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Visual foundations of Euclidean geometry

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ABSTRACT

Geometry defines entities that can be physically realized in space, and our knowledge of abstract geometry may therefore stem from our representations of the physical world. Here, we focus on Euclidean geometry, the geometry historically regarded as “natural”. We examine whether humans possess representations describing visual forms in the same way as Euclidean geometry – i.e., in terms of their shape and size. One hundred and twelve participants from the U.S. (age 3–34 years), and 25 participants from the Amazon (age 5–67 years) were asked to locate geometric deviants in panels of 6 forms of variable orientation. Participants of all ages and from both cultures detected deviant forms defined in terms of shape or size, while only U.S. adults drew distinctions between mirror images (i.e. forms differing in “sense”). Moreover, irrelevant variations of sense did not disrupt the detection of a shape or size deviant, while irrelevant variations of shape or size did. At all ages and in both cultures, participants thus retained the same properties as Euclidean geometry in their analysis of visual forms, even in the absence of formal instruction in geometry. These findings show that representations of planar visual forms provide core intuitions on which humans’ knowledge in Euclidean geometry could possibly be grounded.

1. Introduction

Most humans would agree that there is only one straight line through two given points (Izard, Pica, Spelke, & Dehaene, 2011). Where does this knowledge come from? While geometric laws can be stated in a purely symbolic form, as we just did, the objects of geometry can be physically realized in space: one can draw a triangle, measure the sides and the angles of a figure, or picture the effect of various geometric transformations. Accordingly, it is possible that our geometric knowledge stems, at least in part, from our ability to represent space (Calero, Shalom, Spelke, & Sigman, 2019; De Cruz, 2007; Gallistel, 1990; Hart et al., 2018; Hatfield, 1998; Hohol, 2019; Newcombe & Uttal, 2006; Shepard, 2001). For example, perhaps our representations of space are constrained in such a way that we cannot envision two different straight lines through two points – much like our representations of the physical world prevent us from seeing two solid entities occupying the same place at the same time (Spelke, 1994). More generally, perhaps our mind is prepared to analyze our visual environment in terms of geometric entities, e.g. points, lines, angles – much like it is prepared to analyze our physical environment in terms of solid objects.

This paper examines the possibility that representations of space, as they are recruited to analyze our environment, play a

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foundational role for geometric knowledge. This foundational role may take several forms. In a strong sense, it is possible that our experience of space provides justification for abstract geometric knowledge, or in other words, that we sometimes rely on perception[1] to convince ourselves that abstract geometric statements are true (e.g. Giaquinto, 2005; Hart et al., 2018; relatedly, for a discussion of the role of diagrams in Euclid's geometry, see Mumma, 2010). It is also possible that perception contributes to shape our theories of geometry in a weaker sense, by supporting the emergence and/or transmission of forms of geometry that are compatible with our intuitions about space (for a related argument in the domain of language, see Strickland, 2017). While the causal mechanisms implied by these hypotheses are different, they both rely on a common presupposition: that geometric abilities, as they partake in the analysis of our environment, can precede formal geometric knowledge. This is the question assessed in the present experiments: we examine whether people can analyze forms in terms of their geometric properties, even in the absence of formal education in geometry.

Importantly, not all properties of space are relevant in geometry. Here, we focus on Euclidean geometry, which has historically been described as the "natural" form of geometry (Descartes, 1637 [2001]; Kant, 1781 [2005]; Plato, ca. 380BCE; Poincaré, 1902). In Euclidean geometry, figures are analyzed in terms of their size and shape, while information about their position, orientation, and sense is disregarded[2] (see Fig. 1 for an illustration of the different geometric terms used in this paper). In other words, if two students draw figures of the same size and shape, their drawings are equivalent from the point of view of Euclidean geometry, even if the two drawings are mirror images of each other and occupy different positions on the page. Other geometries operate with different definitions: from the point of view of projective geometry, for example, the shadows projected by different light bulbs make various instances of the same form, even though these shadows can differ vastly in size and shape.

To probe the Euclidean abilities of our participants, we thus tested whether they are able to analyze forms according to the definition of Euclidean geometry: in terms of their size and shape, disregarding variations of position, orientation, and sense. Finding that people can do so, even in the absence of formal education in geometry, would indicate that visual perception can serve as a foundation for geometric knowledge. In contrast, we may find that children start displaying Euclidean judgments only after being taught geometry at school, or more generally that analyses of visual forms fall short of implementing Euclidean geometry, even in mathematically sophisticated people. If such was the case, our findings would imply that Euclidean geometry cannot be fully grounded on perception; yet humans may still be equipped with start-up tools (Piazza, 2010) allowing access to a restricted set of Euclidean properties.

While humans possess a variety of cognitive systems encoding information about space (e.g. systems planning movements and gestures: Berthoz, 2000; Calero et al., 2019; or supporting navigation in large spaces: Cheng & Gallistel, 1984; Gallistel, 1990), here we chose to focus on the perception of planar visual forms, for two reasons. Planar forms have been used to support abstract geometric reasoning in a tradition dating back from ancient Greece; and even today, they are the primary spatial entities that serve in the explicit teaching of geometry in school. Across generations, these practices should favor the development and transmission of theories of geometry that are compatible with our perception of visual planar forms. Consequently, focusing on planar visual shapes in the present study should increase our chances to detect Euclidean abilities, compared to other spatial displays. Moreover, if explicit teaching of geometry were to have an impact on spatial representations (as happens in the case of numeracy and approximate numerosity representations, Elliott, Feigenson, Halberda, & Libertus, 2019; Nys, Ventura, Fernandes, Querido, & Leybaert, 2013; Piazza, Pica, Izard, Spelke, & Dehaene, 2013), these effects would likely be most detectable when children analyze planar forms similar to the kind of material they encounter in geometry classes. Focusing on planar visual forms should thus also increase our chance to detect developmental changes as they relate to formal education in geometry.

In what follows, we first review the literature on visual form perception from infants to adults, with a focus on three properties diagnostic of Euclidean geometry: size, shape, and sense. Specifically, if people rely on Euclidean representations, they should respond to rotated forms according to their size and shape, while disregarding variations in sense. Next, we present the results of two experiments probing people's judgments about rotated visual forms varying in size, shape and/or sense, in children and adults from the U.S. (Experiment 1) and from an Amazonian group, the Mundurucu (Experiment 2).

1.1. *Preamble: Some representations of visual forms do NOT align with Euclidean geometry*

Before we embark on our search for Euclidean representations, it is important to remark that 2-dimensional forms and pictures clearly elicit representations that are *not* in line with Euclidean geometry. For instance, people's judgements on visual forms are often affected by changes in orientation (Arguin & Leek, 2003; Clements, Swaminathan, Hannibal, & Sarama, 1999; Jolicoeur, 1985;

[1] In this paper, we use the term "perception" in a broad sense, referring to the sum of all the cognitive processes and representations that are recruited to analyze the perceptual input. Thus, our usage encompasses both the fast, automatic, and encapsulated processes that are classically thought of as "perception", as well as conceptual representations exerting a top-down influence on people's judgments about forms. Importantly, both types of cognitive mechanisms are relevant to understanding the origins of geometric knowledge. In particular, if a geometric property was easily detectable but usually dismissed as irrelevant to categorize forms, it would be unlikely for this property to be retained in a theory formalizing our intuitions about forms.

[2] In our earlier writings, we have characterized Euclidean geometry in terms of shape only, size being irrelevant (Izard, Pica, Dehaene, Hinchey, & Spelke, 2011). This characterization was informed by Euclid's axioms (see discussion). Here, we adopt a different characterization and follow the terminology that prevails in the fields of mathematics and psychology in defining "Euclidean geometry" based on the transformations preserving size as well as shape (the "Euclidean group"; see e.g. Cheng & Gallistel, 1984; Shepard, 2001). We thank an anonymous Reviewer for noting this discrepancy.

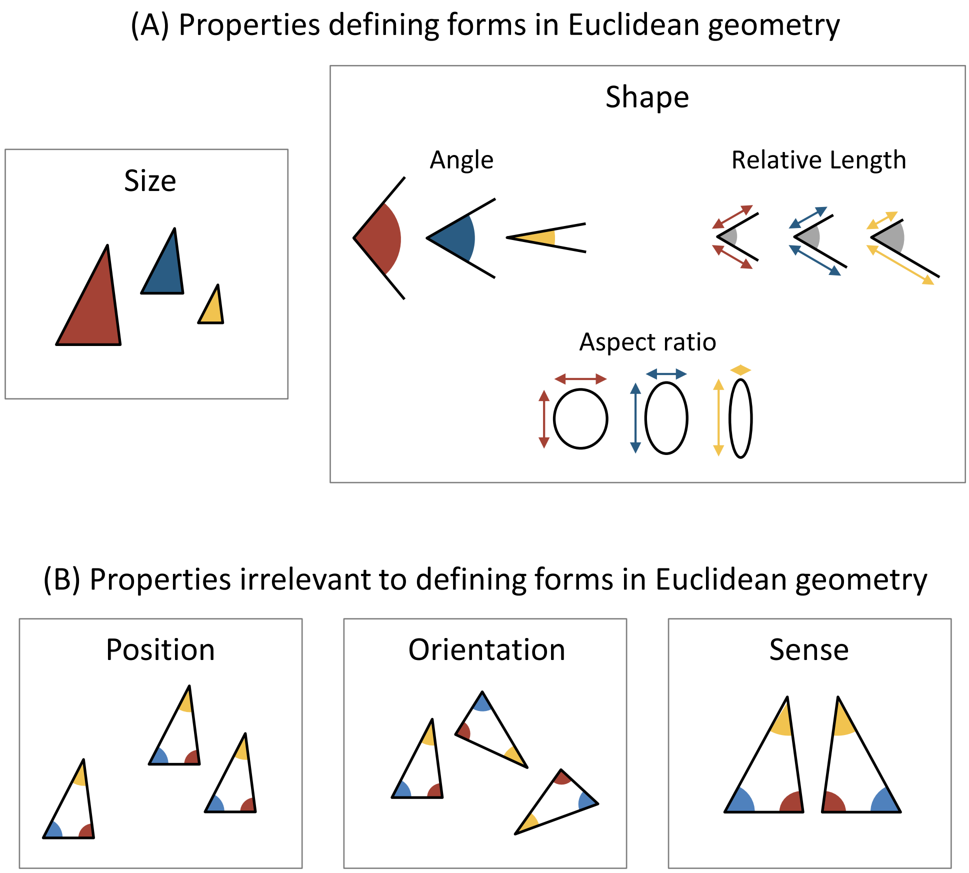

Fig. 1. Illustration of the geometric properties discussed in this paper. (A) In Euclidean geometry, forms are defined by their size and shape. In the left vignette, the three triangles differ in size: the yellow triangle is smaller, the red triangle is larger. All the triangles are however equated in shape. Forms varying in shape (right vignettes) may have different internal angles, and/or they may present with different proportions. Colors are used to highlight differences in angle or length. Relative length refers to a ratio between two lengths (here the lengths of the two branches). The two properties of angle and relative length are deeply interconnected: in a closed figure, it is not possible to change angles without also changing relative lengths, nor is it possible to change relative lengths without also changing angles. The ellipses on the bottom vignette vary in aspect ratio: the ratio between the lengths along their two main axes. (B) In Euclidean geometry, the three properties of position, orientation and sense are irrelevant to defining forms. In this part of the figure, colors are used to highlight the similarity of the triangles, and to help readers identify each corner. Sense is the property that differs between two mirror images: in the rightmost vignette, the red-blue-yellow corners run in the clockwise direction for the triangle on the left, and in the counterclockwise direction for the triangle on the right.

Kalénine, Cheam, Izard, & Gentaz, 2013; Leek, 1998; Mach, 1914; Rock, 1973; Tarr, 1995; Tarr & Pinker, 1989), in sharp contrast to the fundamental Euclidean assumption of orientation invariance. As one striking example, people are willing to consider that a square and a diamond are two different forms, even though these two figures are related by a 45° rotation (Mach, 1914). On the other end, in some tasks observers' judgments can be driven by properties that abstract over the Euclidean properties of size and shape. When identifying objects in pictures, for example, people rely largely on so-called "non-accidental" properties that, unlike size or shape, are preserved across changes in viewpoints: cues such as junctions, collinearity, parallelism, or curvilinearity (Amir, Biederman, & Hayworth, 2011; Amir, Biederman, Herald, Shah, & Mintz, 2014; Biederman, 1987; Biederman, Yue, & Davidoff, 2009; Gibson, Lazareva, Gosselin, Schyns, & Wasserman, 2007; Lazareva, Wasserman, & Biederman, 2008; Szwed, Cohen, Qiao, & Dehaene, 2009; Todd, Chen, & Norman, 1998), and more generally, structural properties describing relations between object parts (Ayzenberg & Lourenco, 2019; Hoffman & Richards, 1984; Hummel & Biederman, 1992; Marr & Nishihara, 1978). What's more, when present, these abstract cues tend to take priority over variations in size and shape in people's judgments about forms (Amir et al., 2014; Biederman et al., 2009; Lazareva et al., 2008; Lowet, Firestone, & Scholl, 2018).

At first view, these findings could be taken to indicate that people do not represent forms according to their Euclidean properties. However, most of the studies reviewed so far used complex images as stimuli, raising the possibility that the abstract cues displayed in these images weighted heavily on people's responses and masked the existence of Euclidean representations. In what follows, we will thus focus our review of the literature on studies employing simple 2-dimensional forms, where high-level "non-accidental" and structural cues were not available. We will further restrict our focus on studies presenting rotated form, lest participants may have solved the tasks using orientation-specific representations that do not implement Euclidean geometry. A first section reviews studies examining which types of variations people are able to *detect* under rotation: when relying on Euclidean representations, people should be able detect variations in size and shape, but not in sense. We then turn to studies examining which types of variations people are able to *disregard*: relying on Euclidean representations would enable people to easily disregard sense, but not size or shape.

1.2. Sensitivity to size, shape, and sense under planar rotation

1.2.1. Detecting variations of size, shape and sense

Decades of research attest that people are sensitive to size and shape in rotated forms, in line with the existence of Euclidean representations. Educated adults reach high levels of precision when discriminating between lines of different lengths (75% accuracy for a 5% difference in length with lines spanning all planar orientations: Norman, Todd, Perotti, & Tittle, 1996, experiment 1), figures of different aspect ratios (75% accuracy for a difference in aspect ratio of 10% and less, on squares and ellipses of variable area: Regan & Hamstra, 1992), and angles of different apertures (53% accuracy for angle differences of 7° in a 6-alternative-forced-choice task, on stimuli varying in size and orientation: Dillon, Duyck, Dehaene, Amalric, & Izard, 2019). Even infants possess the ability to perceive size and/or shape in rotated figures (Cohen & Younger, 1984; Dillon, Izard, & Spelke, 2020; Huttenlocher, Duffy, & Levine, 2002; Lindskog, Rogell, Kenward, & Gredebäck, 2019; Lourenco & Huttenlocher, 2008; Schwartz, Day, & Cohen, 1979; Slater, Mattock, & Brown, 1990; Slater, Mattock, Brown, & Bremner, 1991). In particular, infants can detect variations of shape, above and beyond size variations. For example, Dillon et al. (2020, Experiment 2) presented 7-month-olds with two streams of L-figures, one presenting both size and shape variations (i.e. the figures differed in the relative length of the two branches, as well as in implied area), and another presenting changes in size only (i.e. figures differed in area but the relative length of the branches was fixed). Infants looked longer at the stream presenting shape variations, thus providing evidence that they had registered the shape changes despite the variations in size[3].

People are also able to detect sense differences in rotated forms, as demonstrated for example by research using mental rotation tasks. For instance, in the original version of the task (Shepard & Metzler, 1971), participants were presented with images of 3-dimensional objects made of blocks. The blocks were piled to make segments, and several segments were assembled to form a succession of 90° turns in various directions (right, left; up, down). Crucially, in each pair of images the objects to be compared were made of the same segments, while one of the turns could go in opposite directions. The images were presented at various viewpoints, and participants were simply asked whether the objects depicted were identical or different: this is a sense discrimination task, in a context where the forms are presented at varying orientations.

Adult participants achieve near-perfect accuracy in this task, both with 3-dimensional forms rotated in depth, as in Shepard and Metzler (1971)'s original study, and, most relevant to our purpose, with 2-dimensional forms rotated in the picture plane (e.g., Cooper, 1975; Cooper & Shepard, 1973; Tarr & Pinker, 1989). Children are able to solve planar mental rotation tasks by the age of 5 years (Ehrlich, Levine, & Goldin-Meadow, 2006; Estes, 1998; Frick, Daum, Walser, & Mast, 2009; Frick, Ferrara, & Newcombe, 2013; Funk, Brugger, & Wilkening, 2005; Kosslyn, Margolis, Barrett, Goldknopf, & Daly, 1990; Marmor, 1975), and they can contrast rotated mirror images at even younger ages, when tested in more ecological conditions (Frick, Hansen, & Newcombe, 2013) or given appropriate training (Krüger, 2018; Marmor, 1977). For example, after two training sessions where they could actively rotate pictures on a touch screen, 3-year-olds proved able to associate novel pictures presented at various orientations with a sense match presented upright (Krüger, 2018). Even infants have proven sensitive to sense distinctions in rotated figures, from the early age of 3 months (Frick & Möhring, 2013; Lauer, Udelson, Jeon, & Lourenco, 2015; Quinn & Liben, 2008; for reviews, see Frick, Möhring, & Newcombe, 2014; Moore & Johnson, 2020). For example, Lauer et al. (2015) found that infants aged 6 to 13 months prefer to look at a stream of rotated L-shapes changing in sense, over a stream where sense is held constant.

The studies reviewed above show that adults, children and even infants are sensitive to variations in all three properties of size, shape and sense in rotated forms, so far failing to reveal a distinction between size/shape vs. sense. However, a particular pattern emerges in the case of sense: response times and error rates are proportional to the difference between the orientations of the two forms to be compared. In order to detect differences in sense, thus, participants first rotate the images in their mind to realign them[4]. This finding argues against the existence of orientation-independent representations of sense – and fits well with the Euclidean property of invariance by sense. Of note, children display the classical orientation effect in their response times already at the age of 5 years (Estes, 1998; Frick, Ferrara, et al., 2013; Frick, Hansen, et al., 2013; Kosslyn et al., 1990; Marmor, 1977).

Few studies have employed the mental rotation paradigm to compare distinctions based on sense with other types of distinctions (Takano, 1989; Tarr & Pinker, 1990; Vanrie, Beatse, Wagemans, Sunaert, & Van Hecke, 2002; Vanrie, Willems, & Wagemans, 2001; Wagemans, Van Gool, Lamote, & Foster, 2000). For example, Vanrie et al. (2002, 2001) contrasted two conditions with 3-dimensional objects made of blocks: one where the distracter and standard objects differed in terms of sense, as in the classical mental rotation paradigm, and one where the variations were introduced at the level of the angles formed by the blocks (introducing informative differences in the distances between segments as well as the angle at which the segments met). Interestingly, and in contrast to the sense condition, participants' performance in this condition did not show the classic effect of orientation difference that is

[3] Infants do not always respond to shape changes in the presence of size variations. For instance, in the same paper, Dillon et al. (2020, Experiment 3) found that infants failed to detect variations in another shape property: angle.

[4] With extensive training, adults can learn to discriminate rotated mirror images without relying on mental rotation (Kaushall & Parsons, 1981); they may use cues such as the position of different figure parts, properties of symmetry in all or parts of the figures, or direction of simulated movement (see Corballis, 1988 for a possible mechanism based on movement simulation).

characteristic of mental rotation[5]. Furthermore, compared to the angle/distance condition, the sense condition recruited an additional network of parietal and frontal areas typical of mental rotation tasks. Although people are able to detect variations in sense or in the Euclidean properties of angle/distance, the psychological mechanisms underlying these abilities thus appear to be different: contrary to sense, angle/distance seem to be perceived independently from orientation, allowing direct comparison of angles and/or distances between rotated forms[6].

In summary, studies employing simple forms indicate that adults, children and infants are sensitive to the properties of size, shape and sense in rotated images. However, sense has a special status in that participants usually apply a process of mental transformation to first align the forms in their mind, before they can compare them. In contrast, they seem to perceive differences in size and shape directly, without relying on mental rotation.

1.2.2. Disregarding variations of size, shape and sense

We now turn to studies addressing our second set of questions: can humans easily disregard variations in size, shape and sense in rotated forms; or do these variations affect the perception of other form properties?

First, many studies suggest that sense can be easily disregarded – and perhaps even fully ignored. Children often make symmetry errors in writing (Cornell, 1985; Dehaene et al., 2009; Terepocki, Kruk, & Willows, 2002) or confuse mirror images (Dessalegn & Landau, 2013; Gibson, Gibson, Pick, & Osser, 1962; Huttenlocher, 1967), as do illiterates (Danziger & Pederson, 1998; Dehaene, 2009; Kolinsky et al., 2011; Serpell, 1971). Moreover, mirror image confusions can also be observed in literate adults, in tasks where participants are not instructed to look for sense distinctions, when the retention delays are large, or under conditions of high executive load (Biederman & Cooper, 1991, 2009; Gregory & McCloskey, 2010).

In contrast to variations of sense, variations of size and shape appear to affect people's representations of forms, even when they are not relevant to the task at hand. For example, the width of a rectangle or an ellipse is perceived to be smaller if the height happens to be smaller as well, and vice versa (Morgan, 2005). Moreover, estimations of angle are systematically distorted by variations in branch length. Hence, angles with longer branches – and a larger enclosed area – are estimated to be larger in aperture (Gibson, Congdon, & Levine, 2015; Wenderoth & Johnson, 1984; Werkhoven & Koenderink, 1993). These biases can be quite pronounced: for example, when comparing angles with sides of 5 vs. 30 mm, adult observers overestimate the 30 mm branch angles by more than 25% (a bias of 5.35° for a reference angle of 20°: Wenderoth & Johnson, 1984). Other studies report that angle perception is also affected by the relative length of angle branches: an angle embedded in an isosceles triangle (two branches of equal length) may be judged as much as 30% larger than the same angle shown in a scalene triangle (branches differing by a 1:3 ratio), independently of the global size of the triangle (Kennedy, Orbach, & Loffler, 2008). In sum, when judging the aperture of angles, variations of size and length proportions cannot be easily disregarded – at least in adults.

1.3. Summary and overview of the experiments

The findings reviewed above partially answer our questions concerning humans' sensitivity to the properties of shape, size and sense in rotated visual forms. First, in a context where orientation varies, sense *cannot* easily be perceived: we can distinguish forms from their mirror images only when they are aligned physically or mentally, and even then we are apt to confuse them. As a consequence, disregarding sense should be easy as it should suffice to refrain from applying mental rotation. Second, perceiving size or shape in rotated figures is easy. These properties also seem hard to disregard, at least in the case of size or relative length.

The studies reviewed so far have several limitations. First, few studies compared judgments on different geometric properties directly, such that there were vast differences in the tasks, stimuli, and performance measures used to characterize the perception of size, shape and sense. Second, development of sensitivity to these properties has not been studied systematically, and in particular, children and adults have rarely been tested with the same displays and tasks (for exceptions, see Kosslyn et al., 1990; Odic, 2018). Yet, large and protracted developmental differences have been observed in tasks such as object recognition and form categorization (e.g. Abecassis, Sera, Yonas, & Schwade, 2001; Augustine, Smith, & Jones, 2011; Jüttner, Petters, Wakui, & Davidoff, 2014; Pereira & Smith, 2009; Satlow & Newcombe, 1998), suggesting that developmental effects may exist for sensitivity to geometry as well. Moreover, most studies on size, shape or sense included only participants from industrialized countries, and it is thus unclear whether their results would generalize to societies where children do not receive formal instruction on geometry, and people live in non-

[5] In their studies, Vanrie et al. (2001, 2002) used images of 3-dimensional objects rotated in depth, not 2-dimensional forms. Given that 3-dimensional rotations imply rotations in the picture plane, it feels safe to assume that the absence of orientation effects evidenced in their studies would extend to planar rotations.

[6] Some studies applied the mental rotation paradigm to the perception of forms of different sizes, and reported so-called 'mental scaling' effects (Besner, 1983; Bundesen & Larsen, 1975; Howard & Kerst, 1978; Larsen, 1985; Larsen & Bundesen, 1978; see also Möhring, Newcombe, & Frick, 2014; Möhring, Newcombe, & Frick, 2016). These studies were initially taken to indicate that to cancel size differences, people apply a process of mental scaling, much like the process of mental rotation involved in Shepard and Metzler (1971)'s task. Unlike rotation costs, scaling costs are not specific to the perception of sense differences: besides sense (Larsen, 1985), the effect was observed when participants compared the aspect ratio of rectangles of different sizes (Sekuler & Nash, 1972), and more generally, when they contrasted random polygons and line drawings that differed along many dimensions (Bundesen & Larsen, 1975). The interpretation of these effects in terms of mental scaling has been disputed, however: rather than mental imagery processes, scaling costs likely reflect the dynamics of attention tuning to stimuli of different sizes (Fiser, Subramaniam, & Biederman, 2001; Larsen & Bundesen, 1978).

carpentered environments. Third, in most cases participants were explicitly instructed to look for variations in size, shape or sense. Thus, while these studies provide evidence that these properties of visual forms *can* be encoded, they do not test whether people would spontaneously register these variations, and whether they would view these properties as relevant to categorize forms. These studies may thus provide a distorted picture of the possible contribution of visual perception to geometric knowledge: properties that are not spontaneously encoded or are viewed as irrelevant to the categorization of forms are unlikely to be retained in a theory formalizing our intuitions of space.

A deviant detection task introduced by Dehaene, Izard, Pica, and Spelke (2006) holds promise to overcome these limitations. In this task, participants are presented with arrays of 6 images. Five images (standard) share a target geometric property, which is absent in the last (deviant) image. No verbal description of the target property is provided; instead, participants are simply instructed to find the image that is "most" or "very different". The task can therefore be administered to young children (Chalmers & Halford, 2003; Hu & Zhang, 2019; Izard & Spelke, 2009) or to adults lacking formal instruction in geometry (Dehaene et al., 2006; Sablé-Meyer et al., 2021; van der Ham, Hamami, & Mumma, 2017). Furthermore, given that the instructions do not direct participants' attention to particular geometric properties, this task can test both whether participants spontaneously detect the difference between the standard and deviant images, and also whether they consider these differences to be relevant to define "different pictures".

In our study, participants searched for deviants amongst simple figures, composed of two line segments that were joined to form an L. The figures varied in position and orientation, as well as in size, shape (manipulated by varying the angle at which the lines met), and/or sense. To probe the existence of Euclidean representations, we tested both which types of variations participants could detect, and which types of variations they could not easily disregard. We administered the same test to children and adult participants from the U.S. (Experiment 1), and to a group of Mundurucu children and adults who had not received formal instruction in geometry (Experiment 2).

2. Experiment 1: Children and adults from the U.S.

Experiment 1 used Dehaene et al. (2006)'s intruder task with simple images that varied in size, shape and/or sense. Each image was composed of two lines that intersected at their endpoints to form an open L figure, and we manipulated the length of the lines, the angle between the two lines, and the respective position of the longer and shorter lines to present variations of size, shape, and sense. In addition, in each trial, the 6 figures differed in orientation and position, such that these two parameters could not be used to isolate a deviant.

In a baseline condition ('baseline trials'), we tested whether participants selected size, shape, and sense deviants in this orientation- and position-invariant context. In an interference condition ('interference trials'), we added irrelevant variations of size, shape or sense to the distractor images. Importantly, just like in the case of orientation or position, these variations did not define a clear deviant. When applying interference of size for example, the six images had different sizes, varying in linear increments. By comparing interference to baseline conditions, we tested whether irrelevant variations of size, shape or sense impaired the detection of geometric deviants.

We made the following predictions. If participants' responses are guided by Euclidean representations, (i) they should select both size and shape deviants easily in baseline trials, and (ii) variations of sense should not interfere with this ability. In addition, (iii) they may fail to select sense deviants, and (iv) variations in size or shape may be difficult to disregard, yielding impaired performance in size and shape interference trials compared to baseline. Predictions (i) and (ii) are sine qua non signatures of Euclidean representations. Whether predictions (iii) and (iv) are confirmed depends on the availability of other representations of visual forms, besides Euclidean representations. Sense deviants will be selected, contra prediction (iii), if participants can rely on representations that retain information about sense through rotations. Similarly, variations of size or shape will have no cost on performance, contra prediction (iv), if participants can rely on representations that abstract over these properties.

The present report of Experiment 1 updates and corrects a preliminary report of this study (Izard & Spelke, 2009, Experiment 3). The sample presented here is larger (reaching predefined sample sizes of at least 40 participants in the group of preschoolers, and at least 20 participants in the other groups) and follows a more rigorous analysis approach (see section 2.1.4.1) – the same that will be used to analyze Mundurucu's responses in Experiment 2.

2.1. Methods

2.1.1. Participants

A total of 112 participants were tested in four age groups: 44 3- to 5-year-olds (23 girls, mean age 3.98 years, range 3.08–5.92 years; hereafter "preschoolers"), 26 6- to 8-year-olds (13 girls, mean age 7.57 years, range 6.32–8.92 years; hereafter "school children"), 22 9- to 12-year-olds (11 girls, mean age 10.51 years, range 9.42–12.79 years; hereafter "preteenagers") and 20 adults (10 women, mean age 24 years, range 18–34 years). The data of six additional participants were excluded from the analyses, because they quit before the end of the experiment (3 preschoolers), because they did not speak English fluently (2 preschoolers), or because a parent reported learning difficulties in mathematics (1 preteenager). All participants resided in the greater Boston area. Children were recruited through mailing and phone by the laboratory, and received a small gift in exchange of their participation. Adults were recruited among students at Harvard Summer School or from the Cambridge community, and were offered $5 or course credit. Written informed consent was obtained from adult participants, or from a legal tutor in the case of children participants. Children assented to participate orally.

Our previous report (Izard & Spelke, 2009) included 87 of these 112 participants: 40 preschoolers, 17 school children, 14 preteenagers, and 16 adults. Note that in this report, one school child had been misplaced in the group of adults.

2.1.2. Displays and task

In each trial, an array of six images was displayed on a laptop screen, and participants were instructed to find the "picture" that was "the most" or "very different". As participants were tested over the course of an extended period and at different lab locations, several laptops were used and they had different screen sizes.

All the figures presented were made of two branches, one shorter than the other, with a fixed ratio of 0.7 between the lengths of the two branches. Within each trial, the orientation of the figure varied across the six images in 60° increments, so as to span evenly 360° of variation. The exact position of the figures within each frame was determined randomly by the program, with the only constraint that the figure should fit in the space allotted. In addition, across different conditions we manipulated the figure size (by varying the length of the branches, yet always keeping a fixed ratio of 0.7 between the length of the two branches), the figure shape (by varying the angle between the two branches), and/or the figure sense (whether the smallest branch was in the clockwise or counter-clockwise direction with respect to the longer branch).

Trials were of two kinds. In *baseline trials*, all the distractor figures were identical, varying in position and orientation only. In *interference trials*, the distractor figures differed in size, shape, or sense (in addition to position and orientation), but not in a way to define a deviant (Fig. 2).

There were three types of *baseline trials*, with deviants defined in terms of size, shape, or sense. In *baseline size* trials, the deviant differed in size from the distractors. Size variations were implemented by changing the length of the two branches, yet always keeping their relative length at a ratio of 0.7. The deviant figure differed in size from the others by a ratio of 1:3 (on a wide 17″ screen for example the longer branch in the standard and deviant figures measured 1.27 vs. 3.81 cm). The smaller or larger size served as deviant, counterbalanced across trials. In *baseline shape* trials, the deviant image was created by manipulating the angle between the branches. Three pairs of angle values were used, all differing by a ratio of 1:3: 18° vs. 54°, 30° vs. 90°, and 45° vs. 135°. Again, whether the smaller or the larger angle served as deviant was counterbalanced across trials. Finally, in *baseline sense* trials the deviant image was opposite in sense from the distractors. The absolute sense direction of the deviant and standard images varied across trials: in half of the trials, the shorter branch was located clockwise from the longer branch in the standard figures, in the other half it was located counter-clockwise.

In all baseline conditions, the standard and deviant figures were equated in the parameters that were not manipulated. For

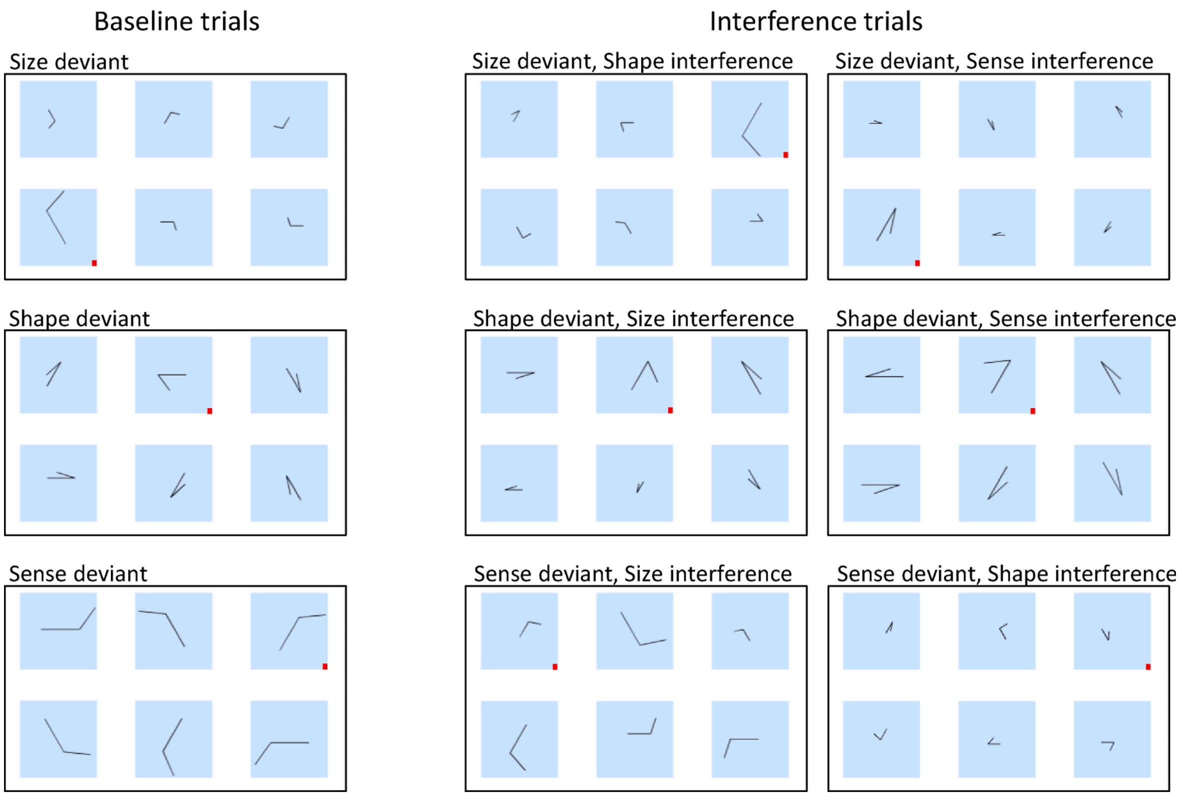

Fig. 2. Illustration of the stimuli. In Baseline trials, all the distractors were identical, except for their position and orientation. Size deviants were created by manipulating the length of the branches (yet keeping the relative length of the two branches at a ratio of 0.7), shape deviants by manipulating the angle between the two branches, and sense deviants by manipulating the respective position (clockwise or counterclockwise) of the short and long branches. In Interference trials, in addition to the manipulation of size, shape or sense defining the deviant as in baseline trials, the six figures varied in size, shape or sense, but not in a way to define a deviant. For illustration purposes, the position of the deviant stimulus is indicated here by a red dot.

example, in size deviant trials, the angle between the two branches, and the respective position of the two branches (clockwise or counter-clockwise) were the same across all six figures. The value of these equated parameters were randomly chosen by the program amongst the values used in interference trials (see below).

In *interference trials*, in addition to the manipulation of size, shape or sense that defined the deviant image as in baseline trials, one of these dimensions was varied in the distractor images. By crossing systematically the dimensions of size, shape and sense, two types of interference trials were created for each deviant dimension. For example, on trials where the deviant was defined in terms of shape (*shape deviant*), the distractor images could carry irrelevant variations either in size (*size interference*) or in sense (*sense interference*). When interfering variations of size or shape were applied, the sizes or angles of the figures were spread evenly across an interval of values. In *size interference* trials, figure size spanned the whole interval between the values used as smallest and largest size deviants. In *shape interference* trials, angles spanned one of three intervals: 18° to 90°, 36° to 126°, or 90° to 150°. Finally, in *sense interference* trials, 3 images were drawn for each direction.

2.1.3. Procedure

The participants were instructed to find the "picture" that looked "most" or "very different". Accuracy was emphasized over speed. Preteenagers and adults used the mouse to indicate their answer and advanced the trials by pressing the space bar. Younger children pointed to indicate their response while an experimenter advanced the trials and clicked on the response chosen. Some of the children who requested to use the mouse themselves were allowed to do so, if the experimenter judged that it would be beneficial to data collection.

Before the actual test, participants received 4 practice trials where the deviant differed in color or in orientation. In the first two trials, all five distractors were strictly identical. In the two next trials, irrelevant variations of size or shape were introduced on the distractors. If the participant made an error on one of the training trials, the four trials were repeated a second time. Participants proceeded to the main test even if they made an error in the second round of practice trials, with the one exception of a 4-year-old girl, who received a third round of practice trials[7]. On average, in the last round of 4 practice trials, performance was above 75% in all groups (preschoolers: $M = 77.3$, $SD = 24.0$; school children: $M = 91.3$, $SD = 14.0$; preteenagers: $M = 95.5$, $SD = 12.5$; adults: $M = 98.8$, $SD = 5.6$).

Following the practice trials, participants received either 27 (preschoolers), 54 (school children and preteenagers) or 162 (adults) test trials, with the exception of one 6-year-old boy, who was erroneously administered only 27 trials. Every 54 trials, a screen appeared to signal a break. At the first break screen, children were told that the game was done, but some of them did one (1 school child, 2 preteenagers) or two (1 school child, 1 preteenager) extra blocks of trials as they requested to keep playing. These extra blocks of data were included in the analyses. Adults continued the experiment when ready.

As detailed above, participants received trials in 9 different conditions (Fig. 2): the deviant figure was defined along one of 3 geometric dimensions (size, shape or sense), and this was crossed with 3 different conditions of interference (baseline and 2 types of interference for each deviant dimension). For example, shape deviants appeared in *baseline* trials (all distractors being identical but for their position and orientation), *size interference* trials (distractors varied in size, in addition to position and orientation), and *sense interference* trials (distractors varied in sense, position and orientation). The order of the trials, the position of the distinctive figure on the panel, and the exact parameters of the display were generated randomly by the program for each participant.

2.1.4. Analyses

2.1.4.1. General approach. Our analyses were conceived to address two series of questions: (1) whether participants would select deviants varying in size, shape and sense, and (2) whether they could disregard variations of size, shape and sense when these variations did not define a deviant. We wished to assess these questions at several stages of development, and also to study how behavioural signatures varied with age. We thus recruited a sufficient number of participants in order to analyze responses within each of our four age groups.

For each of our research question, we applied the same sequence of analyses. First, we performed within-group analyses using *t*-tests, with *p*-values corrected for multiple comparisons within group using Holm's method. Second, we assessed the presence of developmental effects using a global ANOVA with all the groups. Significant main effects and interactions were explored by computing the associated simple contrasts, Holm-corrected for multiple comparisons (using R package emmeans, Lenth, 2022).

The data and the analysis script are available at https://osf.io/kn9je/.

2.1.4.2. Detailed presentation of analyses. Our first analyses tested whether participants selected the size, shape and sense deviants in baseline trials. To do so, for each age group and deviant condition (size, shape, sense) we compared accuracy against chance (1/6 = 16.7%) by means of a *t*-test. Next, to analyze developmental effects, we submitted accuracy to a 4x3 ANOVA with one between-subject variable for age group (4 levels: preschoolers, school children, preteenagers, adults), and one within-subject variable for deviant condition (3 levels: size, shape and sense).

We then tested whether participants could or could not disregard irrelevant variations of size, shape and sense, by comparing interference with baseline trials. First, for each interference dimension (size, shape, sense) and each age group, we compared

[7] All the results reported in this paper remain unchanged if this child is excluded from the analyses.

interference trials with their corresponding baseline trials by means of *t*-tests. Hence, for example, to test whether irrelevant variations of size interfered on participants' judgments, we used two separate *t*-tests to compare baseline shape trials to shape trials with size interference and to compare baseline sense trials to sense trials with size interference. *P*-values were corrected using Holm's method to compensate for the fact that we performed two comparisons per group and interference dimension. Second, to test for developmental effects we conducted a 4x6 ANOVA on the interference effects (difference in performance between interference trials and baseline trials), with one between-subject variable for age group (4 levels) and one within-subject variable for interference condition (6 levels: size-on-shape, size-on-sense, shape-on-size, shape-on-sense, sense-on-size, sense-on-shape).

2.2. Results

2.2.1. Detection of size, shape and sense deviants in baseline trials

First, we analyzed baseline trials to test whether participants selected deviant figures varying in size, shape, or sense. Accuracy was high on average ($M = 55.2\%$), much higher than the chance level of 16.7%. In detail (Table 1, Fig. 3), performance was above chance for size and shape deviants in all groups, but did not depart from chance for sense deviants except in the group of adults.

In order to formally compare performance across conditions and groups, we performed a 4x3 ANOVA with the two variables of age group (preschoolers, school children, preteenagers, adults) and deviant dimension (size, shape, sense). Performance differed significantly across deviant dimensions ($F(2,216) = 239.3$, $p < .001$, $\eta_p^2 = .69$), in line with the observed discrepancy between size and shape trials on one hand, and sense trials on the other (post-hoc contrasts: size or shape vs. sense: $p_{corr}s < 0.001$; size vs. shape: $p_{corr} = 0.15$). Furthermore, the ANOVA also revealed a main effect of age group ($F(3,108) = 44.7$, $p < .001$, $\eta_p^2 = .55$; post-hoc contrasts identified significant differences between all groups, $p_{corr}s < 0.05$, except for the contrast between school children and preteenagers, $p_{corr} = 0.15$), as well as an interaction between age group and deviant dimension ($F(6,216) = 12.9$, $p < .001$, $\eta_p^2 = .26$). A post-hoc exploration of the simple contrasts associated to this interaction (30 simple contrasts, Holm-corrected) indicated that size and shape deviants were detected better than sense deviants in all groups (all $p_{corr}s < 0.001$), while pre-schoolers also showed an advantage for size over shape deviants ($p_{corr} < 0.001$; no difference between size and shape deviants in any other group: $p_{corr}s > 0.80$). Moreover, developmental trajectories differed between size and shape vs. sense. Detection of size and shape improved early during childhood (differences between preschoolers and all other age groups: $p_{corr}s < 0.001$; no difference between the groups of school children, preteenagers and adults: $p_{corr}s > 0.27$). In contrast, detection of sense deviants yielded group effects only in comparisons of adults to the other groups (difference between adults and all other age groups: $p_{corr}s < 0.01$; no difference between the groups of preschoolers, school children and preteenagers: $p_{corr}s = 1.00$).

In summary, these analyses provide evidence that U.S. children can contrast forms varying in size or shape from a very young age. In contrast, children did not analyze figures in terms of sense in our task. In addition, our findings reveal subtle differences in the development of the sensitivity to size and shape, with an early advantage for size in preschoolers.

2.2.2. Interference effects

Next, we examined interference effects to test whether participants were able to disregard irrelevant variations of size, shape, or sense (Table 2, Fig. 4). By and large, these analyses identified significant effects of size or shape interference, but no effect of sense interference. In all groups (except preschoolers), size interfered with the detection of shape deviants, and shape interfered with the detection of size deviants. In addition, shape variations – but not size variations – disrupted adults' ability to detect sense deviants. Indeed, while adults performed above chance on sense deviants in baseline trials (see above) or in the presence of size interference ($M = 38.9$, $SD = 36.9$; comparison to chance: $t(19) = 2.7$, $p = .014$), their performance did not depart from chance on sense deviants with shape interference ($M = 23.9$, $SD = 19.8$, comparison to chance: $t(19) = 1.6$, $p = .12$). In the groups of children, we generally did not find effects of size or shape interference on sense, which could be expected given that children already performed at chance on sense baseline trials, leaving little room for interference effects. As one exception, preschoolers showed a significant effect of size interference on sense, reaching below-chance performance in the presence of size interference ($M = 6.1$, $SD = 13.0$; comparison to chance: $t(43) = -5.4$, $p < .001$). We do not have an explanation for this unexpected finding.

To formally compare interference effects across age groups, interference dimension and target dimension, we ran a 4x6 ANOVA with the two variables of age group (preschoolers, school children, preteenagers, adults) and interference condition (size-on-shape, size-on-sense, shape-on-size, shape-on-sense, sense-on-size, sense-on-shape). The ANOVA yielded a main effect of interference condition ($F(5,540) = 13.8$, $p < .001$, $\eta_p^2 = .11$): post-hoc contrasts exploring this effect indicated stronger effects of interference in the size-on-shape and shape-on-size conditions than in the other four conditions of size-on-sense, shape-on-sense, sense-on-size and sense-on-shape (size-on-shape or shape-on-size vs. any of the four conditions involving sense: $p_{corr}s < 0.05$; all other comparisons $p_{corr}s = 1.00$).

Table 1
Accuracy at detecting size, shape and sense deviants in baseline trials in each group, compared to chance (16.7%) by means of *t*-tests.

	Baseline Size	Baseline Shape	Baseline Sense
Preschoolers (3-5yo)	$M = 56.4$, $SD = 32.4$, $t(43) = 8.1$, $p < .001$	$M = 36.4$, $SD = 32.0$, $t(43) = 4.1$, $p < .001$	$M = 15.9$, $SD = 20.9$, $t(43) = -0.2$, $p = .81$
School children (6-8yo)	$M = 86.4$, $SD = 22.1$, $t(25) = 16.1$, $p < .001$	$M = 82.2$, $SD = 16.7$, $t(25) = 20.0$, $p < .001$	$M = 12.6$, $SD = 15.1$, $t(25) = -1.4$, $p = .19$
Preteenagers (9-12yo)	$M = 87.6$, $SD = 19.6$, $t(21) = 17.0$, $p < .001$	$M = 97.0$, $SD = 6.6$, $t(21) = 57.2$, $p < .001$	$M = 15.0$, $SD = 11.0$, $t(21) = -0.7$, $p = .49$
Adults	$M = 94.8$, $SD = 12.4$, $t(19) = 28.1$, $p < .001$	$M = 95.2$, $SD = 8.1$, $t(19) = 43.5$, $p < .001$	$M = 42.8$, $SD = 36.7$, $t(19) = 3.2$, $p = .005$

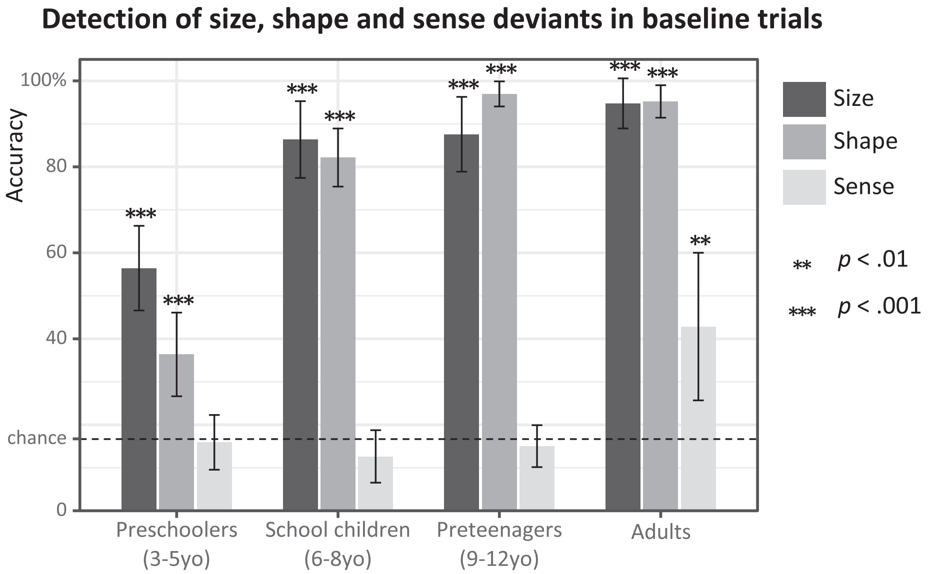

Fig. 3. Accuracy at detecting size, shape and sense deviants in baseline trials. Error bars are 95% Confidence Intervals; *p*-values from *t*-test comparisons to chance (16.7%).

Table 2
Effects of size, shape and sense interference, as assessed by *t*-tests comparing interference to baseline trials. All *p*-values are corrected for multiple comparisons within group and interference dimension (2 comparisons) using Holm's method.

	Size interference	Shape interference	Sense interference
Pre-schoolers (3-5yo)	*on shape:* $t(43) = 1.2$, $p_{corr} = 0.23$	*on size:* $t(43) = 1.5$, $p_{corr} = 0.28$	*on size:* $t(43) = 1.3$, $p_{corr} = 0.42$
	on sense: $t(43) = 2.7$, $p_{corr} = 0.021$	*on sense:* $t(43) = 0.2$, $p_{corr} = 0.86$	*on shape:* $t(43) = -0.0$, $p_{corr} = 1.00$
School children (6-8yo)	*on shape:* $t(25) = 5.6$, $p_{corr} < 0.001$	*on size:* $t(25) = 2.4$, $p_{corr} = 0.046$	*on size:* $t(25) = -0.6$, $p_{corr} = 1.00$
	on sense: $t(25) = -0.2$, $p_{corr} = 0.87$	*on sense:* $t(25) = -1.5$, $p_{corr} = 0.15$	*on shape:* $t(25) = -0.0$, $p_{corr} = 1.00$
Pre-teenagers (9-12yo)	*on shape:* $t(21) = 3.3$, $p_{corr} = 0.006$	*on size:* $t(21) = 7.3$, $p_{corr} < 0.001$	*on size:* $t(21) = -0.3$, $p_{corr} = 1.00$
	on sense: $t(21) = 0.8$, $p_{corr} = 0.41$	*on sense:* $t(21) = -1.0$, $p_{corr} = 0.31$	*on shape:* $t(21) = -0.4$, $p_{corr} = 1.00$
Adults	*on shape:* $t(19) = 3.1$, $p_{corr} = 0.012$	*on size:* $t(19) = 7.4$, $p_{corr} < 0.001$	*on size:* $t(19) = 1.1$, $p_{corr} = 0.60$
	on sense: $t(19) = 1.2$, $p_{corr} = 0.25$	*on sense:* $t(19) = 3.1$, $p_{corr} = 0.007$	*on shape:* $t(19) = -0.2$, $p_{corr} = 0.85$

There was no main effect of age group ($F(3,108) = 1.2$, $p = .30$, $\eta_p^2 = .03$); however, age group interacted significantly with interference condition ($F(15,540) = 4.4$, $p < .001$, $\eta_p^2 = .11$).

We explored the interaction between age group and interference condition by examining contrasts between interference conditions within each age group, as well as contrasts between age groups within each interference condition (96 simple contrast comparisons, Holm-corrected). Different profiles of interference were identified in each age group. Preschoolers showed no difference across the six interference conditions ($p_{corr}s > 0.25$), in line with the absence of meaningful interference effects in this group. In school children, size-on-shape yielded stronger interference effects than the four low-interference conditions of sense-on-size, sense-on-shape, size-on-sense and shape-on-sense ($p_{corr}s < 0.05$; other comparisons $p_{corr}s > 0.29$). In preteenagers and adults, in contrast, the four sense-related interference conditions contrasted with shape-on-size ($p_{corr}s < 0.05$; other comparisons $p_{corr}s > 0.91$). As one exception, in adults shape-on-size did not differ from shape-on-sense ($p_{corr} = 1.00$), in line with the finding of a significant effect of interference of shape on sense in this group. Similarly, developmental trajectories differed across interference conditions, with significant developmental effects only in the two conditions of size-on-shape and shape-on-size (size-on-shape: preschoolers vs. school children $p_{corr} = 0.037$; shape-on-size: preschoolers vs. preteenagers $p_{corr} = 0.023$; all other comparisons between groups for these and other interference conditions $p_{corr}s > 0.08$).

In summary, interference effects mirrored the results obtained on the baseline trials: just as the detection of size and shape deviants was much easier than the detection of sense deviants, we found that irrelevant variations of size and shape impaired the detection of other dimensions, whereas irrelevant variations of sense had no effect. Moreover, again the results revealed subtle differences between the developmental trajectories of size and shape. Size interference effects appeared to develop earlier than shape interference effects, with an increase between the groups of preschoolers and school children in the case of size, and only between the groups of preschoolers and preteenagers in the case of shape interference.

With a total of 96 holm-corrected contrasts, and children performing at chance on sense trials, the exploration of the interaction between age group and interference condition may have lacked sensitivity to identify differences in the development of size and shape interference. We thus ran a follow-up analysis of the two conditions testing interference of size or shape on each other, focusing on the

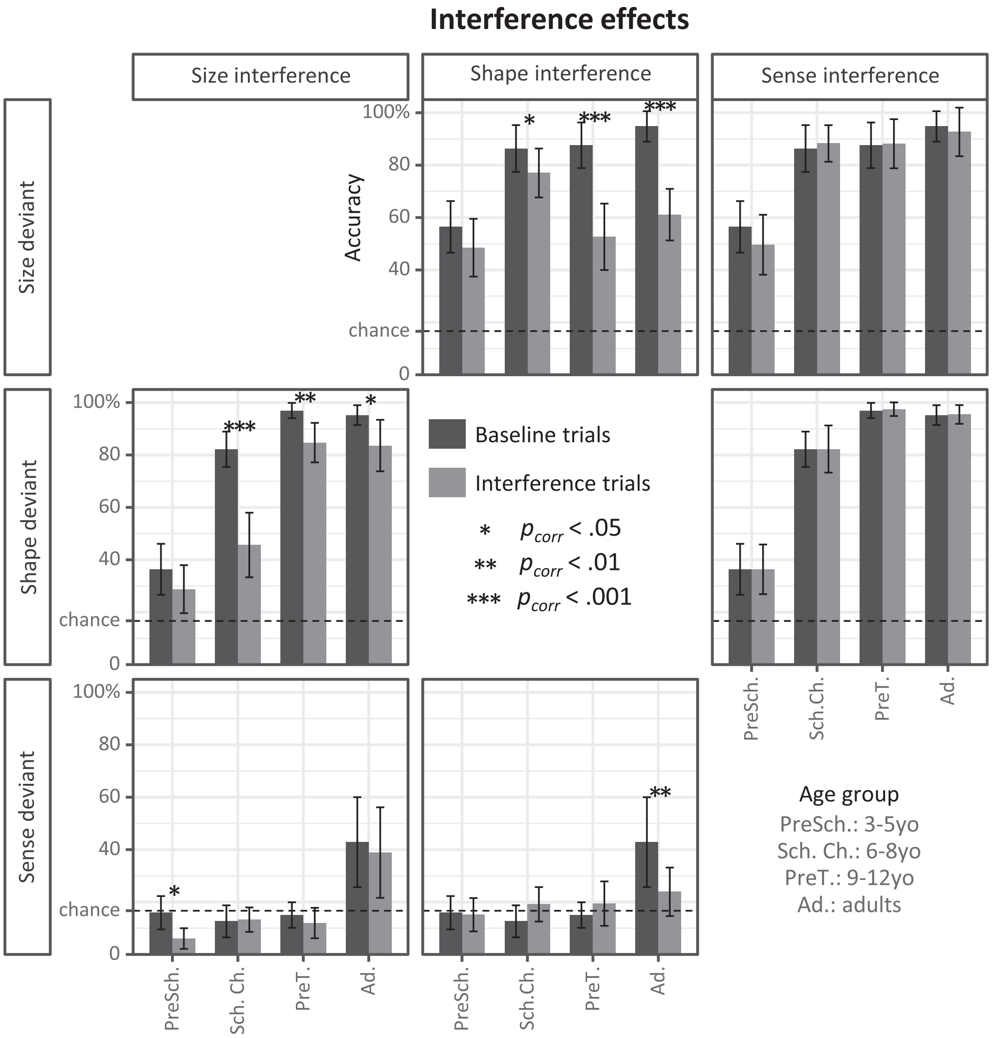

Fig. 4. Effects of interference. Error bars are 95% Confidence Intervals; *p*-values from *t*-tests comparing interference trials with the corresponding baseline trials, Holm corrected within group and interference dimension.

three groups that had proven sensitive to interference in our main analyses: school children, preteenagers and adults. This analysis found no difference between the two conditions of size-on-shape or shape-on-size interference, and no main effect of age group (interference condition: $F(1,65) = 1.8$, $p = .18$, $\eta_p^2 = .03$; age group: $F(2,65) = 0.0$, $p = .98$, $\eta_p^2 = .00$). However, the analysis yielded a significant interaction between interference condition and age group ($F(2,65) = 15.8$, $p < .001$, $\eta_p^2 = .33$), attesting that size and shape interference followed different developmental trajectories. Exploring this interaction (9 simple contrast comparisons, Holm-corrected) revealed a markedly different profile in the group of school children compared to preteenagers and adults, who behaved similarly. Interference of size on shape deviants was stronger in the group of school children compared to preteenagers and adults (p_{corr}s < 0.01; no difference between preteenagers and adults, $p_{corr} = 1.00$) while in reverse, interference of shape on size was stronger in preteenagers and adults than in school children (p_{corr}s < 0.01; no difference between preteenagers and adults, $p_{corr} = 1.00$). Accordingly, school children showed stronger interference of size than shape ($p_{corr} = 0.002$), while preteenagers and adults showed the reverse effect (preteenagers: $p_{corr} = 0.015$; adults: $p_{corr} = 0.021$).

2.3. Discussion

Our study assessed whether children and adults possess Euclidean representations of visual forms by testing four predictions. First, if participants rely on Euclidean representations in our task, they should be sensitive to variations of size and shape, and thus detect size and shape deviants amongst forms that vary only in position and orientation (baseline trials). Second, participants should also be able to disregard sense variations at no cost when selecting a size or shape deviant. Third, detection of sense deviants may be poor, including in baseline trials – unless participants can switch to alternative form representations that retain sense information through rotations. Fourth, variations of size and shape may interfere on the detection of shape and size deviants – unless participants can switch to yet other alternative form representations that abstract over size or shape.

Our first and second predictions – the sine qua non signature of Euclidean representations – were borne out in all age groups: Participants of all ages, including preschoolers, proved capable of analysing size and shape in line drawings, even in the presence of variations of sense. More specifically, participants from all age groups could detect size or shape deviants amongst distractor images that varied in position and orientation well above chance level, and moreover, performance was not altered when the distractor figures also varied in sense.

Our results also yielded evidence in line with our third and fourth predictions. In most of the groups, adding irrelevant variations of size or shape to the distractor figures systematically impaired the detection of shape or size deviants. Participants thus still relied on Euclidean representations in these trials, even though this impeded the detection of deviants. Moreover, detection of sense deviants was poor and contrasted with the detection of size or shape deviants in all the groups. This difference was most marked in the groups of children: no group of children – not the preschoolers, the school children, nor the preteenagers – selected sense deviant figures above chance level, even in baseline trials where distractors varied in position and orientation only. Adults too showed poor performance, with a sizeable proportion of our adult participants (40%) performing at or below chance level on baseline sense deviants. When they attended to sense, adults reported using a strategy of mental rotation, unlike in size or shape trials. Accordingly, adults were faster at solving the baseline size ($M = 4.0$ s, $SD = 2.4$) and shape ($M = 4.0$ s, $SD = 2.0$) trials, compared to sense trials ($M = 10.2$ s, $SD = 6.3$). The detection of size and shape deviants was also fast in children ($M = 4.8$ to 6.8 s across ages and conditions).

This result should not be taken to indicate that our participants lacked the ability to perceive sense differences in rotated 2-dimensional drawings. Indeed, this ability has been repeatedly documented in adults, children and even infants (e.g. Cooper, 1975; Kosslyn et al., 1990; Krüger, 2018; Lauer et al., 2015). The tasks employed in this previous research differed from our task in several respects, potentially explaining the difference in performance (see general discussion). Most importantly to our current purpose, none of these studies compared sensitivity to sense with other geometric properties. By presenting variations in sense alongside variations in size or shape, our study demonstrates that children and adults largely prefer to rely on the Euclidean properties of size and shape rather than on sense to compare visual forms.

Lastly, our findings also revealed unexpected developmental differences in the sensitivity to size and shape. Preschoolers showed better performance at detecting size than shape deviants; and moreover, at the age where we see the first effects of interference, school children suffered greater interference from size than from shape. Compared to the school children, preteenagers and adults relied less on size, in several respects. Irrelevant variations of shape had a larger impact on preteenagers' and adults' detection of size deviants than size variations on their detection of shape deviants, contrasting with the effects observed in school children. Moreover, variations of shape impaired adult participants' ability to compare sense across figures, while variations of size did not. Some of the adult participants even appeared reluctant to use size as a defining feature, defaulting to it only because they saw no other option. For example, one adult participant once protested that the figures were "all the same" in front of a baseline size trial, despite an easily perceptible three-fold difference in size. The participant eventually chose the size deviant by lack of other choices, but still stated that this did not count as a true difference to him/her.

In summary, while the performance of all age groups generally showed the signatures of Euclidean representations (with a dissociation between size and shape vs. sense), we also detected subtle distinctions in how preschoolers, school children, preteenagers and adults analyzed forms. Young children showed greater sensitivity to size, and ignored sense differences. Adults, in contrast, picked out differences of sense, and were also more adept at disregarding size differences. This shift may be universal, based on experience shared by all humans. For example, in a 3-dimensional environment children may progressively learn to disregard size differences because apparent size varies with distance. Alternatively, developmental differences in sensitivity to size, shape and sense may be specific to cultures where people receive formal education in geometry. Geometry curricula indeed emphasize size-invariant concepts, such as the concepts of *square*, *circle*, *right angle*, or *parallelism*. Moreover, pupils are trained to analyze properties of symmetry in figures and to apply transformations such as rotation, which could have bolstered attention to sense in geometry-educated U.S. adults. To assess the universality of our results, in Experiment 2 we administered the same task to child and adult participants from a culture radically different from the U.S.: the Munducucu from the Amazon.

3. Experiment 2: Children and adults from the Amazon

In Experiment 2, we administered the same geometric deviant detection task to a group of Munducucu adults and children. Just like people from the U.S., Munducucu people rely on geometric properties in the course of everyday life, but they engage in different activities and live in a very different environment. For example, the Munducucu navigate in the jungle, construct artisanal houses and artefacts such as baskets, necklaces of miniature sculpted animals, or ornamented headdresses; they are also renowned for their body painting art. In contrast to English, their language contains few words to describe size or shape (a list of the most relevant lexical items can be found as supplementary online material). The Munducucu nonetheless possess a rich vocabulary to describe other types of spatial properties, such as orientation (e.g. contrasting forms with a horizontal vs. vertical main axis), or abstract properties invariant to large classes of transformations (e.g. classifiers for flat, flexible objects; Pica et al., 2022). None of the participants tested in the present study had received formal education in geometry, but their level of instruction varied for other matters. Participants also differed in their exposure to Portuguese. Finally, some of the participants came from traditional villages, while others lived in carpentered areas where houses are aligned with the streets that they border.

3.1. Methods

3.1.1. Participants

Twenty-five participants (15 children, 7 female, age 5–15 years; 10 adults, 3 female, age 24–67 years) from the upper Cururu region were tested during a field trip taken by P.P. in 2009–2010. Most of them participated in other experiments testing their intuitions about numeric quantities during this same field trip (McCrink, Spelke, Dehaene, & Pica, 2013; Piazza et al., 2013). Of the 25 participants, 19 were monolingual in Munduruku (14 children, 5 adults), 8 had received no school instruction or instruction in early literacy only (3 children, 5 adults), and 15 lived in non-carpentered areas (Missao Velha, Miussu, Santa Maria, and Wariri; 10 children, 5 adults).

In accord with the Munduruku customs, in each village the researchers first solicited consent from the leader of the village, and then from the assembly of all villagers. If both the leader and the villagers had consented to participate in the research, the researchers settled in the village, and individual participants came to find them at their own convenience and on a voluntary basis. The village received food and oil to reimburse the costs incurred by the presence of researchers, and individual participants were given small gifts (e.g. fishing nets, hooks). Care was taken to offer objects that Munduruku would normally possess in their ordinary life, so as to avoid undue coercion. Participants were tested by a native speaker of Munduruku who was trained and supervised by author PP.

3.1.2. Procedure

The procedure was identical to Experiment 1, except as follows.

Stimuli were displayed on a wide 17″ screen MacIntosh laptop computer (900x1440 resolution). Participants indicated their response by pointing to the image chosen, while an experimenter recorded the response and advanced trials.

Just like in the U.S., Munduruku participants were first administered 4 practice trials with a color or orientation deviant, which were repeated a second time in case of an error (16/25 participants). If a participant was struggling during this practice phase, the experimenter tried different phrasings for the instructions, describing the target as 'ugly' or 'beautiful'– as had proved helpful in our past research (Dehaene et al., 2006). On average, in the last round of 4 practice trials, performance was above 75% (children: $M = 83.3$, $SD = 18.1$; adults: $M = 87.5$, $SD = 17.7$). Following practice, participants performed 162 test trials with size, shape or sense deviants, presented in 3 blocks of 54 trials (except for one adult who quit the experiment after 2 blocks).

3.1.3. Analyses

The analyses were the same as in Experiment 1. All test trials were included, except for a few trials that were not recorded by the program. Participants contributed 108–162 trials each (average 159.8).

As fewer children could be tested in the Amazon, we simply divided Munduruku participants into two groups of children (age 5 to 15 years) and adults (age 24 to 67 years). Each of our questions of interest was assessed in three steps. First, as we had done in Experiment 1, we performed *t*-tests within each of the two groups of children and adults (baseline trials: comparison to chance; interference effect: comparison of interference trials to baseline trials). Second, we applied these *t*-test analyses to three subgroups of participants who differed most from the U.S. participants of Experiment 1: participants who were monolingual in Munduruku, participants who had received no schooling beyond basic alphabetization, and participants who lived in non-carpentered areas. Third, we performed a global ANOVA to test for developmental differences between children and adults (baseline trials: 2x3 ANOVA on accuracy with two variables for age group and deviant dimension; interference effects: 2x6 ANOVA on the difference in accuracy between baseline and interference trials, with two variables for age group and interference condition).

The data and analysis script are available at https://osf.io/kn9je/.

3.2. Results

First, we assessed Munduruku's ability to detect size, shape and sense deviants in baseline trials. As detailed in Table 3 and illustrated on Fig. 5, both groups of adults and children were able to detect size and shape deviants well above chance. For sense deviants, children performed at chance level while adults performed slightly below chance. Thus, some of the Munduruku adults were able to detect the relation between forms of opposite senses but, contrary to U.S. adults, they assimilated symmetric forms instead of opposing them. The same pattern of results hold when restricting analyses to the subgroups of participants who were monolingual in Munduruku, who had not received schooling beyond basic alphabetization, or who lived in non-carpentered areas: in all three subgroups, performance was well above chance on size and shape trials, and at or slightly below chance on sense trials.

In line with these results, the global ANOVA analysis yielded a significant effect of deviant dimension ($F(2,46) = 844.3, p < .001, \eta^2_p = .97$): participants performed better on size or shape deviants compared to sense deviants (post-hoc contrasts: size or shape vs. sense, $p_{corr}s < 0.001$; size vs. shape, $p_{corr} = 0.47$). Performance did not differ in general between Munduruku children and adults ($F(1,23) = 1.9, p = .18, \eta^2_p = .08$), but the analysis identified a significant interaction between deviant dimension and age group ($F(2,46) = 3.6, p = .037, \eta^2_p = .13$). Post-hoc exploration of simple contrasts did not confirm the interaction pattern, however. There was no difference between groups in either condition ($p_{corr}s > 0.38$). Also, both adults and children showed the same pattern of performance across deviant dimensions: they were better at detecting size or shape deviants than sense deviants (all $p_{corr}s < 0.001$), but accuracy on size and shape deviants did not differ ($p_{corr}s > 0.56$).

In summary, Munduruku responses in baseline trials followed the same pattern as the responses of our U.S. participants overall, with higher sensitivity to size and shape deviants than to sense deviants. Fig. 5 illustrates the convergence between the two populations: for all three types of deviants, performance levels of individual Munduruku participants fell within the range of performance observed in same-age U.S. participants, except for one Munduruku adult who systematically avoided all sense deviants.

Table 3
Accuracy in baseline trials for each condition and group, compared to chance (16.7%) in each group by means of *t*-tests. [a] Significant below chance performance.

	Baseline Size	Baseline Shape	Baseline Sense
Children (5-15yo)	$M = 92.7$, $SD = 7.4$, $t(14) = 39.8$, $p < .001$	$M = 90.0$, $SD = 12.6$, $t(14) = 22.5$, $p < .001$	$M = 15.9$, $SD = 6.3$, $t(14) = -0.5$, $p = .65$
Adults	$M = 97.4$, $SD = 3.6$, $t(9) = 70.3$, $p < .001$	$M = 97.2$, $SD = 4.7$, $t(9) = 54.0$, $p < .001$	$M = 11.7$, $SD = 6.7$, $t(9) = -2.4$, $p = .041$[a]
Monolingual	$M = 94.2$, $SD = 7.2$, $t(18) = 47.0$, $p < .001$	$M = 92.1$, $SD = 11.9$, $t(18) = 27.6$, $p < .001$	$M = 13.7$, $SD = 7.0$, $t(18) = -1.8$, $p = .086$
Unschooled	$M = 94.6$, $SD = 6.5$, $t(7) = 34.2$, $p < .001$	$M = 91.0$, $SD = 15.1$, $t(7) = 13.9$, $p < .001$	$M = 14.6$, $SD = 5.9$, $t(7) = -1.0$, $p = .35$
Non-carpentered	$M = 95.1$, $SD = 5.7$, $t(14) = 53.0$, $p < .001$	$M = 93.0$, $SD = 11.8$, $t(14) = 25.1$, $p < .001$	$M = 13.0$, $SD = 5.8$, $t(14) = -2.5$, $p = .027$[a]

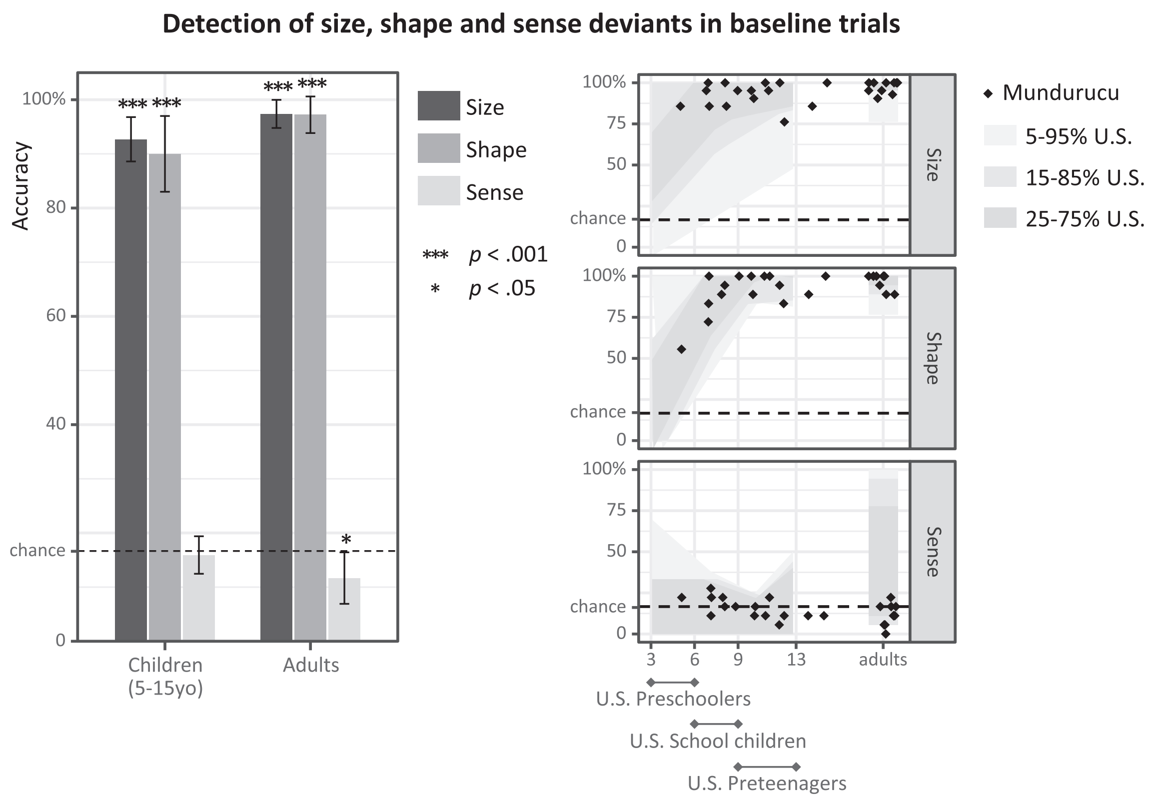

Fig. 5. Performance of Mundurucu children and adults in baseline trials. Left: Average performance by group. Error bars are 95% Confidence Intervals; *p*-values from *t*-tests with respect to chance. Right: Accuracy of individual Mundurucu participants is overlaid on the distribution observed in the U.S. population (computed using the function rqss in R package quantreg, Koenker, 2022). The age axis marks the limits of the age groups used in the U.S. population.

Second, we analyzed interference trials and assessed whether Mundurucu participants were able to disregard irrelevant variations of size, shape and sense (Table 4, Fig. 6). The groups of children, adults, participants who were monolingual in Munducuru, participants who had not received schooling beyond basic alphabetization, and participants living in non-carpentered areas all showed the same pattern of results. Specifically, all the groups showed interference of size on the detection of shape deviants, and interference of shape on the detection size deviants. In contrast, there was no effect of interference by sense, in any of the groups. Lastly, size and shape did not interfere with the detection of sense deviants in either group. This last result was expected for children, for monolingual participants, or for participants who had not received schooling beyond basic alphabetization, given their chance performance at baseline sense deviants. Recall that adults and participants from non-carpentered areas performed slightly below chance on baseline sense deviants; even though these groups did not display significant effects of size or shape interference on sense, they performed strictly at chance on the sense trials with size or shape variations (adults: sense with size variations, $M = 17.2$, $SD = 6.7$, comparison to chance $t(9) = 0.3$, $p = .80$; sense with shape variations, $M = 15.8$, $SD = 9.4$, comparison to chance $t(9) = -0.3$, $p = .79$; participants from non-carpentered areas: sense with size variations, $M = 16.4$, $SD = 7.5$, comparison to chance $t(14) = -0.1$, $p = .89$; sense with shape variations, $M = 16.9$, $SD = 9.4$, comparison to chance $t(14) = 0.1$, $p = .94$).

An ANOVA with two variables for age group (children, adults) and interference condition (size-on-shape, size-on-sense, shape-on-size, shape-on-sense, sense-on-size, sense-on-shape) was conducted to formally compare interference effects across ages and conditions. This analysis confirmed that the strength of interference effects varied between conditions (main effect of interference condition,

Table 4
Effects of size, shape and sense interference, as assessed in *t*-tests comparing interference to baseline trials. All *p*-values are corrected for multiple comparisons within group and interference dimension (2 comparisons) using Holm's method.

	Size interference	Shape interference	Sense interference
Children (5-15yo)	*on shape:* $t(14) = 4.1$, $p_{corr} = 0.002$	*on size:* $t(14) = 9.6$, $p_{corr} < 0.001$	*on size:* $t(14) = -1.1$, $p_{corr} = 0.30$
	on sense: $t(14) = 0.7$, $p_{corr} = 0.49$	*on sense:* $t(14) = -0.7$, $p_{corr} = 0.48$	*on shape:* $t(14) = -1.7$, $p_{corr} = 0.24$
Adults	*on shape:* $t(9) = 2.9$, $p_{corr} = 0.035$	*on size:* $t(9) = 7.9$, $p_{corr} < 0.001$	*on size:* $t(9) = 1.0$, $p_{corr} = 0.67$
	on sense: $t(9) = -2.0$, $p_{corr} = 0.074$	*on sense:* $t(9) = -1.1$, $p_{corr} = 0.31$	*on shape:* $t(9) = 0.8$, $p_{corr} = 0.67$
Monolingual	*on shape:* $t(18) = 4.4$, $p_{corr} < 0.001$	*on size:* $t(18) = 11.0$, $p_{corr} < 0.001$	*on size:* $t(18) = -0.7$, $p_{corr} = 0.95$
	on sense: $t(18) = -0.7$, $p_{corr} = 0.47$	*on sense:* $t(18) = -1.4$, $p_{corr} = 0.19$	*on shape:* $t(18) = -0.7$, $p_{corr} = 0.95$
Unschooled	*on shape:* $t(7) = 3.3$, $p_{corr} = 0.027$	*on size:* $t(7) = 7.6$, $p_{corr} < 0.001$	*on size:* $t(7) = 0.0$, $p_{corr} = 1.00$
	on sense: $t(7) = -1.3$, $p_{corr} = 0.22$	*on sense:* $t(7) = -0.8$, $p_{corr} = 0.47$	*on shape:* $t(7) = 0.0$, $p_{corr} = 1.00$
Non-carpentered	*on shape:* $t(14) = 3.5$, $p_{corr} = 0.007$	*on size:* $t(14) = 10.8$, $p_{corr} < 0.001$	*on size:* $t(14) = -0.4$, $p_{corr} = 0.70$
	on sense: $t(14) = -1.4$, $p_{corr} = 0.18$	*on sense:* $t(14) = -1.2$, $p_{corr} = 0.25$	*on shape:* $t(14) = -1.0$, $p_{corr} = 0.67$

$F(5,115) = 41.7$, $p <.001$, $\eta_p^2 =.64$). Post-hoc contrasts revealed stronger interference effects in the two conditions testing interference of size or shape on each other (size-on-shape, shape-on-size) than in any of the four conditions involving sense either as an interference or as a target dimension (i.e. size-on-sense, shape-on-sense, sense-on-size, sense-on-shape; comparisons with size-on-shape and shape-on-size, $p_{corr}s < 0.01$; no difference between the four sense-related conditions, $p_{corr}s = 1.00$). In addition, the Mundurucu showed greater effects of interference of shape on size than interference of size on shape ($p_{corr} < 0.001$).

The ANOVA also identified a significant interaction between age group and interference condition ($F(5,115) = 2.5$, $p =.034$, $\eta_p^2 =.10$), in the absence of a main effect of age group ($F(1,23) = 2.2$, $p =.16$, $\eta_p^2 =.09$). Exploring this interaction (36 simple contrast comparisons, Holm-corrected) found little evidence of group differences, however. There was no difference between children and adults in any of the conditions ($p_{corr}s > 0.41$). When looking at the profile of interference effects within each group, both adults and children showed stronger interference effects for size-on-shape and shape-on-size than for sense-related conditions (adults: size-on-shape stronger than shape-on-sense, size-on-sense, or sense-on-shape, $p_{corr}s < 0.05$, but not different from sense-on-size, $p_{corr} = 0.63$; shape-on-size stronger than every sense-related conditions, $p_{corr}s < 0.001$; children: size-on-shape stronger than shape-on-sense or sense-on-shape, $p_{corr}s < 0.05$, but not different from size-on-sense or sense-on-size, $p_{corr}s > 0.08$; shape-on-size stronger than every sense-related conditions, $p_{corr}s < 0.001$). Adults also showed asymmetric effects of interference between size and shape (shape-on-size stronger than size-on-shape, $p_{corr} = 0.023$), while this last comparison did not reach significance in children ($p_{corr} = 0.27$).

In summary, effects of interference again mirrored the findings observed in the U.S. population: variations of size or shape interfered with the detection of shape and size deviants, while variations of sense had little to no effect. Moreover, irrelevant variations of size or shape did not affect performance to the same extent, with stronger effects of shape than size interference, especially in Mundurucu adults. As illustrated on Fig. 6, individual size-shape interference effects were within the ranges of values observed in U.S. participants of similar ages – with the exception of two Mundurucu adults respectively showing very strong interference of size or shape.

3.3. Discussion

Experiments 1 and 2 revealed highly similar patterns of results across the U.S. and Mundurucu populations. As was the case in the U.S., the responses of the Mundurucu showed all the signatures of Euclidean representations. Specifically, in baseline trials Mundurucu participants detected size or shape deviants with near-ceiling accuracy, and variations of size or shape also impeded the detection of one another in interference trials. In contrast, variations of sense had no systematic effect on the ability of the Mundurucu participants to detect a size or shape deviant, and the Mundurucu also did not use sense to define deviant images in baseline sense trials. These convergences are remarkable, given the differences between the two cultures in terms of spatial navigation practices, vocabulary for geometry, and formal education in mathematics.

Unexpectedly, the Mundurucu adults sometimes avoided the sense deviants, performing below chance in sense baseline trials. Like U.S. adults, some adult Mundurucu participants thus proved sensitive to the relation between figures of opposite sense; however, unlike U.S. adults, the Mundurucu tended to assimilate mirror images, instead of distinguishing between them. U.S. and Mundurucu adults seemed to employ different cognitive strategies to process the forms in baseline sense trials. In particular, while U.S. adults still selected sense deviants in the presence of size interference, in the Mundurucu population sensitivity to sense was only evident in the baseline sense trials, when all the distractor figures were equivalent in size and shape. Rather than using mental rotation like our U.S. adult participants, we suspect that Mundurucu adults analyzed properties of symmetry in the stimuli, to find that two of the figures were mirror reflections of each other (see Dehaene et al., 2006 for evidence that the Mundurucu can detect symmetry in 2-dimensional figures). Indeed, since the orientations of the six figures were incremented by 60°, two of the distractors were always related to the deviant by vertical or horizontal symmetry. This relation is visible on the baseline sense trial presented in Fig. 2: the deviant (top right) is the vertical reflection of the distractor figure presented next to it (middle of the top row), and it is also related to the bottom left distractor by a horizontal reflection. In size and shape interference trials, where Mundurucu performed strictly at chance level, these relations of perfect symmetry were disrupted, as the figures were of different sizes, or had different angles.

As a second conclusion, we found that Mundurucu participants resembled same-age U.S. participants in their sensitivity to size vs. shape. In the U.S., young children responded more to variations of size than to variations of shape, while the reverse was true in

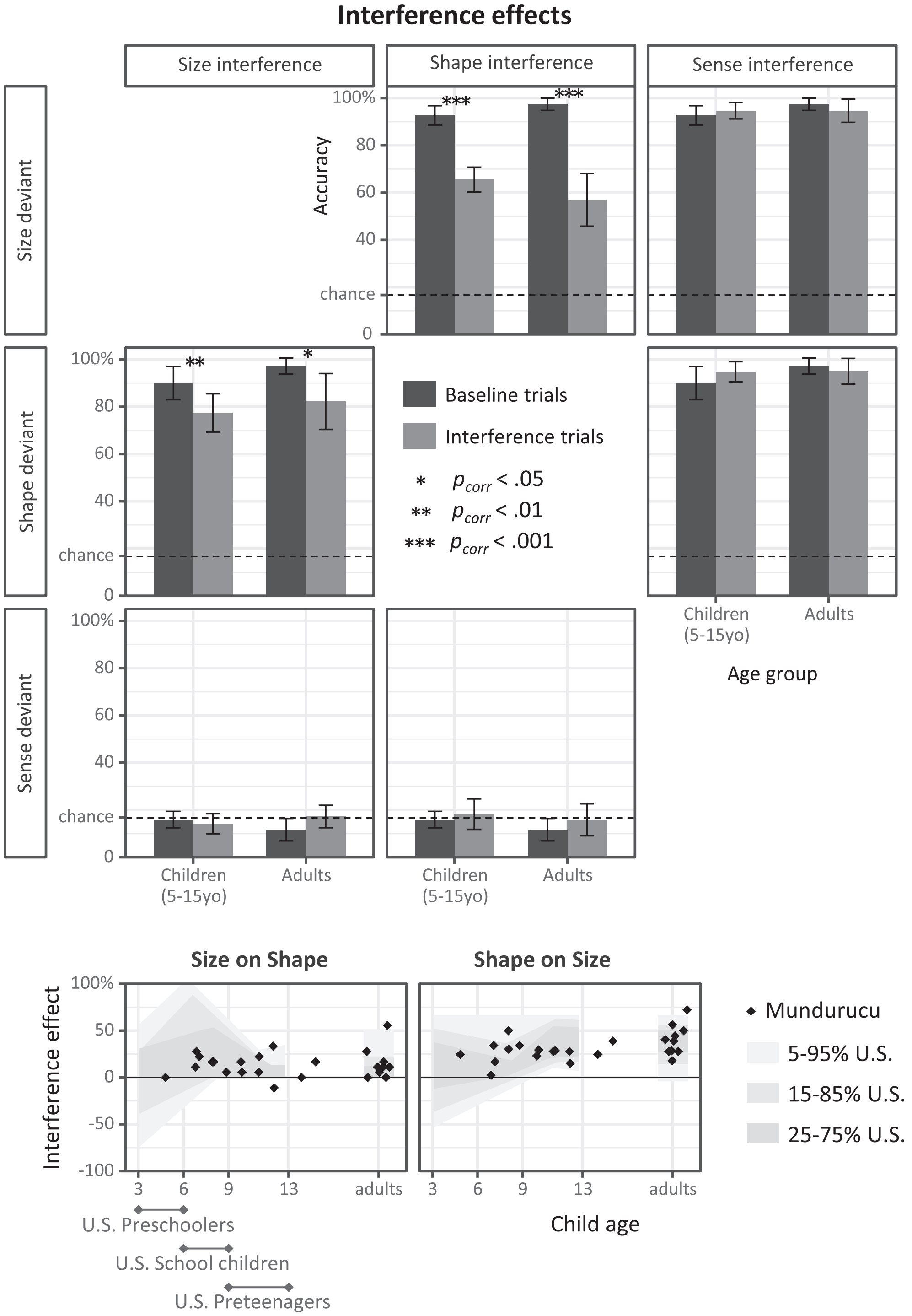

Fig. 6. Effects of interference in the Mundurucu population. Top: Average performance in baseline and interference trials. Error bars are 95% Confidence Intervals; *p*-values from *t*-tests comparing interference trials with the corresponding baseline trials, Holm corrected within group and interference dimension. Bottom: size-on-shape and shape-on-size interference effects in individual Mundurucu participants overlaid on the distribution of interference effects observed in the U.S. population (computed using the function rqss in R package quantreg, Koenker, 2022). The age axis marks the limits of the age groups used in the U.S. population.

preteenagers and adults. In general, the Mundurucu reached comparably high levels of performance in size and shape baseline trials – except for the youngest child, aged 5 years, who performed better at size than at shape baseline trials, in line with his U.S. counterparts. Moreover, similarly to U.S. adults and preteenagers, the Mundurucu generally showed a greater effect of interference from shape than from size. Just like our U.S. adults and preteenagers, the Mundurucu thus converge intuitively towards a geometry with hints of scale invariance, where shape is more relevant than size to define forms.

In summary, Experiment 2 confirms that children and adults can possess Euclidean abilities even in the absence of formal education in geometry. Furthermore, Experiment 2 suggests that all humans eventually view scale-invariant geometric properties as more important than size to define different forms. Cultural differences emerged in the case of sense, however, in that only U.S. adults selected sense deviants in our task.

4. General discussion

4.1. Overview of the results

We probed judgments on 2-dimensional simple visual figures composed of two connected line segments in U.S. adults and children covering a wide range of ages (Experiment 1) and in a population from the Amazon, the Mundurucu (Experiment 2). Participants were simply asked to detect the "most different" picture in panels of six forms. To enforce the fundamental Euclidean assumption that space is homogeneous in all directions and positions, each trial presented the six forms at varying orientations and positions, such that neither orientation nor position could be used to select a deviant. Our experiments thus ask what properties children and adults use to analyze 2-dimensional visual forms, in a context where orientation and position are posited to be irrelevant.

The forms presented varied along three dimensions relevant to characterizing Euclidean geometry: size (varied by changing the scale of the figure), shape (varied by changing the angle between the two lines), and sense (varied by applying a reflection to the figure). In general, responses followed a highly convergent pattern across all the groups studied: size and shape variations were both easy to detect and hard to disregard, while sense variations were both hard to detect and easy to disregard. More specifically, participants from all groups, including U.S. preschoolers and Mundurucu children and adults, were able to detect a deviant form defined in terms of size or shape in an array of identical forms presented at various orientations; and moreover, irrelevant variations of size or shape interfered with the detection of deviant forms. In contrast, participants did not select the sense deviants, except for the group of U.S. adults who still performed with markedly lower accuracy than for size or shape deviants. Moreover, in all groups sense variations did not affect the detection of size or shape deviants. In a context where orientation and position are irrelevant, people's analysis of visual forms thus generally aligns with Euclidean geometry: a geometry where figures are defined by the properties of size and shape, but are invariant by rotation, translation, and reflection.

In detail, our results also revealed some asymmetries in the responses to size and shape. In the U.S. population, sensitivity to size developed earlier than sensitivity to shape: preschoolers performed better at detecting size than shape deviants, and moreover, school children (the youngest U.S. group to display detectable interference effects) suffered greater effects of interference from size than from shape. In contrast, shape exerted a stronger effect of interference than size in older U.S. children and adults, as well as in Mundurucu participants. Starting from middle childhood, therefore, participants' judgments bore the marks of a scale-invariant geometry, where figures are defined by their shapes but not by their size. U.S. adults in particular proved able to navigate flexibly between different geometric analyses. When searching for sense deviants, they disregarded variations of size but not shape: the signature of a scale-, orientation- and position-invariant geometry that analyzes forms according to their shape and sense properties. When searching for shape deviants, they showed no effect of sense interference, and for some individuals, little to no effects of size interference: the signature of a sense-, scale-, orientation- and position-invariant geometry that analyzes forms solely according to their shape properties. Lastly, when searching for size deviants, U.S. adults showed interference of shape but not sense, thus displaying the signature of Euclidean geometry: a sense-, orientation- and position-independent geometry that analyzes forms by their size and shape.

In what follows, we first present the implications of our findings with respect to the main question that motivated our study: whether perception may provide foundations for Euclidean knowledge. We then discuss two particular findings: participants' generally low performance at detecting sense deviants, and the developmental shift in the responses to size and shape.

4.2. Perception of 2-dimensional visual forms as a foundation for Euclidean knowledge

Is knowledge of geometry – and in particular Euclidean knowledge – embedded in our visual perception? Our experiments provide a positive answer to this question. At all ages, and in two drastically different cultures, child and adult participants proved able to analyze 2-dimensional visual forms in the same manner as Euclidean geometry, i.e. in terms of the properties of size and shape, disregarding variations in position, orientation, and sense. Importantly, this was true even of participants who had not received any formal education in geometry, thus demonstrating that Euclidean abilities can precede formal instruction. Given our findings, it is thus *possible* for perception of visual forms to play or have played a foundational role for Euclidean knowledge.

More specifically, given our findings, we can imagine that visual form perception provides foundations for humans' geometric knowledge via two different mechanisms. First, form perception may support learning of geometry in individual learners: since children can access Euclidean descriptions of forms through vision, they may call on their visual representations to associate the concepts they are taught with a Euclidean interpretation. Of course, the interpretation selected may not always be Euclidean, given that perception analyzes forms in different manners, at several levels of geometric abstraction. Hence, for example, young children initially fail to generalize shape names such as "triangle" or "rectangle" to figures that are not presented in an upright orientation (Clements et al., 1999): evidence that they initially associate these labels with orientation-specific representations (and thus not with orientation-invariant Euclidean representations). Still, our findings indicate that Euclidean geometry stands as a possible candidate amongst all the possible levels of geometric analysis; and accordingly, in the course of development children's interpretations of shape names are also constrained by shape and size, the properties relevant in Euclidean geometry. For instance, initially children consider that the word “triangle” applies only to prototypical isosceles triangles, but not to other, unusual triangles that differ in shape from the isosceles prototype (Clements et al., 1999; Satlow & Newcombe, 1998).

Second, besides guiding geometric learning in individual people, it is also possible that visual form perception played a role in the history of mathematics and contributed to establish Euclidean geometry as the primary theory of geometry. Theories that align with our core intuitions could have initially appeared more appealing to the first mathematicians, thus favoring their emergence. In addition, intuitive theories are easier to learn and can therefore be transmitted more efficiently across generations, thus perhaps explaining the persistence of Euclidean geometry through History.

If indeed people’s visual perception contributes to shape mathematical theories via the process of cultural transmission, we should observe that theories of geometry evolve to become increasingly aligned with our perception. Interestingly, one aspect of our findings appears in line with this suggestion. As described above, modern mathematics defines ‘Euclidean geometry’ as the geometry where figures are characterized in terms of both size and shape, but not sense, orientation or position. Euclid’s axioms, however, describe a slightly different geometry, where size is irrelevant and only shape matters (see Klein, 1893 for a converging description of ‘elementary geometry’ as based on the property of shape only). Indeed, while Euclid drew distinctions based on the two shape properties of angle (e.g. definitions for right angle, parallel lines) or relative length (e.g. definition for the center of a circle, Pythagor’s theorem relating parallelism to distance ratios), his theory did not introduce tools such as a standard unit of length to distinguish between figures differing in size. In other words, if two students drew figures with the same shape but at different scales, their drawings would be equivalent with respect to Euclid’s definitions and axioms: the theorems that apply to these two figures are exactly the same. Interestingly, Euclid’s scale-invariant geometry – which was presumably produced by adult mathematicians – aligns better with intuitions displayed by older children and adults in our experiments, while size-dependent modern Euclidean geometry is most consistent with the judgments of young children. Perhaps, and in line with the putative role of core intuitions and cultural transmission in the history of mathematics, teaching geometry to young pupils contributed to propel a version of Euclidean geometry that is size-dependent as the most natural or elementary form of geometry.

4.3. Why did participants generally perform poorly on the detection of sense deviants?

In our task, most groups of participants failed to point out sense deviants amongst distractor images that varied in orientation and position only. Even U.S. adults, the only group selecting sense deviants above chance in our experiments, achieved lower levels of performance on sense than on other deviants. Yet, all our participants certainly possessed the ability to detect sense differences in rotated forms. In industrial societies, this ability has been evidenced in adults and children from an early age, including in infants (Estes, 1998; Frick et al., 2014; Kosslyn et al., 1990; Krüger, 2018; Marmor, 1975; Marmor, 1977; Moore & Johnson, 2020). Similarly, in our past research we found that Mundurucu adults and children are sensitive to sense information in a map reading task (Dehaene et al., 2006), which entails that they successfully encoded sense from the 2-dimensional diagram that served as a (rotated) map.

Why did participants ignore sense variations in our task, in contrast to this previous research? As a first hypothesis, perhaps our task made perceiving sense particularly difficult. For example, in our task each trial presented six different figures to compare, and applying mental rotation to these six figures may have been particularly costly. A few observations suggest that task difficulty alone cannot explain our participants’ poor performance with sense deviants, however. First, while finding a deviant in a 6-item panel may require a greater number of comparisons than performing a sense discrimination task on two images, increasing the number of standard images can also make the task easier, because it reduces the orientation difference between the deviant and its closest standard distractor. In our stimuli, each panel presented two distractors that differed from the deviant by only 60° – which would qualify as an easier orientation difference in a classic mental rotation task. Second, children and teenagers generally achieve good levels of performance in classic mental rotation tasks. In Kosslyn et al. (1990)’s study, for example, groups of 4- to 9-year-old children reached 65 to 90% accuracy when comparing sense in two planar forms rotated by 72 to 180°, while adults solved the task with 80% accuracy. If our U.S. school children or preteenager participants had engaged in mental rotation, they could thus have shown a level of performance comparable to our group of U.S. adults. Lastly, task difficulty cannot explain why Mundurucu adults performed *below* chance with sense deviants. Below chance performance indicates that the Mundurucu were actually able to detect that the sense deviant figure had a different status – though they considered that it should be assimilated with the standard figures rather than opposed to them.

As a second hypothesis, perhaps the U.S. children and Mundurucu participants did detect sense differences in our task (or at least they were aware that they could engage in mental rotation and look for sense differences) but they disregarded sense as irrelevant to the question asked (for a related argument see Li, Abarbanell, Gleitman, & Papafragou, 2011). Suggestively, compared to literate adults, illiterate adults have difficulties judging that two mirror images are “different” (Kolinsky et al., 2011), but are facilitated to say that these images are the “same” (Pegado et al., 2014): an indication that calling two mirror images “different” may be non-intuitive in the absence of formal education. In addition, in the particular context of our task, it is possible that some participants chose to disregard sense because they construed the forms presented as 3-dimensional objects, and mirror images as the two sides of a same object[8]. The Mundurucu culture in particular emphasizes the relation between 3-dimensional objects and their images (e.g. “biogpuk”: a classifier for lake reflections, shadows, silhouettes, mirror images, photocopies, and generally images representing an object after its disappearance). Accordingly, our Mundurucu participants may have been particularly inclined to think that our stimuli stood for 3-

[8] We thank an anonymous Reviewer for raising this theoretical possibility.

dimensional objects.

This explanation raises two questions. First, why would Munducuru adults' performance depart from chance on sense deviant trials? If participants disregarded mirror flips as another type of orientation change, or more generally if they discarded sense variations as irrelevant to our task, they should have performed strictly at chance in trials presenting sense deviants amongst rotated images, not below chance. Second, if children and Munduruku participants detected sense differences in our stimuli, or at least if they knew that they could search for these kinds of differences, why did not they do so as a last resort strategy, when they could not find any other alternative? Previous research using similar intruder tasks indicates that U.S. children and Munduruku people of all ages would select a sense deviant when contrasted with standard forms that are all aligned in the same orientation (Cronin, 1967; Dehaene et al., 2006; Izard & Spelke, 2009), indicating that people from these populations are inclined to consider sense contrasts as valid differences in the context of an intruder task, at least when there is no other alternative[9].

These considerations lead us to favor a different explanation for the present findings: we propose that U.S. children and Munduruku participants did not detect sense differences in our stimuli, and more generally did not attempt to apply mental rotation and look for sense differences. Under this view, U.S. children and Munduruku failed to select sense deviants not because they believe that sense as a dimension is irrelevant to the task, but because they failed to consider mental rotation as a potential strategy, and perhaps even failed to consider sense as a potential dimension of contrast.

There may be several reasons why participants did not use mental rotation in our task. First, perhaps some of our participants came to our task with the strong assumption that all trials ought to be solved with the same approach, and were therefore reluctant to switch strategies across trials. In our study, sense deviant trials were indeed intermixed with other types of deviants, which could be detected without mentally aligning the objects – and this aspect of our design contrasted with the design of all previous studies of mental rotation abilities in children, as well as with the very vast majority of the studies conducted with adults. Under this view, U.S. adults participants would perform better at detecting sense deviants in our task simply because they were more inclined to switch strategies across trials, owing either to enhanced executive functions and cognitive flexibility (Huizinga, Dolan, & Van der Molen, 2006; Legare, Dale, Kim, & Deák, 2018), or to greater familiarity with the particular context of cognitive psychology experiments.

Again though, this interpretation cannot fully account for our findings. Munduruku adults at least were keen on using different strategies across trials; however, rather than applying mental rotation as an alternative strategy, they chose to analyze the relations of symmetry between the figures. Indeed, we found that Munduruku adults performed below chance in baseline sense trials, but not when the relations of symmetry between the deviant and standard forms were disrupted by variations of size or shape. This pattern of responses – and, thus, evidence for a switch in strategies across trials – was only attested in the group of Munduruku adults, and may reflect a particular sensitivity to symmetry in the Munduruku culture. Still, we cannot exclude that some participants in the other groups took the same approach to the task. Children are indeed adept at recognizing relations of symmetry between figures from a young age (Hu & Zhang, 2019); and we found participants performing below chance on baseline sense deviants in all the groups (including U.S. adults; see Fig. 5).

While they may have been willing to switch strategies across trials, we suggest that U.S. children and Munduruku people failed to rely on mental rotation in sense trials because they were not aware that doing so would grant them access to information they could not perceive otherwise. Absent such metacognitive knowledge, mental rotation will not be a strategy of choice when other alternatives fail – and as a consequence, children should only rarely resort to mental rotation spontaneously. Interestingly, and in line with this suggestion, a recent study hinted that young children (younger than 10 years) may not rely spontaneously on mental simulation to solve geometry problems, in contrast to older children or adults (Hart, Mahadevan, & Dillon, 2022). Relatedly, many of the previous studies that have reported discrimination of rotated mirror images in children or infants have either presented moving displays animated with a rotation movement (e.g. Frick & Möhring, 2013), or trained participants to physically rotate stimuli (e.g. Frick, Hansen, et al., 2013; Krüger, 2018). We suspect that these aspects of the studies' design effectively primed the activation of mental rotation processes, thus enabling participants to succeed at detecting sense contrasts.

Under this view, U.S. adult participants differed from all the other groups in that they had identified mental rotation as a relevant strategy for analyzing figures. Where did they gain this knowledge? Although previous studies have highlighted literacy as an important determinant in the perception of mirror contrasts (Danziger & Pederson, 1998; Dehaene, 2009; Kolinsky et al., 2011; Pegado et al., 2014; Serpell, 1971), in our case literacy is unlikely to account for the results, given the poor performance of our group of (literate) U.S. preteenagers. Instead, formal instruction in geometry probably played a key role in U.S. adults' performance. Students attending advanced mathematics and geometry classes are taught to make sense-related distinctions, for example to distinguish between positive vs. negative angles, or to tell apart a translation from a reflection (see Hu & Zhang, 2019 for evidence that young children initially do not make this distinction, and instead analyze translations and reflections as instances of one overarching relation of general symmetry). Instruction on geometric transformations in particular might be an effective way to induce change in children's sensitivity to sense. Working with geometric transformations can highlight sense as an important dimension of form analysis, by associating different sense outcomes with different physical movements (e.g. translations preserve the sense of figures, reflections flip figures and reverse sense). In addition, teaching children to imagine the outcome of geometric transformation may also help them identify mental rotation as a successful strategy for accessing information about sense. More generally, instruction on geometric

[9] In particular, the Munduruku proved willing to select aligned (but not misaligned) sense deviants in a previous study where participants could receive the same instructions variants as in the present study, describing the target either as "ugly" or "beautiful" (Dehaene et al., 2006). It thus appears unlikely that our Munduruku adult participants avoided sense deviants and responded differently from U.S. participants just because some of them had received different versions of the instructions.

transformations can contribute to reinforce the connections between children's visual and motor representations of forms, enabling them eventually to leverage the power of mental simulation in the service of geometric reasoning.

4.4. Why did sensitivity to size and shape change during childhood?

Our results reveal that sensitivity to size vs. shape follows slightly different developmental trajectories in the U.S. Specifically, school children showed greater effects of shape than size interference, while these effects were reversed in preteenagers and adults. Moreover, U.S. preschoolers performed better at size than shape deviants in baseline trials.

How should we interpret these differences? In our task, interference effects may arise for two main reasons. Perhaps, interference arose at the response selection stage: for example, upon seeing two figures of different sizes in a size interference trial, a participant may decide to analyze this trial in terms of size, selecting either a small or a large figure. Asymmetric interference effects would then reflect participants' different prioritization of size and shape. In particular, young children may prefer to respond to size because size is easier or faster to encode than shape[10].

Alternatively, it is also possible that interference effects arose because participants did not analyze the stimuli in terms of the specific parameters we manipulated (i.e. branch length for variations of size, and angle for variations of shape), but rather focused on other parameters, which vary both with branch length and with angle. For example, if participants analyzed the figures in terms of the distance between the branches, they should show particularly strong interference of size on shape in our task, and some interference of shape on size as well. Under that second interpretation, children and adults would show different interference effects because they analyzed different aspects of the stimuli.

Did young U.S. children respond to different cues compared to the other groups? Interestingly, several recent findings converge to suggest that young children's ability to perceive angle in figures varying in size is, at best, limited (Devichi & Munier, 2013; Dillon & Spelke, 2018; Gibson et al., 2015; Izard, Pica, Spelke, et al., 2011; Lehrer, Jenkins, & Osana, 1998). Hence, at the age of 4 years children spontaneously compare L-figures based on their spatial extent, rather than on angle (Gibson et al., 2015). Children prefer to focus on distances and lengths over angles even when detrimental to the task at hand. For example, when asked to estimate the missing vertex angle of a truncated triangle (Izard, Pica, Spelke, et al., 2011), children aged 5 to 6 years were consistently misled by the length of the base of the triangle: they produced larger angles for triangles with larger bases, and smaller angles for triangles with shorter bases (see also Dillon & Spelke, 2018 for converging evidence in a verbal task). Even after entering elementary school, children still confuse angle with distance: asked to produce a method for measuring an angle, they proposed to measure the distance between the branch ends, without correcting for branch length (Lehrer et al., 1998); and asked to draw small and large angles, they varied the length of the branches rather than the angle aperture (Devichi & Munier, 2013). In our study, it is thus possible that young children did not represent figure variations in terms of angle, but instead selected shape deviants based on other cues. Possible candidates could be the figure's aspect ratio (another cue related to shape), its area, and/or the distance between the branches.

Children improve in their understanding of angles during the elementary school years, and in all the tasks described above, confusions between distances and angles decline progressively during these years (Baldy et al., 2005; Devichi & Munier, 2013; Dillon & Spelke, 2018; Izard, Pica, Spelke, et al., 2011; Lehrer et al., 1998; Piaget, Inhelder, & Szeminska, 1960). For example, while the majority of 6-year-olds state that the angle at the vertex of a truncated triangle increases if the base length is made longer, few 10- or 12-year-olds make this mistake (Dillon & Spelke, 2018). In the present study we observed a shift in interference effects in the U.S. around the same ages: size exerted a greater effect of interference than shape in a group of young elementary school children aged 6 to 8 years, and this effect was reversed in a group of 9- to 12-year-old preteenagers. In line with the literature, the shift observed in the present study may indicate that children started analysing angles in our stimuli only around the age of 8 or 9 years.

This developmental shift coincides with the introduction of angle at school, suggesting that formal teaching plays a crucial role to promote responses to angle in U.S. children. Interestingly though, in our task we observed that Mundurucu children and adults prioritized shape over size to the same extent as their same-age U.S. counterparts. This convergence is striking, given that children's education in geometry and engagement with space vary drastically across these two cultures. In the U.S., children have access to regular-shaped manufactured toys from the very first months of life; later they learn to name shapes, and receive formal instruction in geometry. Mundurucu children, in contrast, navigate autonomously in large spaces much earlier than U.S. children, engage in manual activities, and learn to manufacture objects. The Mundurucu and U.S. languages also differ in their geometric lexicons. In particular, angles cannot be quantified in the Mundurucu language, beyond coarse categories of pointy tips or wide corners. In reverse, the Mundurucu language distinguishes between corners with long sides ('*yabipaya*') vs. corners whose sides are distant from each other ('*yabiweka*'), a distinction not lexicalized in English. The development of visual representations of forms appears surprisingly impervious to all these differences, an observation compatible with two different explanations. Perhaps, the shift evidenced here in U.S. children is not triggered by the introduction of angle in school but rather by experiences shared by these two populations. Perhaps too, people from the U.S. and Mundurucu populations converge on shape properties in their analysis of forms via different developmental trajectories. In particular, the shift documented in our U.S. participants may not be universal.

[10] We thank Renée Baillargeon, who served as an Editor for this manuscript, for raising this theoretical possibility.

4.5. Conclusion

In our experiments, participants from the U.S. (including a group of preschoolers) and from the Amazon (children and adults; all devoid of formal education in geometry) detected variations in size or shape on visual forms while disregarding irrelevant variations in orientation and position. They also ignored variations in sense. Their analysis of visual forms thus aligned with Euclidean geometry: the geometry describing figures in terms of the properties of size and shape only, up to a rotation, translation, and/or symmetry. These findings show that humans can possess Euclidean intuitions prior to any formal education in geometry, lending support to the idea that visual perception provides foundations for Euclidean knowledge.

Why does perception encode Euclidean geometry – what computational role may Euclidean representations play for vision? At first view, high-level "non-accidental" cues (Biederman et al., 2009) seem more informative than size or shape to process 3-dimensional objects, as these cues are invariant across viewpoints. However, while descriptions based on non-accidental properties provide sufficient information to recognize object kinds, they are often too coarse to support the recognition of token objects (Ons & Wagemans, 2011). Encoding information about size and shape could provide finer description of object forms, supporting discriminations between objects that are similar in structure. Sense, in contrast, may not be as informative: given that many animate and inanimate objects often have two symmetric sides, two images differing only in sense most likely emanate from the same object viewed from different angles, rather than from two different objects. In summary, we propose that sensitivity to Euclidean geometry evolved to solve the task of token identification[11], as a complement to the analysis of non-accidental cues supporting the recognition of object kinds.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.cogpsych.2022.101494.

References

Abecassis, M., Sera, M. D., Yonas, A., & Schwade, J. (2001). What's in a shape? Children represent shape variability differently than adults when naming objects. *Journal of Experimental Child Psychology, 78*(3), 213–239. https://doi.org/10.1006/jecp.2000.2573

Amir, O., Biederman, I., & Hayworth, K. J. (2011). The neural basis for shape preferences. *Vision Research, 51*(20), 2198–2206. https://doi.org/10.1016/j.visres.2011.08.015

Amir, O., Biederman, I., Herald, S. B., Shah, M. P., & Mintz, T. H. (2014). Greater sensitivity to nonaccidental than metric shape properties in preschool children. *Vision Research, 97*, 83–88. https://doi.org/10.1016/j.visres.2014.02.006

Arguin, M., & Leek, E. C. (2003). Orientation invariance in visual object priming depends on prime—target asynchrony. *Attention, Perception, & Psychophysics, 65*(3), 469–477. https://doi.org/10.3758/BF03194576

Augustine, E., Smith, L. B., & Jones, S. S. (2011). Parts and Relations in Young Children's Shape-Based Object Recognition. *Journal of Cognition and Development, 12*(4), 556–572. https://doi.org/10.1080/15248372.2011.560586

Ayzenberg, V., & Lourenco, S. F. (2019). Skeletal descriptions of shape provide unique perceptual information for object recognition. *Scientific Reports, 9*(1), 1–13. https://doi.org/10.1038/s41598-019-45268-y

[11] More specifically, Euclidean representations may be involved in the recognition of objects that are often viewed in various orientation, such as tools. In practice, many objects are commonly viewed at a fixed canonical orientation (e.g. tables, lamps, faces), waiving the need to implement orientation invariance. These objects are indeed better recognized when presented in their canonical orientation (e.g. Jolicoeur, 1985; Leek, 1998; Tarr, 1995), unlike poly-oriented objects (Leek, 1998). It is possible that, when it comes to mono-oriented objects, the mind represents shape and size in an orientation-specific fashion. It is also possible that shape and size are not represented for these objects and are replaced by other properties (e.g. the position of each part of the object in the visual field). Our findings cannot speak of the existence of orientation-specific representations, since we systematically rotated stimuli and forced our participants to discard orientation.

Baldy, R., Devichi, C., Aubert, F., Munier, V., Merle, H., Dusseau, J.-M., & Favrat, J.-F. (2005). Développement cognitif et apprentissages scolaires: L'exemple de l'acquisition du concept d'angle. *Revue française de pédagogie*, 49–61. https://www.jstor.org/stable/41202066.

Berthoz, A. (2000). *The brain's sense of movement*. Harvard University Press.

Besner, D. (1983). Visual pattern recognition: Size preprocessing re-examined. *The Quarterly Journal of Experimental Psychology Section A, 35*(1), 209–216. https://doi.org/10.1080/14640748308402126

Biederman, I. (1987). Recognition-by-components: A theory of human image understanding. *Psychological Review, 94*(2), 115–147. https://doi.org/10.1037/0033-295X.94.2.115

Biederman, I., & Cooper, E. E. (1991). Evidence for complete translational and reflectional invariance in visual object priming. *Perception, 20*, 585–593. https://doi.org/10.1068/p200585

Biederman, I., & Cooper, E. E. (2009). Translational and reflectional priming invariance: A retrospective. *Perception, 38*, 809–825. https://doi.org/10.1068/pmkbie

Biederman, I., Yue, X., & Davidoff, J. (2009). Representation of shape in individuals from a culture with minimal exposure to regular, simple artifacts: Sensitivity to nonaccidental versus metric properties. *Psychological Science, 20*(12), 1437–1442. https://doi.org/10.1111/j.1467-9280.2009.02465.x

Bundesen, C., & Larsen, A. (1975). Visual transformation of size. *Journal of Experimental Psychology: Human Perception and Performance, 1*(3), 214. https://doi.org/10.1037/0096-1523.1.3.214

Calero, C., Shalom, D. E., Spelke, E. S., & Sigman, M. (2019). Language, gesture, and judgment: Children's paths to abstract geometry. *Journal of Experimental Child Psychology, 177*, 70–85. https://doi.org/10.1016/j.jecp.2018.07.015

Chalmers, K. A., & Halford, G. S. (2003). Young children's understanding of oddity: Reducing complexity by simple oddity and "most different" strategies. *Cognitive Development, 18*(1), 1–23. https://doi.org/10.1016/S0885-2014(02)00140-5

Cheng, K., & Gallistel, C. R. (1984). Testing the geometric power of a spatial representation. In H. L. Roitblat, H. S. Terrace, & T. G. Bever (Eds.), *Animal Cognition* (pp. 409–423). Hillsdale, NJ: Erlbaum.

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education, 30*(2), 192–212. https://doi.org/10.2307/749610

Cohen, L. B., & Younger, B. A. (1984). Infant perception of angular relations. *Infant Behavior and Development, 7*, 37–47. https://doi.org/10.1016/S0163-6383(84)80021-1

Cooper, L. A. (1975). Mental rotation of random two-dimensional shapes. *Cognitive Psychology, 7*(1), 20–43. https://doi.org/10.1016/0010-0285(75)90003-1

Cooper, L. A., & Shepard, R. N. (1973). Chronometric studies of the rotation of mental images. In W. G. Chase (Ed.), *Visual Information Processing* (pp. 75–176). New York: Academic Press.

Corballis, M. C. (1988). Recognition of disoriented shapes. *Psychological Review, 95*(1), 115. https://doi.org/10.1037/0033-295X.95.1.115

Cornell, J. M. (1985). Spontaneous mirror-writing in children. *Canadian Journal of Psychology, 39*(1), 174–179. https://doi.org/10.1037/h0080122

Cronin, V. (1967). Mirror-image reversal discrimination in kindergarten and first-grade children. *Journal of Experimental Child Psychology, 5*(4), 577–585. https://doi.org/10.1016/0022-0965(67)90051-3

Danziger, E., & Pederson, E. (1998). Though the looking glass: Literacy, writing systems and mirror-image discrimination. *Written language and literacy, 1*, 153–167. https://doi.org/10.1075/wll.1.2.02dan

De Cruz, H. (2007). An enhanced argument for innate elementary geometric knowledge and its philosophical implications. In B. Van Kerkhove (Ed.), *New perspectives on mathematical practices. Essays in philosophy and history of mathematics*. World Scientific.

Dehaene, S. (2009). *Reading in the Brain: The Science and Evolution of a Human Invention*. New York: Penguin Viking.

Dehaene, S., Izard, V., Pica, P., & Spelke, E. S. (2006). Core knowledge of geometry in an Amazonian indigene group. *Science, 311*(5759), 381–384. https://doi.org/10.1126/science.1121739

Dehaene, S., Nakamura, K., Jobert, A., Kuroki, C., Ogawa, S., & Cohen, L. (2009). Why do children make mirror errors in reading? Neural correlates of mirror invariance in the visual word form area. *Neuroimage, 49*(2), 1837–1848. https://doi.org/10.1016/j.neuroimage.2009.09.024

Descartes, R. (1637 [2001]). *Discourse on method, optics, geometry, and meteorology*. Hackett Publishing.

Dessalegn, B., & Landau, B. (2013). Interaction between language and vision: It's momentary, abstract, and it develops. *Cognition, 127*(3), 331–344. https://doi.org/10.1016/j.cognition.2013.02.003

Devichi, C., & Munier, V. (2013). About the concept of angle in elementary school: Misconceptions and teaching sequences. *The Journal of Mathematical Behavior, 32*(1), 1–19. https://doi.org/10.1016/j.jmathb.2012.10.001

Dillon, M. R., Duyck, M., Dehaene, S., Amalric, M., & Izard, V. (2019). Geometric categories in cognition. *Journal of Experimental Psychology: Human Perception and Performance, 45*(9), 1236. https://doi.org/10.1037/xhp0000663

Dillon, M. R., Izard, V., & Spelke, E. S. (2020). Infants' sensitivity to shape changes in 2D visual forms. *Infancy, 25*(5), 618–639. https://doi.org/10.1111/infa.12343

Dillon, M. R., & Spelke, E. S. (2018). From map reading to geometric intuitions. *Developmental Psychology, 54*(7), 1304–1316. https://doi.org/10.1037/dev0000509

Ehrlich, S. B., Levine, S. C., & Goldin-Meadow, S. (2006). The importance of gesture in children's spatial reasoning. *Developmental Psychology, 42*(6), 1259–1268. https://doi.org/10.1037/0012-1649.42.6.1259

Elliott, L., Feigenson, L., Halberda, J., & Libertus, M. E. (2019). Bidirectional, longitudinal associations between math ability and approximate number system precision in childhood. *Journal of Cognition and Development, 20*(1), 56–74. https://doi.org/10.1080/15248372.2018.1551218

Estes, D. (1998). Young children's awareness of their mental activity: The case of mental rotation. *Child Development, 69*(5), 1345–1360. https://doi.org/10.2307/1132270

Fiser, J., Subramaniam, S., & Biederman, I. (2001). Size tuning in the absence of spatial frequency tuning in object recognition. *Vision Research, 41*(15), 1931–1950. https://doi.org/10.1016/S0042-6989(01)00062-1

Frick, A., Daum, M. M., Walser, S., & Mast, F. W. (2009). Motor processes in children's mental rotation. *Journal of Cognition and Development, 10*(1–2), 18–40. https://doi.org/10.1080/15248370902966719

Frick, A., Ferrara, K., & Newcombe, N. S. (2013). Using a touch screen paradigm to assess the development of mental rotation between 3½ and 5½ years of age. *Cognitive Processing, 14*(2), 117–127. https://doi.org/10.1007/s10339-012-0534-0

Frick, A., Hansen, M. A., & Newcombe, N. S. (2013). Development of mental rotation in 3-to 5-year-old children. *Cognitive Development, 28*(4), 386–399. https://doi.org/10.1016/j.cogdev.2013.06.002

Frick, A., & Möhring, W. (2013). Mental object rotation and motor development in 8- and 10-month-old infants. *Journal of Experimental Child Psychology, 115*(4), 708–720. https://doi.org/10.1016/j.jecp.2013.04.001

Frick, A., Möhring, W., & Newcombe, N. S. (2014). Development of mental transformation abilities. *Trends in Cognitive Sciences, 18*(10), 536–542. https://doi.org/10.1016/j.tics.2014.05.011

Funk, M., Brugger, P., & Wilkening, F. (2005). Motor processes in children's imagery: The case of mental rotation of hands. *Developmental Science, 8*(5), 402–408. https://doi.org/10.1111/j.1467-7687.2005.00428.x

Gallistel, C. R. (1990). *The Organisation of Learning*. Cambridge, Massachussets: MIT Press.

Giaquinto, M. (2005). From symmetry perception to basic geometry. In *Visualization, explanation and reasoning styles in mathematics* (pp. 31–55). Springer.

Gibson, B. M., Lazareva, O. F., Gosselin, F., Schyns, P. G., & Wasserman, E. A. (2007). Nonaccidental properties underlie shape recognition in mammalian and nonmammalian vision. *Current Biology, 17*(4), 336–340. https://doi.org/10.1016/j.cub.2006.12.025

Gibson, D. J., Congdon, E. L., & Levine, S. C. (2015). The effects of word-learning biases on children's concept of angle. *Child Development, 86*(1), 319–326. https://doi.org/10.1111/cdev.12286

Gibson, E. J., Gibson, J. J., Pick, A. D., & Osser, H. (1962). A developmental study of the discrimination of letter-like forms. *Journal of Comparative and Physiological Psychology, 55*, 897–906. https://doi.org/10.1037/h0043190

Gregory, E., & McCloskey, M. (2010). Mirror-image confusions: Implications for representation and processing of object orientation. *Cognition, 116*(1), 110–129. https://doi.org/10.1016/j.cognition.2010.04.005

Hart, Y., Dillon, M. R., Marantan, A., Cardenas, A. L., Spelke, E., & Mahadevan, L. (2018). The statistical shape of geometric reasoning. *Scientific Reports, 8*(1), 12906. https://doi.org/10.1038/s41598-018-30314-y

Hart, Y., Mahadevan, L., & Dillon, M. R. (2022). Euclid's Random Walk: Developmental Changes in the Use of Simulation for Geometric Reasoning. *Cognitive Science, 46*(1), Article e13070. https://doi.org/10.1111/cogs.13070

Hatfield, G. (1998). *The Natural and the Normative: Theories of Spatial Perception from Kant to Helmholtz*. Cambridge, Massachussets and London, England: The MIT Press.

Hoffman, D. D., & Richards, W. A. (1984). Parts of recognition. *Cognition, 18*(1–3), 65–96. https://doi.org/10.1016/0010-0277(84)90022-2

Hohol, M. (2019). *Foundations of geometric cognition*. Routledge.

Howard, J. H., & Kerst, S. M. (1978). Directional effects of size change on the comparison of visual shapes. *The American Journal of Psychology, 491–499*. https://doi.org/10.2307/1421695

Hu, Q., & Zhang, M. (2019). The development of symmetry concept in preschool children. *Cognition, 189*, 131–140. https://doi.org/10.1016/j.cognition.2019.03.022

Huizinga, M., Dolan, C. V., & Van der Molen, M. W. (2006). Age-related change in executive function: Developmental trends and a latent variable analysis. *Neuropsychologia, 44*(11), 2017–2036. https://doi.org/10.1016/j.neuropsychologia.2006.01.010

Hummel, J. E., & Biederman, I. (1992). Dynamic binding in a neural network for shape recognition. *Psychological Review, 99*(3), 480. https://doi.org/10.1037/0033-295X.99.3.480

Huttenlocher, J. (1967). Discrimination of figure orientation: Effects of relative position. *Journal of Comparative and Physiological Psychology, 63*(2), 359–361. https://doi.org/10.1037/h0024398

Huttenlocher, J., Duffy, S., & Levine, S. (2002). Infants and toddlers discriminate amount: Are they measuring? *Psychol Sci, 13*(3), 244–249. https://doi.org/10.1111/1467-9280.00445

Izard, V., Pica, P., Dehaene, S., Hinchey, D., & Spelke, E. S. (2011). Geometry as a universal mental construction. In S. Dehaene, & E. M. Brannon (Eds.), *Space, Time and Number in the Brain. Searching for the Foundations of Mathematical Thought* (Vol. 24, p. pp. 374):). Academic Press.

Izard, V., Pica, P., Spelke, E. S., & Dehaene, S. (2011). Flexible intuitions of Euclidean geometry in an Amazonian indigene group. *Proceedings of the National Academy of Sciences, 108*(24), 9782–9787. https://doi.org/10.1073/pnas.1016686108

Izard, V., & Spelke, E. S. (2009). Development of sensitivity to geometry in visual forms. *Human Evolution, 23*(3), 213–248.

Jolicoeur, P. (1985). The time to name disoriented natural objects. *Memory & Cognition, 13*(4), 289–303. https://doi.org/10.3758/BF03202498

Jüttner, M., Petters, D., Wakui, E., & Davidoff, J. (2014). Late development of metric part-relational processing in object recognition. *Journal of Experimental Psychology: Human Perception and Performance, 40*(4), 1718–1734. https://doi.org/10.1037/a0037288

Kalénine, S., Cheam, C., Izard, V., & Gentaz, E. (2013). Adults and 5-year-old children draw rectangles and triangles around a prototype but not in the golden ratio. *British Journal of Psychology, 104*(3), 400–412. https://doi.org/10.1111/j.2044-8295.2012.02129.x

Kant, I. (1781 [2005]). *Critique of Pure Reason* (P. G. a. A. W. Wood, Trans.). Cambridge: Cambridge University Press.

Kaushall, P., & Parsons, L. M. (1981). Optical information and practice in the discrimination of 3-D mirror-reflected objects. *Perception, 10*(5), 545–562. https://doi.org/10.1068/p100545

Kennedy, G. J., Orbach, H. S., & Loffler, G. (2008). Global shape versus local feature: An angle illusion. *Vision Research, 48*(11), 1281–1289. https://doi.org/10.1016/j.visres.2008.03.003

Klein, F. C. (1893). A comparative review of recent researches in geometry. *Bulletin of the New York Mathematical Society, 2*, 215–249. https://doi.org/10.1090/S0002-9904-1893-00147-X

Koenker, R. (2022). quantreg: Quantile Regression. *R package version, 5*, 99. https://CRAN.R-project.org/package=quantreg.

Kolinsky, R., Verhaeghe, A., Fernandes, T., Mengarda, E. J., Grimm-Cabral, L., & Morais, J. (2011). Enantiomorphy through the looking glass: Literacy effects on mirror-image discrimination. *Journal of Experimental Psychology: General, 140*(2), 210–238. https://doi.org/10.1037/a0022168

Kosslyn, S. M., Margolis, J. A., Barrett, A. M., Goldknopf, E. J., & Daly, P. F. (1990). Age differences in imagery abilities. *Child Development, 61*(4), 995–1010. https://doi.org/10.1111/j.1467-8624.1990.tb02837.x

Krüger, M. (2018). Three-year-olds solved a mental rotation task above chance level, but no linear relation concerning reaction time and angular disparity presented itself. *Frontiers in Psychology, 9*(1796). https://doi.org/10.3389/fpsyg.2018.01796

Larsen, A. (1985). Pattern matching: Effects of size ratio, angular difference in orientation, and familiarity. *Perception & Psychophysics, 38*(1), 63–68. https://doi.org/10.3758/BF03202925

Larsen, A., & Bundesen, C. (1978). Size scaling in visual pattern recognition. *Journal of Experimental Psychology: Human Perception and Performance, 4*(1), 1–20. https://doi.org/10.1037/0096-1523.4.1.1

Lauer, J. E., Udelson, H. B., Jeon, S. D., & Lourenco, S. F. (2015). An early sex difference in the relation between mental rotation and object preference. *Frontiers in Psychology, 6*. https://doi.org/10.3389/fpsyg.2015.00558

Lazareva, O. F., Wasserman, E. A., & Biederman, I. (2008). Pigeons and humans are more sensitive to nonaccidental than to metric changes in visual objects. *Behavioural Processes, 77*(2), 199–209. https://doi.org/10.1016/j.beproc.2007.11.009

Leek, E. (1998). Effects of stimulus orientation on the identification of common polyoriented objects. *Psychonomic Bulletin & Review, 5*(4), 650–658. https://doi.org/10.3758/BF03208841

Legare, C. H., Dale, M. T., Kim, S. Y., & Deák, G. O. (2018). Cultural variation in cognitive flexibility reveals diversity in the development of executive functions. *Scientific Reports, 8*(1), 16326. https://doi.org/10.1038/s41598-018-34756-2

Lehrer, R., Jenkins, M., & Osana, H. (1998). Longitudinal study of children's reasoning about space and geometry. *Designing learning environments for developing understanding of geometry and space, 1*, 137–167.

Lenth, R. V. (2022). emmeans: Estimated Marginal Means, aka Least-Squares Means. *R package version, 1*(7), 2. https://CRAN.R-project.org/package=emmeans.

Li, P., Abarbanell, L., Gleitman, L., & Papafragou, A. (2011). Spatial reasoning in Tenejapan Mayans. *Cognition, 120*(1), 33–53. https://doi.org/10.1016/j.cognition.2011.02.012

Lindskog, M., Rogell, M., Kenward, B., & Gredebäck, G. (2019). Discrimination of small forms in a deviant-detection paradigm by 10-month-old infants. *Frontiers in Psychology, 10*. https://doi.org/10.3389/fpsyg.2019.01032

Lourenco, S. F., & Huttenlocher, J. (2008). The Representation of Geometric Cues in Infancy. *Infancy, 13*, 103–127. https://doi.org/10.1080/15250000701795572

Lowet, A. S., Firestone, C., & Scholl, B. J. (2018). Seeing structure: Shape skeletons modulate perceived similarity. *Attention, Perception, & Psychophysics, 80*(5), 1278–1289. https://doi.org/10.3758/s13414-017-1457-8

Mach, E. (1914). *The analysis of sensations, and the relations of the physical to the psychical*. Chicago, London: Open Court Publishing Company.

Marmor, G. S. (1975). Development of kinetic images: When does the child first represent movement in mental images? *Cognitive Psychology, 7*(4), 548–559. https://doi.org/10.1016/0010-0285(75)90022-5

Marmor, G. S. (1977). Mental rotation and number conservation: Are they related? *Developmental Psychology, 13*(4), 320–325. https://doi.org/10.1037/0012-1649.13.4.320

Marr, D., & Nishihara, H. K. (1978). Representation and recognition of the spatial organization of three-dimensional shapes. *Proceedings of the Royal Society of London. Series B. Biological Sciences, 200*(1140), 269–294. https://doi.org/10.1098/rspb.1978.0020

McCrink, K., Spelke, E. S., Dehaene, S., & Pica, P. (2013). Non-symbolic halving in an Amazonian indigene group. *Developmental Science, 16*(3), 451–462. https://doi.org/10.1111/desc.12037

Möhring, W., Newcombe, N. S., & Frick, A. (2014). Zooming in on spatial scaling: Preschool children and adults use mental transformations to scale spaces. *Developmental Psychology, 50*(5), 1614–1619. https://doi.org/10.1037/a0035905

Möhring, W., Newcombe, N. S., & Frick, A. (2016). Using mental transformation strategies for spatial scaling: Evidence from a discrimination task. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 42*(9), 1473. https://doi.org/10.1037/xlm0000240

Moore, D. S., & Johnson, S. P. (2020). *The development of mental rotation ability across the first year after birth*. Advances in Child Development and Behavior.

Morgan, M. J. (2005). The visual computation of 2-D area by human observers. *Vision Research, 45*(19), 2564–2570. https://doi.org/10.1016/j.visres.2005.04.004
Mumma, J. (2010). Proofs, pictures, and Euclid. *Synthese, 175*(2), 255–287. https://doi.org/10.1007/s11229-009-9509-9
Newcombe, N. S., & Uttal, D. H. (2006). Whorf versus Socrates, round 10. *Trends in Cognitive Sciences, 10*(9), 394–396. https://doi.org/10.1016/j.tics.2006.07.008
Norman, J. F., Todd, J. T., Perotti, V. J., & Tittle, J. S. (1996). The visual perception of three-dimensional length. *Journal of Experimental Psychology: Human Perception and Performance, 22*(1), 173–186. https://doi.org/10.1037/0096-1523.22.1.173
Nys, J., Ventura, P., Fernandes, T., Querido, L., & Leybaert, J. (2013). Does math education modify the approximate number system? A comparison of schooled and unschooled adults. *Trends in Neuroscience and Education, 2*(1), 13–22. https://doi.org/10.1016/j.tine.2013.01.001
Odic, D. (2018). Children's intuitive sense of number develops independently of their perception of area, density, length, and time. *Developmental Science, 21*(2), Article e12533. https://doi.org/10.1111/desc.12533
Ons, B., & Wagemans, J. (2011). Development of differential sensitivity for shape changes resulting from linear and nonlinear planar transformations. *i-Perception, 2*(2), 121–136. https://doi.org/10.1068/i0407
Pegado, F., Nakamura, K., Braga, L. W., Ventura, P., Nunes Filho, G., Pallier, C., … Dehaene, S. (2014). Literacy breaks mirror invariance for visual stimuli: A behavioral study with adult illiterates. *Journal of Experimental Psychology: General, 143*(2), 887–894. https://doi.org/10.1037/a0033198
Pereira, A. F., & Smith, L. B. (2009). Developmental changes in visual object recognition between 18 and 24 months of age. *Developmental Science, 12*(1), 67–80. https://doi.org/10.1111/j.1467-7687.2008.00747.x
Piaget, J., Inhelder, B., & Szeminska, A. (1960). *Child's Concept Of Geometry*. Basic Books.
Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences, 14*(12), 542. https://doi.org/10.1016/B978-0-12-385948-8.00017-7
Piazza, M., Pica, P., Izard, V., Spelke, E. S., & Dehaene, S. (2013). Education Enhances the Acuity of the Nonverbal Approximate Number System. *Psychological Science, 24*(6), 1037–1043. https://doi.org/10.1177/0956797612464057
Pica, P., Izard, V., Rooryck, J., Tonda, A., Dehaene, S., Spelke, E. S., & Saw, J. (2022). *Mundurucu terms with arithmetical, geometrical and spatial content*. https://doi.org/10.23668/psycharchives.6634.
Plato. (ca. 380 B.C.). *Meno*. available online at classics.mit.edu.
Poincaré, H. (1902). *La Science et l'Hypothèse*. Paris: Flammarion.
Quinn, P. C., & Liben, L. S. (2008). A sex difference in mental rotation in young infants. *Psychological Science, 19*, 1067–1070. https://doi.org/10.1111/j.1467-9280.2008.02201.x
Regan, D., & Hamstra, S. J. (1992). Shape discrimination and the judgement of perfect symmetry: Dissociation of shape from size. *Vision Research, 32*(10), 1845–1864. https://doi.org/10.1016/0042-6989(92)90046-L
Rock, I. (1973). *Orientation and form*. New York: Academic Press.
Sablé-Meyer, M., Fagot, J., Caparos, S., van Kerkoerle, T., Amalric, M., & Dehaene, S. (2021). Sensitivity to geometric shape regularity in humans and baboons: A putative signature of human singularity. *Proceedings of the National Academy of Sciences, 118*(16), Article e2023123118. https://doi.org/10.1073/pnas.2023123118
Satlow, E., & Newcombe, N. (1998). When is a triangle not a triangle? Young children's developing concepts of geometric shape. *Cognitive Development, 13*, 547–559. https://doi.org/10.1016/S0885-2014(98)90006-5
Schwartz, M., Day, R. H., & Cohen, L. B. (1979). Visual shape perception in early infancy. *Monographs of the Society for Research in. Child Development, 44(7), serial No. 182*. https://doi.org/10.2307/1165963
Sekuler, R., & Nash, D. (1972). Speed of size scaling in human vision. *Psychonomic Science, 27*(2), 93–94. https://doi.org/10.3758/BF03328898
Serpell, R. (1971). Discrimination of orientation by Zambian children. *Journal of Comparative and Physiological Psychology, 75*(2), 312–316. https://doi.org/10.1037/h0030832
Shepard, R. N. (2001). Perceptual-cognitive universals as reflections of the world. *Behavioral and Brain Sciences, 24*(04), 581–601. https://doi.org/10.1017/S0140525X01000012
Shepard, R. N., & Metzler, J. (1971). Mental Rotation of Three-Dimensional Objects. *Science, 171*(3972), 701–703. https://doi.org/10.1126/science.171.3972.701
Slater, A., Mattock, A., & Brown, E. (1990). Size constancy at birth: Newborn infants' responses to retinal and real size. *Journal of Experimental Child Psychology, 49*, 314–322. https://doi.org/10.1016/0022-0965(90)90061-C
Slater, A., Mattock, A., Brown, E., & Bremner, J. G. (1991). Form perception at birth: Cohen and Younger (1984) revisited. *Journal of Experimental Child Psychology, 51*, 395–406. https://doi.org/10.1016/0022-0965(91)90084-6
Spelke, E. (1994). Initial knowledge: Six suggestions. *Cognition, 50*(1), 431–445. https://doi.org/10.1016/0010-0277(94)90039-6
Strickland, B. (2017). Language reflects "core" cognition: A new theory about the origin of cross-linguistic regularities. *Cognitive Science, 41*(1), 70–101. https://doi.org/10.1111/cogs.12332
Szwed, M., Cohen, L., Qiao, E., & Dehaene, S. (2009). The role of invariant line junctions in object and visual word recognition. *Vision Research, 49*(7), 718–725. https://doi.org/10.1016/j.visres.2009.01.003
Takano, Y. (1989). Perception of rotated forms: A theory of information types. *Cognitive Psychology, 21*(1), 1–59. https://doi.org/10.1016/0010-0285(89)90002-9
Tarr, M. J. (1995). Rotating objects to recognize them: A case study on the role of viewpoint dependency in the recognition of three-dimensional objects. *Psychonomic Bulletin & Review, 2*(1), 55–82. https://doi.org/10.3758/BF03214412
Tarr, M. J., & Pinker, S. (1989). Mental rotation and orientation-dependence in shape recognition. *Cognitive Psychology, 21*(2), 233–282. https://doi.org/10.1016/0010-0285(89)90009-1
Tarr, M. J., & Pinker, S. (1990). When does human object recognition use a viewer-centered reference frame? *Psychological Science, 1*(4), 253–256. https://doi.org/10.1111/j.1467-9280.1990.tb00209.x
Terepocki, M., Kruk, R. S., & Willows, D. M. (2002). The incidence and nature of letter orientation errors in reading disability. *Journal of Learning Disabilities, 35*(3), 214–233. https://doi.org/10.1177/002221940203500304
Todd, J. T., Chen, L., & Norman, J. F. (1998). On the relative salience of Euclidean, affine, and topological structure for 3-D form discrimination. *Perception, 27*(3), 273–282. https://doi.org/10.1068/p270273
van der Ham, I. J. M., Hamami, Y., & Mumma, J. (2017). Universal intuitions of spatial relations in elementary geometry. *Journal of Cognitive Psychology, 29*(3), 269–278. https://doi.org/10.1080/20445911.2016.1257623
Vanrie, J., Beatse, E., Wagemans, J., Sunaert, S., & Van Hecke, P. (2002). Mental rotation versus invariant features in object perception from different viewpoints: An fMRI study. *Neuropsychologia, 40*(7), 917–930. https://doi.org/10.1016/S0028-3932(01)00161-0
Vanrie, J., Willems, B., & Wagemans, J. (2001). Multiple routes to object matching from different viewpoints: Mental rotation versus invariant features. *Perception, 30*(9), 1047–1056. https://doi.org/10.1068/p3200
Wagemans, J., Van Gool, L., Lamote, C., & Foster, D. H. (2000). Minimal information to determine affine shape equivalence. *Journal of Experimental Psychology: Human Perception and Performance, 26*(2), 443–468. https://doi.org/10.1037/0096-1523.26.2.443
Wenderoth, P., & Johnson, M. (1984). The effects of angle-arm length on judgments of angle magnitude and orientation contrast. *Attention, Perception, & Psychophysics, 36*(6), 538–544. https://doi.org/10.3758/BF03207514
Werkhoven, P., & Koenderink, J. J. (1993). Visual size invariance does not apply to geometric angle and speed of rotation. *Perception, 22*(2), 177–184. https://doi.org/10.1068/p220177