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Artin's Characters Table of the Group (Q_{2n}×D_3) When n=$p_1.p_2 \dots .p_n$, and $p_1, p_2, \dots, p_n$ are Primes Number

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Abstract: *The main purpose of this paper is to find Artin's characters table of the group (Q_{2n}×D_3)when n=$p_1.p_2 \dots .p_n$,and $p_1, p_2, \dots, p_n$ are primes number, which is denoted by Ar(Q_{2n}×D_3) where Q_{2m} is denoted to Quaternion group and D_3 is the Dihedral group of order 6 .*

Keywords: Artin, characters,group, Q_{2n} , D_3 ,prime.

1. INTRODUCTION

Let G be a finite group, two elements of G are said to be Γ-conjugate if the cyclic subgroups they generate are conjugate in G and this defines an equivalence relation on G and its classes are called Γ-classes [3].

Let H be a subgroup of G and let ϕ be a class function on H, the induced class function on G, is given by: $\phi'(g) = \frac{1}{|H|}\sum_{h\in G} \phi^{\circ}(hgh^{-1})$, ∀g∈ G, where ϕ° is defined by:

$$\phi^{\circ}(x) = \begin{cases} \phi(x) & \text{if } x \in H \\ 0 & \text{if } x \notin H \end{cases} \quad [2].$$

Let H be a subgroup of G and ϕ be a character of H, then ϕ′ is a character of G, and it is called the induced character on G[7].

In 1976 ,David.G[3] studied "Artin Exponent of arbitrary characters of cyclic subgroup ", Journal of Algebra,61,p 58-76.

In 1996, Knwabusz .K[9] studied "Some Definitions of Artin's Exponent of finite Group", USA, National foundation Math,GR.

In this work we find Artin's characters table of the qroup (Q_{2n}×D_3) when n=$p_1.p_2 \dots .p_n$,and $p_1, p_2, \dots, p_n$ are primes number.

2. PRELIMINARIES

2.1The Generalized Quaternion Group Q_{2m} [7]

For each positive integer m,the generalized Quaternion Group Q_{2m} of order 4m with two generators x and y satisfies $Q_{2n}=\{x^h y^k$,$0\le h \le 2n-1$,k=0,1} which has the following properties

$\{x^{2n}=y^4=I, yx^ny^{-1}=x^{-n}\}$.

2.2The Dihedral Group D_3 [9]

The Dihedrael Group D3 is generate by a rotation r of order 3 and reflection s of order 2 then 6 elements of D3 can be written as: $\{1,r,r^2,s,sr,sr^2\}$.

2.3The Rational valued characters table:

Definition(2.3.1) [5]

A rational valued character θ of G is a character whose values are in Z,which is θ(g)∈Z for all g∈ G.

Theorem (2.3.2)[9]

Every rational valued character of G be written as a linear combination of Artin's characters with coefficient rational numbers.

Corollary (2.3.3)[5]

The rational valued characters $\theta_i = \sum_{\sigma\epsilon Gal(Q(\chi_i)/Q)} \sigma(\chi_i)$Form a basis for $\bar{R}(G)$,whereχ_i are the irreducible characters of G and their numbers are equal to the number of conjugacy classes of a cyclic subgroup of G.

Proposition(2.3.4)[9]

The number of all rational valued characters of finite G is equal to the number of all distinct Γ-classis.

Definition (2.3.5)[5]

The information about rational valued characters of a finite group G is displayed in a table called a rational valued characters table of G.We denote it by ≡*(G)which is $l \times l$ matrix whose columns are Γ-classes and rows are the valuses of all rational valued characters where l is the number of Γ-classes.

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The rational character table of Q_{2m} where m is an odd number(2.3.6) [7]

Table(1)

	Γ-classes of c_{2m} X^{2r}	X^{2r+1}	[y]
Θ_1	1 1 … 1	1 1 … 1	1
Θ_2	≡*(C_m)	≡*(C_m)	0
⋮			⋮
$\Theta_{l/2}$			0
$\Theta_{(l/2)+1}$	1 1 … 1	1 1 … 1	-1
⋮	≡*(C_m)	H	0
Θ_{l-1}			⋮
Θ_l			0
Θ_{l+1}	2 2 ⋯ 2	-2 -2 ⋯ -2	0

Where $0\le r\le l$, l is the number of Γ-classes C_{2m} ,θj such that $1\le j\le l+1$ are the rational valued characters of group Q_{2m} and if we denote Cij the elements of ≡*(C_m) and hij the elements of H as defined by: Hij= $\begin{cases} C_{ij} & if \quad i = l \\ -C_{ij} & if \quad i \neq l \end{cases}$

The rational character table of Q_{2n} when n=$p_1.p_2 \dots .p_n$,and $p_1,p_2,\dots,p_n$ are primes number (2.3.7)[7]

Table(2)

Γ-classes	[1]	[x^2]	[x^n]	[x]	[y]
Θ_1	1	1	1	1	1
θ_2	n-1	-1	n-1	-1	0
θ_3	1	1	1	1	1
θ_4	n-1	-1	1-n	1	0
θ_5	2	2	-2	-2	0

The rational character table of D_3(2.3.8)[6]

≡*(D_3)

Table(3)

classesΓ-	[I]	[r]	[s]
$\lvert CL_\alpha \rvert$	1	2	3
$\lvert C_{D_3}(cl_\alpha) \rvert$	6	3	2
θ_1	2	-1	0
θ_2	1	1	1
θ_3	1	1	1

3. ARTIN'S CHARACTER TABLES:

Theorem(3.1):[5]

Let H be a cyclic subgroup of G and $h_1,h_2,\dots,h_m$ are chosen representatives for the m-conjugate classes of H contained in CL(g) in G,then:

$$\varphi'(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{m}\varphi(h_i) & if\ h_i \in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

Propostion(3.2)[5]

The number of all distinct Artin's characters on a group G is equal to the number of Γ-classes on G.Furthermore , Artin's characters are constant on each Γ-classes.

Theorem(3.3) [10]

The Artin's characters table of the Quaternion group Q_{2n} when m is odd number is given as follows:

Table(4)

	Γ-classes of C_{2m}								
Γ-classes	x^{2r}				x^{2r+1}				[y]
$\lvert CL_\alpha\rvert$	1	2	⋯	2	1	2	⋯		2n
$\lvert C_{Q_{2n}}(CL_\alpha)\rvert$	4n	2n	⋯	2n	4n	2n	⋯		2
Φ_1		2Ar(C_{2n})							0
Φ_2									0
⋮									⋮
Φ_l									0
Φ_{l+1}	m	0	⋯	0	n	0	⋯	0	1

Where 0≤r≤m-1 , l is the number of Γ-classes of C_{2m} and Φj are the Artin characters of the Quaternion group Q_{2m},for all 1≤j≤l+1.

Artin characters table of Q_{2n} when n=$p_1.p_2 \dots .p_n$,and $p_1,p_2,\dots,p_n$ are primes number (4.4)[8]

The general form of Artin's characters of Q_{2n} when n=$p_1.p_2 \dots .p_n$,and $p_1,p_2,\dots,p_n$ are primes number

Table(5)

Γ-classes	[1]	[x^2]	[x^n]	[x]	[y]
$\lvert CL_\alpha\rvert$	1	2	1	2	2n
$\lvert C_{Q_{2n}}(CL_\alpha)\rvert$	4n	2n	4n	2n	2
Φ_1	4n	0	0	0	0
Φ_2	4	4	0	0	0
Φ_3	2n	0	2n	0	0
Φ_4	2	2	2	2	0
Φ_5	n	0	n	0	1

The Artin characters of D_3 (4.5)[9]

Table(6)

Γ-classes	[I]	[r]	[s]
$\lvert CL_\alpha\rvert$	1	2	3
$\lvert C_{D_3}(CL_\alpha)\rvert$	6	3	2
Φ_1	6	0	0
Φ_2	2	2	0
Φ_3	3	0	1

4. The Main Resulte

<u>Propostion(4.1)</u>

If n $=p_1.p_2 \dots .p_n$,and $p_1,p_2,\dots,p_n$ are primes number, then The Artin's character table of the group ($Q_{2n}\times D_3$) is given as: The general form of the Artin characters of the group($Q_{2n}\times D_3$)when n$=p_1.p_2 \dots .p_n$,and $p_1,p_2,\dots,p_n$ are primes number

Table(7)

Γ-classes	[1,I][x^2,I][x^n,I][x,I][y,I]	[1,r][x^2,r][x^n,r][x,r][y,r]	[1,s][x^2,s][x^n,s][x,s][y,s]
$\lvert CL_\alpha\rvert$	1 2 1 2 2n	1 2 1 2 2n	1 2 1 2 2n
$\lvert C_{Q_{2n}\times D_3}(CL_\alpha)\rvert$	24n 24n 12n 12	24n 12n 24n 12n 12	24p 12n 24n 12n 12
$\Phi_{(1,1)}$ $\Phi_{(2,1)}$ ⋮ $\Phi_{(l+1,1)}$	6Ar(Q_{2n})	0	0
$\Phi_{(1,2)}$ $\Phi_{(2,2)}$ ⋮ $\Phi_{(l+1,2}$	2Ar(Q_{2n})	2Ar(Q_{2n})	0
$\Phi_{(1,3)}$ $\Phi_{(2,3)}$ ⋮ $\Phi_{(l+1,3)}$	3Ar(Q_{2n})	0	Ar(Q_{2n})

which is (5×5) square matrix .

<u>Proof:</u> Let g∈ ($Q_{2n}\times D_3$); g=(q,d),q∈Q_{2n},d∈D_3

<u>Case(I)</u>:

If H is a cyclic subgroup of ($Q_{2n}\times\{I\}$),then 1- H=<(x,I)> 2- H=<(y,I)>

And φ the principle character of H, Φ_j Artin's characters of Q_{2n},1≤j≤l+1, then by using theorem (4.1)

$$\Phi_j(\mathrm{g}) = \begin{cases} \dfrac{|C_G(\mathrm{g})|}{|C_H(\mathrm{g})|}\displaystyle\sum_{i=1}^{n}\varphi(hi) & if\ hi \in H\cap CL(\mathrm{g}) \\ 0 & if\ H\cap CL(\mathrm{g}) = \phi \end{cases}$$

1- H=$<(x,I)>$

(i) If g=(1,I)

$\Phi_{(j,1)}(1,I)=\frac{|C_{Q_{2n}\times D_3}(g)|}{|C_H(g)|}.\ \varphi\ (g)=\frac{24n}{|C_H(g)|}.1=\frac{6.4n}{|C_H(g)|}.1=\frac{6|C_{Q_{2n}}(1)|}{|C_{<x>}(1)|}.\varphi(1)=6.\ \Phi_j(1)$ since H∩ $CL(1,I)=\{(1,I)\}$

(ii) If g=(x^n,I),g∈ H then

$\Phi_{(j,1)}(g)=\frac{|C_{Q_{2n}\times D_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{|C_H(g)|}.1=\frac{6|C_{Q_{2n}}(x^n)|}{|C_{<x>}(x^n)|}\ \varphi(x^n)=6.\ \Phi_j(x^n)$ since H∩ $CL(g)=\{g\},\varphi(g)=1$

(iii) If g=(x^2,I) or g=(x,I) and g∈ H then

$\Phi_{(j,1)}(g)=\frac{|C_{Q_{2n}\times D_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12n}{|C_H(g)|}(1+1)=\frac{3.4n}{|C_H(g)|}.2=\frac{3|C_{Q2n(q)}|}{|C_{H(q)}|}.2=6.\ \Phi_j(q)$

since H∩ $CL(g)=\{g,g^{-1}\}$ and φ(g)=ø(g^{-1})=1 and since g=(q,I),q∈Q_{2n},q≠x^n

(iv) if g∉H then

$\Phi_{(j,1)}(g)=0$ since $H\cap CL(g)=\phi$

2- If $H=<(y,I)>=\{(1,I),(y,I)(y^2,I)(y^3,I)\}$ then

(i) If $g=(1,I)$ then

$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2n}{}^{x}D_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{4}.1=6.n=6.\Phi_{l+1}(1)$ since $H\cap CL(1,I)=\{(1,I)\}$

(ii) If $g=(x^n,I)=(y^2,I)$ and $g\in H$ then

$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2n}{}^{x}D_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{4}.1=6.n=6.\Phi_{l+1}(x^n)$ since $H\cap CL(g)=\{g\},\varphi(g)=1$

(iii) If $g\neq(x^n,I)$ and $g\in H$,i.e.$\{g=(y,I)$ or $g=(y^3,I)\}$ then

$\Phi_{(l+1,1)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12}{4}(1+1)=3.2=6.\Phi_{l+1}(y)$since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

Otherwise

$\Phi_{(l+1,1)}(g)=0$ since $H\cap CL(g)=\phi$

Case(II):

If H is a cyclic subgroup of $(Q_{2n}x\{r\})$ then:

1- $H=<(x,r)>$ 2- $H=<(y,r)>$

1-$H=<(x,r)>$

and φ the principle character of H, then by using theorem (4.1)

$$\Phi_j(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{n}\varphi(h_i) & if\ h_i\in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

(i) If $g=(1,I),(1,r)$ then

$\Phi_{(j,2)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24.n}{|C_H(1,I)|}.1=\frac{6.4n}{|C_H(1,I)|}.1=\frac{6|C_{Q_{2n}}(1)|}{3|C_{<x>}(1)|}\varphi(1)=2.\Phi j(1)$

since $H\cap CL(g)=\{(1,I),(1,r),(1,r^2)\}$

(ii) $g=(1,I),(x^n,I),(x^n,r),(1,r);\ g\in H$

if $g=(1,I),(1,r)$ then

$\Phi_{(j,2)}(g)=\frac{|C_{Q2n}xD_3(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{|C_H(g)|}.1$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

$=\frac{6.3n}{|C_H(g)|}.1=\frac{6|C_{Q2n}(1)|}{3|C_{<x>}(1)|}\varphi(1)=2\Phi_j(1)$

(iii) if $g=(x^n,I),(x^n,r)$ then

$\Phi_{(j,2)}(g)=\frac{|C_{Q2n}xD_3(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{|C_H(g)|}.1=\frac{6.3p}{|C_H(g)|}.1=\frac{6|C_{Q2n}(x^n)|}{3|C_{<x>}(x^n)|}\varphi(1)=2\Phi_j(x^n)$

(iv) if $g\neq(x^n,I),(x^n,r)$ and $g\in H$ then

$\Phi_{(j,2)}(g)=\frac{|C_{Q2n}xD_3(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12n}{|C_H(g)|}(1+1)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

$=\frac{3.4n}{|C_H(g)|}(1+1)=\frac{3|C_{Q2n}(q)|}{3|C_{<x>}(q)|}.2=2\Phi_j(q)$

Since $g=(q,r),q\in Q_{2n},q\neq x^n$

(v) $if\ g\notin H$ then

$\Phi_{(j,2)}(g)=0=\Phi_j(q)$ since $H\cap CL(g)=\phi$

2- if $H=<(y,r)>=\{(1,I),(y,I),(y^2,I),(y^3,I),(1,r),(y,r),(y^2,r),(y^3,r)\}$

(i) if $g=(1,I),(1,r)$ then

$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{12}.1=2n=2\Phi_{l+1}(g)$

(ii) if $g=(y^2,I)=(x^n,I),(y^2,r)$ and $g\in H$ then

$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{12}.1=2n=2\Phi_{l+1}(g)$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

(iii) if $g\neq (x^n,I)$ and $g\in H$ i.e. $g=\{(y,I),(y,r)\}$ or $g=\{(y^3,I),(y^3,r)\}$ then

$\Phi_{(l+1,2)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12}{12}(1+1)=2\Phi_{l+1}(g)$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

otherwise $\Phi_{(l+1,2)}(g)=0$ since $H\cap CL(g)=\phi$

case(III):

if H is a cyclic subgroup of $(Q_{2nx}\{s\})$ then

1- H=<(x,s)>, 2- H=<(y,s)>

and φ the principle character of H, then by using theorem (4.1)

$$\Phi_j(g)=\begin{cases}\frac{|C_G(g)|}{|C_H(g)|}\sum_{i=1}^{n}\varphi(h_i) & if\ h_i\in H\cap CL(g)\\ 0 & if\ H\cap CL(g)=\phi\end{cases}$$

1- H=<(x,s)>

(i) If g=(1,I) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{|C_H(1,I)|}.1=\frac{6.4n}{|C_H(1,I)|}.1=\frac{6|C_{Q2n}(1)|}{2|C_{<x>}(1)|}.1=3\Phi_j(1)$ since $H\cap CL(g)=\{(1,I)\}$

If g={(1,s)} then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8n}{|C_H(1,s)|}.1=\frac{2.4n}{|C_H(1,s)|}.1=\frac{2|C_{Q2n}(1)|}{2|C_{<x>}(1)|}.1=\Phi_j(1)$ since $H\cap CL(g)=\{(1,s)\}$

(ii) If $g=(1,I),(x^n,I),(x^n,s),(1,s)$; $g\in H$ then

If g=(1,I) then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{|C_H(g)|}.1$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

$=\frac{6.4n}{|C_H(g)|}.1=\frac{6|C_{Q_{2n}}(1)|}{2|C_{<x>}(1)|}\varphi(1)=3\Phi_j(1)$

If g={(1,s)} then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8n}{|C_H(g)|}.1=\frac{2.4n}{|C_H(g)|}.1=\frac{2|C_{Q2n}(1)|}{2|C_{<x>}(1)|}.1=\Phi_j(1)$ since $H\cap CL(g)=\{g\}$ and $\varphi(g)=1$

(iii)If $g=(x^n,I)$ then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{|C_H(g)|}.1=\frac{6.4n}{|C_H(g)|}.1=\frac{6|C_{Q_{2n}}(x^n)|}{2|C_{<x>}(x^n)|}\varphi(1)=3\Phi_j(x^n)$

If $g=(x^n,s)$ then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8n}{|C_H(g)|}.1=\frac{2.4n}{|C_H(g)|}.1=\frac{2|C_{Q_{2n}}(x^n)|}{2|C_{<x>}(x^n)|}\varphi(1)=\Phi_j(x^n)$

(iv)If $g\neq (x^n,I),(x^n,s)$ and $g\in H$

If $g\neq (x^p,I)$ then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12n}{|C_H(g)|}(1+1)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

$=\frac{3.4n}{|C_H(g)|}(1+1)=\frac{3|C_{Q_{2n}}(q)|}{2|C_{<x>}(q)|}.2=3\Phi_j(q)$

Since $g=(q,I), q\in Q_{2n}$, $q\neq x^n$

If $g\neq (x^n,s)$ then

$\Phi_{(j,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{8n}{|C_H(g)|}(1+1)$ since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

$=\frac{2.4n}{|C_H(g)|}(1+1)=\frac{2|C_{Q_{2n}}(q)|}{4|C_{<x>}(q)|}.2=\Phi_j(q)$

Since $g=(q,s), q\in Q_{2n}$, $q\neq x^n$

(v) if $g\notin H$ then

$\Phi_{(j,3)}(g)=0=\Phi_j(q)$ since $H\cap CL(g)=\phi$

2-if $H=\langle(y,s)\rangle=\{(1,I),(y,I),(y^2,I),(y^3,I),(1,s),(y,s),(y^2,s),(y^3,s)\}$ then

(i)If g=(1,I) then

$$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{8}.1=3.n=3\Phi_{l+1}(g)$$

If g=(1,s) then

$$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8n}{8}.1=n=\Phi_{l+1}(g)$$

(ii)If $g=(y^2,I)=(x^n,I)$ and $g\in H$ then

$$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{24n}{8}.1=3.n=3\Phi_{l+1}(g)$$ since H∩CL(g)={g} and φ (g)=1

If $g=(y^2,s)$ and $g\in H$ then

$$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8n}{8}.1=n=\Phi_{l+1}(g)$$ since H∩CL(g)={g} and φ (g)=1

(iii)If $g\neq(x^n,I)$ and $g\in H$ i.e. g={(y,I),(y,s)} or $g=\{(y^3,I),(y^3,s)\}$ then

$$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{12}{8}(1+1)=3\Phi_{l+1}(g)$$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

(iv)If $g=(y^2,s)$, $g\epsilon H$ then

$$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}\varphi(g)=\frac{8n}{|C_H(g)|}.1=\frac{8n}{8}.1=\Phi_{l+1}(g)$$

(v)If g=(y,s) then

$$\Phi_{(l+1,3)}(g)=\frac{|C_{Q_{2n}xD_3}(g)|}{|C_H(g)|}(\varphi(g)+\varphi(g^{-1}))=\frac{4}{|C_H(g)|}.(1+1)=\frac{4}{8}.2=1$$

since $H\cap CL(g)=\{g,g^{-1}\}$ and $\varphi(g)=\varphi(g^{-1})=1$

otherwise $\Phi_{(l+1,3)}(g)=0$ since $H\cap CL(g)=\phi$

<u>Example (4.2)</u>: To find Artine's character table of the group $(Q_{66}xD_3)$.

$Ar(Q_{66}xD_3)=Ar(Q_{2.3.11}xD_3)=$

Table(8)

Γ-classes	[1,I]	[x^2,I]	[x^{33},I]	[x,I]	[y,I]	[1,r]	[x^2,r]	[x^{33},r]	[x,r]	[y,r]	[1,s]	[x^2,s]	[x^{33},s]	[x,s]	[y,s]
$\|cL_\alpha\|$	1	2	1	2	2n	2	2	2	2	2n	3	3	3	3	6n
$\|c_{Q_{2n}xD_3}(cL_\alpha)\|$	792	396	792	396	12	396	396	396	396	12	264	264	264	264	4
$\Phi_{(1,1)}$	792	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	264	264	0	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	396	0	132	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	24	0	0	24	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	8	8	0	8	8	0	0	0	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	12	0	4	12	0	4	0	0	0	0	0	0	0	0	0
$\Phi_{(2,2)}$	396	0	0	0	0	0	396	0	0	0	0	0	0	0	0
$\Phi_{(3,2)}$	132	132	0	0	0	0	132	132	0	0	0	0	0	0	0
$\Phi_{(4,2)}$	198	0	66	0	0	0	198	0	66	0	0	0	0	0	0
$\Phi_{(5,2)}$	12	0	0	12	0	0	12	0	0	12	0	0	0	0	0
$\Phi_{(1,3)}$	4	4	0	4	4	0	4	4	0	4	4	0	0	0	0
$\Phi_{(2,3)}$	6	0	2	6	0	2	6	0	2	6	0	2	0	0	0
$\Phi_{(3,3)}$	198	0	0	0	0	0	198	0	0	0	0	0	6	0	0
$\Phi_{(4,3)}$	66	66	0	0	0	0	66	66	0	0	0	0	2	2	0
$\Phi_{(5,3)}$	99	0	33	0	0	0	99	0	33	0	0	0	3	0	1

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