Closure on Knowability

Mark Jago
Forthcoming in *Analysis*, 2010

Abstract: The Church-Fitch argument, or 'paradox' of knowability, apparently shows that, if all truths are knowable, then all truths are known. As some truths are unknown, anti-realists who hold that truths must be knowable have been at pains to block the argument. Here, I consider two such approaches: denying that knowledge distributes over conjunction, and moving to a typed logic. I argue that neither approach works. I first show that a Church-Fitch argument can be run with an assumption weaker than distribution of knowledge over conjunction. I then argue that a modified Church-Fitch argument can be constructed in a typed logic. So neither approach will help those who hold that all truths are knowable.

1 Introduction

The history of philosophy is replete with accounts on which truth is epistemically constrained, or essentially knowable. The idea underlies much idealist, phenomenalist and other anti-realist thought: Berkeley (1710/1995), Kant (1781/1998, A146/B185),[1] Mill (1889), Ayer (1936) and Dummet (1978) all seem to subscribe to the *knowability principle*, usually formalised as[2]

(KP) $p \to \Diamond Kp$

where 'K' means 'it is known by someone at some time that'. But a simple argument, due to Alonzo Church[3] but first published by Frederic Fitch (1963), shows that if every truth is *knowable* then every truth is in fact *known*.[4] That is, (KP) collapses to

($\star$) $p \to Kp$

which is surely false: some truths are not (and never will be) known. This is frequently run as a *reductio* of (KP) and anti-realism along with it.

The argument, which I set out in §2, assumes that knowledge is closed under conjunction elimination (i.e., knowledge *distributes* over conjunction): if a conjunction is known, then so are the conjuncts. The anti-realist might try to block the argument by denying this. I show (§3) that the argument goes through with a weaker premise. Another strategy for the defender of (KP) is to move to a typed system (Linsky 2009). I show (§4) that a revised Church-Fitch argument goes through even in the typed system, with the addition of two assumptions. I then (§5 and §6) discuss and reject two possible responses on behalf of the defender of (KP).

* I would like to thank Alison Fernandes, Tama Coutts and an anonymous referee for Analysis for suggesting improvements to the paper, and Joe Salerno for discussing some of the above arguments with me. This work was supported by the Australian Research Council [DP0880437, 'Rationality and Resource Bounds in Logics for Intentional Attitudes'].

1. At least, Kant seems to support (KP) with respect to appearances (as opposed to things in themselves).
2. Throughout, the inset sentences should be read as axiom schemes, asserting that each of their instances is an axiom.
3. Fitch attributes his theorem 4 to an 'anonymous referee' (1963, 138), who turns out to be Alonzo Church (see Salerno 2009). The resulting argument is often called 'Fitch's paradox', but since the reasoning is all Church's, I'll call it the 'Church-Fitch argument'. Interestingly, Church suggests two ways to maintain (KP) but avoid ($\star$): denying that knowledge distributes over conjunction, and moving to a typed logic (Church 2009). These are the two responses to the Church-Fitch argument I discuss and reject in this paper.
4. Brogaard and Salerno (2009) provide a good overview of the literature on the argument.

2 The Church-Fitch Argument

The Church-Fitch argument can be run in a classical modal logic with the following two assumptions:

(F) $Kp \rightarrow p$

(D) $K(p \wedge q) \rightarrow Kp \wedge Kq$

The first says that knowledge is *factive*, the second that knowledge distributes over conjunction. In normal modal logics, the necessitation rule allows us to infer '$\Box p$' from any theorem 'p'. Given the duality of '$\Diamond$' and '$\Box$' ($\Box\neg p \equiv \neg\Diamond p$), this gives us the rule:

$$\text{(N)} \quad \frac{\vdash \neg p}{\vdash \neg\Diamond p}$$

Using these principles, here is one way of running the Church-Fitch argument.[5]

1.	$K(p \wedge \neg Kp)$	assumption
2.	$Kp \wedge K\neg Kp$	1, (D)
3.	$Kp \wedge \neg Kp$	2, (F)
4.	$\neg K(p \wedge \neg Kp)$	1, 3, *reductio*
5.	$\neg\Diamond K(p \wedge \neg Kp)$	4, (N)
6.	$(p \wedge \neg Kp) \rightarrow \Diamond K(p \wedge \neg Kp)$	(KP)
7.	$\neg(p \wedge \neg Kp)$	5, 6, *modus tollens*
8.	$p \rightarrow Kp$	7, classical logic

We first prove that $K(p \wedge \neg Kp)$ is inconsistent (lines 1–3), and infer that it's impossible (line 5). (KP) then allows us to infer $\neg(p \wedge \neg Kp)$ (line 7), which is classically equivalent to ($\star$). So (KP) entails ($\star$).

There is a huge literature of responses and counter-responses to the argument.[6] Williamson (1982) suggests an intuitionistic approach to block step 8 and Priest (2009) discusses a paraconsistent approach, which blocks step 4.[7] There are also many suggestions for weakening or restricting (KP) (e.g. Tennant 2002). All of these options are well-represented in the literature. Two moves which are not widely discussed are *(i)* rejecting (D); and *(ii)* switching from an untyped to a typed logical language (Linsky 2009).[8] I focus on option (i) in the next section and option (ii) in the remainder of the paper.

3 Weakening the Distribution Premise

If we read 'Kp' to mean that someone knows that p *right now*, then it is at least plausible that knowledge does not always distribute over conjunction.[9] Suppose that Fred is writing

5. To shorten the proof, I've combined the steps of asserting an instance of (F) or (D) with a *modus ponens* inference. In effect, I'm treating (F) and (D) as rules, rather than axioms.

6. Brogaard and Salerno (2009) survey many of these.

7. Percival (1990) argues against Williamson's approach; I argue (Jago 2009) that the paraconsistent move won't help the anti-realist defender of (KP).

8. Incidentally, these are the two options for blocking the argument suggested originally by Church (2009) (see footnote 3). A further move would be to reject (F) but, as the factivity of knowledge is widely accepted, I will not discuss that move here.

9. Church makes this suggestion to Fitch in his original referee report. He argues that (D) may be dropped on the grounds that there is no psychological law linking knowledge of conjunctions to knowledge of the conjuncts (Church 2009, 14).

a proof, and has just concluded and hence come to know that $p \wedge q$ via *reductio*. He might not have realised that his conclusion is a conjunction and, if so, he need not believe and hence need not know either that p or that q. And it might be that no one else has ever realised that p or that q before, in which case, we have a counterexample to (D).[10] The anti-realist's hope is that the Church-Fitch argument will not go through without (D).

What someone who takes this line of argument has missed, however, is that Fred *could* easily come to know both that p and that q. He need only add two steps to his proof to do so. Whatever reasons there may be for Fred's knowledge failing to satisfy (D) are highly contingent. More generally, take any purported counterexample to (D). Then we have some agent knowing that $p \wedge q$. But surely, even if that agent does not in fact know that p and that q, she could very easily perform some simple reasoning and so come to know both that p and that q. So we have

(D$^\Diamond$) $K(p_1 \wedge \cdots \wedge p_n) \to \Diamond(Kp_1 \wedge \cdots \wedge Kp_n)$.

We can then run a Church-Fitch argument as follows:

1.	$Kp \wedge K\neg Kp$	assumption
2.	$Kp \wedge \neg Kp$	1, (F)
3.	$\neg(Kp \wedge K\neg Kp)$	1, 2, *reductio*
4.	$\neg\Diamond(Kp \wedge K\neg Kp)$	3, (N)
5.	$K(p \wedge \neg Kp) \to \Diamond(Kp \wedge K\neg Kp)$	(D$^\Diamond$)
6.	$\neg K(p \wedge \neg Kp)$	5, *modus tollens*
7.	$\neg\Diamond K(p \wedge \neg Kp)$	6, (N)
8.	$(p \wedge \neg Kp) \to \Diamond K(p \wedge \neg Kp)$	(KP)
9.	$p \to Kp$	7, 8, *modus tollens*, classical logic

So even if we drop distribution in favour of (D$^\Diamond$), the argument goes through and (KP) entails that every truth is known. More generally, we can strengthen Fitch's original theorem 5 (Fitch 1963, 139), which says that for any factive operator 'O' which distributes over conjunction, '$p \to \Diamond Op$' entails '$p \to Op$'. We have shown that this result also holds for all factive operators 'O' which satisfy $O(p \wedge q) \to \Diamond(Op \wedge Oq)$.

This point is similar to one made by Williamson (1993), who argues that a Church-Fitch argument can be run in the absence of (D) by strengthening (KP). His suggested strengthening of (KP) is

(KP$_2$) $p_1 \wedge \cdots \wedge p_n \to \Diamond(Kp_1 \wedge \cdots \wedge Kp_n)$

which says that, for any true conjunction, it is possible for each conjunct to be known simultaneously. The dialectical worry here is that the anti-realist may flatly refuse to accept any premise stronger than (KP) as her statement that truth is epistemically constrained. One needs to show that the anti-realist is committed to (KP$_2$) if she is committed to (KP). But this is easy, given (D$^\Diamond$) and the assumption that '$\Diamond$' satisfies the S4 axiom:[11]

(S4) $\Diamond\Diamond p \to \Diamond p$.

The argument from (KP) to (KP$_2$) is then simple for, in any normal modal logic, we have:

10. Note that I'm not subscribing to this argument; I'm suggesting that it might be offered by someone as a reason for denying (D).
11. Most philosophers assume that '$\Diamond$' satisfies the S5 axioms and hence also the S4 axioms.

$$(\mathrm{N}_2)\ \frac{\vdash p \to q}{\vdash \Diamond p \to \Diamond q}$$

Here is the argument.

1.	$p_1 \wedge \cdots \wedge p_n$	assumption
2.	$\Diamond K(p_1 \wedge \cdots \wedge p_n)$	(KP), *modus ponens*
3.	$K(p_1 \wedge \cdots \wedge p_n) \to \Diamond(Kp_1 \wedge \cdots \wedge Kp_n)$	$(\mathrm{D}^{\Diamond})$
4.	$\Diamond K(p_1 \wedge \cdots \wedge p_n) \to \Diamond\Diamond(Kp_1 \wedge \cdots \wedge Kp_n)$	3, (N_2)
5.	$\Diamond\Diamond(Kp_1 \wedge \cdots \wedge Kp_n) \to \Diamond(Kp_1 \wedge \cdots \wedge Kp_n)$	(S4)
6.	$\Diamond(Kp_1 \wedge \cdots \wedge Kp_n)$	2, 4, 5, *modus ponens*
7.	$p_1 \wedge \cdots \wedge p_n \to \Diamond(Kp_1 \wedge \cdots \wedge Kp_n)$	1, 5, *→-introduction*

4 The Typing Approach

Linsky (2009), following a suggestion of Church's (2009, 17), attempts to show that the Church-Fitch argument is invalid in a typed logic. If so, then a typed approach promises to deliver a uniform solution to the liar (and other semantic paradoxes) and the Church-Fitch argument. In this section, I argue that it is not so, by showing how a Church-Fitch argument can be run in the typed system. In the system, each sentence 'p' is assigned a type n and each sentential operator 'O' a type (n). A sentence '$O^{(m)}p^n$' is well-typed only if $m \leq n$, in which case, it is of type $m+1$.[12] In particular, '$p^1 \wedge \neg K^{(1)}p^1$' has type 2 and so '$K^{(1)}(p^1 \wedge \neg K^{(1)}p^1)$' isn't well-typed. As a consequence,

$$p^1 \wedge \neg K^{(1)}p^1 \to \Diamond K^{(1)}(p^1 \wedge \neg K^{(1)}p^1)$$

is not an admissible instance of (KP).[13] In the typed system, (KP) should be rendered as

$$(\mathrm{KP_t})\ \ p^n \to \Diamond K^{(n)}p^n$$

and the relevant instance of $(\mathrm{KP_t})$ is

$$p^1 \wedge \neg K^{(1)}p^1 \to \Diamond K^{(2)}(p^1 \wedge \neg K^{(1)}p^1).$$

But as '$K^{(2)}p^1 \wedge \neg K^{(1)}p^1$' is not strictly a contradiction, the Church-Fitch argument from §2 is blocked in the typed system (Linsky 2009, 172).

To devise a new Church-Fitch argument for the typed system, we need two additional premises. The first is:

(1) One knows that p iff one knows that it is true that p.

For now, I will assume that (1) is correct and see how to use it in a Church-Fitch argument.[14] I will consider whether this is justified in §5. We cannot capture (1) for the case of knowledge$^{(1)}$ as

$$K^{(1)}p^1 \leftrightarrow K^{(1)}T^{(1)}p^1,$$

12. I follow Linksy's notation for type assignments, which is a simplification of Church's system of *r-types* (Linsky 2009, 168).

13. I will follow Linsky in not typing the '◇' operator. To do so would further clutter our syntax, but would not affect the points I make in this section.

14. In particular, I'm assuming that the argument against (D) in §3 doesn't work against (1), even if it does work against (D), for it is plausible that truth is essential to knowledge in a way that conjunction isn't. But a proper argument for (1) will have to wait until §5.

for '$K^{(1)}T^{(1)}p^1$' is not well-typed.[15] Instead, it must be that knowledge$^{(1)}$ that p amounts to knowledge$^{(2)}$ that it is true$^{(1)}$ that p^1:

(KT) $K^{(1)}p^1 \leftrightarrow K^{(2)}T^{(1)}p^1$.

The second additional premise is a general point about the closure of knowledge:

(C) If there is a valid (single-premise) rule of inference from 'p^n' to 'q^m', then $K^{(l)}p^n \to \Diamond K^{(l)}q^m$ (for all $l \geq m, n$).

The argument I gave involving Fred in §3 shows why this principle is sound. If one knows$^{(l)}$ that p, then one can apply the relevant rule of inference from p to q, and so come to know$^{(l)}$ that q. We will use an instance of this principle given by the (unusual, but nonetheless valid) rule of inference:

(T) $$\frac{p^n \wedge q^m}{T^{(n)}p^n \wedge q^m}$$

which, when combined with (C), gives us:

(C$_1$) $K^{(l)}(p^n \wedge q^m) \to \Diamond K^{(l)}(T^{(n)}p^n \wedge q^m)$.

We can then run a Church-Fitch argument as follows.[16]

1.	$K^{(2)}(T^{(1)}p^1 \wedge \neg K^{(1)}p^1)$	Assumption
2.	$K^{(2)}T^{(1)}p^1 \wedge K^{(2)}\neg K^{(1)}p^1$	1, (D)
3.	$K^{(1)}p^1 \wedge K^{(2)}\neg K^{(1)}p^1$	2, (KT)
4.	$K^{(1)}p^1 \wedge \neg K^{(1)}p^1$	3, (F)
5.	$\neg\Diamond K^{(2)}(T^{(1)}p^1 \wedge \neg K^{(1)}p^1)$	1, 4, *reductio*, (N)
6.	$K^{(2)}(p^1 \wedge \neg K^{(1)}p^1) \to \Diamond K^{(2)}(T^{(1)}p^1 \wedge \neg K^{(1)}p^1)$	(C$_1$)
7.	$\neg\Diamond K^{(2)}(p^1 \wedge \neg K^{(1)}p^1)$	5, 6, *modus tollens*, (N)
8.	$p^1 \wedge \neg K^{(1)}p^1 \to \Diamond K^{(2)}(p^1 \wedge \neg K^{(1)}p^1)$	(KP$_t$)
9.	$p^1 \to K^{(1)}p^1$	7, 8, *modus tollens*, classical logic

We can thus reject the typing response to the Church-Fitch argument if (1) is justified. In the next section, I consider whether we should accept (1) and whether the typing approach can be maintained if we do not accept it.

5 Should We Accept (1)?

In the previous section, I gave an argument against the typing approach which rests on principle (1). In this section, I give an argument for (1) and discuss how the typing approach stands if one rejects (1). But before that, I want to dismiss an argument which holds that (1) is shown to be false by agents who have no concept of truth. For even if that is the case, the

15. My argument requires 'T' as well as 'K' to be a typed operator. It would be completely *ad hoc* to type the knowledge operator but not the truth operator. Besides, if one uses a typed system for anything, then one will also want to use it to avoid the liar paradox, and so will require typed truth *predicates*. My argument would go through just as well with truth predicates (and suitable name-forming operators for sentences) in place of truth operators.
16. I use (D) rather than (D$^{\Box}$) as a premise as this shortens the argument. Linsky accepts (D); but regardless, the argument would go through even with (D$^{\Box}$) in place of (D), in the way shown in §3.

argument still goes through, as follows. Suppose that 'p^1' is an unknown truth; then so is '$p^1 \wedge (p^1 \leftrightarrow T^{(1)} p^1)$'. So, rather than running a Church-Fitch argument on 'p^1', we run the argument on '$p^1 \wedge (p^1 \leftrightarrow T^{(1)} p^1)$', and arrive at

$$K^{(3)}(p^1 \wedge (p^1 \leftrightarrow T^{(1)} p^1)) \wedge \neg K^{(2)}(p^1 \wedge (p^1 \leftrightarrow T^{(1)} p^1)).$$

As argued above, we can derive an explicit contradiction from this if we accept premise (1) *for the agents in question*. But, given (D), the agents in question know$^{(3)}$ that $p^1 \leftrightarrow T^{(1)} p$, and so must have a concept of truth. Hence, even if we think there are agents without a concept of truth for whom (1) fails, the argument will go through just as well when restricting (1) to those agents who *do* have a concept of truth.

What is the argument in favour of (1)? Note that the *true that* operator is transparent to whatever kind of justification, safety or reliability is required for a true belief to count as knowledge. So (1) is entailed by:

(2) One believes that p iff one believes that it is true that p.[17]

For if one knows that p and believes that it is true that p, then that belief is justified, safe or reliable in just the way required for it to count as knowledge. Similarly, if one knows that it is true that p and also believes that p, then by the same argument, one thereby knows that p.[18] So we should accept (1) if we accept (2).

Should we accept (2)? Here are two arguments in its favour, one concerning truth, one concerning accounts of belief. The first assumes the deflationist's line, that the cognitive content (or sense) of 'it is true that p' coincides with that of 'p'. Thus, if belief is a relation between an agent and some cognitive content (or sense), then one stands in that relation to the content of 'it is true that p' iff she stands in that relation to the content of 'p', and (2) is validated. And even if one is not a deflationist about truth, the idea that the cognitive contents of 'p' and 'it is true that p' coincide is a persuasive one.[19]

The second argument in (2)'s favour locates truth as essential to what makes a belief a *belief*, so that there is no gap between believing that p and believing that it is true that p. Just consider the awkwardness in saying, 'she believes it, but doesn't believe it to be true'. Indeed, many philosophical accounts of belief, including holistic accounts in which the principle of charity plays a role, will validate (2). On such views, 'a believes that p' is true iff that ascription to a is part of a holistic ascription of certain attitudes (e.g., beliefs and desires) which best makes sense of her behaviour.[20] The only evidence which could support an ascription of 'a believes that p' but not of 'a believes that it is true that p' (or vice versa) is linguistic behaviour: a's assenting to 'p' but not to 'it is true that p' (or vice versa). But then, given the principle of charity, we should not interpret a's use of 'true' to mean what we mean by *true*, for what we mean is governed (in part or in totality) by the instances of the T-scheme. Then that behaviour will not count as evidence against ascribing the belief that p iff we ascribe the belief that it is true that p. So (2), and hence (1), is supported by (what I take to be) the most popular and plausible philosophical accounts of belief.

I now want to argue that rejecting (1) will not help the typing approach. I'll give two arguments to this effect. First, assume for argument's sake that (1) is false. Nevertheless, if one knows that it is true that p^1, it is certainly *possible* for that agent to come to know (in the regular, first-order way) that p^1:

17. Note that the principle is about one's relation to (the truth of) the content *that p*, and not to whether some specific *sentence* 'p' is true.
18. I'm assuming here that knowledge entails belief.
19. Indeed, Wright (1992) and Lynch (2009) seem to accept the principle, although neither is a deflationist.
20. See, e.g., Bennet 1964, Davidson 1973, Lewis 1974, Stalnaker 1984 and Dennett 1987.

(3) $K^{(2)}T^{(1)}p^1 \to \Diamond K^{(1)}p^1$.

Moreover, it is plausible that coming to know this should not affect one's level-2 knowledge in any way. That is:

(KT$_\Diamond$) $(K^{(2)}T^{(1)}p^1 \wedge K^{(2)}q^2) \to \Diamond(K^{(1)}p^1 \wedge K^{(2)}q^2)$.

If we accept this principle, which seems plausible, then the following Church-Fitch argument can be run:

1.	$K^{(2)}(T^{(1)}p^1 \wedge \neg K^{(1)}p^1)$	Assumption
2.	$K^{(1)}p^1 \wedge K^{(2)}\neg K^{(1)}p^1$	Assumption
3.	$K^{(1)}p^1 \wedge \neg K^{(1)}p^1$	2, (T)
4.	$\neg\Diamond(K^{(1)}p^1 \wedge K^{(2)}\neg K^{(1)}p^1)$	2, 3, *reductio*, (N)
5.	$\neg(K^{(2)}T^{(1)}p^1 \wedge K^{(2)}\neg K^{(1)}p^1)$	4, (KT$_\Diamond$), *modus tollens*
6.	$K^{(2)}T^{(1)}p^1 \wedge K^{(2)}\neg K^{(1)}p^1$	1, (D)
7.	$\neg\Diamond K^{(2)}(T^{(1)}p^1 \wedge \neg K^{(1)}p^1)$	1, 5, 6, *reductio*, (N)

The derivation then goes through as in §4, with the conclusion that $p^1 \to K^{(1)}p^1$.

Here is the second argument. We are interested in (KP$_t$) only because we are interested in the viability of the anti-realist theories about meaning that entail (KP$_t$).[21] So rather than focusing on (KP$_t$), we may focus on a related consequence of those theories. Consider a concept of knowledge, $\mathcal{K}$, for which (1) *does* hold. Then we have

($\mathcal{K}$T) $\mathcal{K}^{(1)}p^1 \leftrightarrow \mathcal{K}^{(2)}T^{(1)}p^1$.

I argued above that all that is required for this to hold is for (2) to hold for the associated notion of belief. Even if our concept of belief does not fit a model which validates (2), there are nevertheless possible linguistic communities whose concept of belief does, and hence whose concept of knowledge validates ($\mathcal{K}$T). Moreover, those linguistic communities may be indistinguishable from ours in nearly every respect, save perhaps their use of 'believes that'. In particular, there will be unknown truths in such communities. Fix on one such possible community, C.

Now suppose that one supports a knowability principle because of one's (broadly Dummettian) anti-realist theory of meaning. That theory of meaning, if true of our language, should also be true of C's language. After all, the Dummettian argument for a theory of meaning based on verification-conditions does not depend on what the precise truth-conditions of belief ascriptions are. If the Dummettian argument works at all, then it works in C's case too. So if one's theory of meaning entails (KP$_t$), then it also entails the corresponding knowability principle for $\mathcal{K}$,

($\mathcal{K}$P$_t$) $p^n \to \Diamond\mathcal{K}^{(n)}p^n$.

But ($\mathcal{K}$P$_t$) cannot be correct. Suppose it were: then, given ($\mathcal{K}$T), we could run the argument from §4, with '$\mathcal{K}$' in place of 'K', concluding that there could not be any unknown (with respect to $\mathcal{K}$) truths. But C is a possible community in which there are unknown (w.r.t. $\mathcal{K}$) truths. Hence we must reject ($\mathcal{K}$P$_t$) and any anti-realist theory of meaning which supports it.

21. Alternatively, one might support (KP$_t$) because of one's anti-realist theory of *truth*. My argument will go through just as well in this case.

[redacted] is that I have been too quick in [redacted] into the typed system. Why should the demand [redacted]pe

So even if we reject (KP_t) and replace it with (KP'_t), a Church-Fitch argument still goes through. I conclude that neither denying (D) nor moving to a typed logic will help the defender of the knowability principle (KP) to block the Church-Fitch argument.

References

Ayer, A. (1936). *Language, Truth and Logic*, Victor Gollancz, London.

Bennet, J. (1964). *Rationality*, Routledge & Kegan Paul, London.

Berkeley, G. (1710/1995). *A Treatise Concerning the Principles of Human Knowledge,* ed. K.P. Winkler, Hackett, Indianapolis.

Brogaard, B. and Salerno, J. (2009). Fitch's paradox of knowability, in E. N. Zalta (ed.), *The Stanford Encyclopedia of Philosophy*.
URL: *http://plato.stanford.edu/archives/sum2004/entries/fitch-paradox/*

Church, A. (2009). Referee reports on Fitch's 'A Definition of Value', in J. Salerno (ed.), *New Essays on the Knowability Paradox*, Oxford University Press, pp. 13–20.

Davidson, D. (1973). Radical interpretation, *Dialectica* 27: 313–328.

Dennett, D. C. (1987). *The Intentional Stance*, MIT Press.

Dummet, M. (1978). Realism, *Truth and Other Enigmas*, Duckworth, pp. 145–165.

Fitch, F. (1963). A logical analysis of some value concepts, *Journal of Symbolic Logic* 28(2): 135–142.

Jago, M. (2009). Review of New Essays on the Knowability Paradox, ed. J. Salerno, *Journal of Logic, Language and Information* pp. DOI: 10.1007/s10849–009–9107–8.

Kant, I. (1781/1998). *Critique of Pure Reason,* ed. P. Guyer and A.W. Wood, Cambridge Univ Press, Cambridge.

Lewis, D. (1974). Radical interpretation, *Synthese* 27(3): 331–344.

Linsky, B. (2009). Logical types in arguments about knowability and belief, in J. Salerno (ed.), *New Essays on the Knowability Paradox*, Oxford University Press, pp. 163–179.

Lynch, M. P. (2009). *Truth as One and Many*, Oxford University Press, Oxford.

Mill, J. S. (1889). *An examination of Sir William Hamilton's philosophy: and of the principal philosophical questions discussed in his writings*, Longmans & Green, London.

Percival, P. (1990). Fitch and intuitionistic knowability, *Analysis* 50: 182–187.

Priest, G. (2009). Beyond the limits of knowledge, in J. Salerno (ed.), *New Essays on the Knowability Paradox*, Oxford University Press, pp. 93–104.

Salerno, J. (2009). Introduction, in J. Salerno (ed.), *New Essays on the Knowability Paradox*, Oxford University Press, pp. 1–10.

Stalnaker, R. (1984). *Inquiry*, MIT Press, Cambridge, Mass.

Tennant, N. (2002). *The taming of the true*, Oxford University Press, New York.

Williamson, T. (1982). Intuitionism disproved?, *Analysis* 42(2): 203–207.

Williamson, T. (1993). Verificationism and non-distributive knowledge, *Australasian Journal of Philosophy* 71(1): 78–86.

Wright, C. (1992). *Truth and Objectivity*, Harvard University Press, Harvard, MA.