Conjunctive and Disjunctive Parts

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Abstract: Fine (2017a) sets out a theory of content based on truthmaker semantics which distinguishes two kinds of consequence between contents. There is *entailment*, corresponding to the relationship between disjunct and disjunction, and there is *containment*, corresponding to the relationship between conjunctions and their conjuncts. Fine associates these with two notions of parthood: *disjunctive* and *conjunctive*. Conjunctive parthood is a very useful notion, allowing us to analyse partial content and partial truth. In this chapter, I extend the notion of disjunctive parthood in terms of a structural relation of *refinement*, which stands to disjunctive parthood much as mereological parthood stands to conjunctive parthood. Philosophically, this relation may be modelled on the determinable-determinate relation, or on a fact-to-fact notion of grounding. I discuss its connection to two other Finean notions: vagueness (understood via precisification) and arbitrary objects. I then investigate what a logic of truthmaking with refinement might look like. I argue that (i) parthood naturally gives rise to a relevant conditional; (ii) refinement underlies a relevant notion of disjunction; and so (iii) truthmaker semantics with refinement is a natural home for relevant logic. The resulting formal models draw on Fine's (1974) semantics for relevant logics. Finally, I use this understanding of relevant semantics to investigate the status of the *mingle* axiom.

1 Introduction

What I say, in making an utterance, is closely connected to the ways in which my utterance would be true or false, depending on how things might be. You might say the same as me, but in a different way; you might believe what I say, or hope or worry that it's true. *What* I say – the sharable and repeatable element of meaning – is the *content* of my utterance. The truth of the utterance across a range of possible situations, and how it entails and is entailed by other utterances, is down to its content. The *possible worlds* analysis of content, due to Lewis (1986), Kratzer (2012), and Stalnaker (1976a;b), is simple and powerful but coarse-grained, as is widely acknowledged by defenders and detractors alike.

Truthmaker semantics places what's good about possible world semantics in a far less coarse-grained account. Content is understood in terms of *how* (and

not merely *whether*) an utterance would be true in possible situations. Sentences are evaluated relative to the possible states *in virtue of which* they would be true; and content is analysed in terms of those possible states. Kit Fine has developed the approach (2015; 2016; 2017a; 2017b) and investigated its philosophical and logical applications (2012a; 2014). In a short space of time, on the basis of Fine's work, truthmaker semantics has become a thriving field of research. (Fine (2017c) discusses the progress so far; Jago (2018) and Moltmann (2015) pursue related approaches.)

A key component of Fine's account is the distinction between a proposition's *conjunctive* and *disjunctive* parts (Fine 2017a), which correspond to the propositions it contains and the propositions it entails, respectively. My aim in this paper is to extend the notion of disjunctive parthood and put it to metaphysical and logical use. After introducing Fine's approach (§2) and his conjunctive/disjunctive parts distinction (§3), I'll argue that his treatment of disjunction needs revision. This is best accomplished by introducing a structural relation of *refinement*, which can be used to analyse disjunctive parthood, much as the usual part-whole relation is used to analyse conjunctive parthood (§4). *Refinement* may have applications to other ontological categories, delivering disjunctive properties and particulars (§5). The picture that emerges allows us to see connections between truthmaker semantics and three other Finean notions: grounding, vagueness, and arbitrary objects.

The second half of the paper investigates what a logic of truthmaking with refinement might look like. I argue that, in truthmaker semantics, (i) parthood naturally gives rise to a relevant conditional; (ii) refinement underlies a relevant notion of disjunction; and so (iii) truthmaker semantics with refinement is a natural home for relevant logic (§6). The resulting formal models draw on Fine's (1974) semantics for relevant logics. §7 discusses the handling of conjunction in this setting. §8 relates the discussion to the Routley-Meyer semantics and the status of the *mingle* axiom. For background on relevant logic, see Anderson and Belnap 1975, Bimbó 2006, or Dunn and Restall 2002.

2 Truthmaker Semantics and Content

The characteristic logical feature of truthmaker semantics, which sets it apart from just about every other approach to content, is its treatment of conjunction. It's a lazy Sunday morning. Anna is knitting in the back room, whilst Bertie is snuffling in the garden. What makes it true that Anna is knitting and Bertie is snuffling is the conjunctive state of affairs, the sum of *that Anna is knitting* and *that Bertie is snuffling*. Each conjunct truth is made true by one of these states of affairs, taken individually, so that the conjunction is made true by the combined state of affairs.

One remarkable feature of this simple picture is this: what makes the

conjunction true is not what makes the conjuncts true. What makes it true that Bertie is snuffling is not the conjunctive state of affairs, *that Anna is knitting and Bertie is snuffling*. Rather, its truthmaker is just *that Bertie is snuffling*. This point is made forcefully by Rodriguez-Pereyra (2006):

> The truthmaking relation obtains between a portion of reality and a proposition if and only if the proposition is true *in virtue of* the portion of reality in question. But then what makes a conjunction true need not make its conjuncts true. … it is not the case that ⟨Peter is a man⟩ is true in virtue of the fact *that Peter is a man and Saturn is a planet*. What ⟨Peter is a man⟩ is true in virtue of is simply the fact *that Peter is man*. (Rodriguez-Pereyra 2006, 970)

Based on these considerations, we should reject the *conjunction thesis*:

(Conjunction) x is a truthmaker for $A \wedge B$ iff x is a truthmaker for A and x is a truthmaker for B.

In general, the left-to-right direction does not hold. Van Fraassen (1969, 485) notes a similar point.

As a consequence, the 'in virtue of' conception of truthmaking is not monotonic: it may be that s truthmakes A and is a part of y, which does not truthmake A. (This is to deny the *heredity* principle along the parthood order.) This 'in virtue of' notion is what Fine calls *exact truthmaking*. Whereas Rodriguez-Pereyra takes it to be definitive of the concept of truthmaking, Fine is more ecumenical. We may recognise an *inexact* as well as the exact notion of truthmaking, with the former understood in terms of the latter:

(Inexact truthmaking) x is an *inexact truthmaker* for $A =_{\mathrm{df}}$ there is some part y of x ($y \sqsubseteq x$) which is an exact truthmaker for A.

This discussion is foreshadowed by van Fraassen (1969), who provides a semantics for two notions of entailment in terms of set-theoretic representations of worldly 'facts'. The first notion, 'a very tight relationship', is understood as 'whatever makes A true, also makes B true' (van Fraassen 1969, 485). The second notion is looser: all the facts containing a fact that makes A true contain a fact that makes B true. The former notion corresponds to *exact truthmaking entailment* (Fine and Jago 2019), the latter to *inexact truthmaking entailment*, which, as van Fraassen shows, coincides with Anderson and Belnap's *First Degree Entailment* (Anderson and Belnap 1962; 1963). Van Fraassen's discussion says very little about the former relation, but he does note its central feature: $A \wedge B$ does not exactly entail A (van Fraassen 1969, 485).

A *state space* (or *frame*) for truthmaker semantics is a pair $\langle S, \sqsubseteq \rangle$, where S is a set of states (the truthmakers) and $\sqsubseteq$ is a partial order (reflexive, transitive and

anti-symmetric), such that each pair $s, t \in S$ has a least upper bound in S, denoted $s \sqcup t$. A state space is *complete* when each subset $S' \subseteq S$ has a least upper bound, $\bigsqcup S'$. Complete spaces always contain a *null state* $\square$, defined as $\bigsqcup \varnothing$, and a *full state* $\blacksquare$, defined as $\bigsqcup S$. In the presence of $\square$, arbitrary subsets of states $S' \subseteq S$ are also guaranteed to have a greatest lower bound, $\bigsqcap S'$. For pairs s, u, we denote this state $s \sqcap u$. Intuitively, it is the overlap between s and u (and if there is no overlap in the intuitive sense, then $s \sqcap u$ is $\square$). In his account of content, Fine (2017a;b) works with *distributive spaces*, in which $s \sqcap (t \sqcup u) = (s \sqcap t) \sqcup (s \sqcap u)$. Models $\langle S, \sqsubseteq, V \rangle$ add a valuation V to a state space.

Fine (2017a) identifies the content of a sentence with the set of its possible truthmakers. This is a version of the *truthmaker account of propositions* (Jago 2017; 2018; forthcoming). Propositions ('P', 'Q', 'R' and so on) are subspaces of the state space and are:

Closed iff $\bigsqcup Q \in P$ for every nonempty $Q \subseteq P$;

Convex iff $s, u \in P$ and $s \sqsubseteq t \sqsubseteq u$ implies $t \in P$; and

Regular iff both closed and convex.

This notion is *unilateral*, in contrast to *bilateral* pairs of subspaces of S, thought of as possible truthmakers and possible falsemakers, respectively. Bilateral propositions are useful for dealing with negation (Fine 2017a, §3). Since my focus here is on conjunction and disjunction only, I'll focus on unilateral propositions; but much of what I say carries across to the bilateral case. I'll use $P^{\sqcup}$ to abbreviate $\bigsqcup P$. If P is regular and nonempty, we always have $P^{\sqcup} \in P$.

If we were dealing with sets of possible worlds, we would at this point make note of two special propositions: the empty set and the set of all worlds. On that approach, these correspond to the (one and only) necessary falsehood and the (one and only) necessary truth, respectively. The truthmaker approach offers a more sophisticated picture. It retains the empty set $\varnothing$ and the set of all states, S, as propositions, which Fine denotes $F_{\square}$ and $T_{\blacksquare}$, respectively. The former is trivially false because it has no possible truthmakers and is associated with $\square$ in the sense that it is about nothing: its closure $F_{\square}^{\sqcup}$ is $\square$. The latter is trivially true in the sense that is it made true by any state and is associated with $\blacksquare$ in the sense that it is about everything: its closure $T_{\blacksquare}^{\sqcup}$ is $\blacksquare$. The truthmaker approach offers two further 'extremal' propositions: the singletons $T_{\square} = \{\square\}$ and $F_{\blacksquare} = \{\blacksquare\}$. The former is trivially true in that it needs nothing substantive to make it true; the latter is trivially false in that its only truthmaker is the sum of all possible states.

We can use these to interpret the Church Ackermann truth constants, $\top$ and t:

$$\top = T_{\blacksquare} \qquad\qquad t = T_{\square}$$

$\top$ is the weakest proposition entailed by every proposition, whereas t is the strongest provable truth. (t is often characterised by taking it as an axiom plus adding a rule from $\vdash A$ to $\vdash t \to A$. In systems with PERMUTATION (§6), such as **R**, t may be characterised via the axiom $(t \to A) \leftrightarrow A)$.) Although $\top$ and t coincide classically, they are distinct in various non-classical systems, as in §6. (We might also align $\top$'s and t's negations, $\bot$ and f, with $F_{\square}$ and $F_{\blacksquare}$, respectively. This makes good sense of $\bot$, the strongest proposition which entails everything. But f will not be a negation of t suitable for the relevant logics I'll discuss from §6 onwards.)

Note that $T_{\square}$ and $T_{\blacksquare}$ are not the only necessary truths (and not even the only logical truths). Whenever P and Q differ, so will $P \vee \neg P$ and $Q \vee \neg Q$. In general, 'the structure of verification and falsification for necessary truths and falsehoods can be as rich and varied as it is for the contingent truths' (Fine 2017a, 631).

3 Conjunctive and Disjunctive Parts

On the possible worlds account of propositions, there is just one way for a proposition to be part of another: when the former is a subset of the latter. And in that case, the former entails the latter, since any world where the former is true is thereby a world where the latter is, too. So parthood aligns with entailment, on this classical picture. Entailment is indifferent between conjunction and disjunction, in the sense that P entails Q when $P = P \wedge Q$, or equivalently when $Q = P \vee Q$.

The truthmaking approach, by contrast, draws an important distinction between the relations between P and $P \vee Q$, on the one hand, and between $P \wedge Q$ and P, on the other (Fine 2017a, §5). This is brought out by considering their truthmakers. Going from P to (distinct) $P \vee Q$, we simply add additional states to P, broadening the ways in which the proposition may be true. But going from $P \wedge Q$ to P is a matter of selecting, from each truthmaker for the conjunction, the part which is relevant just to P. These are a very different operations, corresponding to the different ways in which truth is preserved from disjuncts to disjunctions and from conjunctions to conjuncts. The former, from P to $P \vee Q$, weakens the proposition, in the sense that it provides more grounds for truth. The latter, from $P \wedge Q$ to P, weakens the proposition in a very different way, by reducing each ground for truth. On the classical approach, it's hard to draw this distinction, since P is classically identical to $(P \wedge Q) \vee P$. As a consequence, the move from $P \wedge Q$ to P simply involves adding more worlds, just as in the move from P to $P \vee Q$.

What independent reasons are there to distinguish these two forms of consequence? One is that they each make a different contribution to Anderson and Belnap's (1962) *primitive entailments*. A primitive entailment is a sentence of the form $C_1 \wedge \cdots \wedge C_n \to D_1 \vee \cdots \vee D_m$, where each C_i and D_i is either a primitive sentence letter or the negation of one, and is valid (or *explicitly tautological*)

iff some C_i is identical to some D_j. Various rules then extend the notion to *tautological entailment*. Although the validity of the primitive entailments is 'obvious' (1962, 11), it is not conceptually basic. It involves two basic principles: first, the shared sentence is a consequence of the conjunction and, second, the disjunction is a consequence of it. This suggests that the primitive relationships underlying the tautological entailments are conjunction-to-conjunct, on the one hand, and disjunct-to-disjunction, on the other.

The picture is similar if we think in terms of the basic structural rules which underlie inference. The most basic rule is the axiom of identity, $A \vdash A$. Perhaps next in importance, in *classical* reasoning at least, come the two forms of *weakening*:

$$\frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta} \qquad \frac{\Gamma \vdash \Delta}{\Gamma \vdash A, \Delta}$$

Using these three rules, we can build sequents corresponding to all the primitive entailments. But the left and right versions of weakening are conceptually different rules with different conceptual justifications (in settings where they are justified at all). Since we typically think of the premises conjunctively and the conclusions disjunctively, weakening on the left corresponds to conjunction-to-conjunct reasoning and weakening on the right corresponds to disjunct-to-disjunction reasoning.

Fine (2017a, §5) offers an understanding of these two basic forms of consequence in terms of two notions of parthood. There are two senses in which P may be a part of Q. One, the most familiar, is when P is a subset of Q, so that all truthmakers for P are thereby truthmakers for Q. This is the sense in which P is a part of $P \vee Q$, and so we call this *disjunctive parthood*. On the second sense, by contrast, each P-state is a part of a Q-state and each Q-state has a P-state as a part. This is the sense in which P is a part of $P \wedge Q$, and so we call this *conjunctive parthood*. It is hard to distinguish these notions on the classical picture, which identifies $(P \vee Q) \wedge P$ with P. So the conjunctive connection between $P \vee Q$ and $(P \vee Q) \wedge P$ is identified with (the converse of) the disjunctive connection between P and $P \vee Q$. If we align (truthmaker) entailment with preservation of truthmakers (P entails Q when $P \subseteq Q$), then entailment aligns with disjunctive parthood.

Conjunctive parthood, by contrast, is a form of content inclusion (Fine 2017a, §5). Suppose you say that Berkeley is an English philosopher. You're partly right, partly wrong. And if you also say that Stephen Fry is an English philosopher, you're also partly right and partly wrong; but you're partly right in a different way in each case. We can sometimes isolate a true part of a false proposition (which must itself be a proposition); and we can compare whether two partly true propositions are partly true in virtue of the same true parts. This notion of parthood clearly isn't entailment, for entailment must preserve (full) truth. Saying that Berkeley is English isn't partly true, even though that proposition is a disjunctive part of the

proposition that Berkeley is either English or a philosopher. The relevant sense of parthood aligns naturally with conjunctive parthood: a false proposition is partly true when it has a true conjunctive part. (Not every falsehood is partly true, even though each A is classically equivalent to the partly true $A \wedge \top$.)

In saying that Berkeley is an English philosopher, one says, in part, that Berkeley is a philosopher. In general, proposition Q *contains* P when P is a conjunctive part of Q. Thus the consequence-like relationship between conjunctions and their conjuncts is (the converse of) containment, not entailment. Returning to our example, in saying that Berkeley is a philosopher, you do not say in part that he is either English or a philosopher (although that is entailed by what one says). What you say does not contain the content of *Berkeley is either English or a philosopher*, for the latter content goes beyond the subject matter of *Berkeley is a philosopher*.

Formally, P is a conjunctive part of Q when:

(i) Each $s \in P$ is part of some $u \in Q$; and

(ii) Each $u \in Q$ has a part $s \in P$.

Conjunctive parthood takes us from mereologically *lesser* to *greater* propositions, with $\{\square\}$ the least and $\{\blacksquare\}$ the greatest. Disjunctive parthood, by contrast, takes us from logically *stronger* to *weaker* propositions, with $\varnothing$ the strongest and S the weakest. For regular nonempty propositions, $P_1 \wedge P_2 \wedge \cdots$ is the least regular nonempty proposition to contain each of $P_1, P_2, \ldots$ (Fine 2017a, theorem 12) and $P_1 \vee P_2 \vee \cdots$ is the strongest regular proposition entailed by each of $P_1, P_2, \ldots$ (2017a, theorem 14).

4 Refining Disjunctive Parthood

I'm going to argue that more can be done with disjunctive parthood, both in terms of its underlying analysis and its applications. First, let's see why the current understanding is limited. Fine (2017a;b) handles disjunction extensionally, just as in classical and intuitionistic logic:

($\vee$EX) $s \Vdash A \vee B$ iff $s \Vdash A$ or $s \Vdash B$

One worry with this approach is that a truthmaker for $A \vee B$ may be a whole comprising a truthmaker for A and a truthmaker for B (as in Fine and Jago 2019). For intuitively, $A \vee B$ has three routes to truth: A, B, and both. Since a truthmaker for $A \wedge B$ (the *both* case) need not be a truthmaker either for A or for B, this case is missed by ($\vee$EX). But this omission is benign, for regular propositions are closed under summation and so $s \sqcup u \in P \vee Q$ whenever $s \in P$ and $u \in Q$.

The problematic case occurs if there are $A \vee B$-states which are neither A-states nor B-states. Here is one reason to think there are such states. The state of my

having a (binary) sibling makes it true that I have either a sister or a brother, but it doesn't make it true that I have a brother and it doesn't make it true that I have a sister. We should not identify the former 'disjunctive' state of affairs with either of the more determinate 'disjunct' ones, for they have different identity and persistence conditions. They involve different properties and, were my brother to change his gender to female, the *sibling* state would remain but the *brother* state would not.

Here is a second reason. Let *red*, *scarlet*, and *maroon* be the propositions that the box is red (scarlet, maroon) respectively. Since *scarlet* truthmaker-entails *red*, *scarlet* is a disjunctive part of *red*. The same goes for *maroon*: it too is a disjunctive part of red, and so on, for all the other determinates of redness. This suggests that *red* has the form *scarlet* $\vee$ *maroon* $\vee \cdots$, where each disjunct corresponds to a determinate of redness. So by ($\vee$EX), any truthmaker for *red* must be a truthmaker for *scarlet* or for *maroon* or .... But then there can be no single state of *the box's being red*, for such a state would make *red*, but none of its 'disjuncts', true. We may repeat the argument for the determinates of *scarlet*, of *maroon*, and so on, concluding in each case that there is no corresponding state of affairs. The only genuine states of affairs, given this line of reasoning, are those featuring properties with no determinates. These are fundamental states of affairs, admitting no further grounds. There are no truthmakers but the fundamental ones, sitting at the ground floor of reality.

This is an unappealing metaphysics, however. I exist and thereby make the proposition that I exist true. It's true in virtue of my existence. Yet I'm not a fundamental entity. Moreover, I have properties which are not fundamental, like being a philosopher. So there is a state of my being a philosopher which is not a fundamental state. Not all truthmakers are fundamental entities. (To think otherwise, one must deny that non-fundamental entities such as you and I exist at all: one must be a mereological nihilist (Sider 2013), believing in nothing but part-less atoms.) *That the box is red* is a state as good as any other and is perfectly suited to make *red* true. Since it makes true none of *red*'s disjunctive (proper) parts, ($\vee$EX) needs revision.

Here is a third argument for that conclusion. After many washes, my Levi's are no longer the deep black they were. But neither are they clearly grey. It's indeterminate what colour they are, although they are clearly on the black-to-grey spectrum. Let *black* and *grey* be the corresponding propositions. Then, it seems, my jeans's colour makes *black* $\vee$ *grey* true, without making either *black* or *grey* true.

This may be too quick. *Epistemicists* like Williamson (1994) hold that a proposition may be true but indeterminately so. Barnes and Williams (2010) concur, from a very different metaphysical standpoint. Either *black* is true or *grey* is, but it's indeterminate which. These are minority views, however. Most

philosophers agree with Fine (1975, 266) that 'any (extensionally) vague sentence is neither true nor false'. But still, it does not follow that there is a proposition, *black*, which is indeterminate in truth value. Fine (1975, 265) holds that 'vagueness is deficiency of meaning'. It is words, rather than worldly states of affairs, that are vague. (Lewis (1988) takes a similar line.) If states of affairs are determinate and propositions are sets of such states, then propositions too are determinate. Vagueness exists in the relation tying sentences to propositions. So, on this picture, it would be wrong to say that there is a proposition *black* which is indeterminate in truth.

Nevertheless, it is hard to avoid the conclusion that there are indeterminate states of affairs. There are worldly facts about how we use words, giving rise to semantic states of affairs, *that sentence S expresses proposition P*. If it is indeterminate whether S expresses P, then we seem to have an indeterminate semantic state of affairs. If it is determinate which entities S and P are, then the indeterminacy is in the *expressing* relation. Perhaps it is indeterminate whether the state *that sentence S expresses proposition P* actually exists. I don't wish to argue here that we should understand vagueness in this or that way. The point is far more modest: we should not rule out models on which, due to vagueness, a disjunction is true but neither of its disjuncts are.

To see what we should put in place of (∨EX), let's go back to our starting point with truthmaker semantics. States, unlike possible worlds, are typically *incomplete*, capturing a particular portion of some possible reality. This incompleteness of states manifests in two dimensions. To bring these out, consider a toy world comprising a 3×3 grid of coloured tiles, labelled 1 (top-left) to 9 (bottom-right). We can describe that world by describing the colour of each tile (and then saying: that's it). Here are three incomplete states of the grid:

s_1: Tile 1 is red;

s_2: Tile 1 is red and tile 2 is red;

s_3: Tile 1 is scarlet.

States s_2 and s_3 each add extra information to state s_1. In different ways, they are more complete than it. State s_2 adds information about tile 2, a portion of reality not mentioned in s_1. It goes beyond s_1's ontology. State s_3, by contrast, stays within s_1's ontology but conveys more precise information about it. The information in s_3 is more determinate (about the same tile) than the information in s_1. It specifies *how* tile 1 is red: by being scarlet.

It is useful, from a metaphysical perspective, to separate these two dimensions of incompleteness, for each corresponds to a way in which reality is structured. The first is mereological summation. The second we might call *refinement*, *precision*, or *determination*. Completeness amounts to a fixed point of summation and

refinement. We can conceptualise the mereological relation in part-whole terms, or as a construction operation, taking states and building less fundamental ones. Similarly, we can conceptualise the refinement relation in two ways. In one way, we consider an imprecise state and a refinement of it. In the other, we think in terms of an operation, taking states and building something less precise from them. The inputs in this case need not all (and in some cases, cannot) be actual states. If the box is scarlet, it cannot also be maroon, or crimson, or cherry, but any of these suffices for its being red. We might think of *that the box is red* as a construction from the actual state *that the box is scarlet* and the possible states *that it is maroon*, *that it is crimson*, and so on. (Or we could take the operation to be primarily on properties: from *being scarlet*, *being maroon*, and so on, to *being red*.)

The determinable-determinate relation gives a flavour of the refinement relation, but they should not be identified. Determinates of a given determinable are usually taken to exclude one another: if something is scarlet, it isn't crimson. Refinements of an imprecise state may co-exist, by contrast: my having a sibling may be refined by my having a brother, by my having a sister, or by my having both a brother and a sister. (*Determinates* of a determinable are then its refinements at the 'same level' which exclude one another.) Moreover, many cases of refinement are not naturally treated as involving determinables. *That a is F or b is G*, where a and b and F and G have nothing to do with one another, is not naturally treated as a determinable state, for there is no determinable property involved. The relevant building operation builds refinable states from any possible inputs, just as mereological summation builds wholes from parts.

Is *refinement* just the *grounding* relation? It certainly seems that all cases of s refining u are cases of s grounding u. But grounding may be many-one: the parts together ground the whole, as the conjuncts together ground the conjunction. In Fine's (2012b) terminology, the binary proper-part-to-whole relation is a case of *partial* grounding, whereas the binary refinement-to-refined relation is a case of *full* grounding. I will take *refinement* as a conceptual primitive, along with *parthood*, and understand *grounding* (at least for concrete material matters) in terms of them. On this approach, grounding is a disjunctive concept. A concrete case of grounding is either of the part-to-whole kind or of the refinement-to-refined kind. (We might align existential cases with disjunctive cases. If we do not, we should add a further category. Either way, there is a further entity-to-set case, which I shan't consider further.)

Mereological summation is naturally aligned with conjunction (Fine 2017a, §3). It delivers conjunctive states from conjunct states and conjunctive parthood on propositions (§3). In each case, summation underwrites the properties we expect of conjunctive entities. Similarly, refinement is naturally aligned with disjunction. When s is refined only by t and u, it's natural to think of s as the state $t \vee u$. The

state *that the box is red* is the state *that it is scarlet or crimson or maroon or* . . . . This is not a claim about concept or their meanings: one can have the concept of redness without the concept of scarlet. The claim is metaphysical, about the states (and the underlying properties) themselves. I will call such states *ontic disjunctions* and take them to be unstructured, just as conjunctive entities are.

5 Refinement Across Categories

Truthmaker semantics is usually presented in terms of state spaces, leading to a focus on part-whole relations between states. But there's no reason to restrict our attention to states. The notion of a truthmaker should not be restricted to any ontological category, for any entity x is a truthmaker for the proposition that x exists. Similarly, 'there is nothing in the account of the [Boolean] operations or operators which requires that their application be limited [to propositions]' (Fine 2017a, 635).

Linking conjunction to summation suggests a general account of 'and' in terms of summation. When Anna, Bec, and Cath form a circle, something forms the circle. That thing is not Anna, not Bec, and not Cath. It is the whole of which each is a part: an ontic conjunction of particulars, itself a particular, denoted by the conjunctive term 'Anna, Bec, and Cath'. This use of 'and' cannot be understood in terms of sentential conjunction, for 'Anna forms a circle' is false. On this basis, Fine (2017a, 638) rejects the idea that 'the Boolean operators are essentially logical or truth-theoretic in character'. Rather:

> The Boolean operators have equal and equable application to all expressions whatever, no particular application – whether to sentences or names or some other kind of expression – being privileged. Thus far from being of central significance, the application to sentences or sentence-like expressions is merely one application among others. Moreover, the ur-use of 'and' is essentially mereological rather than logical in character; 'and', whether used to connect nominal or sentential expressions, will signify fusion [i.e. summation]. (Fine 2017a, 638)

Can the same be said of *refinement*? If not, the argument from §4 is called into question. So let us make the case that refinement operates across ontological categories. One thing Anna and Cath have in common is that they always take either the train or the bus. *Going by train or bus* is something they have in common: a disjunctive property. The existence of the property is, in some sense, nothing over and above its disjuncts. Yet its nature and possession conditions differ from the mereological sum of *going by train* and *going by bus* in being disjunctive, not conjunctive.

Or consider again the relation between properties *being red* and *being scarlet*, which is much like the relation between disjunction and disjunct. In §4, I argued

that there is a genuine state *that the box is red* and hence that the corresponding proposition is not merely the union of its disjunctive parts. It is a short step to understanding the property *being red* disjunctively, as *being scarlet, or maroon, or* ... (Rosen 2010, 128). *Being scarlet* must somehow be a constituent of *being red* without being a mereological part of it, since something may possess *being red* but not *being scarlet*. The apt model is disjunction, not conjunction. *Being red* is constructed from its determinates via disjunctive summation, the operation which stands to the usual concept of summation as refinement stands to parthood.

It is harder to make the case for disjunctive particulars, in parallel to the summed particular *Anna, Bec, and Cath* which forms the circle. 'Or' in 'Anna or Cath' seems always to distribute: 'Anna or Cath will be here' means 'Anna will be here or Cath will be here', and so we seem unable to use term-disjunction to argue for disjunctive particulars. Nevertheless, I think we can find a philosophical home for disjunctive particulars, drawing on two Finean themes.

The arbitrary Northerner is a Northerner and so (unlike the set of Northerners) is a concrete human being. It is not identical to any particular Northerner, but neither is it independent of them all. If each particular Northerner were to move to the south, there would be no arbitrary Northerner. Nor is the arbitrary Northerner the stereotypical Northerner: it does not wear a flat cap or keep ferrets. We may take 'the arbitrary Northerner' to denote the ontic disjunction of all particular Northerners, such that each refines it. This entity exists so long as some Northerner does, without entailing that any specific Northerner exists. Its properties are those common to those disjoined individuals (excepting properties like *being a specific person* and *not being an arbitrary object*). Its accent is neither classic Yorkshire nor classic Lancashire, but is clearly distinguishable from crisp Home Counties.

(Fine (1983, 58) treats arbitrary objects as 'a kind of abstraction', which seems in tension with my saying they can be concrete. This may be terminological. A crowd of people is, in a sense, an 'abstraction' from particular people and yet is spatiotemporally located. So too for arbitrary people. They are located in what Fine (1983, 63) calls the 'generic' sense but need not be in the 'classical' sense: the arbitrary Northerner lives north of Nottingham, but neither east nor west of the Pennines.)

This treatment explains the role of arbitrary objects in formal reasoning. Since disjunctive entities have the properties common to all their disjunctive parts, showing that the arbitrary F is G entitles us to conclude that all Fs are G. Fine (1983; 1985) takes arbitrary objects to exist but not in an 'ontologically significant' sense (1983, 57), whereas I say they genuinely exist as dependent entities. (They are not *ordinary* objects, which is to say, *we* don't care much about them.) The arbitrary Northerner stands to particular Northerners much as *being a coloured object* stands to particular colours. We don't include the former in the census just as we don't count the latter as a colour.

Another case, tentatively suggested in §4, aligns vague entities with ontic disjunctions of precise entities. (We might restrict the vague entities to 'semantic' ones: the meanings of terms, for example. But we need not.) Their refinements are their admissible precisifications. Such entities have the properties common to all their admissible precisifications. If there genuinely is a property of *redness*, it is surely a vague entity. (Fine (1983, 61) briefly draws a parallel between arbitrary entities and vagueness.) This approach explains how vague entities may arise from determinate fundamental entities: they are disjunctive constructions from (more) determinate entities. (Even if quantum indeterminacy is a kind of ontic vagueness, it is surely not the reason *redness* is vague.)

Fine's (1975) models for vague languages have a structural similarity to state spaces with refinement. Formally, both are based on a truth-preserving partial order. If a state makes A true, then so does any state which refines it; and similarly, if A is true on a 'specification' s in Fine's supervaluationism, then A is true on any admissible precisification (or *extension*) of s. It is certainly not the case that refinements and admissible precisifications coincide (even if we allow for ontically indeterminate states). *Radicchio* (a specific shade of scarlet) refines without being an admissible precisification of *red*, for the post box is clearly red but not radicchio. The claim is that various notions may be understood, structurally, in terms of refinement, not that we have just one notion.

6 From And to If

We've been discussing the metaphysics of truthmaking with refinement. Now it's time to change course and think about the *logic* of truthmaking with refinement. For the remainder of the paper, we'll see what happens when we add a conditional to our language, interpreted using the resources of truthmaking with refinement. Spoiler alert: we'll be entering the territory of *relevant logics*. If that's new ground for you, you may find Anderson and Belnap 1975, Bimbó 2006, or Dunn and Restall 2002 helpful background material.

Conditionals are not yet well understood in truthmaker semantics. The logic of containment (that is, conjunctive parthood) is captured by Fine (2016). Fine and Jago (2019) characterise exact truthmaker entailment. Neither approach analyses an embeddable object-language conditional. They stand to a truthmaker semantics for a full language roughly as FDE stands to full relevant logic. There's a conditional in Fine's (2014) truthmaker semantics for intuitionistic logic, understood in terms of *conditional connection* states. That conditional is intuitionistic under the *inexact* notion of consequence; its exact behaviour is undiscovered countryside. I won't pursue that option here.

I will investigate a notion of conditionality which is *already* contained in standard truthmaker semantics (that is, even before we add refinement). The

guiding idea is that (i) the defining feature of the exact semantics is its treatment of conjunction and (ii) conjunction, so understood, delivers a natural conditional via *residuation*:

(Residuation) $A \wedge B \vDash C$ iff $A \vDash B \to C$

Suppose we take this relationship to be an analytic truth relating 'and', 'if', and 'entails'. Then an understanding of 'and' and 'entails' provides an understanding of 'if'. In particular, the road is open to an exact truthmaker analysis of the conditional. That is my aim in this section.

What features will this conditional have, given our exact understanding of conjunction? First, since conjunctions do not in general entail their conjuncts, we will not in general have $A \vDash B \to A$. If we also accept a deduction theorem, allowing us to move from A's entailing B to the validity of $A \to B$ (and vice versa), then:

(PP) $A \to (B \to A)$

will not be a valid implication. This is *positive paradox*, a classically and intuitionistically valid principle which is rejected by relevant logics. So we seem to be in the territory of relevant implication.

Second, premises and antecedents will permute, in the sense that $A \vDash B \to C$ iff $B \vDash A \to C$. A deduction theorem would then give us the permutation axiom:

(Permutation) $(A \to (B \to C)) \to (B \to (A \to C))$

Third, repeating a premise as an antecedent is redundant: if $A \vDash A \to B$ then $A \vDash B$. For the former implies $A \wedge A \vDash B$ and our treatment of conjunction gives $A \vDash A \wedge A$. A deduction theorem would then gives us the contraction axiom:

(Contraction) $(A \to (A \to B)) \to (A \to B)$

We can go on, deriving all the axioms for $\mathbf{R}^+$, the positive fragment of the most prominent relevant logic, **R**.

These arguments assume a deduction theorem. To assess whether this is warranted, we first need to say what we mean by *validity*. We don't want a sentence to be valid only when every state makes it true, for we will require states which don't make $A \to A$ true. Otherwise, any B would entail $A \to A$, giving us $B \to (A \to A)$ and hence (PP). Instead, we can designate a unique state to record validity. The null state $\square$ is an obvious candidate.

Somewhat surprisingly, this choice allows us to derive the deduction theorem via residuation. To simplify the argument, let's initially consider models without a refinement ordering. Now let p be a new sentence made true only by the null state, so that $s \Vdash A \wedge p$ iff $s \Vdash A$. Then $A \to B$ is valid iff $\square \Vdash A \to B$ iff $p \vDash A \to B$

iff $p \wedge A \vDash B$ (by residuation) iff $A \vDash B$. (We'll consider the more complex case of models with a refinement ordering below.)

What should the truthmaker clause for $\rightarrow$ be, on this approach? Reflexivity ($B \rightarrow C \vDash B \rightarrow C$) and residuation imply $(B \rightarrow C) \wedge B \vDash C$: summing a conditional-state with an antecedent-state results in a consequent-state. This gives us a necessary condition on truthmaking the conditional: if $s \Vdash B \rightarrow C$ and $u \Vdash B$ then $s \sqcup u \Vdash C$. We can (with a bit more effort) show that it is a sufficient condition too. Again, to simplify the argument, we will initially consider models without a refinement ordering. Suppose $s \sqcup u \Vdash B$ whenever $u \Vdash A$ (to show $s \Vdash A \rightarrow B$). This time, let p be a new sentence made true only by s, so that $s \Vdash A \wedge p$ iff $s \Vdash A$ and let $t \Vdash p \wedge A$. Then by definition, $t = t_1 \sqcup t_2$ where $t_1 \Vdash p$ and $t_2 \Vdash A$, so t_1 must be s. But then $t \Vdash B$, and hence $p \wedge A \vDash B$. By residuation, $p \vDash A \rightarrow B$ and, since $s \Vdash p$, we have $s \Vdash A \rightarrow B$.

These arguments together give us the following truthmaker clause for $\rightarrow$:

($\rightarrow$) $s \Vdash B \rightarrow C$ iff, for any $u \Vdash B$, $s \sqcup u \Vdash C$.

This clause was first given by Urquhart (1972a;b) in a semantics for relevant implication. (He made no mention of truthmakers, of course! He worked with a 'union' operator on 'bodies of information', which nevertheless has the semilattice properties of $\sqcup$.)

Having introduced a conditional and noted its resemblance to the relevant conditional of **R**, can we go further and develop a full relevant semantics? Suppose we try, interpreting disjunction in the usual extensional way, via ($\vee$EX). This move results in a logic strictly stronger than $\mathbf{R}^+$ (Urquhart 1972a). For ($\vee$EX) validates inferences such as

(P) $A \rightarrow B \vee C, B \rightarrow D \vDash A \rightarrow D \vee C$

which are not valid in **R**. (Urquhart (1972a) attributes the example to Dunn and Meyer. Fine (1976) axiomatizes the resulting logic containing extensional (inexact) conjunction.)

Given our treatment of implication, inferences like (P) are related to the idea of states being *prime*: making true a disjunct of each disjunction they make true. For we may read (P) as saying that: if A-states lead to $B \vee C$-states and B-states lead to D-states, then A-states will lead to $D \vee C$-states. On our semantics, this will be so when each $B \vee C$-state is either a B state (and hence a D-state) or a C-state, in other words, when all states are prime. One can avoid (P) if there are non-prime states in the domain. But then, how is disjunction to be understood?

We may appeal to *prime refinements*, those refinements u of a state s which are also prime states, notated $u \leq_P s$. A disjunction is made true by a state when each of its prime refinements makes a disjunct true:

($\vee$) $s \Vdash A \vee B$ iff, for all $u \leq_P s$, $u \Vdash A$ or $u \Vdash B$.

This is, in essence, Fine's own approach to relevant logic (Fine 1974). He adopts Urquhart's (1972a; 1972b) semilattice approach to the conditional but allows for non-prime states, banned by both Urquhart and Routley and Meyer (1972a;b; 1973), with disjunction understood as in ($\vee$). Fine's interpretation of his semantics is quite different than the truthmaker one offered here, however. He interprets states as *theories*: 'sets of propositions closed under commitment' (Fine 1974, 348) and $s \sqcup u$ (here thought of as the sum of s and u) in terms of commitment: it is the set of all propositions P such that s commits us to u committing us to P (1974, 349). Yet unless we have a prior understanding of propositions and their commitments, this tells us little about meaning. If theories contain sentences and commitment is implication, for example, then the interpretation says that $s \sqcup u = \{B \mid \exists A \in u : A \to B \in s\}$. The interpretation is none other than the canonical model used in the completeness proof.

Another precedent for ($\vee$) is Humberstone's (1988) semantics for $\mathbf{R}^+$. He introduces an algebraic operation $+$ (in addition to Urquhart's semilattice $\sqcup$), with a disjunction true at s iff there are t, u such that A is true at t, B is true at u, and $t + u = s$. Notice the similarity with the truthmaker clause for conjunction. Humberstone (1988, 65) even suggests a mereological interpretation of $+$. Thinking of the points s, t as regions of logical space, the sum of s and t will behave disjunctively (just as union on sets of possible worlds does). But this interpretation leaves the semilattice operation mysterious: how might one combine two regions of logical space, other than by summing them?

(It may at first seem strange that Humberstone's condition for $\vee$ is existentially quantified whereas ours is universally quantified: surely if one works the other is incorrect? In fact, they amount to much the same. If we reconceptualise states as regions and prime states as points, then any region s may be exhaustively decomposed into an A-region an a B-region (as in Humberstone's condition) just in case all points in it are A-points or B-points (as in ($\vee$)).)

Let's return to the truthmaker interpretation. Truthmaker semantics with refinement, I suggest, provides a natural and philosophically well-motivated interpretation of relevant disjunction. But having added the refinement ordering to our semantics, we should check whether the arguments used to justify the clause for $\to$ still stand up. The issue with the right-to-left justification of ($\to$) is that refinements should make true whatever the states they refine make true: if $s \leq u$ and $u \Vdash A$ then $s \Vdash A$ too. But then, we cannot assume there's a sentence p made true only by state s, as we did above: p must also be made true by any state which refines s. Nevertheless the argument will go through so long as refinement is preserved by summation, as in the following principle:

($\leq/\sqcup$) If $s \leq u$ then $s \sqcup t \leq u \sqcup t$

Now we can run a modified right-to-left argument for ($\to$). We suppose: (i) $s \sqcup u \Vdash B$ whenever $u \Vdash A$ (to show $s \Vdash A \to B$); (ii) p is a primitive sentence made true

only by s and every state it refines; and (iii) $t \Vdash p \wedge A$. Then $t = t_1 \sqcup t_2$ where $t_1 \Vdash p$ and $t_2 \Vdash A$. Given (ii), $s \leq t_1$ and so, by ($\leq/\sqcup$), $s \sqcup t_2 \leq t_1 \sqcup t_2 = t$. From (i), $s \sqcup t_2 \Vdash B$ and so $t \Vdash B$. Now we can reason as in our previous argument: $p \wedge A \vDash B$, so $p \vDash A \to B$, and hence $s \Vdash A \to B$.

Given ($\leq/\sqcup$), we have a sound and complete semantics for the $\to, \wedge, \vee$ fragment of **R** (with $\wedge$ read as intensional conjunction (Jago 2020). If we introduce extensional conjunction & via its classical clause,

(&) $s \Vdash A \,\&\, B$ iff $s \Vdash A$ and $s \Vdash B$

we have a sound and complete semantics for $\mathbf{R}^+$ (Fine 1974). I'll address whether this bifurcated treatment of conjunction is acceptable in §7.

7 The Conjunction Problem

There is an issue facing any truthmaking interpretation of relevant logic. On the exact notion, conjunctions do not entail their conjuncts. In relevant logics, they do. So, an exact interpretation of relevant logic seems to be blocked. The inexact notion of truthmaking will not help. For although conjunctions inexactly entail their conjuncts, the inexact treatment of the conditional is not a relevant one. For if any addition to an A-state results in an A-state, as in the inexact semantics, then each state will truthmake $A \to A$, thereby validating the irrelevant $B \to (A \to A)$.

The picture is more complex, however, for relevant logics contain two notions of conjunction. Classically and intuitionistically, conjunction is that connective which licenses inferences from a pair of sentences to their combination and back again and which allows us to move between a finite list of premises and their single conjunction (which amounts to satisfying *residuation*). In relevant logics, these roles split into two. *Extensional* conjunction, '&', is governed by the axioms $A \,\&\, B \to A$, $A \,\&\, B \to B$ and the *adjunction* rule: from A, B, infer $A \,\&\, B$. But we cannot move from conjoined premises $A \,\&\, B$ to individual premises A, B. This would license the move from the valid $A \,\&\, B \vDash A$ to $A, B \vDash A$ and hence to the irrelevant $B \vDash A \to A$. *Intensional* conjunction is that connective which interacts with implication via *residuation*. It is the conjunction of the exact semantics, in other words. (Relevant logicians typically use '$\circ$' and for intensional conjunction and call it 'fusion'. I'll stick with '$\wedge$' for continuity with §§2–6, adding an 'I' subscript as a reminder: $\wedge_I$.)

Which of these symbols best captures reasoning with English 'and'? Which is *real* conjunction and which the imposter? Given how familiar extensional conjunction is, one might be tempted to ignore its intensional variant. Mares warns his fellow relevant logicians against this: 'we cannot just ignore [$\wedge_I$]' for 'it has a central place in the proof theory of relevant logic' (Mares 2009, 15). (Read (1988) also gives a central place to intensional conjunction in his understanding

of relevant logic.) It is incumbent on relevant logicians such as Mares to make sense of $\wedge_I$.

Mares interprets relevant logic in general in terms of situations carrying information, and in particular:

> a situation s carries the information that $A \wedge_I B$ if and only if s carries the information that all the consequences of there being in the world a situation in the same world which carries the information that A and there being a situation in the world which carries the information that B. (Mares 2009, 15)

In other words, s 'tells us' that there exists (in the same world) a situation t with the information that A and a situation u with the information that B. So we can infer t's existence (and u's existence) from s's existence. We can also infer A from t's existence, allowing us to infer A from s's existence. But s is an arbitrary $A \wedge_I B$-state and so we appear to be able to infer A from $A \wedge_I B$, contrary to the intended intensional reading of $\wedge_I$. So I do not think we can accept Mares's interpretation of $\wedge_I$.

The problem goes deeper for relevant logicians. Consider the basic claim that relevant logic contains two notions of conjunction (irrespective of how the intensional version is to be understood). The standard explanation from the relevant logician has it that the inferential patterns we intuitively associate with 'and' split, in relevant logic, between the intensional and extensional operators. On that view, we are wrong to treat 'and' as a unified concept governed by these inferential patterns. There must be two 'and'-concepts in common usage in English, a fact hidden to introspection and conceptual reflection but uncovered relatively recently by relevant logicians. That is deeply implausible.

It is equally implausible to deny a genuine link between '$\wedge_I$' and our use of 'and'. That move would leave us unable to account for our use of 'and' in contexts such as these:

Case 1: We ordinarily read lists of premises *conjunctively*: '$A, B \vDash C$' as 'A and B together entail C'. For the relevance logician, the 'and' here must be read intensionally. (If read extensionally, we could infer from $A \mathbin{\&} B \vDash A$ to $A, B \vDash A$ and hence to $\vDash A \to (B \to A)$, which all relevant logics must avoid.)

Case 2: We take the combination of two good arguments, for conclusions A and B respectively, as a good argument for their conjoined conclusions, 'A and B'. Again, 'and' here cannot be understood extensionally. If it were, it would permit a derivation of *positive paradox*, as follows:

A	Assumption
$\quad B$	Assumption
$\quad A$-and-B	And-introduction
$\quad A$	And-elimination
$B \to A$	Discharge assumption
$A \to (B \to A)$	Discharge assumption

Combining good arguments in this way with 'and' must be understood via $\wedge_I$ (for which and-elimination is not valid).

One radical move is to ban extensional conjunction from relevant logics altogether. Avron (1992) argues that relevant logics should 'avoid extensional connectives altogether and use instead a purely relevant logic with purely relevant [by which he means, intensional] connectives' (Avron 1992, 262). His argument for this radical conclusion is that extensional conjunction 'is responsible for the problems of **R** and **E** and the inconsistencies in the relevantists' work'; and in particular, 'in the presence of truth-functional conjunction the positive 'paradox of implication' [i.e. (PP)] is valid, and there seems to be no convincing method to avoid this conclusion' (1992, 261). (Case 2 above is very much in line with Avron's argument.)

My worry with Avron's stance is that, on the concept of conjunction we express in English with 'and', it is clear that conjunctions entail their conjuncts. That's simply not the kind of inferential move we can be confused about, since it is in part constitutive of what we mean by 'and'. If 'and' is banned from relevant logic, then Avron's line must be that ordinary reasoning isn't captured by a relevant logic. This is what makes Avron's line so hard for relevant logicians to adopt.

The deep problem for the relevance logician, then, is that the following are all true and yet seem incompatible:

(1) There is one and only one genuine concept of conjunction in English;

(2) Conjunctions entail their conjuncts;

(3) There are inferential patterns governing 'and' which are invalidated in relevant logic by extensional conjunction.

The problem does not arise for classical or intuitionistic logic, on which $\wedge_I$ and & coincide.

The truthmaker interpretation resolves the problem. There is an exact and an inexact notion of truthmaking, giving rise to exact and inexact notions of

inference. The former preserves specific grounds for truth and hence relevance; the latter preserves just the existence of a ground for truth, which need not maintain relevance. Then 'and' may be understood *uniquely* as $\wedge_I$, thereby satisfying (1) and (3). (2) is understood as a claim about *truth-preservation*: a true conjunction has true conjuncts. This is underwritten by $A \wedge_I B$'s *inexactly* entailing both A and B. The problem is resolved by splitting the notion of entailment, rather than the concept of conjunction.

On this reading, Avron's proposal becomes tenable. Relevant logic would contain a single conjunction connective, $\wedge_I$, and the extensional behaviour of 'and' would be explained by the inexact notion of entailment. But what if we follow the standard approach and include &? Its exact semantics suggests an 'overlapping content' interpretation: 'A & B' says that A and B have a truthmaker in common. That state is an exact truthmaker for A & B. We may be interested in whether some mental statement A has a physical realiser, for example. This will be the case if A and the disjunction P of all physical statements have a common truthmaker (Fine and Jago 2019), that is, if A & P (as understood here) is true. (Indeed, this interpretation is analogous to our treatment of premises in an exact entailment: $A_1, A_2, \ldots$ exactly entail C when any exact truthmaker for each A_i is thereby an exact truthmaker for C (Fine and Jago 2019). Any such state is thereby part of the common content of the A_is.) Since a state may inexactly truthmake both A and B without their having an exact truthmaker in common, we cannot reason inexactly from A, B to A & B. Unlike $\wedge_I$, & does not coincide with classical conjunction within inexact reasoning.

8 Routley-Meyer and Mingle

With a philosophically interpreted semantics for relevant logic in hand, we can make progress on whether certain controversial axioms should be accepted. This situation stands in contrast with our understanding of the standard Routley-Meyer semantics (Routley and Meyer 1972a;b; 1973). On that approach, many candidate axioms correspond to structural conditions on the ternary relation R: *pseudo-modus ponens* to $Rsss$, *assertion* to symmetry in the first two places ($Rstu$ iff $Rtsu$), the E-axiom $(((A \to A) \to B) \to B)$ to $Rs0s$, and *positive paradox* to $R00s$. (Here, 0 is a state used to record validity, not necessarily $\Box$.)

Whether R has these structural features depends on how we interpret it. Typical philosophical interpretations (such as Mares 1996; 2004 or Beall et al. 2012) do not settle the issue unambiguously (Jago 2013). They are designed to accommodate many relevant logics, precisely by *not* deciding these issues. The truthmaker interpretation is more definite (and, by that token, it is far less flexible). Given what we understand of parthood and refinement, *assertion* and *pseudo-modus ponens* are clearly mandated whereas *positive paradox* is clearly rejected.

We may interpret R by setting $Rstu$ iff $u \leq_P s \sqcup t$ (thereby dispensing with non-prime states altogether). So defined, R satisfies *idempotence* ($Rsss$), *identity* ($R{\square}ss$) and *commutativity* (if $Rstu$ then $Rtsu$). It satisfies *Pasch's postulate*: if $Rstx$ and $Rxuv$ for some x then $Rsuy$ and $Rytv$ for some y. For if $x \leq s \sqcup t$ and $v \leq x \sqcup u$, then $v \leq s \sqcup u \sqcup t$, so setting $y = s \sqcup u$ gives the result. And it satisfies *monotony*: if $Rstu$ and $R{\square}s's$ then $Rs'tu$. For if $u \leq s \sqcup t$ and $s \leq s'$ then $u \leq s \sqcup t \leq s' \sqcup t$ so $u \leq s' \sqcup t$ by transitivity of $\leq$.

The truthmaker interpretation gives us a (positive) logic at least as strong as $\mathbf{R}^+$. Does it give us anything stronger? Consider the *mingle* axiom:

(M) $A \to (A \to A)$

This extends **R** to the system **RM**. Its addition may be acceptable, given the requirements of relevance. On the Routley-Meyer approach, the condition corresponding to (M) is:

(4) $Rstu$ implies $R0su$ or $R0tu$

It is hard to see any *philosophical* justification for this principle, in terms of any of the usual interpretations of R.

On our truthmaking definition of R, (4) becomes:

(5) $u \leq_P s \sqcup t$ implies $u \leq_P s$ or $u \leq_P t$

A precisification of a whole is a precisification of one of its parts. This will not be true in general. For suppose s and t concern quite different topics, take $u = s' \sqcup t'$ where $s' \leq s$ and $t' \leq t$, and suppose u is prime. Then, intuitively, s' refines the content of s without going beyond that subject matter, and likewise for t' and t. But $s' \sqcup t'$ goes beyond the content of s and t, taken on their own and so refines neither of them. So we have $u \leq_P s \sqcup t$ and yet neither $u \leq_P s$ nor $u \leq_P t$.

There is, however, an alternative route to validating (M) within the truthmaker semantics. (M) is equivalent to $A \wedge A \to A$, which says that the sum of A-states is an A-state. *Closure* (§2) will validate this principle, for if the proposition *that A* is the set of A's truthmakers and is closed under $\sqcup$, then the sum of any truthmakers for A is itself a truthmaker for A. Given closure, (M) is valid. We may impose closure on just the primitive sentences, which will imply the general principle. (The proof in each case is simple.)

Closure is a very plausible truthmaking principle. Taken together, you and I are no less a truthmaker for 'at least one person exists' than taken separately. Summing us preserves whatever is common to our truthmaking abilities. But closure is not uncontroversial. Rodriguez-Pereyra (2009), in response to Jago (2009), rejects it:

> ⟨There is at least one muse⟩ … is made true by Thalia and is also made true by Clio. It is made true by them separately, not collectively. But … the conjunction ⟨There is at least one muse and there is at least one muse⟩ is collectively made true by Thalia and Clio. (Rodriguez-Pereyra 2009, 441)

We disagree on whether Thalia and Clio taken together ('collectively') make it true that there is at least one muse. I say they do. For they clearly (collectively) make it true that there are at least two muses, and one way for there to be at least one muse is for there to be two muses. If one way to be X is to be Y and s makes it true that Y, then s makes it true that X too. There is room for metaphysical disagreement here, just as there is room for logical disagreement (even between relevant logicians) over (M).

9 Conclusion

The aim of this chapter was twofold. First, to extend the notion of disjunctive parthood using *refinement*, placing it on a par, structurally, with conjunctive parthood (§§3–5). And second, to investigate what the resulting logic might look like. The conditional covertly lurking in truthmaker semantics is a relevant conditional, obtained from (exact, intensional) conjunction through residuation (§6). I took this as evidence that relevant logic's intensional conjunction, rather than its extensional version, best corresponds to English 'and' (§7). Finally, I discussed the status of the Mingle axiom within truthmaker semantics (§8).

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