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Plato on Geometrical Hypothesis in the *Meno*

Abstract: This paper examines the second geometrical problem in the *Meno*. Its purpose is to explore the implication of Cook Wilson's interpretation, which has been most widely accepted by scholars, in relation to the nature of hypothesis. I argue that (a) the geometrical hypothesis in question is a tentative answer to a more basic problem, which could not be solved by available methods at that time, and that (b) despite the temporary nature of a hypothesis, there is a rational process for formulating it. The paper also contains discussion of the method of analysis, problem reduction and a diorism, which have often been ambiguously explained in relation to the geometrical problem in question.

Keywords: Plato, Socrates, Hypothesis, Analysis, Geometry

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At *Meno* 86eff. Socrates proposes to consider 'whether virtue is teachable', which is the pivotal question throughout the dialogue, on a hypothesis such as geometers often employ. On this occasion, as previously in the recollection passage, he introduces a geometrical problem in order to illustrate how the method of hypothesis works. This geometrical problem has aroused scholars' interest mainly for two reasons. First, it provides historians with an important source on the early stages of Greek mathematics, evidence for which is rather scarce by comparison with post-Euclidean materials. Second, it is expected to shed light not only on Socrates' application of the method to his own argument on virtue but also on Plato's more substantial use of the method in the *Phaedo* and the *Republic*; those philosophical arguments have been highly controversial. Despite such attention, its mathematical contents remained an unsolved riddle for a long time, due mainly to Plato's obscure language. However, thanks to the laborious accumulation of research on the mathematical side, the basic structure of the geometrical problem, although not decisively determined, seems to have achieved a broad consensus by now.

The purpose of this paper is not to offer an alternative interpretation of the geometrical problem or to elucidate how Socrates uses the method of hypoth-

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esis for his philosophical arguments in the *Meno*, let alone in the other dialogues. I limit discussion to the implication of the favoured interpretation; for it seems to me that some fruitful outcomes of such long-term research on the geometrical problem have not been fully appreciated yet in relation to the nature of hypothesis in the *Meno*. If my evaluation is correct, the following notable conclusions can be drawn: (a) the geometrical hypothesis in question is a tentative answer to a more basic problem which cannot be solved by available methods, and (b) despite the temporary and intuitive nature of the hypothesis, there is a rational process for discovering that hypothesis. My suggestion has to be eventually compared with Socrates' successive arguments on virtue, and it will be worthwhile to be aware of these results, because his hypothetical inquiry into virtue is itself debatable and said to be modelled upon the very geometrical hypothesis I shall discuss here.

With this picture in mind, I start by giving the outline of the geometrical problem based on the standard interpretation (section I), and then consider its historical background in relation to two important mathematical techniques used at that time: the method of analysis and problem reduction (section II). Finally, it is discussed what the observation so far implies regarding the nature of hypothesis in the *Meno* (section III), with some concluding remarks (section IV).

I. The geometrical problem

In the first half of the dialogue (down to 80d4) Socrates requires Meno to define 'what virtue is' (the τί question) because it is impossible to answer 'what kind of thing virtue is' (the ποῖον question) before settling the τί question. After his three unsuccessful attempts to answer the τί question, Meno sets up a paradox of inquiry which infers that it is impossible to search for something in the first place (80d5–8). In order to convince Meno of the possibility of inquiry, then, Socrates appeals to the theory of recollection (80e1–86c6). Despite Socrates' passionate persuasion, however, Meno adheres to pursuing the ποῖον question before dealing with the τί question (86c7–d2), and Socrates ends up consenting to his request on the condition that Meno allows him to investigate it 'on a hypothesis' (ἐξ ὑποθέσεως, 86d3–e4).

Before discussing the teachability of virtue, Socrates illustrates the procedure of the method of hypothesis by reference to a geometrical problem in the following way:

> By 'on a hypothesis' I mean the following. Take the way in which geometers often consider a question someone asks them, for example, whether it is possible for this area to be inscribed as a triangle in this circle. One of them might say: 'I don't know yet whether the area is like that, but I think I have a sort of hypothesis, so to speak, which will be serviceable for our task. It is the following. If this area is such that, when someone has placed it alongside its given line, it falls short by the same sort of area as the very one that has been placed alongside, I think one result follows, and a different one if it is impossible for it to do this. So by making a hypothesis (ὑποθέμενος) I am willing to tell you the result regarding its inscription in the circle, namely whether it is impossible or not.' (86e4–87b2, tr. Sedley and Long, modified)

The geometrical problem described here is whether a given area can be inscribed as a triangle in a given circle. When a geometer does not know the answer to the question, according to Socrates, he will try to answer by considering whether the given area meets a certain condition or not. But what that condition is is extremely ambiguous and has therefore puzzled many scholars. After the long-term accumulation of research in the 19th century, the most promising interpretation was presented by Cook Wilson at the beginning of 20th century and has been followed by many scholars down to the present day.[1]

According to Cook Wilson's interpretation, the original problem is equivalent to the inscription of a rectilinear figure (S) in a given circle as an isosceles triangle BDE.[2] And the condition Socrates gives above is whether the area S is such that when it is applied as a rectangle BCDA to the diameter BF, it falls short by a rectangle DCFG similar to the applied one BCDA (see Fig. 1).

1 Cook Wilson 1903. The essentials of his interpretation were given by August (1829) and Butcher (1888), according to Heath 1921, vol. 1 299–300, who reached the same view independently of Cook Wilson. The list of its proponents is Heath 1921, vol. 1 298–303; Knorr 1986, 71–6; Mahoney 1968, 334–7; Menn 2002, 209–15; Scott 2006, 133–7; Vitrac 1990–2001, vol. 1 380; Wolfsdorf 2008, 46–57. Bluck 1961, 441–61 conveniently reviews other major interpretations.

2 Cook Wilson 1903, 233 actually starts with the following discussion presumably to prevent the introduction of an isosceles triangle from losing generality: 'If a scalene triangle is inscribed in a circle, an isosceles triangle, equal to it, can obviously be inscribed in the same circle.' As Menn 2002, 209–10 n. 22 points out, however, this might still be problematic because the *Elements* proves no method of transforming a scalene triangle inscribed in a circle to an isosceles triangle which is equal to it and inscribed in the same circle. To this I answer that, as Mahoney 1968, 335 n. 35 explains, the area of the isosceles triangle having vertex B varies continuously from the minimum (zero) to the maximum area inscribable in the circle (the equilateral triangle), and therefore that Socrates could have answered that an area below that limit can be inscribed as an isosceles triangle, but an area over that limit cannot be inscribed as any triangle at all; if one triangle equal to the area can be inscribed, it is enough to answer that the inscription of a triangle is possible, so we do not need to think of the inscription of a scalene triangle.

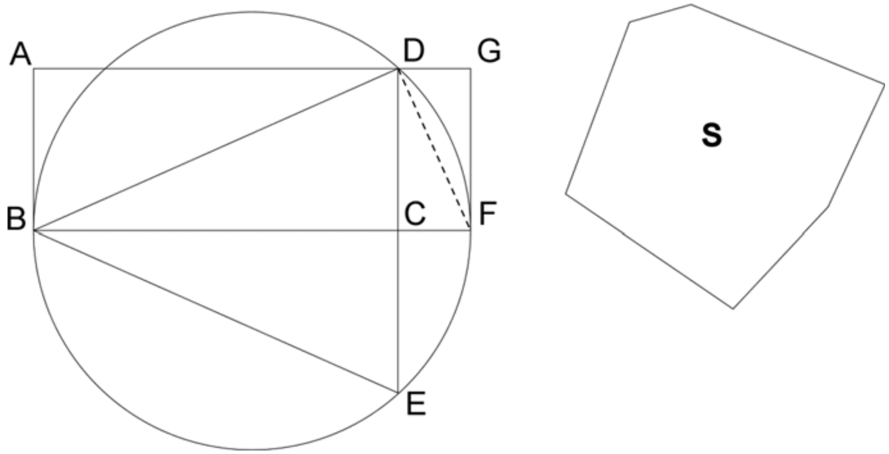

Fig. 1 The application of area S to diameter BF as rectangle BCDA which is equal to isosceles triangle BDE and falls short by rectangle DCFG similar to BCDA.

Before analysing the details we can put forward a prime reason in favour of his interpretation. The decisive advantage of his construction is its striking similarity to Euclid VI. 27, 28 (cf. II. 5), which use one of the most distinctive techniques in Greek geometry, 'application of areas (in the case of their falling-short)'. According to Proclus,[3] Eudemus and his school said that the Muse of the Pythagoreans discovered the technique of application of areas (I. 44), including the two special cases, their exceeding (VI. 29; cf. II. 6) and their falling-short. This technique was necessary in order to construct a regular pentagon (IV. 10, 11), which is also attributed to the Pythagoreans,[4] by dividing a given line into the 'extreme and mean ratio' or 'golden section' (VI. 30; cf. II. 11).[5] It is doubtful, to be sure, that any mathematical achievement is historically attributable to Pythagoras himself or the Pythagoreans, but the existence of the reports of its attribution at least suggests that these geometrical discoveries are as old as the early 4th century B. C., and perhaps as the latter part of the 5th century B. C. We may judge, therefore, that the technique was already well known by the time of the *Meno*. This circumstantial evidence strongly supports Cook Wilson's interpretation.

3 Procl., *in Euc.*, 419. 15–420. 23 [Friedlein].
4 For example, Proclus says that the Pythagoreans discovered the construction of the cosmic figures (Procl., *in Euc.*, 65. 20–21 [Friedlein]). Among the five regular solids, the dodecahedron needs the regular pentagon to construct it by putting together plane figures.
5 On the relationship between application of areas and the construction of a regular pentagon by the Pythagoreans, see Heath 1921, vol. 1 150–4 and 158–62; Knorr 1986, 66–71.

It is true that his interpretation cannot be totally free from difficulties, from linguistic and mathematical points of view, as many point out.[6] Concerning linguistic drawbacks, for example, the above interpretation assumes that the rectangle BCDA is applied to the diameter of the circle, but the Greek τὴν δοθεῖσαν αὐτοῦ γραμμήν (*its* given line) is said to signify a side of the given figure rather than the diameter of the circle, because αὐτοῦ naturally corresponds to τοῦτο τὸ χωρίον, with which we are concerned.[7] Another assumption which is regarded as precarious is that the applied figure and the deficient figure are similar; it is insisted that the Greek τοιούτῳ χωρίῳ οἷον ἂν αὐτὸ τὸ παρατεταμένον ᾖ, especially the addition of αὐτό, suggests congruence rather than similarity.[8] However, if we bear in mind that mathematical terminology was somewhat fluid in Plato's time[9] and that he tends to avoid fixed diction, these objections are not strong enough to overturn the historical evidence above in favour of another interpretation.

6 Bluck 1961, 448–52; Gaiser 1964, 272–5; Karasmanis 1987, 105–7; Lloyd 1992, 171–3; Meyers 1988, 176; Sharples 1985, 158.

7 It seems to me, however, that Cook Wilson 1903, 235–7 is more convincing when saying that it would be odd to speak of some figure as applied to its own side or base, and that if taken that way, to apply the given figure would be in no relation to the circle. Also, Bluck 1961, 451 claims that since the circle has already been given (τόνδε τὸν κύκλον, 86e6), 'given' (δοθεῖσαν, 87a4) becomes redundant on Cook Wilson's view. However, his objection should be directed rather at the view that the line in question is a side or the base of the given figure, because the figure itself has already been given (τόδε τὸ χωρίον, 86e6). Since, on the other hand, the circle has already been given while its diameter has not, I suppose, Plato would have needed the word 'given'. Cook Wilson reasonably concludes that even though αὐτοῦ refers to χωρίον, the phrase τὴν δοθεῖσαν αὐτοῦ γραμμήν can be taken as meaning 'the line given for it' in the sense of 'the line given in respect of which some operation is to be performed on the given figure', namely the diameter of the circle. The same view is endorsed by Wolfsdorf 2008, 51–2.

8 Bluck 1961, 450 says, by referring to Heijboer, 'Of a figure similar to another figure we might vaguely say that it is like the other, but we could not possibly say that it is "like the other figure itself"'. However, if 'equality' or 'sameness' had been what Plato meant, he would have written τῷ ἴσῳ or αὐτῷ χωρίῳ τῷ παρατεταμένῳ instead. In VI. 27, 28 the figure to which a deficient figure has to be similar is different from the applied one. Therefore, Plato might have added αὐτό to emphasise that the figure made similar to a deficient figure is not another but the applied one itself. That rather suggests that Plato might have had in mind those propositions of Euclid's.

9 There is no doubt that Plato meant inscription by ἐνταθῆναι here, but Euclid uses ἐγγράφεσθαι or ἐγγράψαι for inscribing a rectilinear figure into a circle in IV. def. 3, props. 2, 6, 11, 15, 16. Therefore, although Euclid uses παραβάλλειν for application of areas (Plato uses παρατείνειν; cf. *Republic* 527a) and ὅμοιος for similarity in VI 27, 28, the slight imprecision of wording in itself does not necessarily exclude the suggested interpretation.

On the other hand, the mathematical objections are worthy of careful consideration because of their connection with the nature of the hypothesis we are investigating. The major objections are (1) that since whether the given area meets the condition Socrates gives was impossible to find out in Plato's time, the geometer could not give the fixed answer to the original problem, (2) that there is no indication how Socrates reached the condition he gave, and therefore that Meno and readers must have been unable to understand the suggested interpretation, and (3) that there is no distinction between a diorism (διορισμός) and an actual solution, or no clear reference to a diorism at all. It seems to me, however, that these objections are not genuine problems for the standard interpretation but are resolved with a closer analysis of its relationship with another important technique in Greek mathematics: the pair of 'analysis' and 'synthesis' (which I call simply the method of analysis). The method of analysis was almost certainly used to find how to construct a regular pentagon, which I pointed out above is historically attributed to the Pythagoreans. And Diogenes Laertius and Proclus report that Plato explained this method to Leodamas of Thasos, a contemporary mathematician.[10] Even though their reports do not necessarily prove that Plato *invented* the method of analysis,[11] it is not unreasonable to infer from those pieces of evidence that he at least knew this method. Before answering the questions above, then, let us consider the method of analysis and its application to our geometrical problem.

II. The method of analysis and problem reduction

The earliest informative source for the description of the general procedure of the method of analysis appears in the *Treasury of Analysis* written by Pappus of Alexandria, who lived at the end of the third century A. D.[12] According to Pappus, 'analysis' starts by assuming the theorem or problem in question to be true or solved. Then from that assumption one deduces another proposition, from which another is deduced, and so forth; this deductive process continues until one reaches the theorem or problem which is judged to be true or solvable independently of the first assumption and the other propositions deduced from that assumption. Although there is an objection that the process from the first

10 D. L. III. 24 (207. 12–15, Marcovich); Procl., *in Euc.*, 211. 19–23 [Friedlein].

11 Heath 1908, vol. 1 134 n. 1; 1921, vol. 1 291–2.

12 *Collection* VII. 634. 3–636. 14 [Hultsch].

assumption to the end-point is not deductive but intuitive,[13] the deductive view is more favoured by the extant geometrical materials, on which we should place priority not only because we cannot deny numerous later interpolations in Pappus' explanation,[14] but also because Pappus' examples themselves support the deductive view.[15] On the other hand, 'synthesis' starts with the end-point in analysis and deduces from it other propositions in approximately reverse order of analysis; therefore synthesis plays the role of establishing the equivalence between the first assumption and the end-point in analysis, and completes the demonstration of solving the original problem.[16] In sum, the peculiar advantage of the method of analysis lies in its heuristic approach to finding a starting-point of proof and general guidelines for reaching the conclusion from that point.[17]

Bearing such a heuristic aspect in mind, let us have a closer look at Cook Wilson's interpretation (modified), which represents the basic procedure of the method of analysis.[18] Analysis is as follows: let the isosceles triangle BDE equal to the given rectilinear figure S be inscribed in the circle; obviously, the triangle BDE is equal to the rectangle BCDA; since the angle BDF is a right angle (III. 31), BC : CD=DC : CF (VI. 8); therefore, the rectangle BCDA is similar to the rectangle DCFG. And this is the same as the condition Socrates gave in the passage cited above. Synthesis is the following: the similarity of the rectangles BCDA

13 Cornford 1932, 46–8 put forward the intuitive interpretation, but Robinson 1936 rightly defended the traditional deductive view, followed by Cherniss 1951, 414–9. Gulley 1958 insists that Pappus' general exposition can include both intuitive and deductive analyses, but admits that in actual geometrical examples analysis is deductive.

14 Mahoney 1968, 324–6.

15 For example, *Collection* VII. 107, which is taken up as a typical model of analysis by Behboud 1994 and Heath 1908, vol. 1 141–2.

16 As Hintikka and Remes 1974 rightly argue (see particularly ch. 4), the process of analysis does not necessarily consist of linear deductive inferences from the first assumption to the end-point, but of deductive transformations of the complex of given geometrical objects. Since each step of analysis does not necessarily establish equivalence, the process of synthesis is indispensable (cf. Robinson 1936, 471–2; *pace* Mahoney 1968, 326–7). For more detailed accounts, see Behboud 1994, 65–6 and Saito and Sidoli 2010, 587.

17 Hintikka and Remes 1974, 1. Although Knorr 1986, 348–60 doubts that analysis has a heuristic aspect in the case of theorems, we do not need to worry too much about his doubt for our purposes because we are now paying attention to problematic analysis but not theoretic one. In contrast, Netz 2000, 139–45 questions a heuristic role of analysis in general, including problematic analysis, but still admits it to the extent that analysis reveals 'the idea behind the solution', following Hintikka and Remes.

18 Cook Wilson 1903, 233. Cook Wilson himself does not mention the method of analysis although that does not mean that he was not aware of it. Similar reconstructions by analysis can been seen in Knorr 1986, 72; Mahoney 1968, 335; Menn 2002, 212–3.

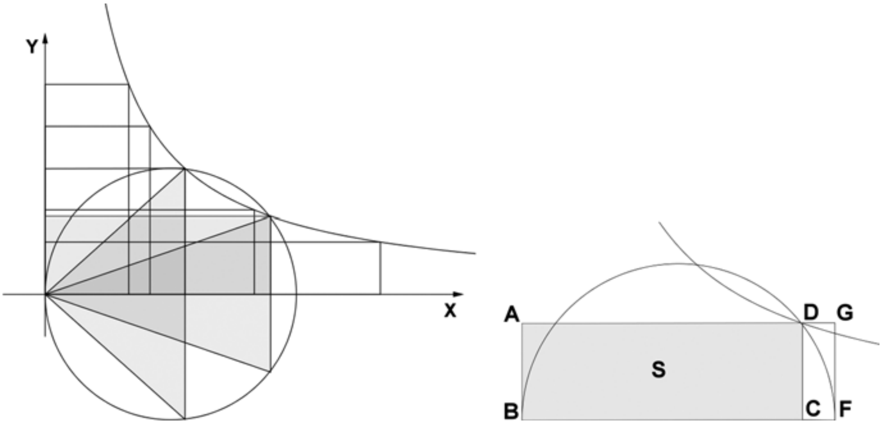

Fig. 2 Intersection point D of the given circle with a hyperbola drawn based on the pointwise construction.

and DCFG entails BC:CD=DC:CF; thus, since the angle BDF is a right angle, the point D is on the circle and the isosceles triangle is inscribed in the circle. In this way the equivalence between the similarity of the two rectangles and the inscription of a triangle in the circle is easily confirmed.

On this interpretation, there is no doubt that a mathematician is supposed to have found or inferred the condition Socrates gave by assuming first that the isosceles triangle in question is inscribed; the intermediate procedure of identifying the condition is carried out not with a flash of intuition but in a reasoned fashion based on the method of analysis. As opposed to Pappus' explanation of analysis, however, an important feature here for our purposes is that the goal of analysis or the construction of the similar rectangles BCDA and DCFG was not possible at the time of the *Meno*. For in order to construct the rectangle BCDA which is both equal to the area S and similar to the rectangle DCFG it is indispensable to be able to draw a hyperbola, because the upper right point (D) of the rectangle BCDA equal to the area S draws a hyperbola (equivalent to XY=S) and the point D of the rectangle BCDA similar to the rectangle DCFG draws a semicircle BF (see Fig. 2). The problem specified here is regarded as equivalent to the problem of finding (an) intersection point(s) of a hyperbola and a given circle. On the other hand, Menaechmus is said to be the first to have discovered the property of a hyperbola as well as parabolas,[19] although whether the rudi-

19 See Eratosthenes' letter to Ptolemy III preserved in Eutocius' commentary on Archimedes' *On Sphere and Cylinder* [Heiberg, vol. 3 96. 17–8] and Procl., *in Euc.*, 111. 20–3 [Friedlein]. Knorr 1986, 62 notices that 'Menaechmean triads' refer to a hyperbola and two parabolas, which were mentary theory of conic curves was available to him is disputable.[20] It is reported that he was a pupil of Eudoxus, who lived about 390–340 B. C., and was associated with Plato,[21] so the time of his activity is presumed to be around the mid-4th century B. C. It is almost certain, therefore, that at the time of the *Meno*, which is likely to have been written around 385 B. C.,[22] Greek geometers did not identify the existence of such curves.

It emerges from this historical fact that Socrates here does not give us the proof of the inscription of the triangle in the circle, but only a different problem equivalent to the original question. We may say, in other words, that he 'reduces' the original question to the other, whether the rectangle BCDA equal to the area S can be constructed on the diameter BF in such a way that it becomes similar to the rectangle DCFG; if the given area meets the condition, the inscription of a triangle into the given circle is possible, but if not, not. This is called the method of 'reduction'.

Problem reduction is one of the significant results of the method of analysis because equivalence established by a pair of analysis and synthesis enables one problem to be examined as another.[23] When they confronted an unknown problem, Greek mathematicians tried to reduce it to another problem they could already solve. Problem reduction was often made from a particular problem to a more general one, to which the method of analysis made a great contribution, but, more importantly, in the early stages of Greek geometry such a reduced problem lacked any solution.[24] One of the most famous intractable problems involving it is the problem of doubling a cube. It is reported that the problem of doubling a cube was reduced to that of finding two mean proportionals between two given lines by Hippocrates of Chios, who worked in Athens in the second half of the 5th century B. C.[25] Hippocrates presumably carried out this problem

used to solve the problem of doubling a cube, rather than the three forms of conic section: parabola, hyperbola and ellipse.

20 While Heath 1921, vol. 1 251–5 and vol. 2 110–6 explains his achievement by means of conic sections, Knorr 1986, 61–6 suggests 'the pointwise construction', which is a slight development of application of areas. To take the construction of a hyperbola for example, which is essential to solving the *Meno* problem, the coordinates (X, Y) of a hyperbola (XY= the given area S) can be attained by repeatedly applying to the diameter of the circle the area S as a rectangle whose base (X) increases from zero little by little (see Fig. 2).

21 Procl., *in Euc.*, 67. 9–10 [Friedlein]. For the date of Eudoxus' life, see Lasserre 1966, 137–9.

22 Bluck 1961, 108–20.

23 Knorr 1986, 71–3; Menn 2002, 203 and 209–15; Wolfsdorf 2008, 56–7.

24 Cf. Mahoney 1968, 331–4.

25 See Eratosthenes' letter in Eutocius [Heiberg, vol. 3 88] and Procl., *in Euc.*, 212. 24–213. 11 [Friedlein].

reduction by the method of analysis.[26] After his discovery of the equivalence between them, the duplication of a cube was studied as the problem of constructing two mean proportionals, and Eutocius transmits to us many solutions to the latter, including that of Menaechmus, which employs a hyperbola and two parabolas.[27] With the help of his method, as we saw above, geometers would have been able to solve the reduced problem in the *Meno* for the first time.[28]

The observation so far gives us a significant insight into the geometrical example in the *Meno*: the problem to which Socrates reduced the original problem was impossible to solve at that time, but revealed a mathematically more important and basic problem. At the time of the composition of the *Meno*, the duplication of a cube was still a live problem. Although no other source refers to the *Meno* problem of inscribing a triangle equal to a given area in a circle, it would not be unreasonable to suppose that the reduced form of those problems considered together encouraged mathematicians at that time to pay more and more attention to constructions beyond the plain method of ruler and compass; that would have paved the way for Menaechmus' discovery of conic curves. We may conclude from this, therefore, that a notable product of the problem reduction in the *Meno* was the revelation of a more basic problem hidden in the particular problem.

III. The nature of the geometrical hypothesis

What then did Socrates posit as a hypothesis? And what was its nature? The historical analysis so far should most naturally lead us to the conclusion that the geometrical hypothesis Socrates posited in order to consider the original problem is a tentative answer to the question whether or not it is possible to

26 Saito 1995 offers the meticulous observation that Archimedes' reduction of the problem of finding a sphere equal to a given cone or cylinder in *On Sphere and Cylinder* II. 1 is a reproduction of Hippocrates' reduction of doubling a cube. Archimedes there uses the methods of both analysis and application of areas.

27 Eutocius [Heiberg, vol. 3 56–107]. Cf. Heath 1921, vol. 1 244–70; Knorr 1986, 50–66.

28 Wolfsdorf 2008, 50 wrongly says 'the problem (in the *Meno*) is equivalent to that of finding two mean proportionals between two given lengths'. The reduced problem in the *Meno* can be solved by applying Menaechmus' construction of a hyperbola, but is not equivalent to the problem of finding two mean proportionals itself. Although, therefore, Archytas, a contemporary of Plato, solved the latter, that does not mean that the *Meno* problem was also solved by Archytas' method.

construct a rectangle meeting the condition he rationally inferred. It is sometimes said that the main aim of problem reduction is to transform a less tractable problem into a more tractable one,[29] but this is, strictly speaking, not accurate, as far as the *Meno* case is concerned.[30] Since no solution was available to contemporary mathematicians, it is unlikely that Socrates aimed to solve the reduced problem; even though the reduced problem is objectively closer to solution from our viewpoint, therefore, the tractability of the reduced problem should be irrelevant. Rather, the problem reduction in the *Meno* can be regarded as constructive only in the sense that it helped Socrates to find a tentative starting-point or hypothesis of the argument by specifying a more basic feature of the mathematical problem.

Now is the time to answer the mathematical questions we postponed. The first objection was that the reduced problem was incapable of solution by the methods available to mathematicians at the time of the *Meno*. It is certain, as we have discussed, that constructing a rectangle meeting the given condition was impossible because drawing a hyperbola is indispensable. However, why on earth is its impossibility problematic in the first place? I would like to argue on the contrary that that very impossibility is Socrates' point here. What should be remembered is that the reason for Socrates' introduction of the method of hypothesis was Meno's requirement for Socrates to consider whether virtue is teachable before knowing what virtue is. Throughout the dialogue Socrates constantly stands his ground, that to answer any ποῖον question one needs to know the answer to the τί question first (71b3–4, 86d8–e1, 100b4–6). Since the τί question has not been answered, therefore, it is no wonder that the reduced problem revealed here through the method of analysis was unsettled as yet and that Socrates introduced the method of tackling the original problem by positing such an unstable hypothesis as an intuitive and tentative answer to the reduced problem.[31]

After formalising the reduced problem, Socrates says 'by making a hypothesis (ὑποθέμενος) I am willing to tell you the result regarding its inscription in the circle, namely whether it is possible or not (87a7–b2)'.[32] I take it that by

29 Ebrey 2013, 92; Lloyd 1992, 173; Menn 2002, 212; Wolfsdorf 2008, 56–7. But Menn points out the aspect of reducing to a more basic problem as well, for which I am arguing.

30 To take another example, Eratosthenes says that the duplication of a cube was converted into no smaller puzzle (Eutocius [Heiberg, vol. 3 88. 22–3]).

31 Menn 2002, 211 and 216 argues that the hypothetical investigation here means the reduction of the ποῖον question to the τί question with the hope of answering the latter in the end. The possibility that their investigation goes beyond the area of ποῖον question is never suggested in the hypothetical passage in the *Meno*.

32 Ebrey 2013, 94 maintains that 'whether it is impossible or not' is not an indirect question, and that 'it' means 'for the given area to meet the specified condition' (P), not 'the inscription 'making a hypothesis' Socrates means assuming temporarily either that the construction of a required rectangle can be achieved or that it cannot. One thing to notice is that this problem reduction certainly has some merits as well in terms of answering the original question. For, even if a hyperbola could not be drawn at that time, considering the problem from the point of view of applying the given rectilinear figure S to a segment of diameter BF allows one to get a rough idea of where the upper right point (D) of an applied rectangle BCDA could intersect with the given circle, or of the impossibility of finding such an intersection point because the given area is too large. Therefore, mathematicians could have still answered that the inscription in question, or finding the intersection point(s) of an applied rectangle with the given circle, is possible or not, even though such an intuitive answer could not be rigorously proved. As I have emphasised, however, Socrates' intention lies in a mathematician's postponing the problem which was impossible to solve at that moment. The gist of his introduction of the method of hypothesis is to give to the original problem an answer which is temporary and unstable. In this sense the method of hypothesis contains such a provisional aspect.

Let us move on to the second objection: that Plato does not indicate or justify how the original question is reduced to another, so that Meno and readers could not understand the process of reducing the problem or the content of the reduced problem. It is true that all he did in the text was to bring forward the condition without any argument for it, but there is an ample reason in the particular case of the *Meno* why Socrates omitted to explain the actual process of the problem reduction: its intermediate steps are straightforward enough.

After assuming that the isosceles triangle equal to the area is inscribed, as we discussed above, the propositions Socrates employed were only two, that the angle in a semicircle is right (III. 31) and that if in a right-angled triangle a perpendicular be drawn from the right angle to the base, the triangles adjoining the perpendicular are similar (both to the whole and) to one another (VI. 8); both of them were famous and important theorems in Greek mathematics at that time. When it comes to proposition III. 31, Diogenes Laertius reports that Thales

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of the area as a triangle in the given circle' (T); he takes Socrates as saying 'whether P is impossible or not, I am willing to tell you the result concerning T', namely '*if* P is true, T is true, and *if* P is not true, T is not true'. However, I do not think that this can be the right reading. For just after that passage Socrates says, 'So *too* regarding virtue (καὶ περὶ ἀρετῆς), (…) let us consider by making a hypothesis whether or not virtue is teachable' (87b2–4), clearly treating those two passages as parallel; in the case of virtue, 'whether or not …' is difficult not to read as an indirect question, and what is at stake in the question is not 'whether or not virtue is knowledge' (the reduced problem) but 'whether or not virtue is teachable' (the original problem).

was the first to inscribe the right-angled triangle into a circle.[33] Even if the attribution of this achievement to Thales lacks credibility, the existence of such a report supports the view that proposition III. 31 was a long-standing theorem. On the other hand, proposition VI. 8 was indispensable to constructing a mean proportional between two given lines. And highly advanced forms of this technique were used by Archytas, who was a contemporary of Plato, and by someone who was presumably in the Academy but wrongly identified with Plato, for solving the problem of doubling a cube, which was reduced, as we saw above, to the problem of constructing two mean proportionals between two given lines.[34] Even if the solution attributed to Plato was dated considerably later than the time of the *Meno*, it is notable for our purposes that Archytas already treated proposition VI. 8 as self-evident in his elaborate application of that theorem. Therefore, Archytas' treatment of VI. 8 strengthens the view that the original problem in the *Meno* was expected to be easily reduced to the problem in the way which I have argued for. We may say, therefore, that it is quite natural to believe that those who had learned even a little geometry were able to anticipate how the problem reduction was carried out.[35]

However, people might not be able to dispel their worry completely yet, because even if the intermediate steps of the problem reduction were consequences natural enough to be expected, they could still object that it must have been difficult for Meno and readers to recognise what kind of mathematical problem Socrates meant in the first place by his simple remark: could they understand that Socrates referred to 'applying to the diameter of the given circle a rectangle which enables an isosceles triangle equal to the given area to be inscribed in the given circle'? To this objection I would like to answer that Socrates illustrated this geometrical example by drawing actual figures and pointing at them, just as he showed the first geometrical example to the slave boy earlier.[36] When he introduced the problem, he repeatedly used demonstrative pronouns like '*this* circle' (τόνδε τὸν κύκλον) and '*this* area' (τόδε τὸ χωρίον) in 86e6, which make more sense when we suppose that Socrates was indicating actual figures in front of them. Although it is less certain whether the wording in the reduced condition also presupposed an actual diagram, the frequent use of pronouns in it (τοῦτο τὸ χωρίον, τοιοῦτον οἷον, αὐτοῦ, τοιούτῳ χωρίῳ οἷον, 87a3–5) seems to support its presence, which would have helped Socrates' ca-

33 D. L. I. 24 (18. 9–10, Marcovich).

34 See Heath 1921, vol. 1 246–9 and 255–8; Knorr 1986, 50–2 and 57–61.

35 My thanks to Ken Saito for the point in this paragraph.

36 I do not think that the diagram Socrates drew for the slave boy has to be reused for the one in question. Cf. Meyers 1988, 178; Sharples 1985, 160.

sual use of words to convey the accurate meaning. Accordingly, there would be fewer grounds for believing that Meno did not understand what was going on here. Then, how about Plato's contemporary readers and us? Presumably, Plato might not have wished all the details to be understood, but might have been satisfied with a successful conveyance of only the main point of the procedure: reducing the original problem to some more general and basic problem which remained unresolved at that time, but from which he wanted to start an argument by positing a hypothesis as a tentative answer to it. If the inscription of a triangle equal to a given area in a given circle was widely recognised as a baffling problem just as the duplication of a cube was, then it would not have been impossible for Plato to have made himself understood to the general public as well as to someone like Meno. Therefore, we need not conclude that Plato was deliberately obscure here.[37]

Finally, why did Plato not state a diorism (διορισμός)? A diorism is a special way in Greek mathematics to specify a general insight into the circumstances of the solutions to a problem, such as their limitations, number and arrangement;[38] it is closely related to the method of analysis and usually placed between analysis and synthesis. This is because since analysis assumes the existence of some actual solution, that assumed solution necessarily falls within the limits of the possible solutions, but in synthesis a mathematician needs, first of all, to consider when actual solutions are possible, how many solutions a problem has, and in what arrangement those solutions hold; based on the different cases clarified by a diorism, synthesis proceeds separately.[39] Therefore, a diorism is made explicit only when a synthetic proof is about to be formulated. To take the *Meno* case for example, when a hyperbola intersects the given circle at one point (when the given area is equal to the equilateral triangle), only one solution is possible; when it intersects at two points (when the given area is smaller than the equilateral triangle), two solutions are possible; when it does not intersect (when the given area is greater than the equilateral triangle), no

37 The scholars who regard Socrates' conciseness as intentional obscurity are Heath 1921, vol. 1 302; Klein 1965, 206–7; Lloyd 1992, 178–82; Menn 2002, 215.

38 In contrast with this strict sense of a diorism, Proclus, *in Euc*., 203. 9–10 [Friedlein] seems to use the word διορισμός in a looser sense which, according to Heath 1908, vol. 1 130, means 'a closer definition or description of the object aimed at, by means of the concrete lines or figures set out in the ἔκθεσις instead of the general terms used in the enunciation'.

39 For a detailed explanation of the role of a diorism and its relationship to the method of analysis, with consideration of ample examples from Archimedes, Apollonius and Pappus, see Saito and Sidoli 2010, 588–612. Procl., *in Euc*., 202. 1–5 [Friedlein] broadly stipulates the strict sense of diorism. Cf. Heath 1908, vol. 1 130–1; 1921, vol. 1 319–20.

solution is possible. And based on these distinctions, actual constructions are separately made and proved.

Despite such specific roles of a diorism in a synthetic proof, many scholars suppose that Socrates is trying to speak of a diorism here,[40] and some of them complain that he nevertheless does not give a clear diorism and further confuses it with problem reduction. It seems to me, however, that Socrates' silence on a diorism does not lead to any critical counterargument against my suggestion in accordance with Cook Wilson's interpretation that he deliberately introduced a geometrical problem which was reduced to a problem unsolvable at that time. For since the full contents of a diorism are revealed in order to formulate synthesis, it is only natural that Socrates had no intention of discussing a diorism, because it was impossible in the first place to give a synthetic demonstration of the reduced problem with methods available to contemporary mathematicians. His aim, as I have argued, lay only in postponing its treatment by positing a hypothesis.

It is true that even though such a complete analysis of a diorism could not be performed by discussing whether the hyperbola intersects the given circle, one might object that Plato was still able to show a simpler form of diorism before a synthetic proof was found, and that Cook Wilson's interpretation fails to indicate even that type of diorism. For, as many point out, if Plato was trying to lay down just the condition for the inscription in question being possible, as many point out, he could have done so by demonstrating, in a much easier way, that the maximum area which can be inscribed in the given circle as a triangle is an equilateral triangle inscribable in the circle;[41] if the given area is smaller than that, the inscription is possible, but if not, not. However, that fact surely rather strengthens the view that Plato did not attempt to specify just the possibility of solution but was concerned with the actual inscription.[42] That So-

40 Bedu-Addo 1984, 6 n. 23; Cherniss 1951, 419; Gaiser 1964, 264; Gulley 1958, 7 n. 1; Heath 1921, vol. 1 303; Knorr 1986, 73–4; Lloyd 1992, 173, 177–8 and 180.

41 Knorr 1986, 73 and 92–4 n. 58. He illustrates there that this fact can be proved only via an elementary method of construction. Despite his belief that Plato's effort in the text must have been directed towards constructing the inscribed triangle in question, he concludes that Socrates discussed such an advanced construction problem in an attempt to specify diorism because of his oversight of its elementary methods. But his conclusion is hardly convincing.

42 Menn 2002, 211–2 and 214 rightly discusses the relationship between a diorism and problem reduction although I cannot agree with his emphasis on a positive side of the problem reduction with the comment, 'he (Plato) would have seen the problem as part of a promising program for finding διορισμοί of any given construction-problem and for solving any problem when it can be solved'.

crates undoubtedly refers here to application of areas,[43] which was a relatively advanced technique, can be more reasonably explained by supposing that his reference was directed at the explicit construction of the triangle in question. If his aim was not to show a diorism, the objection to Cook Wilson's interpretation simply disappears. To be sure, the form of Socrates' question 'whether an area is inscribable as a triangle' appears to suggest his consciousness of the possibility of solution. At the same time, however, the question can be taken as asking whether one can show how to construct the triangle in question.

In addition, we should not overlook Proclus' report that Leon, who was presumably much younger than Plato, invented diorisms (διορισμοὺς εὑρεῖν).[44] If Proclus' reports are correct, Plato could not have had a diorism in mind when he wrote the *Meno*.[45] Even if he had recognised it, the essential role of a diorism, as we noticed above, is to give a guideline for a synthetic proof or solution. The ascertained fact is, however, that since mathematicians then did not know how to draw a hyperbola, they could not solve the inscription-problem even when the maximum area inscribable into the circle as a triangle was recognised in some way as equal to the equilateral triangle, which Plato would surely have known. Therefore, Plato's aim in using the method of hypothesis is far from giving a diorism or a solution, but rather still lies in starting the argument with a tentative hypothesis which enables one to leave aside an unresolved issue.

IV. Conclusion

Before winding up this paper, I need to deal with a potential question about my interpretation of the method of hypothesis. I have argued that based on analytic reduction Plato would have posited a tentative answer – whether positive or negative – to the solvability of the reduced problem (the construction of a rectangle meeting the condition) as a hypothesis. Against this proposal, however,

43 Most commentators agree on this irrespective of whether or not they follow Cook Wilson's interpretation.

44 Procl., *in Euc.*, 66. 18–22 [Friedlein]. Leon is called a pupil of Neoclides, who was younger than Leodamas. According to Proclus, *in Euc.*, 211. 19–23, Plato taught Leodamas the method of analysis. These reports probably show that Plato was older than Leon.

45 Heath 1921, vol. 1 303 suggests 'Leon may have been the first to introduce the term or to recognize formally the essential part played by διορισμοί in geometry', followed by Cherniss 1951, 419 n. 59, who says that Proclus' report 'may mean only that in his compilation of "the elements" he formulated many new διορισμοί'. See also Bluck 1961, 79–80. Their remarks are clearly based on the assumption that Plato actually introduced a diorism here in the *Meno*. However, we do not need to impose an error on Proclus.

there might be an objection because, whereas the procedure of 'analysis' is said to hypothesise the original problem to be solved first, my explanation hypothesises the solvability of the reduced problem, not that of the original one.[46] A natural response to this objection would be that having tacitly hypothesised the answer to the original problem first, from that Plato would have deduced the hypothesis we suppose.[47]

In this regard there is an important fact which seems to have escaped the objectors' notice. For the objection is mainly based on Pappus' general exposition of the method of analysis, because in actual examples of 'analysis' mathematicians did not use the word 'hypothesis' for a starting point of 'analysis'. What should be remembered here is that Pappus' explanation presupposes that 'analysis' finally reaches a proposition which can be judged to be true independently of the first assumption, and that 'synthesis' starts with a stable point and completes the rigorous demonstration of the original problem from it; he does not handle the other important case of the method of analysis, where 'analysis' ends with another unknown problem and 'synthesis' accomplishes a successful reduction to that unknown problem. I have argued in this paper that the duplication of a cube was a representative example of such problem reductions, and that the *Meno* problem can be regarded as analogous to it. In contrast to Pappus' description, therefore, it is no wonder here that the end point of 'analysis' or the starting point of 'synthesis' was called a 'hypothesis'.

In the *Meno* Socrates reduced the original problem to the one which was incapable of solution at that time. In this case, as we have discussed, it is natural to suppose that a tentative answer to that reduced problem was posited as a hypothesis. Although the eventual solution of the construction problem could not help being guided by intuition, the intermediate process of reaching the hypothesis was carried out through rational arguments based on the method of analysis. In this sense, the geometrical hypothesis Socrates posited in the *Meno* has an unstable and reasoned nature.[48]

46 Cf. Robinson 1953, 121.

47 Bluck 1961, 82–3.

48 My thanks to David Sedley, Nicholas Denyer, Geoffrey Lloyd, Ken Saito, Reviel Netz, David Ebrey and Matthew Duncombe for valuable written comments, and to audiences at the 8th London Ancient Science Conference and the international conference 'Revolutions and Continuity in Greek Mathematics' for helpful discussion.

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