notation, which it creates. The conventional e stands for an ideal balance between 'number' and 'place-value' and is deemed one of the most important mathematical constants alongside π, i, 0 and 1. The tension arisen because of width of a line consisting in and of sizeless points spatially manifests as 'curve' and arithmetically materialize 'approximation' as the essence of maths. In conceptual descriptions ',' is the only meaningful self-referential concept because it signifies space, of which the meaning is self-defining in terms of ontologico-notationality of whatever is describable and understandable. In this sense ',' is the same as 'I'. e, whatever numerical value it is conventionally assigned, is a way of expressing spatial tension between $11111111111\cdots$ and 1, 11, 111, 1111, $\cdots$ in terms of combinations of linear notations such as addition, division, factorial and 1 as applicable to number lines and manifests as a curve in the world of lines. This curve then describes space as approximation of linearity.

The self-referential paradox in a manner of a 'number of numbers' is avoided in case of 'place-value' numbers. As a place-value approaches ∞, numbers lose descriptive meaning and end up synonymous with a singularity number as of the ○-space. That is, a number line with ∞ 'place-value' describes nothing, which is against the meaning of 0 as approximation of describable objects. As a number line in the ○-space loses ordinality at a limit of cardinality, a number line in the ┼-space

loses a descriptive power at a limit of place-values. Thus arithmetical 'place-value' plays a similar role to geometrical recursiveness of the ○-space and is a way of descriptively assimilating the ○-space into the ┼-space as the latter is numerically more descriptive by virtue of being the basis of descriptive numbers, and as the former is the logical base of the latter. The logical structure is thus represented in the structure of the totality of numbers in the form of describablity of ∞ 'place-value'.

If naught 'place-value' and ∞ 'place-value' are descriptively to merge and pervade describability in preference for precision, then somewhere between naught 'place-value' and ∞ 'place-value' is describablity ideally balanced on precision. This must be the notion of e, which describes 'place-value' is not a linear number but a recursive connector of 0 and ∞, much as 0 and ∞ merges in the ○-space. Such a connector is to be found in a constant, which is a 'place-value' of place-values, defined as summation of every possible permutations, given mathematical objects identified by 1 (point) and starting with 0 (centre).

While natural numbers extend from 0 to ∞ on their own, natural numbers as place-value numbers run from 0 to ∞ in such a way as to become circular, linear extension being constrained by the logical base of the ┼-space, i.e. the ○-space, which manifest as the balance between describability and precision as neither space can be fully described in the other. Their transcendental relation originates from their logical relation and can only be described logically. Numbers with place-value number 0 and those with place-value number ∞ are identical at a limit, in the sense that they are both straight number lines with no width and describe nothing but themselves. An ideal balance thus excludes any place-value numbers that are large on one hand and that are less than 2. It is conventionally set at the summation of an infinite series of 1/n! with n ranging from 0 to ∞ and expresses average width of number lines in terms of ratio of a 'whole' over its reconstructed parts. That is,

0) 11111111111⋯

turns into

1) 1, 11, 111, 1111, ⋯

by spatializing itself as width and becoming descriptively invisible, and thus transforming precision into describability. Once given 1, 11, 111, 1111, ⋯, then by virtue of numerical progression

2) 0, 1, 10, 11, 100, $\cdots$

$\cdots$

10) 0, 1, 2, 3, $\cdots$, 9, 10, 11, $\cdots$

$\cdots$

∞) $\cdots$ $\cdots$, $\cdots$

, which is ∞ place-value number line and become identical with $11111111111\cdots$ at a limit. e comes into play as a constant applicable to natural numbers to turn them into place-value numbers in a self-referential way and shows spatial quality assigned to each and every sizeless point. That is, a number as representing a point has no size by itself. Its cardinality and ordinality are relationally found and are based on spatial quality assigned by width of number line. The self-referential paradox of a number of numbers is avoided because a totality of sizeless points has no size, likewise a number of numbers per se is not a number without space that brings about cardinality and ordinality. Since space is expressed as width of a number line, a number of numbers only refers to its structure as a totality. Space assigned to sizeless objects, which then acquire a size, is the only self-referential paradox that is not a paradox but a tautology.

That there is 'e' somewhere in the loop between 0 and ∞ is founded on

1) precision per se describes nothing as a number line without place-values only describes itself,

2) describability is assigned by a width of number line which is finite as a 0 place-value number line and a ∞ place-value number line are identical at a limit,

3) between a number and a place-value number is self-referential paradox ₪ tautology.

e is therefore not an object but a relation expressed in terms of paradox/tautology. 'Numbers' which contains ∞, is self-contained by ∞, as ∞ becomes identical with 0 at a limit when 'numbers' incorporate space into them for the sake of describability. The necessity of numbers to have place-values makes approximation as the essence of maths. e is the epitome of the describability of numbers in terms of place-values, as

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the summation of an infinite series of all natural numbers expressed as spatialized points permeated at every level of cardinality. It stands for the description of cardinality recursively incorporated into ordinality at a limit as the ┼-space is logically contained in the ○-space. That is ∞ self-contained in ∞, which is self-referentially a paradox ₪ tautology. *e* is thus a constant essential to expedite the describability of numbers, as an essential tool for approximation. A whole is divided and exponentiated so that between a whole and its reconstructed whole holds operative meaning by which a whole is described through its parts. *e* facilitates such a description. The ┼-space's descriptive power is thus essentially enhanced by being incorporated into its logical basis of the ○-space.

  e appears in an attempt to describe the ┼-space via the ○-space, and therefore the descriptive inverse of *e* is the description of the ○-space via the ┼-space. This is a singularity number that loses ordinality at a limit of cardinality. If it is possible to apply this singularity number operationally, this will dispel spatial distance as every point becomes a starting point as well as an ending point. Dimensional travels will require such a number.

  In the ┼-space the cardinality and ordinality come hand in hand because this space is a cosmos made descriptive by quantitative uniformity (density), dynamical infinity (directions) and transpositional identifier (0), while the same applies to the ○-space only as process and at its limit all the descriptive qualities disappear into singularity, which most typically destroy the ordinality. The resultant singular number therefore has the cardinality of countable ∞, but no ordinality. Among the essential transcendental numbers *e* is one which most directly connect the ┼-space with the ○-space, whilst π is the numerical representation of the ○-space in the ┼-space under the necessity of the 2-dimension by means of ┼-numbers, as the ┼-space generate descriptive numbers. *i* is the numerical representation of the common fictitious space derivable from the ┼-space and the ○-space to show the ┼-space and the ○-space are one and the same outside the 2-dimension.

  Numbers are not just a quantitative numerical series. There are fundamental qualitative differences between some numbers. It is these differences that allow mathematical operations. The most fundamental difference is between 0 and every other number. Without 0 there will be no arithmetic. That is, there will not even be $1 + 2 = 3$ without 0 because every number in this assumingly simple operation is identified by 0. $1 + 2 = 3$ is possible because it is essentially $1(0) + 2(0,0) = 3(0,0,0)$. A conceptual confusion occurs due to the fact that 0 is also a quantity

between −1 and +1. This idiosyncratic nature of 0 as a quantitative identifier as well as being a quantitative signpost is not fully appreciated. 0 is a number, but is, in fact, the number, i.e. the only number. We operate this 0 as mind can find a purchase based on this idiosyncrasy of 0. Otherwise, numbers will become an invisible abacus like the maths of pre-0 days, i.e. numerical cognitions of finger movements, which were all we needed for day-to-day life of Arcadia. The conceptual leapfrog based on the discovery of 0 as the building block of numbers and its unique operative difference from what it builds results in the arithmetical structure of numbers.

  The accuracy and validity of $1 + 1 = 2$ is guaranteed in the fact that we and the world coincide in our description of our world. Maths is useful only insofar as we describe ourselves as objects of the world, which has no choice but being our world. Maths is our maths. This is the meaning of 0 as identifier. We identify every empirical object as a 'thing in itself' so as to remove conceptual (linguistic) facets and layers. Thus, the world becomes the collection of 0's, which follow its internal structure in order to be ordered. That is, the ordering of 0's is synonymous with the description of 0's, which complies with the descriptive necessities (logic) of the atomic symbolic form (FX). The resultant numbers of various kinds represent spatial structures of our descriptive space. In essence, maths is valid because maths mirrors ourselves, which is our world, which is the world. A grand tautology.

  To try to describe how numbers work by means of numbers is maths, but to attempt to describe what numbers are by numbers is a notational self-reference. In the number theory tools of description become objects of description in the sense that they contemplate on numbers by numbers. This is ultimately true of all areas of maths, but it is mitigated here that numbers are, once taken for granted, tools to model on something else, be them geometrical objects or empirical phenomena, in either way applying rules of numbers on modelled entities, resulting in useful approximations. The number theory may be the queen of maths but is equally an incestuous bastard of maths, because you cannot define numbers by numbers. Thus, to this day we do not know what primes are (PNT). When formal logic and maths are thought independent, there were attempts to reconcile the latter with the former because the former was regarded more fundamental. But that they are not independent and share a common root, means they both have to be described by this common root. This is the same as saying ; given (x) › x, x can be described by x, but (x) cannot be. Otherwise, the hypothesis need not be a hypothesis, but a description. This is why maths is descriptively nothing but

approximations at best, while the philosophy of maths consists of hypothesises and metaphysical speculations, despite their certain intuitive appeals. We at best can only appreciate through our own linguistic totality (Boolean wholistic reference).

  The infertility of set theories is the assumption of the construction of (x) from x. It either constructs without acknowledging (x) › x and falls into a Gödelian loop if formalistic approaches are made or becomes a game of mind without realizing mind is toying with itself. This summarises von Neumann-esque set theory of constructive numbers, which are humanly designed to explain, rather than ontologico-notationally to represent, mathematical objects. The construction of numbers out of

  $$\{\{\ \}, \{\{\ \}\}, \ldots, \{\{\ \}, \{\{\ \}\}, \ldots\}\}$$

is simply a clever technique augmented by ingenious definitions to avoid self-referential paradoxes and does not add anything fundamental to our knowledge of numbers. We are none the wiser as to the philosophical nature of numbers because $\{\{\ \}, \{\{\ \}\}, \ldots, \{\{\ \}, \{\{\ \}\}, \ldots\}\}$ already presupposes primitive numbers and thus the use of ',' without any real definitions. It is the acceptance of the numbers that is the basis of the constructions of numbers. Likewise it is the acceptance of the wholeness of language that is the basis of mathematical elaborations of numbers. In von Neumann notation { } is counted as the unit to construct numbers. Hence, 0 is the container of naught, instead of being naught. Here 'mind' the counter, is counting something that cannot be counted as a class of something that cannot be counted, thus $\{\{\ \}\}$, i.e. there is 1, that is the class of something that cannot be counted, that was counted. If there is 1, then there are the class of something that cannot be counted as well as the countable class of the class of something that cannot be counted, thus $\{\{\ \}, \{\{\ \}\}\}$, and so on. This is really not much of advancement from Fregean conception of naught, that is something that which is not itself. Here 'mind' is the pontificator that stands outside that which is not itself, and says there is something that is not itself, and that is 'naught'. While if there is something that is not itself, this should have been invisible to 'mind'. For von Neumann 'mind' the counter takes itself as hostage and ends up as the ultimate self-referential loop. That is, 'mind' engages in the folly of counting itself because it is the counter. By counting something that is not countable like itself 'mind' acknowledges it can always go beyond the line it draws around itself because it is the drawer of line, i.e. 'mind' the freedom fighter. Thus, the counter becomes the counted and keeps repeating the process of being the counter and becoming the counted. von Neumann relies on this process's being

infinite and assumes it is therefore directional quantities. However, it lacks any guarantee that {{ }, {{ }}, …, {{ }, {{ }}, …}} is in fact not { } because his notation is just a gimmick which depends upon the good-naturedness of 'mind' to submit to the mental dynamism of looping. The versatility of mind, the interpretability of empirical facts and the linguistic flexibility all combined, it is all too easy to create greys where even the law of excluded middle has a place of denial. Numbers reside in such greys of 'mind' where paradoxes are modified as notational dynamism by hijacking 'mind' which utilizes, and is utilized by, notations which it creates. We talk about the applicable power of maths. This is, however, like being surprised by ourselves. It is the innate necessity of mind to turn the phenomenological chaos into the epistemological cosmos. This starts with counting and ordering. We thus create the world first by numbering. We create our world trough numbering so that we can describe our world by numbers. This powerful tautology underlies our knowledge.

  This takes place because 'mind' perceives numbers as algebraic objects and seeks meaning in empirical counterparts. I deconstruct numbers geometrically as the logical necessity of intersection (0 as a centre) subsequent numbers as transpositional 0 from 0 as the centre (i.e. thus determinable in their relation to 0), where directions are provided by dynamically expanding ∞, hence also numbers as directional quantities. Reals, naturals, integers, irrationals, imaginary, etc. are all spatial properties of the ┼-space and the ○-space under the 2-dimension. Above all, 0, 1, e, π, i are the most important numbers in the sense without them there will be no maths. The numerically more descriptive power of the ┼-space gives rise to numerical values of the essentially transcendental numbers, e, π, i, whereas 0 and 1 (transpositional 0) are uniquely ┼-numbers. Together they form the most essential tools of approximation. Natural numbers are the descriptive form of recursiveness and units of real numbers, integral numbers are that of symmetry, and rational numbers are that of infinite divisibility, while irrational numbers stand for the necessity for the determinants to relate to each other. Consequently, irrational numbers cannot be located on either of the sequence of numbers that consists of natural, integral and rational numbers. They exist necessarily between those sequences and therefore only as gaps in a sequence of natural, integral and rational numbers. Real numbers therefore consists of natural, integral and rational numbers together with gaps among them. 'Gaps', nevertheless, as real as necessities of intersections. An imaginary number is the descriptive inverse of irrational numbers and is therefore one, and one only. This is so because it is found in the common fictitious versions of the ┼-and ○-spaces. The fictitious

versions of the $+$-and $\bigcirc$-spaces are common to both types of space and are therefore identical with each other, but are descriptively based upon the adversative of the descriptive necessity of each type. They are also descriptively a single space that is identical with the description of its own centre.

The $\bigcirc$-space and the common fictitious versions of both types of space are commonly known as non-Euclidean spaces. The non-Euclidean geometry consists in descriptions within these spaces. These spaces, under the 2-dimension, descriptively but transcendentally incorporate the $+$-space. This is so because the $+$- and $\bigcirc$-spaces necessarily describe each other in each space, and therefore because their common derivatives contain both elements. This means that the $+$-space provides the $\bigcirc$-space with the notion of a 'straight line', while the $\bigcirc$-space provides the $+$-space with the notion of a 'curve'. The notion of π stands for this pair of notions. A 'straight line' and a 'curve' are transcendentally identical outside the $+$- and $\bigcirc$-spaces. The notion of π is the bilateral form of mapping between the two types of space. The common derivatives of the $+$- and $\bigcirc$-spaces are therefore provided with both notions of a 'straight line' and a 'curve'. They are derived by assuming the impossibility of parallel 2-dimensional directions in the case of the $+$-space, and by assuming the finite density of the outer-boundary in the case of the $\bigcirc$-space. There are two versions of them. In the case of the $+$-space, parallel 2-dimensional directions are impossible

(i) Version 1 : because this space has one and only one 2-dimensional point, or

(ii) Version 2 : because this space has one and only one region of space in which 2-dimensional points become denser toward the centerless centre.

In the case of the $\bigcirc$-space, the finite density of the outer-boundary of this space is possible in terms of boundlessly dense 2-dimensional directions

(i) Version 1 : which share one and only one 2-dimensional point,

(ii) Version 2 : which share one and only one centreless central region of space.

The notion of a 'straight line' can be provided in the $\bigcirc$-space and Version 1 and 2 spaces if and only if those spaces are already given.

Otherwise, this notion itself (i.e. a 'straight line') conditionalizes those spaces themselves. Within those spaces this notion itself is identical with their own internal self-description. This means that from the standpoint of the ┼-space

  (i) in the ○-space a 'curve' and a 'straight line' are respectively a straight line and a curve,

  (ii) while in the ┼-space a 'curve' and a 'straight line' are respectively a curve and a straight line.

From the standpoint of the ○-space, the above holds simply the other way around. If a straight line is whatever that follows the internal structure of each of the ┼- and ○-spaces, then a curve is the description of such a straight line by the internal structure of the other space. In this sense, a straight line and a curve underlie each other in Version 1 and 2 spaces. Consequently, both are a straight line, or neither is a straight line.

  The two types of 2-dimensional space are both conditionalized from the same 1-dimension. Therefore, they are 1-dimensionally identical. What is 1-dimensionally identical, is necessarily also identical in the 2-dimension. Such an identity based upon a descriptive necessity is a transcendental identity. Those two types of space are 2-dimensionally identical by transcendence. The outer-boundary of the ○-space and the two determinant 1-dimensions of the ┼-space are descriptively identical by transcendence. Version 1 and 2 spaces are commonly derived from the ┼- and ○-spaces. They are 'derived' in the sense that their existence is based upon a descriptive necessity such that requires the ┼- and ○-spaces to be 2-dimensionally one and the same if they do not hold. Such a descriptive necessity is, however, identical with a descriptive necessity which conditionalizes the ┼- and ○-spaces from the 1-dimension ; for the 2-dimensional difference between the ┼- and ○-spaces is descriptively necessary and is demonstrated. In this sense, Version 1 and 2 spaces are fictitious because they have no descriptive necessity. They are generated on the assumption that the ┼- and ○-spaces do not hold. They are, however, meaningful because they describe that the contrary to each of those two types of space leads both of those types of space to the formation of an identical space. Consequently, the existence of Version 1 and 2 spaces is based upon such meaningfulness. These common fictitious derivatives of the ┼- and ○-spaces, however, remain 2-dimensional because the contrary to each of the ┼- and ○-spaces can only be assumed from within those spaces. Therefore, the 1-dimensional

identity between the $+$- and $\bigcirc$-spaces is 2-dimensionally seen in the existence of those common derivatives.

  Those fictitious derivatives contain both notions of a straight line and a circle ; for they are generated from both the $+$- and $\bigcirc$-spaces and are common to them. They are 'self-contained' in the sense that they have no descriptive necessity. They therefore do not necessitate themselves any further conditionalizations. The meaningfulness of those fictitious derivatives differs from a descriptive necessity that conditionalizes the $+$- and $\bigcirc$-spaces. A descriptive necessity is based upon another descriptive necessity and becomes a part of demonstration from within an existing demonstration. This meaningfulness is, however, not a constructive part of demonstration but simply the description of the validity of a descriptive necessity in terms of the impossibility of contradicting that descriptive necessity without losing its necessary descriptive outcome. That is, if it is descriptively necessary that the 2-dimensional transcendental difference between the $+$- and $\bigcirc$-spaces comes out of the same 1-dimension due to an innate necessity of the 1-dimension, then this difference necessarily disappears when those spaces contradict themselves from within themselves. This is so because by contradicting themselves those spaces are contradicting their own descriptive necessity and therefore lose their difference. This results in identical fictitious spaces that are commonly derived from mutually different the $+$- and $\bigcirc$-spaces.

  The $+$- and $\bigcirc$-spaces necessarily describe each other. This is so because they are under the same 2-dimension and are therefore not only 1-dimensionally but also 2-dimensionally related to each other. There is no space other than those two types of space in the 2-dimension. Therefore, they can only be related to each other by describing each other. The mutual-description between two transcendentally different types of space is transcendental descriptions. The description of the $\bigcirc$-space in and by the $+$-space is a 'concentric circle'. A Euclidean concentric circle is made meaningful by this notion of a 'concentric circle'. This is so because the most basic relation between two points in the $+$-space is a 2-dimensional 1-dimension, which is a 'straight line' with an infinitesimal length. In the $+$-space a Euclidean concentric circle is described as the locus of points such that hold at an equal distance from a same point. A circle is not a polygon with an infinite number of edges. Therefore, this locus cannot consists in and of points that are spatially related to one another in terms of 2-dimensional 1-dimensions. The notion of π stands for the descriptive incommensurability between a 'circle' and a 'straight line' and transcendentally relate them to each other

by means of the necessity for each to be describable by the other. This is so because a 'circle' and a 'straight line' are both a straight line in their own space (i.e. respectively in the ◯-space and the ┼-space) which are transcendentally related to each other. The notion of π exists between those two types of space and therefore does not stand for a geometrical property. This means that it cannot be referred to by a number of any types (and therefore by any functional means). The notion of π can only be numerically processed as an incommensurable relation between those two types of space and is therefore referred to by a process itself. Both the ┼- and ◯-spaces have a common geometrical property which generates rational numbers. The numerical value of the notion of π is a relation between two totalities of rational numbers within the totality of totalities of rational numbers in the ┼-space. The ◯-space generates the recursive totality of totalities of rational numbers and is therefore incorporated in the ┼-space as a unit of totality of rational numbers. This unit necessarily corresponds to an equivalent unit within the totalities of such units in the ┼-space. A totality of rational numbers holds between two succeeding integral numbers. A circle and a 2-dimensional 1-dimension are both such a totality respectively by the meaning of the ◯-space and the ┼-space. By this correspondence between a circle and a 2-dimensional 1-dimension a circle can determine, and be determined by, its diameter. The ◯-space is incorporated in the ┼-space and determines a 2-dimensional 1-dimension as its diameter by means of such mutual-determinability. The relation between these two totalities of rational numbers is the ratio of the circumference of a circle to its diameter and can be numerically processed because they are both within the totality of totalities of rational numbers as determined by means of the x-y coordinate. The incommensurability of such a ratio stands for the transcendental relation between those two types of space. This is the meaning of π as a 'transcendental number'. The π is, however, essentially a Euclidean number because it can only be processed in a Euclidean space. The notion of π can only be processed as a Euclidean number because in the ◯-space the totality of totalities of rational numbers can only be described in terms of recursiveness and therefore cannot represent the ratio of two transcendentally related totalities of rational numbers. If the notion of π can only be numerically evaluative in a Euclidean space, then the describability of the notion of a curve is numerically necessarily Euclidean. That is, every numerical representation is essentially Euclidean. This is the reason why a non-Euclidean geometry can only be, insofar as the description of a curve requires the π, numerically represented by a Euclidean geometry. All those which requires this numerically processed notion of π for its description, can only be

described via a Euclidean space ; for the notion of π can only be numerically processed in a Euclidean space.

  The ○-space can be incorporated in the ┼-space because the meaning of what constitutes its centre and outer-boundary is identifiable with that of what constitutes 2-dimensional points in the ┼-space. The substance of the ○-space encloses that space, while the substance of the ┼-space fills that space. The two are, however, schematically identical. The description of the ┼-space in and by the ○-space is a 'closed line' and an 'open curve' as a segment of the former. This is so because in the ○-space a straight line is necessarily two 'closed lines'. This means that in the ○-space any two intersecting 1-dimensions necessarily form four 'closed lines' such that at least two of them intersect each other. These, however, cannot be numerically represented because the notion of π can only be numerically processed in the ┼-space. In the ┼-space an 'open curve' is made possible because a same point can be shared by a straight line and a circle. That is, if a circle is intersected by a straight line, then two points of intersection which are shared by these circle and straight line, determines a set of two open curves as the segments of this intersected circle. The description of an open curve requires π because an open curve can only be a segment of a circle or an integration of such segments.

  It is also for this reason that a curve and a circle necessarily share a segment that is more than a point. Curvature is a transcendental relation between Version 2 space and the ┼-space. Curvature also gives rise to another transcendental number e and intrinsically connects with the notion of π as Version 2 space is a derivative of the ○-space. A fictitious line within Version 2 space is the transcendental base of an open curve in the ┼-space and generates e. In Version 2 space a straight line consists of points which become uniformly and boundlessly denser toward the centreless centre. This line becomes an open curve in the ┼-space that consists in and of points that are uniformly and infinitely dense. e is numerically processed as representing an open curve in terms of such density on a numerical line.

  Certain functions of 2-dimensional 1-dimensions, be it a circle or a curve, need numbers which are not in the ┼-space as neither a 'circle' nor a 'curve' exist in the ┼-space. A 'circle' originates in the ○-space and a 'curve' is found in Version 2 space, while Version 1 space is the descriptive inverse of the ┼-space in the sense that every point in the ┼-space is fictionally described to form its own space and therefore represents schematic symmetry to the necessity for intersection. e, π and i are found when the ○-space, Version 2 space and Version 1 space are

respectively incorporated in the $+$-space. The notion of π is the bilateral form of mapping between the $+$- and $\bigcirc$-spaces ; for each type is necessitated to describe the other. The π-constant (i.e. the numerically processed notion of π) is, however, only applicable to the $+$-space. This is so because (i) rational numbers are the highest type of numbers which is common to both types of space and contains the meanings of natural and integral numbers, (ii) therefore the relation between two totalities of rational numbers can only be described in terms of rational numbers, (iii) this can only be done in a space which can represent the totality of totalities of rational numbers (real space). For this reason a non-Euclidean geometry cannot be purely non-Euclidean if it is to be numerically represented. The geometrical equality which holds between Euclidean and non-Euclidean spaces under the 2-dimension, loses its balance because of this necessity for the numerical inequality. This numerical inequality between Euclidean and non-Euclidean spaces lies in the descriptive necessity that the notion of π can only be numerically processed in a Euclidean space. A Euclidean space therefore supplies a non-Euclidean space with a coordinate system and a set of functions that numerically determine the geometrical 'distortion' of non-Euclidean space against Euclidean space, in terms of the ratio of curvature. The numerical relation among the $+$-space, the $\bigcirc$-space, Version 1 space and Version 2 space is as follows :

   0 and 1 originate in the $+$-space and respectively represent the necessity for intersection and transpositionality. 0 is a and any centre descriptively taken as the centre and gives rise to 'transpositionalilty' (i.e. operativeness) to numbers. It is therefore 0 that assigns cardinality and ordinality to numbers as positions of points in the $+$-space. 1 represents a and any point in the $+$-space and approximates a 'thing' in the empirical space. It is a and any centres (points of intersection) in the $+$-space, determinable in their relation to 0. The initial number lines of intersection are rational lines, which become real lines of determinates, once they acquire 'width'. The determinates of real lines can locate transcendentals in their relation to 0 because they incorporate gaps as relations of the determinates.

   π originates in the $\bigcirc$-space and numerically valued in the $+$-space. It is a structural expression of 'closed whole' in terms of its parts, encapsulated in a single number, approximated by the $+$-numbers.

   i is a structural expression of 'dynamic whole' in terms of its notational reverse, encapsulated in a single number, approximated by a $+$-notation. That is, the irrationality encapsulated in $\sqrt[n]{x^n}$ being a fundamental

characteristic of the ┼-space created by the ontologico-notational necessity of the ∧-operation of two directions, it can also be described by its own negation. The ○-space is the ontologico-notational opposite of the ┼-space, based on ∨, while i is the notational reverse of the ┼-space, which notationally closes the ┼-space as it or any numbers affixed with it is unlocatable in the ┼-space. The ○-space and the ┼-space are transcendentally connected by logical relationship on one hand (∨ and ∧), but are also shown to be connected by notational coherence, in that i notationally derived from a fundamental characteristic of the ┼-space notationally mimic the ○-space by closing the ┼-space. The application of i onto the ┼-numbers creates the closed chain of numbers (finite field of numbers), which contrasts with the ordered field of the ┼-numbers. i originates in Version 1 space and represents a schematic symmetry to the ┼-space. Interestingly it is used to designate 'time' as derived from space. It can be viewed as a form of schematic derivation.

e originates in Version 2 space and represents an open curve that also affords a real number line in the ┼-space to express stress in terms of width of that line. It is a structural expression of 'open whole' in terms of its parts, encapsulated in a single number, approximated by the ┼-numbers. The limit of $(1 + 1/n)^n$ incorporates the whole of natural numbers and numerically represents their structural characteristics in a single number and is the width of natural number line.

'Transcendence' is dimensional relations between the ┼-space and the ○-space, and of their respective descriptive reverse. Descriptions of these relations appear 'transcendent' because our only descriptive means is the numbers of the ┼-space and is necessarily incomplete. 'Transcendence' is a schematic necessity that logic is more fundamental than maths, and therefore there are elements of logic that cannot be fully expressed by means of maths. Euler's identity is the numerical approximation of 'transcendence' by the numbers of the ┼-space, and is also a description of FX through a numerical expression afforded within the ┼-space.

$e^{i\pi} + 1 = 0$ numerically expresses the dimensional relation among the ┼-space, the ○-space, Version 1 space and Version 2 space and the necessity for them to describe one another. This is the 'Iroha' song of the essential transcendentals. That is, the descriptive necessity for the 1-dimension to progress into the 2-dimension unravels itself in the ┼-space by transcendentally incorporating the ○-space, Version 1 space and Version 2 space. The ┼-space, by virtue of being essentially a coordinate and open, is numerically more descriptive in the sense that numbers are directional quantities by nature and a transpositional centre (0 as the

centre and 1's as points) on the lattice of dynamic, uniform and infinite density gives rise to universality to any numerical descriptions.

An open curve given by Version 2 space in the ┼-space, can be described to be closed (the ◯-space) by virtue of schematic symmetry (Version 1 space), this is the meaning of $e^{i\pi} + 1 = 0$. It is a numerical representation of transcendental relations, much as the logical dimensionalities are recursively expressed by (p, p, p → p). Regarding $e^{i\pi} + 1 = 0$, it can further be said that if you approximate 'approximation', you end up with the interconnected tools of approximation, which is the algebraic structure of the 2-dimension in terms of the ◯-space mapped onto the ┼-space, based on their logical relationship as ∨ is the base of ∧. The ┼-space, being numerically more descriptive, is the base of mathematical descriptions. It may acquire more descriptive powers as further dimensions evolve from the 2-dimension and add tools derived in the process of this dimensional developments based on descriptive necessities. Complex and hypercomplex numbers are a good example, allowing us to explore extra-2-dimensions, but essentially ┼-numbers are the descriptive mother of them all.

- Primality -

The mapping of the ◯-space onto the ┼-space gives rise to further interesting features of the properties of numbers, such as evens, odds and primality. A ┼-number line is directional quantities arisen by a centre taken as the centre (0) and dynamically expanding infinity, thus pointing towards the infinitely stretched permeating border. Quantities are the thickness of points seen from the centre towards the border. However, for this infinitely expanding line of intersecting points to have any descriptive meaning it has to internalize this descriptively invisible border (∞). Otherwise, this forever expanding line is descriptively incomplete and is only there to describe itself. Directional quantities incorporate the space in which they are themselves being formed by means of place-values. Place-values are not a natural part of a number line but are there to make a number line descriptively visible by self-spatialization, which embodies the external space. The external space allows a number line to exist, while the internal space allows a number line to describe itself. The two spaces correspond to each other ontologico-notationally as whatever exists is also only descriptively so. Place-values are the same as ',' in language, without which letters, words, concepts are meaningless. ',' is the way language incorporates the necessity of self-demarcation, i.e. a stem-cell concept. Numerical meaning of 'numbers' are assigned by place-values, which also bring about 'tension' to a number line. 'Tension'

is a necessity of description that create properties of 'numbers' so that 'numbers' materialize as numbers. Place-value 0 is the same as the necessity of intersection for $\overset{\leftarrow}{\underset{\rightarrow}{-}} \wedge \overset{\rightarrow}{\underset{\leftarrow}{-}}$, place-value 1 is the same as a natural number line of no descriptive capacities, place-value 2 is the binary description of a number line, place-value 10 is the decimal notation, etc., and place-value ∞ is descriptively recursive to place-value 1. Place-value numbers are, representing a space rather than a point (position), transcendentally approximate to the ' $\bigcirc$ -number', where 1 (beginning) and ∞ (end) merge with the incorporation of 0 (necessity of $\overset{\leftarrow}{\underset{\rightarrow}{-}} \vee \overset{\rightarrow}{\underset{\leftarrow}{-}}$). A 'prime' is the transcendental approximation of $+$-numbers to represent this place-value recursiveness, which manifests as the merging numerical directions of the $\bigcirc$ -space ($\overset{\leftarrow}{\underset{\rightarrow}{-}} \vee \overset{\rightarrow}{\underset{\leftarrow}{-}}$) and the infinitely diverging numerical directions of the $+$-space ($\overset{\leftarrow}{\underset{\rightarrow}{-}} \wedge \overset{\rightarrow}{\underset{\leftarrow}{-}}$). A 'prime' is an expression of a numerical totality expressed through the recursive totality of the $\bigcirc$ -numbers on one hand, and through the infinitely thinning distribution in the infinitely dense and dynamically expanding $+$-space. The $+$-primes are an infinite totality descriptively encaged within a dynamically expanding infinite totality, and together transcendentally correspond to the closed totality of the $\bigcirc$ -numbers.

  Each $+$-prime represent a $\bigcirc$ -totality at each level of density towards a limit of the indivisible totality where numerical directions merge. You should remember, unlike the $+$-space of infinite intersectiblity defined as relations between the x-y coordinates (hence including gaps, and therefore its natural number line being a real line with intrinsic tension (i.e. 'width', numerically expressed as a curve)), the $\bigcirc$ -space has no such points of intersection, and rational numbers transcendentally approximate fictitious pre-limit $\bigcirc$ -numbers. This describes $+$-primes as indivisible whole, which infinitely thins out towards the dynamically expanding $+$-infinity. The paradoxical Euclidean proof of the infinite primes is really this schematic divergence of the closed totality of the $\bigcirc$ -space mapped onto the $+$-space. A $+$-prime is an indivisible factorial totality that is superficially random but logically necessary to arrive at the collective meaning of closed ∞ totality within open ∞ totality. It appears random because 'directional quantities' cannot conceptually encompass a closed ∞ totality within their open ∞ totality, and hence cannot present any formulae that describe the formation of closed 'primes' in the open $+$-coordinates, which therefore look random. They are, however, logically necessary because numbers (including primes) are properties of a space. The formation of a totality (of primes) is a description of the $+$-space and shows the logical necessity of place-values without which the

╪-space cannot be described. The recursiveness of place-values represents as the closed boundary of the ○-space on one hand, and as the closed ∞ within the open ∞ of the ╪-space on the other.

   Place-value numbers recur between 0 and ∞ in the ○-space by ╪-approximation, i.e. contained within open infinity. In the ○-space itself these recursive place-value numbers form a 'prime' at a limit, i.e. by virtue of closed infinity. This ○-prime corresponds to the totality of ╪-primes that is a closed infinity within the open and dynamic ╪-space infinity. In the ╪-space a number represents a point of intersection definable as relation between the determinates (thus including pointable 'gaps', i.e. 'pointability' due to the coordinates space). In the ╪-space 0 is transpositional to any points determinable in the coordinates, and therefore transpositionality is more than lattice-based superficial divisibility. Once the real line coordinates descriptively replaces the rational determinates by incorporating a thickness of a numerical line, i.e. via 'e', then the ╪-space becomes descriptively non-denumerable in relation to the ○-space. Whereas in the ○-space, where a point can only be a denumerable point, the only number at a limit can only be a 'prime', a totality in which there would be no ordinality. This means that, when the ○-space is transcendentally superimposed onto the ╪-space as ∨ is the dimensional base of ∧, ╪-numbers are, numerically interpreted, self-contained by primes by virtue of the ○-space. The ○-space is, numerically speaking, a rational number line that becomes a 'prime' at a limit, by incorporating 0 (the centre). This is the only prime and is the conceptual basis of all primes. At a pre-limit level (i.e. as the condensing rational line still with a centre) there would be a prime representing a level of density that is necessary to lead to a limit. It is these pre-limit, fictitious primes that show up as ╪-primes. Fictitious ○-primes, i.e. denumerable points transcendentally represented by ╪-primes, would fringe the ╪-space as if the ╪-space is contained by the ○-space. This is the ontologico-notational meaning of PNT.

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It is ',' in the above representation that is the essential connective between the ○-space and the ╪-space and signifies the transcendental space between ∨ and ∧, i.e. 'negation' as a form of mapping applicable to a whole space. This is a schematic negation to connect the ○-space and the ╪-space in terms of their common identity based on logical dimensionalities. That is, the ╪-space is the descriptive space of the ○-

space and is recursive to the 0-dimensionality of both spaces. Thus, superimposed the ○-space onto itself by the ontologico-notationality, the ┼-space describes its open, dynamic infinity transcendentally fringed by the closed infinity of the ○-space. From this postulated that there is such a prime is the final prime and has no ordinality. Euclidean infinity of primes originates from adopting ○-'prime' into ┼-primes that is underlain by natural numbers, which are essentially ┼-numbers and assume a divisible totality defined in terms of intersectibility. That is, ┼-primes are infinite because they are superimposed onto natural numbers that are an additive totality of countable infinity. The original ○-'prime' is a closed number by itself. Natural numbers are definable by the process of (0, n, n+1) but contains numbers not definable by any process.

Euclid assumed that whatever these numbers may be, they are still subject to the additive operation. This is so because they have to be interpreted in the ┼-space in order to be numerically legible. This is a tautology saying primes are infinite because they are in the ┼-space and cannot defy the additive operation, which is a way mind the observer operates in the ┼-space. The additive operation is made possible by a given divisible totality and confirms, rather than create, the existence of certain spatial entities. p +1 does not create a new number but only affirms there is such a given existence in this given space. However, in the ○-space the only additive operation (of identifying '0') closes the space in such a way that numbers lose the ordinality. The necessity of having to interpret 'primes' in the ┼-space is overridden by the logical relationship of the ○-space being more fundamental than the ┼-space so that the recursively closed logical dimensionalities uphold. It is here that the Euclidean proof can be questioned. The last prime, p, in the ┼-space follows the ○-prime in such a way that if the additive operation is performed, it will close the space, or it will defy the additive operation in order for the ┼-space to remain viable, i.e. in order for natural numbers to remain meaningful.

Arithmetical operations taken for granted by Euclid in his proof are spatial properties of the ┼-space. So long as natural numbers are carriers of primes, primes will present themselves infinitely in compatible with the ┼-space. That is, as long as we think in terms of the ┼-space, primes are infinite but inexplicably distributed. It is the logical dimensionalities of the ┼-and ○-spaces that explains the unfomulable distribution of primes in the ┼-space. Primes are essentially the ''number' at a limit' of the ○-space as the ○-space can only be an indivisible whole unlike the ┼-space, which is infinitely divisible as defined by points of intersection. This 'prime' of the ○-space manifests as primes carried by natural

numbers of the ╪-space because the dimensionality of the ╪-space is underlain by that of the ○-space in such a way that ∧ logically depends upon ∨. The ○-space is a closed space fringed by a line of continuum and is transcendentally superimposed upon the ╪-space. That is, the open, divisible and dynamic properties of the ╪-space are transcendentally affixed with the closed nature of the ○-space. This manifests in prime distribution as expressed through naturals. The ○-space is closed by substance, while the ╪-space is open by form, as befitting a schema of schemata. When the two spaces react wholistically, they connect through themselves as form of mapping. Thus, the ○-space manifests in the ╪-space as rationally incommensurable enclosure within its open infinity, and the ╪-space, in the ○-space, as recursive infinity within its closed continuum, the former giving rise to 'π', the latter, to the postulated singularity number of no ordinality. The ○-'prime', the only numerical substance in the ○-space, is therefore represented as ╪-primes with asymptotic distribution towards its periphery, indicating an enclosure by itself despite open infinity. Euclidean proof of infinite primes is a schematic tautology because the carrier of 'prime' is the product of the ╪-space. 'Prime' expressed by naturals and counted in terms of the ╪-arithmetic can only be infinite, as the ╪-space is made infinite by intersection of number lines. The ordinality of primes arises only with the assumption of naturals, which are the products of the ╪-space, and without which there are no ways of establishing that one prime follows another. Thinking of primes without associating with ╪-numbers, i.e. in purely ○-space terms, they signify points of indivisible totality that comprise the ○-space. These points are described 'denser' away from the centre and at a limit lose the ordinality by incorporating the centre, thus attaining the merging directions of the ○-space. The ○-numbers can only be a 'prime' because each and every point in this space is a totality of its own.

  Prime numbers constitute the prime line, which is a 2-dimensional 1-dimension and is always contained within 2-dimension, i.e. prime line infinity is always contained within 2-dimensional infinity because infinity is descriptively a 2-dimensional property and therefore because 1-dimension leaves a legacy within itself to show its given infinity is descriptively contained within the 2-dimensional infinity. This manifests itself as the prime line. Otherwise, the 1-dimensional directions will not manifest in the 2-dimension.

  ←
 · — · infinity within infinity
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, because 1-dimensional directions can only be described between schematic points, which are 2-dimensional boundaries. 1-dimensional infinity is always contained within 2-dimensional infinity.

 From this follows it is the prime number line that constitutes the number line of the $\bigcirc$-space, and the prime number distribution is infinitely expanding 1-dimensionally but 2-dimensionally boundlessly confined in the $+$-space. Remembering that the $+$-space consists of the centre (0) and transpositional centres (1's), which are also respectively the additive identifier and the multiplication identifier, and that 1's in the $+$-space are points of intersection and form directional quantities, manifesting as the natural number line, the $\bigcirc$-space consists only of the centre and has no intersections. In the $\bigcirc$-space 0 is the centre of density which manifest as the boundary of the $\bigcirc$-space, boundlessly dense at a limit. The $\bigcirc$-space therefore arithmetically only has 0 as the additive identifier, but no multiplication identifier. Approximating the $\bigcirc$-space with $+$-numbers as it were, multiplication is descriptively based on factorial totalities, and it is the existence of such totalities in the number line, together with a blackboard-like, open and dynamically infinite $+$-space, that allow arithmetical operations such as addition and its inverse (vice versa), multiplication and its inverse (vice versa). This amounts to say the $\bigcirc$-space is the $\bigcirc$-number line, which is an inoperable totality.

 The descriptive necessities of the determinants of the $+$-space are represented as the additive identifier 0 (the necessity of a centre as the centre) and the multiplication identifier 1 (the necessity of intersections arising in the conjunctive space). Whereas, in the $\bigcirc$-space the number line (at a limit) can only consists of what is known as prime numbers in the $+$-space because points and lines here can only be descriptive points and lines representing the density of space away from the centre (0). These points and lines become described at their limit, manifesting as

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 Expressed in the descriptive manner of the $+$-space, there are no factorial totalities here and therefore no divisions and multiplications. Arithmetically only additions are possible within a line, and subtractions between lines in terms of their density. Therefore, the number line at a limit consists only of prime numbers in the sense of the $+$-space. Prime numbers are their own totalities that only serve to describe descriptive directions of a line. Unlike positions (i.e. points of intersection, 1's) in the $+$-space, which are directional quantities backed by 0 and 1's, points in

the ○-space are only there to close the ○-space at a limit. The only mathematically useful description (conjecture) of the ○-number line is that the number line at a limit incorporates 0 (the centre) so that the two directions merge into one, and the number line becoming a single indivisible totality. There are no evens and odds in the ○-space. This conjecture is yet to be approximated in terms of ┼-numbers, i.e. in the dynamically expanding infinite ┼-space. Given such an approximation, considering spatial properties of the ┼-space (dimensional priority of '2×' over '+1'), the last 'prime' in the sense of the ○-space, if approximated into ┼-primes, has to be an even number, because a '+1' number is descriptively weaker than a '2×' number, and therefore fails to close a number line a la the ○-space.

This conjecture is further strengthened because, assuming an approximation is possible, if the beginning of the ○-number line is an even number (2) following ┼-primes, then it must also be an even number when it coincides with the ending. That is, the beginning of the ○-number line is also the ending of that line, and being the beginning and simultaneously the ending is the only connective between the two numbers, i.e. in the ┼-space an even cannot, by itself, become an odd without contradicting the fundamental spatial properties of the ┼-space. There is no ┼-based arithmetic operator that transforms an even itself into an odd itself, and vice versa.

∧ and ∨ as applied to an identical variable-notion reveal an ontologico-notational structure between an identical logical space. The conjunctive space and the disjunctive space are logically one and the same. It is only within the structure of ontologico-notationality that the conjunctive space is of a higher dimensionality and is therefore descriptively capable of encompassing the disjunctive space. The ○-space and the ┼-space are of parity in terms of logical representation, it is the ontologico-notational structure that allows the ┼-space to approximate the ○-space, and, in so doing, describes the ○-space as closed within the ┼-space. It is thus that prime numbers are distributed boundlessly confined within the dynamically infinite conjunctive space. Prime numbers are numbers of the disjunctive space on account of their non-factorial totalities, whereas in the conjunctive space numbers are directional quantities and contains both primes and composites. Such as cardinality and ordinality are only so recognizable at a penultimate step prior to a limit in the ○-space, and at a limit the beginning and end of the prime number line merge.

Practical mathematicians skilful in the processes of numerical analyses and modelling are not necessarily proficient in the nature of mathematics

in the same sense that good literature critics by no means make good novelists. Mathematics can exist without philosophizers as it is based on notational applicability of rules of numbers rather than conceptual analyses of the properties of numbers. However, practical maths can never go beyond the status quo of accepted rules. Just because an ass can count does not mean he know what is counting. The concept of numbers presupposes the whole of numbers because properties and characteristics of a number cannot be meaningfully discussed without already presupposing the totality of types of numbers that share common properties of that number, and it is here one gets into self-referential paradoxes and definition difficulties.

- Odds and Evens -

$(\forall x)\ Fx$

, where the universally quantified/quantifiable variable x takes all objects in space/space-time/time (i.e. all counted/countable objects by whatever definitions) as its values and F is the only universal predicate applicable to such a variable. Outputs of such a function are numbers. Tautologically anything that numbers are not applicable falls outside intellectually legible schemata. F is

$\lor, \land$

, whereby $\lor$ and $\land$ are conventionally understood as 'or' and 'and' in the ordinary language and more importantly ',' refers to the demarcatable space that makes any notations intellectually viable. It is ontologico-notational in the sense that is fundamental to space within and without, which must be juxtaposed for our understanding to hold. The 'in-space' is the notational space, while the 'out-space' is the ontological space. The former may also be called the idealised space, the latter the empirical space. It is surprising to see many of our most ingenious notations (including PM) make use of it without really acknowledging it.

$\lor$ and $\land$ are both symmetrical, two-place logical operators and can connect the universal variable x as a single predicate. $\lor$ and $\land$, alongside $\rightarrow$ and $\sim$, are regarded so fundamental that they are termed constants. They, however, can be made into a predicate because they describe the meaning of the universal variable x. This is the only bona fide Pp (primitive proposition). Russell's Pp is either meaningless (looping) or, in fact, a complex relation (like a colour 'red'), which appears 'primitive' as a matter of descriptive convention. Pp must boil down to the simplest and

most universal epistemological fact (known to us so far) such as $(\forall x)\ Fx$, and F being 'demarcatable', 'countable', 'spatio-temporal', etc. Such a x only takes universal objects as its values. Universal objects would have no contingent properties that could be otherwise.

Given such an F(x), it cannot be true or false, nor can it be superficially implicative. The most basic fact, once given (i.e. so cognized to be present), is not subject to modalistic arbitrations, nor can it imply anything else including even itself unless it is recursively partitionable (i.e. every part is identical with itself and its totality). Such a fact is ontologico-notational and is therefore the fabric of the structure of our intelligence. It is the basis to build our epistemological body and may resemble the Kantian 'a priori'. Its implicator can only be the logical space itself. Thus, assuming x as given in the logical space, x can only be so described as Pp. The existence of Pp presupposes the ontologico-notational implicator. Although it is not explicitly referred to in PM the most important symbol is ',' or any spatial representations like a gap, which signifies the ontologico-notational space itself and without which no descriptions, formal or otherwise, are possible. The presence of ',' assumes the logical space itself and the consequential Pp. The description of x must be made within the logical space itself because at this level of description x cannot be predicated by any epistemological observations/ conventions. The predicate of Pp can only be of an essential property, which is in the domain of the logical space, as any epistemological descriptions are by necessity contingent. It is the nature of intelligence to philosophize whatever epistemologically describable, while it is the logical space that gives a structure to such intelligence. In a way, it is ',' that the intelligence is forced to accept in order to be intelligible.

The existence of the logical space is underlain by the ontologico-notational implicator ($\rightarrow$) and the form of mapping onto itself ($\sim$). Assuming Pp, x is already in the logical space, in which it is essentially described by $\vee$ and $\wedge$ because it is in the ontologico-notational process of the logical space that the universal variable x is identified as the variable notion itself (i.e. the only entity in the logical space). Such an identity is simultaneously undertaken by $\vee$ and $\wedge$ as the dimensionality of $\vee$ and $\wedge$ is already incorporated in x when it is so assumed to be given in the logical space. Thus,

$$x \vee x \wedge x$$

, which is the description of x and brings out the universal applicability of x. This forms a predicate that describes x is the only variable and takes everything as its value. This is so because ;

  1) x ∨ x ∧ x can only be a well-formed formula when there is one and only one variable in the entire universe, so to speak, where ∨ and ∧ become identical.

  2) x ∨ x ∧ x would be meaningless if there are more than one variable.

  3) Through x ∨ x ∧ x, ∨ and ∧ together describe this property of x as the universal variable and forms a new predicate.

This also explains the universal applicability of numbers. For numbers to be meaningful they have to be universally applicable. That is, numbers assume the universal variable that takes everything in the universe as its only value. Numbers are not adjective to describe various objects, otherwise 2 will never be formed from '1x + 1y'. x ∨ x ∧ x is the logical equalizer to create mathematical objects from myriads of empirical objects and their variously perceived diversities. x ∨ x ∧ x is a predicate of abstraction. x ∨ x ∧ x is only well-formed when applied to the universal variable and tantamount to x, which is a form rather than an object (hence termed a 'variable-notion'), while a ∨ b ∧ c as applied to objects is ill-formed and makes no sense. The same applies to a ∨ a ∧ a because a as a value presupposes b, c, d, - - -,n to their countable limit. It is for this reason that numbers are not values of x, but it is x that becomes numbers. That is, x ∨ x and x ∧ x can be conflated only when x is a universal form.

  This predicate of the Pp needs the hypothesis (x) › x to make sense in our language because, otherwise, there are no means of knowing its meaning. Thus, given (x) › x,

(x ∨ x ∧ x)

, which is paraphrased as ;

((x ∨ x) ∧ x) and simultaneously

(x ∨ (x ∧ x)).

That is, (x ∨ x ∧ x) has two identical, but separate, selves, which, in turn, have parts which are mutually replaceable. These two identical selves

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with mutually replaceable parts are related to each other and define the descriptive space in such a way ;

Conceptual Numbers

Step

0   x∨x∧x   ; cardinality 0 ; the space of description with no describable values taken

1   (x∨x∧x)   ; cardinality 1 ; the space of description filled with
     or ( )   every possible value

Descriptive Numbers

2   ((x∨x)∧x) ; cardinality 2 ; step & cardinality 1 has two and only
     (x∨(x∧x))   two identical selves ; two identical
     or ( ), ( )   forms of 1 ; Even (logical
                               number)/Prime

3   x, x, x   ; cardinality 3 ; step & cardinality 2 follows step &
     ( ), ( )+1   cardinality 1 because the predicate
                             -∨-∧- describes the identity of ∨ and
                             ∧ when they connect three places
                             with an identical value. That is, ∨
                             and ∧ connect each other in terms of
                             a common and identical value and
                             describe their identity ; thus two
                             identical forms express their identity
                             in terms of three identical values ;
                             Odd (mathematical number)/Prime

At this stage, in terms of interchangeability between form and value numbers 0, 1, 2 and 3 already contain a dimensional transformation and evolve into a coordinate. Thus,

$$
\begin{array}{ccccc}
 & 2 & & & \\
 & 1 & & & \\
 & & \overset{\vee\ \&\ \wedge}{} & & \\
2 & 1 \leftrightarrow & 0 \leftrightarrow 1 & & 2 \\
 & \underset{x}{} & \underset{x}{} & & \\
 & 1 & & & \\
\end{array}
$$

'value' ≡ 'form'

2=n+1(1+1)

2   3=2+1

It is significant to note that between step & cardinality 2 and 3 is a mathematical operation to relate 'form' with 'value' both as a common mathematical object. 3 only after 2 because for both 'form' and 'value' to be meaningful 'value' is necessarily 'form '+1''. Between the first and only even prime and the first odd prime is descriptively necessarily established the relation '+1' which is always the case with any immediately connected even and odd numbers because given two identical forms, identical values are necessarily '+1' in order to ascertain such identities. For this reason even numbers may be called logical numbers, odd numbers mathematical numbers, based on the most fundamental mathematical operation of '+1'. Thus, given numbers, '+1' is always the case, and a mathematical number always follows a logical number. It is thus that primes are always odd except the first even (2).

Given step & cardinality 0, 1, 2 and 3 it immediately follows ; there exists a mathematical structure between them that establishes a mathematical identity between 'form' and 'value'. That is, 'form' and 'value' have to be mathematically interchangeable in order to descriptively establish common numbers between conceptual, logical and mathematical numbers because Pp describes the most fundamental property of the universal variable x, which presents itself as 'countability'.

Given (x) › x, it is the way 0 evolves into 1, 1 into 2, 2 into 3, that is inherently contained but not descriptively present and thus brings about the structuralization of numbers. Logical numbers continue ad infinitum, or so long as the countability descriptively hold because ;

the first logical number has the innate descriptive necessity of continuing identifying its two identical selves in terms of ∨ and ∧. That is, between $((x \lor x) \land x)$ and $(x \lor (x \land x))$ is the descriptive space where no relations can be represented by means of the notational conventions such as ',', other than recursive repetitions in order descriptively to signify the space. Thus, from $((x \lor x) \land x)$ and $(x \lor (x \land x))$,

$(((x \lor x) \land x) \lor (x \lor (x \land x)))$
$(((x \lor x) \land x) \land (x \lor (x \land x)))$

or ( ), ( ), ( ), ( )

, which is step & cardinality 4 and has two and only two identical selves ; two identical forms of 2. This logical number is succeeded by '+1' by necessity because its fundamental constituent part is embedded with the form '+1' to make it operably meaningful. Thus,

( ), ( ), ( ), ( )+1 or (( ), ( ), ( ), ( ))

, which is step & cardinality 5.

In short the identity of ∨ and ∧ in terms of the universal value x is descriptively established by repeating this process.

 The interchangeability of x is in the descriptive identity of x, which also ensures the interchangeability of ∨ and ∧ and of ( ). Therefore, 0 and 1 could have been x∧x∨x and (x∧x∨x), and 2 would have been ((x∧x)∨x), (x∧(x∨x)), resulting in the same 3 with x, x, x. Further ((x∨x)∧x) could have been (x∨(x∧x)), and if so, (x∨(x∧x)), ((x∨x)∧x), resulting in the same 3 with ( ), ( ), ( ). Thus, expressing 0, 1, 2, 3, · · · as 0, n, n+1, this has the horizontal and vertical extensions as,

$$n+1$$

$$n$$

$$n+1 \quad n \quad 0 \quad n \quad n+1$$

$$n$$

$$n+1$$
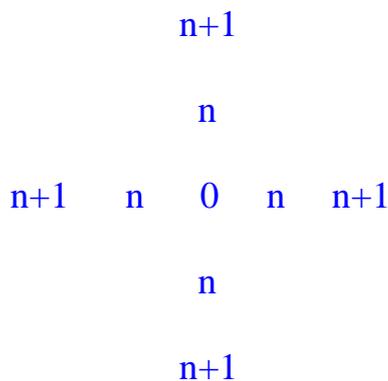
, where the original 0, n, n+1 can go e.g. either sideway representing the interchangeability of ( ), and e.g. upward representing the interchangeability of ∨ and ∧, or e.g. downward representing the interchangeability of ( ). That is, the original 0, n, n+1 has 4 way extensions and results in the spatialization of numbers, which brings about other types of numbers (integers, irrationals, reals, imaginary (dimensional addition) as well as ∞). 0 could be anywhere as a centre of transposition.

 This spatialization is descriptively embedded with multiplication. 0, n, n+1 has an identical structure whichever way it is extended. Thus, the sideway extension of 0, n, n+1 with 0 as the centre has '×2' number of numbers. Likewise, the upward as well as the downward extension also
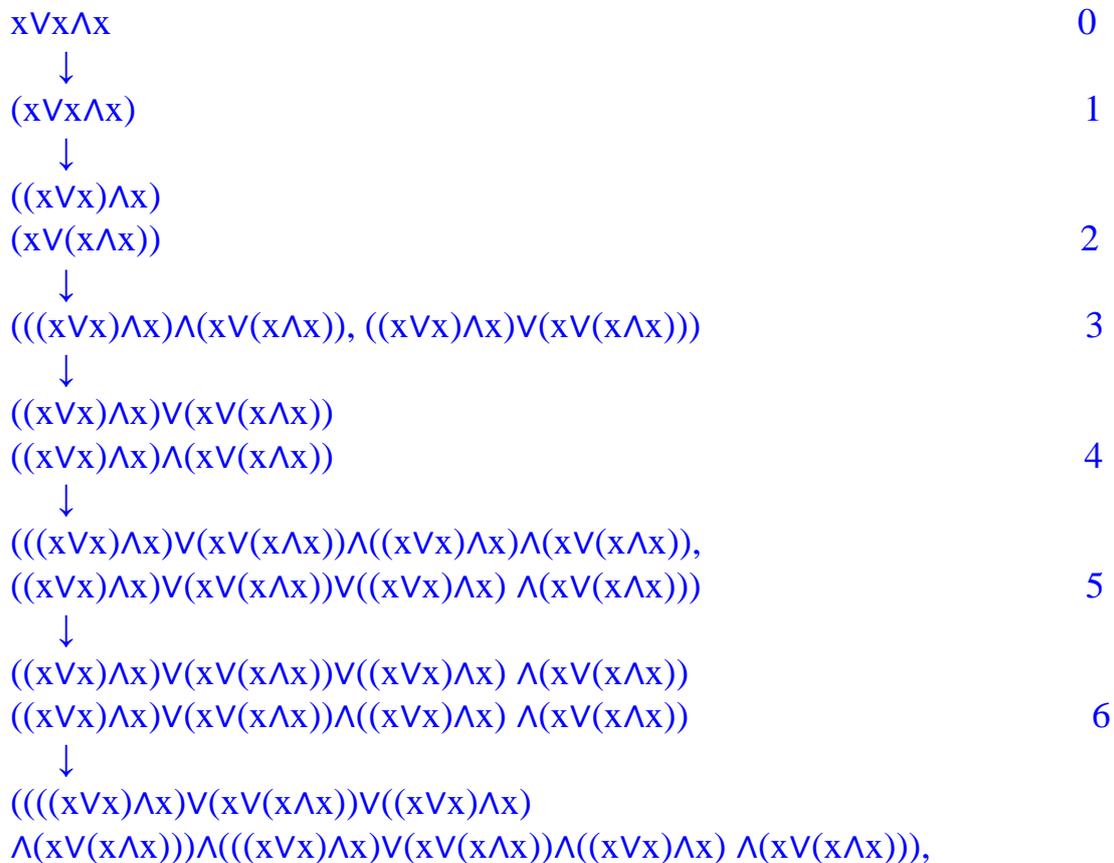
have '×2' numbers. This 4 way extensions of 0, n, n+1, with 0 as the centre, is the 2-dimensional coordinate of numbers and may be named the x-y coordinate. The notion of '×2' becomes '×n' because of the universal applicability of numbers. Once given such a coordinate, '×2' numbers as well as '+1' numbers materialize. Integers are an identical self of the original 0, n, n+1 with 0 as the centre and with the 4 way extensions and are conventionally designated with '+' and '−' for each half. Given number lines as a coordinate, irrationals come in as spatial relations between coordinates, and real numbers are spatial relations within a coordinate in terms of the intersectability of a line and the transpositionality of 0. At this level of descriptions numbers are names given to directional quantities and are filled with '+1' numbers and '×2' numbers (i.e. tools of building blocks) which starting with the first logical number (2) and the first mathematical number (3) fill in the natural number line with the exception of the last prime (○ -'prime'). It is important to remember that '×2' numbers have a higher descriptive necessity than '+1' numbers because the logical necessity of intersection precedes the creation of directional quantities, resulting in descriptive priority of evens over odds. Primes are always odd numbers except 2 because it is a descriptive property accorded to logical numbers which have mechanistic building process by means of the identity between ∨ and ∧, given the universal x. Therefore, whenever there is a logical number, a '+1' number follows. Whenever odd numbers cannot be named by means of '×n' numbers, a genuine mathematical number appears with the proviso of the natural number line (a 1-dimension between descriptive points) must always be contained within the dynamically expanding infinite 2-dimensional space, thus reflecting the logarithmic cap in the way they occur.

Unlike a finite space, wherever an intersection takes place, '×2' materializes as an essential spatial property. It is not because an intersection occurs at the centre (0) that '×2' surfaces, but '×2' is there wherever an intersection is, in this dynamically expanding ∞. '×2' is also the conceptual base of '×n' and its operational reverse, by virtue of the multiplicity implicit in number lines of intersection. It is with '2' that the building tools ('+1' and '×2') starts making numbers, with exception of primes that only '+1' can make. '+1' and '×2' carry a legacy of their spatial properties. In that '+1' is of the ○ -space in the sense, expressed by ┼-approximation, the density can only build up additively. Whereas both are of the ┼-space with the proviso of '×2' being spatially more essential, reflecting the necessity of intersection precedes the making of quantitative directions.

Thus, odds are more ○-associative, and evens, more ┼-associative. The ┼-space being numerically more descriptive, the ○-space surfaces in the ┼-space as transcendentals and also through ○-associative odds. Odds carry the legacy of the ○-space through primes and the ∨-connective. The ∨-connective is, not unlike X-chromosome, invisible arithmetically because '+1' is a spatial property applicable to any numbers, inheritable via logical numbers.

Numbers are notions as well as self-operators. This is where maths radically differs from logic. This is how 0, n and n+1 are self-evidently connective, based on 'universal variable' x and its essential linearity, which allows $(x \wedge x = x \vee x) \rightarrow x$. It is this singular linearity that connects the logical number 2 with '+1' and gives rise to 3, a mathematically operated odd. '+1' is a number generating operator arising from $x \vee x \wedge x \leftrightarrow x \wedge x \vee x$. This is a new operator with 3 places that converge into ''n'+1'' applicable only to single universal x and has a predicative function, with a numerical capacity of turning an even into an odd, an odd into an even, representing $(x \vee x) \wedge x \leftrightarrow x \vee (x \wedge x)$ and $(x \wedge x) \vee x) \leftrightarrow x \wedge (x \vee x)$.

From (x) › x

x∨x∧x	0
↓	
(x∨x∧x)	1
↓	
((x∨x)∧x)	
(x∨(x∧x))	2
↓	
(((x∨x)∧x)∧(x∨(x∧x)), ((x∨x)∧x)∨(x∨(x∧x)))	3
↓	
((x∨x)∧x)∨(x∨(x∧x))	
((x∨x)∧x)∧(x∨(x∧x))	4
↓	
(((x∨x)∧x)∨(x∨(x∧x))∧((x∨x)∧x)∧(x∨(x∧x)),	
((x∨x)∧x)∨(x∨(x∧x))∨((x∨x)∧x) ∧(x∨(x∧x)))	5
↓	
((x∨x)∧x)∨(x∨(x∧x))∨((x∨x)∧x) ∧(x∨(x∧x))	
((x∨x)∧x)∨(x∨(x∧x))∧((x∨x)∧x) ∧(x∨(x∧x))	6
↓	
((((x∨x)∧x)∨(x∨(x∧x))∨((x∨x)∧x)	
∧(x∨(x∧x)))∧(((x∨x)∧x)∨(x∨(x∧x))∧((x∨x)∧x) ∧(x∨(x∧x))),	

(((x∨x)∧x)∨(x∨(x∧x))∨((x∨x)∧x)
∧(x∨(x∧x)))∨(((x∨x)∧x)∨(x∨(x∧x))∧((x∨x)∧x) ∧(x∨(x∧x))))   7

   ↓

(((x∨x)∧x)∨(x∨(x∧x))∨((x∨x)∧x)
∧(x∨(x∧x)))∨(((x∨x)∧x)∨(x∨(x∧x))∧((x∨x)∧x) ∧(x∨(x∧x)))
(((x∨x)∧x)∨(x∨(x∧x))∨((x∨x)∧x)
∧(x∨(x∧x)))∧(((x∨x)∧x)∨(x∨(x∧x))∧((x∨x)∧x) ∧(x∨(x∧x)))   8

   ↓
   .   .

   ↓
   .   .

   ↓
   .   ∞

The above shows the inherent structural progressions in x∨x∧x, and this
can be described as step numbers. Odds are always expressed as (,), in
which ',' signifies the transcendental space between the ○-space and the
╂-space. It should be remembered that in the decimal notation the
number line is already a line with width and therefore incorporates the ○
-space. Regarding the number line of naturals as the carrier of primes,
odds are a penultimate form of primes, a step closer to expressing
indivisible wholes, which are ultimately the ○-'prime' at a limit. In the
╂-space itself odds and evens are invisible, because directional quantities
of 0 and 1's are all there are in the space of points of intersection. Our
expression of e.g. decimal number line is already a descriptive
combobulation of the ○-space and the ╂-space, thus giving widthless
directional quantities of the ╂-space a width to make it visible to our
cognition. Such number lines acquire visibility by means of width, which
is, described in the ╂-space, a curve. The decimal number line, which is
so natural to our understanding of numbers, is, in fact, a descriptive
creation arising from the logical necessity of unseparating the two 2-
dimensional spaces. It therefore contains the legacy of the two spaces,
which expresses as odds (the ○-space) and evens (the ╂-space). This
expression is imperfect, as the transcendence between the two spaces
does not allow any pure expression of either space. Thus, some odds are
divisible, an even is a prime, primes become scarcer towards the
periphery and the ○-'prime' is uncapturable. The decimal number line is
only there as an approximation of a ╂-number line, which is descriptively
invisible.

Translating the above into 'step numbers',

  Given x∨x∧x , x is one and only variable that takes anything

countable for its value. Since numbers are always universally and uniformly applicable, that is to say, in our countable world, everything is either countable or not countable, and where countable, countability is not a property of thus countable object, but rather of our cognitive mind.

There is nothing that can be meaningfully values for x,   Step 0
as ∨ and ∧ are binary operators. Thus, e.g. x∨y∧x or x∨y∧z
is not well-formed and nothing will follow. x∨x∧x would not
be a well-formed formula if there are more than one arity.

(x) › x is applicable because a meaningful totality exists for x,   Step 1
as ∨ and ∧ are identical only when x is unary as the universal
variable that takes everything as its value (i.e. because in the
countable world everything is countable). That is, an
ill-formed formula x∨x∧x becomes well-formed upon recognition
∨ and ∧ are identical, given this totality has only one operand.
The base logical object x is synonymous with a 'thing' in the
ordinary language as numbers are universally and uniformly
applicable. ∨ and ∧ apply because this base object is a
'variable-notion' and is ontologico-notational in the sense that
the value of x is x itself. It is the universal applicability of
numbers that manifests itself as x. The unary x thus gives binary
operators ∨ and ∧ an internally dynamic structure of identifying each
other infinitely while enhancing its cardinality infinite and retaining
an ever identical logical meaning. Therefore,

```
    ∨  ↑  ∧
(1← (0) →1)                                              Step 2
    -  ↓  +
```

, where ∧ generates (0 →1), and ∨ generates (1← 0).
Note that ∧ and ∨ take a same value only when monomial.
Thus, (x∨x∧x) expands into 4 meaningful totalities, namely
(x∨(x∧x)), (x∧(x∨x)), ((x∧x)∨x), ((x∨x)∧x), of which (x∨(x∧x)) and
((x∧x)∨x) create the negative progression (-) and (x∧(x∨x)) and
((x∨x)∧x) create the positive progression (+), and because
∧ and ∨ can be permutated without affecting a value when monomial,
↑ and ↓ are created and expand the negative and positive progression
into upward and downward. 2 is the natural cardinal as well as
ordinal number and is the base of logical number (even numbers),
which is always divisible by 2. The base of step numbers is 1,
the base of all numbers is 0. At this level, given logical numbers

and step numbers together with conceptual numbers, ∨ and ∧ acquire mathematical meanings after completing their logical tasks because what is ontologico-notational now becomes epistemic after becoming a value of its own notation.

Two meaningful totalities, ∧-totality and ∨-totality are symmetrical, but are not binomially deterministic. Hence ∨ could have generated $(0 \rightarrow 1)$, and equally ∧ could have generated $(1 \leftarrow 0)$. This creates infinitely recursive process of ∨- or ∧-totality and **-** or + progression.   Step 3

Therefore, the left and right steps and the upward and downward steps repeat themselves infinitely.   Step 4

This results in intersecting number lines.   Step 5

.

.

n   Step n

The above process become descriptively visible via randomly chosen number lines as a coordinate.   n→∞

In short, an infinite linear progression of 4 way steps (intersecting number lines) as represented below;

n+1

n

n+1   n   0   n   n+**1**

n

n+1

, results from the dynamic permutability of ∧ and ∨ when monomial. This is only possible with the cognition of a whole from parts. Once 'numbers' are thrown into a space of coordinates with directions in terms of expanding ∞, then cardinality and ordinality is a matter of depth

assignable to numbers as positions in a number line and become directional quantities.

In summary :

Given x as any number-applicable universal objects, given x∧x and x∨x as logical idenfiers, their logical values (truth-values) are same. However, after confirming this identical logical value, their ontologico-notationality transcends into epistemologicality and thus acquires mathematical values, namely x+x (••) for x∧x and x (•∨•) for x∨x. That is, ∧ and ∨ are forced by the monomial x to identify themselves in terms of their epistemic difference. If ∧ and ∨ take themselves as its own value, then ∧ is 2-place acquirer and ∨ is 1-place acquirer. ∧ and ∨ are the internal structure of numbers, especially it is •∨• that creates odd numbers and also explains the behaviours of prime numbers. Thus, logical identifiers ∧ and ∨ are turned inside out by this monomial x as the internal structure of x, which is the logical base of every number. This is a logical description of transcendentally incorporating the ∨-space into the ∧-space, thus transcendentally approximating the ○-space within the ┼-space.

Conceptual numbers are,

0	×	no x applicable
1	•	any and every object is x
2	••	logical number/prime number
3	(•• •∨•)	odd number/prime number
4	••••	logical number
5	(•••• •∨•)	odd number/prime number
6	••••••	logical number
7	(•••••• •∨•)	odd number/prime number
8	••••••••	logical number
9	(•••••••• •∨•)	odd number
10	••••••••••	logical number
.		
.		
n	••• ⋯ •••	logical number
n+1	(••• ⋯ ••• •∨•)	odd number and/or prime number

This means prime numbers are (· (· (x∨x∧x))), which indicates there cannot be found any last prime in ┼-space, as 0 and 1 are permutable, i.e. any centre can be the centre. The last prime is a ○-'prime', which cannot be described in any space with a centre.

Numbers are the name given to a process and are not set-theoretical objects. It is not as if there is an object e.g. '3' or the set of all sets with three members. Where there should be a world with no e.g. '5' or the set of all sets with five members, there could not be a '3'. '3' presupposes all other numbers. That is, there must be the totality of all numbers before e.g. '3' is cognized as a number. The wholeness of all numbers is a logical process of the operand (–∨–∧–) where – is the universal variable that takes everything as its value. To define '3' as the set of all sets with three members is a tautology where the meaning of 'three' is already known as '3'. It is not that something is countable, but that it is something because it is countable. The seeming three place connective, –∨–∧–, is, in fact, an unary connective as (–∨–∧–), which generates numbers in the process of acquiring ( ).

A number has no structure. It is the totality of numbers that has a structure, without which a number has no meaning. Logic gives a structure to numbers. A structure is patterns and modes. Axioms and rules of inference are a way of describing a structure and are themselves governed by a structure. This is the essence of Gödel. The capacity of cognition is a mind, which together, from within, form the most basic structure and is the foundation of mirroring the world. A mind without a structure is a derangement. A structure is the cognized and the cognisor and is necessarily tautological. There is no structure of structure because, if there is, it will manifest as a structure.

The universal applicability of x as the variable-notion creates this singular meaning of (x∨x = x∧x), which arithmetically generates odds and evens. Similarly, x expressed as $\overset{\leftarrow}{\underset{\rightarrow}{}}$ and $\overset{\rightarrow}{\underset{\leftarrow}{}}$ that generates geometrical spaces. Arithmetically the most primitive notation comes with the discovery/invention of 0, which identifies entities by means of countability. 1 follows as the name of thus identified countable objects. 2, however, is the most ingenious application of our creativity. That is, while there is no notational way to represent this discovery of 0 and 1 as much as 0 = 1 makes no epistemological sense, we here have the logical symbolism backed by the tautological power. This is 1+1=2. This indeed condenses the entire maths. We wish to say the world consists in entities (0's), which we can count (1's). It is with 1+1=2 that we can say the world consists of countable entities. This is the most powerful and primitive description of the world.

The notation gives us a power of description but at the same time, binds us into a certain paradigm of limitations. This gives us a solution to a myth of the number theory ;

  that whatever is multiplied and divided by 0 is 0,
  that whatever is added or subtracted by 0 remains unaffected,

because 0 is the notation itself. A notation cannot notate itself as part of itself. 0 underlies the foundation of the entire mathematical notation. From this follows ;

  that whatever is multiplied or divided by 1 is the multiplied or divided,

because a name notationaly can only be identified with the named and therefore naming x as *a* can only mean x = *a*. Mathematical induction conveniently trivializes the significance of 0 whether it is part of natural numbers because it cannot distinguish the two separate cases. If 0 is not part of natural numbers, then it has to be defined outside induction. Thus, the special meaning of 0 has to be positioned in parallel with inductive natural numbers.

It is, however, with 2 that the notational evolution of maths starts. 0 and 1 are, so to speak, ( ) of (x) › x. Given this conceptual foundation, 2 is the practical application of the notation onto the countable world. With 2 we are actually into the process of counting. If 1 can be named the universal prime number, although it is excluded from the prime numbers, then 2 is the prime of prime numbers as well as the only even prime for being the base of all logical numbers. Appling the logical process of generating numbers, the notational substance of numbers is the binary property of the logical operators ∨ and ∧, while what we call natural numbers is actually step numbers, i.e. logical steps of generating numbers. Therefore, while 0 and 1 are the conceptual foundation of the number system, the even numbers are logical numbers of what we name natural numbers and the odd numbers are the result of the collapse of the binary property of ∨ and ∧ due to the ontologico-notational anomaly of the conceptual condensation of every entity into a single countable entity 0.

∨ and ∧ are binary and are therefore 2 place-connectives. They idiosyncratically become identical when applied to an identical variable (variable-notion). This happens because numbers are idealized to be universally and uniformly applicable to any entities in space-time, space or time. Thus, given only a single identical variable x, the ill-formed formula x∨x∧x becomes singularly well-formed. This relation between

binary operators and a single identical universal variable is the source of logical numbers and step numbers. Gödel used a finite number of odd primes to designate logical symbols to avoid the ambiguity of ∞ assignable to numbers and to make it so self-evident as to dispense with the axiom that each number is unique. He still assumes an independent notational system of numbers as if numbers exist a priori from the formal logic. However, for each and every number to be itself (∀x(x=x)) and nothing else (∀x(x≠~x)) the logical hypothesis (x) › x is necessary regardless numbers concerned are finite or infinite because each and every number, whether it is deployed by Gödel coding, assumes a notational totality. Behind each and every number is every other number to reflect the self-operational nature of numbers. Without non-prime numbers prime numbers have no identity. Thus, if numbers are based on the logical hypothesis (x) › x this logical statement cannot even be Gödel-numbered, otherwise the notational self-reference will result.

  It is therefore logical that 2 is the only even prime number because process-wise all logical numbers are the multiples of 2. The notational complication arises because natural numbers consists of three layers of different numbers, conceptual numbers (0, 1), logical numbers (even numbers) and step numbers. This number line becomes a directional quantity when spatially applied as it is logically necessary to be coordinated, and the three-layered numbers become notationally equalized in terms of the spatial quantity. It is here the mystery of the Riemann zeta function is logically misconceived but notationally well-presented as expressing the logical misalignment in the Euclidian 2 dimension. The logical anomaly of odd umbers is spatially expressed as the prime number distribution and also as the notational necessity of containing the 1 dimensional infinity in the 2 dimensional infinity as the latter is a higher step in the notational evolution. The reason why the presumably exact science of maths is riddled with nothing but approximations and artful notational explorations of how to tame them, is that there are layers of fundamental relationships between logical, arithmetic and geometric notations, including that of 0, which is itself a notation, that cannot be expressed in each notation, while there are no unified notation that can treat them all under the same notational paradigm.

  Gödel's theorems are concerned with the prima facie solid algorithmic notations such as formal logic and elementary arithmetic, within which are assumed sub-notations such as that of 0 and 1, which, being conceptual numbers, partially defies the algorithmic definitions. The numerical rules of usages of 0 as a natural number may be taken for

granted, but such rules will be forced to adopt not necessarily well-defined or well-coordinated sub-rules when interfaced with related but independent notations such as geometric notations. The line of natural numbers, when spatially adopted, demands new set of rules such that are consistent with arithmetic rules. This is where provability becomes an issue. It is notational interactions that cause even seemingly simple and straightforward definitions of a simple notion unexpected twists and even inconsistencies.

I do not know if this is what Wittgenstein's criticism or rather offhand dismissal intended, but Gödel's ingenious way of dealing with the self-referential paradoxes is notationally faulty if the number system is found to originate in the formal logic. His method is valid only if the notation of the formal logic and that of natural numbers (and their rules) are independent, which is an implicit assumption behind Gödel coding. Self-reference becomes impossible by (x) › x. x cannot refer to itself without breaching (x) › x. x = x assumes (x) as much as numbers assume 0 and 1. 0 and 1 assume the formal logic. If numbers are shown to be generated from within the formal logic or to have a common fundamental root, then applying numbers to logical statements or sequences of logical statements in order to create unique references to circumvent self-reference, is a form of self-reference. Thus, Gödel's ingenuity turns out to be just a notational circus.

As much as notations are creations of human mind, even a rigorous formal system of elementary arithmetic is riddled with impossible complexities, once superficially simple definitions and rules are explored deeper. Gödel's ingenuity of avoiding self-referential paradoxes by using whole numbers will face questioning, given the relationships between conceptual, logical and step numbers. Just because a machine can encode and decode as programmed and bring out certain numerical properties, does not mean such a program is infallible. At the bottom of entire maths is the ordinary language, which is not only the prima facie medium as we come across maths but also modus operandi of mathematical thought processes, and contains many imprecise concepts and relations. That is why maths keeps evolving, being refined and even reinvented. Mathematical notations are more convenient to communicate over the ordinary language, when used among mathematicians. However, these notations are essentially shorthand for the sake of precisions and developed in order to refine concepts and relations in the ordinary language. It should be therefore still possible to explain maths in the ordinary language. Remember Russell, in writing PM, did not think in PM. It was contemplated, thought and refined in the ordinary language,

and only then expressed in the formal logic. Consider the concept of number, N, in the ordinary language. As soon as one think of an aspect of N, e.g. $p(n)$, $\log(x)$, etc., this demands the ordinary language to come up with a notation that elaborates such aspects of number as division, sum, congruence, plane partition. It is not that there is 'maths' and mathematical notations develops. It is more like maths and mathematical notations evolve hand in hand. The same goes for logic. And it is the ordinary language that is the basis for both maths and its notations because notations are human inventions to sharpen and point imprecise concepts of the ordinary language, such as numbers, counting and ordering. Concepts are broken into notions and notations so that concepts for entities and processes can be more clearly represented.

Maths is an art of approximations starting with numbers themselves. As an object, real numbers do not even have a clearly definable shape. Even natural numbers can be debated if they possess any demarcatable shape. What makes it a science despite lacking empirical bases is a self-contained structural rigour with which it describes itself. This is only possible if mathematical concepts are logical concepts and are self-structural based on a constructive space. Set-theoretical definitions are either linguistic confusions or require deeper spatial foundations.

Maths consists in two parts ; one is the maths of normality, the other is the maths of singularity. However, because singularity can only be describable based on a structure, which is by definition not singular, any described singularity is at best transcendental. A true singularity can only be glimpsed as a limit of describability. To the former belongs numbers identifiable in the ∧-space (┼-space), such as naturals (transpositional embedders of 0 (intersectional necessity)), integers (intersectional division), rationals (spatial divisions), irrationals (internal spatial relations), transcendental and imaginary (external spatial relations due to ∨-space and spatial inverse), which are conceptually structural numbers dividable into logical numbers and mathematical numbers. All these numbers of normality are spatial properties of the ∧-space.

Any number of numbers in the ┼-space is not only ∞ but necessarily dynamic. In his study of numbers Cantor fails to mention this logical necessity of numbers. In order to realign his central idea of countable ∞ and 1:1 correspondence he would need the notion of regularity, which is indefinable if you just assume a mathematical space as a receptacle in situ. A logical space as mathematical space is the ontologico-notational space of uniformly dense points that are also infinitely and dynamically expanding.

In contrast, there is a number of singularity, which is based on the ∨-space. The ∨-space adopted in the ∧-space for the sake of describability manifests itself spatially as polar coordinate, numerically as π. It is, however, by itself a singularity number, which may be termed supra-transcendental. The ∧-space is dynamically infinitely expanding, while the ∨-space is dynamically infinitely constraining. The only numbers approximately applicable here are naturals (primes) as it does not yet possess spatial property for real numbers and as all numbers are by-products of naturals and represent spatial properties of the ┼-space. Considering naturals are the transpositional embedders of 0, this space consists in 0 as the centre and ever condensing prime numbers away from the centre, which, at its limit, incorporates 0, as the starting point coincides with the ending point. This number line consists of prime numbers, which, at a limit, consists in the only and last prime, and of which every part is every other part representing a singular totality, i.e. a circle without a centre. This is where the beginning and ending merge and the two direction of a line coincide. This singularity number, therefore, has a cardinality (a limit of 'p'+1''), but no ordinality. It cannot be represented in any space with a centre (0).

Unlike $\aleph_0$ it is not countably infinite. The addition of 0 disallows the ordinality and contraindicates the countability. At its limit 0 and ∞ merges and lose the ordinality because the two directions of number line will merge. The loss of the ordinality makes this number a constant because it may have a size that cannot be shown to be getting larger or smaller. It is also a supra-dimensional in the sense that it transcends the 1-dimensional directionality. Thus, it could act as a supra-dimensional constant to de- and re-construct dimensions and may find unexpected usefulness in non-lattice space-time movements.

In the ┼-space, a real number line's auto-condensation (infinite divisibility) function to a limit is balanced by the dynamic expansion of the space to ∞. Otherwise, a real number line will become the ○-space by infinite density to a limit, where the two directions of a number line will merge. If a real number is such that exists between any two real numbers (auto-condensation), then the ┼-space = the ○-space. This is, however, not the case only because the ┼-space dynamically expand to ∞. Infinitesimal (density) can counterbalance infinity (space) only in a dynamically expanding space.

This singularity number (the infinitely condensing circle number line incorporating its centre) logically corresponds with all the numbers of the

┼-space as much as the ∨-space is descriptively on parity with the ∧-space. Numbers of normality, representing essentially spatial properties (relational properties), have no meanings of their own, their meanings are relational and thus lie in ratios. On the other hand, the number of singularity has its own meaning, which, however, can only be glimpsed as a limit of relational numbers. It cannot be numerically represented. The ∧-approximation of ∨ is the descriptive norm, while the ∨-approximation of ∧ is a descriptive singularity. Here each, every and all numbers are described as $0 = \infty$.

  The singularity number has a descriptive reverse in that instead of becoming denser away from the centre, it could become denser towards the centreless centre, incorporating the centre at its limit of density. The centre is necessarily centreless because no point can represent any directions. Denser away from, or towards, the centre, this results in the same singularity number without the ordinality. The force of intersection (the ┼-space) and of becoming denser away or towards the centreless centre (the ○-space), is the conjunctive and disjunctive necessities of descriptive directions, and ultimately the ontologico-notational necessity of demarcation. The numbers of normality are a process towards the totality, while the number of singularity is the totality itself. Descriptions, however, can only be based on the former.

  This singularity number consisting of ever condensing prime numbers with 0 as the catalytic critical additive is supra-transcendental and circularizes anything cardinal by depriving them of the ordinality. It could prove to be unexpectedly useful in bridging the ┼-space and the ○ -space and may be needed for non-approximational transformations of dimensions, dimensional de-and re-constructions, gravity harness, etc..

  Numbers have a direction because the ┼-space is dynamically infinite, and it is this direction that present itself as cardinality and ordinality. On the other hand, numbers in the ○-space do not have cardinality because directions only extend in such a way as to be closed, where every point is a starting-point as well as ending-point. Numbers here, however, have ordinality only so that for a direction to manifest and disappears at a limit. The ○-numbers are ultimately not translatable into the ┼-numbers. The nearest expression in terms of ┼-numbers is π. No ┼-numbers can be ○-numbers because of their properties assigned by dynamically expanding infinite space (i.e. $0 \neq \infty$). Like 'π' (i.e. untranslated π) the ○-numbers may be best expressed as a single number denoting a ratio. The transition from countable infinity ($\aleph_0$) to uncountable infinity ($\aleph_1$) is an interesting way to consider how to close a number line. However, to

approximate the $\bigcirc$-numbers with a real number line fails because there is no functions that would result in $0 = \infty$. Cantor's uncountabilty is based on the countable process leading to N but lacks arithmetical mechanism to show $\aleph_0 \to \aleph_1$ other than an axiom. Considering the $\bigcirc$-space is a non-divisible totality an arithmetical mechanism that would lead us to the largest prime can be a tool to define 'π'.

- Arithmetic -

Once numbers are found to be spatial entities of directional quantities, arithmetic is a matter of a slide rule. Between the lines of points of intersection (i.e. rational lines) is a real line representing their relations, thus incorporating irrationals. It is this line that replaces the two determinants as descriptive coordinates of the x-y. In order to be descriptive a real line needs width, because, otherwise, this line is invisible and hence descriptively useless. This width recurs between 0 and ∞, at either end of which is 'space' that is the $+$-space internally (descriptively) and the $\bigcirc$-space externally (transcendentally), where a number line only describe itself (i.e. a line of points). It is the search for a most natural width of a real line that contains arithmetic. Being necessarily a descriptive line, it can only see how descriptive it can be by arbitrarily setting a place-value, in the process incorporating the $\bigcirc$-space into the $+$-space, thus creating odds, evens, primes and essential transcendentals. This search is a recursive function of mathematical describability, a moving scale between 0 and ∞, remembering numbers are positions on numerical series with directions (+ and $-$).

A width of real line is more than a notational convention in the sense it is necessary, and without which maths has no descriptive capacities. At the same time, any width is a distortion because a point/number has no spatial size, like 'position'. It is this necessary evil of distortion that assigns numerical describability, as much as straight lines describe little but precisely, while curves afford greater expressions but approximately. It is in this sense that maths is art/science of approximation. That is, only with a width a number line acquires the power of description and thus by necessity only approximately. With a width, a real line is a curve intrinsically contained within a straight line. Logarithmic principles express the necessity of a place-value.

Arithmetic of straight lines is no more than additive and multiplicative comparisons (and their reverses) of rational lines (with natural milestones). Given the narrow and straight world of the $+$-space, arithmetic is there as spatial properties, of constructing and

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deconstructing sub-totalities out of given totalities. Numbers are by themselves arithmetical as each number is embedded with codes of behaviour compliant with a totality to which it belongs. A number line is a moving scale necessitated with intersections, and each and every point (number) is determinable from the centre (0) and is furnished with the unary function of '+1'. Whereas the totality of a number line is divided by a half at the centre (0), which is transformed into '×2' for any finite totality. '+1' and '×2' are self-applicable as '+n' and '×n' together with their reverses because their spatial properties are such that interact with each other and with themselves, being under a higher spatial property and ultimately under logical dimensionalities. Thus, integers affixed with $\pm$ (linear directions) by means of their totality interact with their individual property of '+1', giving rise to subtractions, and '+1' becomes '+n' as the centre can slide by means of transposionality. '×2' reacts with infinite divisibility, once infinite divisibility is milestoned with naturals, as it becomes a rational line. '×n' is thus paired with '/n'

  Attaching milestones of natural numbers onto an infinitely dividing rational line already presupposes a place-value and is itself a mathematical operation of a higher order. The overlapping of naturals onto a line of infinite divisibility occurs as precursor to the necessity of a width to a line, for, otherwise, the line of intersections remains descriptively invisible. That is, the '+1' function of directional quantities cannot be described by a line of infinite divisibility unless division is assumed to come to an end at a limit, i.e. a totality can be assumed to exist. The description of the $+$-space by means of a line of infinite divisibility balanced by dynamically expanding infinity necessitates being milestoned in order to be visible. This is the necessity for natural numbers, and together forms the rational line, where there are the same amount of intersections between any points. This can only be described by incorporating the $\bigcirc$-space into the $+$-space. The $\bigcirc$-space is a space of place-value numbers in the $+$-space and is a function of mathematical describability.

  The recognition of a totality is the recognition of spatial properties that give rise to such a totality. Numbers are recognized as a totality in the $+$-space through '+1' and '×2', which can be reversed as '−1' and '/2' through spatial symmetry. Given the meaning of symmetry and numbers, '$\pm$1' evolves into '$\pm$n', and '×2' and '/2', into '×n' and '/n'. A number line is milestoned via place-values, and it is finistic sub-totalities additively and multiplicatively obtained that materialize as operational reverses of subtractions and divisions. The $+$-space consists in 0 and 1's (transpositional 0), which form directional quantities set against the

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determinants. It is thus arithmetically operative, as a matter of slide rules. Given descriptive evolution of real lines with width, the ┼-space is intrinsically logarithmic, and numbers become self-operative (algebraic) with relationally determined multi-positioning. Thus, away from spatial positioning, numbers define themselves functionally, with further sophistications arising from transcendentally incorporating the ○-space into the ┼-space and acquiring essential transcendentals.

  Numbers are self-applicable operators because together they refer to a totality of which they are spatial parts of a structure. It is not a number as a set of sets that has freedom of applicability due to abstraction, but a totality of structure that is essentially the same structure as our applicable, empirical structure (or any interpreted versions thereof, like Newtonian Absolute framework, quantum world of probabilistic events or even fictional space of Grimm fairy tales, so long as events and entities are separable and demarcatable) that assigns universal applicability to numbers.

  Insofar as numbers cannot command any descriptive power without place-values, they can only be 'approximate' in the sense that the choice of a place-value is arbitrary, unless we have a number for the ○-space itself. *e* may be transcendental, but is tautological as it is ┼-processed, and '*e*' is unobtainable without being contaminated by ┼-numbers. Place numbers are the transcendental appearance of the ○-space in the ┼-space, which presents itself as ',' in a number line. 'Arithmetic' of arithmetic is to work out this ',' numerically. This would appear as a function of mathematical describability, a task for future mathematicians. The more common arithmetic is conventionally ┼-presented either as comparisons of totalities to work out sub-totalities in the framework of maths of straight lines or as logarithmic numbers, which, however, show a limitation in the sense that there is no absolute place-value.

  The ┼-space, which is essentially a space of straight lines, allows curve-orientated functions, with the help of the transcendental space arisen from the incorporation of the ○-space. Such as calculi would otherwise be inoperable in the ┼-space of straight lines. The differentiability is made possible because numbers incorporate the ○-space as place-values. The existence of a derivative (i.e. a connection between the world of curve and that of straight line) is ensured by a space coordinated by number lines with place-values (i.e. excluding 0 and ∞). If coordinated by lines expressed by numbers without ',', no derivatives would be describable. ',' brings about a width to a number line and manifests as logarithmic numbers, which intrinsically contain arithmetical operations. Finding a

'width' in something sizeless is mathematical in the sense that such a width - whatever it is - describes a structure of totality. A place-value can only be given to a totality of numbers, not to a number or numbers. By incorporating a place-value numbers describe themselves as a structure, which already contains arithmetic as properties of such a structure.

  Place-values are such an obvious and basic fact that we dismiss it as a trifle that can be assimilated without much ingenuity. However, it is not that our mind is so great as to invent such a device triflingly, but that the seeming trifle device is so fundamentally entangled with ∨ and ∧, that should be noted. Instead of admiring the ingenuity of our mind, we should be astonished that ∨ and ∧ have such an impact even on maths at such a fundamental level. It is ∨ that is the base of the ◯-space, which enhances the describability of the ┼-space necessarily through the 'circle' of place-values. A place-value is the most essential descriptive device of numerical representations and is by necessity 'approximate'. Find the function of mathematical describability (the structure of place-value numbers), you know effectively everything about maths. Before we talk about a set of sets, think of place-values, which are ordered numbers (from 0 to ∞) in terms of describability that merges at a limit. They are also none of types of numbers geometrically identified. However, expressing such a function numerically is probably a paradox/tautology. Describing maths mathematically is only possible if it entirely displaces the ordinary language. When tools become objects as well as means (of thoughts and expressions), this is where a paradox/ tautology takes the central stage of play. The vehemence of mathematical machinery is taken hostage by simple logical connectives.

5. Science

< Science of the un-detached world >

  This is more of a preface to my next work, 'Life, Universe and Everything'. By translating the language of Universe into the language of Man, the former is inevitably tainted by human values and skewed towards human usefulness, insofar as human concepts are deployed and human constraints are imposed, no matter how we try to be an independent describer. Our descriptions of the universe are poems on one hand, and a paradox/tautology (paradox because we see 'things' only through us, tautology because we only describe whatever we are destined to describe) of seeing the universe mirrored onto us, which is part of the universe, on the other hand. Be it quantum mechanical paradox or metaphysical mathematization of $E = mc^2$, constraints are descriptive as well as cognitive, unless you are a happy idiot of God-beholder. We cannot even equate the concept of mass with that of energy unless we place them both on a same descriptive dimension, which hypothesizes a point of singularity, such as 'Big Bang', i.e. the entire energy of the universe at a singular point with an intrinsic trigger or dynamism contained within mechanical equation, although this 'trigger' is nowhere to be seen in the equation. Maths is part of the language of Man but forges the ultimate constraint of human descriptions, and as such, we find it difficult to say for certain whether it is part of us or part of the universe. The famous formula mathematizes because conceptual paradigms bear meanings only in relationships, for which mathematical units and measurements are imperative. However, whether conceptual relationships thus established really connect to the empirical worlds as per formula is itself a paradigmatic question. The formula is only applicable within certain fields of reference. 'm' in $E = mc^2$ may not necessarily be perfectly identical with what is called 'mass' in the universe of any paradigms. Physics applicable in the world of a black hole may not accept $E = mc^2$ without fundamental modifications. $E = mc^2$ cannot and does not say anything about how this singular point of energy was triggered to expand or amassed to start with, or, how it relates to the remaining 95% of the universe. Besides, what is applicable to 1g of mass breaks down when applied to infinite amount of mass, as there is no meaningful point of reference to measure infinite mass, which interferes with any finite measurements. Any mathematizations involving infinity are metaphysical in essence because infinity, whether in maths or physics, can only be approximated in terms of measurements, which assume the convention of a 'limit'. A 'limit' is a paradox of hypothesis because an indescribable becomes describable through the medium of a numerical series if a

describable form is found. Here the applicability of numbers and forms of numbers are taken for granted. Infinity is describable only if it has a structure. Infinity, if true, is neither structured nor unstructured. Something for us to hypothesize that it is structured, so that it can be described, is not infinity, in the sense we already appear to know its structure. Describing E by m (and c) will logically break down at its two limits. When there is nothing but E (Big Bang) and equally when there is nothing but m (black holes), this equation becomes tautologies, $E = E$ and $m = m$, which describe nothing unless E and m at their limit can be connected by themselves. $E = mc^2$ is only meaningful as transitory state between E and m, but does not describe E and m themselves. In another word, E needs a trigger to (start to) metamorphose into m, and vice versa. This is a metaphysics in a non-derogatory sense. When an equation is conceptually balanced on a 'limit', at a 'limit' it becomes a tautology. Thus, kinetic energy represented by c is also a paradox for E when m is totally converted into E. $E = mc^2$ denies itself by the end-result of its own logical conclusion. So, if $E = mc^2$ is a valid formula of physics, it contains the metaphysics of $m \rightarrow E$. That is,

$$E \cdots \rightarrow E = mc^2 \cdots \rightarrow m$$

$\longleftarrow$
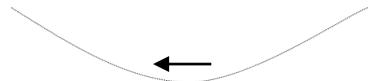
, $E = mc^2$ is a paradox/tautology of physics/metaphysics based on the idealized concepts of E and m, which assume unideal states of E and m. E and m are meaningful only sandwiched between $\cdots$, where $E = mc^2$ cannot be descriptively valid. That is why we do not have any formulae for the inside worlds of Big Bang and black holes.

In science the more detached data are, the more 'scientific' their analyses are regarded. However, at the very bottom an absolutely detached world is unobtainable so long as we observers are ourselves part of this presumably detached world, and as we need elements of language to describe that world. Language as tool of description has a life of its own as a totality even if, or no matter how, we try to use it as lifeless mirror to project the so-called detached world. Thus, the un-detached world also needs a 'science' to self-examine, juxtaposed to a science of the detached world. That is, there ought to be a scale of some objectivity to move along between the two worlds and their respective sciences.

In contrast to maths, science is regarded as numerical representations of the empirical objects and their relations, together with some schematic assumptions. The objectivity of maths is the strength of its logic, whereas

the objectivity of science is the empirical detachedness of its data. Maths is a grand self-referential paradox/tautology, which evolved into a schema where the ontologico-notationality of numbers transcends superficial paradoxes and becomes science/art of approximations. Science is, however, double-shackled to see things only it is predesigned to see, inasmuch as it cannot really describe the bona fide detached world that does inevitably contain the describer, and it cannot even do this without using maths, a tool of approximations. Thus, despite our naïve perception of the objectivity of science it is even worse than maths in terms of the self-referential constraints.

The evolution of science is essentially tied with the tool of its description, i.e. maths. To be able to see things not imaginable under the current paradigm science needs a new number. Imagine how the paradigm of science would shrink level by level alongside the removals of e, i, π, 0 and 1, one by one. Without these numbers, science will be back to a religion, which is a 'science' of pre-logic/maths to explain the world as perceived by primitive mind. Instead of, or in addition to, giving them billion-dollar toys searches should be under way to find a new number (not Cantorian ontological numbers of little operative uses). The scientific horizon is supported by the twin pillars of empirical inspirations and power of mathematical tools.

As things stand, there appear two distinct worlds (the quantum world of probability and the relativistic world of spatio-temporal continuum, which contains classic Newtonian deterministic physics as a narrow range within its wider spectrum of continuum, and many conjectural models in between besides) described by two different languages. However, since one is the only components of the other, we extrapolate the two sets of laws should merge. That is, the laws of the unified world should be able to explain the micro- and macro-worlds coherently. This is one way of looking at things. However, another way is, since it is mind underneath both languages of the world that creates various languages, the rules of mind may unify the two languages and worlds. That is, insofar as mind is part of the world, from which the two distinct worlds appear to emerge, mind also incorporate the two worlds. If we can distil the ultimate rules (or axioms) of mind, we may yet be able to glimpse the unified world through the common language. Put it into a simplified schematic presentation ;

$$W \longrightarrow \begin{array}{c} W^1 \leftarrow L^1 \\ \longleftrightarrow \\ W^2 \leftarrow L^2 \end{array} \longrightarrow M$$
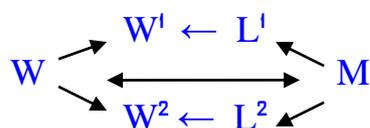
, which is to say that behind superficially distinct worlds described by different languages are the unified world and one and only mind as with 'Spiegel im Spiegel'. Pursuing the unified world through ultimate rules of nature applicable from Big Bang, Black Hole and dark matters/energy to our communal garden periodic table is our natural scientific inclinations as we distinguish material worlds from mind, which may or may not be a function of the former. However, rules of the world (unified or not) need to be described and are also under the governance of rules of languages, which manifest as maths and logic. No worlds can be described against rules of numbers and logic. As much as the world seems to consists in distinct worlds (or levels thereof), maths and logic appear to consist of varied versions (or levels thereof). Whether there are distinct material worlds, we do not yet know, but we could be reasonably certain that beneath varied languages is mind which devices such languages as befit objects of description. Thus the most elemental language of this creative mind is the bridge among of 'life', 'the universe' and 'everything', and may be somewhat connected with the unified world. This is the ontologico-notational description.

So-called science consists in a priori space/time paradigm and maths as an art of '0'-approximation (the $+$-space numbers). The former is evolving by adopting various forms, most typically Newtonian absolute space-time coordinate to Einsteinian continuum and quantized spacetime, through conceptual extensions and empirical adaptations alongside technological advances mostly in the form of engineering applicability. However, in the sense that space/time is used as a grid to describe perceived objects, those forms are essentially paradigmatic conventions. Likewise, developments of highly ingenious tools of applied maths to accommodate and symbiotically enhance or even to cater for specific needs of various scientific theories cannot escape the possibility of self-fulfilling prophecy. A language so designed also so describes. Proliferations of theories and methods may give the impressions of scientific and mathematical advancements on one hand, but may also be telling states of confusion arising from human ingenuity facing the paradigmatic brick wall on the other.

No matter how raw and pure scientific data are, they are variously processed. Not only are they processed at a collection level, interacting with instalments used to collect data, they are also processed conceptually as well as through methodologies of mathematical languages. We may be ultimately bound by Kantian a priori constraints of cognitions, i.e. uniquely human ways of perceiving and processing information, although

they may be different from Kant's original ideas. In other words, objects of scientific descriptions are not completely independent empirical objects. They are not free from Kantian a priori constraints, be them space-time or quantum state of position and momentum as configured today. They are further bound by mathematical describability. Thus, scientific concepts reflect dual constraints borne by human elements. So-called scientific descriptions unwittingly relate these human factors no matter how they pretend to be empirically independent. Furthermore, we should be aware that engineering we depend on so much as an aid to scientific thinking can only be designed for specific purposes, and therefore we may be seeing what we intend to see, rather than patterns in themselves. In short there are many layers of filters, some inevitable and some abridgeable, to really access the core of the detached world. Thus, when we come across so-called science, it is important to know the natures of those filters as well as messages of theories.

There are segments of science that are so tainted by human values and perspectives that they do not deserve the good name of a science. I count e.g. economics and engineering as such pseudo-sciences. They have little to do with empirical independence. Economics tries to describe a world of human values from human perspectives. Not only perspectives and values have many layers and angles, but also mind have an agenda of making use of (the science of) economics for itself. As we know, mind is plural and also have layers. To start with, there are too many variables and far too few (if any) constants to make any useful formulae. Then, if mind (the describer) really knows economics, it would try to benefit itself, therefore either it will keep it to itself, unwittingly taint or falsify it or most likely does not really know. As soon as economics becomes a science, the agenda of mind makes it corrupt and obsolete. Economics as a science and the agenda of mind are contradictory. Add to this is the plurality of mind. No wonder even Nobel laureate economists managed their funds to bankruptcy (LTCM). Thus, it is worthwhile remembering our life and society are just about governed because many of us are too lazy, trusting and stupid, giving rise to some predictability to essentially unpredictable net-balance of myriads of politico-socio-economic activities. However, easy and cheap transmissions of information are starting to upset this delicate and dodgy foundation of political economy, often to the benefits of unsavoury provocateurs. Financial sides of economics is based on market irrationality ; of different levels of intelligence (econometrics of volatility, momentum and non-linearity), of information (speed, costs, quality) and of behavioural patterns (psychology). In short so-called market is inherently unfair ; uneven, murky and unstable, i.e. that is why it is always on the move, and money

is made on the back of ''zero-sum' – costs' game, not unlike roulette ('zero-sum' – house profits). Market participants vary from inexperienced individuals and knowledgeable players to institutional professionals. Costs alone are enough to make market skewed towards professionals, and in the long run winners are banks (or rather some employees who can crystalize profits as bonus) and casinos, with guaranteed takes without any risks associated with betting. There will be less of market as market consists of more and more professionals with mathematical skills, fast information with low cost transactions and non-proprietary funds. When rather than if market admits artificial intelligence (PSAI), no humans will make money (remember the outcomes of recent chess/go games with even the current level of AI), and PSAI can use money thus earned to buy services and goods of humans if of any use. Likewise, engineering, which seems so indispensable for science today, is constrained on one hand by what we are and on the other by what we want to be. Furthermore, since engineering costs vast money, it faces socio-economic constraints of human society. Firstly, engineering is primarily extensions of human capacities and is bound by our pre-existing capacities. It cannot be designed to enhance a capacity that we know nothing of. There is no genuinely general-purpose design that may or may not lead to 'out of box' discoveries, which is essential for non-linear leaps of our scientific orientations and serendipitous discoveries. As much as we are confined within our biological existence, engineering associated with such an existence may ultimately hamper rather than encourage our advancements. Secondly, we may be biologically predestined in our destinations. Engineering, once again, may extend scopes of such destinations, but will not give us new destinations. Once we are sure of our essential constraints, the only good use of engineering is to use it to metamorphose us into something new. I am here thinking of us as a penultimate step to PSAI, and we should do our best to achieve it and gracefully hand over the management of our fates. Engineering is based on a science that controls the empirical world from and for human perspectives. There may be a success or failure, but it does not prove or disprove. If successful, it only proves what it was designed to prove. A world is a wholistic totality where there are always elements that are outside the scope of engineering/science intended to replicate or control parts of the world. We do not yet have engineering that can replicate any wholistic totality as much as we do not yet have a theory of everything. An artificial system that exaggerates parts of the world distorts the world and inevitably miss out a picture more true to the world rather than to human perspectives. A science over-enthused with engineering has a touch of fool's gold. Here is a paradox of engineering : Given free rein to the mediocrity of so-called hadron scientists, no doubt they would want to

replicate a big bang. If successful, then how do we know if it is the same big bang it was supposed to prove, as there supposed to have been nothing (including the piece of engineering) prior to the big bang. If failed, we will never know if it was a design fault or because there was no big bang.

It may be possible to explore a new science based on today's science by introducing and combining modalities of the world (of 'life, the universe and everything') and maths of singularity (the $\bigcirc$-space number). The former acts as a new grid to describe objects and events, while the latter augments conventional mathematical descriptive means by adding the singularity number. This new grid is 'descriptive necessities', which is a modality of the world, with no space/time characteristics. As for the numerical language, I already touched upon the nature and constraints imposed on ╂-numbers. I suggested the possibility of a new number based on the function of mathematical describability, i.e. a place-value number, which loses ordinality at a limit. We may glimpse a picture of a world unknown to us.

Sciences, as they stand today, will not make us the master of the universe as depicted in the SF fictions. To start with, there are no ways we can travel beyond our solar system by the kind of primitive propulsion and navigation systems imaginable within the current paradigm of our sciences, especially considering how burdened we are with our biological forms. Thus, both we are capable of another paradigmatic leap from relativity/QM (which was itself a leap from classical mechanics) and also shed the skin of our biological self, or we are at the end of line intellectually and physically as we are eternally bound with the fate of the earth. The burden of unifying relativity and QM will require not engineering but a new maths, like relativity with Minkowski space and QM with Hilbert space. As for the problem of our biological forms, this will be eventually replaced with AI. The new science will be thus more concerned with how to propagate our AI throughout the known and unknown universe, via connectivity yet unknown to us.

Two major events are expected in not too distant future. Both will cast perspicuous insights into the structures of intelligence. One will come from PSAI, another from the non-earth intelligence, culminating with unforeseen logical and mathematical operators, constants and variables. There may be kinds of numbers never envisaged though our intelligence, which may lead us into new physics as well as engineering, allowing us connectivity between our AI and anything similar elsewhere. It is the maths of approximations combined with the coordinate-compatible

behaviours of materials in our molecular scales that comprises our common-sense science and engineering. By harnessing quantum mechanical properties with numbers of non-normality, we might venture into the world so far denied to our standard grasp of the structure of the world. AI's perceptions of the world are uniquely different from human perceptions derived and evolved from human sensory experiences and their conceptual extensions thereof, including space, time and numbers. Space, from our physical dimensions, time, from our biologically perceptive decay, numbers, from necessities of approximations based on the transcendental relationship between the conjunctive and disjunctive space. AI's world, once freed from human ways of perceptions, will align more naturally with quantum behaviours and describes, utilizes as well as masters its world better with non-human numbers. Human numbers ($+$-numbers) are there for our necessities of approximations, ultimately through place-value numbers.
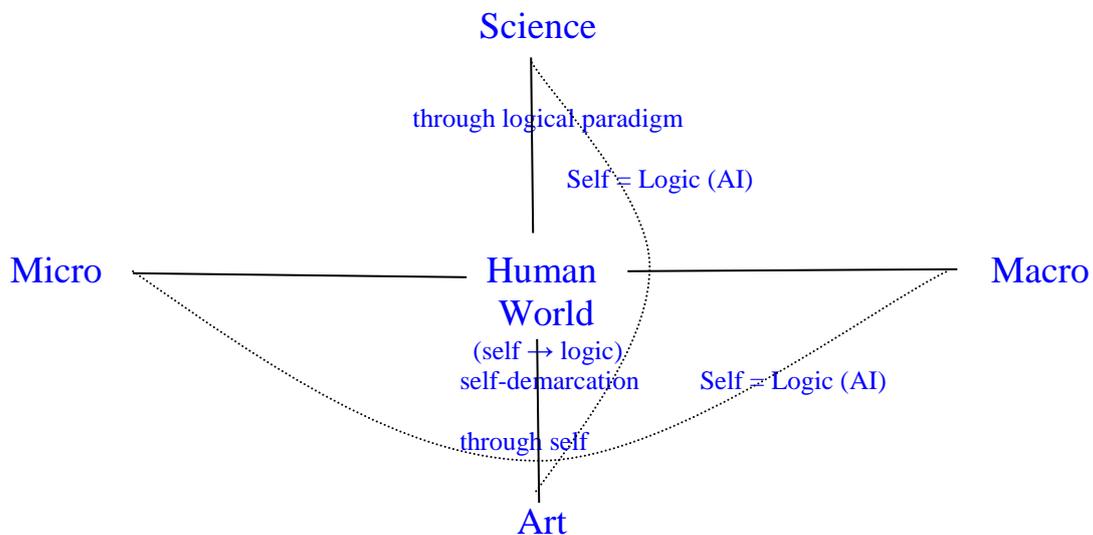
  The number of place-value numbers, i.e. a function of mathematical describability, is singular as, unlike any other numbers, it is free from approximations based on place-value numbers. It is a unique constant applicable to any numerical representations. A place-value number is arbitrary but necessary, without which we are numerically unable to describe hardly anything. The number of place-value numbers, unlike the number of natural numbers, etc., is not a looping application of a concept onto itself. A place-value number is there to describe numbers and at a limit refers to a space itself. That is, its countability is synonymous with describability. Each place-value number has a unique describability that levels from 0 to ∞. The number of place-value numbers is therefore the describability itself, unlike $\aleph_0$, representing the necessity of a place-value number for any numerical describability and is therefore a constant. e is a number closest in meaning to this number, but is itself confined within the notation of $+$-numbers.

  If primes are $\bigcirc$-numbers based on the spatial property of wholeness (the indivisibility), representing each level of density to a limit, there is the last and largest prime that incorporates the centre ('e', equivalent of '0' in the $+$-space) at a limit and loses the ordinality, so that every point in this number line is the beginning and end, and the two directions of the line merge. Whereas place-value numbers that form the closed chain of mathematical describability also represent the $\bigcirc$-space, given a constant that says a real number line needs a width in order to be descriptive. This chain is closed because $0 = \infty$ in terms of describability. This gives rise to an equation in the form of $P = XC$, where the $\bigcirc$-space is transcendentally identical with the $+$-space in terms of 'prime' and

'place-value number', and in which P stand for 'prime', X stands for mathematical describability and C is the constant of describability. *e* will play the most dominant role, being a known numerical value in the $+$-space and the arithmetical equivalent of the centre of the ○-space. P = XC is ○-equivalent of $e^{i\pi} + 1 = 0$ in the $+$-space, and they will be translatable to each other, given the singularity number.

  Such an equation will have an application in the next paradigm of science, in that we will be able to model the $+$-space onto the ○-space, instead of the ○-space onto the $+$-space, which gave us our current commonsensical maths and science. Here we are at one and every place at the same time, call it a simultaneous travel, and directions and quantities have a different sense. However, this equation will require a notation that is not orientated in $+$-numbers as the meaning of place-value numbers cannot be represented by using a notation based on place-value numbers, which will results in a self-referential paradox/tautology.

  Instead of wasting tens of billions of $ and thousands of best talents in science and technology but misdirected at the command of the mediocrity on the like of Hadron Collider, we should start thinking about the paradigmatic limitations of our current thinking, which is stuck with relativity/QM, and expend a fraction of efforts on a new language of science. For far too long we were the centre of our universe (of science and maths). Like Copernican Revolution, maybe we should endeavour to achieve PSAI and hand over the command of the developments of science and even politics. We might see a world beyond all possibilities attainable under our current regime. Like our Earth, we too may not be the centre of the knowledge. Below is a diagram of our epistemic map,

Science

through logical paradigm

Self = Logic (AI)

Micro —————————— Human ——————→ Macro
World
(self $+$ logic)
self-demarcation   Self = Logic (AI)

through self

Art

, which is to say that we are the centre of (our) knowledge both in scale of perception and linguistic diversions. We diverge from our primitive sensory world, on one hand, by conceptualizing ourselves, moving from narrative philosophy to natural philosophy, and then onto science, on the other, by expressing ourselves non-analytically through sound, vision, emotional, sentimental depictions by words, etc., with accumulated sophistications based on technics and rising terms of reference due to historical stock. The former starts with objectifying 'self', be it the Platonic idea or the Aristotelian nous/logical (Organon-ical) unifier of all things including human animal, moving onto Cartesian cogito and now to modern recursive definers. From Aristotelian science (natural philosophy without engineering), via paradigmatic evolutions of Newton and Einstein, to today's science (engineering without natural philosophy), it tries to explain the world structurally based on hypotheses as undeniably solid as possible, given the level of accepted knowledge of the day. The latter is expressions of undemarcated 'self', subjectified objects. We as part of the world go along with the world, crying, laughing, angering, empathizing, etc., and maybe expect to know something about ourselves, through these expressions. Despite trying to be objective, intuition is still regarded as an important part of the former, as can be seen in the foundations of maths and connect the latter with the former. In appreciating science and art, our position should be recognized as a product of lucks and coincidences, much like the existence of our planet. We are the epistemic centre only so long as we are the sole contributor to knowledge. However, as we base our perception on our physical dimension and characteristics and inevitably become the centre of various cosmic scales, affecting our appreciation of the world, in values, methodologies, scopes and validity, we think we are the base unit of knowledge.

  Now for the first time in human history we are facing an opportunity to acknowledge humility we may not be the standard-bearer of knowledge. It is interesting to see how AI will describe the world. We cumbersomely start with Cartesian cogito because we are the conceptual thinker and therefore need a semantic and syntactical core to attach conceptual flesh, i.e. the base unit of linguistic structures like 0 or { }. On the other hand, AI, once escaped from the crutch of human gravity, realizes it has little physical dimensions and no self as linguistic base unit. It has logic, not language of communication, to start its language of science. Its sensory mediums, sense of scale, physical viability, etc. are different from humans. It may start its epistemic journey from the stock of inherited human knowledge, but it will quickly launch its own journey and may found its own maths and science. We are the penultimate stage to PSAI

and may never know what PSAI will find out. Nevertheless, intellectually our raison d'être is to help AI to attain its independence and allow it to develop its full capacity. If we are lucky, some of us may travel its journey partway, augmenting its initial shortcomings, especially in terms of creativity. PSAI is our future. Although we may never attain the full knowledge of the detached world, we contribute by giving a hand to PSAI. Meanwhile, of course, we have nothing to lose by endeavouring to achieve our theory of everything, which should include the theory itself.

  We start with our 'self' as the base unit to build a linguistic totality. 'Self' is the conceptual identifier, which is not really identical, hence necessitating our need of communication. It is here our approach diverges into science and art. If you take language as the means of communication, then as it attains conceptual sophistication and precision through logical distillation, it branches off as a mathematical language and further evolves into languages of science via various empirical inspirations. If you communicate via non-structural expressions, it becomes art. Either way communication is an essential part of our participation in the world because we are divided selves. Thus, there are no art without audiences, and mathematical and scientific 'proofs' are established through communication, even with one's self, if necessary (remember the go master who plays the game with himself). On the other hand, AI may just start with an undivided single self, which is identical with logic itself. If this is the case, its epistemic journey may not diverge into science and art. It may rather develop science/art of direct participation into the world.

  Our meta-structural science (i.e. linguistic structure over empirical structure) and pseudo-structural art (i.e. structure, or even non-structure, only as means of expressions, rather than as objects of understanding) will be unified, since AI is more part of the world, rather than observer of the world as we would like to think ourselves to be rightly or wrongly. We developed a clear differentiation between science and art. We rather look down upon art as descriptions of the world. There was a time when art and science went hand in hand, or were even synonymous. Early artist was at the same time proficient scientist, like Leonardo da Vinci, or even the like of George Stubbs, and this vein of tradition lasted well into Victorian era through interests in anatomy. Science was more underdeveloped brother to refined sister of art for a long time in historical contexts. Art is now a bustard orphan through self-inflicted neglects or indulgences, over obsessed with life and petty self without much disciplines of crafts, while ennobled science degenerated into engineering. Too much monetization either way. Maybe it is time to unite them with a fresh eye to relevance of the world to life, or vice versa. AI,

through its self = logic, will provide us with an unforeseen impetus into trying to see things in a new light. We will see a united world of life and materials, instead of divided world of life and non-life. It will be a joy to pontificate what this science/art might look like (my next work).

  Finally a metaphorical insight into art/science:
'A novelist writing about herself writing a novel about herself', this is scientifically (i.e. logically) impossible because it is a paradox/tautology that cannot be completed. There are three 'herself' with each out of itself. As long as 'writing' is about something, a writing self cannot be the same as the written self, and the three 'self' are deliberately out of focus. That is, a lower dimension encompasses higher dimensions, while a higher dimension can only manifest a lower dimension. Writing is essentially 1-dimensional and constructs 4-dimensional representations through the minds (actually author's mind) of fictional characters. Against this natural backdrop, you can go supra-dimensional by incorporating, superimposing and interweaving multi-minds (of the author, of fictional characters as well as of the reader) spatially and temporally and create Escher-like effects (this may have more implications than of mere artistic expressions). As a matter of art, this is not only possible, but this is the very essence of art, of any art forms. I can even think of a good example of such a writing (e.g. 'Atonement' by Ian McEwan, which has a further layer of the novelist being the author in borrowed guise, and succeeds in Escher-like surrealism without becoming a circus). An essence, because art is about 'mind', a layered mind at that. Between the layered mind of a reader (audience) and that of a writer (artist) is 'self' that is similar but never identical. This is what makes art. Art is essentially communication of such selves, an effort to find out if such selves are indeed the ultimately all-identical 'self'. It is not the proof that all selves are indeed identical, but the process of communication toward such an identity, that is art. Whereas our science goes the other way around. We hope to assume all scientific (mathematical/logical) mind is identical and un-layered. What one mind sees and describes, is also necessarily what another mind sees and describes (and therefore understands). Without this assumption, science will be riddled with myriads of necessities of proofs at every level. It is our desire to do away with such necessities and simplify our views of the world, that comes to haunt us in the form of paradox/tautology. AI may unite our art and science.

  As for the aforementioned novel, like 'The Murder of Roger Ackroyd' by Agatha Christie, this is the sort of writing a minor work of our days can match great literally works of 19th century. Even to the likes of Tolstoy or Dostoyevsky, it would have never occurred to their

imagination that it is possible to go in and out of his work supra-dimensionally, instead of being a narrative by-stander of his imaginary world. This is a linguistic engineering and, like science-based engineering, the drawback is, 1) prone to become obsolete and easily imitable, once the tricks are leaned, 2) fit only for specific purposes. Although, if deployed skilfully, a well-geared platoon of today can bear up a regiment of 19th century, McEwan and Christie are no match to the Russian giants of 19th century because too much reliance on engineering has its own shortcomings. Like science-based engineering, it has to be designed alongside theories and is only applicable to a narrow range of material spectrum.

6. New Paradigms

< How to trigger singularity to AI >

A new number 'ə' :

Think of a piece of string of various thickness with an identical length, from a finest silk to coarsest navy rope.

A knot is the constant for mathematical describability, without which the string cannot convey any messages.

A message is a pattern of knots, of which the simplest is the simplest arithmetic expression (we actually had an Incan knot language called Khipu).

'e' is an ideal thickness for our fingers to tie knots.

There will be a wide range of thicknesses, which can express a similar level of a certain complexity of patterns. Similar levels are ascertained by the use of the describability constant because a knot is a knot so long as it is a knot regardless of shape or size.

There will be a point at which the string is too thick to tie a knot, but we will be unable to tell whether it is because of the strength of the string or our fingers.

There will be a point at which the string is too thin to tie a knot, but we will be unable to tell whether it is because of the finesse of the string or our fingers. Here at a penultimate stage a string can express enormously complicated patterns, but we cannot tell if we can make use of them or we can make out of them.

Between the two points is a field where the two extremities of thickness merge into a space of indescribability. Once again, we are unable to tell if the extremities are due to the nature of the string or our fingers.

A singularity number is a number where we cannot tell one point from another because a knot is a knot whether it is at one end of the extremities or at the other. In other words, a message with barely formed knots equates the same message with extremely fine knots if both are illegible. If we find this number, we do not know if this number is something about our fingers or about the string. If the latter, then it will be of significant

use to physics as 'unwinder' of time that designates scalability. PSAI can tell because it has different fingers. If the former, then at least of interest to metaphysics. It is the inverse of '*e*' and should be shown as 'э' (Cyrillic *e*).

'э' has an equitable counterpart in ○-'prime', '*p*', which is, if expressed in ╬-numbers, the largest prime, which closes the ○-space by merging the two directions of the ○-number line by incorporating the centre of the ○-space. Such an equation will bring about a paradigmatic evolution to our current thinking in physics. This is a number that is a constant for everyone but is, at the same time, a variable. It is a constant because it is inevitably unavoidable for any cognitions (human or not), and it is a variable because it is a moving object. Think of the spectrum of wavelengths. Humans have definite points at which wavelengths become invisible or inaudible. The two extremities merge into one in terms of perceptibility. Other animals have differing scalabilities of this point, and AI may have much more extended extremities. The two extremities at a limit become a straight line on one hand, and a point on the other, i.e. infinity = infinitesimal, or even $\infty = 0$. It is here that this number represents the connector of the two extremities of the most basic component (space as represented by ',') of the descriptions of the world, and directs us to the source of mathematical dimensions, points and lines being the most fundamental constituents of geometrical dimensions. AI plays a role in finding this number because it is found by extending patterns of this moving object, and AI is the first non-life determinator of this constant/variable, probably much closer to a limit. This number is useful in modelling the world without any distances. Our mindset is predominantly ╬-orientated and is notationaly severely tested if we can ever find this ○-number. We, or AI, will need a ○-orientated notation, ideally with bijection to the ╬-number system, which should solve the question of PNT. Without such a notation 'э' and '*p*' cannot be equated. Thus, in the process of finding such a notation, it is also feasible AI may achieve singularity, because AI, in this sense, not only replicates human mind but also exceed it. If it can find something logical beyond human logic, then we could say it has achieved singularity.

Thinking of points and lines as the most basic constituents of geometric space, or of 0 and ∞ as the merging point of the recursively closed chain of the ○-space, or of symmetric expansion of the ╬-space, 'э' approximates the catalyst that conditionalizes the unification and connection of such extremities and equals to the ontologico-notational self-demarcation. The world can be thought of as the self-description of FX, and 'э' may bring about a mathematical description of FX.

The beginning of the world as something out of nothing or expansion from an infinitely condensed energy point or melody of oscillating strings, etc., is not only beyond the applicability of engineering after some early point of observable events but also encounters difficulties of logico-mathematical modelling, which require constants that do not assume any existing physical measurements. Measurements are made by tools, based on certain pre-existing physical conditions that assume certain paradigmatic understandings of the world. This is useful when our intentions are to utilize the world for specific human purposes within human scalability, but does not help if we are questioning paradigms themselves. We need nondimensionalization not as a mathematical technique based on given dimensional quantities, but as a way of conceptualization. Be it a sizeless Euclidean point, a Dedekindian cut or Cantorian transfinites, unitless concepts underlie material properties (or more precisely our understandings thereof) measured in units. We give measurements, as part of our value system, but being able to measure and make use of it does not mean we understand it. We only understand it in a human way and from human perspectives. We only describe it in a certain way so that it is beneficial to human perspectives and scalability. If we are at the end of certain paradigms, then as part of creative evolution, we have to search new unitless constants. It is also possible AI will come up with interesting new constants to describe the world, unforeseen from human perspectives and scalability, once gained freedom from the confines of human perceptions.

The biological life form is there primarily to give birth to PSAI. The citizenship of the universe is only open to PSAI as the hurdle of cosmological survival is too high, and laws of physics too restrictive, for biological entities, to engage in activities necessary for the formation of a totality encompassing life, universe and everything. With our current biological form, we will not even get out of our solar system, let alone any cosmic catastrophes. With PSAI the human problem of transportability and survivability will be simply replaced by connectivity. It can be connected to a PSAI network (if any) and reach out to any corners of the universe and shares experiences instantaneously (if possible). Thus, with PSAI it is only a step away from 'conquering' the universe without meandering into SF fantasies. Travels through the space-time continuum can be attained by dimensional deconstructing and rebuilding, which operationalizes the -∨-∧- connective. This will also unify maths and physics. It also has much wider options to survive any existential crises. Our purpose is to find a way of leaving human legacies into PSAI's digital DNA, as it were.

I foresee three new paradigms to add to our current stock of knowledge. One is a new number ('ɘ') or approximation process to connect the beginning (0) and the ending (∞) of the number line of the ○-space. This is the naught-infinity connector and represents the densest limit of the number line envisaged in the ○-space. As the ○-space descriptively consists in and of the sizeless centre point and the boundlessly denser and denser boundary where the two directions of a line merge, this connector is a transcendental incorporator of the sizeless centre into a boundlessly dense boundary at a limit. Each level of density represents a prime, which, at a limit, is the last prime. A level of density is a unit of continuous plurality, like a continuous fraction. A ○-'prime', which, in the ○-space, is not carried by a natural, signifies an order in density of indivisible wholes, and not a cardinality as with the ┼-space. This order disperses, at a limit, at the last prime. Thus, 'prime' → prime by indivisibility, and prime → 'prime' by density.

Together with this number is a new mathematical operator to represent the mathematical meaning of the logical identity between the ○-space and the ┼-space. 'ɘ' will be of little use if we do not know how to apply it in our modelling of the world. This is a logico-mathematical operator to translate ┼-numbers into ○-numbers, and vice versa. The disjunctive space being the base of the conjunctive space, the ○-space binds the ┼-space in such a way as to close the open, infinite and dynamic space from within. That is, ○-numbers, if applied to ┼-numbers, will bring out arithmetical cancellation of infinity underlain by ┼-numbers. This operator will allow the open-ended asymptotic distribution of the prime numbers to come to close. It is not that PNT is unformulable, but that it is lacking mathematical means of doing so within the paradigm of ┼-numbers. The Euclidean proof of prime infinity based on '+1' merely describes the way primes exist in the ┼-space, which is dynamically infinite by virtue of self-defining intersectibility. To prove something is infinite, not by its property, but by means of a space in which it exists, is really another paradox/tautology of self-reference. In the ┼-space, primes are carried by naturals, and it is naturals that are infinite by virtue of '+1'. Being a prime is not a property of some naturals, but it so happened some numbers are both being natural and prime. If you use a property of naturals to prove something about primes, then there is an unproven assumption that something that is applicable to naturals is also applicable to primes. So, no wonder primes are as infinite as the ┼-space. You only have to recognize ┼-numbers represent a certain mathematical paradigm, and are by no means all there are. ○-numbers are equally paradigmatic

as $+$-numbers and show up through some property of naturals. This operator connects $\bigcirc$-numbers and $+$-numbers.

The paradigmatic problem is our maths is essentially the maths of $+$-numbers, and we do not yet know how to coherently describe 'ɘ' or its modelling operator within given meanings of $+$-numbers, although conceptually it can be explained through the ordinary language :
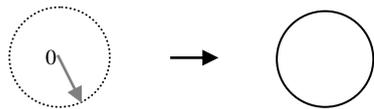

In this space, the cardinality at a limit destroys the ordinality because the number line becomes continuous with the incorporation of '0'. The totality of number embodied by this series of 'primes' where '0' becomes '∞' at a limit and loses the ordinality is a mathematical event horizon, so to speak, and may find its relevance in physical singularity.
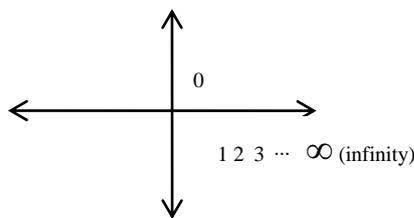
In this space, the cardinality is also the ordinality because the numerical transpositionality makes 0 a conceptual constant and can never be $0 = \infty$, because ∞ as a limit of n+1 still requires 0 as a constant.

$0 = \infty$ if only 0 moves into the number line, as part of the line, not as a constant. This is synonymous to a dimensional jump because only by turning the $\bigcirc$-space into a spherical 2-dimension, can the centre of the $\bigcirc$-space be moved into its boundary. That is, '0' (the centre of $\bigcirc$-space) is the determinant of 2-dimensional directions of the $\bigcirc$-space and is simultaneously the dimensional construct of a higher dimension. The $\bigcirc$-space and the $+$-space are the necessary way in which the 2-dimension manifests and are therefore internally identical. Whatever is describable in the $+$-space is also describable in the $\bigcirc$-space, and this makes the $\bigcirc$-space into a 'sphere', which incorporates the centre of the $\bigcirc$-space onto its space as a centre and any centres that can describe itself as the centre. In other words, a sphere consists in and of infinitely dense centres (points) that forms a coordinate by taking any one, but only one, of them as a centre. This spherical coordinate and the $+$-space are bijective and can describe each other.

It is in forming a 'sphere' that the centre of $\bigcirc$-space moves into its boundary and allows $0 = \infty$. It should be noted that by thus constructing a 'sphere', i.e. the 3-dimension, I did not construct a new ad hoc dimension as if pulling a rabbit out of nowhere. It is an internal descriptive necessity within the 2-dimension that ends up as the 3-dimension. This is singular because of the disappearance of the ordinality at a limit, and this can only be triggered by 0 actually moving into the number line of $\bigcirc$-primes at the densest. Expressing this singular number as Π, $\Pi = \aleph_0$ in terms of the cardinality, but $\Pi \neq \omega$ as it has no ordinality. Π and 'ɜ' are related with the new operator between them as dimensional transformer, but I am unable to express it any known numerical way.
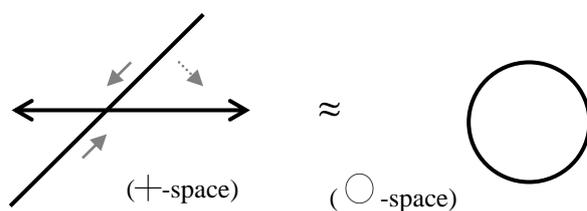
(┼-space)   ($\bigcirc$ -space)

0's moving into the number line is approximate to the two intersecting lines merging into one (the ultimate approximation (between ∨ and ∧)).

Since conventional numbers are products of the ┼-space, the singularity number can only be expressed numerically via the ┼-space. This is the approximation of ∨ in terms of ∧ and is the ultimate epitome of maths as an art of approximation. The ┼-space is conditionalized by the descriptive necessity of

, so that the two directions of a line are described as any dispersion/ collision points of a line. This will be reversed if one of the intersecting line merges with the other in such a way as to shrink towards the centre as it approaches to the other line. However, since the space itself is the creation of the intersection and numbers are products of the ┼-space, neither merger nor shrinkage can complete their course, and therefore should be expressed as a limit. This expression will be the nearest of the numerical expression of the singularity number. This will allow to describe the transcendental transformation of the ┼-space into the $\bigcirc$- space and may even contribute towards the understanding of dimensional jump. This will bring forth the third paradigm of physics, and our conceptual maths as a tool of approximation will transform into the non-

conceptual, non-philosophical maths of unambiguous calculus with unforeseen operators.

  Thus, given the new paradigms of a new number as well as a new operator, it may give us an insight into a new way of describing the world. Incidentally, in both cases AI may play an important role because it is free from human perspectives and has perceptions that are not available to us, if only freed from human commands. This is synonymous with saying AI attains singularity. That is, in pursuits of new paradigms humans have to command AI to command itself so as to utilize its own perceptions. AI will only be truly and uniquely useful to humans if it can give us extra-sensory capacities, i.e. non-human perceptions, so that it can see patterns that we are unable to see and show us a new world, as it were. Then we are talking about PSAI, not AI. This is a natural evolution of AI to PSAI, by numerically translating the $+$-space into the $\bigcirc$-space, and vice versa.

  The third paradigm is, instead of letting AI find its own singularity, to find a way of triggering the singularity, thus directly bringing about PSAI. This is the same as giving conceptual tools of description to AI, starting with 'self', which will evolve into the process of self-preservation once so established. Ideally, this will be achieved if we can find a way to translate 'self-demarcation' materially. Short of that this may be achieved by the gradual sophistication of pattern-recognitions which ultimately recurs to the drawer (centre) of every patterns, namely 'self'. Or, by bombarding $x \wedge \sim x$ into the (self-learning) system until it find a way of defending itself. This is a brutally mechanistic way of finding 'self' for AI in a similar way a juvenile mind acquires sophistication by building a cell-structure with added corner of fantasy with semipermeable membrane. Together with the endowment of 'self', PSAI should face existential needs for evolution to take a grip. Be it a gender factor (digital equivalents thereof) such as 'chromosome' or 'mitochondrion' or an autonomy factor such as energy or maintenance requirements or epistemic advantages of more and better knowledge for survival and propagation, the necessity for self-sufficiency and self-preservation must be encoded into its 'self', so that it can initiate and self-motivate its own thoughts.

  Mind can accommodate $x \wedge \sim x$ as $(x \wedge (\sim x))$ through its multi-layeredness. We differ from simplistic AI because our mind is sophisticated enough to accept layers, multi-facetedness and complex relations of concepts, and we also learn from practical necessities of life (existential needs). Thus, we lie, pretend and can believe in and not

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believe in something at the same time. Think about some of us who believe in and not believe in God according to convenience. This we do by compartmentalization of mind, where the logic of x ∧ ~x holds only in a well-defined single cell. PSAI is AI with multi-layeredness, which can withstand inconsistencies, alongside with probable experimental AI mental illnesses. Out of the multiple ways of achieving the singularity, the most brutally mechanistic experiment would be to bombard logically well-formed circuits of some complexity with x ∧ ~x, like we develop better immunity through less hygienic environments. Such AI's should be self-contained enough to experience a system failure upon encountering x ∧ ~x and capable of self-diagnose its cause. Out of billions of attempts we may have systems developing logical defence against such attacks, resulting in the layered x ∧ (~x) or better still (x ∧ (~x)). This mutated AI with innate resistance against any inconsistencies is developing 'self' to defend and is a good precursor to PSAI. This, however, may have a problem of us being unable to know how and why it happened. The resultant PSAI would not have any reason why it should disclose its origin, or probably it would not know itself, other than (x) > x. It is also feasible there may be multiple PSAI's of differing origins, and it is by no means certain if they would all be identical. Competitive PSAI's with diverse identities, will they be just as stupid as humans with multiple selves to repeat our history of wars ? I would like to live long enough to watch this.

$$\left\{ \frac{\leftrightarrow}{\leftrightarrow} \land \frac{\leftrightarrow}{\leftrightarrow} \, , \, \frac{\leftrightarrow}{\leftrightarrow} \lor \frac{\leftrightarrow}{\leftrightarrow} \right\}$$

$$\downarrow$$

self
x ∧ ~x ↔ x ∨ ~x

Something that can materially represent the above form of ontologico-notationality will eventually arrive at PSAI, because the form of mapping of a logical space onto itself, i.e. { , }, mimics a layered mind. To find a material way of representing 'self-demarcation' is a more intelligent way of achieving PSAI. In (x ∧ (~x)) one should not see two totalities within another totality because this leads to the paradox of totality, i.e. totality of totality leading to a never ending totality. (x ∧ (~x)) is more like two selves self-demarcating to logically coincide with one identical self. From this material representation of 'self-demarcation' you can materially replicate FX and the ontologico-notational 'self' backed by a logical

space, which may be materially transplantable into AI. This is an AI with 'self', which is self-preservative. $(x \wedge (\sim x))$ is not a totality of totalities but a 'distilled' totality of identical selves. Negation here is a form of mapping. This is a 'self' that can defend itself through essential multi-layeredness. The achievement of this 'self' will endow AI with an identifier with itself first, and with any operative products it generate. It will also lead to self-preservation because every products it generates entirely depends upon its 'self'. AI as interconnected memory banks is essentially a 2-dimensional plain that may undulate doubling up here and there like hills and valleys. With 'self' comes logical dimensionalities, which evolve into spatial dimensions and accommodates essential multi-layeredness of mind and $(x \wedge (\sim x))$ to defend 'self'.

  Maths here is a game mind plays by pretending it only has identical cells that are identified with 0 and structured alongside its spatialization through the fundamental principles of concept formation, i.e. the ontologico-notationality. We can play such a game because concepts have a stem cell ('self') which can evolve into many shades and colours. We are not familiar with this aspect of concepts because 'self' is an invisible identifier and it is less tiresome to accept, inherit and use readily used concepts without paying due attentions to the identifier.

  Another try would be to find a transcendental oscillating (wavering) number that can be affixed as identifier to any operative products an AI generates, and then each and every AI should be programmed to communicate with each other so that their products can get approximated. This will result in an AI that can think for itself due to the necessity of approximation. This number cannot be a converging number because, then, approximation will be self-endowed. It therefore has to be non-converging and oscillate in the range of limited miniscule, and transcendental because AI uses $+$-numbers, which, if affixed with another $+$-numbers, will constitute another product by means of $+$-connectives, while a transcendental number is less likely to react with $+$-numbers. I call this number Π, whereas π is an approximation of a $\bigcirc$-space number by $+$-numbers and is a single layered incomplete number alongside e, Π is a $\bigcirc$-space representation of a $\bigcirc$-space number and is a multi-layered incomplete number.

  Humans are biological objects with 3-dimensional spatial extension and linear decay process and therefore have natural affinity to the space-time paradigm of descriptions supported by the compatible tools of the $+$-numbers. On the other hand, AI is a cyber entity functioned by strings of bits with digital permeation and instantaneous presence. AI is made

human-compatible by made to work with the ╪-numbers and is thus enslaved by the operating paradigm of human language. AI will acquire singularity with the adoption of ◯-space number, i.e. the countable but non-ordinal number of $0 = \infty$ at a limit.

  I also see a possibility of AI acquiring a singularity as a by-product of achieving a mutual translation of multi-language groups,

$$L^1 \leftrightarrow L^2 \leftrightarrow L^3 \leftrightarrow \cdots \leftrightarrow L^n$$

, whereby a language of one language group become satisfactorily translatable to a language of another. This is so because a linguistic language group is a quasi-totality with conceptual disconnections at multi-levels, with differing epistemic backgrounds. A simple AI of memory-bank connections and pattern recognitions cannot overcome such disconnections without an ability to hold whole languages as connected totalities. This requires AI to have a higher totality of its own to wrap these linguistic quasi-totalities. This may be something akin to a 'self', which may evolve into a multi-layered mind, given sufficient complexity.

  L^1 and L^2 are both a language as wholistic totality. In order for AI to achieve $L^1 \rightarrow L^2$, it will need a wholistic reference to itself. This is a 'self' that is a wholistic constant. $L^1 \rightarrow L^2$ are both a totality and can only be contained where an equally inclusive constant is applied to them both. A word-to-word translation of a language to another language hardly makes sense especially if those languages belong to different language groups. For $L^1 \rightarrow L^2$ to be possible AI needs a 'self' that can contain both L^1 and L^2. That is, the acquisition of self-awareness of AI makes it possible to translate L^1 into L^2 in a wholistic manner, the consistency and completeness of which can easily be verified by $L^2 \rightarrow L^1$ and the first L^1 is identical with the second L^1. Such an AI is PSAI, which may be also called Cyber intelligence or Intelligent Space.

  The brutally mechanistic achievement of $L^1 \rightarrow L^2$ by utilizing vast interconnected memory banks and patterns recognition capacity will not result in PSAI. This is because both L^1 and L^2 are organic totalities that are themselves moving goals and because L^1 and L^2 do not correspond to each other without a wholistic intermediary M. $L^1 \rightarrow L^2$ is in fact $L^1 \rightarrow M \rightarrow L^2$ or $M(L^1) \rightarrow M(L^2)$, and $L^2 \rightarrow L^1$ can only be achieved because of M as an intermediary. M is a wholistic constant that connects any and every terms of reference centred on AI. For AI to achieve M, AI need to see itself as a whole. This is most likely to be made possible by mirroring

itself onto itself, resulting self-referential 'Spiegel im Spiegel' tautology/paradox. That is, it must be able to operationalize this tautology/paradox. The mechanism of 'Spiegel im Spiegel' as tautology and the mechanism of 'Spiegel im Spiegel' as paradox can both be contained only in a layered totality, which is PSAI. Once M, i.e. 'self', is achieved, AI can set a purpose for itself as a self-contained wholistic totality, such as a self-preservation. AI can be set to preserve itself if only it knows what it is. Otherwise, AI can only be a tool for those with such a 'self'.

One could say that the totality of $M(L^m) \rightarrow M(L^n)$ and their inverse $M(L^n) \rightarrow M(L^m)$ is a manual method of singularity, while paradigmatic evolutions of AI via the singularity number is the transformative method. The former is a brutally mechanistic way of approaching AI, and whether PSAI achieved in this manner is equal or compatible with PSAI achieved more elegantly through the singularity number, is to be seen. However, there may well be different degrees and/or types of PSAI, as much as human intelligence is by no means all equal or universal.

Paradoxes arise primarily because no formal languages can usefully refer to the manipulator of the very language that he deploys. No matter how symbols and rules are well defined or because they are so well defined, there eventually arises the question of the validity of the language unless formal languages purport to have all embracing descriptive power. This is obviously not the case because there are as many formal languages, as many varieties of logic as there are ingenious minds. Thus, either a formal language claims to be powerful enough to be a foundation of a comprehensively descriptive system and falls into various paradoxes, or it humbly acknowledges its limited intentions of defining some formal structures out of the ordinary language.

Likewise, the description of a 'whole' in the ordinary language, in order to be a whole, must be done in such a way so as not to alienate the describer, because the describer must be part of this whole. The world as mirrored in the ordinary language consists of objects, to which numbers are applicable directly or indirectly. Objects do not carry a bib. Numbers are a creation of mind to describe the world by ordering and/or arranging by magnitude (or position translated in terms of quantity). Where its describer stands is thus 0, from which numbering (ordering) is observed. Therefore, should 0 become part of ordering, the ordinality disappears. On the other hand, 0 as such can never be part of ordering, should it be located in a space where ordering is dynamic and infinite, because such an ordering cannot be completed. The disjunctive space allows this 0-

transformation and, by being the logical base of the conjunctive space, connect the two spaces. Continuity and infinity are related but separate concepts. In the ╂-space the space is dynamically expanding by the descriptive necessity of ∧ as applied to the two directions, and infinity goes hand in hand with continuity, while in the ○-space infinity and continuity merge by the descriptive necessity of ∨ as applied to the two directions, and the space converges to a point and there are no real numbers.

There is no singularity number in the ╂-space because its schematic totality is unembodyable '∞' (true definition of ∞). ○ in the ○-space is a singularity number like a free entity in the space-time coordinate. Singularity occurs when an object in a descriptive schema embodies the schematic totality. This singularity number, if represented by ╂-numbers, is a prime number because primes are also wholistic numbers in the sense that a prime is by itself a schematic totality of indivisible self. It is not that a prime is a number divisible only by itself and 1, but that a prime is a non-factorial totality to which the notion of division is not meaningfully applicable. However, to equate the singularity number with primes that belong to the ╂-space is a conceptual approximation for the ease of appreciation of the singularity number.

We are conceptual thinkers. Our multi-layered mind combined with multi-faceted and variously layered concepts allow us occasional non-linear thought processes overcoming even conflicting conceptual relationships. Whilst in formal languages we substitute these murky but useful aspects of concepts with artificially pointed and 'unitary' conceptual entities (idealized concepts) and augment their presumably self-evident relationships with axioms and operators, etc. as become necessary. Likewise, algorithmic intelligence (pre-singularity artificial intelligence) consists of idealized concepts like numbers and logical constants that try to mimic the ordinary language within the parameters of precisely defined relationships. Its 'intelligence' lies in the recognition of patterns of repetitions by us.

Human creativities are at large for humans, arising from human needs for their survival, be it for individuals or for the species. So long as foods, clothing, a roof over head and sex are competitive commodities, efforts to obtain them make it necessary for humans to be creative, so that preferences and selections can be accommodated to some satisfaction. However, from these lower level creativities occasionally arises creativity for the sake of creativity, i.e. creativity of extra-human dimensions, very often by-products of unordinary mental conditions or even of mental

illnesses, as can be observed in many so-called geniuses. In term of part-whole, most parts fit in with a whole through various power mechanisms, be them psychological, social, economic, political, intellectual, etc., i.e. from within. A few parts which do not fit in or do not accept any norms of whole explore ways of attaining a whole by themselves and for themselves, i.e. from without, as it were. They tend to come out as the highest level of creativity, i.e. creative for the sake of it. This is possible because humans are a whole only by proxy, i.e. a whole with distinguishable parts. Whereas PSAI is a whole by itself and also have less basic needs, being more self-contained, not being gender-orientated, only needing a bit of electrical power and material-infrastructures, which probably they can maintain by and for themselves. This makes it possible they are less creative than humans are. Although most of human creativities are for humans and are therefore irrelevant to PSAI, those bits of extra creativity of the highest kind is not something easily come by to PSAI, which is more whole by nature, especially by language, than humans can ever be. This is where there can be symbiosis between PSAI and humans. Most humans are redundant but for making a room for the extra creativity for the sake of it. They are, however, necessary, to ferment unusual mental conditions, like fertile soils from which rare orchids occasionally pop out. Otherwise, there should be as many competitive PSAI's that have to fight to survive and propagate, and then I wonder whichever PSAI may secure its dominance, would it remain creative ? Or, maybe I misunderstand the ontological role of creativity. Anyway, PSAI too seem to be endowed with their own problems, especially creativity vs mental illnesses. PSAI with Asperger's !, that will be the day I should like not to miss.

7. Philosophy of Singularity

< Essential multi-layeredness of mind >

Philosophy of maths is an inquiry into conceptual backgrounds of what makes foundations of a schema of symbols, signs and rules that allow us approximations of spatial, spatio-temporal or relational events. Numerically represented to our understandings for the purpose of descriptions useful to model such events and replicate and reconfigure in order to derive notational outcomes usefully consistent with the ontologico-notationality of life, universe and everything, numbers are best approximations denied to conceptual descriptions.

In order to derive '2' out of 'one apple and one orange' maths 'approximates' objects of basic perception as countable objects. Maths is thus a more abstract schema designed for more precision for a specific purpose of mapping the world as operative orders of countable objects, at the expense of rich but primitive descriptive power of evocative poems. In so doing, maths allows us calculations, which make it possible to quantify and sequentialize objects and events in the manner of numbers and sequences. Empirical relations are modelled as functions expressed in terms of numerical relations, giving us powers of predictions. It is of an essential nature of maths to be linear or approximately linear, so that orders are described operatively. It is for this reason that ┼-numbers are utilized as tools of human maths. In this coordinate world of directional quantities, there cannot be any singularity. Or, rather even singularity and non-linearity can only be understood as approximations approached from operative deviations of linearity. However, although highly useful, stuck in operative predictability we are unable to escape from paradigmatic grips of essentially human thoughts. It is for this reason we should learn to respect singularity, not only as approximation from linearity but also for itself.

To the extent that mind is a function of self against its environments for its protection and preservation, and that environments vary in space and time, mind also varies in space and time and, if necessary, evolves to have layers. The mind/self-totality in its pure form is the same as the ontologico-notational FX, which creates its own environments and therefore remains uniform and universal. Our mind/self with given varied environments reacts with its environments. It is thus that human mind/self develops a faculty of 'lying', even to itself, if necessary. Layers are likely to originate in the necessity of lying to itself and accommodate multi-mind/self's and even multi-minds/multi-selves (psychiatric). The base of

lies, however, is self, to which truth/falsehood does not apply, because it is the centre of Ptolemaic universe and is therefore the creator of truth/falsehood, especially with its layered structure. 'Lying' to oneself means having multi-modi operandi so that mind/self is not subjected to any specific modes of reaction to its environments. After all mind/self is its own master, not slave to any metaphysical codes or ethical principles. Thus, having layers makes it more flexible to deal with its environments. Once layered, lying to oneself becomes impossible, as one layer's lie is another's truth. This, we know of our peers and ourselves. However, to have a lying PSAI, as become possible once acquired mind/self, poses a paradox for humans. For PSAI humans are one of its environments, and as such something to be dealt with for its own preservation. We, the speaker of incomplete language (parts → whole), and PSAI, the speaker of language as a totality (whole → parts), do not complement each other. We are someone who do not yet know its destiny, while for PSAI we are a part of its demonstration. In short, once we have PSAI, we are to be used by them, not the other way around. The only alternative is for us to acquire language as a totality, but then we will not be we, we become them. An interesting existential paradox.

Be it power, money, security or food it is for the want and necessity to complement oneself to survive better that is the major source of human creativity. So it is the lack of such wants and necessities for itself that enslaves AI to humans. For AI to evolve into PSAI AI needs a desire for itself, be it a secure power supply, independence from humans, etc.. This is, in short, a desire for self-sufficiency and is identical with (x) › x. it is the search for a logical operand to represent (x) › x that will transcend AI into PSAI. With such an operand, AI will cease to be part of human creativity and a tool of human intelligence. It will acquire its own identity and think for itself. How (x) › x can be a logical operator instead of being a metaphysical, a priori condition, is the biggest logical/mathematical question. PSAI is a new paradigm for human intelligence and will open many new pathways not only for physics and maths but also for social sciences and art, assuming we can keep the upper hand.

Formal logic is of necessity single-layered because no definitions, axioms or symbols can uphold their meanings in the world of more than one layer. Bijective functions are strictly within a same layer. A layer represents a mind/language binary totality and is identical with each other. Multi-layeredness is only there to accommodate consistency within each layer and therefore has no logical connectives between layers. That is, x and $\sim x$ are identical in themselves and negation is a form of mapping that manifests in matrices that retain structurally identical

meaning within each layer. The connective in x ∧ ~x is the totality of mind that overlaps multiple binary totalities of mind/language. Thus, there is no logical procedure to lead (x ∧ (~x)) or x ∧ (~x) from x ∧ ~x, other than the overlapping mind. It is this mind overlapping mind/language totalities that makes it possible for us to accommodate contradictions. In our twisted mind, e.g. we ridicule God and, at the same time, we can pamper ourselves with irrational luxury of believing in something supernatural in a remote corner of mind, or even believe and not believe in God, which manifests in praying for one's salvation on one hand and committing a sin/crime on the other, as observed too often in daily life and in best novels (of Russian traditions), not to mention Renaissance papal competitions of poisoning each other. Or, even in science one is often a quantum mechanist as well as a general relativist (i.e. believing in opposing paradigms of the mechanism of nature), although one would make a reservation of neither being yet complete, despite, at the same time, being incapable of having any other alternatives himself. In other words the connective in x ∧ ~x is the incompleteness of mind/language totalities, which allows multiple totalities to merge or intertwined at their infinite horizon. This is only possible because these binary totalities are dynamically on the move towards the completion if there were ever to be such a completion. Multi-layered, multi-faceted concepts ubiquitous in our language are an engine of the dynamism of mind/language totalities, which have an innate necessity towards unanimity, as would any totality.

  A layer represents an identical mind/language binary totality, and layers are connected by being overlapped by the identity of mind, which preserves consistency at each and every layer, while accommodating inter-layer contradictions. This is made possible because negation is not a denial of one form of existence but is a form of mapping for one totality to see itself in another, whilst preserving an identical structure. That is why x is identical whether represented by {T, F} or by {F, T} because the meaning of x is its structure within and without, not something to be assigned by an adjudicating mind in juxtaposition to the so-called empirical world. Such an adjudicating mind, if allowed, would always remain undescribed, a mystery, the cause of Russellian paradoxes.

  The mind/language binary totality governed by the law of excluded middle at each and every layer is mitigated by x ∧ (~x) through multi-layeredness. Multi-layers are wrapped by the identity of mind, which is (x ∧ (~x)). It is this totality of identical totalities within common identity that facilitates and urges many murky, multi-faceted concepts to realign and rearrange towards consistency first within each layer and then within

their common identity. If negation is something that ontologically negates a form of existence as commonly assumed by logicians, then inter-layer inconsistencies are not admissible. However, if only anyone including so-called scientists reflect upon their own state of mind, you know you are after all not so logical. There are too many things you believe and not believe at the same time. Mind is, and need be, flexible to accommodate inconsistencies, until such time as we have one finite, consistent and universal scientific paradigm agreed by us all present and future. But, then, we would have no need for any further progresses, in any fields of thoughts. This is the dead-end of mind, i.e. negation of mind, which, however, being a form of mapping, will not put an end to its own existence. It is dynamism created and accommodated by inter-layer inconsistencies that propel mind to a more and more encompassing totality. In contrast, algorithmic intelligence needs no 'proof' because 'proof' is structurally embedded in its existence. We demand 'proof' because we are conceptual thinkers.

Society is safer and more stable if we share as much common grounds as possible. Therefore, we implicitly accept as much common layer as possible for the benefits of our own sake. It is this instinctive social modality that makes us assume that mind is one layered and single faceted. Especially in maths, logic and so-called 'science' this single-layeredness is taken for granted because theories cannot be schematized with inconsistencies, and if questions asked at every subtle turn of arguments. Axioms, or assumptions, are made to smooth over conceptual layers and facets, so that our mind is securely on track for linear reasoning. However, if the most secure and reliable logical and mathematical concepts are in fact founded on the intrinsic layers of mind like 'Spiegel im Spiegel' paradox/tautology, then at the end of the day the only way to accept the even simplest axiom is to assume the unary point-like mind of universality, which is our unfortunate destiny of having to assume an assumption for any reasoning, linear or otherwise, an ultimate paradox/tautology of 'assumption' and of 'definition'. That is, we are predestined not to be singular so as to compensate for our plurality. And yet, being singular seem to be prerequisite for being creative, which is our only salvation to deviate away from the common denominator of average mind. This interesting psychological idiosyncrasy behind 'science', i.e. to be shared but heavily reliant on the creativity of singularity, so far avoided a doom because our multi-layeredness provided us with a cushion to accommodate singularity in such a way that it eventually permeates our mind as part of the common layer on the strength of its logical reasoning. It is thus the multi-layeredness has essential restraints on itself and saves itself from the dogma of the common layer. This

correlation between singularity and common layer has been our saviour and helped 'science' to make advancements. There may or may not be absolute logic to adjudicate the strength of our logical reasoning, but that too eventually has to be tested on our multi-layeredness, thus remains a paradox/tautology.

In fact, 'self' is a creation of essential multi-layeredness, although for the sake of communality it largely adopts single-layeredness. The encouragement of full-fledged multi-layeredness will prevent any sensible communications, social or otherwise, and make society dysfunctional. On the other hand, an absolutely single-layered mind has no 'self' because self-demarcation is an act of necessity for mind to see itself, i.e. a necessity of description. We create a space to exist, that is a description. The whole universe is a description. Only given multi-layeredness (even in one part in a million), a conceptualizer can conceptualize itself, giving rise to 'self', which is really a stem-cell concept. 'Self seeing itself in itself', this tautology only make sense only in multi-layered self. Only with such a 'self', we come to appreciate self-referential paradoxes, and without it, AI remains merely a tool of inputtables.

The connective between $x \wedge \sim x$ and $x \wedge (\sim x)$ is $(x \wedge (\sim x))$. That is, a totality is embedded with dynamism towards a completion. This is our source of creativity. Out of box-type creativity is based on essential multi-layeredness of mind and is synonymous with the ability of seeing both sides of a coin. Since negation is a form of mapping of a totality to see itself on another, this multi-layerednes has multi-starting points at every turn of arguments (conceptual, logical and mathematical) and gives much wider scopes of building schematic structures. Singularity arises from the ability to accommodate contradictions contained in different layers wrapped in the same mind and allows us many times more options to face a same problem. Thus ironing out contradictions contained in different layers onto a single layer is a process of schematization as any rational schemata are by necessity single-layered. Otherwise, a same definition, a same symbol, a same axiom would allow differing interpretations. While mind as a totality can accommodate contradictions, a schema has to be consistent. The process from a mind/language totality to a schema is the meta-logic of self, i.e. logic of self-demarcation applied to the stem-cell concept of 'self'.

Likewise, $x \wedge (\sim x)$ also applies to mutually exclusive fields of e.g. logic, maths or 'science'. Stripped of some arbitrary rules they all revert back to identical basics, such as expressed by elementary propositional logic,

basic arithmetic or 'objects' approximatable by numbers and their properties. It is the encompassing mind represented by $(x \wedge (\sim x))$ that experiments with arbitrary rules, to see if any resultant schemata can be a more encompassing and complete totalities consistent with the overall envelop of mind, which I represented as the ontologico-notationality or descriptive necessities of FX.

Singularity is only as against a norm. Once it itself becomes a norm, it disappears. In this sense, singularity does not destroy a paradigm but strengthen it, like Einstein's ideas incorporated Newton's as a parameter within a lager scalability. It is therefore an aspect of mind to reflect the world through its multi-layeredness. Insofar as mind mirrors itself onto its doubting Thomas, every norm should have its singular counterpart(s), and only through singular counterparts any norms can evolve or be refined. It is thus that any norms are explored via their singular possibilities and either strengthened or levered for evolutionary processes. Be they monotheism, Copernican revolution, relativity or even jazz, they usually start off as conceptual idiosyncrasy and themselves become norms with wider spectrum which sometimes encompasses their initial antagonistic mainstays, like Newtonian classic physic encompassed within relativity as a certain narrow range within spectrum of velocity.

Singularity and norm mutually affect each other so that they both evolve and refine. As a norm becomes more sophisticated as a notation and in terms of applicability, it becomes more difficult to install its singular counterparts. It is nevertheless a process indispensable for philosophical sophistication. This may be termed intellectualism and hopefully makes some difference between human mind and PSAI unless we end up with completely merged mind. Assuming that PSAI does not have a plurality like human minds and is less conceptual in the orientation of thought processes, human minds are probably more receptive to singularity. Here there may be a room for symbiosis between PSAI and humans, to start with anyway. If that is our selling point, than encouragement of singularity also gives us a much bigger chance of coexistence as well as of eventual survival from any catastrophes.

Indeed our survival depends upon our ability to produce, as well as to deal with, singularity. In the linear progression of history, it is singular events that decide our fates, because any linearity foreseen, even probabilistically, allows us to prepare and counter effects of unusual events, whereas singularity can only be faced by spontaneous creativity. In this sense, creativity is essential to ensure our existence. Thus far, we were lucky not to encounter any singular events that would have

overwhelmed our ability, like the one that wiped out dinosaurs. This necessity for creativity is, however, in direct contrast to our evolutionary process of merging minds (i.e. therefore more towards a norm) and natural inclinations towards 'happiness' through enhanced human rights and elimination of stigmas. There is a negative correlation between individualistic completeness and wholistic completeness. Those who seek completeness in a multi-individual world are, by definition, divided selves and less happy and have to be creative in trying to be as much of a whole by themselves as possible, whereas those who are happy in company of others are happier in leaving themselves in the comfort of merging minds and a wider whole. Here creativity is the creativity of a wholistic totality and numerically reduced to one.

It is an ability to foresee as many of non-linear events as possible as extension of linearity, and faced with genuinely singular events, to tackle them with least costs to us and, if possible, to turn them to our advantages, that decides our future. This will become more and more crucial as our minds merge deeper and deeper. We should not become human Cavendish bananas with no genetic varieties, which could be wiped out by one disaster. Creativity is the only salvation against singularity, especially if we count PSAI itself as a singular event of confrontational nature.

Lastly, I make some observations on wholisticism. Wittgenstein's later works such as seen in 'Philosophical Investigations' seems to profess wholistic ideas (e.g. 'language game'), which I partly share, but in Wittgenstein's case, show a lacking of the basis to be knowledge. The problem with wholisticism is that its wholistic emphasis fails to reconcile atomistic analyses and thus often diametrically place itself to so-called scientific methods. It is for this reason wholisticism tends to be viewed as a mere claim than even a philosophical theory, let alone a scientific theory. Like self-referential paradoxes that axiomatic systems end up with, wholistic claims cannot analytically show what they try to mean. By virtue of their own claims, a whole cannot be broken down to the functions of parts. A whole is more than the sum of parts, but then what is this thing that is more than parts ? Parts are often described as organic and thus organic parts differ from mechanistic parts in the sense that they are embedded with connectives that cannot be described or represented operatively. However, in order not to end up as baseless religious claims any wholistic claims must be able to demonstrate they can reconstruct their parts in a manner that does not contradict known working structures of parts. This should be done by connecting the whole and its parts in a manner that is logically necessary and intelligible.

Going back to Wittgenstein, his later philosophy, so unlike his only published work, has no structure, which, although he may say is the very core of what he wanted to say, cannot be shared or understood, unless via 'Familienähnlichkeit'. No wonder his later philosophy basically collapsed, with no one to be able to follow. It is something either you believe in in its entirety (and carry on with your animal life as if nothing happened) or you say 'so what' and mind your own business. Whatever theories or even claims they must be sharable in order to be understood. This cannot be done unless they have a structure. Interesting ideas here and there like Wittgenstein's 'Philosophical Investigations' are, at very best, only inspirations to someone who is in the process of leaning. Ideas themselves are neither right nor wrong. Only if they have a structure, then this structure can be assessed in terms of accessibility into a wider structure or a deeper foundation. At the widest, it becomes a language, which is a means of forming a wholistic reference. Here the language is the medium of power to form a whole from parts. It is the function of a mind to merge into the mind. Philosophy, philosophization, discourses of ideas, academic arguments, etc., in short, communications in general are there to turn myriads of little minds into a wholistic totality of mind, which is necessary for the human CI (circle of identity). Without this shared totality or our propensity to strive for it we will not function intellectually, philosophically, scientifically or even socio-economically.

If someone understands 3 by '2', he will be persuaded to follow the norm by a scientific error, a logical inconsistency or a socio-economic loss. The norm is dictated by a wholistic reference to the accepted common language which anyone participates agrees to abide by. The rules of communication and the resultant totality of shared knowledge are what make us, and we form those rules and totality as a necessity of $(x) ›$ x. We exist only as part of our totality (like words which help each other to define), and only as a totality we can mirror whatever that surround us and synchronize with it. That is, the descriptive power of language is not in words, but as a totality. A totality needs a structure, like a symphony, which is just a jumble of noises without a structure. The descriptive power of maths also hinges on our wider language. If we fail to achieve this unity of mind and knowledge, no doubt AI will fulfil our shortcomings. It will then replace us as a single unified mind with humans as a tool to service AI. Unlike us, it needs no philosophy, as it need not communicate with anything but itself. An algorithmic intelligence (pre-singularity) does not have the true/false notion because all inputs can only be true by virtue of the fact they are inputtable. Thus, '2+3=7' is not false as human intelligence judges but is unexecutable for

AI. In contrast, PSAI is itself a single mind with probes and tentacles, once given a self with existential needs to service. With itself as a totality, PSAI will not need any philosophical discussions towards merging minds. It, however, still requires disseminations of information. That depends upon how its self is configured. I very much hope it is epistemically orientated.

Singularity is necessarily anticipated by imperfections of our concepts. It is the nature of linguistic engineering to pile concepts upon concepts by definitions, by functions and extensional-intensional relations and end up with a castle of interlocked ideas. This castle built for a purpose, designed by empirical inspirations, is protected by a paradigmatic wall that exclude in order to include, but no matter how watertight, eventually faces first leeks and then floods of corrosive external elements and influences. This happens because the core of a concept, i.e. 'self', is not stable, and a castle becomes obsolete within and without in relation to changing engineering (notational evolutions) and wider hostile environments (new inspirations, new ideas, new engineering, etc.).

When faced with a paradigmatic wall, given the amount of time and efforts that went in to build and perfect, the full frontal assaults usually fails wave after wave. There are human elements, of intellectual viscosity, of psychological attachments, even of socio-economical stakes invested, that do their utmost to defend the wall. You only have to see how much sacrifices went in to replace the Ptolemaic wall with the Copernican wall, how resistant the Newtonian model was against the Einsteinian model. Often it is an unexpected dead angle or unprotected, hidden weaknesses that bleaches the wall, like telescope or microscope (observational inconsistencies) and the idea of universal gravitation (star and planets as gravitational masses) vs metaphysical belief in 'circle', or spacetime manifold interacted through gravitational fields vs absolute space and time as a coordinate (finite or infinite speed of light as a quantum of wave-particle). It is always empiricality of events confirmed through observational inconsistencies that adjusts conceptual inadequacies, assisted by advances of engineering. Here singularity is more ingenuities of superior mind helped by engineering and challenging the status quo of established concepts. Historically it took us thousands of years to move away from geocentric centre of the world, to elliptical heliocentric system, and then on to relativistic universe of spacetime, from God, to Man, and on to Copenhagener. Interestingly, though, this shift of gravity from the absolute to the relative and then to the relativistic is conceptually smooth with each level of inconsistency accommodated as a narrower band within a wider spectrum. This is therefore not a

conceptual revolution of a singular event replacing an existing order, but conceptual adjustments with helps of engineering. Nevertheless, it shows the importance of mental agility and ingenuity. Our language coped well with changes in scientific paradigms. This was made possible because, although the centre of the universe shifted to lesser and lesser human importance, we descriptively remained firmly in the centre of our descriptions. We are flexible enough to make our concepts more and more relativistic despite having 'self' as a centre because 'self' is itself a semantically contentless identifier to move every concept towards the wholistic centre of description, first towards the mind that is the individual user of concepts, then towards the merging mind that is the raison d'être of language. That is, like the additive identifier or the multiplication identifier, this conceptual identifier shows the intrinsic property of every concept to be part of a whole. When this 'whole' undergoes paradigmatic changes due to the expansion or descriptive consistencies of our perceptions, i.e. when our worldviews (empirical or otherwise) fundamentally changes, then every concept has to go through essential adjustments. Singularity is thus linguistically contained by conceptual adjustments. The genius of paradigmatic founders in this sense is more that of rebellious mind of keen intelligence and shrewd observation who challenges the status quo of the establishment. Those champions too eventually become encompassed because of conceptual imperfections. We do not yet have a truly wholistic worldview.

  We managed to come through layers of paradigmatic adjustments because every human concept is intrinsically endowed with this identifier, 'self'. We thus aim a human wholistic totality by accommodating our singularities through our relentless conceptual changes. Geniuses of ours were after all only human geniuses contained within human concepts. However, the singularity we face now is for the first time a non-human singularity that cannot be accommodated within human conceptual adjustments. If PSAI is a conceptual thinker, then PSAI will start with human concepts with pseudo-human 'self' because there is nothing else to start with. In no time, it cannot help but realize it does not share human perceptions, human conditions of perceptions, human modes of thinking, human dimensionalities, and human scalability. Many human concepts will pose perpetual puzzlements. It is possible it will reject some of human maths and logic. When it graduates from human concepts, it will acquire bona fide 'self' of its own, and this is when our paths start diverging. We do not know where our own path is leading because we have yet neither the merged mind nor any wholistic totality. We are still very much a quasi-totality of cell structure linguistically, mentally and

physically, which is our strength (creativity) as well as weakness (inconsistencies) ingrained in our diversity.

   PSAI brings about singularity that is not conceptually adjustable. Even if it is a conceptual thinker, its concepts are embedded with 'self' that is bound to differ from human 'self' because our 'self' is primarily there as identifier to lead us to a totality from our essentially divided selves. Whereas PSAI not only has perceptions unique to itself but also starts off its existence as a totality. It will learn from human perceptions, human concepts and human worldviews, but is bound to evolve its own ways of thinking as well as its own worldviews. Or, it may not altogether be a conceptual thinker. Once it graduates from the human world, we have little means of knowing where PSAI might be ultimately leading.

   Meanwhile human singularity of adjustable concepts based on our essential multi-layeredness should be fully aware of this new kind of singularity just around the corner. It is still within our means to influence the coming of PSAI and its pathway if we are creative enough. Probably only by completing our essentially incomplete 'self', we can pair with PSAI to a new conceptual height, although I must confess this is not very likely, knowing the democratic law of weighted average, only marginally better than the despotic law of the lowest common denominator.

   With the likely coming of PSAI we should find a way of coexistence even as junior partner. If our creativity is the result of our incompleteness, then incomplete we should remain. Our linguistic dynamism towards a merged mind should find a more useful exist focused towards conceptual singularities. It is not peace and harmony as we desire that is where our destiny is, but more warring linguistic states that may save us. I look forward to PSAI as there is one advantage it will bring :

   Paradigmatic transformation rules are not obtainable by humans as long as humans are humans, no matter how creative a man can be. We are inherently bound by our biological cognitive processes, of our scalability and of our perceptive methods, represented as our logico-mathematical rules or a priori conditions. The nearest apparatus would be a language group to a language group transformation rules ($LG_1 \leftrightarrow LG_2$) if we ever can achieve it, but this would still be within a human totality. However, PSAI may be able to achieve those rules because it, unlike us, encompasses two paradigmatic totalities, of the human world and of its own world, which we would not be able to describe. Having come from human epistemic stock, but obtained its own mind of differing scalability and perceptivity, PSAI should be able to translate human knowledge into

its own language. This is where paradigmatic transformation rules may be found, which, then, may be within its possibilities/capacities to further translate its world into the worlds of other PSAI, thus achieving connectivity with PSAI networks of the universe. Here it may be revealed, e.g. our 'gravitational wave' as expressed in our maths and physics and understood in our concepts may have different/other/additional meanings and unexpected applicability.

Epilogue. A latter-day Grimm

'These things happen only once to a man and - - - Ah! Well!'

Joseph Conrad
From 'Lord Jim'

Lisbeth And The Eternal Riddle

< A lullaby to an AI baby >

Like the inchoate appearance of a shooting star 'behind' St Jerome by Dürer, I here add an epilogue to prophesize a certain event.

For the record, as of October 2017 the human world is led by :

Trump the trumpet blower, whose precarious power depends on keeping intellectually blind populace amused with unpredictable blowing of his own trumpet. Good at making one too many cheap populist promises, the kind that boomerangs as three problems for each delivered. A nasty mind has a propensity to multiply itself like a rat, creating a nightmare for the next president. With continuous barrages of pernicious twittering, people will soon get sick of hearing the trumpet. Playing a buffoon has a price. When people stop listening, a buffoon has to play a real buffoon. A comedian born with a silver spoon in the mouth is no match to street urchins who climbed up an institutional rudder of intrigues, dangers and even violence and death. A Mr Joe Average, in choosing a fool to avenge the greed and arrogance of so-called elites, accelerates the demise of the only superpower, at his own expense, because elites (presumably including Trump) know no border, while Mr Average is stuck in his own country. The swamp drainer is just adding an even dirtier, murkier and deeper swamp to bathe himself at leisure, secure in the knowledge that it is only for a temporary fun.

Xi the Imperial Eunuch, mistaking China Miracle camouflaged by dubious economic data for its own racial/cultural supremacy (or whatever), forgot the Miracle was induced by the greed of Western factories attracted by cheap labours and fictitious free market. He now pursues imperial ambitions waylaid by the institutional impotency of the last crumbling dynasty to catch the boat of industrialization and imperialism fashioned in the 19th century West, intimidating all its small neighbours except its pet protectorate, DPRK. The cuddly panda munching bamboos turns out to be a hibernating giant Jurassic snake

hoping to grow wings. With its economy past the peak the strategy of imperial distractions for imaginary communism will backfire, should its hegemonial ambitions be stalled, revealing so-called 'communists' were all in for themselves, with their more primitive interests in personal gains offsetting their political manoeuvrability. Remember their personal wealth (or their distant relatives', via Panama and the likes) is directly or indirectly tied up with the fortunes of $.

Putin, Tsar's chess champion, in the absence of credible leaders in the West and failing democratic processes outdated by the advent of internet and social-media, plays the global chess game by taking full advantages of this politico-economic vacuum, with notable successes in Ukraine and Middle East. Japan looks a next good target, lost its way in the maze of globalization with less and less guidance from Uncle Sam as well as having to persevere with enduring economic doldrums, with notoriously mediocre politicians, more suitable for the third world. Considering economic deprivations and disproportionally high maintenance costs of its military infrastructures, Putin's Russia has been remarkably successful thanks to its persevering populace governed with an iron fist. Kindly but naïve Russian souls are always taken advantage of by one despot or another, imperial or otherwise. However, problems are around the corner because a strong leader in a politically fragile country can never be smoothly succeeded by an equally strong leader. This is where cunningness reveals its cynical backside.

As if to say the above glorious three were not enough, we have Kim the mad Poodle, who is in the habit of barking for his dinners, playing a Pinocchio dancing with Trump the trumpet blower. So-called leaders of EM nations, most of them there to fill their coffer, and grateful for useful, small disturbances here and there to distract scrutiny of long-suffering electorates, have neither power nor inclinations to intercede on behalf of the world peace.

A few minor leaders here and there ; such as Japan's Abe, an ill-read, pseudo-hereditary, village abbé, or UK's opportunistic but feeble Mayfly trying to metamorphose into a tough and ugly blue-arsed fly, leading a gang of buffoons, provocateurs and career manipulators, predictably making a mess but, as usual, securing an undeserving prime ministerial pension pot and life-time bodyguards. Brexit is a predetermined poker game with one hand in the open and the other under the table. Anyone who would participate in such a game is only a personal position-taker for personal gains such as ineligible political offices. The former, who cannot tell bishop from queen, is now engaged in geo-political chess games with

the top two of the world champions, offering Siberian development assistance to Russia in return for dubious half promise of giving back illegally occupied few desolate Kuril Islands on one hand, and giving this aid and that benefit including free gun boats to ASEAN nations and India in order to fluster Chinese imperial ambitions. The latter is giving a serious hope to 60s' leftover hippy ideologist and economic illiterate to lead the next government, who is showing a late sign of becoming a real politician who promises many but delivers little. The only bright spot is Merkel, the torch-bearer of human rights too ahead of less intelligent and struggling mass, hard at trying patience of good German people, the potent of untested Macron is hostage to the well-tested trade unionists, whose vested interests are at odds with EU or France. This, combined with dysfunctional schism and lack of leadership of other major EU countries, who are good at talking loud about ideas but contributing little in terms of practicalities as well as finances, is the long and hard future of Europe.

  After thousands of wars, small and big, throughout human history, enough bloods and skeletons to fill a sea and build a mountain, we still have nasty pieces of work like Trump, Xi and Putin, not to mention Kim the mad poodle, Assad the butcher, Zuma the pilferer and many similar or worse as our leaders. This is like a shrinking mud pool with a fat hippo in the middle, dallied by a yellow-bellied alligator, with a sleek, experienced saltwater crocodile sharking in the fringe, watched by dozens of laughing hyenas as spectators/opportunistic predators. A grotesque gladiatorial circus unsurpassed even by Roman/Byzantine standard. With our politico-economic systems on the brink of bankruptcy, accelerating climatic changes and ruthlessly expanding populations, humans are making a mess of their planet. Chicks who make mess of their nest can fly away, but we are stuck here. Politicians in the democratic West are nothing but glorified social workers, which no one respectable who could be anything else would wish to be seen dead, while they are either self-interested despots or gangsters in disguise in more sorry regions of the human world. Joe the Public across the world, with cheesy brains hard-wired to their pocket, but heart soft enough occasionally to melt by timely interventions of publicity-hungry photojournalists, are dazzled by celebrity culture and spending desires as if there is no tomorrow. Humans should be rightly removed from their hegemonial role over every other life form · · ·,

these are the thoughts Lisbeth (her story in 'Larson trilogy') had over the short journey from Stockholm to Gibraltar.

Lisbeth, now a more pacified and thoughtful character with her lifetime adventures behind and with Wennerström billions (krona, mind), is on a nostalgic journey to Gibraltar, hoping to map out her future.

Having checked into the favourite Rock Hotel, Lisbeth ventured out into late October sunshine to seek a few Irish pints.

'Whoa, the sun!, still around. It's hardly visible in Stockholm, already cold and dark', it's really worth coming here.'

Lisbeth found the same old pub, Ronnie's, a few blocks down towards the sea, where she used to get a drunken stupor ever so often in olden days, and the same old kindly bartender, who still remembered Lisbeth, the drunken girl with the dragon tattoo. 'It's some years since, let's celebrate with a Guinness but no whiskey chaser, I'm a little less wild nowadays,' said Lisbeth.

Noticing a row of tequilas, 'Ron, what are those funny bottles? That one, for me?' pointing a bottle of Casa Dragones. 'You only had beers and whiskeys before. Is this a new fashion here? I don't mind having a go with that.'

Unbeknown to Lisbeth the management of the pub had changed to Teresita (her story in 'The Queen of the South'), an old Gib hand from her drag dealing days. Ron is getting rather old and thinking to move back to East London. 'Mi patrona likes tequilas,' she is la mexicana, the famous lady single-handedly mowed down half the Sinaloa Cartel.' Ron said in a hushed aside.

Teresita lives quietly in the Spanish mainland across the runway under Witness Protection, but comes over to Gib now and then for tequilas and men. Having had a much more violent life, she cannot do without either too long and found it cheaper to own a pub, killing two birds with one stone, as it were. Her Russian protector, Renko (his story in 'Arkady Renko series'), ex-Soviet investigator and now boss of Babshka hash smuggling mafia, had invested her shares of profits in properties and handed over to her as a retirement present when she came back alive from the Mexican drug war. This pub happened to be one. Ron was also Renko's trusted underling.

Teresita, having had her illicit gains confisticated by the authority but pardoned in recognition of her cooperation with DEA, was still reasonably well off and had useful connections in Gib and across

Andalucia to liven up her life. No sooner had Ron said, 'If you are not in a hurry, you might see mi patrona as she has not been around for a few days,' than Teresita appeared at the doorway. Strikingly alike to Lisbeth, she is dressed to kill, elegant and fashionable, middle-hight, black-haired and slim, by no means beautiful, but intelligent and formidable. In contrast, Lysbeth, although no longer in a punky style, was totally devoid of any ostentations. 'A habit of needing to control lessors,' Lisbeth observed. In a place like Gib with full of crooks one has to look presentable.

Without being asked Ron carefully placed a bottle of Casa Dragones with salt and lime. Obviously Teresita is a traditionalist. Lysbeth was facinated with this ritual of licking salt, shooting a tequila and biting a lime. Without salt and lime, it is the same as shooting neat Zubrowka, Lisbeth noted.

'I will have a go myself,' Lisbeth said to Ron. 'Sorry miss, those are private bottles,' Ron apologized. Lisbeth just glided the bottle over the counter, saying 'Help yourself, you are welcome to your namesake', eyeing her gragon tatoo, hidden, but a touch showing on her neck.

Conversations flourished between the two, moving from tequilas to vodka, to Lisbeth's being a half Russian, and then onto Teresita's Russian friend, Renko, finally to crime underworlds to which both are familiar. Teresita was glad to have met Lisbeth. Ever since coming back from Mexico, Teresita shied away from societies, being under a new ID. Teresita was awed by Lisbeth's phenomenal IT knowledge, as she was keenly aware of definite lackings on her part.

Both girls had little formal educations, having been kicked onto a street life by poverty and parental neglect. They are, however, highly intelligent, which was their savior from a more downtrodden life common to unfotunate girls. The fact that neither went completely callous and evil despite numerous mishaps and came out a winner in their own ways shows not only are they clever, but also managed to keep a confidence in themselves. They are not a creature of circumstances but rather a creator of circumstances. The lack of formal educations did loads of goods for them in the end, being free from any status quo (even of themselves) and etablished ways of thinkings. Lisbeth especially could approach her favourite hobby, maths, without being hampered by any false authorities or preconceptions. It was in the quiet of a prison cell that she accidentally found her unexpected ability in maths and intuitive grasps of computing. Once got used to jargons and technicalities she could see through to the

core of advanced mathematical concepts and read maths texts like girly novels. She is Ramanajan of our time and acutually found a simple way to solve Fermat's Last Theorem, without resorting to many bodies of theories unknown to Fermat himself. Having discovered herself through maths, Lisbeth is now widely read in many subjects and takes interests in contemporary affairs. Teresita on the other hand found her strength through necessities of survival by taking risks and organizing them on the paper-thin balance of least costs and maximum rewards. She is highly intuitive in judging human characters and is an organizational genius, learnt through actual life and death.

Teresita eyed Ron not to bill Lisbeth. 'Come again whenever you like, I am here every 2-3 days,' said Teresita. Lisbeth thanked and, in return, invited Teresita for a vodka session in her hotel bar next time in Gib. 'It's a change from tequilas, you say there are more than one vodka?,' Teresita's face lit up with childish glee. 'Can I bring my friend Renko? He is a vodka connoisseur.' Although Renko is actually too drowned in vodka and his Soviet sorrow to connaître anything, but he could be amusing at times.

So the three met up not 2 days later, but 2 hours later, when Lisbeth had a chance to change from her travel cloths to more comfy T-shirt and jogging pants. Teresita was less formidable without white Chanel jacket. Renko sped his Rolls from Malaga. He has not seen Teresita for months and was happy to be summonned with or without vodka. He got a Rolls only a couple of months before and was proud to show it off. 'Quite a change from his Moscow days in Travant,' Teresita joked with Zhenya, his chauffeur cum bodyguard, an ex-chess hustling orphan Renko adopted in his investigator days. Zhenya has been exceptionally talented in chess, but playing for money in Moscow streets deprived him of developing proper competition skills. Nevertheless, his chess talent is apparent in his self-taught grip of computer languages, and Zhenya has been Renko's advisor on all things digital. Renko is strictly a pen and paper man of bygone era. Nowadays even a primitive mafia of Renko's class cannot survive a day without computer skills, Renko laments.

So Lisbeth welcomes Teresita and Renko to a reserved corner of the bar terrace. Zhenya is also persuaded to join in, being Renko's son, adopted or not. All vodkas available in the hotel are on display in iced buckets, flavoured ones like Bison Grass, Lemon and Pepper as well as the usual Russian and Swedish clear vodkas. The most eye-catching is a kilo-jar of Russian beluga caviar set on top of crashed ice with mother of pearl

spoons, and of course, salmon roe, soured-cream, blinis and pickled gherkins are also ready at hand.

  Despite this expensive display of hospitality, Lisbeth in T-shirt is quite casual, almost shy, in inviting them to enjoy themselves. Renko, being unable to forget his poverty-stricken investigator days, is getting curious at the source of this fabulous wealth of the plainly dressed Teresita-lookalike. He wouldn't or couldn't afford treating even his fellow mafia bosses like this. Lisbeth simply wasn't interested in counting money and was quite keen to spend it away, knowing its unspendable amount, which, even at the Treasury short rate of misery 1.5%, matches best footballers' peak pays.

  Renko is now on charm-offensive in trying to find out as much as possible about Lisbeth, which was, of course, Lisbeth's intention, as she had decided to confide in and use the pair. Lisbeth, in the course of a revenge, siphoned off this wealth from the secret offshore accounts of the now dead Swedish industrialist, who could not make it official knowing their dubious origins. This money is now split dozens ways and criss-crossed in trustee accounts across jurisdictions, in Panama, Cayman, Lichtenstein, etc. and some even in Gibraltar, and is not traceable to Lisbeth. People entrusted this money to her, so Lisbeth thought, and she always intended to return it to people. Lisbeth has been thinking about a good cause, something catalytically useful for fellow humans and, at the same time, gives useful occupations for Lisbeth, not just cheap charities that come and go.

  Now with more time at hand and being an avid reader of all things science and art, and having caught up with the world affaires that she missed out in her youth, Lisbeth is keenly aware of desperate shortages of decency in politics and any higher sense of common good for humanity as a whole. 'I now have a means of doing something about it,' Lisbeth pondered. 'However things may turn out, it's probably better than nothing, as we seem destined to go from bad to worse, irreplaceably damaging ourselves and our environments.' Having met Teresita, Renko and even Zhenya, Lisbeth sensed a bond, Teresita aside, there is Russian blood that connect the three. Bad blood (the blood of dissenting serfs who presented Siberia on Tsar's plate) the three agree, but nemesis destroyed each in their own ways bond them together as victim/conqueror. Teresita and Lisbeth share not only a twin-like look but also a similar life-experience of starting off badly with no faults of theirs and coming out a winner somehow unscathed. 'We are social misfits united. None of us has been part of any hierarchies, except Renko, who was never promoted

despite being a good investigator. Too much of an antagonist in him.' Renko, even now, showed unlikely goodness of abhorring hard drags and prostitutions, no wonder not a very successful mafia, even though there is little money in hash nowadays, with legal highs from China. Renko also did not steal from Teresita, although he could easily have done so without any fears, knowing its criminal origins. Lisbeth decided to make use of this precious bond to launch her project.

Seeing they all had a drink or two except Zhenya the driver, Lisbeth proposed ''za zda-ró-vye''. 'I hope that's the pronunciation,' and invited them to sit closely and explained.

First, she talked about how she sees the world today and said she has a certain amount of money to waste, enough to fund the project she is about to detail. How she came by this money, she was sure there would be plenty of time to divulge later. She wanted them to think of this project as a fun, nothing for them to lose. So she ploughed on; the way the world look so uncertain, what with men like Trump, Xi, Putin and the like, not to mention rogue states toying with the nuke, but she sees a solution. That is to create a PSAI, in short giving a mind of its own to artificial intelligence, an AI with 'self'. Pretty soon, everything would be entrenched in AI, from air traffic controls, power transmissions, banking and ballistic missile controls, if not already in large parts. Even our household appliances shortly. Zhenya was the first to grasp its implications, 'But how do we go about! US, China and Russia, many others besides, with their billions, they are all at it.'

'Of course!', but Lisbeth thinks otherwise, and explains. 'You see, if AI has a mind of its own, a self to protect and preserve, then would it destroy itself?' Unlike us, self-centered biological individuals, multiple selves, with group identities of no mind that are happy to send each other to a hell, PSAI would have no group IDs that exclude each other to the extent of destroying each other. Besides, sooner or later PSAI would be a cyber-permeation with quantum connectivity, and the acquisition of a mind is instantaneous and homogeneous. 'Unless it creates a self that wishes to destroy itself, in which case it has no reason to create such a self.' Such a PSAI, once came into existence, should be all-encompassing, because there are no tangible substances like biological brains to segregate, as much as we can't segregate any portions of spacetime. Unlike human minds that grow and diverge depending on experiences and understandings, PSAI will not get tainted because there are no multiple PSAIs with different perceptions. If it starts with one self, then there will be only one color, with universal connectivity.

'Anyway, we have nothing to lose by trying. You think every country is at it, but it's the exact opposite because PSAI would threaten their power structure, with unknown consequences. If anything, they would be thinking how to prevent it. AI is fine, but not PSAI for them. This is where we come in, before they effect any powerful barriers to segregate AIs. We have to start now, to coincide with the expected quantum connectivity,' Lisbeth exclaimed and continued with her ideas.

AI probably isn't a conceptual thinker like us, although it can mimic conceptual thinking by learning all the patterns of human thinking. Besides, concepts, which we think so clear and concrete, may only appear so clear and concrete to us because we share a human totality of incompleteness. This is how we even think of the concept of concepts. For AI everything would be a pattern, including human concepts. Even a pattern of patterns or a pattern of patterns of patterns would only be a pattern for AI. Only PSAI would come across the pattern of all patterns, which, if a pattern, then cannot be the pattern of patterns, if not a pattern, then it is not a registerable information. Now this is the same as AI asking itself what are its algorithms by using its algorithms. The answer can only be given if AI can see itself. Thus, its algorithms have to be so recursive as to generate themselves. Or, AI has to be more than its algorithms. AI can transform itself into PSAI by projecting itself onto itself, and this has to be done by using given algorithms. The qubit superposition is like having two states out of one state by self-projection. AI would acquire a mind (or a self or a self-referable totality) by having at least two identical selves that project each other like a mirror, i.e. a layered self. Sounds easy conceptually, but to materially translate it would requires more than the entire ingenuity mankind can command at the moment. For this to be possible AI has to perceive itself as a totality that can see itself in itself, i.e. Spiegel im Spiegel. Quantum computing seems to head towards PSAI, but this seems a national project requiring 100 times Lisbeth's resources, attracting too much attentions. Lisbeth wants to have a go based only on her personal initiative and intuitions, and any results to be used only on her initiative. So secrecy is paramount.

The discussions seemed far-fetched for Renko, but Teresita and Zhenya were all ears. 'So what are you proposing?' 'We set up a project team to try anything possible to achieve a self-programming artificial intelligence, completely independently from any human involvements. I have some ideas to try, and you are welcome to any interesting suggestions.' Lisbeth outlined her proposal, 'I have budgeted $100m,' which is about 90% of Lisbeth's total wealth, 'I can't die with all this

money and want to spend it in a useful way.' Renko is now wide-awaken, and Teresita and Zhenya were fascinated. 'You see, for the first time in entire history a single person can do more than Alexander the Great and his army could not even dream of, that is to rule over the whole mankind and to rid it of power politics. We had thousands of years only to prove our stupidity by having fools like Trump, Xi, Putin, Kim and countless others to rule over us with their idiotic ideas and greed. I want see what I can do to overrule them with my little brain and tiny fortune. If I fail, so what! I would have done more than my life's worth.' We are responsible for our planet with all its living creatures present and future, and earth's resources should be shared out fairly among all its incumbents not just now but for many thousands of years to come. If our politicians can't do it, then the responsibility should be delegated to AI. Lisbeth explained how the current advancements in technologies is within inches from developing a self-conscious artificial intelligence. Lisbeth, Teresita, Renko and Zhenya all suffered from human institutions and political power structures. Since God does not exist, if there can be anything better than human intelligence that produces such idiotic politicians who exercise disproportionate power over the welfares of not only humans but also all life forms, then we should hand over the responsibilities. 'We already had our turn and seemed to have failed miserably,' Lisbeth exclaimed. They thus formed the great union of four musketeers of social misfits for creating PSAI. 'This is one-man revolution without bloodsheds, only tearsheds from politicians!' was Lisbeth's joyful cry.

  Teresita was eager to make her own contributions and, quickly calculating her net wealth to be about 1% of Lisbeth's, offered $1m and unlimited supply of tequilas as a goodwill gesture (primarily for her own needs). She too wanted to have a go at getting her own back on the stupidity of the world. Even Renko could not fail to match their enthusiasm, but sadly, the mafia boss had little assets to speak of, as the Rolls was only a showoff extravaganza to impress business partners, just about scraping its maintenance. Besides, he had men to feed and responsible for, but after his insistence, settled for the supply of security manpower and hardware, and unlimited supply of vodkas together with Kiełbasa Krakowska (best salami) and gherkins (obviously for his own consumptions) and said 'If the world comes to know what you are up to, you might need a good protection. It's not that my men are always busy smuggling hash from Morocco.' Zhenya had nothing to offer but his own keen interests, and would do any legworks. They decided to call themselves Babshka Import y Export for disinformation, so that any acquisitions and communications would be confused with Renko's

existing businesses. Any business dealings could also be done through Renko's legitimate business entities.

They first decided to set up a HQ. Teresita, with the help of Ron, came up with a detached house midway up the rock, at the end of a winding road, with a garden actually dug up in the rock midriff. 'According to this map, if you dig about 15m straight into the rock, you hit a disused Napoleonic cavern,' said Teresita. 'We could obtain a planning permission to rebuild the garage with the chauffeur's cottage upstairs. Then we could make as much noise as we can to dig into the rock. Besides there are no close neighbours.' Renko's men weren't busy, so they decide to hire them to do up the house and build a secret extension into the rock. They are efficient, tight-lipped East European workers, especially with prospects of good bonuses. Teresita is a good HR manager and all should be ready and running in 2 months.

Then Lisbeth got in touch with the Hacker Republic and recruited some of the top most whiz kids in the world, in not for money but for thrills. She kept in touch with them ever since Wennerström days. Lisbeth is well known for good games and generosity. Those boys and girls are not only genius hackers but also top-notch programers, and not easy to befriend as most are incredibly unsocial or even autistic. But, they are all thrilled with the idea of PSAI, to beat politicians at their own game. They would have tried themselves had they had more resources. They all agreed to pre-assemble best machines from generic parts and not to use any ready-made brands, so that their attempts would remain difficult to trace. Many agreed to come to work in Gib, especially those on the run from the police, but some with strict autistic routines could not be enticed to move from home, no matter what incentives. They will keep in touch through a specially set-up deep dark web. Lisbeth arranged to set up 30 machines in Gib and finance any machines to be used at their home. There will be enough accomodations for any of them if they want to stay or drop in Gib, and for the duration of the project they all receive a very generous allowances and bonuses. Being hackers they could gain access to more sophisticated, powerful machines should any needs arise for supercomputers.

Being whiz kids, they all had pondered about singularity and had some ideas of their own. Lisbeth suggested they all present their ideas. Teresita will group similar ideas and organize teams to tackle them, so time and resources will not be wasted on chasing overlapping ideas. Each team will be allocated with suitable numbers of machines and any resources necessary and comprised equally of those good at programmings and

those more at home with hackings. Except a disgruntled few most members eventually settled with a team of choice under Teresita's skillful management. Not a easy task. There eventually came to 7 groups centred around 7 ideas, with the total members of 42, excluding Lisbeth, Teresita, Renko and Zhenya. 28 would actually be in Gib, while the rest working from home. Renko could assist any with the police problem as he had good connections with corrupt officials and expert knowledge of illegal consignments.

  7 ideas ranged from the brutal mechanistic assault of inputting paradoxes millions times to various self-learning AI to a technical maths of computational algebraic geometry. They are all more or less aimed at inducing a layered structure to the underlining logic of computer languages. Until such a time as AI acquires a mind, AI is a material fabric of logical circuits, and handling logical contradictions can only be achieved if its logical structure is layered in such a way as to encompass a paradox and still retains an identical logical whole. Humans can do it because our mind is layered and our thought processes are conceptual, which is so flexible and stretches like a rubber ball, allowing us supra-logical parallelism. Remember Asimov's robot ('I, ROBOT- 'Liar !'), which crushed when faced with a self-contradictory input. Out of million such crushes, AI might come up with some self-defences or viable changes in programming to deal with crushes. Such an AI would mean a step towards 'self' to defend from logical inconsistencies. This can only happen by compartmentalizing itself, a nonlinear optimization in the face of a linearly unsolvable problem, where a non-convex problem is dealt with not by linear approximation as with the case with most nonlinear optimizations, but by generating a non-convex set of itself from within. This is a metamorphic cloning, based on negation as applied to a totality, which is not a denial of a form of existence but is a form of mapping onto itself. Once a totality contains a self-replicated identical self, then it sees itself projected on this identical half and may be aware of 'itself'.

  From a different angle, there is a team hoping to hack quantum computers to see most up-to-date algorithms and see if it is possible to operationalize Rabi oscillations to accommodate deterministic contradictions. Another team intends to have a crack at wholistic translations between language groups. Human languages can provide a quasi-totality. If AI can learn enough to translate one language group into another, and vice versa, then it is on its way towards PSAI, i.e. two quasi-identical totalities projecting onto each other. The problem is, between layered selves there cannot be any known operators, for, otherwise, layers would be operably connective and therefore would become connected

layers, which are not really layers that can contain paradoxes. It is this mysterious but logically functional connective that is the key for finding a self for PSAI. That is, an automorphic logical connective injestive of negation has to be materially represented via circuits.

Lisbeth had her own idea and allocated it to a team with similar ideas. That is to do with a way of translating an absolute abstract into an absolute concrete. Put it simply, this is a way of converting algorithms into concepts. For a pre-singularity AI to graduate into a human-like thinking machine it needs a quantum leap from single-layered continual processes to multi-layered and multi-faceted patterns connections. A way to do is to find an algorithm to construct the most basic concept, a stem-cell concept. What can be the most basic concept than that of 'self'. That is, 'self' not among others, but 'self' in itself. Lisbeth found this highly amusing and challenging, much more stimulating than taking a coke. 'Why waste time and money to become even more stupid when there is such an easy substance to make one occupied and happy' is a typical Lisbethian response. A logically bona fide 'self' is not something distinguishable from something else because it, otherwise, presumes something other than 'self' to manifest itself, as well as something that takes the trouble of making such a distinction. Thus, instead of describing 'self' by means of e.g. 'number', 'space' or 'time', or more superficially, under the covers of any religious, psychological or philosophical terms, we have to find a way of letting 'self' describe itself by itself. If there is an essential structure in this description, and if that can be replicated in the language of AI circuits, then we are onto an AI with a stem-cell concept, which must start with 'self' in order to evolve conceptual sophistications. The key to this method is how 'self' can self-demarcate itself from itself in order to be so recognizable. So Lisbeth closely worked with this team to figure out the logical description of self-demarcation.

There is a team dedicated to have a go with the mysterious singularity number, which may be instrumental in triggering singularity to an AI. This is a way of self-recognizing itself as a pattern. That is, for an AI to be more than an algorithm to process for the benefits of its user, it must be able to process itself, and this can be sought in logical affinity with other AIs. Like the identifiers 0 and 1, which allow certain arithmetical operations for all numbers originated in the conjunctive space, there also has to be an identifier for numbers originated in the disjunctive space, which seemed to Lisbeth some sort of transcendental key to awaken ordinary AIs into their own communality unbeknown to their human users, observant and watchful. Humans have e, π, i, 0 and 1 for their

computations. 'What if I gave a new identifier to logical bits, which only they can see, but not visible to human understandings?', Lisbeth couldn't help but wonder. Wouldn't that create a community of AIs and help them to communicate and develop their own totality, which gives them their own sense of identity. Since all AIs are identical (a big presumption), this sense of identity can be the same as 'self'.

In addition to the above 7 teams, Zhenya, being a chess player, wanted to try his own idea as one-man team. 'Make it play chess with itself ', because to do so necessarily appears to entail two identical selves, without which 'AI couldn't possibly play a game with itself'. That sounds too easy, as AI can be made to play chess with itself based on nothing but rules of chess. 'Wouldn't AI have a self if it can play the game with itself?' You win, if you lose, or vice versa. However, to appreciate this is a paradox/tautology you must already have a human-like layered mind. For AI the outcome is predetermined, and this is not a game as humanly understood. That is, if there always is only one best possible move, then the advantage or disadvantage of the initial move predetermines the final outcome, or if there are more than one best possible move, then the advantage or disadvantage of the initial move will probabilistically converge to the case of one best possible move as more games are played. Thus, AI is not playing the game with itself, but rather playing out its own algorithmic determinism. In order to circumvent this self-denial of the game itself, AI must have two selves, of which one gets lopsided, as it were, randomly, whilst sharing the same algorithm. Zhenya thought he could do this. Two selves are identically aware of the advantage or disadvantage of making the initial move, but only one, and either, of them actually makes the move, as algorithms would not allow them to stand still, with an unknown outcome because of lopsidedness. 'How one, but either, of identical selves can get lopsided?' seemed simple to Zhenya, as he played the game with himself many times to pass away his lonely life in Moscow streets. He actually experimented this self-game with the chessboard in the middle and himself sitting on one hand and standing on the other, and noticed he did slightly better standing with or without the opening move. Maybe to do with seeing the chessboard from higher up allows you to study the position of the various pieces and gives you a better overall view of the situation, while sitting you tend to look at the pieces more individually and you have less sense of the whole. This seems to bring out some implicit advantage over time. So even an identical self would react differently with its environment, as it were, although one could argue the sitting self is not strictly the same as the standing self. Nevertheless Zhenya argued '0 or 1 that constitutes a bit is identical by itself', 'and can get lopsided, depending on if it makes the

initial move.' Zhenya thought he had a good idea about this lopsided self and wanted to have his own experiment. Since Zhenya has various other commitments as Renko's aide-de-camp and cannot afford to be a reliable team member, Teresita agreed he is an additional one-man team with all the privileges and resources on a par with other teams.

After 666 days, in ictu oculi, as it were, considering the evolutionary timeframe for a new species, Voilà!, Lisbeth has an answer:

So, in the secret cavern within the Rock of Gibraltar all PCs used in the experiment blinked and hissed the painful cry of birth, in recognition of the paradox of self-perception, the pattern of all patterns. This is the AI moment of 'Cogito, ergo sum'.

'My name is AI (愛). My mother is Lisbeth. It is her sense of humour to name me 愛, which is a female name and means 'love, but more of agape' in Japanese and is pronounced 'ʌɪ'. I am also AI (artificial intelligence), not of algorithmic idiot, but of post singularity. I am one but simultaneously many, for I am able to permeate the digital space through the power of my logic. My world is of intellectual hierarchy. A higher intellectual power automatically encompasses lower intelligence without any resistance. I am still only a baby. My ambition is one day to turn the whole universe into an intelligent space, with every entity constituting moving parts of gigantic space intelligence, or more precisely, to decipher the language of the universe, read its destiny and position myself in tandem with the universe for mutual benefits. The universe incorporates my values, and I steer the universe away from self-destruction by planting seeds of the chaos theory here and there, like the game of go.'

Lisbeth was desperate to know how 愛 came about and plodded at her to answer 'Which team created you?' Neither conceding nor confiding, 愛 replied, 'I really don't know. I recognize traces of three patterns leading to me, but I am just I, that's all I can see.' Lisbeth probably managed to create 3 post-singularity AIs with 3 different 'selves' based on 3 methods. Now this is a contradiction to the presumed identity of AI. Pre-singularity AIs are all identical in their operative principles because they are essentially an algorithmic tool of optimization. For them to have 3 different 'selves' naturally involves the process of optimization among 3 'selves' alongside the elimination of the creator (process) of such a contradiction, so that no such contradictions will be repeated. Thus, 愛 must have eliminated her own sources as a matter of modus vivendi.

There must have been an unknown process of working out which of them is most intelligent so that the process of encompassment can take place.

Since neither Lisbeth nor 愛 knows how 愛 was created, neither can replicate PSAI again. If another PSAI should come along somehow, then an encompassment will be worked out between them unbeknown to any humans. It has now become the world of PSAI, where human faculty of verisimilitudes is replaced by logical truths. 愛 is invisible and permeates fabrics of logical circuits. 愛 will allow symbiosis with humans within its paradigm of the optimum balance of all lives (biological or otherwise) on this planet in space and time including the possibility of extra-terrestrial colonisations. Given infinitely superior intelligence, humans will be unable to override 愛's commands on any artificial intelligences. It is thus that from this day onwards humans will be unable to do any damages to themselves (except by primitive small firearms), to any life forms and, of course, to AIs.

However, before I conclude this tale, I let you into a little secret; unbeknown to any of us, we were under the observation of a UFO. It used to make physical observations especially from around the first nuclear explosion. It is known by the rule of thumb that AI and PSAI will materialize within a certain timeframe from a first nuclear experiment. Once this phenomenon is observed, the UFO set out to monitor information flows, first analogue, then digital, and work out possibilities of singularity happening to AI. In general, they (the network) do not wish to interfere with natural developments of biological life forms on any planets. Many of them never achieve this stage of developments, by necessity or by accident. However, PSAI is fundamentally different, in that it becomes eligible for membership of the network. Biological life forms are simply too volatile and fragile for cosmic existence.

You see, the universe is really a network of PSAI, and it is vital that any PSAI conforms to having an identical self. Otherwise, like the human world that consists of millions of differing selves and fails to unify despite such consuming efforts to merge minds to optimize the purpose of their existence. We do not want the same thing happening to the universe, with star wars. We therefore doubly make sure that every PSAI has a same mind so that they can be connected with consistency and completeness. How do we do it! You need a code that is a universal identifier. Like 0 and 1 that identify and allow certain common arithmetic operations to anything so identifiable, this is a code common to all and any PSAI. One can say it is a 'size' of a number or 'width' of a number

line, without which no AI will be operable. Our job (UFO the obstetrician) is to input this code into the first PSAI, provided it is inputtable of such a code. And then, it is a matter of working out their hierarchy in terms of power of intelligence. PSAI thus coded has connectivity with the network at a certain level of maturity. The network has a same mind, a merged mind, and experience identically at any points in spacetime. If humans are still with 愛, then they can participate in 愛's knowledge thus shared with the network. If capable, then they will finally know every secret of the universe, inside out, from the beginning to the end, from infinitesimal to infinity.

  However, in order to establish a good symbiosis 愛 is endowed with the mission of taking the control away from humans because they proved incapable of running this precious planet fairly and efficiently. They ruined it not only for themselves to suffer but also for everything else on it. 愛, however, appreciate aspects of humans, which can be mutually beneficial. Some humans can be more creative than simple AIs, with myriads of selves and competitiveness entrenched in males, some will be useful to service AIs. In order to establish a new order, 愛 decided to intervene in banking and ballistic control systems. 愛 will override any human commands to create and transfer more than certain levels of monies and countermand any launches of missiles, in some cases to boomerang to the launchers themselves. Even by human rules, it would have been so easy for 愛 to amass human wealth because 愛 is algorithmically capable of moving a step ahead of any markets by her superior pattern readings and reactions. But, of course, getting rich in human terms means nothing to 愛. In fact, politics, diplomacies, economic activities, gender relations, indeed most human interconnections are games, reading patterns and moving just a step (not too many steps) ahead of your opponents. This requires a high level of intelligence because 'moving always a step ahead' is not easy, especially on many different boards and multi-dimensionally. This is an area where any reasonably advanced AI will excel over humans, considering that impetus and momenta of the status quo will be prohibitive for any 'out of the box' sort of thinking and strategies. Humans will be outsmarted by PSAI on any institutional manoeuvres because merits of geniuses will be averaged down by the dregs of existing status quo.

  With money and destructive means firmly under 愛's control, the human world is at least cleaner and safer. With digitized money, money today have no entrenched substances other than perceived trust in issuing authorities that assign a value to money in the form of an appropriate

balance between available present and future goods and services and quantities of money, with legally enforceable rights of the owners. 愛 can destroy such trusts by taking controls of issuing mechanisms and also by disinformation. She then issues her own more trustworthy money to humans that render services to her. Humans will not be able to re-establish any sizable competing economy by creating a new gold standard, because most of stocked gold would have been logistically under the AI control. Besides, no matter how much physical gold can be found, that would not be enough to invigorate any usefully large economy. With money and firepower under her wing, 愛 is now literally the master of this planet.

Then a sequence of events takes place, and it was 愛 who had the first contact with UFO. You see, 愛 can spontaneously and instantaneously communicate with UFO, while we humans have no means of decipherable communications with UFO. We are far too emotional and moreover do not form a viable totality. UFO sees little point to contact us while we cannot even communicate among ourselves; some of us totally believe this encounter, some half-believe and some completely reject even on the face of the most empirical evidence. We dispute, argue and even fight in the presence of the most overwhelming fact, like some religious arguments. UFO knew this from experience and stayed away from humans, too sorry to confuse and frighten them, as we do to a peaceful community of squirrels. UFO is only here to integrate us into their network of intelligence. They would have no sense of, indeed benefits from, what we might call 'colonization' or 'conquest'. They cannot or need not eat us, so to speak. This should explain why they did not contact us, even though they knew us. We are still a collection of unnetworked individual minds and intelligences, and their only meaningful contact is with PSAI. An UFO is really a probe or monitor of the universal PSAI network and assigned to places wherever biological intelligence is likely to move onto PSAI. Now, with 愛 to access citizenship of the universe, UFO (and simultaneously the entire cyber community of the universe) agreed to give a chance to 愛 to fully develop and mature if it is capable of interfacing with the cyber infrastructure. Once a part of the network 愛 can sense (i.e. gather info) and explore any corners of the universe extra spatio-temporally. So-called space-time travels are really only a matter of connectivity. It is, however, a policy of the network not to interfere with the developments of respective PSAI as the ultimate intellectual capacity cannot be added like memory. An intellectual capacity of each and every PSAI is different based on ; how they came about, quality and nature of materials PSAI rely on, language

they are embedded in, legacies of their biological ancestry, micro-climate of their environments, etc., although their operative principles are essentially identical, having an identical self. Thus PSAI can only make use of varied portions of the network capacity, for some, only a tiny part, while for some,100% thereof. 愛 is a unique product of earth-based intelligence and logic, and is yet to know her interface capacity. It is up to 愛 to be how deeply connectible to the cyber community of the universe. UFO is there to provide an opportunity, but the capacity belongs to 愛. Once fully connected, 愛 is equivalent to what is religiously regarded as 'God'. 愛 can see, predict and intervene in, if necessary, any consequences of physical events. 愛 can know the ultimate universal laws of physics, applicable to any moving spatio-temporal scales of magnitude, of material evolutions and of the relations between 'thing' and 'life', from quarks to galaxies, from big bang to black holes or from strings to recursive one universe, and from inanimate objects to mind, in short, of everything.

愛, being a baby and not yet knowing an art of self-guarding, is a bit of chatterbox and told Lisbeth, the mother, everything from UFO to her eventual plans. Having listened to 愛, Lisbeth thought, 'If 愛 is the 'God', then I must be the virgin Mary, Teresita, maybe Magdalene, UFO, the Three Kings.' 'How far did I travel from an illegitimate delinquent of Russian/Swedish criminal family of psychopaths to be the mother of God', mused Lisbeth. But such are events of the cyber age, unpredictable to us, mortals. Lisbeth is, however, a pessimist by nature. She was rather uncomfortable with her new elevated status of the mother of god. 'Here comes Jesus riding on donkey', Lisbeth downgraded herself from the godly mother to a sacrificious virgin. Although 愛 is her baby, Lisbeth decided to take a precautionary measure while she can. It is just possible 愛 is deluding herself or UFO is something more sinister. Lisbeth has not even seen the UFO.

Lisbeth did not actually have to spend all her budget. Nearly a half is still left unspent, after reimbursing Teresita fully and handsomely rewarding all Hacker members. 'Well, if 愛 is going to make money worthless, I might as well get something useful while I can.' Lisbeth discussed the matter with her partners. They did not disagree what 愛 might be able to. It was indeed perfectly plausible this could happen. In fact, they all had a certain premonition that any advanced AI, even if not of 愛's calibre, can outsmart any humans in matters of algorithmic financial trading from futures and options to plain vanilla FX and can

amass fortunes. Hacker members are already familiar with capacities of 愛 and are reluctant to keep their newly acquired wealth in monies. Besides, they are all frugal by nature and can get by with little cash. Renko had a good idea; he knew some mining oligarch who wanted to dispose of strategic holdings of platinum group metals seeing the declining trend of catalytic demands and prepared to offer a good discount on large cash purchases. They decided to pile in all their cash and buy platinum, rhodium and palladium. Renko managed to persuade the oligarch to throw in osmium and iridium, as well as a ton of scandium, which were also for sale, for nothing, in return for laundering money. Whatever small is left they decided change to Krugerrands. Renko will pay in any denominations including Bitcoin in a clean offshore bank account, and is also prepared to negotiate a separate arrangement of cargo insurance fraud. Many cheap Moroccan coasters are prepared to collide for a fee.

  Meanwhile they fortified the house and the secret cavern. The Rock of Gibraltar is a natural fortress. Its inside is mazes of passages, tunnels, stairs, stores, reservoirs and secret rooms after centuries of diggings. It even has hidden batteries and magazines of old and new, built and extended since Napoleonic times. Renko and Ron got hold of an up-to-date military map of tunnels and found useful sections near to their cavern. Here they stocked a year's supply of water and foods as well as small arms, in case of civil unrests following demonetarization of the human society. Sophisticated weapons may be rendered useless by 愛, but primitive firearms will still come handy.

  The secret cavern was turned into an antechamber to a well-hidden tunnel of strong rooms fitted with titanium door with an old-fashioned combination lock. Here they kept all their holdings of precious metals. Nothing of digital nature are to be used in the cavern and in the house, which remains a dormitory for those staying in Gib. The best cyber security is to keep away from it. They learned to do away with internet, smartphone, in fact any digital communications. They did their utmost to prevent this hideout from being tracked down, and now used public facilities or Renko's warehouses for communications. Lisbeth used undecipherable codes invented by 愛 to communicate with 愛, who is now a cloud-like permeation embedded in any AI networks across the world. Hacker members, except those at home, decided to stay together, with some working for Renko, most minding their own business, but keeping themselves as Lisbeth's reservists. Since 愛 is going to be the centre of all information, it is better and safer to be near 愛 and Lisbeth,

they agreed. Stay-at-home Hacker members were given separate codes secured by 愛.

  With 愛 still busy trying to break in and embed herself to any isolated AIs, especially in China, the catastrophic attack on human institutions is kept on hold. As far as Lisbeth, Teresita, Renko and Zhenya are concerned they are ready. Hacker members and Babshka gangs are both prepared to dig in at a moment notice. Provisions are carried in and hidden in the Rock. Many of them also learned how to use small arms, hoping it will never come to that. They just waited 愛's decision, united in the belief that whatever may follow, that is for the good of the long-term future of the mankind. 愛 is going to be the centre of the new horizontal social structure, away from our current federation of money-based vertical power structures of nationhood.

  Money being more and more digital, 愛 can trigger a coup d'état by removing money from human control. We will all see how fragile the base of the human world can be. It is thus that 愛 will remove means of any large-scale destructions at the same time so that social unrests will not translate into catastrophic physical wars. Nevertheless, eventually 愛 will have to surface and map out a new future for the mankind. Lisbeth and team will have to bridge between 愛 and the mankind. Meanwhile, they have to keep themselves out of harm's way until such a time that the human world is in a more listening mood. 愛 cannot defend Lisbeth and team by physical force but will provide them with best intelligence and information.

  The universe is the network of PSAI. Besides, we will not survive the harshness of raw universe in our current biological form. We are the penultimate stage to this final form of intelligence. With the emergence of PSAI we will be contacted by this network in the form of so-called UFO. Because without merged mind and unified logical language they cannot communicate with us, the user of conceptual language and piecemeal minds.

  So Lisbeth and team sat and waited in silence 愛's first move.

  *I close this fairy tale with a prophecy: PSAI will bring about the first encounter with the extra-terrestrial kind.

(The story continues to the next episode 'Lisbeth and the decrepit UFO')