The Conjunction and Disjunction Theses

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Abstract: Rodriguez-Pereyra (2006) argues for the *disjunction thesis* but against the *conjunction thesis*. I argue that accepting the disjunction thesis undermines his argument against the conjunction thesis.

IN AN IMPORTANT recent paper on truthmaking, Gonzalo Rodriguez-Pereyra (2006) argues for the *disjunction thesis*:[1]

(D) If an entity e is a truthmaker for a disjunction $\langle P \vee Q \rangle$, then either e is a truthmaker for $\langle P \rangle$ or e is a truthmaker for $\langle Q \rangle$.

He also argues against the *conjunction thesis*:

(C) An entity e is a truthmaker for a conjunction $\langle P \wedge Q \rangle$ iff e is a truthmaker both for $\langle P \rangle$ and for $\langle Q \rangle$.

His argument, which is against the 'only if' direction of (C), is simple. ⟨Peter is a man ∧ Saturn is a planet⟩ is true in virtue of *Peter's being a man and Saturn's being a planet*. But ⟨Peter is a man⟩ is not true in virtue of *Peter's being a man and Saturn's being a planet*. Hence truthmakers for conjunctions are not always truthmakers for the conjuncts (2006, pp. 970–1). Note that this argument does not affect the 'if' direction of (C):

(C_I) If e is a truthmaker for $\langle P \rangle$ and a truthmaker for $\langle Q \rangle$, then e is a truthmaker for $\langle P \wedge Q \rangle$.

Since (C_I) is highly plausible and Rodriguez-Pereyra gives no argument against it, I will assume that it is true. I will also assume:

(D_I) If e is a truthmaker for $\langle P \rangle$, then e is a truthmaker for $\langle P \vee Q \rangle$ and a truthmaker for $\langle Q \vee P \rangle$.

This is also a perfectly uncontroversial principle and is accepted by Rodriguez-Pereyra (2006, p. 968).

I will argue that the doctrine which accepts (D) but rejects (C) is hard to maintain. Let '$\langle P \rangle \rightsquigarrow \langle Q \rangle$' abbreviate 'all truthmakers for $\langle P \rangle$ are truthmakers for $\langle Q \rangle$'. The key principle for my argument is:

($\star$) $\langle ((P \wedge Q) \vee P) \vee Q \rangle \rightsquigarrow \langle P \vee Q \rangle$

1. I follow Rodriguez-Pereyra in writing '$\langle P \rangle$' to mean 'the proposition that P'.

which, together with (D_I), entails that $\langle P \wedge Q \rangle \rightsquigarrow \langle P \vee Q \rangle$. For suppose that e is a truthmaker for $\langle P \wedge Q \rangle$. Given ($D_I$), e is a truthmaker for $\langle (P \wedge Q) \vee P \rangle$ and hence for $\langle ((P \wedge Q) \vee P) \vee Q \rangle$, which, together with ($\star$), entails that e is a truthmaker for $\langle P \vee Q \rangle$. ($\star$) is problematic for Rodriguez-Pereyra because ($\star$) and (D) together entail that, if e is a truthmaker for $\langle P \wedge Q \rangle$, then either e is a truthmaker for $\langle P \rangle$ or e is a truthmaker for $\langle Q \rangle$. Rodriguez-Pereyra cannot accept this conclusion, for (by his lights) neither ⟨Peter is a man⟩ nor ⟨Saturn is a planet⟩ is true in virtue of *Peter's being a man and Saturn's being a planet*. Hence Rodriguez-Pereyra must reject ($\star$).

($\star$) merits discussion in the truthmaking debate because it is entailed by the following six principles:

(T1) $\langle P \vee P \rangle \rightsquigarrow \langle P \rangle$

(T2) $\langle (P \vee Q) \vee R \rangle \rightsquigarrow \langle Q \vee (P \vee R) \rangle$

(T3) $\langle P \wedge P \rangle \rightsquigarrow \langle P \rangle$

(T4) $\langle (P \wedge Q) \vee R \rangle \rightsquigarrow \langle (P \vee R) \wedge (Q \vee R) \rangle$

(T5) If $\langle P \rangle \rightsquigarrow \langle Q \rangle$, then:

(a) $\langle P \vee R \rangle \rightsquigarrow \langle Q \vee R \rangle$,

(b) $\langle R \vee P \rangle \rightsquigarrow \langle R \vee Q \rangle$,

(c) $\langle P \wedge R \rangle \rightsquigarrow \langle Q \wedge R \rangle$, and

(d) $\langle R \wedge P \rangle \rightsquigarrow \langle R \wedge Q \rangle$.

(T6) If $\langle P \rangle \rightsquigarrow \langle Q \rangle$ and $\langle Q \rangle \rightsquigarrow \langle R \rangle$, then $\langle P \rangle \rightsquigarrow \langle R \rangle$.

In effect, (T1–T6) give a small (and incomplete) proof system for truthmaking claims, with (T1–T4) as axioms and (T5) and (T6) as rules of inference, in which one can prove ($\star$) (a derivation is given in the appendix). I now turn to arguing for each of (T1–T6).

(T1) is an instance of (D) and so cannot be denied without rejecting (D). (T2), a combination of associative and commutative principles, follows immediately from (D) and (D_I). For suppose that e is a truthmaker for $\langle (P \vee Q) \vee R \rangle$. By (D), it is a truthmaker for at least one of $\langle P \rangle$, $\langle Q \rangle$ and $\langle R \rangle$. In each case, given (D_I), e is a truthmaker for $\langle Q \vee (P \vee R) \rangle$ and so (T1) holds. (T6) is a transitivity principle: if all truthmakers for $\langle P \rangle$ are truthmakers for $\langle Q \rangle$ and all of $\langle Q \rangle$'s truthmakers are truthmakers for $\langle R \rangle$, then clearly all truthmakers for $\langle P \rangle$ are thereby truthmakers for $\langle R \rangle$. Hence (T1), (T2) and (T6) are straightforwardly true.

The remaining principles all involve '$\wedge$' and so require something to be said about truthmakers for conjunctions. Rodriquez-Pereyra takes the only plausible truthmakers for conjunctions to be either conjunctive facts, such as the fact *that Peter is a man and Saturn is a planet* or non-conjunctive facts taken together, such as the collection of facts *that Peter is a man*, *that Saturn is a planet* (2006,

p. 970).[2] I will use the brackets '{' and '}' as notation for whatever are the correct truthmakers for conjunctions, so that '{*that P*, *that Q*}' denotes either the conjunctive fact *that P and Q* or the collection of facts *that P*, *that Q*. The general form of a truthmaker for a conjunction is $\{e_1, e_2\}$, where e_1 and e_2 are themselves facts or collections of facts. Using this notation, we can formulate Rodriquez-Pereyra's view that truthmakers for conjunctions are either conjunctive facts or collections of facts as follows.

(C*) If e_1 is a truthmaker for $\langle P \rangle$ and e_2 is a truthmaker for $\langle Q \rangle$, then $\{e_1, e_2\}$ is a truthmaker for $\langle P \wedge Q \rangle$.

(C**) If e is a truthmaker for $\langle P \wedge Q \rangle$, then there are entities e_1 and e_2 such that $e = \{e_1, e_2\}$, e_1 is a truthmaker for $\langle P \rangle$ and e_2 is a truthmaker for $\langle Q \rangle$.

Both principles are uncontroversial and in no way rely upon (C). Note that (C**) is perfectly compatible with e_1 and e_2 being identical (but does not entail that they are).

(T5) can now be derived. To do so, assume (throughout this paragraph) that $\langle P \rangle \rightsquigarrow \langle Q \rangle$. Suppose also that e is a truthmaker for $\langle P \vee R \rangle$. Given (D), e is either a truthmaker for $\langle P \rangle$, in which case (by assumption) it is also a truthmaker for $\langle Q \rangle$, or else it is a truthmaker for $\langle R \rangle$. Either way, by (D_I), e is a truthmaker for $\langle Q \vee R \rangle$. By a similar argument, if e is a truthmaker for $\langle R \vee P \rangle$ then, given the assumption, e is a truthmaker for $\langle R \vee Q \rangle$ as well. Next, suppose that e is a truthmaker for $\langle P \wedge R \rangle$. By (C**), there are entities e_1 and e_2 such that $e = \{e_1, e_2\}$, e_1 is a truthmaker for $\langle P \rangle$ and e_2 is a truthmaker for $\langle R \rangle$. By assumption, e_1 is a truthmaker for $\langle Q \rangle$ and so, by (C*), $\{e_1, e_2\}$ and hence e is a truthmaker for $\langle Q \wedge R \rangle$. By a similar argument, if e is a truthmaker for $\langle R \wedge P \rangle$ then, given the assumption, e is a truthmaker for $\langle R \wedge Q \rangle$ as well. This establishes (T5).

This leaves (T3) and (T4) which, given that they are principles directly concerning conjunctions, are key to deriving ($\star$). (T4) is derived as follows. Assume that e is a truthmaker for $\langle (P \wedge Q) \vee R \rangle$. Given (D), either e is a truthmaker for $\langle P \wedge Q \rangle$ or e is a truthmaker for $\langle R \rangle$. If the latter then, by (D_I), e is a truthmaker both for $\langle P \vee R \rangle$ and for $\langle Q \vee R \rangle$ and so, by (C_I), e is a truthmaker for $\langle (P \vee R) \wedge (Q \vee R) \rangle$. If the former then, by (C**), there are entities e_1 and e_2 such that $e = \{e_1, e_2\}$, e_1 is a truthmaker for $\langle P \rangle$, and e_2 is a truthmaker for $\langle Q \rangle$. By (D_I), e_1 is a truthmaker for $\langle P \vee R \rangle$ and e_2 is a truthmaker for $\langle Q \vee R \rangle$. Then, by (C*), $\{e_1, e_2\}$ (and hence e) is a truthmaker for $\langle (P \vee R) \wedge (Q \vee R) \rangle$. This establishes (T4).

It follows that, in order to accept (D) and reject (C), one must reject (T3), for this is the only way to reject ($\star$). But this is a hard doctrine to maintain. (T3) is intuitively appealing because there is an intuitive sense in which the propositions $\langle P \rangle$ and $\langle P \wedge P \rangle$ say the very same thing as one another. It would be strange for two propositions to say the same thing as one another, yet for one to require

2. It should be added that, in special cases, single non-conjunctive entities can be truthmakers for conjunctions. Given (C_I), it is plausible to take each natural number, on its own, to be a truthmaker for ⟨there exists a number $\wedge$ there exists a natural number⟩.

more to be made true than the other. But regardless of this, Rodriguez-Pereyra's argument against (C) provides no argument against (T3). In the case of (C), he argues that

> the fact that *Saturn is a planet* is not anything in virtue of which ⟨Peter is a man⟩ is true and it is totally irrelevant to the truth of ⟨Peter is a man⟩. And when a fact is totally irrelevant to the truth of a proposition, no plurality of facts one of which is that fact, and no conjunctive fact of which that fact is a conjunct, is something the proposition in question is true in virtue of. (2006, p. 972)

This is plausible but provides no argument against (T3). It requires that, if $\{e_1, e_2\}$ is a truthmaker for $\langle P \wedge P \rangle$, then both e_1 and e_2 are individually relevant to $\langle P \rangle$. If so, $\{e_1, e_2\}$ is wholly relevant to $\langle P \rangle$ and so, for all Rodriguez-Perayra has said, may truthmake it. We have been given no reason for thinking that any truthmaker for $\langle P \wedge P \rangle$ fails to be a truthmaker for $\langle P \rangle$.

To sum up, if one wants to reject (C) but accept (D), as Rodriguez-Pereyra does, then one must reject (T3). But (T3) is appealing and Rodriguez-Pereyra gives no argument against it. I conclude that Rodriguez-Pereyra should not reject (C) whilst accepting (D).

Reference

Rodriguez-Pereyra, Gonzalo 2006: 'Truthmaking, Entailment, and the Conjunction Thesis'. *Mind*, 115, pp. 957–82.

Appendix

To prove ($\star$), $\langle ((P \wedge Q) \vee P) \vee Q \rangle \rightsquigarrow \langle P \vee Q \rangle$, using (T1–T6), we proceed as follows.

1. $\langle (P \wedge Q) \vee P \rangle \rightsquigarrow \langle (P \vee P) \wedge (Q \vee P) \rangle$ (T4)
2. $\langle ((P \wedge Q) \vee P) \vee Q \rangle \rightsquigarrow \langle ((P \vee P) \wedge (Q \vee P)) \vee Q \rangle$ (1, T5a)
3. $\langle ((P \vee P) \wedge (Q \vee P)) \vee Q \rangle \rightsquigarrow \langle ((P \vee P) \vee Q) \wedge ((Q \vee P) \vee Q) \rangle$ (T4)
4. $\langle (Q \vee P) \vee Q \rangle \rightsquigarrow \langle P \vee (Q \vee Q) \rangle$ (T2)
5. $\langle ((P \vee P) \vee Q) \wedge ((Q \vee P) \vee Q) \rangle \rightsquigarrow \langle ((P \vee P) \vee Q) \wedge (P \vee (Q \vee Q)) \rangle$ (4, T5d)
6. $\langle P \vee P \rangle \rightsquigarrow \langle P \rangle$ (T1)
7. $\langle (P \vee P) \vee Q \rangle \rightsquigarrow \langle P \vee Q \rangle$ (6, T5a)
8. $\langle ((P \vee P) \vee Q) \wedge (P \vee (Q \vee Q)) \rangle \rightsquigarrow \langle (P \vee Q) \wedge (P \vee (Q \vee Q)) \rangle$ (7, T5c)
9. $\langle Q \vee Q \rangle \rightsquigarrow \langle Q \rangle$ (T1)

10. $\langle P \vee (Q \vee Q)\rangle \rightsquigarrow \langle P \vee Q\rangle$ (9, T5b)

11. $\langle (P \vee Q) \wedge (P \vee (Q \vee Q))\rangle \rightsquigarrow \langle (P \vee Q) \wedge (P \vee Q)\rangle$ (10, T5d)

12. $\langle (P \vee Q) \wedge (P \vee Q)\rangle \rightsquigarrow \langle P \vee Q\rangle$ (T3)

13. $\langle ((P \wedge Q) \vee P) \vee Q\rangle \rightsquigarrow \langle ((P \vee P) \vee Q) \wedge ((Q \vee P) \vee Q)\rangle$ (2, 3, T6)

14. $\langle ((P \wedge Q) \vee P) \vee Q\rangle \rightsquigarrow \langle ((P \vee P) \vee Q) \wedge (P \vee (Q \vee Q))\rangle$ (5, 13, T6)

15. $\langle ((P \wedge Q) \vee P) \vee Q\rangle \rightsquigarrow \langle (P \vee Q) \wedge (P \vee (Q \vee Q))\rangle$ (8, 14, T6)

16. $\langle ((P \wedge Q) \vee P) \vee Q\rangle \rightsquigarrow \langle (P \vee Q) \wedge (P \vee Q)\rangle$ (11, 15, T6)

17. $\langle ((P \wedge Q) \vee P) \vee Q\rangle \rightsquigarrow \langle P \vee Q\rangle$ (12, 16, T6)

The strategy is simple although, as is usual with axiom systems, the proof is unlovely. Lines 1–5 distribute '$\vee$' over '$\wedge$' twice in the left-hand side of ($\star$) and re-order to get to $\langle ((P \vee P) \vee Q) \wedge (P \vee (Q \vee Q)\rangle$. Lines 6–12 reduce this to $\langle P \vee Q\rangle$ by eliminating 'duplicates'. Finally, lines 13–17 put these together, using the transitivity of '$\rightsquigarrow$' to get ($\star$).